

# SU(2) symmetry in underdoped cuprates



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**Collège de France, Paris, March 26th, 2015**

**$^{89}\text{Y}$  NMR Evidence for a Fermi-Liquid Behavior in  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$** H. Alloul, T. Ohno,<sup>(a)</sup> and P. Mendels*Physique des Solides, Université de Paris-Sud, 91405 Orsay, France*

(Received 15 May 1989)

We report NMR shift  $\Delta K$  and  $T_1$  data of  $^{89}\text{Y}$  taken from 77 to 300 K in  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  for  $0.35 < x < 1$ , from the insulating to the metallic state. A Korringa law and therefore a Fermi-liquid picture is found to apply for the spin part  $K_s$  of  $\Delta K$ . The spin contribution  $\chi_s(x, T)$  to  $\chi_m$  is singled out, as the  $T$  variation of  $\Delta K$  scales linearly with the macroscopic susceptibility  $\chi_m$ . This implies that  $\text{Cu}(3d)$  and  $\text{O}(2p)$  holes do not have independent degrees of freedom. Their hybridization, which has a  $\sigma$  character, hardly varies with doping. These results put severe constraints on theoretical models of high- $T_c$  cuprates.

PACS numbers: 74.70.Vy, 75.20.En, 76.60.Cq, 76.60.Es

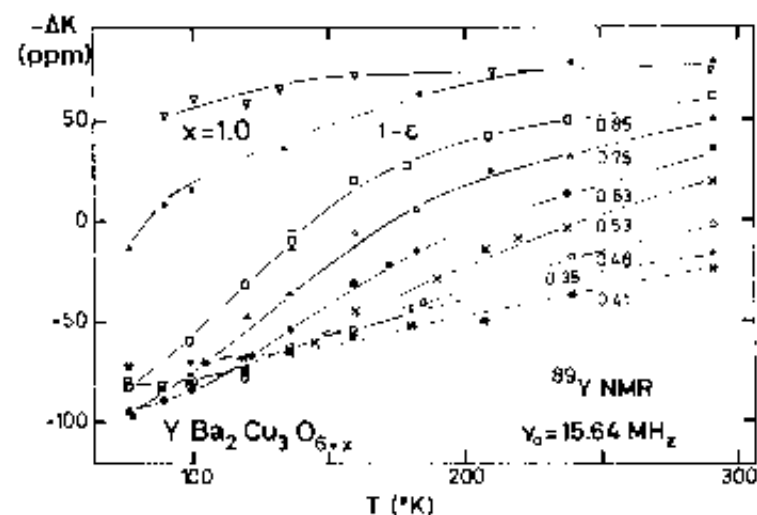


FIG. 1. The shift  $\Delta K$  of the  $^{89}\text{Y}$  line, referenced to  $\text{YCl}_3$ , plotted vs  $T$ , from 77 to 300 K. The lines are guides to the eye.

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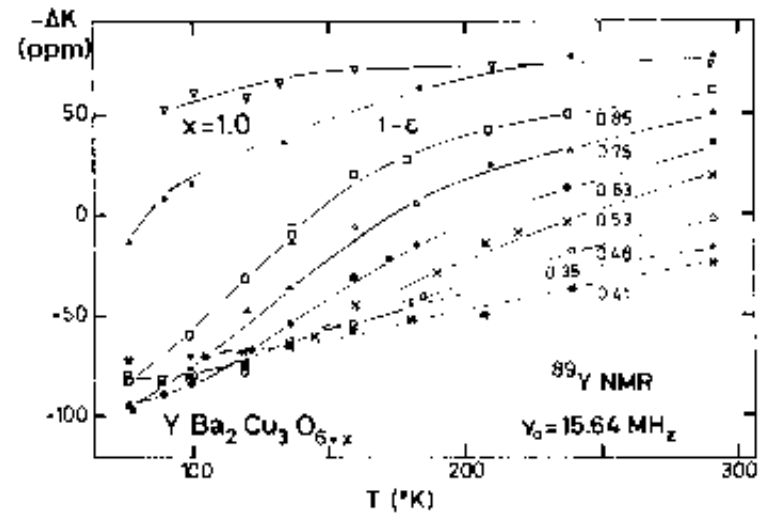
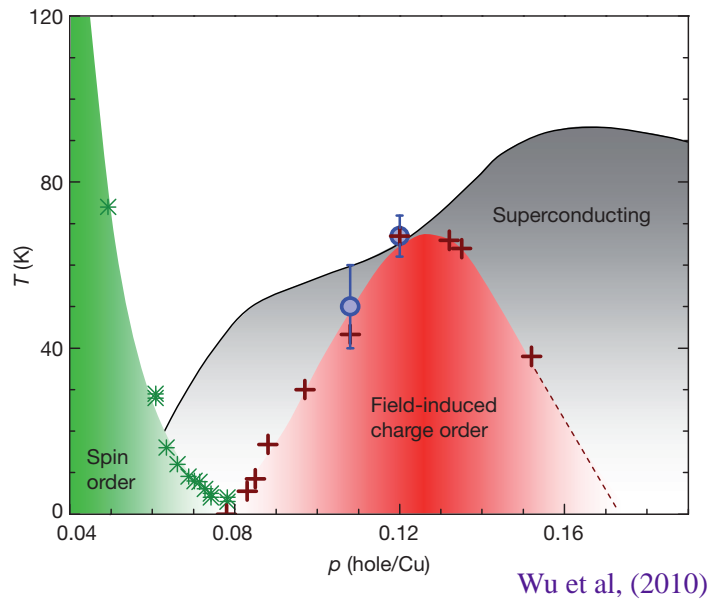
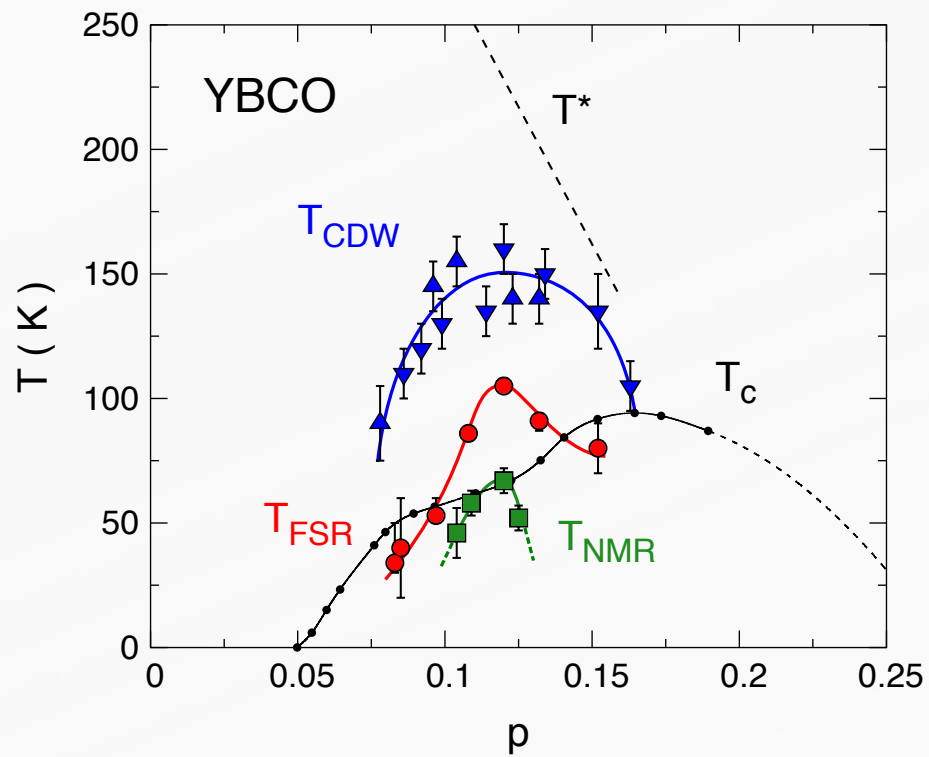
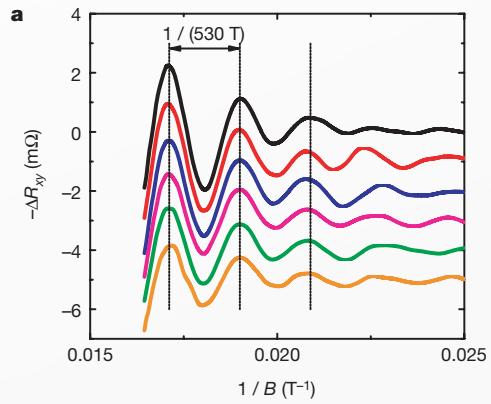


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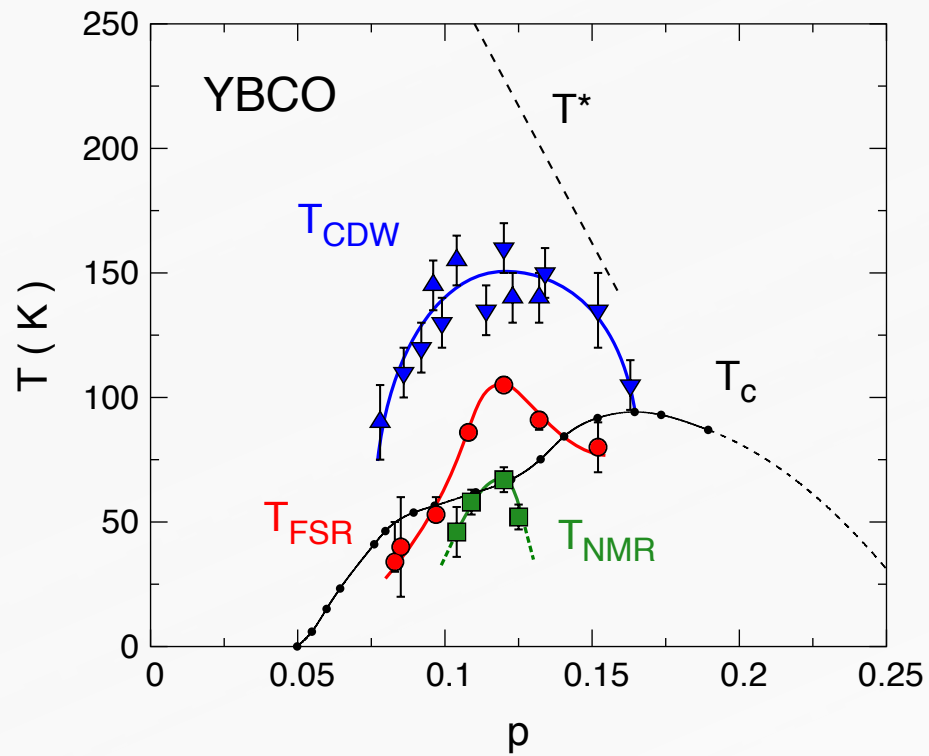
Cyr-Choignière, preprint 2015



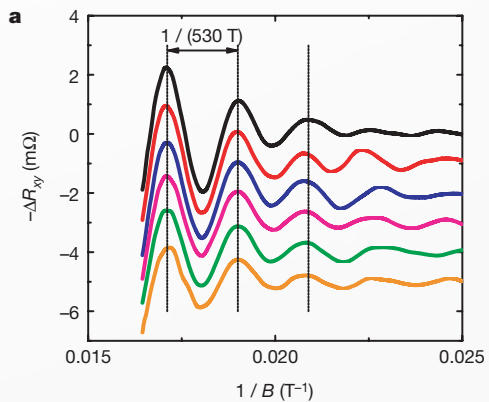
Doiron-Leyraud et al. (2007)  
Sebastian et al. (2011)



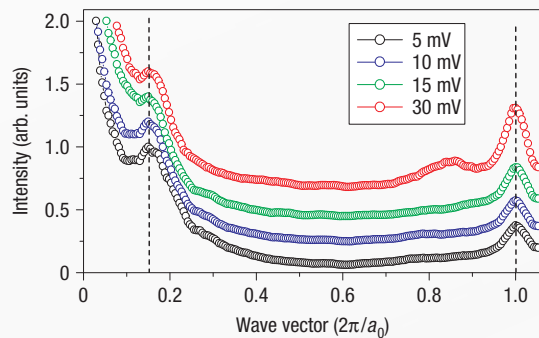
Cyr-Choignière, preprint 2015



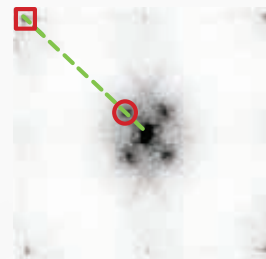
Doiron-Leyraud et al. (2007)  
Sebastian et al. (2011)



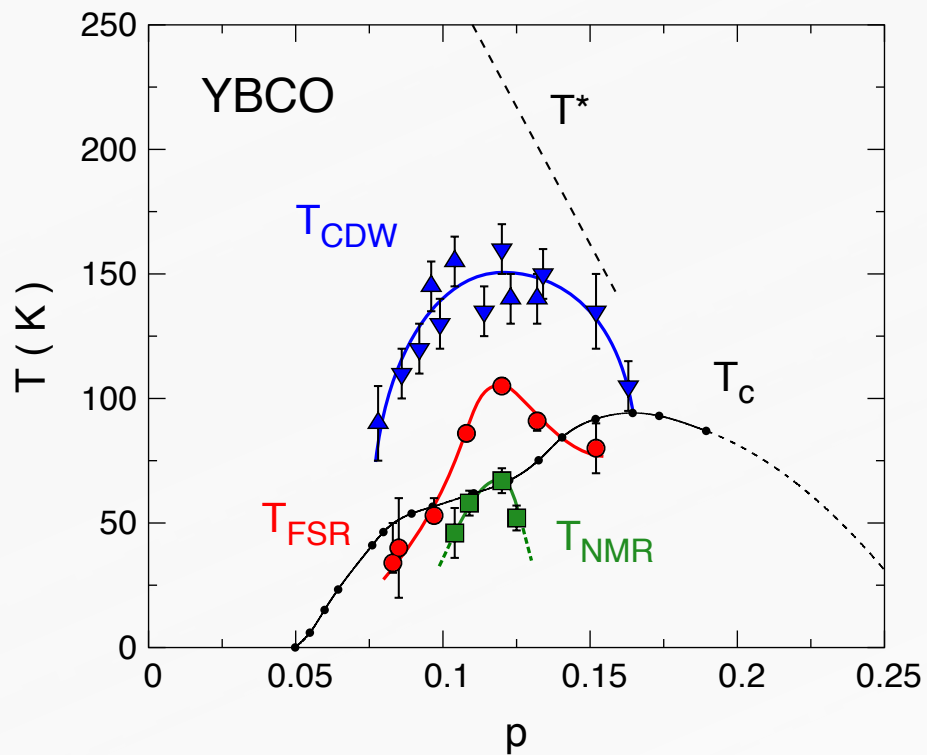
Wise et al, Nat. Phys. (2008)



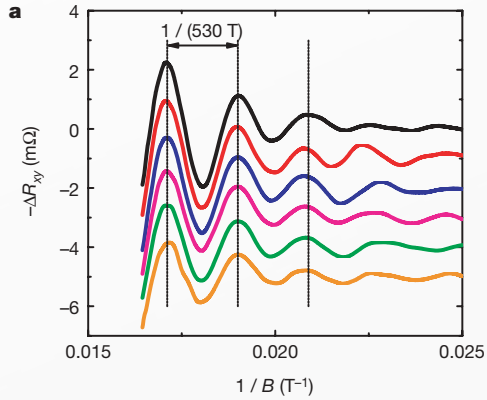
**BSCCO (opt. doped)**



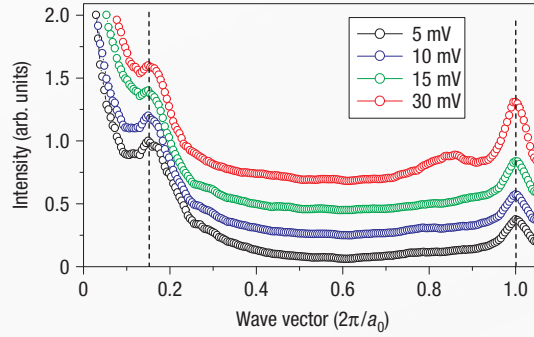
Cyr-Choignière, preprint 2015



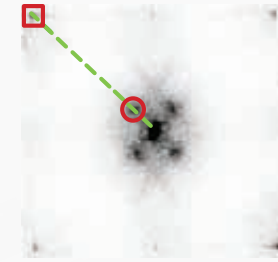
Doiron-Leyraud et al. (2007)  
Sebastian et al. (2011)



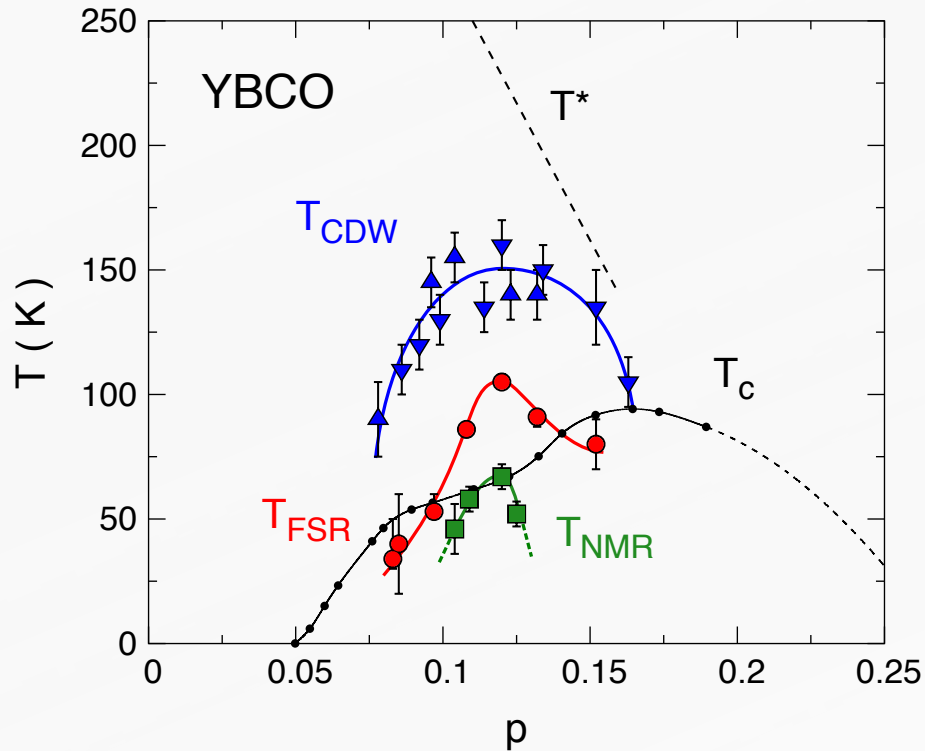
Wise et al, Nat. Phys. (2008)



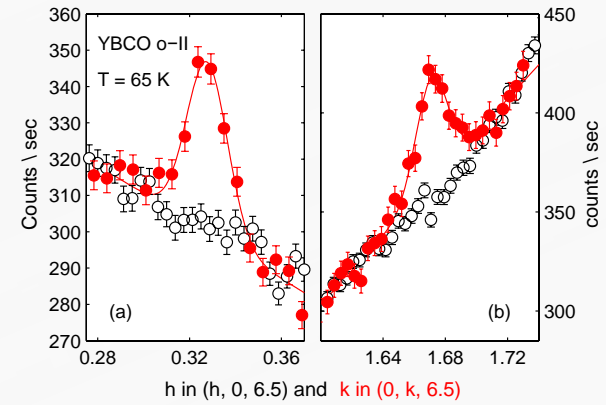
BSCCO (opt. doped)



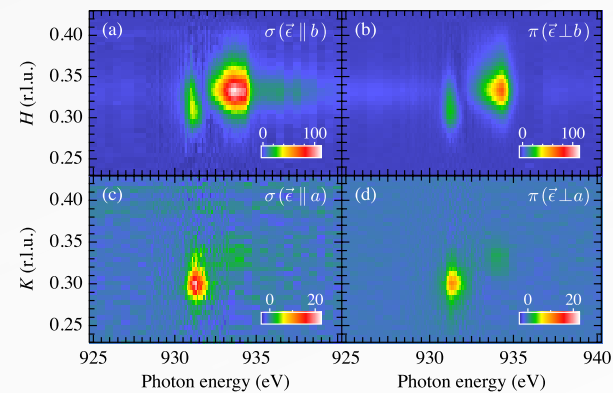
Cyr-Choignière, preprint 2015



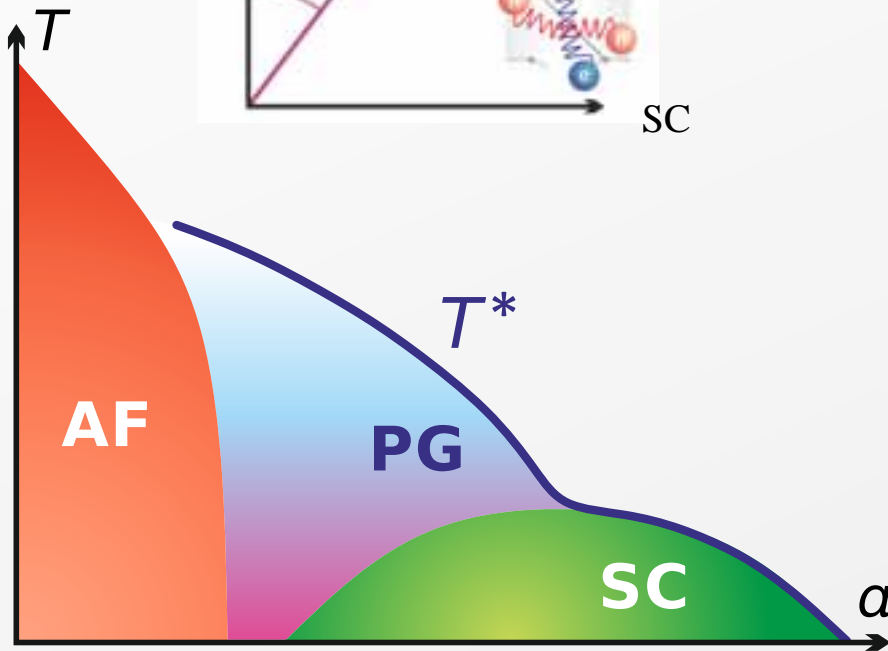
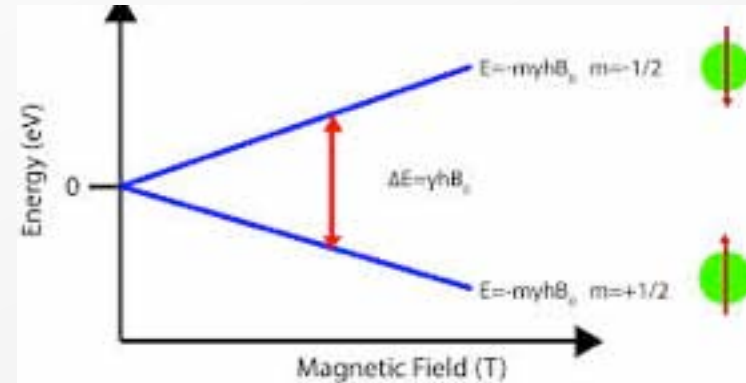
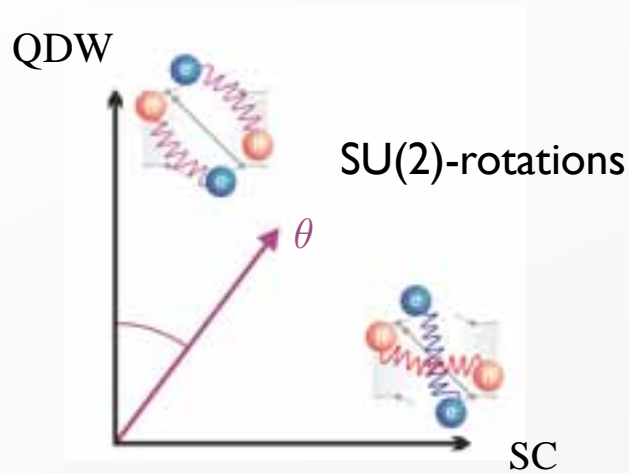
Blackburn et al, PRL (2013)



Achkar et al, PRL (2012)



# Emergent symmetries for the pseudo-gap



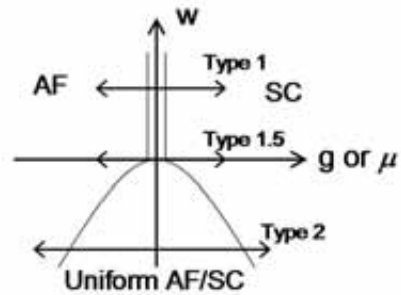
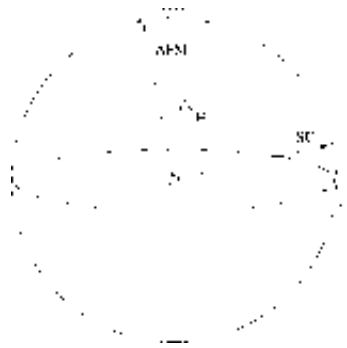
Degenerescence of levels:

accidental ?  
symmetry related ?

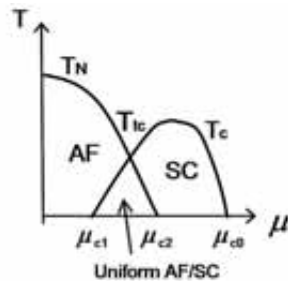
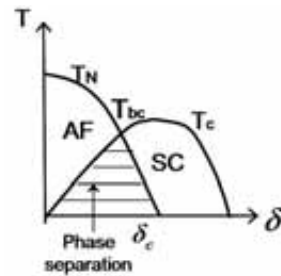
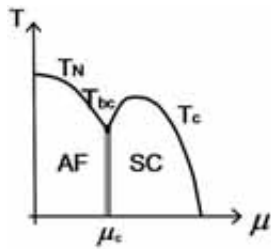
Sachdev et al (2013)  
Efetov, Meier, CP (2013)



### SO(5)-group



Fine-tuning condition ?



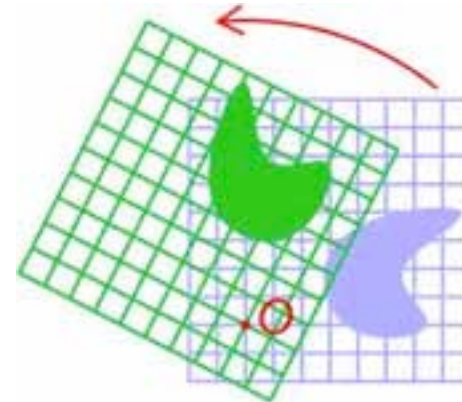
Demler, Zhang, Hanke (2005)

SU(2) symmetry related to the SU(2) symmetry of the superexchange hamiltonian and gauge SU(2) symmetry

$$U_{ij} = \begin{pmatrix} -\chi_{ij}^* & \Delta_{ij} \\ \Delta_{ij}^* & \chi_{ij} \end{pmatrix}$$

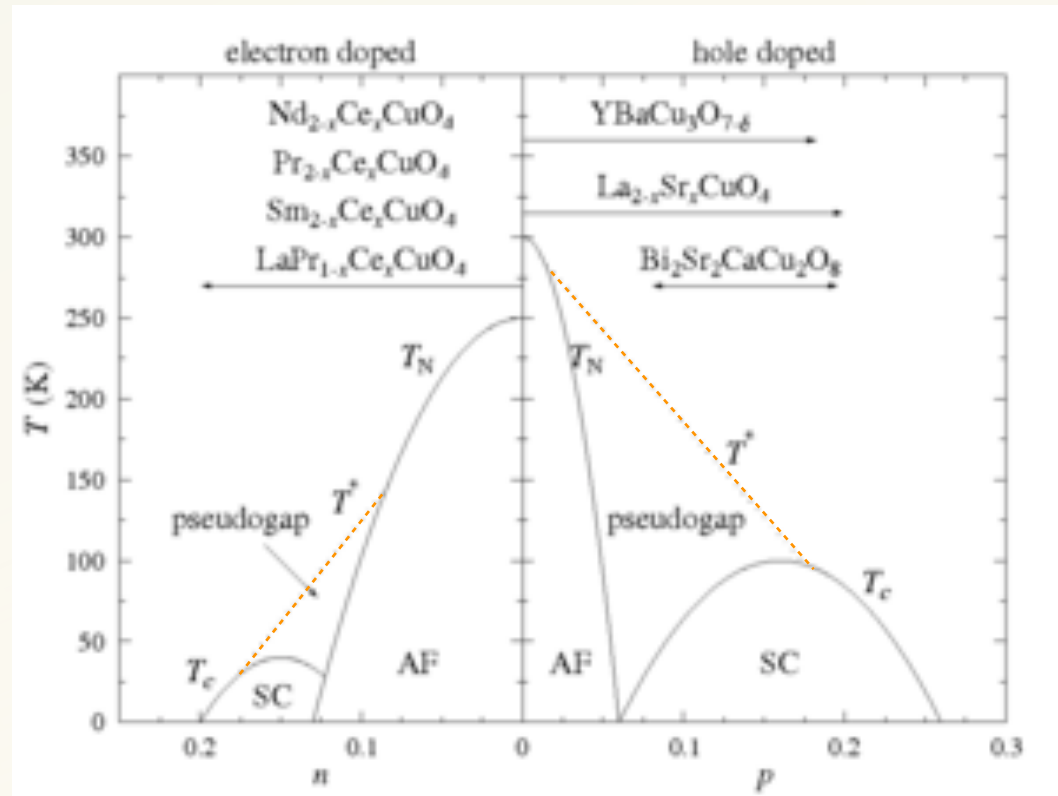
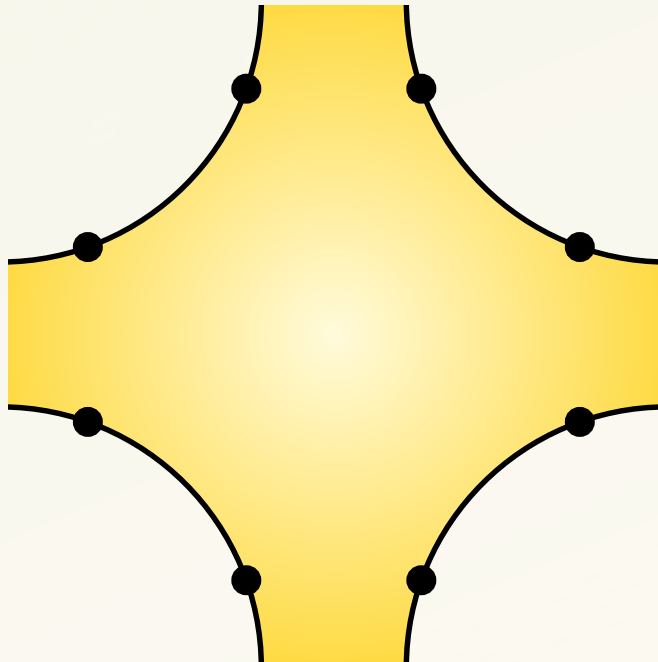
$$\chi_{ij} \delta_{\alpha\beta} = 2 \langle f_{i\alpha}^\dagger f_{j\beta} \rangle, \quad \chi_{ij} = \chi_{ji}^*$$

$$\Delta_{ij} \epsilon_{\alpha\beta} = 2 \langle f_{i\alpha} f_{j\beta} \rangle, \quad \Delta_{ij} = \Delta_{ji}$$

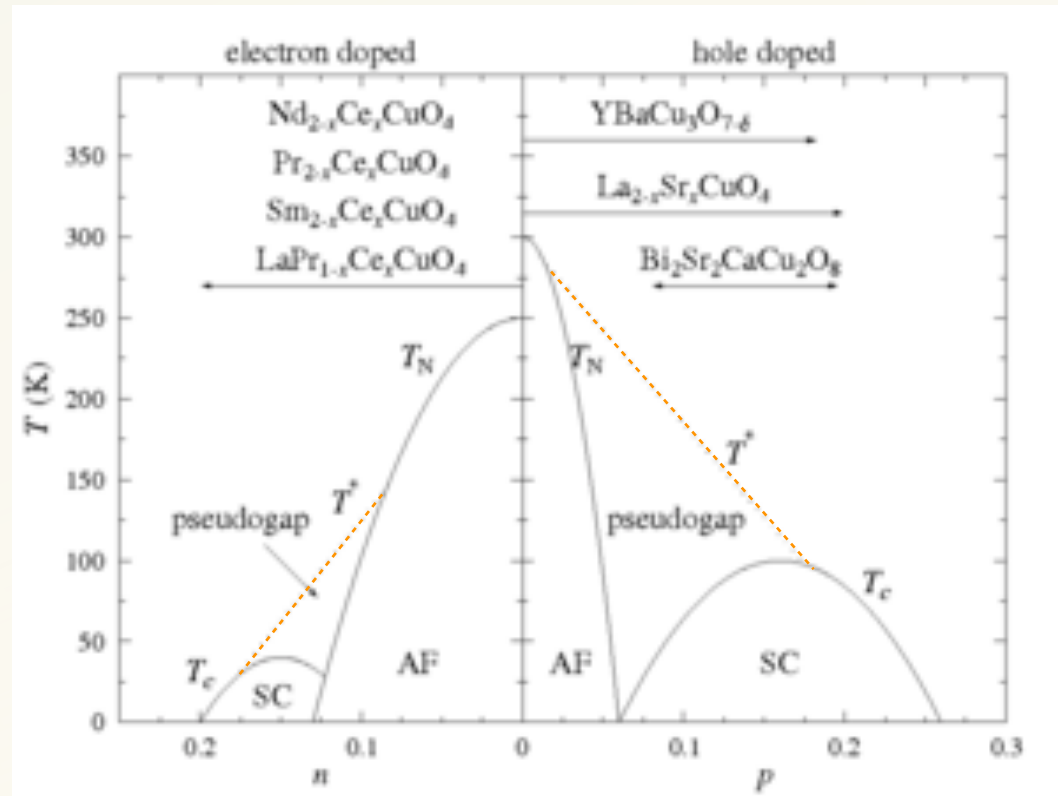
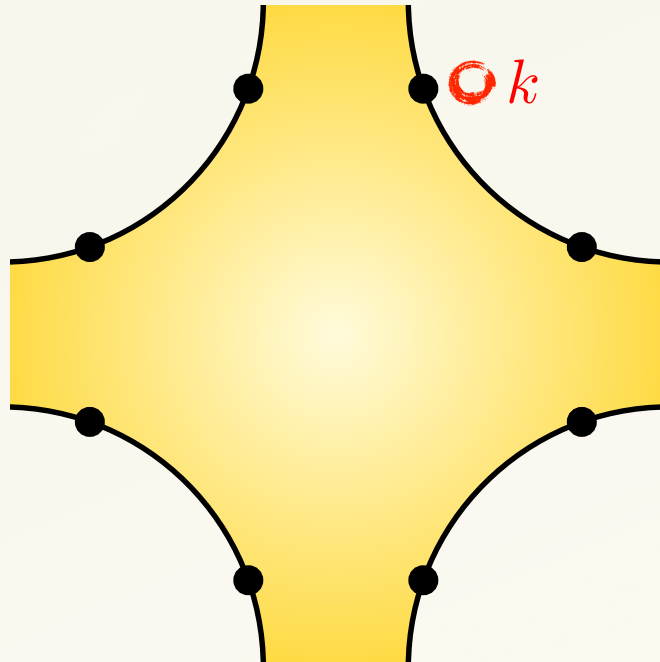


Sachdev et al (2013)  
 Kotliar and Liu (1988)  
 Lee, Wen, Nagaosa, RMP (2006)

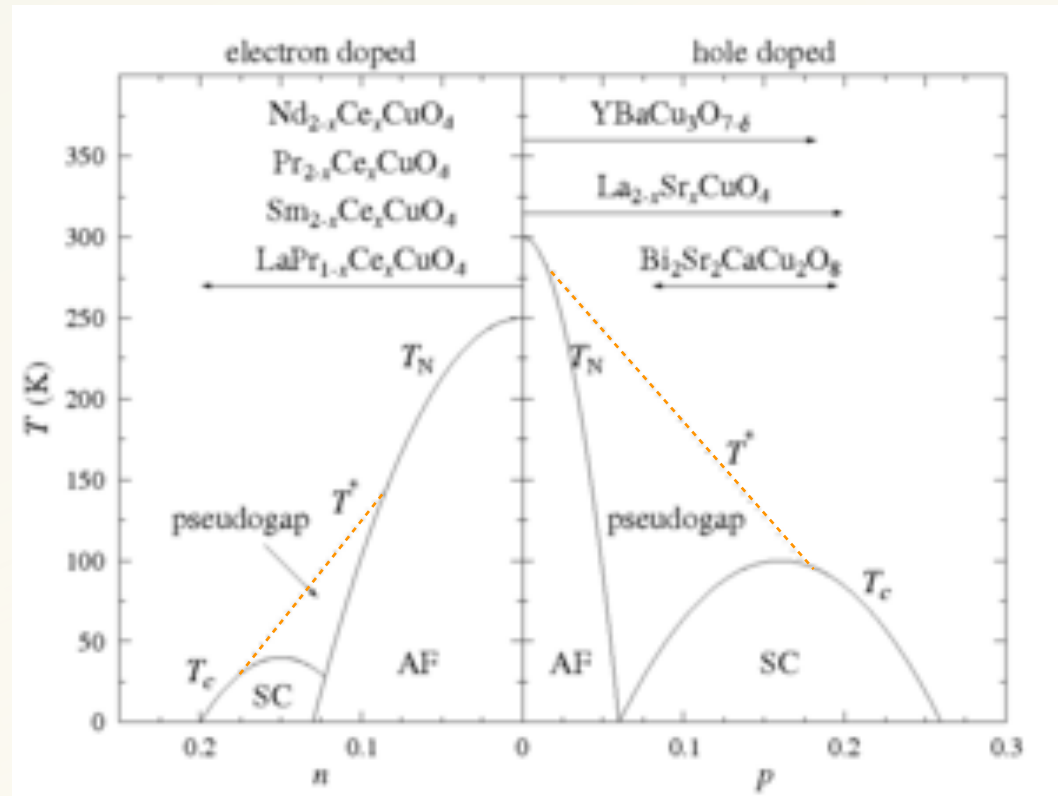
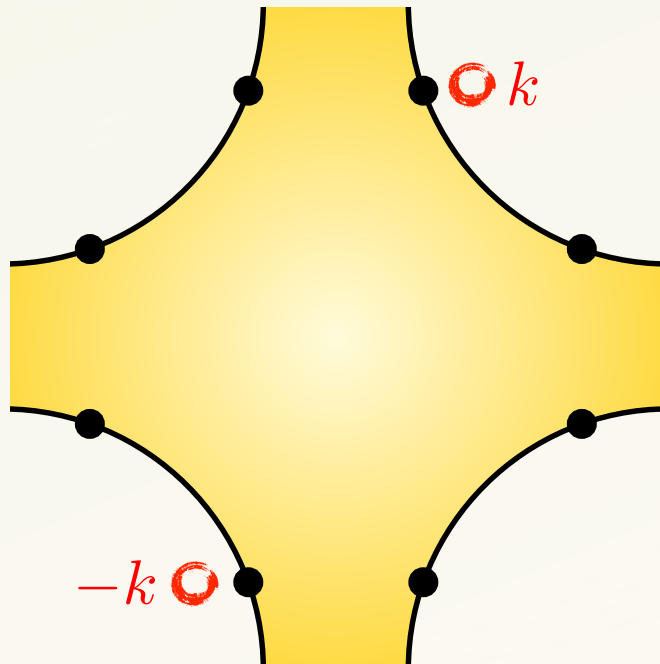
# Picture of the relation between the SC and Charge sector



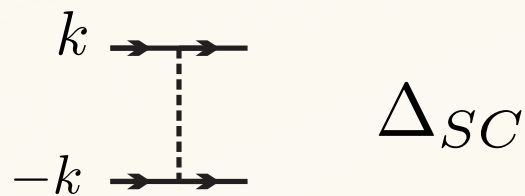
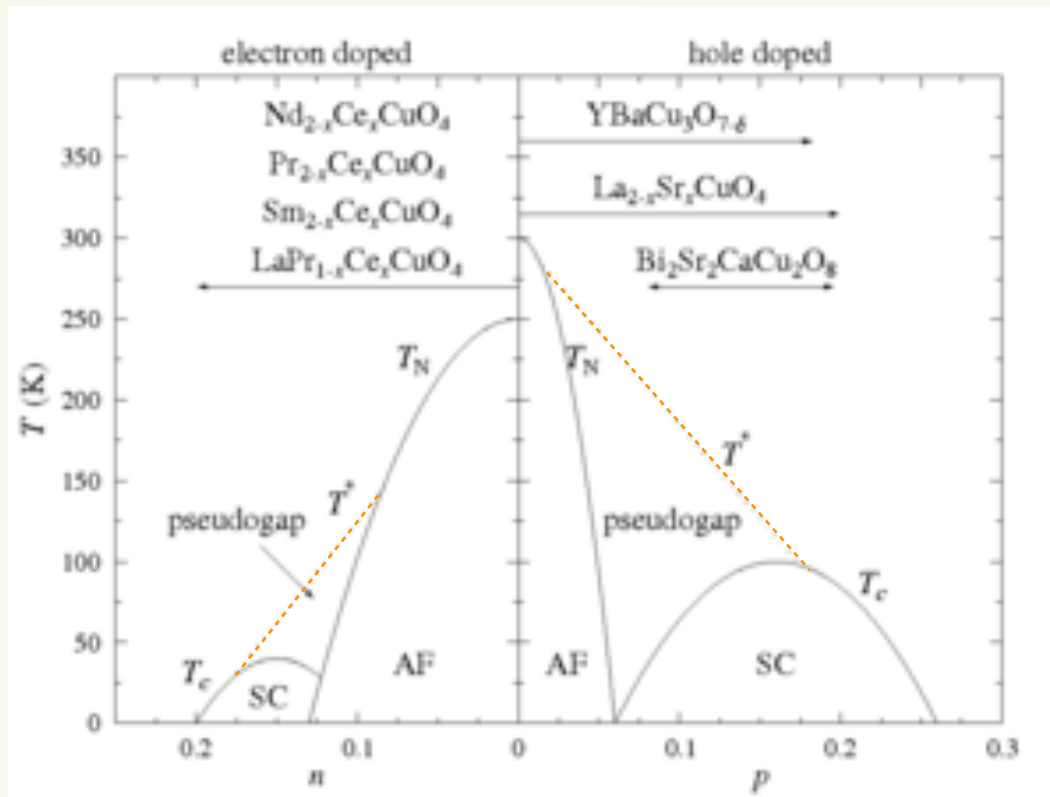
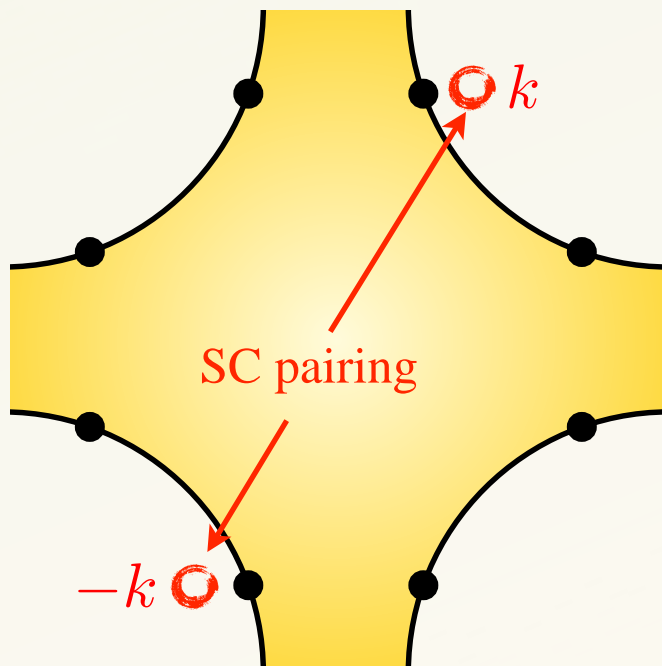
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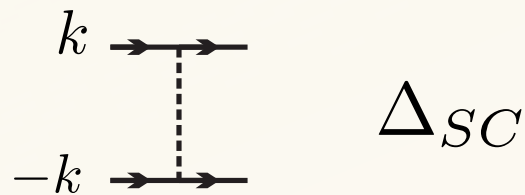
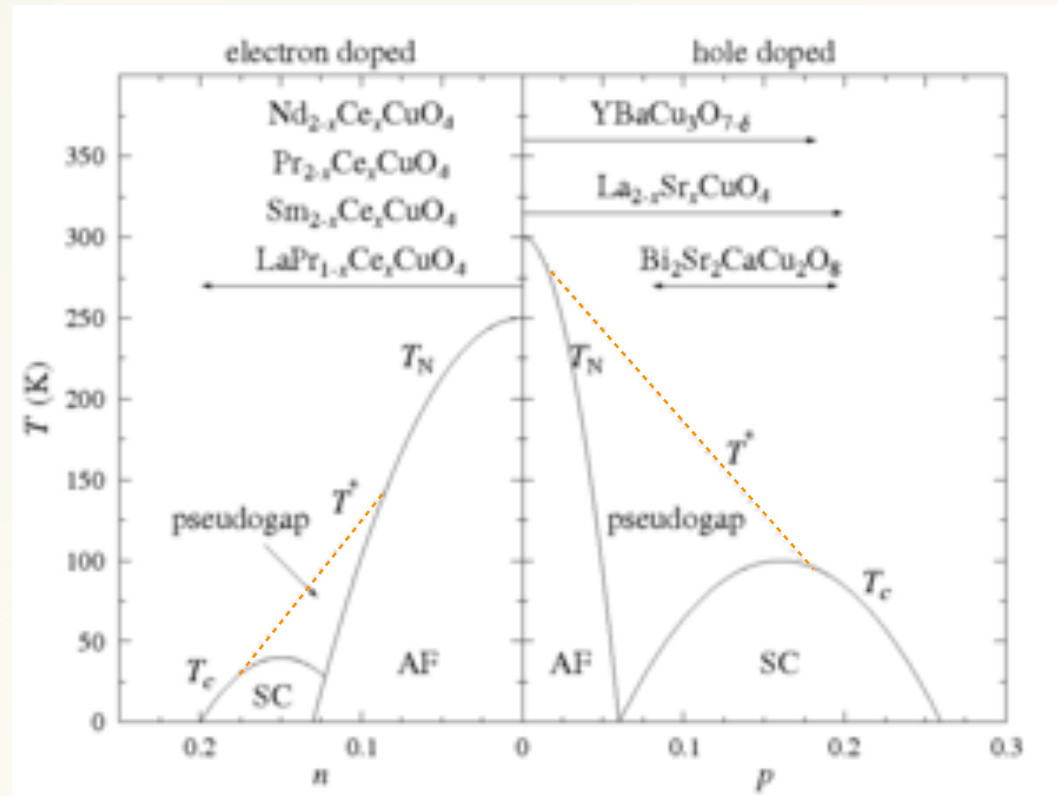
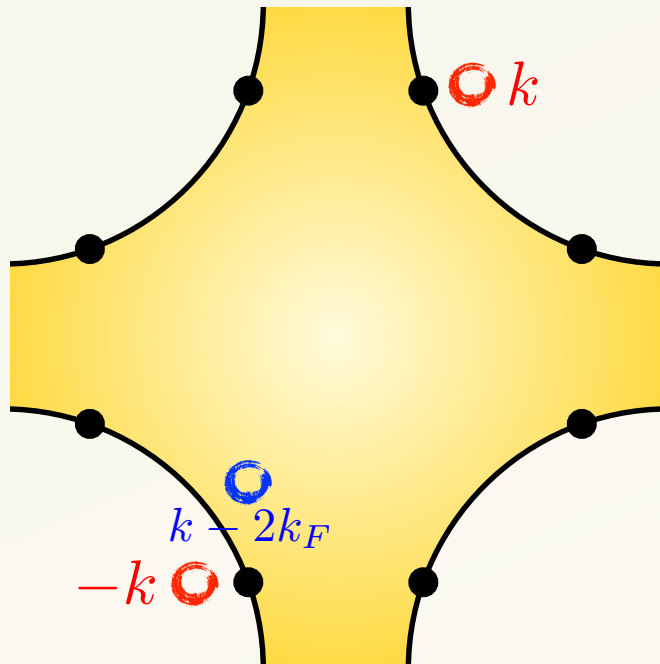
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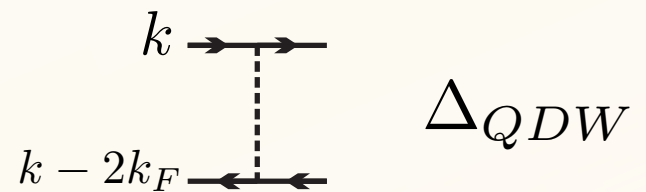
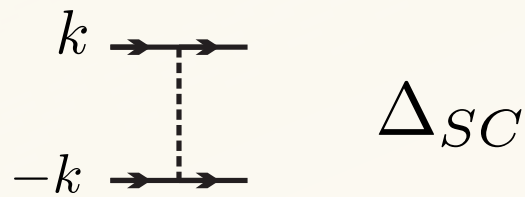
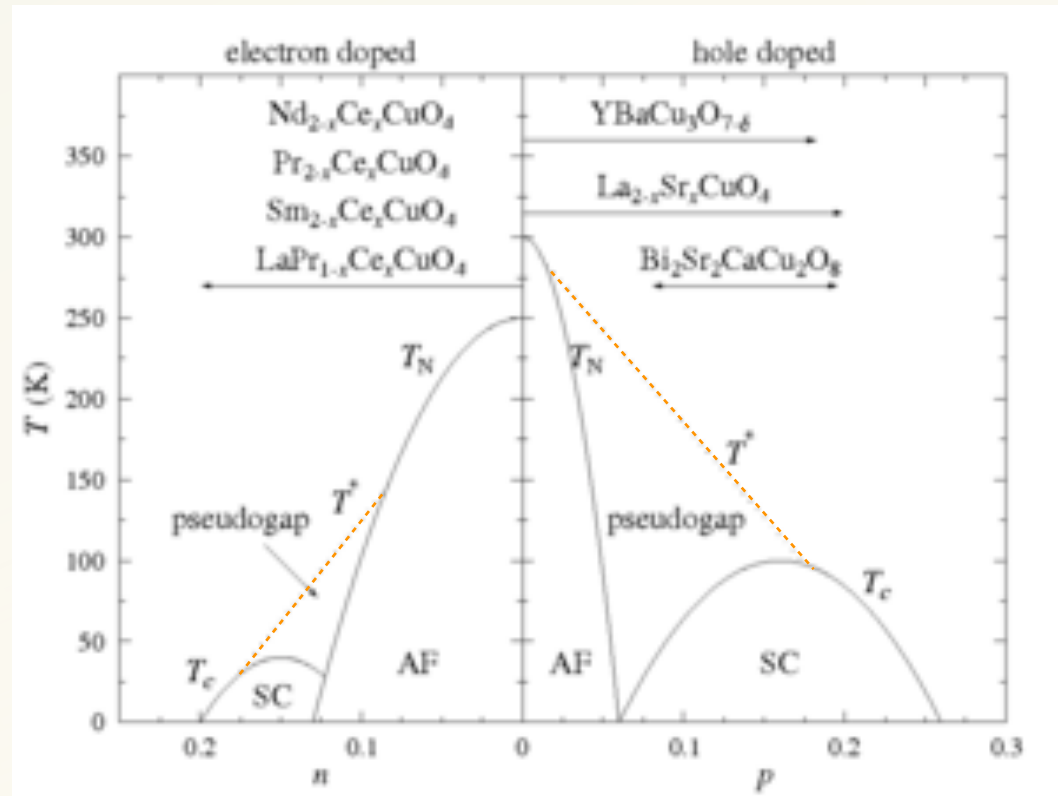
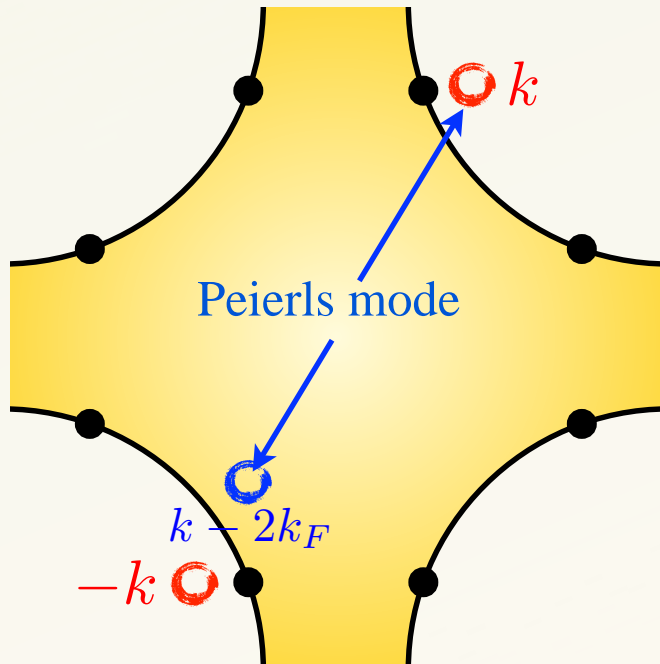
# Picture of the relation between the SC and Charge sector



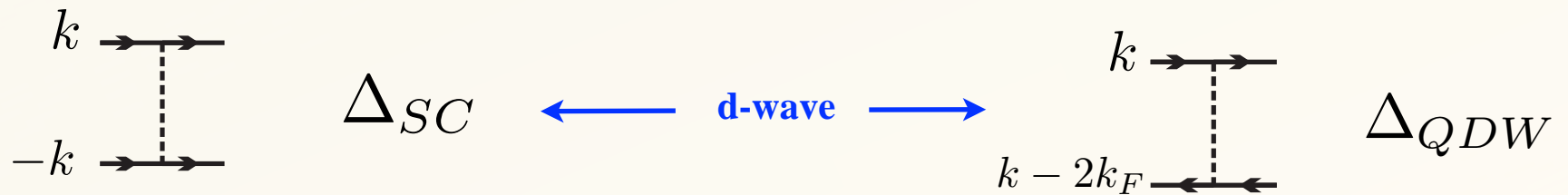
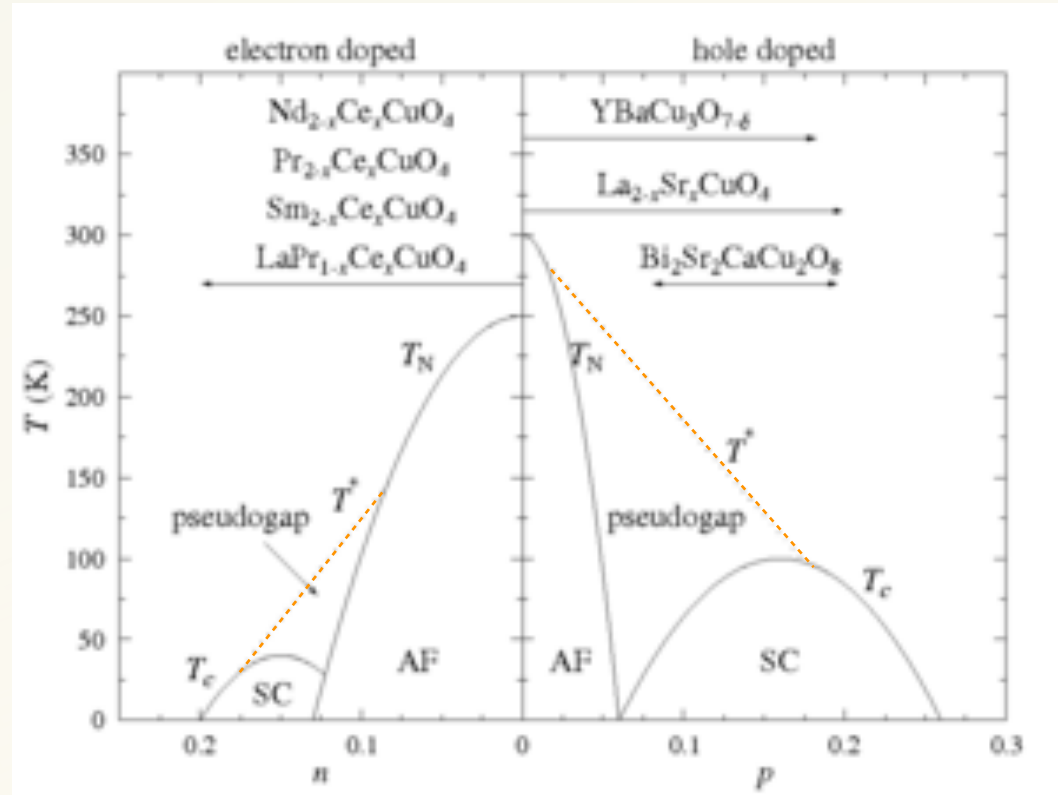
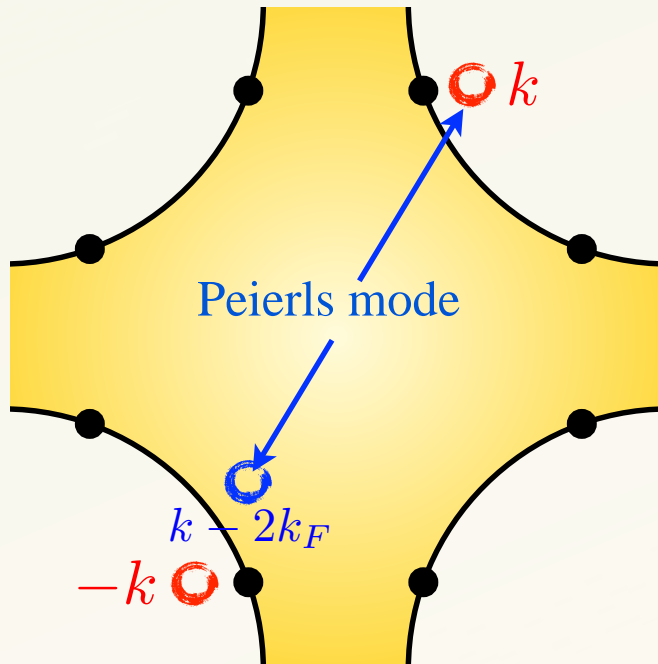
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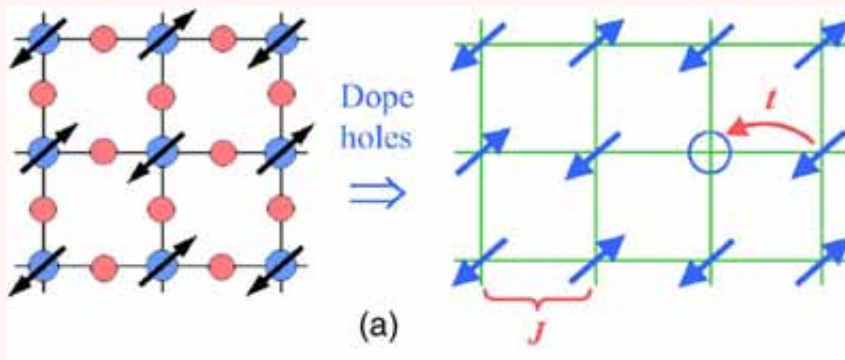


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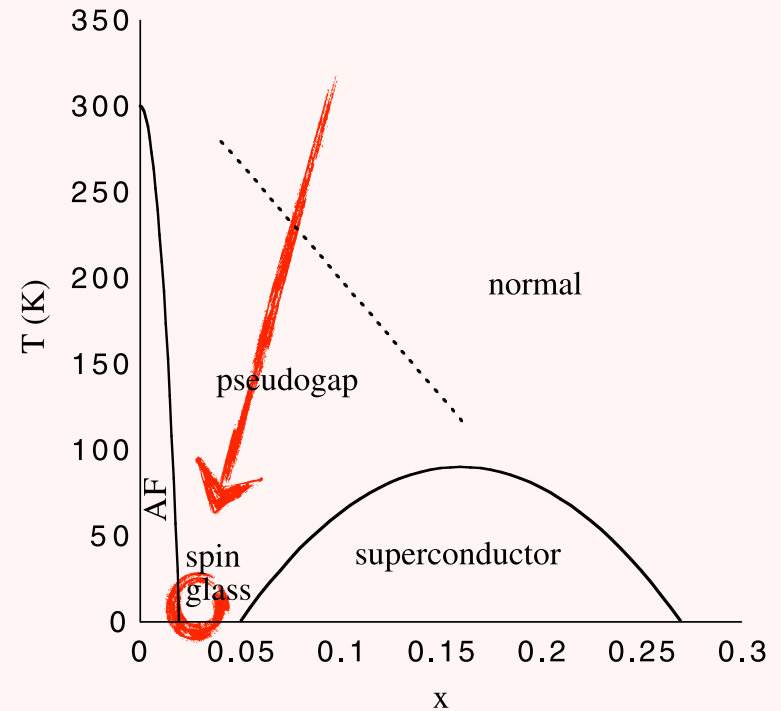




# Exact realization : neglecting Coulomb interactions



Pines, Monthoux, Scalapino



$$H = \cancel{P} \left[ - \sum_{\langle ij \rangle, \sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + J \sum_{\langle ij \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j) \right] \cancel{P}$$

$P$ : projection on no double occupancy

# Exact realization of SU(2) symmetry

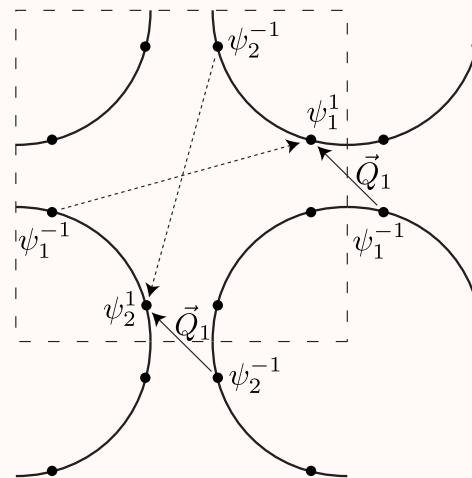
## AFM QCP in d=2

K.B.Efetov, H.Meier, C.P. Nat. Phys. **9**, (2013)

$$\mathcal{L} = \chi^\dagger \left( \partial_\tau + \varepsilon(-i\hbar\nabla) + \lambda\vec{\phi}\vec{\sigma} \right) \chi \quad \langle \phi_{\omega,\mathbf{k}}^i \phi_{-\omega,-\mathbf{k}}^j \rangle \propto \frac{\delta_{ij}}{(\omega/v_s)^2 + (\mathbf{k} - \mathbf{Q})^2 + a}$$

A Abanov, A. Chubukov, Schmalian RMP 2003  
 Belitz, Kirkpatrick, Vojta, RMP 2005  
 J Rech, CP, A Chubukov, PRB 2006

M. Metlitsky and S. Sachdev (2010)

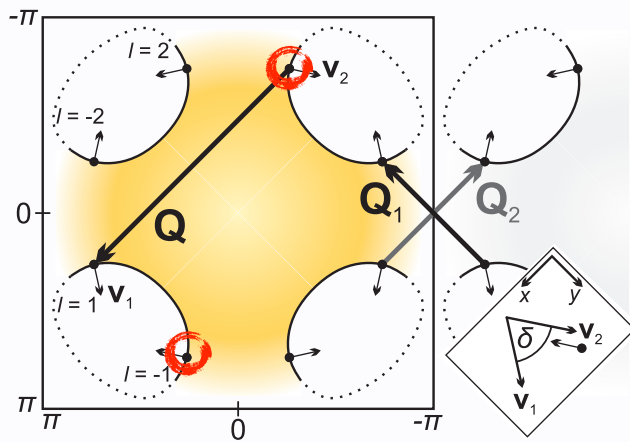


Eliashberg theory : neglect vertices

Chubukov, Morr (2003 ...)

$$i\Sigma(\omega_m) = \text{---} \overbrace{\text{---}}^{\text{wavy line}} \text{---} \\ k, \omega_m$$

$$\chi_0^{-1} \Pi(q, \Omega_m) = \text{---} \overbrace{\text{---}}^{\text{wavy line}} \text{---} \\ q, \Omega_m$$



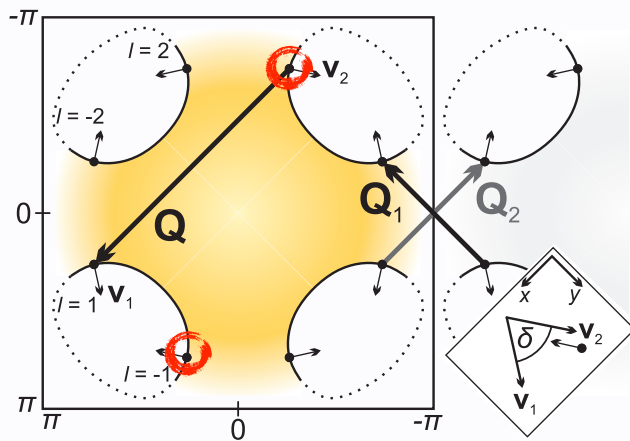
$$\delta \ll 1$$

## Composite order parameter

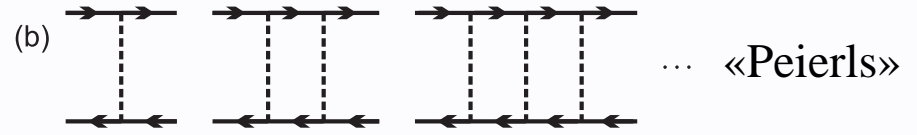
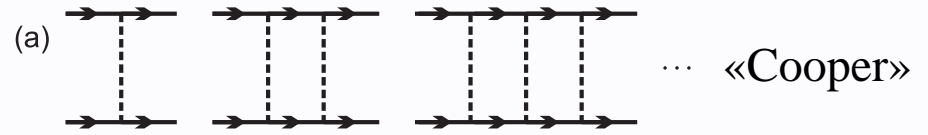
$$c_{\mathbf{p}}^{\text{pp}} \langle (i\sigma_2)_{\alpha\beta} \psi_{\alpha,\mathbf{p}} \psi_{\beta,-\mathbf{p}} \rangle + c_{\mathbf{p}}^{\text{ph}} \langle \delta_{\alpha\beta} \psi_{\alpha,\mathbf{p}} \psi_{\beta,-\mathbf{p}}^* \rangle,$$

## SU(2) symmetry and fluctuations

$$u = \begin{pmatrix} \Delta_- & \Delta_+ \\ -\Delta_+^* & \Delta_-^* \end{pmatrix} \quad \text{with} \quad |\Delta_+|^2 + |\Delta_-|^2 = 1$$



$$\delta \ll 1$$



$$\text{---} D_{\text{eff}} \text{---} = \text{---} D \text{---} + \text{---} \bigcirc \text{---}$$

### Composite order parameter

$$c_{\mathbf{p}}^{\text{pp}} \langle (i\sigma_2)_{\alpha\beta} \psi_{\alpha,\mathbf{p}} \psi_{\beta,-\mathbf{p}} \rangle + c_{\mathbf{p}}^{\text{ph}} \langle \delta_{\alpha\beta} \psi_{\alpha,\mathbf{p}} \psi_{\beta,-\mathbf{p}}^* \rangle,$$

### SU(2) symmetry and fluctuations

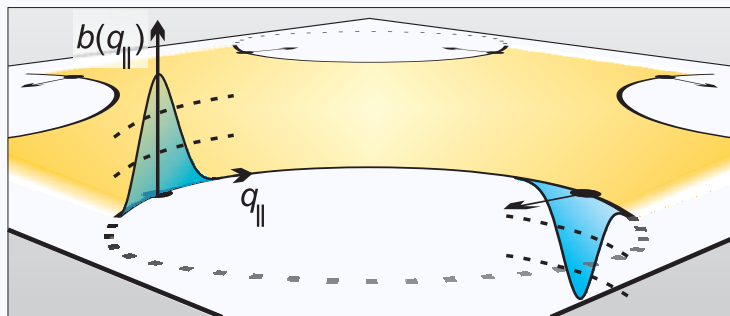
$$u = \begin{pmatrix} \Delta_- & \Delta_+ \\ -\Delta_+^* & \Delta_-^* \end{pmatrix} \quad \text{with} \quad |\Delta_+|^2 + |\Delta_-|^2 = 1$$

# Gap equations around the QCP are universal

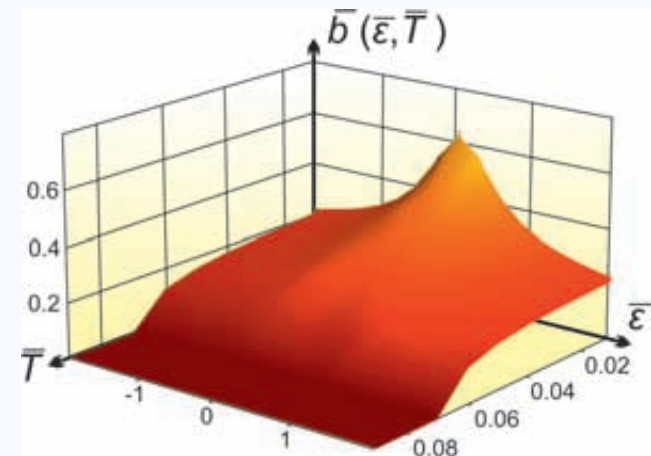
$$b(\varepsilon) = \frac{3\lambda^2}{4N_V} T \sum_{\varepsilon'} \frac{\bar{D}(\varepsilon - \varepsilon') b(\varepsilon')}{\sqrt{f^2(\varepsilon') + b^2(\varepsilon')}} ,$$

$$\bar{D}(\omega) = \frac{1}{\sqrt{\gamma\Omega(\omega) + a}}$$

linear dispersion at the hot-spots



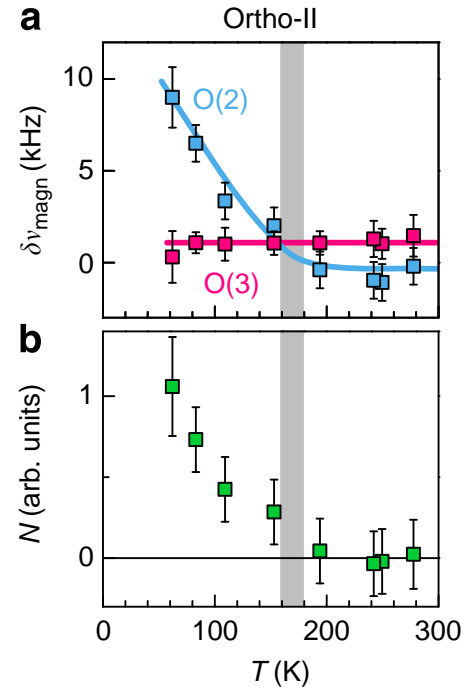
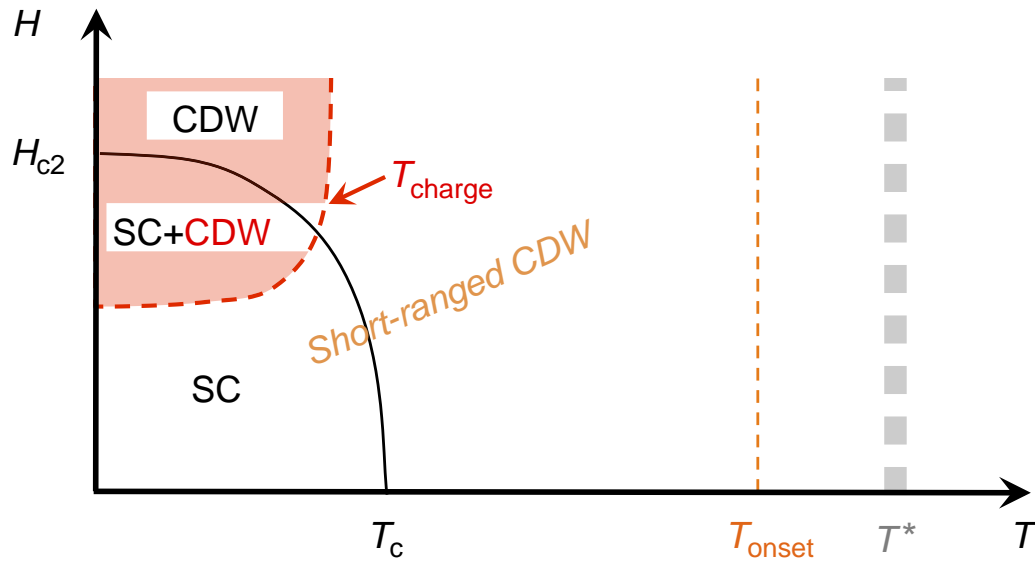
d-wave symmetry



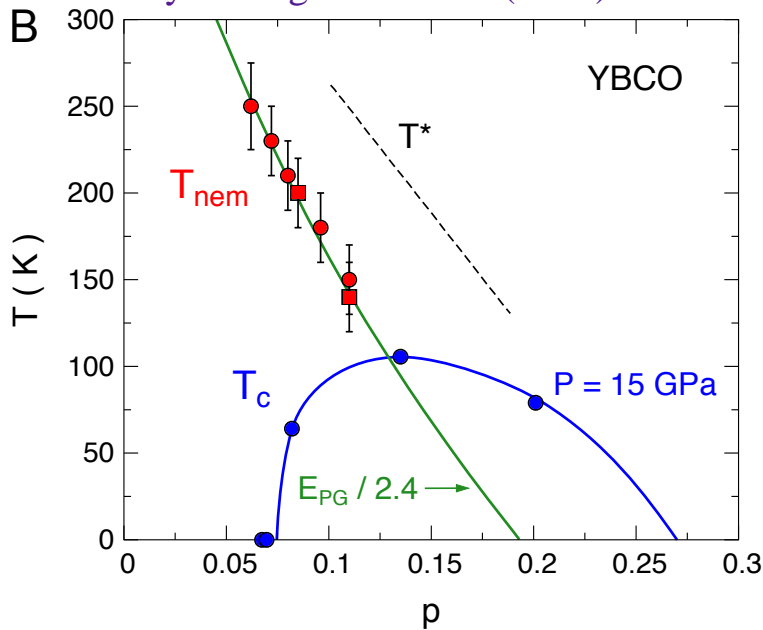
$$b\hat{u} \quad \text{with} \quad \hat{u} = \begin{pmatrix} \Delta_- & \Delta_+ \\ -\Delta_+^* & \Delta_-^* \end{pmatrix}$$

*Non abelian superconductor*

Wu et al. (2015)



Cyr-Choignières et al. (2015)

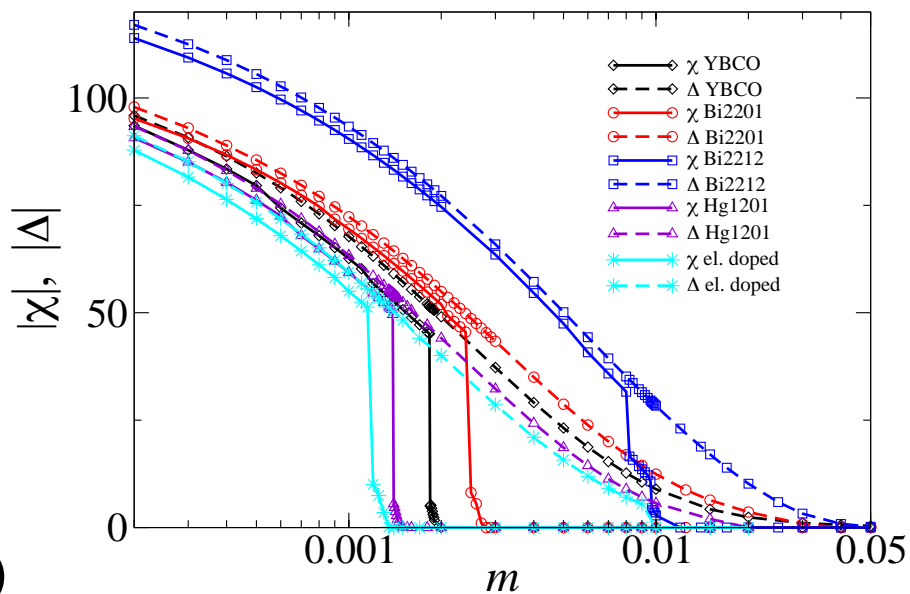


Pseudo-Gap = SU(2)  
 composite order parameter  
 = "non abelian"  
 superconductor

# Curvature breaks the SU(2) symmetry : SC dome

## How much is it broken ?

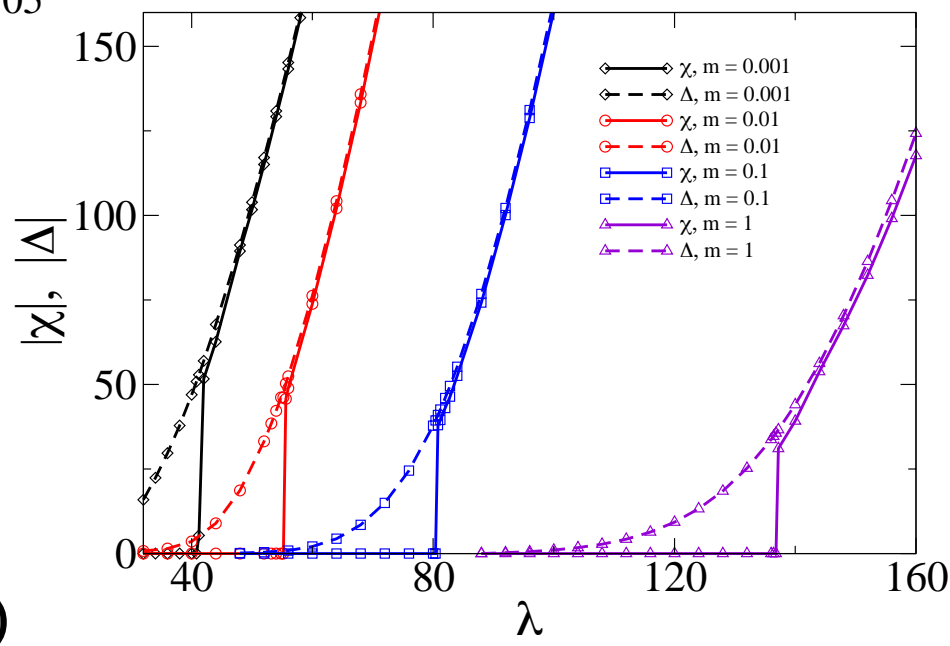
T. Kloss , X. Montiel & C.P. (2015)



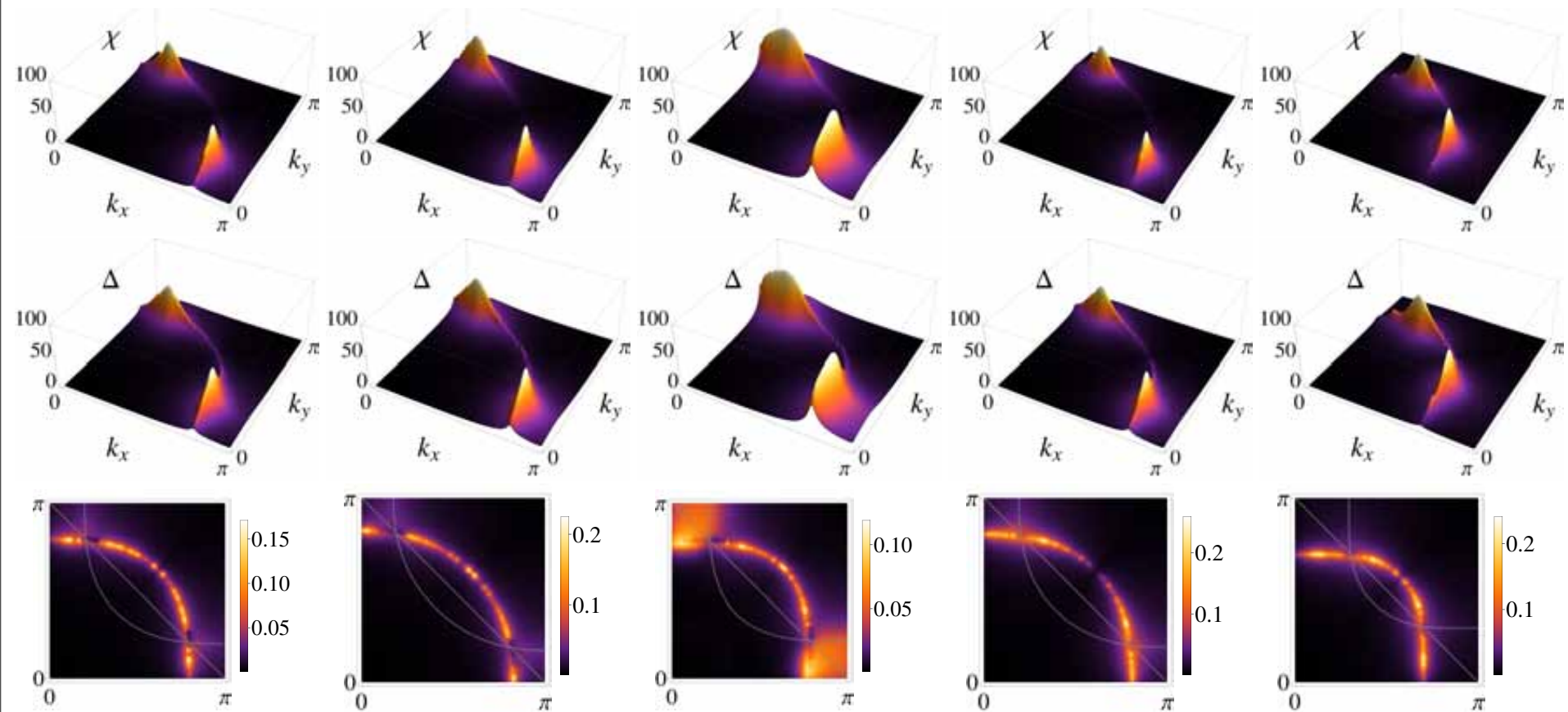
$$\mathcal{L} = \chi^\dagger (\partial_\tau + \varepsilon(-i\hbar\nabla) + \lambda\vec{\phi}\vec{\sigma}) \chi$$

$$\langle \phi_{\omega, \mathbf{k}}^i \phi_{-\omega, -\mathbf{k}}^j \rangle \propto \frac{\delta_{ij}}{(\omega/v_s)^2 + (\mathbf{k} - \mathbf{Q})^2 + m}$$

*SU(2) symmetry is optimal when AF correlations are maximum*



b)



YBCO

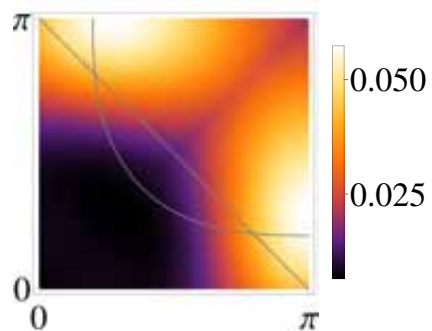
Bi2201

Bi2212

Hg1201

NdCeCu<sub>04</sub>

$$\lambda = 44, m = 10^{-3}$$



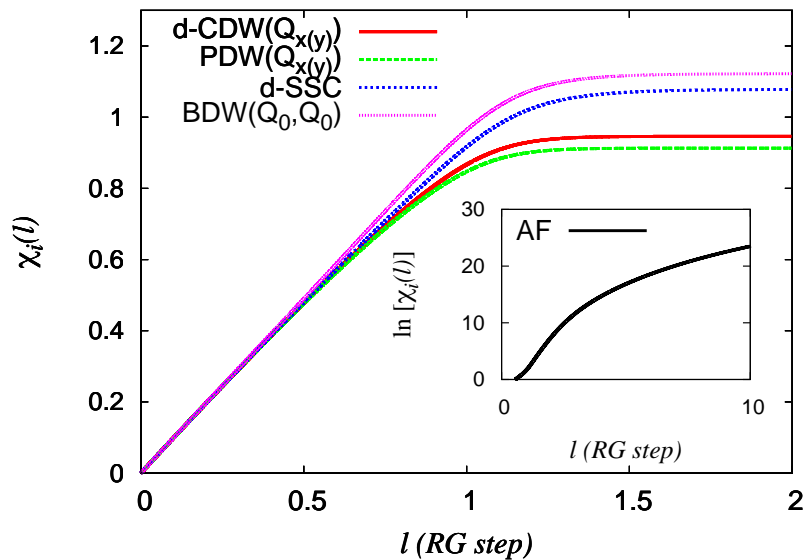
$$\lambda = 160, m = 1$$

$$0.02 \leq \Delta E \leq 0.3$$

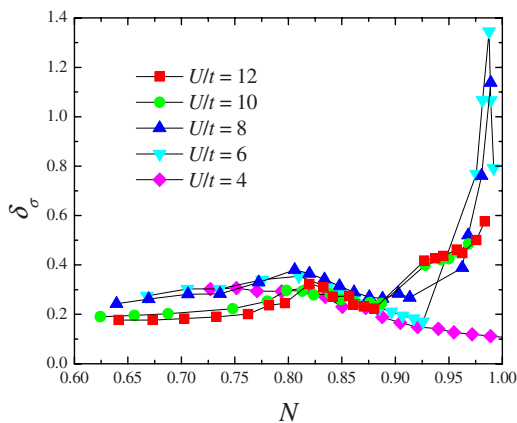


# Confirmation by other techniques ?

2 loops RG



Freire, Carvalho & C.P. (2015)

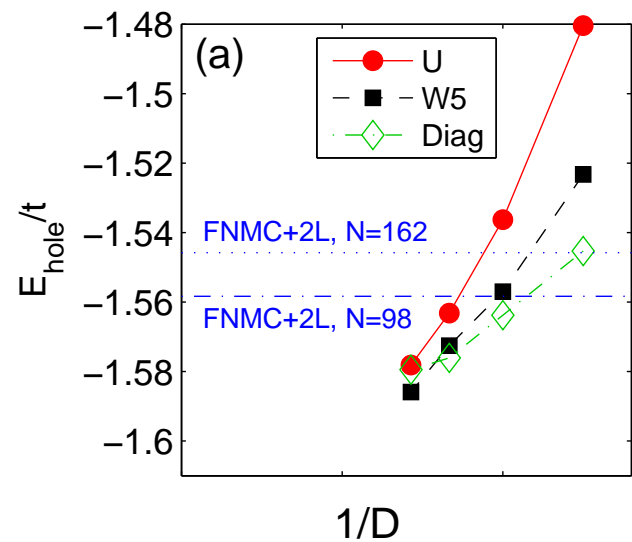


C-DMFT ?  
Nematic response

Okamoto et al (2010)

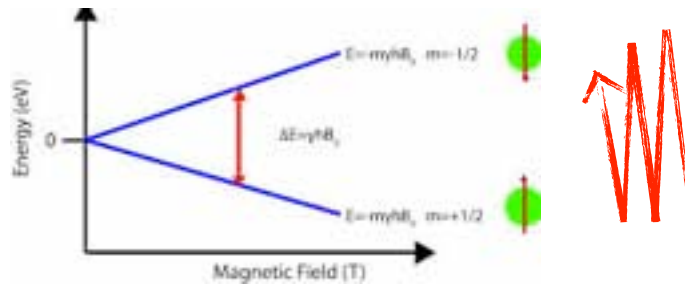
*SU(2) partner of axial CDW  
is PDW*

Exact diag



Corboz et al. (2014)

# In search for the collective mode...



Collective mode

Raman Scattering, X-Rays,  
Electron Energy Loss  
Spectroscopy

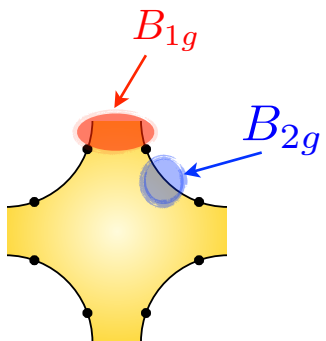
- charge 2
- singlet
- around  $q \simeq 2k_F$

$$| \downarrow \rangle = c_{k,\uparrow}^\dagger c_{k-2k_F,\uparrow} | 0 \rangle$$

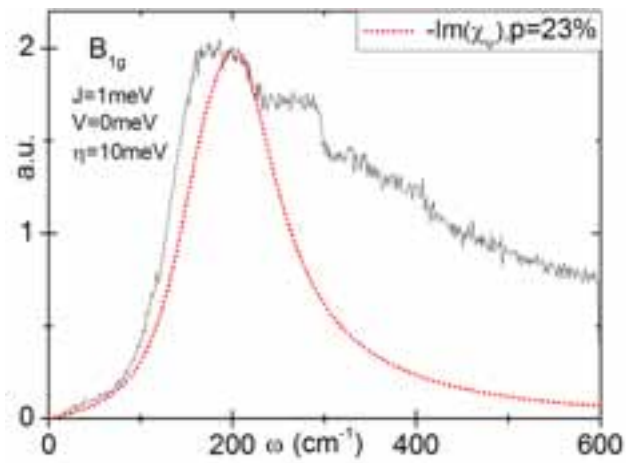
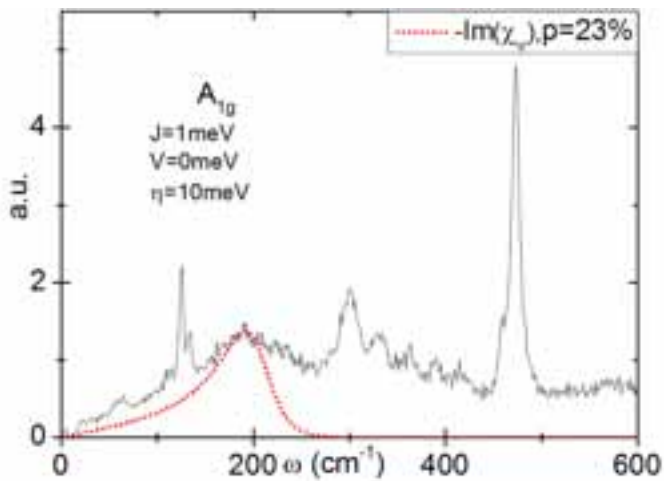
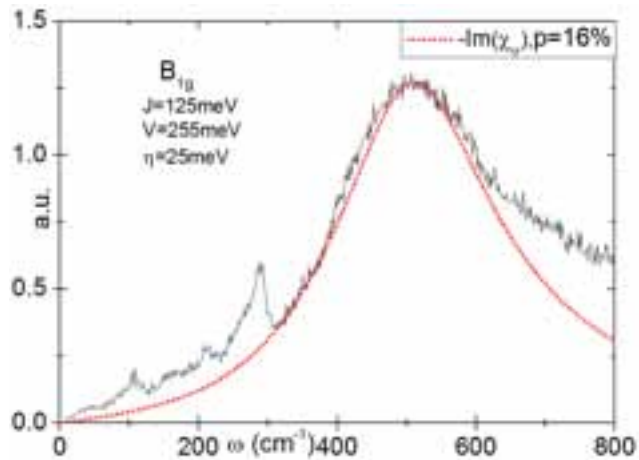
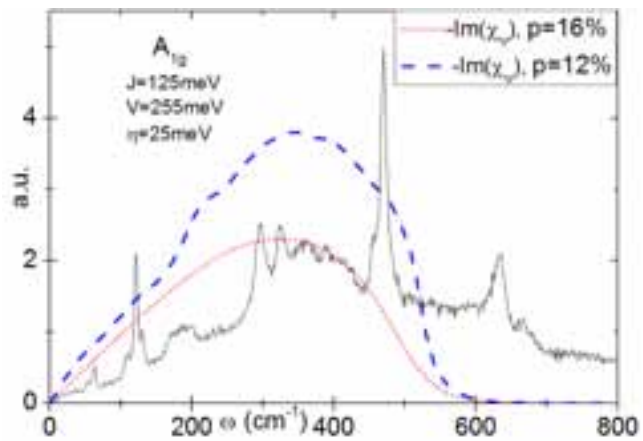
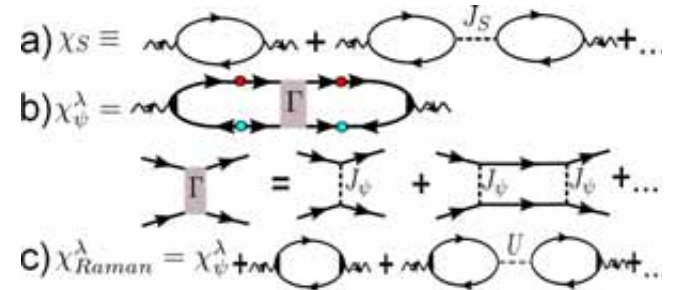
$$| \uparrow \rangle = c_{k,\uparrow}^\dagger c_{-k,\downarrow}^\dagger | 0 \rangle$$

$$\hat{\rho} = \sum_k c_{-k,\downarrow} c_{k-2k_F,\uparrow}$$

X. Montiel, T. Kloss,  
Y. Gallais, A. Sacuto, CP, preprint



$$\begin{aligned}
 H = & \sum_{\mathbf{k}, \sigma} \xi_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} \\
 & + \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} \left( J_{\mathbf{q}} c_{\mathbf{k}, \alpha}^{\dagger} \sigma_{\alpha\beta}^T c_{\mathbf{k}+\mathbf{q}, \beta} c_{\mathbf{k}'+\mathbf{q}, \gamma}^{\dagger} \sigma_{\gamma\delta} c_{\mathbf{k}', \delta} \right) \\
 & + \sum_{\mathbf{k}, \mathbf{k}', \sigma, \sigma'} V_{\mathbf{q}} c_{\mathbf{k}, \sigma}^{\dagger} c_{\mathbf{k}+\mathbf{q}, \sigma} c_{\mathbf{k}', \sigma'}^{\dagger} c_{\mathbf{k}'-\mathbf{q}, \sigma'}
 \end{aligned}$$

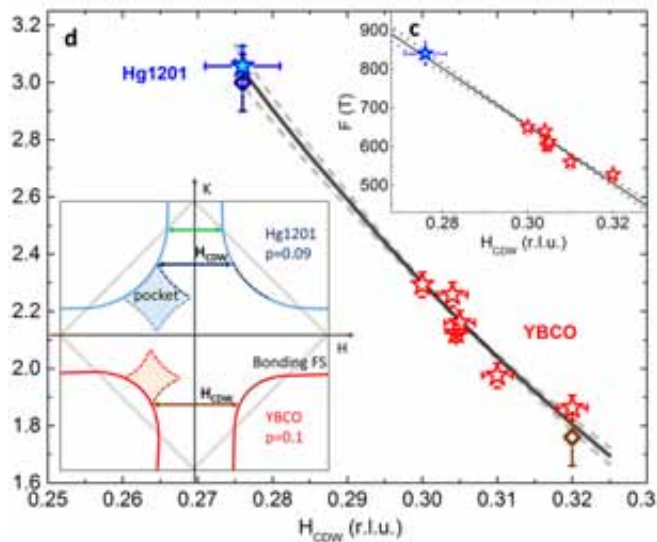
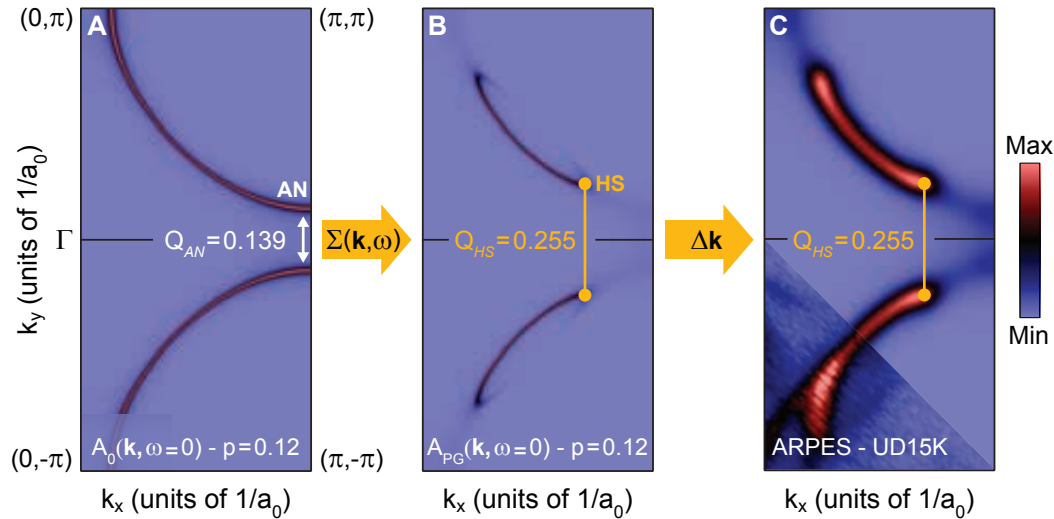


$$V_{\mathbf{q}} = V$$

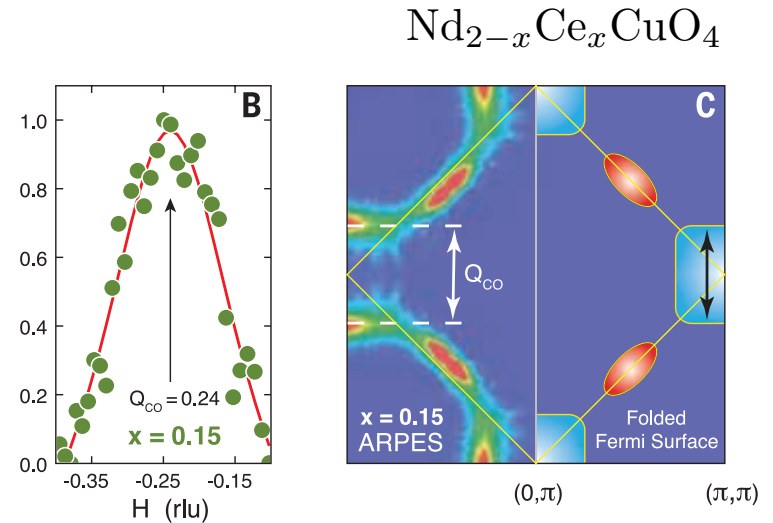
$$J_{\mathbf{q}} = -J$$

# Controversy with the CDW wave vector

Damascelli : (2013)

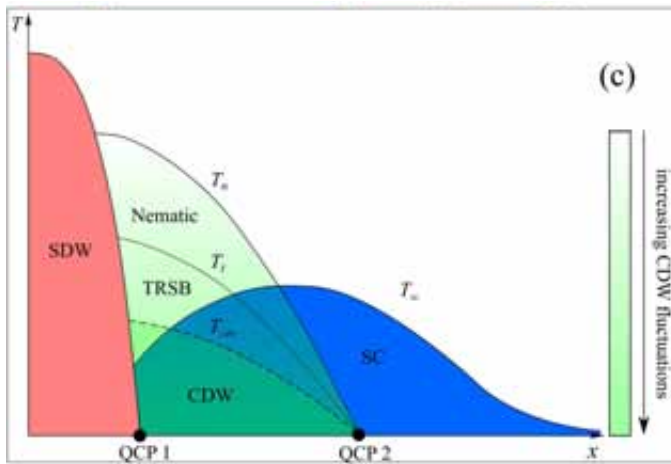


Greven (2014)

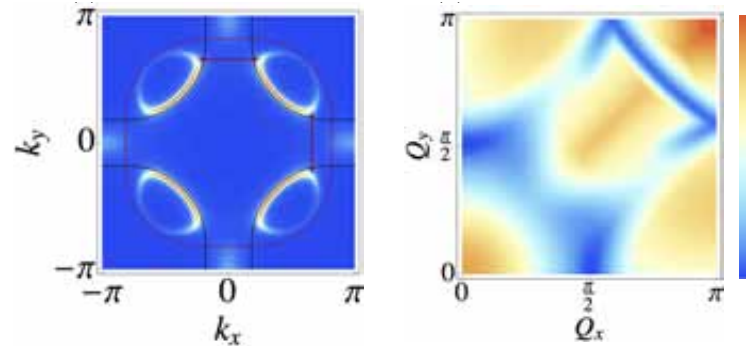


da Silva Neto et al. (2015)

Y. Wang, A. Chubukov (2014)

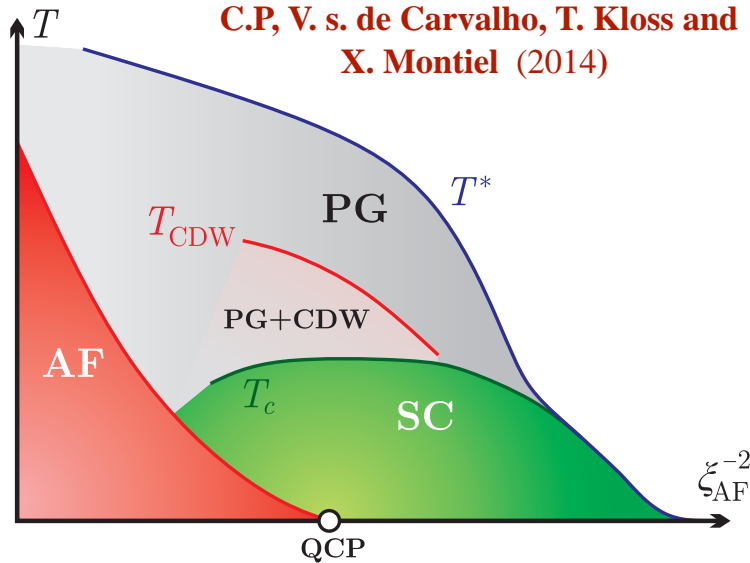


S. Sachdev, D. Chowdhury (2014)

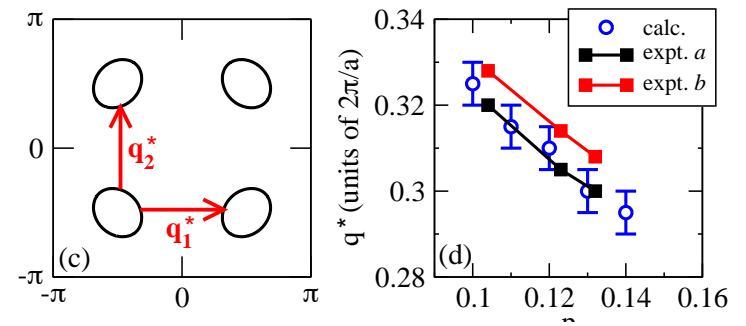


Pre-emptive order breaking TR

C.P, V. s. de Carvalho, T. Kloss and X. Montiel (2014)



Atkinson, A. Kampf (2013)



Secondary order at the tip of the arcs

Co-existence : PG + Q<sub>x</sub>, Q<sub>y</sub> CDW

# Re-examination of the SF model



pure CDW

co-existence ?

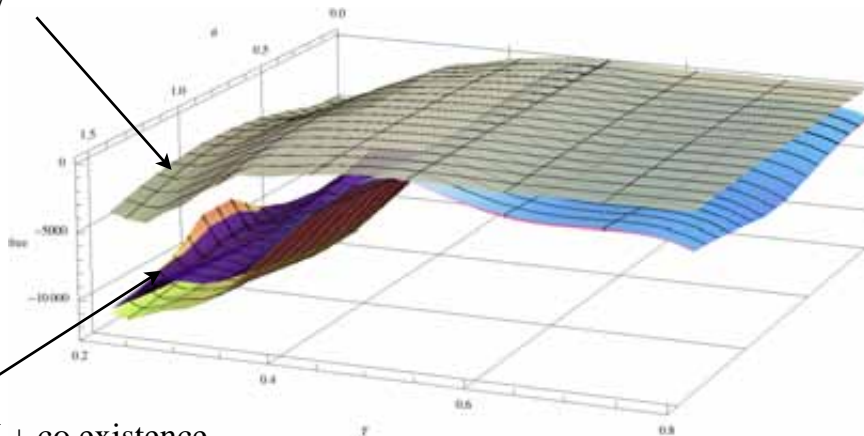
SC/QDW



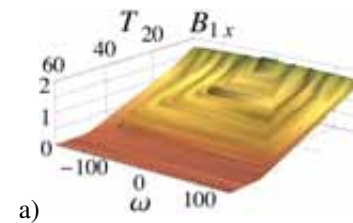
Small nematic term needed to get the CDW below  $T^*$

$$u\rho_{Q_x}\rho_{-Q_x}\langle\rho_{Q_y}\rho_{-Q_y}\rangle$$

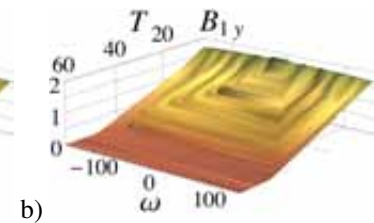
pure CDW



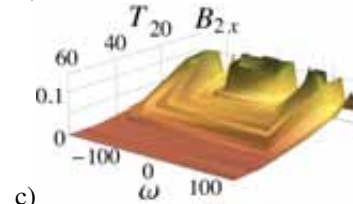
SC/QDW + co existence



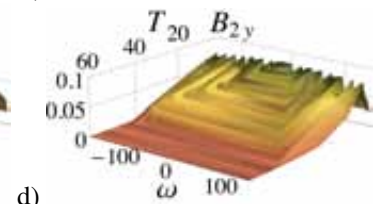
a)



b)



c)



d)

$$B_{1,x/y} = QDW$$

$$B_{2,x/y} = CDW$$

$$QDW \gg CDW$$

Free energy for the three cases

C.P, V. s. de Carvalho, T. Kloss and X. Montiel (2014)

# Conclusions

- Charge orders are a key players in cuprate physics: natural competitor of superconductivity.
- $SU(2)$  symmetry present in the under-doped region of the phase diagram
- Pseudo-gap with  $SU(2)$  symmetry and charge orders are precursors of the AFM order
- One can stabilize axial-CDW in co-existence with composite Peierls-SC phase
- $SU(2)$  rotation of axial CDW=PWD

