

A Few Steps Toward a Science of Mental Life

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ABSTRACT—Under what conditions can a true “science of mental life” arise from psychological investigations? Can psychology formulate scientific laws of a general nature, comparable in soundness to the laws of physics? I argue that the search for such laws must return to the forefront of psychological and developmental research, an enterprise that requires extensive collaboration between psychologists, neuroscientists, physicists, and mathematicians. Psychological laws may arise from at least 3 sources: the anchoring of thought processes in the biophysics of the brain, the computational constraints on possible mental algorithms, and the internalization of physical or statistical laws into our brains during evolution or development. I consider as an illustration the domain of numerical cognition, where a few solid psychophysical and decision-making laws have been established and related in part to their evolutionary precursors and neural bases. From this platform, I tentatively outline a few promising research directions in the domains of infant development, reading acquisition, executive control of multiple tasks, access to conscious report, and the spontaneous flow of conscious thoughts.

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In the fall 2005, I was elected to the Collège de France in Paris, for a newly created chair of Experimental Cognitive Psychology. Shortly after their election, professors at the Collège de France traditionally give a 1-hr inaugural lecture, which both introduces their discipline and sets a program of research and teaching for the years to come. The present article is the translation and adaptation of the text of the inaugural lecture, which was given on April 27, 2006. I am grateful to Kurt Fischer for encouraging the publication of this text, which only partly relates to issues of mind, brain, and education, and to Michel Ferrari and Marc Schwartz for their translation from the French original

“Psychology is the science of mental life.” With these few words, James (1890) outlined the domain that has become cognitive psychology. Psychology is at once seen as an integral part of life sciences, one that exploits the entire panoply of methods of biology—from genetics to brain imaging—but a science of *mental life* that aims to produce general laws of thought, an intimate and subjective domain that one might have thought inaccessible to the scientific method. Its goals are broad: How is the chain of command organized from perception to the motor act? In what form are our memories stored? What is a word, a concept, an emotion, an intention, a decision, an introspection? What rules govern the syntax of cognitive operations? How does one distinguish conscious information from nonconscious information? The challenge of cognitive psychology, in response to each of these questions, is to outline relevant general laws and to understand their origins at the intersection of constraints imposed as much by brain biology as by the environment and the culture within which these flourish.

One might legitimately judge this goal as unattainable. Many still consider psychology as a “soft” science. They doubt that it has methods and experimental results whose quality approach, even distantly, those of physics or chemistry. They see in the diversity of cultures, personalities, and human competencies the proof that no system could ever “put the human soul into equations.”

Yet, recent years testify to the unprecedented progress of cognitive science. In the last 20 years, labs have been created all around the world, wherein the deciphering of mental operations has brought together psychologists, linguists, anthropologists, ethnologists, neuropsychologists, neurologists, physicists, and mathematicians. The confrontation of ancient philosophical questions, often asked with acuity by Plato, Kant, or Descartes, by the new technologies of the behavioral sciences of neuroimaging and of mathematical modeling creates a friction particularly favorable to the

emergence of new knowledge. The expectations of society are equally numerous in this domain of research, so close to our everyday lives, working hand in hand with medicine and education, and in which the possibilities (particularly that of brain imaging) remain poorly known. It is thus urgent to teach cognitive psychology, as much to share the profound implications of some of its findings, as to debate, with full understanding of the causes, the challenges they raise.

IN SEARCH OF UNIVERSAL LAWS IN PSYCHOLOGY

The question that I would like to discuss in this article concerns the nature of the laws that psychology is likely to discover and the very possibility that certain of these laws will be as solid and universal as the laws of physics.¹

In the last 20 years, much of cognitive psychology has focused on the details of a few specific phenomena, rather than on the general architecture of cognition. No doubt it follows, with reason, the example of physics, which has shown us that only the obstinate study of a narrow question gives access to the intimate structure of the natural world—witness Galileo and the laws of falling bodies, Newton and the colors of the rainbow, and Einstein and the origins of the photoelectric effect. Psychology, in equal measure, has adopted Darwin's lesson: The brain evolved under multiple pressures and now comprises a broad collection of specialized functions. Cognitive psychology, as a field of study, has, therefore, preferentially focused on investigating in detail each domain of cognition. There exists, for instance, a psychology of face recognition, of reading, of action planning, of emotions, and of the representation of others.

Should we conclude that it is impossible for our science to establish universal laws? My answer is a clear "no." Over and above the hazards of our species' evolutionary and cultural history, I see at least three possible sources of general laws of cognition.

First, there are the laws of physics, chemistry, and biology. The anchoring of thought processes in brain biology implies that the principles of the organization of biological life constrain our mental life. As Jean-Pierre Changeux has emphasized, the human brain is a formidable chemical machine wherein we find the same molecular mechanisms at work as in the drosophila fly or the torpedo fish. Thus, the speed (or rather the slowness) of our mental operations and of our learning is directly related to the speed of the propagation of electrical signals, and of the state transitions of receptor molecules of our brain. Since 1850, with the help of methods developed by Emil Du Bois-Reymond, the physicist and physiologist Hermann Von Helmholtz established that the speed of nerve impulse is only a few tens of meters per second. Inspired by this work, Franciscus Donders showed, in 1868, that this slow conduction affects mental decisions: the speed of thought is not infinite. It is easy to decompose

mental operations into a sum of slow steps, each step requiring several tens of milliseconds (Donders, 1969; Sternberg, 1969). Today, this speed, which accelerates over the course of development, can be directly related to axonal myelination, which can now be estimated by diffusion tensor imaging in the living human brain. In certain forms of mental retardation, such as fragile X syndrome, the slowing down of cognitive functions is directly related to quantitative anomalies of the genome (Rivera, Menon, White, Glaser, & Reiss, 2002). Thus, the medical and developmental implications of such psychobiological laws are considerable.

A second category of psychological law is found at a level of description that one might qualify as algorithmic. The invention of the computer by Alan Turing and of John von Neumann, but also the work of Noam Chomsky or David Marr, has led to the emergence of a science of computation whose object is to invent and analyze algorithms that can efficiently resolve the most varied sorts of problems: visual recognition, information storage, learning of formal grammars, and so on. The human brain is a superb example of an information-processing system, one often confronted with the same kinds of problems for which there exist but a small number of effective solutions. Thus, the laws that psychology uncovers often answer to universal algorithmic constraints. A large part of cognitive psychology consists of inferring those algorithms of thought.

In the past, however, carried away by their enthusiasm with the computer metaphor, too many functional psychologists have neglected the architecture of the brain. All evidence indicates that this architecture in no way resembles a classical computer. It is a stunning machine where multiple levels are embedded in an architecture that supports massive parallel processing. With a hundred billion processors and a trillion connections, this structure is without equivalence in computer science and it would be a profound mistake to think that the computer metaphor can be applied to the brain in an unmodified fashion. Some have said that neurobiology is interested in the material basis and that psychology focuses on the program of the brain—the hardware and the software. But this reductive dichotomy is totally inadequate. All levels of organization, from molecular to social interactions, collaborate to determine our mental functioning (Changeux & Dehaene, 1989). There is, thus, no watertight compartmentalization between biology and psychology. On the contrary, both psychologists and neurobiologists try to understand, by different routes, how cognitive function emerges from the nested hierarchical architecture of the nervous system. Certainly, the laws of psychology can be temporarily formulated by means of formal algorithms. Nevertheless, they will never be deeply understood until they have been related to all levels of brain organization. For those discouraged by the complexity of this program, I simply recall the quip attributed to Lyall Watson: "If our brain were simple, we would be too simple-minded to understand it." It is precisely because our brain is so complex that we have a small chance of understanding it!

Neuroimaging methods play a central role here. There are still a few psychologists who consider brain imaging as an expensive enterprise of “neophrenology,” whose sole objective is to localize functions. To illustrate why this phrenological perspective is wrong, I would like to refer to an engraving by Mattias Greuter, who seems to have anticipated the future of brain imaging as far back as the 16th century (Figure 1). We see a volunteer being “scanned” in an alchemical oven. From this machine emerges not brain localizations, but mental representations of music, people, animals, houses, and so on. This metaphor strikes me as appropriate. Neuroimaging methods, above all, attempt to decompose the functional architecture of mental representations and offer an access to the mechanisms of thought that is more direct than the analysis of behavior. To the cognitive psychologist who likes to refine

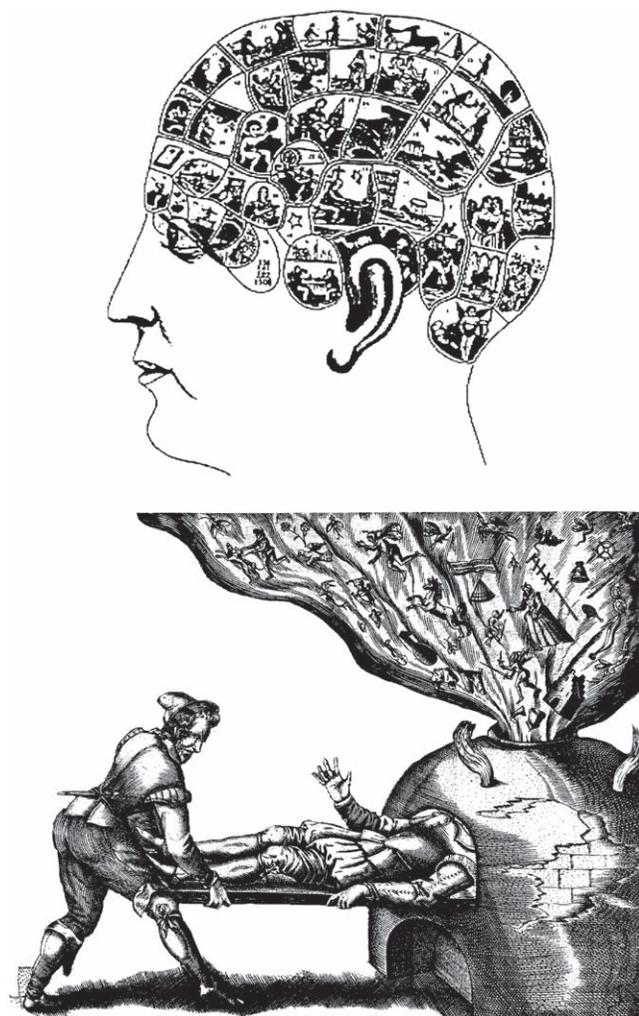


Fig. 1. Appropriate and inappropriate metaphors for functional brain imaging. The main goal of brain imaging is not to localize functions, a project that would resemble phrenology (top), but to analyze the format of representation, the functional architecture, and the internal mechanisms of mental operations, as metaphorically illustrated by this 16th century engraving by Mattias Greuter (bottom). Thus conceived, imaging methods are integral to the tools of cognitive psychology.

his tools, brain imaging provides the sharpest of scalpels. It relies naturally on the development of “virtual anatomists,” programs that identify anatomically the fissures of the cortex, separate the transcortical networks within the white matter above which the cortex is enfolded, and present it in the form of flat, standardized maps.

But cartography is only one step. Afterward, functional properties are projected onto these anatomical maps that are of direct interest to psychology. Let me give a few examples of what research questions lie within reach of current methods; following the temporal sequence of brain activity when reading a word, whose first steps highlight the anomalies in dyslexic children (Helenius, Tarkiainen, Cornelissen, Hansen, & Salmelin, 1999; Marinkovic et al., 2003); identifying retinotopic visual maps, whose surface varies from one individual to another and predicts visual acuity (Duncan & Boynton, 2003); tracking the focus of attention that a person brings to one object or another and that amplifies cortical activity in corresponding cortical areas (Kamitani & Tong, 2005); or, finally, decoding of mental images by visualizing their traces on the cortex (Kosslyn, Thompson, Kim, & Alpert, 1995; Thirion et al., in press). When we imagine a shape in the mind’s eye, the activity in visual areas sketches the contours of the imagined object (Figure 2). Thus, we begin to unravel one of the most ancient questions of psychology, the analogical or propositional nature of mental images, something behavioral analysis alone could never fully resolve.

The question of mental images brings us back to the laws of psychology. In effect, the internalization of images of the outside world within our brain introduces a third category of universal psychological laws, which are, in reality, internalized laws of physics. “The most incomprehensible,” Albert Einstein said, “is that the world is comprehensible.” However, how could we survive if the laws of our environment were totally foreign to us? From a Darwinian perspective, the permanence of the organism cannot be imagined without some minimal ability to make the world intelligible. Over the course of our evolution, but also during development, our nervous system learns to *comprehend* its environment, which literally means to take it into ourselves, to internalize it in the form of mental representations that, by a psychophysical isomorphism, constitute a small-scale reproduction of some of its natural laws. We carry within ourselves a universe of mental objects whose laws imitate those of physics and geometry.

It is in perception and in action that these internalized physical laws manifest themselves most clearly. Our sensorimotor apparatus understands kinematics when it anticipates the trajectories of objects. But these laws continue to be applicable in the absence of any action or perception when we merely imagine a moving object or a trajectory on a map. As Roger Shepard and Steven Kosslyn have shown, the time needed to rotate or explore these mental images follows a linear function of the angle or distance traveled. Mental trajectory imitates that of a physical object.

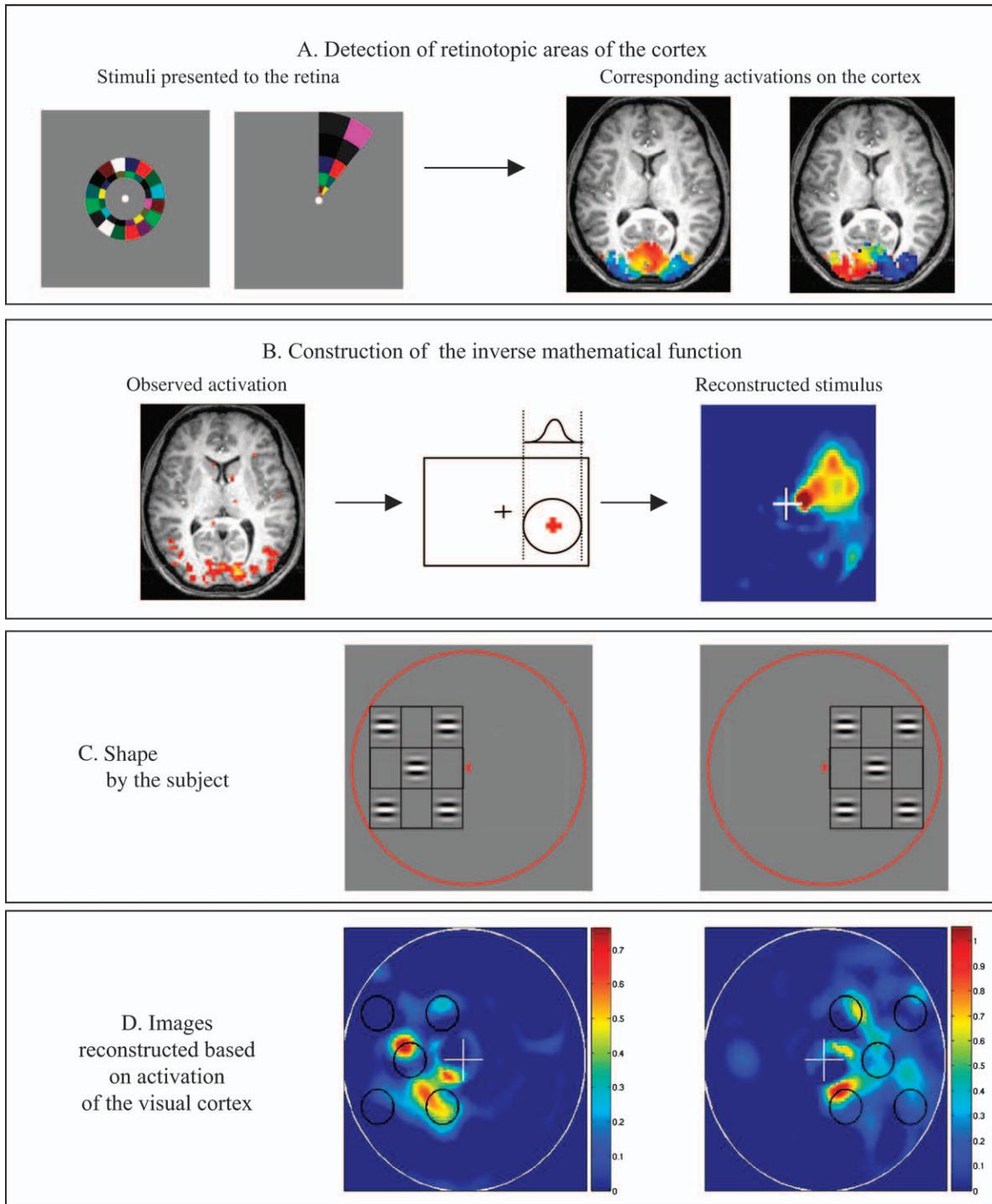


Fig. 2. Reconstructing the contents of percepts and mental images from cortical activity patterns. One of the future directions of brain imaging research lies in the decoding of mental contents from brain activity patterns. This methodological development is illustrated here in the case of retinotopic decoding of visual images (Thirion et al., 2006). Systematic measurement of occipital activity evoked by moving visual stimuli (A) allows us to construct an inverse mathematical function for each participant that links each active voxel to a corresponding part of the perceived stimulus (B). We applied this technique to fMRI data recorded while someone imagined an “X” shape either in the right or in the left visual field. The reconstructed image (C) approximates the contents of the subject’s mental image (D).

Roger Shepard, following Ernst Mach, sees in these internalized physical laws the origin of the mysterious efficiency of “thought experiments” which allow us, by pure reflection,

to draw profound conclusions about these natural laws (Shepard, 2001). Galileo, before even beginning the experiment, used reasoning alone to conclude that two bodies of

different masses must fall at the same speed. The very possibility of such thought experiments show that our mind incorporates some of the laws of physics.

My own research has led me to extend this conclusion to mathematics. I argue, in effect, that mathematical activity finds its ultimate roots in the structured mental representation that we have inherited from our evolution: the sense of space, time, and number. In the lab, we can only study the most elementary of these mental operations, which one might call “protomathematical.” Nevertheless, we are beginning to understand them in enough detail that, step by step, we can infer the processes employed by our brain. In the present article, I briefly review some of the experimental steps that lead to the dissection of one such protomathematical domain, mental arithmetic, and the identification of the different laws that govern it. We will see that the psychology of arithmetic offers a wonderful pretext to review some of the most solid laws of cognitive psychology. By a curious recursive loop, the mathematics produced by the human mind helps us to formulate its laws.

THE ORIGIN OF MATHEMATICAL CONCEPTS

The issue of the origins of concepts, particularly of abstract concepts like number, is among the most central questions that cognitive psychology purports to solve. One might think that arithmetic is nothing but a recent cultural invention, an ensemble of recipes invented by the civilized world to resolve its accounting problems. Yet, ever since the 1950s, the work of Otto Köhler demonstrated that the concept of number is accessible to many animal species. Köhler trained rodents and birds to enumerate a set of dots, then to find which of several boxes had a cover with the same number of dots. More recently, dozens of experiments—conducted notably by Herb Terrace and Elizabeth Brannon—have extended these demonstrations to show that estimation, comparison, and calculation of approximate numerical quantities are accessible not only to nonhuman primates but also to rodents, birds, dolphins, and certain reptiles (Brannon, 2006). Elegant controls show that, in many cases, it really is number and not another variable (like surface area or width) that determines the animal’s choice. A simple criterion allows us to judge the degree of abstraction of this mental representation: generalization across visual and auditory modalities. A primate, for example, recognizes the association between three faces and three voices (Jordan, Brannon, Logothetis, & Ghazanfar, 2005).

Does the concept of number only emerge in laboratory animals after thousands of training trials? On the contrary, research in cognitive ethnology shows that many species spontaneously use arithmetic in the wild. According to the observations of Marc Hauser, before chimpanzees attack an adversary, they evaluate whether their coalition is large

enough, a behavior also observed in dolphins and lions (Hauser, 2005). Other primates, tested in a single trial without training, anticipate the result of the addition or subtraction of food morsels before choosing the larger of two sources of food. Thus, intuitions of approximate numbers—but also of other Kantian categories like space and time—are widespread in the animal world, no doubt because they are essential to survival. Every species needs to evaluate sources and quantities of food or, in the case of social species, the number and quality of its allies or its enemies.

Homo sapiens have inherited these protoarithmetic capacities. Jean Piaget, pioneer of cognitive development, thought that he had discerned a hierarchical construction of logical mathematical operations through careful questioning of children (Piaget & Szeminska, 1941). He concluded that the abstract concept of number emerges only at a late stage in children. However, we know today that, by proposing situations of cognitive conflict that are difficult for the infant brain to manage, his experiments underestimated early numeric competencies. Furthermore, Piaget studies were based principally on a dialogue with the child and did not sufficiently distinguish the explicit and often linguistic formulation of concepts (which is indeed late to develop) from a nonverbal arithmetic intuition that is much more precocious and universal. The new psychology of development, inspired by ethology, designed nonverbal methods to evaluate the competencies of babies only a few months old and without recourse to language. In the domain of arithmetic, the results are incontrovertible: The sense of number exists very early in the infant. A few months after birth a baby already knows the difference between 8 and 16 objects, establishes multimodal links between two sounds and two images, and evaluates a concrete arithmetic operation (Feigenson, Dehaene, & Spelke, 2004). The experiments of Karen Wynn and Elizabeth Spelke show that infants have already internalized basic arithmetic operations of addition and subtraction. When an animation shows five objects disappearing behind the screen, which are joined by five more objects, the baby looks longer at the final scene if, by magic, there are only five objects and not the 10 expected (McCrink & Wynn, 2004).

Twenty years of research on the cognitive development of arithmetic, thus, refute the idea of a slow, logical construction spread out over the entire length of childhood. From birth on, our brain expects to find moving objects in the external world whose combinations obey the rules of arithmetic. Should we, therefore, conclude that the concept of number is “innate”? I object to the use of this term, which, in my opinion, cognitive psychology often uses imprudently and almost as an incantation. To say that a behavior is innate only hides our ignorance of the mechanisms of its development. A vast explanatory gap separates molecular genetics, the only level where one can legitimately speak of an innate code, from the precocious competencies of the child. Genes do not specify behaviors, much less concepts.

At most, they define initial biases—a “learning instinct,” to use the felicitous expression of James Gould and Peter Marler. Although strong initial biases may exist, as is clearly seen in the initial cortical organization of language areas (Dehaene-Lambertz et al., 2006), the initial organization and the development of the infant brain still constitutes a vast terra incognita that will be fascinating to explore in the years to come.

THE PSYCHOPHYSICAL LAWS OF MENTAL ARITHMETIC

The presence of numeric competencies in newborns, before they even acquire their first words, highlights the possibility of an abstract thought without language. On this point, the preoccupations of psychology join those of cognitive anthropology: to uncover universal mental structures over and above the variability of language and culture. In recent years, a series of studies, inspired by the methods used to study young children, has demonstrated the presence of arithmetic and geometrical intuitions without language in remote populations (Dehaene, Izard, Pica, & Spelke, 2006; Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). With our linguist colleague Pierre Pica, Véronique Izard, Cathy Lemer, Elizabeth Spelke, and I examined the mathematical competencies of an Amazonian people, the Mundurucus, who only have words for the small numbers, one to five. If words shape the contents of our thoughts, then the numerical cognition of these people should be limited to small approximate numbers. But it is nothing of the sort. The Mundurucus, children and adults, possess a rich arithmetic intuition. When tests do not use language but present large numbers in the form of animated sets of objects, participants instantly understand the concepts of addition, subtraction, or approximate comparison. They do not know how to count, but they know that the cardinal of a set changes as soon as an object is added or taken away. The *concept* of number, thus, precedes the *word* for number. Similar observations were made for geometry: the fundamental concepts—point, right angle, parallelism, distance, midpoint, and so on—are all present in protomathematical form before we have words for them.

Experiments have allowed us to discern, within this core of competencies, simple and universal laws that are valid in the human adult as well as in babies or animals. To illustrate them, consider a simple test of number comparison (Figure 3). Two sets are presented visually, side by side, and we are asked to decide, without counting, which has the greater number of objects. By systematically varying the number within the two sets, and by collecting several hundred answers, we can establish the laws of numerical decision making. For example, let us fix one of the numbers at 16 and vary the other. We observe the first law, the *distance effect*: The number of errors of comparison decreases as a regular function of the distance between the numbers. Its slope, which varies from one person to another, measures the

precision of numerical judgments: a steep slope indicates a good capacity to detect small numerical differences.

Now, let us change the first number to 32 (double the preceding value). We see that the slope of the discrimination curve is halved. This is one indication of a second law, which bears the name of its discoverer, *Weber's law*: when size increases, the imprecision of judgment increases in direct proportion. In other words, the bigger the numbers, the more approximate their estimation.

We owe an interesting (although continuously debated) reformulation of this observation to Gustav Fechner. According to Fechner's law, physical dimensions, like size or number, are represented on an internal continuum which is not linear but compresses big numbers in direct proportion to their size, according to a logarithmic law. Indeed, if our observations are represented on a logarithmic instead of a linear scale, the error rate becomes a symmetric function, which is very simple, regular, and invariant across the set of numbers tested (Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004).

Where does the sigmoidal discrimination function itself come from? Let us suppose, following Fechner, that each quantity is represented mentally on a logarithmic scale. Let us further assume, following Thurstone, that these mental quantities cannot be encoded with absolute precision and display a Gaussian variability. Thus, each quantity is mentally represented on a given trial by a random Gaussian variable on a logarithmic continuum. The optimum response rule can then be calculated mathematically. To decide if a quantity is smaller or larger than 16, it suffices to fix a response criteria on the continuum and to answer “bigger” each time the random variable exceeds that point. This model does not perform perfectly, but it predicts a level of error equal to the area captured between the Gaussian curve and the response criteria. Indeed, the integral of the Gaussian adapts to the experimental findings with remarkable precision, but only if this function is applied on a logarithmic scale.

This small example shows how a simple mathematical model can account for a complex behavior. We have known for many years that signal detection theory—about which I have just sketched out a few principles—accounts in remarkable detail for most perceptual judgments (size, weight, pitch, etc.). We see today that it also governs abstract dimensions such as numbers. Psychophysical laws, often derivable from Bayesian statistics, suggest that the principles of cerebral encoding of mental objects may be similar at perceptual and conceptual levels. We now see the beginning of a “physics of concepts,” at least the simpler ones such as the concept of approximate number.

NUMBER NEURONS

Let us pursue the parallel between the history of psychology and that of physics for a few moments. Physics had its most

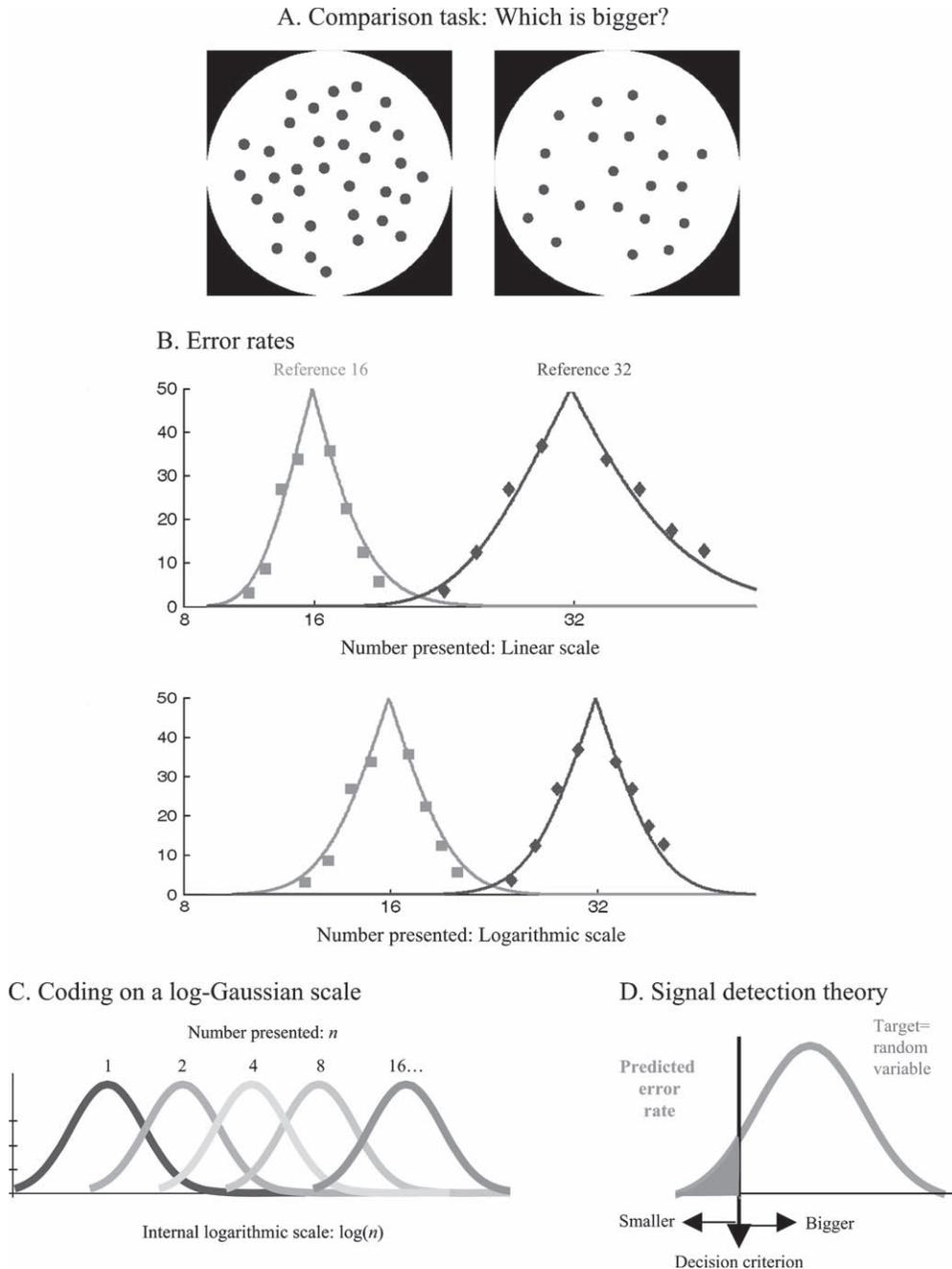


Fig. 3. Basic results of psychophysics in the number domain. When a participant (human or animal) is asked to select the larger of two sets of objects (A), the error rate systematically decreases as the distance between the numbers increases (B). When set on a linear scale, this function is asymmetric and of variable width, but it becomes symmetric and of fixed width when set on a logarithmic scale. Two hypotheses suffice to explain this finding: (a) numbers are coded by Gaussian variables on an internal logarithmic scale (Fechner's law; C) and (b) a fixed decision criterion is applied to this internal continuum (D). This theory predicts that the rate of errors decreases as the integral of a Gaussian, a theoretical function adjusted to the empirical data in panel B (after Piazza et al., 2004).

spectacular success when a theoretical object, initially a pure mental construct, saw itself confirmed years later by experimentation. The most famous example is that of the neutrino; Wolfgang Pauli postulated its existence in 1930, but it wasn't detected until 1956. It is a very encouraging sign of the maturity of cognitive psychology that it is now beginning to

encounter somewhat similar successes in the naturalization of its theoretical objects. Fechner's proposition that number and other dimensions are represented on a logarithmic scale, at first, was nothing but a mathematical formalism. It remained so until the years between 1980 and 1990, when the formal modeling of neural networks allowed for a neurobiological

explanation to be proposed (Dehaene & Changeux, 1993). Quantities could be coded by competing neuronal groups, each attaining a maximum level of discharge when a given quantity is presented. Thus, this theoretical model predicted the existence of “neuronal number detectors” coding for the presence of a set of four or five objects, for example. Explicit modeling of their functioning shows that the bigger the number, the more variable their neural tuning curves. According to this model, it is rational to allocate fewer neurons to bigger numbers, and this compressed neural code leads to a first approximation of the Weber-Fechner law (Dehaene & Changeux, 1993; Verguts & Fias, 2004).

In parallel, the development of brain imaging methods (from 1985 to 2000) allowed us to obtain ever more precise images of the human brain performing calculations, first by single photon imaging, then by positron emission tomography, and finally by functional magnetic resonance imaging (fMRI) (Dehaene, Piazza, Pinel, & Cohen, 2003). Those imaging experiments revealed a strong regularity. All tasks that invoke a sense of quantity—addition, subtraction, comparison, even the simple viewing of an Arabic numeral, or the mere enumeration of a cloud of dots—activate a reproducible network of regions hidden in the depth of the intraparietal sulcus of both hemispheres. This localization is in agreement with the findings of neurologists. As early as the 1920s, based on observations of many wounded soldiers of the first World War, two German neurologists, Henschen and Gerstmann, observed that lesions to the left parietal lobe led to acalculia, where the patient is no longer able to perform operations as simple as 7 minus 2 or 3 plus 5. Those observations have been confirmed by many new cases since then.

Since the year 2000, the continued refinements of neuroimaging techniques have led to a precise specification of the region activated by calculations in the parietal lobe. fMRI shows this activity embedded within a network of regions implicated in the movement of the eyes, of attention, and of the hands (Simon, Mangin, Cohen, Le Bihan, & Dehaene, 2002; Simon et al., 2004). All these visuomotor activities have been studied by electrophysiology in macaque monkeys. Specialized parietal areas have been identified in the macaque brain, and their geometrical layout largely reproduces the organization found in humans. Thus, there exists a plausible homology between human and nonhuman primates. Does this homology extend into the domain of mathematics? As we have seen, the macaque monkey can perform elementary arithmetic operations with a behavioral profile of distance and size effects quite comparable to that of human subjects (Cantlon & Brannon, 2006). So, a very tentative hypothesis was formulated: The intraparietal region, in both human and nonhuman primates, might well house the number-detecting neurons postulated by neural network models of number processing (Dehaene, 1997).

In 2002, Andreas Nieder and Earl Miller put this audacious idea to the test. They trained animals to judge whether

two sets contain the same number of objects. A huge battery of controls confirmed that the performance of the animals was definitely linked to numbers and that their performance followed Weber’s law. Then, recording in the prefrontal region as well as deep within the intraparietal region, they discovered that almost 15–30% of the neurons were sensitive to number at a location quite compatible with human brain imaging studies (Figure 4). Their response profiles closely matched the theory: Each neuron fired maximally to a given number, its rate of firing decreased with numerical distance, and its response curve as a function of the stimulus number traced an almost perfect Gaussian curve when the results were plotted on a logarithmic scale (Nieder, Freedman, & Miller, 2002; Nieder & Miller, 2003, 2004).

All these properties allow us to finally understand the neuronal origin of Weber’s law and the distance effect. Weber’s law stems from the increasing width of the neuronal tuning curves as the numbers get larger. The distance effect comes from the overlap between populations of neurons that code for nearby numbers. Anatomically, a plausible homology seems to relate the parietal regions that are concerned with number in the human and animal brain. This set of results opens a vast program for research. Can the precision of arithmetic operations be deduced from that of the neural code? How does activity move through the neuronal map during a calculation? By what mechanisms do parietal neurons acquire their selectivity? Are they present in untrained animals? Which developmental genes establish the parietal map? What is their degree of conservation from one species to another? Can disorders of arithmetic competence in some children (dyscalculia) be explained by an anomaly in such genes or in their interaction with pathogens such as exposure to alcohol in utero?

THE MENTAL CHRONOMETRY OF NUMERICAL DECISION MAKING

One may object that human arithmetic presents a major difference with the rudimentary competence present in animals. Our species alone has invented a variety of symbols, words, and numbers, whose form varies massively from one culture to another. Thanks to these symbols, do we not have access to precise calculations of an entirely different nature, unrelated to the perception of quantities in animals? Absolutely not. While there is no doubt that the invention of symbolic algorithms has considerably expanded our human mathematical competencies, their foundation remains profoundly entrenched in animal cognition. When we do symbolic calculations, the distance effect and Weber’s law continue to characterize our behavior. In 1967, Moyer and Landauer discovered that the distance effect affects our comprehension of symbols such as Arabic numerals (Figure 5) (Moyer & Landauer, 1967). Suppose, for example, that I present you with several numbers, one

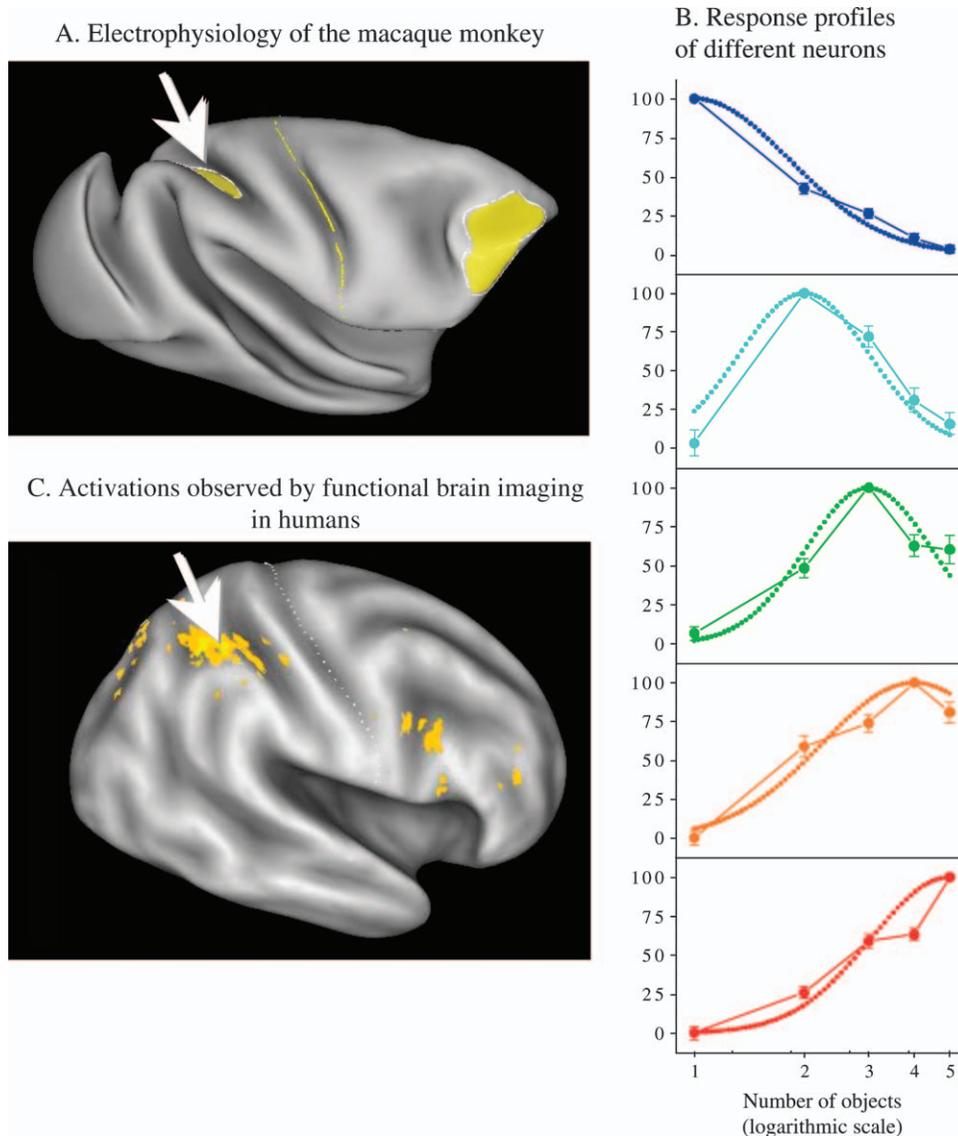


Fig. 4. Number-coding neurons in the macaque monkey (after Nieder & Miller, 2002, 2003, 2004). Neurons have been identified in the depth of the intraparietal sulcus (A) whose response profiles correspond to the predictions of psychophysical theory: each neuron prefers a particular number, and its rate of firing shows a Gaussian profile on a logarithmic scale (B). The deep intraparietal region wherein these neurons have been identified shows a plausible homology with the horizontal segment of the human intraparietal sulcus, where brain imaging shows activations linked to mental calculation and to the processing of quantities (C).

after the other, asking you to decide, as rapidly as possible, if these numbers are bigger or smaller than 65. Although their magnitude is known to you precisely, you would be faster for numbers far from 65 and get increasingly slower as the numbers get closer to 65. Furthermore, your error rate, although low overall, would follow a similar distance profiled (see Figure 5).

How do we make sense of this distance effect? The brain clearly does not base its decision on a manipulation of the digital symbols of Arabic numerals. If that were the case, it would first compare the leftmost number with 6 and then afterward, and only if necessary, the rightmost number with 5. On the contrary, we find that the global quantity is taken into account. A mental conversion is produced: The number

is translated mentally into an internal quantity comparable to that which was evoked by an ensemble of objects, thus variable, fluctuating, and subject to psychophysical laws.

In the presence of such stochastic fluctuations, the brain must behave like a statistician who collects multiple samples before reaching a firm conclusion. The effect of distance can thus be explained by the fact that this collection process, which is necessary to make a decision, lasts longer when the objects compared are close in meaning. But what is the optimal mathematical algorithm by which one should make such a decision? Around 1943, Alan Turing had formalized this problem within the context of cryptography. At Bletchley Park, he received German messages encrypted by the machine,

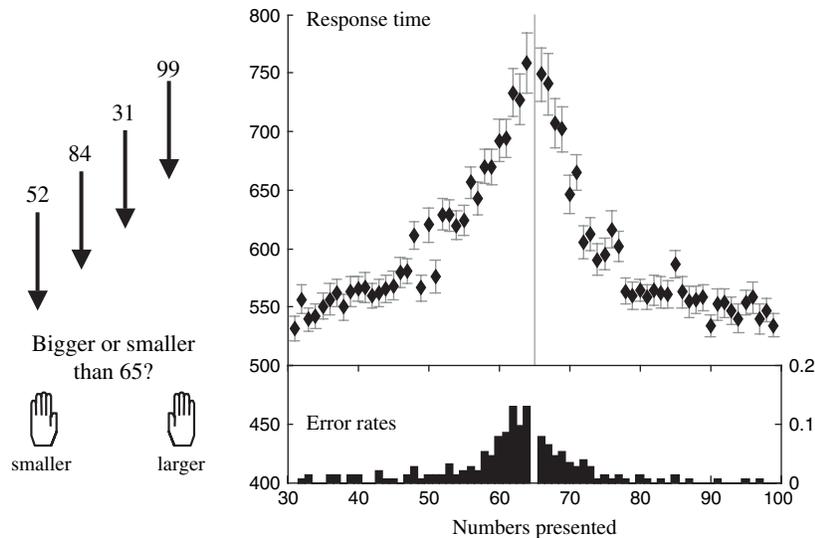


Fig. 5. The distance effect in number comparison, first discovered by Moyer and Landauer in 1967. Even when comparing numbers presented in a symbolic form, the response time and error rate continually decrease as the distance between the compared numbers increases.

Enigma. Each fraction of the message furnished only a few bits of information, too little to exploit. Turing discovered how to combine them so as to obtain more stable information. His theory defined the weight of the information, “I,” in favor of hypothesis “A,” as the logarithm of its “likelihood,” which is the ratio of the probabilities of observing “I” under this hypothesis and under the contrary hypothesis:

$$\text{weight of I in favor of A} = \text{Log} \left[\frac{\text{probability of I if A is true}}{\text{probability of I if A is false}} \right]$$

According to Bayes’ law, the weights furnished by independent pieces of information can be added:

$$\text{total weight in favor of A} = \text{initial bias} + \text{weight(I}_1) + \text{weight(I}_2) + \text{weight(I}_3) + \dots$$

What this equation says is that an essentially arbitrary level of certainty about the final decision can be obtained by adding the weights and waiting for the total to reach a predefined certainty threshold, even if each piece of information is very small. Combined with the use of the first computers, Turing’s algorithm allowed the enigma code to be decrypted. But Turing had invented a general mechanism whose application went far beyond the field of cryptography alone. In 1947, the statistician Abraham Wald rediscovered Turing’s algorithm and demonstrated that it constitutes an optimal mechanism for sequential statistical inference. In the 1960s, psychologists Stone and Laming postulated that the human brain uses this sequential sampling rule. Consider how this idea applies to the comparison of numbers. At each instant, the representation of number is supposed to be drawn from a Gaussian law on a logarithmic continuum. Each sample provides a vote in favor of an answer, greater or smaller than 65. The sum of these

votes grows stochastically, forming what mathematicians call a random walk (Figure 6). The model presupposes that an answer is given as soon as the sum attains one of two preestablished thresholds. The subject decides to answer “greater” if the higher threshold is attained and answer “smaller” if the lower threshold is attained. The thresholds can be adjusted so as to achieve an optimal compromise between response time and error rate.

What is truly remarkable about this model is that, with minimal complications, it can explain in great detail the variability in human decision making (Smith & Ratcliff, 2004). Why are our decisions so variable and full of errors while the symbolic stimulus shows no variability? The random-walk model explains this by highlighting that the decision system must, in a noisy neuronal environment, find and extract the relevant signal somewhat like a specialist in cryptography would. Using this model and its variants, introduced by Link, Ratcliff, or Usher and McClelland, calculating the distribution of response times becomes a purely mathematical problem: that of a diffusion process with absorbing barriers. Its solution is well known by physicists. Ratcliff and his colleagues showed that the predicted distribution quite accurately fits the laws of mental chronometry and, in particular, the precise shape of the known distribution of response times.

If the signal is weak, it may happen that the internal accumulation never reaches the threshold, especially if the accumulator is leaky and fails to faithfully accumulate information across time (Usher & McClelland, 2001). The subject must, therefore, answer with a forced choice after a fixed amount of time. The law of large numbers predicts that, if we stop the accumulation after a fixed time, the internal state of the accumulator, which produces the decision, will be a Gaussian

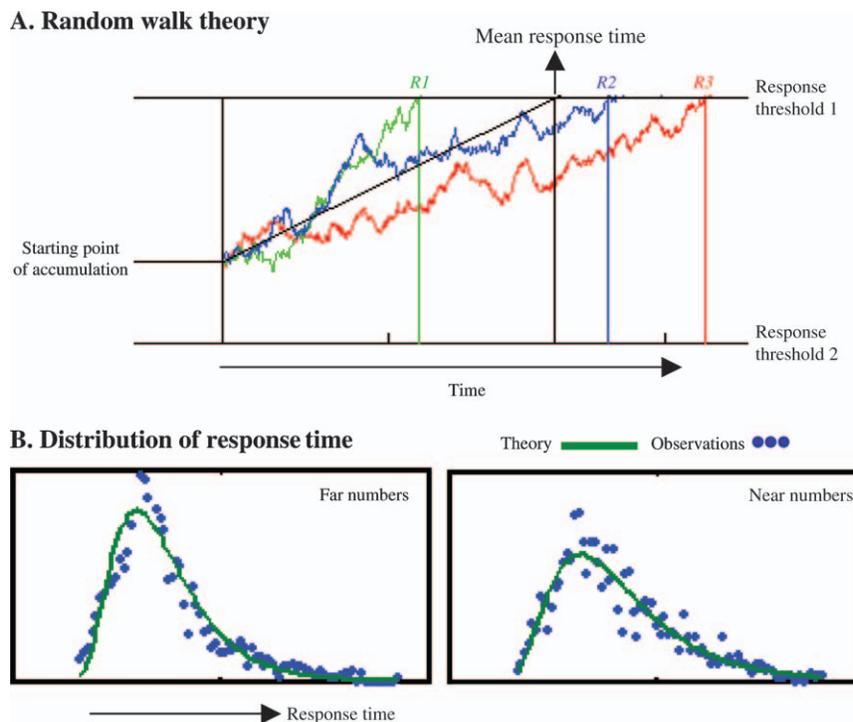


Fig. 6. Modeling simple decision making by a random-walk accumulation process. Each decision results from a stochastic accumulation whose slope depends on the quality of the task-relevant perceptual or cognitive inputs. Accumulation of evidence continues until one of two response thresholds is reached. The model explains response variability from trial to trial and correctly predicts the shape of the response time distribution in many basic cognitive tasks.

random variable. Thus, under those circumstances, we recover the hypotheses of signal detection theory, which I described earlier: The new decision theory encompasses the old one. It also accounts for the fact that, under such response-deadline conditions, performance increases with decision time according to a law precisely predicted by the theory.

THE NEURAL LAWS OF DECISION MAKING

I have already stated that psychological models (at least the good models) should, like the hypothesis of the neutrino, ultimately be verified at the neuronal level. Almost 40 years after its first formulation, the psychological theory of the stochastic accumulator was confirmed by electrophysiology. Michael Shadlen, William Newsome, and their colleagues recorded the activity of neurons in the prefrontal cortex of macaque monkeys while they were making a perceptual decision. They found that the activity of these neurons increased stochastically, with the slope directly related to the quality of the perceptual information (Gold & Shadlen, 2002). Parietal and prefrontal neurons appeared to accumulate the perceptual signals transmitted by the relevant posterior cortical regions in order to make a decision. Their profile of discharge traced a random walk no longer virtual but inscribed in cerebral

activity. This random walk predicted the latency of the response in the animal and even the occurrence of errors.

The last 5 years have seen a remarkable expansion of this research domain. Accumulator neurons have been observed in the lateral parietal region, the frontal cortex, and the superior colliculus. Their behavior has been finely modeled by the physics of dynamic systems. Wong and Wang (2006) have shown how a sequential accumulation emerges, in first approximation, from the dynamic activity of a network with feedback loops, where each decision is represented by a distinct group of neurons. The time of convergence, thus, depends on the characteristics of a “saddle point” that separates the attractor basins corresponding to the different choices. It is a minimalist model, obviously simplified, which tentatively links the faculty of temporal integration to the speed of the NMDA (N-methyl-D-aspartic acid) glutamate receptors, an interesting prediction that needs to be tested in the future.

We also need to add to the model reward mechanisms that may bias decision making toward choices that have led in the past to a favorable outcome. These emotional biases that Antonio Damasio likes to call “gut feelings” regulate our choices, often for the best, but sometimes for the worse. Drug addiction can be explained, at least in part, by a bias in our decision-making apparatus: According to current theories, some drugs directly manipulate the pharmacological

mechanisms of decision making such that they remain frozen in an inescapable choice (Gutkin, Dehaene, & Changeux, 2006; Redish, 2004).

The progressive elucidation of mental decision-making mechanisms thus gives flesh to the vision of Jean-Pierre Changeux, who underscored that “a total compatibility of principle exists between the most absolute determinism and the apparent unpredictability of behavior.” The French poet Mallarmé seemingly anticipated this idea when he famously stated, “All thought emits a roll of the dice.” In conformity with the project defined in Changeux’s *Neuronal Man*, the variability and illusion of freewill associated with human decision making begin to be connected to simple neuronal mechanisms whose dynamics govern our behavior. Folk psychology asks how we make decisions. The new theory shows how decisions form within us by spontaneous symmetry breaking within stochastic neuronal networks. In this emerging theory, the psychological laws of mental chronometry are deduced from the statistical physics of neuronal networks and these implement, to a first approximation, the optimal decision-making algorithm first outlined by Alan Turing. In sum, evolution has given our cerebral networks dynamics that approximate statistical calculations in an ideal observer.

THE DECOMPOSITION OF A MENTAL OPERATION

After having looked at decision making, let’s take a larger perspective and consider the successive steps in processing numeric symbols. Our current model can be separated into three stages: visual recognition of the symbol, conversion into an internal quantity that serves to support decision making, and motor programming of a response. To evaluate the validity of such a breakdown, the psychologist Sternberg (1969) introduced the “additive factors method.” This involves varying independently several experimental variables, each supposed to affect only one processing step, and to study their impact on both response time and cerebral activity (Figure 7). In the case of number comparison, each step can be selectively slowed down by a distinct experimental factor: Visual identification is slower for words than for numbers; decision making, as we have seen, is slower when the difference between numbers approaches zero; and finally, response latency is greater if we increase the complexity of the response, for example, by requiring participants to click twice in a row.

If information processing is serial, the cumulative effects of slowing down each step should be additive. And that is in fact the case: Each step adds a fixed amount of time to the average total time of calculation. fMRI and electroencephalography confirm the presence of three distinct cerebral systems, each affected by a single factor and engaged at a different time (Dehaene, 1996; Pinel, Dehaene, Riviere, & LeBihan, 2001). Visual analysis begins after 110 ms in the occipito-temporal

region of the left hemisphere for written words and in both hemispheres for Arabic numerals. At this stage, there is no trace of meaning: Processing is exclusively concerned with the recognition of the symbols’ shape. However, after about 190 ms, the intraparietal region becomes active and the code changes: This region is only interested in numeric quantities not in the particular notation used to denote them. Finally, after 250 ms, there appear the first effects of motor programming in the premotor and motor cortex. At this point, the only thing that counts is the side of the upcoming response, left or right.

MECHANISMS FOR VISUAL WORD RECOGNITION

The serial nature of these steps is only approximate. At a fine temporal scale, the analyses initiated by Jean Requin, and extended notably by the work of Alexa Riehle and Jeff Miller, show a gradual transmission from one step to the next compatible with the model of cascade propagation proposed by Jay McClelland. However, it remains true that each cerebral region contributes to a very specific operation. I have already described the parietal region, which encodes the meaning of numbers, but the left occipito-temporal region is just as interesting to psychologists. This region appears to be an essential node in the cerebral network for reading. Already in 1892, the French neurologist Joseph Déjerine had noticed its role in identifying letters and words. Déjerine first described the syndrome of pure alexia: One of his patients with a lesion in this region was no longer able to read even a single word, although he could still write and speak. One hundred ten years later, cerebral imaging, conducted in particular at the Salpêtrière Hospital by Laurent Cohen, replicates these observations (Cohen & Dehaene, 2004; Gaillard et al., 2006). A series of fMRI experiments indicates that this region responds to written words independently of their size or position and is also responsible for the invariance for case, the fact that we can recognize words in both UPPERCASE and lowercase.

At the border between nature and culture, the existence of a region specialized for written words poses a beautiful problem for cerebral development. How does one account for the fact that the brain dedicates a region for reading, the same in all individuals, almost in the same place, within a centimeter or so, in all cultures, no matter whether they read in French, English, Hebrew, or Chinese? The proposed explanation is that of cultural “recycling”: The reconversion of a preexisting brain area to a new activity (Dehaene, 2005). In humans, as in other primates, the occipito-temporal region is already used for visual recognition of objects and faces. Work by Miyashita and Logothetis has revealed a considerable degree of neuronal plasticity in this region: Neurons adapt to recognize new forms, including fractals and other arbitrary combinations of

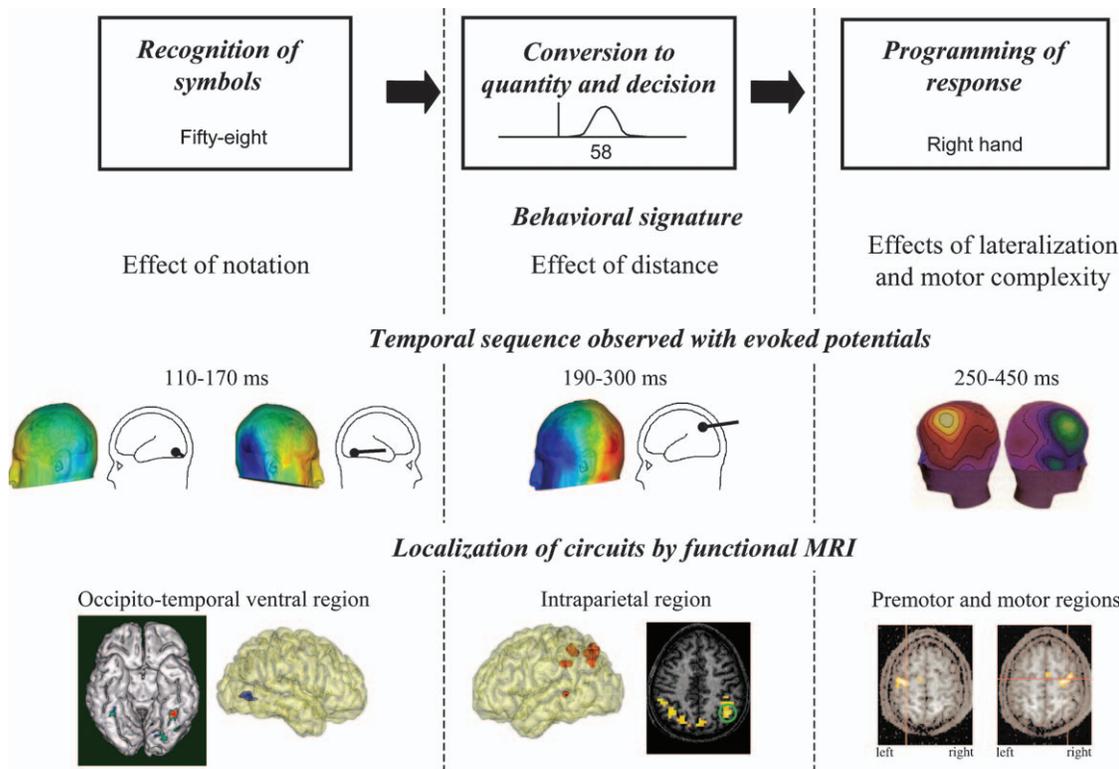


Fig. 7. Decomposition of a cognitive task. The additive factors method rests upon the identification of experimental factors that are supposed to selectively affect distinct stages of a cognitive task. If the task is suitably decomposed, one can verify that (a) the contributions of each factor have additive effects on response time; (b) in event-related potentials, each factor affects a distinct time window, in the appropriate temporal order, and with a distinct topography on the scalp. (c) in fMRI, each factor affects a distinct network of brain areas whose localization agrees with the topography of event-related potentials in the corresponding experimental condition.

features. Several teams, notably those of Tanaka, Tamifuji, and Orban, have recorded neurons that are selective for fragments of objects, some of which already have the approximate form of letters. With exposure to writing, these networks appear to reorganize and recycle themselves in order to form a hierarchical pyramid capable of recognizing letters, their assembly into graphemes and morphemes, and finally words or word fragments. fMRI experiments in children show that the occipito-temporal region progressively acquires its expertise for chains of letters between the ages of 6 and 12. This region also figures prominently among the brain areas whose activity is abnormally weak in dyslexic children (Paulesu et al., 2001; Shaywitz et al., 2002).

Our growing comprehension of the mechanisms of reading is not without consequence in the continuing debate on the optimal methods for teaching reading. It is now clear that the occipito-temporal region does not work by globally recognizing the form of the word. Rather, it learns by decomposing words into letters, graphemes, and morphemes, which have to be connected to the phonemic and lexical units of spoken language. Neuroimaging and behavioral findings thus support the explicit teaching of phoneme-grapheme correspondences, a conclusion that converges with studies of teaching practices and their impact (Ehri, Nunes, Stahl, & Willows, 2001;

National Institute of Child Health and Human Development, 2000). The time seems right to develop, in the domains of reading and of arithmetic, collaborative experiments that would tightly link teachers and specialists of psychology and the brain in order to further test pedagogical techniques and their impact (for instance, does haptic training with letters, as proposed in Maria Montessori's curriculum, truly facilitate reading acquisition? See Gentaz, Colé, & Bara, 2003). A particularly ripe domain concerns the design and testing of rehabilitation paradigms for children at risk of dyslexia or dyscalculia. Rehabilitation software, directly inspired by our cognitive understanding of word recognition and arithmetic, is being designed and can be tested by both behavioral and neuroimaging techniques (Temple et al., 2003; Wilson et al., 2006; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). In both cases, the results seem highly promising.

COORDINATION OF MULTIPLE MENTAL OPERATIONS

Let us now return to the global architecture of arithmetic. I have described a few very simple hypotheses concerning the basic representations of number and how they enter into

decisions. No doubt these hypotheses are too simple. But at least they have the merit of a certain predictive efficacy. The model that emerges suggests that we all possess an intuition about numbers and a sense of quantities and of their additive nature. Upon this central kernel of understanding are grafted the arbitrary cultural symbols of words and numbers. Space precludes a detailed description of all the transformations of this symbolization introduced into our cognitive system and unique to the human species (see, however, Pica et al., 2004; Verguts & Fias, 2004). Let me simply say that each symbol gathers together and condenses scattered information into a small object of thought and of memory, but especially that it *discretizes* the continuum of preverbal analogical representations. The arithmetic intuition that we inherit through evolution is continuous and approximate. The learning of words and numbers makes it digital and precise. Symbols give us access to sequential algorithms for exact calculations. It is at this level, and only at this level, that the brain can be directly compared to a Turing machine, a pretty poor computer, really, a million times slower than the simplest calculator and whose calculations are often marred by mistakes.

The human Turing machine remains very mysterious. How do we link multiple operations? How do we control the execution of each step? The linear, input-output vision of mental activity, which many psychological models still adopt out of convenience, suggests that cognitive processes link automatically without any supervision. However, let someone make an error, be distracted, or engage in multiple tasks, and we see immediately other coordination or executive systems come into play at a higher hierarchical level. Thus, the simplistic input-output processing scheme must be overturned in favor of a model where the brain exerts a strong downward control onto itself (Posner & Rothbart, 1998; Shallice, 1988). Our brain is an intentional organ that fixes goals and actively seeks information and actions that lead to those goals. There is in each one of us a central executive whose role is to control tasks and to manage conflicts or errors. But this mental operator, which has for too long remained a homunculus or *deus ex machina* of psychology, must itself be analyzed in terms of elementary mechanisms.

An understanding of these processes of cognitive control, still in its infancy, can profit from a basic observation: Even if the brain is composed of multiple parallel processors, at a higher cognitive level, the human brain behaves like a surprisingly slow and serial machine that can only do one operation at a time. This observation is very ancient. We find it already in the *Traité de l'Homme* (1664) wherein René Descartes attributes to the famous pineal gland our inability to pay attention to smell and vision at the same time. "While this gland is thus engaged in leaning toward a particular side," says Descartes, "this impedes it from being able to as easily receive the ideas of objects acting upon the other sense organs. [...] From which you see how ideas impede each other

and how it happens that we cannot be very attentive to several things at once."

The study of the mental collisions between several simultaneous operations has become as useful a tool for psychologists as is the particle accelerator for physicists (Figure 8). By smashing a cognitive task into bits, the collision reveals a complex internal organization. Telford (1931) discovered the existence of a phenomenon that he called the "psychological refractory period" or PRP, later characterized by Welford, Broadbent, and Pashler. For example, let's ask participants in an experiment to do two successive tasks, respond to a sound as well as compare two numbers. When a long interval of time separates these two stimuli, each task is processed without difficulty in a fixed amount of time. As the interval diminishes, the latency of the first response remains constant, while the second becomes slower. In the limited case where the two stimuli are presented at the same time, the second task must wait a considerable time, as if it underwent the countereffect of a refractory period established by the first.

However, the work of Al Pashler shows it is not the entire second task that slows down (Pashler, 1984, 1994). Neither the perception of the stimulus nor the execution of the motor response is different when performing the two tasks simultaneously. Only one stage, the so-called central stage, encounters a bottleneck in which mental operations are done in a series and not in parallel. The studies of Mariano Sigman suggest that a direct link exists between this central step and the stochastic accumulation model that I described above: on the stochastic decision-making stage to be responsible for this central bottleneck (Sigman & Dehaene, 2005, 2006). We can recognize several objects or make several responses in parallel but not make several decisions simultaneously. Again, on this point, we find an interesting convergence with psychophysics. In the domain of perceptual decisions, Andrei Gorea and Dov Sagi demonstrate that the brain is not capable of simultaneously adopting two optimal criteria of response while trying to simultaneously make two distinct decisions (Gorea, Caetta, & Sagi, 2005).

Several interpretations of these results have been debated. Pashler, somewhat like Descartes, envisages a passive bottleneck stemming from the very architecture of the nervous system: The operator of the response selection is not able to attend to two tasks at the same time. That is why the first task that it engages is done immediately, whereas the second is held in waiting. For others, such as Meyer, Gordon, or Logan, serial processing results from a strategy adopted to minimize interferences (see, e.g., Logan & Gordon, 2001). Finally, for Navon, Miller, or Jolicoeur, central resources can be shared among several operations, but a coordination process, which remains to be fully specified, evaluates whether it is advantageous to give priority to one operation to the detriment of another.

Empirically, response time costs related to the coordination of multiple tasks have been measured, which brain imaging

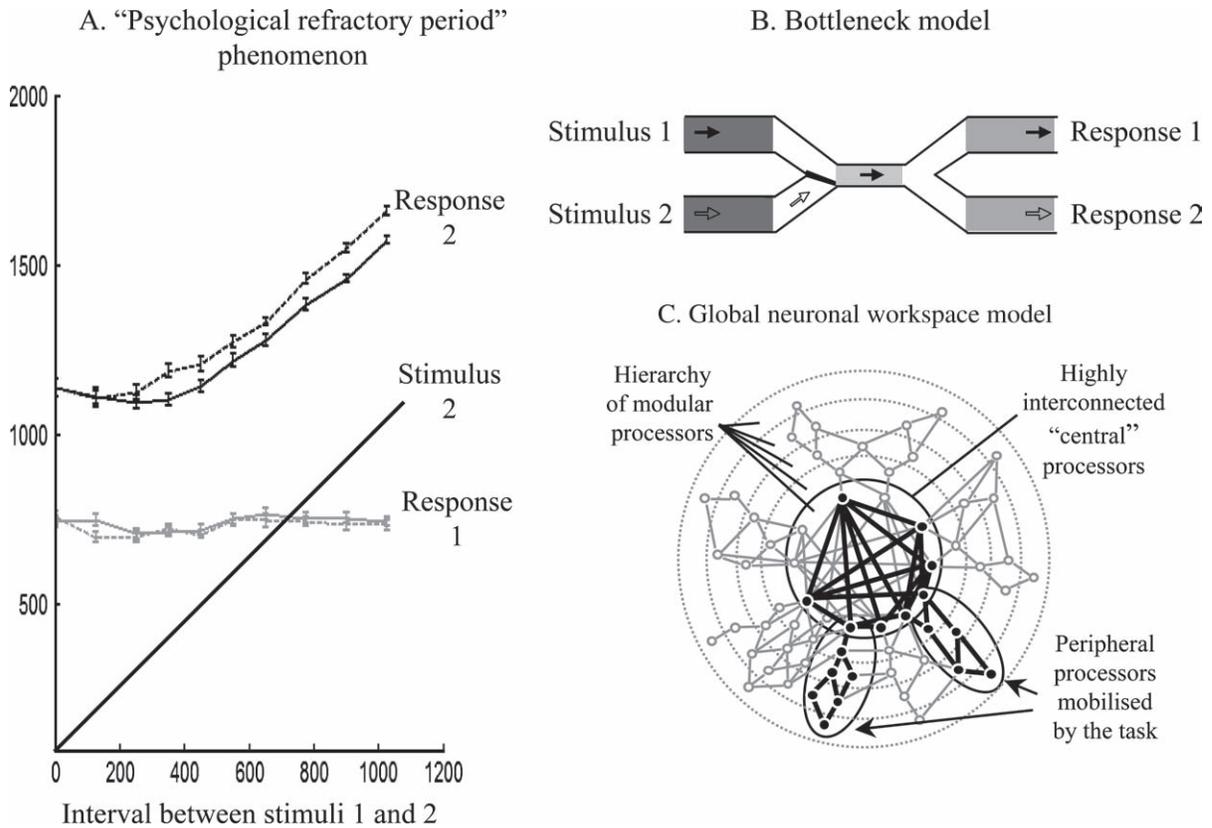


Fig. 8. A central limit in the simultaneous execution of two cognitive tasks. When a subject is asked to execute two tasks in close succession, as the time interval between the two stimuli gets shorter, the response to the second task progressively slows down in the same proportion (A, after Sigman & Dehaene, 2005). The bottleneck model (Pashler, 1984) imputes this slowing down to a central stage where the two tasks are executed strictly serially, while other more automatized perceptual and motor stages are still executed in parallel. According to the global neuronal workspace model (Dehaene & Naccache, 2001), this central bottleneck relates to the architecture of cerebral processors, which comprises, at the highest level, a set of highly interconnected processors, particularly involving frontoparietal cortices, whose activation is associated with serial mental operations performed under conscious effortful control.

has systematically associated with prefrontal and parietal regions (Marois & Ivanoff, 2005). The English psychologist Alan Allport, for instance, has shown that the passage from one task to another creates a measurable cost tied to establishing, abandoning, or switching between task sets. Paul Bertelson, as well as Navon, and Gopher have also described effects of competition between tasks: Even if these operations are distant in time, the simple task of keeping two strategies ready to go slows the execution of both.

CENTRAL SUPERVISION AND ITS LINK TO ACCESS TO CONSCIOUSNESS

All these observations, and many others, point to a very complex system of cognitive control particularly well developed in the human species. It can be fragmented into multiple processes, some responsible for establishing a strategy, others for keeping information in waiting, for temporary branching to another task, for orienting attention, for detecting errors, etc. (e.g., Koechlin, Ody, & Kouneiher, 2003). All these operations appear at a higher hierarchical level than the more automatic

processes of perception, access to sensation, or the execution of a motor response.

Among the more interesting discoveries of recent years is the establishment of a direct link between this hierarchical division of cognition and the distinction between conscious and nonconscious operations. The study of subliminal perception has shown that the set of processes at the first level can be activated in the absence of consciousness. We owe Ken Forster, Anthony Greenwald, and Jonathan Grainger for their elegant demonstrations of nonconscious visual recognition. Furthermore, the work of Tony Marcel, confirmed through the help of brain imaging and electrophysiological techniques by Lionel Naccache and many others, shows that even conceptual and motor representations can be activated without our being the least bit consciousness of them (for a review, see Dehaene, 2004; Kouider & Dehaene, in press). According to Goodale and Milner, the whole dorsal visuomotor chain operates outside any introspective consciousness—Yves Rossetti speaks elegantly of the “automatic pilot” of our gestures.

Inversely, all the operations that rely on cognitive control seem impossible to execute without our being conscious of

them. Acting against our automatic cognitive mechanisms, for example saying “red” when we see the word “green,” necessitates conscious control, and lies at the heart of the inclusion/exclusion process dissociation method for studying conscious processing (Debner & Jacoby, 1994). When a conflict or an error occurs in lower level processes, cognitive control increases during subsequent trials. However, this regain of control only happens when the conflict is consciously detected and not when the stimuli are presented below the level of consciousness (Kunde, 2003).

We can try to sum up these observations with two simple laws. First, a nonconscious stimulus can travel through a series of perceptual, conceptual, and motor steps prepared by the central executive. Second, access to the central processing system is necessarily accompanied by conscious awareness. According to this model, consciousness appears to be associated with a serial cerebral system of limited capacity responsible for controlling other mental operations.

It may seem surprising that we have but one consciousness limited to one object of thought at a time—and it is probable that deeper studies may discern limits to this serial law of centrality. However, the fact that our conscious awareness of the external world is very limited is confirmed by the work of Sperling and extended by Raymond, Shapiro, and Duncan. In the mental collision paradigm, they show that a stimulus presented during the central processing of another goal can be literally erased from consciousness. When this stimulus is followed by a mask, its processing is not followed through to the end, and its conscious perception evaporates: The subject states that no stimulus had been presented. This phenomenon is known as “attentional blink” (Raymond, Shapiro, & Arnell, 1992; Sergent, Baillet, & Dehaene, 2005).

The link between executive attention and consciousness is reinforced by other visual illusions. Kevin O’Regan, with Ronald Rensink, has developed a paradigm of “change blindness,” which consists of presenting two different images in alternation separated by a blank screen that blocks the automatic attraction of attention toward those regions where the image changes. Under these conditions, it is possible to keep watching the changing images for several tens of seconds, without seeing that a major change has happened before our eyes (Rensink, O’Regan, & Clark, 1997; Simons & Ambinder, 2005).

As a further demonstration of the limits that attention imposes on access to consciousness, Irving Rock and Ariën Mack engaged individuals in a difficult task, requiring that they pay attention to the periphery of their visual field. They then presented a contrasted stimulus in the center of the fovea, for durations up to 700 ms, and immediately stopped the experiment to question the subjects: Did they detect anything abnormal or unexpected? Most participants reported not having perceived anything (Mack & Rock, 1998).

Such experiments open a window onto one of the most difficult problems in cognitive psychology: What is consciousness? Some philosophers have highlighted the apparent gap between the subjective character of conscious experience and the objective analysis in the third person that we can engage in using the methods of cognitive science. According to them, conscious experience, by nature subjective, escapes experimentation. I do not share that point of view. Paradigms such as masking, attentional blink, change blindness, and many others are witness to the fact that there exist reproducible experimental conditions within which all subjects agree on the nature of their conscious experience. These phenomena allow us to identify objectively the cerebral basis for subjective consciousness.

In recent years, we have lost count of the number of illusions, errors, or hallucinations that have been modeled and connected to objective neuronal bases. The synthesis that begins to emerge from these studies associates consciousness to a parietal–frontal system that amplifies, gathers, and organizes the information issuing from other cortical regions so as to incorporate them into intentional and controlled behavior (Dehaene & Naccache, 2001; Dehaene, Changeux, Naccache, Sackur, & Sergent, 2006). This idea is not new. James (1890) already anticipated it by highlighting “no matter what the complexity of an object, we conceive it in our thought as a unique and indivisible state of consciousness.” Already in 1921, the neurologist, Leonard Bianchi, spoke of a “field of mental synthesis” especially developed in humans, which he associated with the frontal lobe (Bianchi, 1921). Baars (1989) used the metaphor of a theater stage, the “workspace” of the mind, where conscious information is gathered before being “broadcasted” to a variety of mental processes.

Neuroanatomy and brain imaging are now beginning to give flesh to these metaphors. They confirm that the prefrontal areas are implicated in a vast distributed associative network whose sudden and coordinated activation punctuates each access of information to consciousness. Starting from very different perspectives, the research of neurophysiologists, neuroimagers, and psychologists such as Victor Lamme, Christoph Koch, or Vincent Di Lollo converges to highlight the essential role of top-down neuronal amplification over a long distance in access to consciousness. Jean-Pierre Changeux, Lionel Naccache, and I have defended a similar theory, according to which information represented by the firing of a population of specialized neurons becomes conscious when it reverberates with other distant neurons associated with intentional, mnemonic, and executive processes distributed in the so-called associative areas of the temporal, parietal, and prefrontal cortices. “Neurorealistic” computer simulations, while still rudimentary, confirm that such reverberating networks possess objective properties of phase transition whose characteristics reproduce the most basic phenomena associated with vigilance and conscious access

(Dehaene & Changeux, 2005). In particular, they clarify why a marked nonlinear threshold of consciousness separates two states of activity that correspond to subliminal operations and conscious operations.

While this research, which is very recent, paves the way to a theoretical definition of consciousness, two major obstacles will have to be overcome in order to attain this goal. The first consists in going from simple correlations to a relation of cause and effect. Cognitive neuroimaging can only show a correlation between certain states of activity of the brain and the conscious access to information. Thus, they necessarily leave open what the philosophers Tom Nagel and Ned Block call an “explanatory gap” between the material and the psychological level, which led some philosophers, psychologists, and even physiologists such as Sir John Eccles to the impasse of dualism. In the future, the demonstration of a causal relation and, *in fine*, of identity between neuronal states and conscious mental states will require techniques that interfere with cerebral activity. Transcranial magnetic stimulation (TMS) has become a well-established technique to induce currents in the brain. When it is applied to the visual cortex, this stimulation produces perceptual illusions of light or of movement (Silvanto, Cowey, Lavie, & Walsh, 2005). Conversely, when it is applied to the parietal regions implicated in spatial attention orienting, it can erase the conscious perception of a real stimulus (Beck, Muggleton, Walsh, & Lavie, 2006). Thus, TMS and other interference techniques, if they are applied within rigorous safety and ethics guidelines, are likely to play an essential role in establishing causal links between attention orientation, central integration, and conscious perception.

The second obstacle to the establishment of a theory of consciousness is of a different nature. Consciousness, said William James, is an uninterrupted flow, a permanent train of thought, comparable to a bird that is constantly alternating between flight and perching. I like this metaphor, which connects to the vision of the physiologist, Rodolfo Llinas: The brain functions in an anticipatory mode, ceaselessly active, reassessing the past to better anticipate the future. However, cognitive psychology has all too often neglected this internal state of the conscious subject, most frequently contenting itself with bombarding individuals with stimuli and gathering their responses. Naturally, this unfortunate reliance on reflex-like stimulus-response paradigms also results from an experimental limitation—how could we ever infer the structure of mental representations without gathering any behavioral measures? Here again, neuroimaging techniques may offer a solution. When a person is resting without any particular instruction, the brain shows an intense structured activation, often parietal–frontal, that spontaneously fluctuates between several states correlated across a long distance (Laufs et al., 2003; Raichle et al., 2001). The external stimulus briefly interrupts this flux that begins again once the task is finished. According to Pierre Maquet and Steven Laureys,

this distributed spontaneous activity characterizes the state of conscious alertness: It disappears under anesthesia and in deep sleep, is absent in comatose or vegetative-state patients, but reappears when consciousness is recovered (Laureys, 2005). Thus, this spontaneously activated state may constitute a solid neuronal correlate of conscious vigilance. Spontaneous brain activity is dramatically altered in depression and schizophrenia, thus opening new perspectives in the comprehension of psychiatric illness.

I would, therefore, propose as a major experimental challenge for future years that autonomous mental activity, too often neglected, must regain its status as a central object of study for cognitive psychology. Our experiments often restrict participants to very narrow cognitive tasks. If we hope to understand the spontaneous flux of consciousness, new experimental methods that give much greater freedom to the subject must be imagined.

Such an approach seems particularly essential in the developmental domain. The infant and the young child are never passively submitted to their environment. Passive associative learning is clearly rejected, even by animal neurophysiological experiments, as an accurate model of the cortical changes induced by plasticity and reward (Blake, Heiser, Caywood, & Merzenich, 2006). The vast majority of learning situations, and especially those that occur in our classrooms, require an active, exploring mind that possesses preexisting competences and selects among them by an active process of hypothesis generation and testing. To be sure, automatization and routinization are also important processes of learning that work “behind the scenes” to support fluent processes of reading or arithmetic. This is the part of cognitive development that we are beginning to understand relatively well, thanks to both experimentation and modeling. But we know very little of the other side of the coin: the active learning process that allows children to suddenly “grasp” an idea or a rule, and that allows for one-trial learning and generalization. A very different type of cognitive architecture is probably needed here, one that relies upon internally driven processes where spontaneous activity and selection by reward systems play a central role (Dehaene & Changeux, 2000; Rougier, Noelle, Braver, Cohen, & O’Reilly, 2005).

CONCLUDING REMARKS

Throughout this article, I have tried to show how human cognition follows strict laws that do not spare even the most subjective aspects of our conscious perception. We have traveled a considerable way since Watson (1913) declared, “Psychology as the behaviorist views it is a purely objective experimental branch of natural science ... Introspection forms no essential part of its methods, nor is the scientific value of its data dependent upon the readiness with which they lend themselves to interpretation in terms of consciousness.” On

the contrary, in today's psychology, the subjective data of consciousness are legitimate objects of study that can be directly related, through modeling and neuroimaging, to objective aspects of cerebral architecture.

For my part, I believe profoundly in a renewal of the program of psychophysics of Fechner, Wundt, Ribot, and Piéron, but a program which having become "neurophysical" will go beyond the simple description of formal psychological laws to anchor itself unambiguously at the neuronal level. The ultimate explanation of mental objects, perceptions, illusions, decisions, or emotions must be formulated in terms of dynamic laws of transitions in neuronal networks. We must, therefore, conceive of new theories in order "to explain the complications of the visible in terms of invisible simplicity," as the French physicist Jean Perrin once said. Our good fortune is that we live in a time where the joint advances of psychology and cognitive neuroimaging allow us to begin to make visible, as if the skull were open, the invisible mechanisms of thought.

NOTE

1 I shall not discuss here the difficult issue of the epistemological status of those psychological or physical "laws." Suffice it to say that, in my opinion, they are merely provisional formulations, in an imperfect mathematical language specific to the human species, of part of the regularities that we identify in the natural world. As a consequence, these laws should not be granted an ontological status independent of the mind of the scientist who formulates them (see Changeux & Connes, 1995; Dehaene, 1997, chapter 9). In its rapid progress, cognitive neuroscience temporarily leaves aside two difficult questions: Does the human brain possess enough resources to describe itself? And isn't this enterprise of self-description of the laws of the mind by the mind itself intrinsically limited, or even contradictory or tautological? If cognitive psychologists pay little attention to those questions, it is because they have few doubts that, relative to the variety and increasing objectivity of the scientific exploration methods at their disposal, the possibility that our mental resources impose a "cognitive horizon" to neuroscientific theories, if it exists, seems rather remote. Indeed, as I will illustrate in the present article, even very elementary mathematical tools can bring important progress to our field.

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