Notes for talk on Self-Reference at Yablo Conference (Paris, Dec., 2021)

Yablo on Paradox without Self Reference

(S_k) For all m > k, S_m is untrue.

Two observations

(1) Ill-foundeded reference appears to be necessary for paradox.

(2) 'Not that self-reference suffices for paradox. Such a view is refuted by the work of Gödel and Tarski, and by various commonsense examples'.

However, arguably, self-reference is not even possible and the arguments that tend to show that it is not possible also tend to show that ill-founded reference is not possible.

Quotation from Kripke ([1975], p. 693)

A simpler, and more direct, form of self-reference uses demonstratives or proper names: Let 'Jack' be a name of the sentence 'Jack is short', and we have a sentence that says of itself that it is short. I can see nothing wrong with 'direct' self-reference of this type. If 'Jack' is not already a name in the language, why can we not introduce it as a name of any entity we please? In particular, why can it not be a name of the (uninterpreted) finite sequence of marks 'Jack is short'? (Would it be permissible to call this sequence of marks 'Harry', but not 'Jack'? Surely prohibitions on naming are arbitrary here.) There is no vicious circle in our procedure, since we need not interpret the sequence of marks 'Jack is short' before we name it. Yet if we name it 'Jack', it at once becomes meaningful and true.

One-sided De-Relativization

If the U-name n refers to x under the interpretation I then the I-name **n** = n/I refers (simpliciter) to x.

Two-sided De-Relativization

If the U-name n refers to the U-expression x under the interpretation I then the I-name **n** = n//I refers (simpliciter) to the I-expression x//I.

Definitional Route to Self-Reference

(1) Jack $=_{df}$ 'Jack'	by Definition
(2) Jack = 'Jack'	from (1) by Definitional Success
(3) 'Jack' names 'Jack'	from (2) by Semantic Ascent.

Rule of Ostensive Meaning

Where **μ** is an ostensive meaning function, the I-name **n** = n/**μ** refers to **μ**(**n**) (so when **μ** is the identity function, **n** = n/**μ** refers to **n**.

Interpretational Quotation

Q(**E**) refers to **E** while Q(E) refers to E = b(Q(**E**))

Implicit Definition of '**Jack**'

Jack = '**Jack**'.

A Theory of Semantized Reference

Primitive Predicates

B (basis predicate) used to indicate that the I-expression y is based on the U-expression

R (reference predicate) used to indicate that the I-expression y refers to x.

Definitions

$b(y) =_{df} \iota x.yBx$

$r(y) =_{df} \iota x.yRx$

$\text{U-name}(x) =_{df} \exists y(yBx)$

$\text{I-name}(x) =_{df} \exists y(xBy)$

$\text{Individual}(x) =_{df} \neg(\text{U-name}(x) \vee \text{I-name}(x))$

Axioms (for the minimal theory)

Uniqueness

Unique Basis $\text{I-name}(x) \supset \exists!y(xBy)$

Unique Reference $\text{I-name}(x) \supset \exists!y(xRy)$

Nominal Reference

$xRy \supset \text{I-name}(x)$

Nominal Identity

$b(\mathbf{m}) = b(\mathbf{n}) \wedge r(\mathbf{m}) = r(\mathbf{n}) \supset \mathbf{m} = \mathbf{n}$.

Nominal Existence

$\forall x \forall n \exists \mathbf{n}(b(\mathbf{n}) = n \wedge r(\mathbf{n}) = x)$

Inscriptional Existence

$\exists x(\text{I-name}(x))$

Some Further Definitions

A *referential chain* is a sequence of items $x_1, x_2, \ldots$ (possibly infinite) which is such that:

(i) $r(x_k) = x_{k+1}$ for any item x_k of the sequence (other than the last item if there is one), and

(ii) the last item of the sequence (if there is one) is not an I-name (and so does not refer).

A *proto-referential chain* is a sequence of items $x_1, x_2, \ldots$ (possibly infinite) such that:

(i) each item other than the last (if there is one) is a U-name, and

(ii) the last item of the sequence (if there is one) is not an I-name.

We extend the basis function b so that the basis b(x) of any inscription or individual x is that object itself. We then say that:

the referential chain x_1, x_2, ... *realizes* the proto-referential chain $b(x_1)$, $b(x_2)$, ...

Some Further Axioms

Extended Nominal Existence

Every proto-referential chain is realized

Extended Nominal Identity

Each proto-referential chain is realized in at most one way.

Some Observations

(1) The minimal theory has a minimal model (indeed can be made categorical up to the cardinality of the inscriptions and individuals by adding a suitable closure condition). This minimal model is one in which there do not exist any infinite referential chains. So, in particular, self-reference and the kind of ill-founded reference involved in Yablo's puzzle cannot arise.

(2) There is a minimal model for the extended theory (which can also be made categorical) and it is, of course, one in which free-wheeling self-reference and ill-founded reference is possible. In particular, we will have referential chains of the following two forms:

$\mathbf{j}$, $\mathbf{j}$, $\mathbf{j}$, ...

in which $\mathbf{j}$ refers to $\mathbf{j}$; and

$\mathbf{j}_1$, $\mathbf{j}_2$, $\mathbf{j}_3$, ...

in which each $\mathbf{j}_k$ refers to its successor and the j_1, j_2, j_3, ... are all distinct.