Gravitational Lensing and Dark Matter

Yannick Mellier

IAP Paris and CEA/IRFU Saclay
Multiple images: the «Einstein Cross»

Galaxy 2227+030

Redshift $z=0.0394$
The « Einstein Cross »

CIV emission line at 154.9 nm observed 417.6 nm : \( z = 1.695 \)

4 images with identical spectra
General relativity:
curvature of space time locally modified by mass condensation

Deflection of light, magnification, image multiplication  distortion of objects :  directly depend on the amount of matter

Gravitational lensing effect is achromatic (photons follow geodesics regardless their energy)
Gravitational Lensing

theory, concepts and definitions
Gravitational lensing: fundamental assumptions

- Weak field limit: $\sigma^2 \ll c^2$
- Stationary field: $t_{\text{dyn}} \sim R_{\text{lens}}/v \gg t_{\text{cross-photon}}$
- Thin lens approximation: $R_{\text{lens}} \ll R_{\text{bench}}$
- Small deflection angle:
  \[ b = \text{impact parameter} ; \quad R = \text{Schwarzschild radius} \]
  \[ \alpha = 4G M/bc^2 \ll 2G M/R_S c^2 \]
  \[ \Rightarrow R_S \ll b \]
- Transparent lens
Lens equation and deflection angle

\[ \tilde{\eta} = \frac{D_{os}}{D_{ol}} \tilde{\xi} - D_{ls} \hat{\alpha} \left( \tilde{\xi} \right) \]

; \quad \alpha = -\frac{2}{c^2} \int_S^O \nabla_\perp \Phi \, dl
Deflection angle and mass density

\[ \alpha = -\frac{2}{c^2} \int_S \nabla_\perp \Phi \, dl \]

\[ \alpha (\xi) = \frac{4G}{c^2} \int \frac{(\xi - \xi') \Sigma (\xi')}{|\xi - \xi'|^2} \, d\xi' \]

where

- \( \Sigma (\xi) \) is the projected mass density,
- \( \xi \) is a 2-dimensional vector in the lens plane and
- the integration is done over the lens plane.
Lens equation: spherical lens

- **Lens equation**

\[
\vec{\eta} = \frac{D_{os}}{D_{ol}} \vec{\xi} - D_{ls} \hat{\alpha}(\vec{\xi})
\]

Setting \( \vec{\eta} = D_{os} \vec{\beta} \) and \( \vec{\xi} = D_{ol} \vec{\theta} \),

\[
\vec{\beta} = \vec{\theta} - \hat{\alpha}(\vec{\theta})
\]

- **Spherically symmetric lens**

\[
\beta = \theta - \frac{D_{ls}}{D_{os}D_{ol}} \frac{4GM(\theta)}{c^2 \theta}
\]
Perfect lens configuration

« Einstein ring »

Source-Lens-Observer perfectly aligned
Einstein ring

\[ \theta_E = \left( \frac{D_{ls} \cdot 4GM(\theta_E)}{D_{os}D_{ol} \cdot c^2} \right)^{1/2} \]

Typical values:

• For a lens of 1 solar mass located at 1 AU and a source a 1 kpc
  \[ \theta_E = 0.003 \text{ arc-second} \]

• For a lens of \(10^{11}\) solar masses located at 100kpc and a source at 300 kpc
  \[ \theta_E = 1 \text{ arc-second} \]

• For a lens of \(10^{15}\) solar masses located at 1Gpc and a source at 3 Gpc
  \[ \theta_E = 30 \text{ arc-second} \text{ (sensitive to cosmological parameters)} \]
Convergence and critical density

- **Convergence and critical density** The gravitational convergence is a dimensionless surface mass density:

\[
\kappa(\vec{\theta}) = \frac{\Sigma \left( D_{ol} \vec{\theta} \right)}{\Sigma_{cr}}
\]

where \(\Sigma_{crit}\) is the critical surface mass density.

\[
\Sigma_{cr} = \frac{c^2}{4\pi G} \frac{D_{os}}{D_{ol} D_{ls}}
\]

that defined a "strength" of the lens. Strong lensing cases have \(\Sigma > \Sigma_{cr}\).
Magnification and distortion

\[ \vec{\beta} = \vec{\theta} - \dot{\alpha} (\vec{\theta}) \]
\[ \alpha = -\frac{2}{c^2} \int_s^O \nabla_\perp \Phi \, dl \]

- Jacobian of the lens mapping. Differentiating the lens equation

\[ A (\vec{\theta}) = \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \begin{pmatrix} \delta_{ij} - \frac{\partial^2 \psi (\vec{\theta})}{\partial \theta_i \partial \theta_j} \end{pmatrix} = M^{-1} \]

- Convergence, Shear

\[ \kappa = \frac{1}{2} (\psi_{,11} + \psi_{,22}) \]
\[ \gamma_1 (\vec{\theta}) = \frac{1}{2} (\psi_{,11} - \psi_{,22}) = \gamma (\vec{\theta}) \cos \left[ 2 \varphi (\vec{\theta}) \right] \]
\[ \gamma_2 (\vec{\theta}) = \psi_{,12} = \gamma (\vec{\theta}) \sin \left[ 2 \varphi (\vec{\theta}) \right] \]
Magnification and distortion

- Magnification, Convergence, Shear

\[
A = M^{-1} = \begin{pmatrix}
1 - \kappa - \gamma_1 & -\gamma_2 \\
-\gamma_2 & 1 - \kappa + \gamma_1
\end{pmatrix}
\]

\[
M^{-1} = (1 - \kappa) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos(2\varphi) & \sin(2\varphi) \\ \sin(2\varphi) & -\cos(2\varphi) \end{pmatrix}
\]

where \(\gamma = \gamma_1 + i\gamma_2 = |\gamma|e^{2i\varphi}\)

- Amplification amplitude

\[
\mu = (\det A)^{-1} = \frac{1}{(1 - \kappa)^2 - |\gamma|^2}
\]

- Eigenvalues of \(M^{-1}\):

\[
1 - \kappa + \gamma ,
1 - \kappa + \gamma
\]
Magnification and distortion

- **Image and source** From the magnification matrix,
  
  - $\kappa$ expresses an isotropic magnification. It transforms a circle into a larger/smaller circle.
  
  - $\gamma$ is an anisotropic magnification. It transforms a circle into an ellipse with minor and major axes:
    
    $$b = (1 - \kappa + \gamma)^{-1}, \quad a = (1 - \kappa - \gamma)^{-1}$$
From (reduced) shear to ellipticity

- **Reduced shear** Let us write the magnification matrix as:

\[ A = \mathcal{M}^{-1} = (1 - \kappa) \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix} \]

where

\[ g(\vec{\theta}) = \frac{\gamma(\vec{\theta})}{1 - \kappa(\vec{\theta})} = g_1 + ig_2 = |g|e^{2i\varphi} \]

is the reduced shear. It directly provides the image ellipticity induced by lensing on a circular source:

\[ \frac{b}{a} = \frac{1 - |g|}{1 + |g|} \]

as well as the orientation of the major axis, \( \varphi \).

→ Measuring ellipticity = measuring gravitational shear
Caustic and critical lines

- Amplification amplitude

\[ \mu = (\text{det}A)^{-1} = \frac{1}{(1 - \kappa)^2 - |\gamma|^2} \]

- critical lines corresponds to positions in the lens plane with \( \text{det} A = 0 \)

- the corresponding positions on the source plane are the caustic lines

- the positions of source points with respect to a caustic lines define the number of image multiplication and the source magnification

- when a source crosses a caustic line, its amplification is almost infinity, and image pairs are formed
Caustic lines and critical lines

Circular potential with core

Critical lines

caustics

J120540.43+491029.3
Caustic and critical lines

Elliptical potential with core

Kneib 1993
Caustic and critical lines

Bimodal potential with core
Dark matter with Strong Gravitational Lensing
Abell 370: first gravitational arc discovered

6 Sept. 1985 - A370 arc discovery

Very 1st image at CFHT Cass. focus

RCA 512x320 CCD 0.8'' /pixel,

10mn R-band, seeing 0.8''
Abell 370: first gravitational arc discovered

\[ z_{\text{cluster}} = 0.375 \]

A spiral structure resolved at \( z=0.724 \)
Abell 370: singular isothermal sphere model

Lens equation:

\[ \theta_S = \theta_I - 4\pi \frac{\sigma^2}{c^2} \frac{D_{LS}}{D_{OS}} \frac{\theta_I}{|\theta_I|} \]

Effective potential:

\[ \varphi = 4\pi \frac{\sigma^2}{c^2} \frac{D_{LS}}{D_{OS}} r \]

Magnification matrix:

\[
\begin{pmatrix}
1 & 0 \\
0 & 1 - 4\pi \frac{\sigma^2}{c^2} \frac{D_{LS}}{D_{OS}} \frac{1}{|\theta_I|}
\end{pmatrix}
\]

Deflection angle:

\[ \theta_{SIS} = 4\pi \frac{\sigma^2}{c^2} \frac{D_{LS}}{D_{OS}} \approx 16'' \left( \frac{\sigma}{1000 \text{ km} \cdot \text{sec}^{-1}} \right)^2 \]

Total mass inside the Einstein radius:

\[ M(\theta) = 0.57 \times 10^{14} \ h^{-1} \ M_\odot \left( \frac{\theta}{30''} \right) \left( \frac{\sigma}{1000 \text{ km} \cdot \text{sec}^{-1}} \right)^2 \]
No counter arc: Cluster mass distribution NOT circular

1 radial arc: if clusters are IS, core radius CANNOT be zero.

Tangential + radial arc: Core radius can be derived
Modelling strong lenses

Constraints:

- The angular lens equation sets the relation between each image position, $\theta_i^l, \theta_j^l, \theta_k^l, \ldots$, and the unique position back in the source plane:
  \[
  \theta^S = \theta_i^l - \alpha(\theta_i^l) = \theta_j^l - \alpha(\theta_j^l) = \theta_k^l - \alpha(\theta_k^l) = \ldots
  \]

- The flux ratios between each image is an estimate of their magnification ratio:
  \[
  \frac{F_i^l}{F_j^l} = \frac{|\mu_i^l|}{|\mu_j^l|} ; \quad \frac{F_i^l}{F_k^l} = \frac{|\mu_i^l|}{|\mu_k^l|} ; \quad \ldots
  \]

- When sent back to the source plane, the morphology of each un-lensed image should be identical.

Method: mapping, inversion

Kneib 2012
MS2137-23 mass model
from critical lines analysis

\[ \Phi(r, \theta) = \Phi_0 \sqrt{1 + \left( \frac{r}{r_c} \right)^2 (1 - \epsilon \cos 2\theta)} . \]

\[ M^{-1} = \begin{pmatrix}
1 - \frac{1}{r} \frac{\partial \Phi}{\partial r} - \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} & - \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \Phi}{\partial \theta} \right) \\
- \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \Phi}{\partial \theta} \right) & 1 - \frac{\partial^2 \Phi}{\partial r^2}
\end{pmatrix} . \]

Small ellipticity and small core radius:\[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \Phi}{\partial \theta} \right) \sim 0 \]

Tangential critical line:\[ 1 - \frac{1}{r} \frac{\partial \Phi}{\partial r} - \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} = 0 . \]

Radial critical line:\[ 1 - \frac{\partial^2 \Phi}{\partial r^2} = 0 . \]

Mellier et al 1993
<table>
<thead>
<tr>
<th>System</th>
<th>$\epsilon$</th>
<th>$\theta$</th>
<th>$r_c$</th>
<th>$\sigma_{\mathrm{los}}$ (km s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster model</td>
<td>$0.08^{+0.03}_{-0.03}$</td>
<td>$29^{+5}_{-3}$</td>
<td>$8^{+0.5}_{-2}$</td>
<td>1000</td>
</tr>
<tr>
<td>cD$^a$</td>
<td>$0.07 \pm 0.04$</td>
<td>$29^{+5}_{-5}$</td>
<td>$1.5 \pm 1$</td>
<td>$350 \pm 30^b$</td>
</tr>
<tr>
<td>Galaxy 7 model</td>
<td>0.04</td>
<td>8</td>
<td>0'27</td>
<td>185</td>
</tr>
<tr>
<td>Galaxy 7$^a$</td>
<td>0.07</td>
<td>8</td>
<td>...</td>
<td>$230 \pm 30^b$</td>
</tr>
</tbody>
</table>

MS2137-23
mass model
from critical lines
analysis and 2
sources

Mellier et al 1993
MS2137-23: position and shape of the 5th image: depend on the mass profile

Gavazzi et al 2004
MS2137-23: strong/weak with the 5th image

Even a lens configuration with 2 sources and a [radial+tangential] arc system cannot provide conclusive results on SIS vs NFW

Conclusion: find the 5th image!
But not that obvious...if located under a very bright galaxy

Gavazzi et al 2004
Dark matter with Weak Gravitational Lensing
First giant arc discovered in Abell 370

Zcluster = 0.374, Zarc = 0.720

IHP 28-29 Nov. 2006
First giant arc discovered in Abell 370

$z_{A370}=0.375$

Zcluster = 0.374, Zarc = 0.720

IHP 28-29 Nov. 2006

~1 Mpc

5'$
Beyond gravitational arcs: weak lensing

Simulation, lensing cluster: isothermal sphere at z=0.3

Mellier 1999
From ellipticity to shear

\[ \delta = \frac{2\gamma (1 - \kappa)}{(1 - \kappa)^2 + |\gamma|^2} \]

\[ = \frac{a^2 - b^2}{a^2 + b^2} \]

\[ \delta \sim 2\gamma \text{ (weak lensing regime)} \]

Assume sources orientation is isotropic:

Weak lensing regime: \( \delta \sim 2\gamma = \langle \varepsilon_{\text{Shear}} \rangle_\theta + \text{noise + complications} \)

Reliability of results: depends on PSF analysis
Complication: galaxy ellipticity contaminates gravitational ellipticity

$$\sigma_{\varepsilon} = 0.35$$

$$E(\varepsilon^2) = \sum_{ij} \gamma_t(\theta_i) \gamma_t(\theta_j) + \delta_{ij} \frac{\sigma_{\varepsilon}}{2}$$

$$S/N = 10 \left( \frac{n}{30 \text{ arcmin}^2} \right)^{1/2} \left( \frac{\sigma_{\varepsilon}}{0.2} \right)^{-1} \left( \frac{\sigma_v}{600 \text{ km/s}} \right)^2 \left( \frac{\ln \left[ \frac{\theta_2}{\theta_1} \right]}{\ln10} \right)^{1/2} \left( \frac{D_{ls}}{D_{os}} \right)$$
From shear to mass density

2D mass density map \(=\) Distortion (ellipticity) map

\[
\kappa(\theta) = \frac{1}{\pi} \int \hat{F}^*(\theta - \theta') \gamma(\theta') \, d\theta^2 + \kappa_0
\]

Application to real data. Sampling ellipticities on a grid:

\[
\Sigma(\theta) - \Sigma_0 = \Sigma_{\text{critic}} \frac{1}{\pi} a^2 \sum_{i,j} \Re(\hat{F}^*(\theta - \theta_{i,j}) \bar{\varepsilon}(\theta_{i,j}))
\]

where \(a\) is the distance between grid points
HST Cluster of galaxies Abell 2218, z=0.17
Mass reconstruction: Abell 2218

(Golse 2002)

Bernardeau & Mellier 2002
Getting the absolute mass

- The mass reconstruction provides the shape of the projected mass distribution but not the absolute scale.
- Need the redshift of the sources:

\[
\kappa(\vec{\theta}, z) = \frac{\Sigma(\vec{\theta})}{\Sigma_{\text{critic}}(z)} = \Sigma(\vec{\theta}) \frac{4\pi G}{c^2} D_{ol} \left[ \frac{D_{ls}}{D_{os}} \right]
\]

\[
\gamma(\vec{\theta}, z) = \frac{4\pi G}{c^2} D_{ol} \left[ \frac{D_{ls}}{D_{os}} \right] \gamma(\vec{\theta})
\]

Redshift increasingly important as \(z_{\text{lens}}\) increases.

\(z_{\text{sources}}\)

\(d_{ls}/d_{os}\)

\(z_{\text{lens}} = [0.07-1.0]\)

\(\Omega = 0.3\)

\(\lambda = 0.7\)
<table>
<thead>
<tr>
<th>Name</th>
<th>$\sigma_{WL}$ (km s$^{-1}$)</th>
<th>$M(&lt;0.5h^{-1}$ Mpc)</th>
<th>$M_{2500}$</th>
<th>$M_{500}$</th>
<th>$M_{NFW}^{2500}$</th>
<th>$M_{NFW}^{500}$</th>
<th>$M_{NFW}^{200}$</th>
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<td>A2390</td>
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<td>$5.2 \pm 0.6$</td>
<td>$2.4 \pm 0.5$</td>
<td>$6.8 \pm 1.5$</td>
<td>$2.9 \pm 0.6$</td>
<td>$9.2^{+2.0}_{-1.9}$</td>
<td>$14.6^{+3.1}_{-2.9}$</td>
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<td>$1164^{+151}_{-173}$</td>
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<td>$27.0^{+9.0}_{-8.4}$</td>
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<td>$14.7^{+5.1}_{-4.4}$</td>
</tr>
</tbody>
</table>
Summary mass from cluster WL reconstruction

\[ \sigma_{\text{dyn}} \quad [\text{km/s}] \]

\[ \sigma_{\text{WL}} \quad [\text{km/s}] \]

Hoekstra 2014
The Bullet cluster
Mass reconstruction

Application to clusters of galaxies
Radial mass density profile of dark matter:

NFW, SIS, POW, Generalised-NFW, Moore, Buckert, de Vaucouleur?
Cl0024+1654 + Abell 1689
Weak+strong lensing

Cluster mass profiles from ACS+Subaru lensing

Strong Lensing
Weak Lensing

Surface mass density, $\Sigma_m$ [hM$_\odot$/Mpc$^2$]

A1689 (z=0.183)
C10024+1654 (z=0.395)

NFW fit

Umetsu et al 2010
Stacking 25 lensing clusters

strong lens constraint

- high S/N lensing shear profile by stacking 25 clusters
- follow NFW well

\[
\begin{align*}
\langle \theta_E \rangle &= 14.4^{+10.6}_{-7.0} \text{[arcsec]} \\
\langle M_{\text{vir}} \rangle &= 4.57^{+0.33}_{-0.31} \times 10^{14} h^{-1} M_\odot \\
\langle c_{\text{vir}} \rangle &= 5.75^{+0.70}_{-0.57}
\end{align*}
\]

25 clusters stacked \( \langle z \rangle = 0.469 \)

NFW fit

Oguri 2012
Stacking lensing clusters

$C_{\text{vir}}$ (concentration) vs. $M_{\text{vir}} \ [10^{14} h^{-1} \text{M} \ (\text{cluster mass})$

- Best-fit M-c relation from individual analysis of clusters
- $\Lambda$CDM prediction w/ lensing bias + triaxiality

Oguri 2012
Gravitational lensing by large scale structure: cosmic shear
Cosmic shear: gravitational lensing by Large Scale Structure of the Universe

First Theoretical studies

- Kristian & Sachs 1966
- Gunn 1967
- Blandford et al 1991
- Miralda-Escudé 1991
- Kaiser 1992
Cosmic shear: weak gravitational distortion

→ Projected on the sky: coherent ellipticity field
Most spectacular
Cosmic shear and tomography with HST

Massey et al. 2007
Cosmic shear and tomography with HST

(a) Dark matter

(b) Light (galaxies)

1.2 deg.

Massey et al 2007
Cosmic shear and tomography with HST

Massey et al 2007
CFHTLens - mass maps Wide fields

Van Waerbeke, Heymans et al 2012
Cosmology with WL?

Weinberg et al 2013

After first enthusiastic reactions: skepticism on reliability of WL data and cosmological interpretations: WL is a very hard (too hard?) technique

... What Next?
Future of cosmic shear surveys

- LSST (ground based)
- Euclid (space-based)
Euclid

From Thales Alenia Space Italy, Airdus DS, ESA Project office and Euclid Consortium

Star Trackers

Pointing error along the x,y axes = 25mas over 700 s.

Common VIS and NIR FoV = 0.54 deg²

From Thales Alenia Space Italy, Airdus DS, ESA Project office and Euclid Consortium
Galaxy-scale strong lensing with Euclid

Galaxy-lenses SLACS (~2010 - HST)

SLACS: The Sloan Lens ACS Survey
A. Bolton (U. Hawai‘i IfA), L. Koopmans (Kapteyn), T. Treu (UCSB), R. Gavazzi (IAP Paris), L. Moustakas (JPL/Caltech), S. Burles (MIT)

Image credit: A. Bolton, for the SLACS team and NASA/ESA
Euclid after 2 months
(66 months expected)
Gravitational lensing

- Can probe the distribution of dark matter from galaxies to large scale structures of the Universe almost directly.
- Is an independent method, beside X-ray or dynamical ones
- Show evidence of dark matter in
  - MACHOS in our galaxy (microlensing: no time to discuss here)
  - Other galaxies (Strong lensing, Weak lensing)
  - Groups and Clusters of galaxies (SL, WL)
  - Superclusters of galaxies (WL)
  - Large Scale Structure (WL)
Summary

• Gravitational lensing confirms that a Universe without dark matter can hardly explain observations

• Modified gravity is still an option but not favoured

• Weak and/or Strong lensing data agree with NFW and SIS, but favours NFW-like radial profiles

• All data compatible with Lambda-CDM predictions

• Cosmic shear is detected, favours lambda-CDM, but detection and measurements very hard and systematics still an issue

• Likely : WL still have to improve
  → Much more statistics: number of galaxies, wave number
  → Improve shear measurement → space
  → Numerical simulations for baryon physics (small scales)