Particules Élémentaires, Gravitation et Cosmologie
Année 2008-'09

Gravitation et Cosmologie: le Modèle Standard
Cours I: 9 janvier 2009

Introduction, Programme, Rappel

- Introduction
- Programme du cours 2008-'09
- Rappel: Gravitation Newtonienne et Relativité Restreinte

Introduction to the Course

During 3 academic years (04-05, 05-06, 07-08) we have discussed our present successful theory of non-gravitational (i.e. electromagnetic, weak and strong) interactions: the Standard Model (SM) of elementary particles

Two Nobel prizes were given during that period for the theory behind the SM: to GPW for AF (2004); to Nambu for SSB and to KM for their mechanism for CP breaking (2008)

The SM belongs to the class of theories known as QFTs, a framework combining the principles of QM with those of SR

Actually, it belongs to a special class of QFTs, known as gauge theories: they appear naturally as the way to describe, in a Lorentz-invariant way, spin-1 massless particles.

Three forces, one principle!

Q: How can the same kind of theory describe 3 forces that look so different?

A: The gauge symmetry is realized in 3 different ways (like 3 phases of a stat. mech. system): Coulomb, Higgs, Confining, for EM, Weak and Strong interactions resp.

Q: Can the same kind of reasoning work for gravity?

A: At first sight it does. The only crucial change is that we have to replace massless spin 1 particles (like the photon) with a massless spin 2 particle (called the graviton).

It can be shown that interactions mediated by the exchange of a graviton do look like gravity

Otherwise, gravity appears to be in a Coulomb phase, like electromagnetism, the other long range force of Nature

The bad news

Unfortunately, theorists have been unable so far to extend to gravity the fully quantum framework that led them to the SM: the UV divergences are too strong!

Here we shall follow the traditional approach to gravity via Classical General Relativity (CGR) getting, once more, an amazingly good description of the observed phenomena.

But we should keep in mind that the SMN = SMEP+SMG is limping...(a classical left foot against a quantum right foot)

There are indications that, in order to arrive at a fully consistent quantum theory of gravity, one needs to go beyond the framework of QFT, for instance to string theory (which, incidentally, automatically predicts the existence of massless J=1 and J=2 particles!).

Plan of 2008-'09 course
(only 12 hrs: other topics for next year?)

Date	9h45-10h45	11h-12h
09/01	Reminder of Newtonian gravity and special relativity	From the EP to general covariance
16/01	Some math. tools	Einstein's equations
23/01	Physical consequences (T.D.)	Precision tests (T.D.)
30/01	The cosmological EEs	Dynamics of an expanding U.
06/02	CMB & the early universe	Puzzles of HBB cosmology
13/02	Inflation & cosmological perturbations (J.-Ph. U.)	Testing inflation via CMB anisotropies (J.-Ph. U.)

T.D. = Thibault Damour (IHES)   J.-Ph. U. = Jean-Philippe Uzan (IAP)

Reminder of Newtonian Gravity

$$m_a^{(in)} \frac{d^2 \vec{x}_a}{dt^2} = G \sum_{b \neq a} \frac{m_a^{(gr)} m_b^{(gr)} (\vec{x}_b - \vec{x}_a)}{|\vec{x}_b - \vec{x}_a|^3}$$

Here $G = (6.672 \pm 0.004) 10^{-8} cm^3 gr^{-1} s^{-2}$

is Newton's constant and we know, experimentally, that

$$\eta \equiv 1 - \frac{m_a^{(in)}}{m_a^{(gr)}} < 10^{-14} - 10^{-12}$$

Free-fall is universal (Galileo)!

Newton's law is invariant under the 10-parameter (Galileo) group of coordinate transformations

$$x_i \quad \rightarrow \quad x'_i = R_{ij} x_j + v_i t + a_i$$
$$t \quad \rightarrow \quad t' = t + \tau \quad ; \quad R^T R = 1$$

(Counting of parameters: 3 R_{ij}, 3 v_i, 3 a_i , τ => 10)

However, Maxwell's equations of classical electromagnetism are not (or else c --> c +v). Instead, they are invariant under a deformation of Galileo's group known as the Poincaré group which is at the basis of Einstein's Special Relativity

Reminder of Special Relativity

P = Poincaré group = Lorentz x Translations = LxT

$$x^\mu \to x'^\mu \quad = \quad \Lambda^\mu{}_\nu \, x^\nu + a^\mu \quad , \quad x^0 = ct$$

$$\Lambda \, \eta \, \Lambda^T = \eta \qquad ; \qquad \eta = \mathrm{diag}(-1,1,1,1)$$

raises and lowers indices

NB: repeated index convention

$6\Lambda + 4a$ = 10-parameters

We will often use units in which c=1

$$x^T \eta \, x = \vec{x} \cdot \vec{x} - c^2 t^2$$

or for infinitesimal:

$$ds^2 = dx^T \eta \, dx = dx^2 - c^2 dt^2$$

are invariant (= 0 for light)

Interesting subgroups:

1. Translations (obvious)
2. Spatial rotations

$$\begin{pmatrix} 1 & , & 0 \\ 0 & , & R_{ij} \end{pmatrix}$$

e.g. for rotation around the $x=x^1$ axis:

$$\begin{pmatrix} 1 & , & 0 & , & 0 & , & 0 \\ 0 & , & 1 & , & 0 & , & 0 \\ 0 & , & 0 & , & \cos\theta & , & \sin\theta \\ 0 & , & 0 & , & -\sin\theta & , & \cos\theta \end{pmatrix}$$

3. Boost along x^1-axis

$$\begin{pmatrix} \cosh\alpha & , & \sinh\alpha & , & 0 & , & 0 \\ \sinh\alpha & , & \cosh\alpha & , & 0 & , & 0 \\ 0 & , & 0 & , & 1 & , & 0 \\ 0 & , & 0 & , & 0 & , & 1 \end{pmatrix}$$

where

$$\cosh\alpha = \gamma = \frac{1}{\sqrt{1-\beta^2}}$$
$$\sinh\alpha = \beta\,\gamma\,;\quad \beta \equiv \frac{v}{c}$$

thus:

$$t' = \frac{x'^0}{c} = \gamma\,(t + \frac{\beta}{c}\,x^1) \rightarrow t$$
$$x'^1 = \gamma\,(\beta\,x^0 + x^1) \rightarrow x^1 + vt$$

where the arrows define the non-relativistic limit in which we recover Galileo's transformations

Immediate consequences

1. Time dilation

An observer O measures the duration Δt of an event occurring in a system at rest. Another observer, O', moving with velocity v w.r.t. O, measures the duration $\Delta t'$ of the same event. What is the relation between Δt and $\Delta t'$?

O measures Δt and $\Delta x = 0$ O' measures $\Delta t'$ and $\Delta x' = v\,\Delta t'$

We must have:

$$c^2 \Delta t^2 - \Delta x^2 = c^2 \Delta t^2 = c^2 \Delta t'^2 - \Delta x'^2 = (c^2 - v^2)\Delta t'^2$$

that is: $$\Delta t' = \gamma \Delta t \ ; \ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \equiv \frac{1}{\sqrt{1-\beta^2}} \geq 1$$

Time dilation is seen all the time in particle accelerators!

Immediate consequences

2. Lorentz contraction

An observer O measures the length Δx of a rod at rest. Another observer, O', moving with velocity (-v) w.r.t. O, measures the length $\Delta x'$ of the same rod. What is the relation between Δx and $\Delta x'$?

Let us use the Lorentz transformation:

$$\Delta x' = \gamma(\Delta x + v\Delta t) \quad ; \quad \Delta t' = \gamma(\Delta t + \frac{v}{c^2}\Delta x)$$

O' must find what is $\Delta x'$ when $\Delta t' = 0$, i.e. for $\Delta t = - v/c^2 \, \Delta x$

The answer then is:

$$\Delta x' = \gamma(\Delta x - \frac{v^2}{c^2}\Delta x) = \gamma^{-1}\Delta x \leq \Delta x$$

Imposing the principle of special relativity

All laws can be expressed in a "covariant" way, i.e. as equations among objects that transform in the same way under the Lorentz (Poincaré) group.

Let us introduce some physical objects with nice transformation properties

Both x^μ and dx^μ transform as 4-vectors under L.T. In order to form the equivalent of a velocity we cannot use dx^i/dt (which is not in a rep. of the L.G.) but rather define a 4-velocity, a 4-momentum, and a 4-acceleration, by:

$$v^\mu = \frac{dx^\mu}{d\tau} \;,\; p^\mu = m v^\mu \;,\; a^\mu = \frac{d^2 x^\mu}{d\tau^2}$$

$$d\tau^2 = -ds^2 = c^2 dt^2 - dx^2 \;,\; v^2 = -1 \;,\; p^2 = -m^2 c^2$$

$$\text{Since } p^\mu = (E/c \,, \vec{p}) \Rightarrow E^2 = \vec{p}^{\,2} c^2 + m^2 c^4$$

Example 1: Newton's law

In terms of the quantities we have introduced

$$v^\mu = \frac{dx^\mu}{d\tau}\ ,\ p^\mu = m v^\mu\ ,\ a^\mu = \frac{d^2x^\mu}{d\tau^2}$$

$$d\tau^2 = -ds^2 = c^2dt^2 - dx^2\ ,\ v^2 = -1\ ,\ p^2 = -m^2c^2$$

Newton's law reads:

$$m\,a^\mu = \frac{d\,p^\mu}{d\tau} = F^\mu\ ,\ p_\mu F^\mu = 0$$

In the rest system we have: $F^\mu = (0\ ,\vec{f})$

where f is the non-relativistic Newtonian force

Example 2: Maxwell's equations

The electric and magnetic fields are described by an antisymmetric tensor:

$$F_{\mu\nu} = -F_{\nu\mu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ , \ \ (F_{0i} = E_i \ ; \ F_{ij} = \epsilon_{ijk} B_k)$$

Charge density and current are described by a four-vector:

$$J_\mu \ , \ \ (J_0 = \rho \ ; \ J_i = J_i)$$

Maxwell's equations (in appropriate units) take the following elegant, L.I. form:

$$\begin{aligned} \partial_\nu F^\nu_{\ \mu} &= -J_\mu \ (\Rightarrow \mathrm{div}\vec{E} = \rho \ , \ \mathrm{curl}\ \vec{B} - \partial_t \vec{E} = \vec{J}) \\ \partial_\nu \epsilon^\nu_{\ \mu\rho\sigma} F^{\rho\sigma} &= 0 \ (\Rightarrow \mathrm{div}\vec{B} = 0 \ , \ \mathrm{curl}\ \vec{E} + \partial_t \vec{B} = 0) \end{aligned}$$

Maxwell's equations

$$
\begin{aligned}
\partial_\nu F^{\nu}_{\ \mu} &= -J_\mu \ (\Rightarrow \mathrm{div}\vec{E} = \rho \ , \ \mathrm{curl} \ \vec{B} - \partial_t \vec{E} = \vec{J}) \\
\partial_\nu \epsilon^{\nu}_{\ \mu\rho\sigma} F^{\rho\sigma} &= 0 \ (\Rightarrow \mathrm{div}\vec{B} = 0 \ , \ \mathrm{curl} \ \vec{E} + \partial_t \vec{B} = 0)
\end{aligned}
$$

Particle motion in an EM field

$\frac{dp^\mu}{d\tau} = q \ F^{\mu}{}_{\nu} \ \frac{dx^\nu}{d\tau}$ equivalent to usual $\frac{d\vec{p}}{dt} = q \ \left(\vec{E} + \vec{v}\mathrm{x}\vec{B}\right)$