Synthetic gauge fields and topological effects in optics

From superfluid light towards quantum Hall liquids

Iacopo Carusotto
INO-CNR BEC Center and Università di Trento, Italy

Tomoki Ozawa  Hannah Price  Grazia Salerno
Marco Cominotti (now Grenoble)  Onur Umucalilar (now Antwerp)

In collaboration with:
- C. Ciuti (MPQ, Paris 7)
- M. Wouters (Univ. Antwerp)
- A. Amo, J. Bloch, T. Jacqmin, H.-S. Nguyen, V. G. Sala (LPN, Marcoussis)
- A. Bramati, E. Giacobino (LKB, Paris)
- T. Volz (now Macquarie), M. Kroner, A. Imamoglu (ETHZ)
- D. Gerace (Univ. Pavia)
Why not hydrodynamics of light?

Light field/beam composed by a huge number of photons
- In vacuo photons travel along straight line at c
- (practically) do not interact with each other
- In cavity, collisional thermalization slower than with walls and losses

=> optics typically dominated by single-particle physics

In photonic structure:
- $\chi^{(3)}$ nonlinearity → photon-photon interactions
- Spatial confinement → effective photon mass

=> collective behaviour of a quantum fluid

Many experiments so far:
- BEC of photons, superfluid light, synthetic gauge fields, topologically protected edge states

In this talk:
- Towards fractional Quantum Hall liquid of light
Standing on the shoulders of giants

Laserlight — First Example
of a Second-Order Phase Transition Far Away
from Thermal Equilibrium*

R. Graham and H. Haken
I. Institut für theoretische Physik der Universität Stuttgart
Received April 23, 1970

We solve the functional Fokker-Planck equation established in a previous paper in the vicinity of laser threshold. The stationary solution is obtained explicitly in the form $P = N \exp \left(-\phi(u) \bar{u}^2\right)$. $\phi$ has exactly the same form as the Ginzburg-Landau expression for the free energy of a superconductor, if the pair wave function is replaced by the electromagnetic field amplitude $\bar{u}$. This gives us the key for a thermodynamic reinterpretation of all laser phenomena.

In particular the laser threshold appears as a second-order phase transition in all details. It is indicated that our theory provides a new formalism also for the Ginzburg-Landau theory.

Vortices and Defect Statistics in Two-Dimensional Optical Chaos

F. T. Arecchi, (a) G. Giacomelli, P. L. Ramazza, and S. Residori
Istituto Nazionale di Ottica, Largo E. Fermi, 6, 50125 Firenze, Italy
(Received 1 April 1991)

We present the first direct experimental evidence of topological defects in nonlinear optics. For increasing Fresnel numbers $F$, the two-dimensional field is characterized by an increasing number of topological defects, from a single vortex, up to a large number of vortices with zero net topological charge. At variance with linear scattering from a fixed phase plate, here the defect pattern evolves in time according to the nonlinear dynamics. We assign the scaling exponents for the mean number of defects, their mean separation, and the charge unbalance as functions of $F$, as well as the correlation time of the defect pattern.

And of course many others:
Coullet, Gil, Rocca, Pomeau, Rica, Brambilla, Lugiato...
Part I: BEC and superfluidity in semiconductor microcavities
Planar DBR microcavity with QWs

- **DBR:** stack $\lambda/4$ layers (e.g. GaAs/AlAs)
- Cavity layer → confined photonic mode,
  delocalized along 2D plane:
  \[
  \omega_C(k) = \omega_C^0 \sqrt{1 + k^2 / k_z^2}
  \]

- e-h pair in QW: sort of H atom. **Exciton**
- bosons for $n_{exc} a_{Bohr}^2 \ll 1$ (verified by QMC)
- Excitons delocalized along cavity plane.
  Flat exciton dispersion $\omega_x(k) \approx \omega_x$
- Optical $\chi^{(3)}$ from exciton collisions

**Exciton radiatively coupled to cavity photon at same in-plane $k$**

**Bosonic superpositions** of exciton and photon, called **polaritons**

**Two-dimensional gas of polaritons**

Small effective mass $m_{pol} \approx 10^{-4} m_e$ → originally promising for BEC studies

Exciton → interactions. **Photons** → radiative coupling to external world
How to create and detect the photon gas?

Pump needed to compensate losses: stationary state is NOT thermodynamical equilibrium

- **Coherent laser** pump: directly injects photon BEC in cavity, may lock BEC phase
- **Incoherent** (optical or electric) pump: BEC transition similar to laser threshold spontaneous breaking of U(1) symmetry

Classical and quantum correlations of in-plane field directly transfer to emitted radiation
Mean-field theory: generalized GPE

$$i \frac{d \psi}{d t} = \left[ \omega_o - \frac{\hbar \nabla^2}{2m} + V_{ext} + g |\psi|^2 + \frac{i}{2} \left( \frac{P_0}{1 + \alpha |\psi|^2} - \gamma \right) \right] \psi + F_{ext}$$

Time-evolution of macroscopic wavefunction $\psi$ of photon/polariton condensate

- standard terms: kinetic energy, external potential $V_{ext}$, interactions $g$, losses $\gamma$
- under coherent pump: forcing term
- under incoherent pump: polariton-polariton scattering from thermal component
give saturable amplification term as in semiclassical theory of laser

=> a sort of Complex Landau-Ginzburg equation

To go beyond mean-field theory:

- Wigner representation; exact diagonalization; Keldysh diagrams; functional renormalization...

Interaction constant $g$:

- not known exactly.
- Bosonic picture initially questioned, but fully confirmed by Monte Carlo  (Astrakharchik et al., '14)
- biexciton Feshbach resonance  (Theory: Wouters, PRB '07; IC et al., EPL '10. Expt @ EPFL, '14)
2006 - Photon/polariton Bose-Einstein condensation

But also differences due to non-equilibrium:

- $\text{BEC} @ k \neq 0 \rightarrow \text{volcano effect}$
- T-reversal broken $\rightarrow n(k) \neq n(-k)$
- interesting questions about thermalization

Photon/polariton BEC closely related to laser operation in VCSELs

Many features very similar to atomic BEC

Interference
Richard et al., PRL 94, 187401 (2005)

Momentum distribution

Quantized vortices
K. Lagoudakis et al.

Suppressed fluctuations
A. Baas et al., PRL 96, 176401 (2006)
Figure from LKB-P6 group:

2009-10 - Superfluid hydrodynamics

Oblique dark solitons →

A. Amo, et al., Science 332, 1167 (2011)

Turbulent behaviours ←

A. Amo, et al., Science 332, 1167 (2011)

Hydrodynamic nucleation → of vortices

Nardin et al., Nat. Phys. 7, 635 (2011)

Role of interactions crucial in determining regimes as a function of $v/c_s$
Part I-2

*Quantum hydrodynamics*
Beyond mean-field: quantum hydrodynamics

Quantum fluctuations of hydrodynamical variables

Most fascinating prediction → analog Hawking radiation of phonons from trans-sonic interfaces (so-called analog black holes)

Cond-mat analog models → Unruh PRL '81. Optical BH's → F. Marino, PRA 78, 063804 (2008)

Wigner-QMC calculation

Signature of Hawking radiation in correlation function of intensity noise of emission

Parametric emission of entangled pairs of Bogoliubov quanta
Flow+horizon play role of pump

D. Gerace and IC, PRB 86, 144505 (2012)
Non-separability features of HR discussed in Busch, Parentani, IC, PRA 2014; Finazzi-IC, arXiv 1309.3414
Very recent experimental results @ LPN

BH created! The hunt for Hawking radiation is now open!!

H.-S. Nguyen, Gerace, IC, et al., to appear

Other (not fully conclusive) experiments for HR in artificial BH's: Weinfurtner et al., PRL 2011; Rubino et al. PRL 2010.

low power: no horizon

high power: horizon
Part II: Synthetic gauge fields and Chern insulators for photons
First expt: photonic (Chern) topological insulator

MIT '09, Soljacic group
Original proposal Haldane-Raghu, PRL 2008

Magneto-optical photonic crystals for μ-waves
T-reversal broken by magnetic elements

Band with non-trivial Chern number:
→ chiral edge states within gaps
  - unidirectional propagation
  - immune to back-scattering by defects

Wang et al., Nature 461, 772 (2009)
Synthetic gauge fields for photons

2D lattice of coupled cavities with tunneling phase

\[ H = \sum_i \hbar \omega_0 \hat{a}_i^\dagger \hat{a}_i - \hbar J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j e^{i\phi_{ij}} + \sum_i [\hbar F_i(t) \hat{a}_i^\dagger + \text{h.c.}] \]

- Floquet bands in helically deformed waveguide lattices → Segev (Technion)
- silicon ring cavities → Hafezi/Taylor (JQI)
- electronic circuits with lumped elements → J. Simon (Chicago)

Hafezi et al., Nat. Phot. 7, 1001 (2013)
Lattice periodicity: magnetic Brillouin zone

\[ H = \sum_i \hbar \omega_0 \hat{a}_i^\dagger \hat{a}_i - \hbar J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j e^{i\phi_{ij}} \]

Under a magnetic flux \( \alpha = p/q \) per lattice plaquette:

- Translational symmetry reduced to \( q \) sites. More complex magnetic translation group

- \( q \)-times smaller magnetic Brillouin zone

- non-trivial Berry connection
  \[ A_{n,k} = i \langle u_{n,k} | \nabla_k u_{n,k} \rangle \]

\( \alpha = 1/3 \) Band dispersion

Berry curvature

\[ \Omega_n(k) = \nabla_k \times A_{n,k} = \nabla_k \times [i \langle u_{n,k} | \nabla_k u_{n,k} \rangle] \]
Hofstadter butterfly and chiral edge states

Square lattice of coupled cavities at large magnetic flux

- eigenstates organize in bulk Hofstadter bands
- Berry connection in k-space: \( A_{n,k} = i\langle u_{n,k} | \nabla_k u_{n,k} \rangle \)

Bulk-edge correspondence:

\( A_{n,k} \) has non-trivial Chern number

- unidirectional propagation
- (almost) immune to scattering by defects
- T-reversal not broken, 2x pseudo-spin bands with opposite Chern

Hafezi et al., Nat. Phot. 7, 1001 (2013)
How to observe topological properties of bulk?

Lattice at strong magnetic flux, e.g. $\alpha = 1/3$

**Band dispersion**

**Berry curvature**

\[
\Omega_n(k) = \nabla_k \times \mathcal{A}_{n,k} = \nabla_k \times [i \langle u_{n,k} | \nabla_k u_{n,k} \rangle]
\]

Semiclassical eqs. of motion:

\[
\hbar \dot{k}_c(t) = eE, \\
\hbar \dot{r}_c(t) = \nabla_k \varepsilon_{n,k} - eE \times \Omega_n(k)
\]

Magnetic Bloch oscillations display a net lateral drift
- Initial photon wavepacket injected with laser pulse
- spatial gradient of cavity frequency $\rightarrow$ uniform force

Figures from Cominotti-IC, EPL 103, 10001 (2013).
First proposal in Dudarev, IC et al. PRL 92, 153005 (2004). See also Price-Cooper, PRA 83, 033620 (2012).
Array of many dissipative cavities

Cavity lattice geometry → promising in view of interacting photon gases, but radiative losses.

Short time to observe BO's, but experiment @ non-eq steady state even better

Coherent pumping \( H_d = \sum_i F_i(t) \hat{b}_i + F_i^*(t) \hat{b}_i^+ \) + losses at rate \( \gamma \)

Pump spatially localized on central site only:
- couples to all \( k \)'s within Brillouin zone
- resonance condition selects specific states

In the presence of force \( F \):

motion in BZ → lateral drift in real space by Berry curvature

\[
\begin{align*}
\hat{\mathbf{h}} \mathbf{k}_c(t) &= e \mathbf{E} \\
\hat{\mathbf{h}} \mathbf{r}_c(t) &= \nabla_k \mathcal{E}_{n,k} - e \mathbf{E} \times \Omega_n(k)
\end{align*}
\]

Detectable as lateral shift of intensity distribution by \( \Delta x \) perpendicular to \( F \)

T. Ozawa and IC, Anomalous and Quantum Hall Effects in Lossy Photonic Lattices, PRL (2014)
More quantitatively

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<th>α</th>
<th>1st</th>
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<th>3rd</th>
<th>4th</th>
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Low loss \( (γ < \text{bandwidth} ) \) \quad \rightarrow \quad Δx=F \frac{Ω(k_0)}{2γ} \quad (\text{anomalous Hall eff.})

Large loss \( (\text{bandwidth} < γ < \text{bandgap} ) \) \quad \rightarrow \quad Δx= \frac{q \text{ Chern}}{2 \pi γ} \quad (\text{integer-QH})

**Integer quantum Hall effect** for photons \((\text{in spite of no Fermi level})\)

**Photon phase observable** \(\rightarrow\) expts sensitive to **gauge-variant** quantities!!

Part II-2:

From traps to Landau levels on a torus
Chang-Niu's **semiclassical equations of motion**:

\[
\begin{align*}
    \hbar \dot{k}_n(t) &= e E, \\
    \hbar \dot{r}_n(t) &= \nabla_k \mathcal{E}_{n,k} - e E \times \Omega_n(k)
\end{align*}
\]

Can be derived from **quantum Hamiltonian**

\[
H = E_n(p) + W[r + A_n(p)]
\]

with

\[
W(r) = -e E r
\]

Similar to **minimal coupling**

\[
H = e \Phi(r) + \left[p - e A(r)\right]^2 / 2 m
\]

with \( r \leftrightarrow p \) exchanged

**Physical position** \( r_{ph} = r + A_n(p) \) \( \leftrightarrow \) **physical momentum** \( p - e A(r) \)

**Berry connection** \( A_n(p) \) \( \leftrightarrow \) **magnetic vector potential** \( A(r) \)

**Berry curvature** \( \Omega_n(p) = \text{curl}_p A_n(p) \) \( \leftrightarrow \) **magnetic field** \( B(r) = \text{curl}_r A(r) \)

**Band dispersion** \( E_n(p) \) \( \leftrightarrow \) **scalar potential** \( e \Phi(r) \)

**Trap energy** \( W(r) \) \( \leftrightarrow \) **kinetic energy** \( p^2 / 2m \)

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Harper-Hofstadter model + harmonic trap

Magnetic flux per plaquette $\alpha = 1/q$:
- for large $q$, bands almost flat $E_n(p) \approx E_n$
- lowest bands have $C_n = -1$ and almost uniform Berry curvature $\Omega_n = a^2/2\pi\alpha$

Within single band approximation:

Momentum space magnetic Hamiltonian $H = E_n(p) + k[r + A_n(p)]^2/2$

equivalent to quantum particle in constant $B$: $H = e \Phi(r) + [p - e A(r)]^2 / 2m$

Mass fixed by harmonic trap strength $k$

- Landau Levels spaced by “cyclotron” $\rightarrow k |\Omega_n|$
- And global (toroidal) topology of FBZ matters!! Degeneracy of LLs reduced to $|C_n|$

Of course, if:
- Too small $\alpha$ / too strong trap $\rightarrow$ band too close for single band approx
- Too large $\alpha$ / too weak trap $\rightarrow$ effect of $E_n(p)$ important

Price, Ozawa, IC, Quantum Mechanics Under a Momentum Space Artificial Magnetic Field, arXiv:1403.6041
Numerical spectrum

Landau levels of lowest HH band crossing with Landau levels of second HH band

$\alpha \rightarrow 0$ harmonic trap states
(band gap too small)

Landau levels of lowest HH band

Price, Ozawa, IC, Quantum Mechanics Under a Momentum Space Artificial Magnetic Field, arXiv:1403.6041
9\textsuperscript{th} and 48\textsuperscript{th} state for $\alpha = 1/11$

eigen-functions recover
$\beta = 8$ Landau level on torus
for 1\textsuperscript{st} and 2\textsuperscript{nd} HH bands.

Only difference is Bloch function

Price, Ozawa, IC, Quantum Mechanics Under a Momentum Space Artificial Magnetic Field, arXiv:1403.6041
How to observe and characterize these states?

Does not seem trivial in atomic gases...

Straightforward in optics under coherent pump:
- each absorption peak → an eigenstate
- coherent pump frequency selects a single state
  ▶ Near-field image → real-space eigenfunction
  ▶ far-field emission → k-space eigenfunction

near field

far field

Ozawa, Price, IC, unpublished
Part II-3:

**Photons in honeycomb lattices**
(a kind of photonic graphene)
Arrays of micropillars

Many ways to create lattice:

- lateral patterning during growth (EPFL)
- surface acoustic waves
- metallic electrodes (Stanford)
- mechanical deformation (Pittsburgh)

here → Lateral confinement by etching cavity
All 2D lattice geometries possible with suitable etching masks

Honeycomb lattice of pillars → polariton “graphene”

Jacqmin, IC, et al., *Direct observation of Dirac cones and a flat band in a honeycomb lattice for polaritons*, PRL (2014)
Expt @ LPN, Marcoussis. Theory @ BEC Trento
Band dispersion

Reconstructed from energy- and angle-resolved photoluminescence

Jacqmin, IC, et al., Direct observation of Dirac cones and a flat band in a honeycomb lattice for polaritons, PRL (2014)
Non-equilibrium BEC

Strong pump, honeycomb lattice:
- photon/polariton BEC at top of band
- kept together by repulsion and $m^* < 0$ as in gap solitons
- similar behaviour also in 1D lattices

Planar geometry, $m^* > 0$:
- BEC on \( k \)-space ring for small pump spot
- first observed in Grenoble '05

Generally:
- no thermodynamical need for BEC at \( k=0 \) !!
- free energy not involved in mode-selection
- as in lasers, mode with strongest amplification is typically selected

M. Richard et al., PRL 94, 187401 (2005)
Theory: Wouters, IC, Ciuti, PRB 77, 115340 (2008)
**What new physics with it?**

**Dirac waves** instead of Schroedinger ones
- **Klein tunneling** → suppressed reflection at barrier
- negative refraction
- Goos-Haenchen lateral shift

**Spin-orbit** coupling:
- light polarization ↔ spin degree of freedom
- flat bands originate from P orbitals

**Nonlinear** effects:
- new kinds of solitons and vortices
- flat bands enhance effect of nonlinearity

**Topological wave propagation**
- effect of Berry curvature on linear and nonlinear waves
Spin-orbit coupling observed in “photonic benzene”

6 pillars geometry
- orbital momentum $\rightarrow$ inter-pillar tunneling energy
- visible in incoherent photo-luminescence

Spin-orbit coupling only apparent in BEC:
- linewidth narrows down
- mode competition strongly selective

$\rightarrow$ BEC in $l=1$ mode with azimuthal polarization:
- opposite vortices in $\sigma_{\pm}$ polarizations
- radial polariz. if BEC in $l=2$ mode (occurs at higher power)

Effect in graphene geometry under study

Simulations of Klein tunneling

Honeycomb photons propagating against potential step
Direct access to real space (near field) and momentum (far field) distributions

T. Ozawa and IC, in preparation (2014)
Berry connection in “gapped” honeycomb

Adding site asymmetry:

- gap opens at Dirac points

- strong Berry curvature $\Omega$ at band edges

- $\Omega$ has opposite signs at $K/K'$ points
  $\rightarrow$ Chern number vanishes

Using momentum-selective pump one can extract
Berry curvature around Dirac point from lateral shift of wavepacket

T. Ozawa and IC, Anomalous and Quantum Hall Effects in Lossy Photonic Lattices, PRL (2014)
Part III:
The future:
Strongly interacting photons
**Photon blockade**

Full 3D confinement: microcavity + in-plane confinement

**Bose-Hubbard model:**

\[ H_0 = \sum_i \hbar \omega_o \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{\hbar U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) \]

- single-mode cavities at \( \omega_o \). Tunneling coupling \( J \)
- Polariton interactions: on-site repulsion \( U \)
- Incident laser: coherent external driving \( H_d = \sum_i F_i(t) \hat{b}_i + h.c. \)
- If \( U >> \gamma, J \), coherent pump resonant with \( 0 \rightarrow 1 \) transition, but not with \( 1 \rightarrow 2 \) transition. Effectively impenetrable photons
- Weak losses \( \gamma \ll J, U \rightarrow \) Lindblad terms in master eq. determine non-equilibrium steady-state
- Strong number fluctuations \( \rightarrow \) dramatic effect on MI, but....
**Impenetrable “fermionized” photons in 1D necklaces**

Many-body eigenstates of Tonks-Girardeau gas of impenetrable photons

Coherent pump selectively addresses specific many-body states

**Transmission spectrum** as a function **pump frequency** for fixed pump intensity:

- each peak corresponds to a Tonks-Girardeau many-body state $|q_1,q_2,q_3...>$
- $q_i$ quantized according to PBC/anti-PBC depending on $N=$odd/even
- $U/J >> 1$: efficient photon blockade, impenetrable photons.

N-particle state excited by N photon transition:

- Plane wave pump with $k_p=0$: selects states of total momentum $P=0$
- Monochromatic pump at $\omega_p$: resonantly excites states of many-body energy $E$ such that $\omega_p = E / N$

See also related work D. E. Chang et al, Nature Physics (2008)
Finite $U/J$, pump laser tuned on two-photon resonance

- intensity correlation between the emission from cavities $i_1, i_2$
- at large $U/\gamma$, larger probability of having $N=0$ or $2$ photons than $N=1$
  - low $U<<J$: bunched emission for all pairs of $i_1, i_2$
  - large $U>>J$: antibunched emission from a single site positive correlations between different sites

- Idea straightforwardly extends to more complex many-body states.
Part III-2: \textit{Fractional quantum Hall effect for photons}
Photon blockade + synthetic gauge field = QHE for light

Bose-Hubbard model:

\[ H_0 = \sum_i \hbar \omega \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j e^{i \varphi_{ij}} + \hbar \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) \]

with usual coherent drive and dissipation \(\rightarrow\) look for non-equil. steady state

Transmission spectra:

- peaks correspond to many-body states
- comparison with eigenstates of \(H_0\)
- good overlap with Laughlin wf (with PBC)

\[ \psi_i(z_1, ..., z_N) = N_L F_{\text{CM}}^{(l)}(Z) e^{-\pi \alpha \sum_i u_i^2} \times \prod_{i<j}^N \left( \frac{1}{2} \right) \left( \frac{z_i - z_j}{L} \right)^2 \]

- no need for adiabatic following, etc....


See also related work by Cho, Angelakis, Bose, PRL 2008; Hafezi et al. NJP 2013; arXiv:1308.0225
**Tomography of FQH states**

Homodyne detection of secondary emission

→ info on many-body wavefunction

\[
\langle \hat{b}_i \hat{b}_j \rangle = \langle X_0^{(i)} X_0^{(j)} \rangle - \langle X_{\pi/2}^{(i)} X_{\pi/2}^{(j)} \rangle \\
+ i \langle X_0^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_{\pi/2}^{(i)} X_0^{(j)} \rangle
\]

**Note:** optical signal gauge dependent, optical phase matters!

Non-trivial structure of Laughlin state compared to non-interacting photons

A simpler design: rotating photon fluids

Rotating system at angular speed $\Omega$

- Coriolis $F_c = -2m\Omega \times v$
- Lorentz $F_L = e \mathbf{v} \times \mathbf{B}$

Rotating photon gas injected by LG pump
with finite orbital angular momentum

Resonant peak in transmission due to Laughlin state:

$$\psi(z_1, \ldots, z_N) = e^{-\sum_i |z_i|^2/2} \prod_{i<j} (z_i - z_j)^2$$

Overlap measured from quadrature noise of transmitted light

$$\langle \hat{b}_i \hat{b}_j \rangle = \langle X_0^{(i)}X_0^{(j)} \rangle - \langle X_0^{(i)}X_{\pi/2}^{(j)} \rangle - i\langle X_0^{(i)}X_{\pi/2}^{(j)} \rangle + i\langle X_{\pi/2}^{(i)}X_0^{(j)} \rangle$$

R. O. Umucalilar and IC, Anyonic braiding phases in a rotating strongly correlated photon gas, arXiv:1210.3070
**Anyonic braiding phase**

- LG pump to create and maintain quantum Hall liquid
- Localized repulsive potentials in trap:
  - create quasi-hole excitation in quantum Hall liquid
  - position of holes adiabatically braided in space
- Anyonic statistics of quasi-hole: many-body Berry phase $\phi_{Br}$ when positions swapped during braiding
- Berry phase extracted from shift of transmission resonance while repulsive potential moved with period $T_{rot}$ along circle
  \[
  \phi_{Br} \equiv (\Delta \omega_\infty - \Delta \omega_0) T_{rot} \frac{[2 \pi]}{2 \pi}
  \]
- so far, method restricted to low particle number

Conclusions

Dilute photon gas
2000-6 \rightarrow \text{BEC in exciton-polaritons gas in semiconductor microcav.}

GP-like equation
2008-10 \rightarrow \text{superfluid hydrodynamics effects observed}
2009-13 \rightarrow \text{synthetic gauge field for photons and topologically protected edge states observed.}

Take-home message:
Optical systems are (almost) unavoidably lossy \rightarrow \text{driven-dissipative, non-equilibrium dynamics not always a hindrance for many-body physics, but can be turned into great advantage!}

Many questions still open:
- quantum hydrodynamics, e.g. analog Hawking radiation in acoustic black holes
- critical properties of BKT transition in 2D – peculiar non-equilibrium features anticipated
- topological effects with spin-orbit couplings; non-Abelian synthetic gauge fields

Challenging perspectives on a longer run:
- strongly correlated photon gases \rightarrow \text{Tonks-Girardeau gas in 1D necklace of cavities}
- with synthetic gauge field \rightarrow \text{Laughlin states, quantum Hall physics of light}
- Theoretical challenge \rightarrow \text{how to create and control strongly correlated many-photon states?}
- more complex quantum Hall states: non-Abelian statistical phases.
An integrated platform for \text{topologically protected states for QIP ??}
If you wish to know more...

**Quantum fluids of light**

Iacopo Carusotto*  
INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, I-38123 Povo, Italy

Cristiano Ciuti†  
Laboratoire Matériaux et Phénomènes Quantiques, Université Paris Diderot-Paris 7 et CNRS, Bâtiment Condorcet, 10 rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13, France  
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**CIRCUITNAVIGATING AN OCEAN OF INCOMPRESSIBLE LIGHT**

A JOURNEY ACROSS THE EXCITING PERSPECTIVES OF QUANTUM FLUIDS OF LIGHT

IACOPO CARUSOTTO  
INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, Povo, Italy


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Jan. 12th – 23rd, 2015 @ ECT*, Trento  
school & workshop on  
Strongly correlated quantum fluids of light and matter  
Organizers: IC, C.Ciuti, A.Imamoglu, R. Fazio