

Optical Flux Lattices for Cold Atom Gases

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Artificial Magnetism for Cold Atom Gases

Collège de France, 11 June 2014

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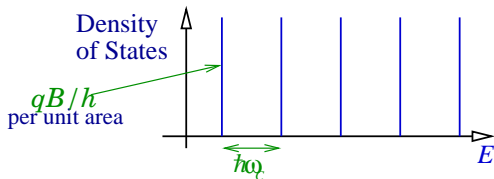
Engineering and Physical Sciences
Research Council



Motivation: fractional quantum Hall regime

2D charged particle in magnetic field \Rightarrow Landau levels

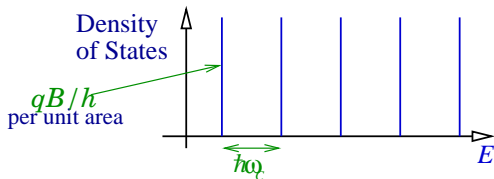
Flux density $n_\phi = \frac{qB}{h}$



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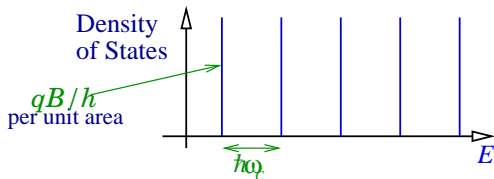


e-e repulsion \Rightarrow fractional quantum Hall states, at certain $\nu \equiv \frac{n_{2D}}{n_\phi}$

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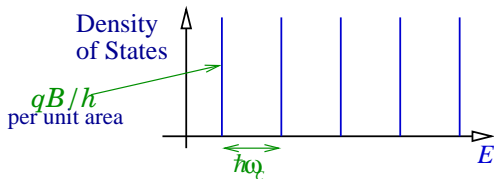
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Bosons? (contact repulsion)

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$$\text{Flux density } n_\phi = \frac{qB}{h}$$



e-e repulsion \Rightarrow fractional quantum Hall states, at certain $\nu \equiv \frac{n_{2D}}{n_\phi}$

Bosons? (contact repulsion)

- $\nu > \nu_c \simeq 6$: vortex lattice (BEC)

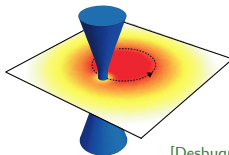
[NRC, Wilkin & Gunn '01; Sinova, Hanna & MacDonald '02; Baym '04]

- $\nu < \nu_c$: FQH states (incl. “non-Abelian”)

[NRC, Wilkin & Gunn '01]

Synthetic Magnetic Field: Rotation

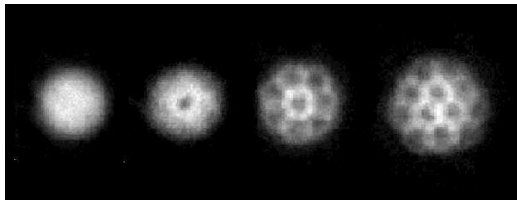
Stir the atomic gas



[Desbuquois *et al.* (2012)]

Rotating frame, angular velocity Ω

Coriolis Force \Leftrightarrow Lorentz Force $n_\phi \equiv \frac{qB}{h} = \frac{2M\Omega}{h}$



[K. W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, *Phys. Rev. Lett.* **84**, 806 (2000)]

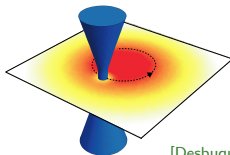
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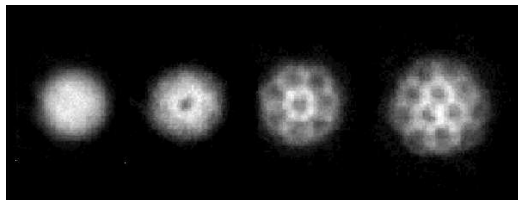
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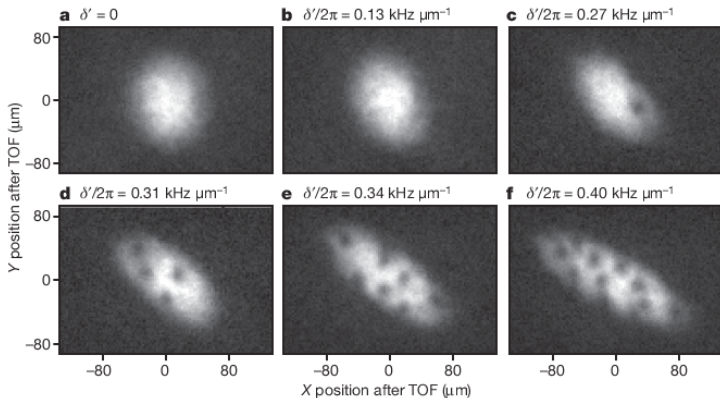
$$\Omega \lesssim 2\pi \times 100\text{Hz}$$

$$\Rightarrow n_\phi \lesssim 2 \times 10^7 \text{cm}^{-2}$$

[K. W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, *Phys. Rev. Lett.* **84**, 806 (2000)]

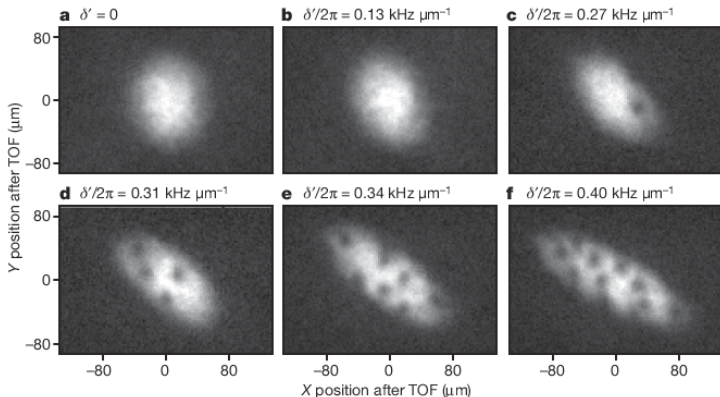
Synthetic Magnetic Field: Optically Dressed States

[Y.-J. Lin, R.L. Compton, K. Jiménez-García, J.V. Porto & I.B. Spielman, *Nature* **462**, 628 (2009)]



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But...
$$n_{\phi} \lesssim \frac{1}{R\lambda} \sim 2 \times 10^7 \text{ cm}^{-2} \quad [R \text{ cloud size}]$$

Outline

Optically Dressed States

Optical Flux Lattices
Design Principles

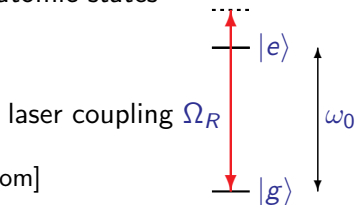
Outlook & Summary

Optically Dressed States

[J. Dalibard, F. Gerbier, G. Juzeliūnas, P. Öhberg, RMP **83**, 1523 (2011)]

Coherent optical coupling of N internal atomic states

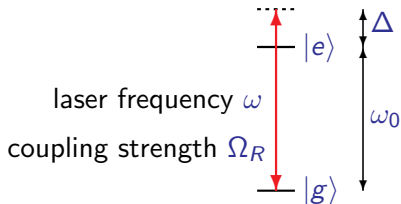
[e.g. 1S_0 and 3P_0 for Yb or alkaline earth atom]



Forms the local “dressed state” of the atom

$$|0_{\mathbf{r}}\rangle = \alpha_{\mathbf{r}}|g\rangle + \beta_{\mathbf{r}}|e\rangle = \begin{pmatrix} \alpha_{\mathbf{r}} \\ \beta_{\mathbf{r}} \end{pmatrix}$$

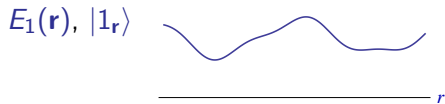
optical coupling $\hat{V}(\mathbf{r})$



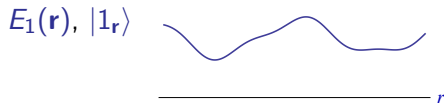
Rotating Wave Approximation $\omega \gg \Delta, \Omega_R$

$$\hat{V}(\mathbf{r}) \rightarrow \frac{\hbar}{2} \begin{pmatrix} \Delta & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & -\Delta \end{pmatrix}$$

In general
$$\hat{V} = \frac{\hbar}{2} \begin{pmatrix} \Delta(\mathbf{r}) & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & -\Delta(\mathbf{r}) \end{pmatrix}$$

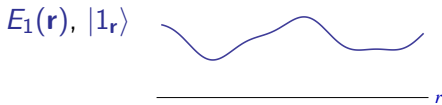


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Adiabatic motion in lower energy dressed state, $|\Psi\rangle = \psi_0(\mathbf{r})|0_r\rangle$

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$$H_{\text{eff}}\psi_0 = \langle 0_r | \left[\frac{\mathbf{p}^2}{2M} + \hat{V} \right] \psi_0 | 0_r \rangle \Rightarrow H_{\text{eff}} = \frac{(\mathbf{p} - q\mathbf{A})^2}{2M} + V_0(\mathbf{r})$$

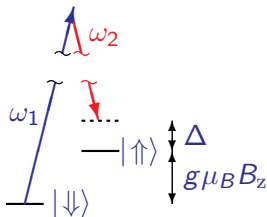
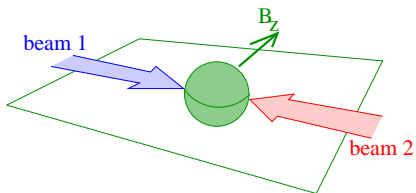
["Berry connection" \Rightarrow vector potential $q\mathbf{A} = i\hbar\langle 0_r | \nabla 0_r \rangle$]

[J. Dalibard, F. Gerbier, G. Juzeliūnas & P. Öhberg, RMP **83**, 1523 (2011)]

Experimental Implementation

Rubidium BEC

[Lin, Compton, Jiménez-García, Porto & Spielman, Nature **462**, 628 (2009)]



$$\Delta k = k_1 - (-k_2) \simeq 2 \times \frac{2\pi}{\lambda}$$

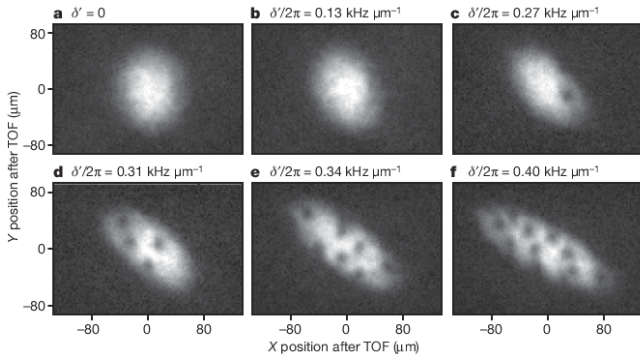
$$\hat{V} = \frac{\hbar}{2} \begin{pmatrix} \Delta & \Omega_R e^{-i\Delta k x} \\ \Omega_R e^{i\Delta k x} & -\Delta \end{pmatrix}$$

field gradient $B_z(y) \Rightarrow \Delta \propto y$

Experimental Implementation

Rubidium BEC

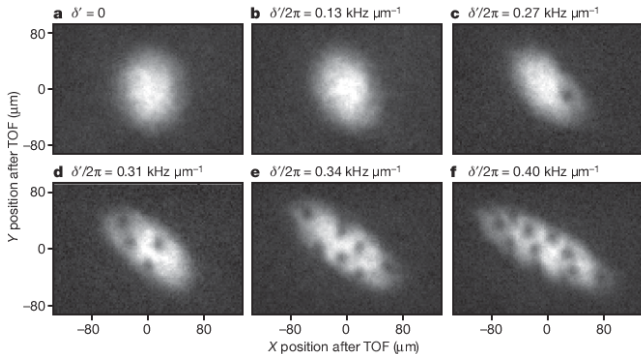
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Maximum flux density: Back of the envelope

Vector potential $q\mathbf{A} = i\hbar\langle 0_r | \nabla 0_r \rangle \Rightarrow |q\mathbf{A}| \lesssim \frac{h}{\lambda}$

Cloud of radius $R \gg \lambda$

$$N_\phi \equiv \int n_\phi d^2r = \frac{q}{h} \int \nabla \times \mathbf{A} \cdot d\mathbf{S} = \frac{q}{h} \oint \mathbf{A} \cdot d\mathbf{r} \lesssim \frac{1}{\lambda} (2\pi R)$$

$$\Rightarrow \bar{n}_\phi \equiv \frac{N_\phi}{\pi R^2} \lesssim \frac{1}{R\lambda} \simeq 2 \times 10^7 \text{ cm}^{-2} \quad [R \simeq 10\mu\text{m} \quad \lambda \simeq 0.5\mu\text{m}]$$

Maximum flux density: Carefully this time!

Vector potential $q\mathbf{A} = i\hbar\langle 0_r | \nabla 0_r \rangle \Rightarrow |q\mathbf{A}| \lesssim \frac{h}{\lambda}$

\mathbf{A} can have *singularities* – if the optical fields have vortices

e.g. $\Omega_R(\mathbf{r}) \sim (x + iy)$

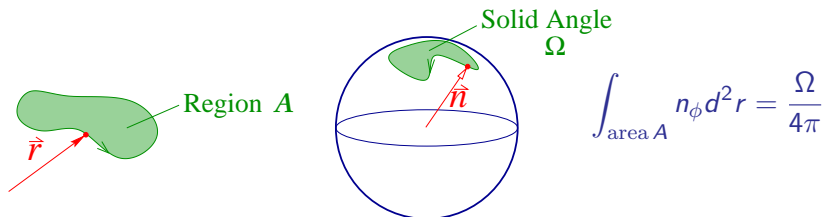
Vanishing net flux. Can be (re)moved by a gauge transformation.

[cf. “Dirac strings”]

Gauge-Independent Approach (two-level system)

Bloch vector $\vec{n}(\mathbf{r}) = \langle 0_r | \hat{\sigma} | 0_r \rangle$

Flux density $n_\phi = \frac{1}{8\pi} \epsilon_{ijk} \epsilon_{\mu\nu} n_i \partial_\mu n_j \partial_\nu n_k$



The number of flux quanta in region A is the number of times the Bloch vector wraps over the sphere.

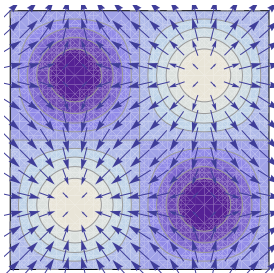
“Optical flux lattices”

[NRC, Phys. Rev. Lett. **106**, 175301 (2011)]

Spatially periodic light fields for which the Bloch vector wraps the sphere a nonzero integer number, N_ϕ , times in each unit cell.

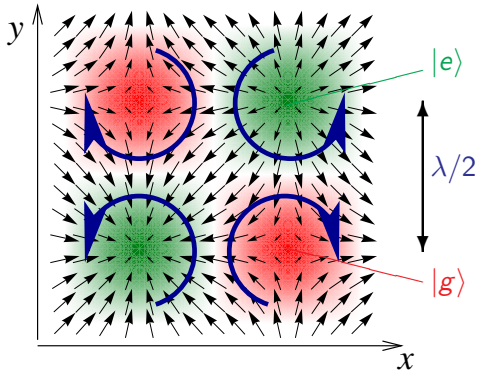
$$\bar{n}_\phi = \frac{N_\phi}{A_{\text{cell}}} \sim \frac{1}{\lambda^2} \simeq 10^9 \text{cm}^{-2}$$

vectors (n_x, n_y)
 contours n_z
 $N_\phi = 2$

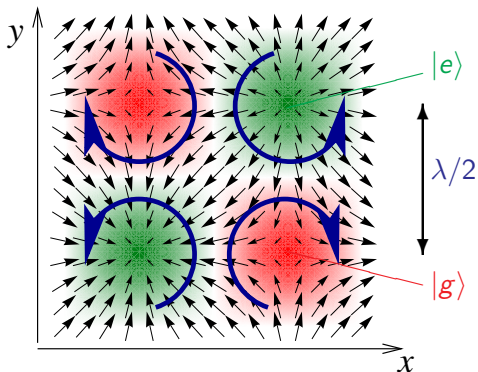


$$\hat{V} = \mathcal{V} \begin{pmatrix} \sin \kappa x \sin \kappa y & \cos \kappa x - i \cos \kappa y \\ \cos \kappa x + i \cos \kappa y & -\sin \kappa x \sin \kappa y \end{pmatrix}$$

Lorentz Force: Semiclassical Picture

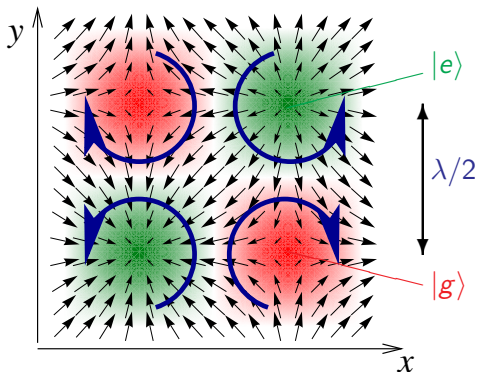


Lorentz Force: Semiclassical Picture



$$p_x \sim (\hbar k) \frac{y}{\lambda/2} \quad \Rightarrow \quad F_x \equiv \dot{p}_x \sim \frac{\hbar k}{\lambda/2} v_y$$

Lorentz Force: Semiclassical Picture

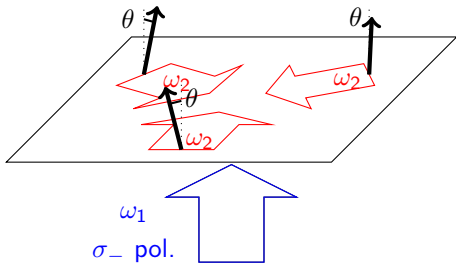
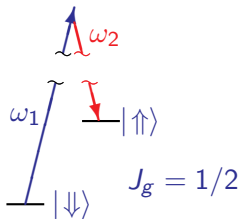


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Lorentz force, with $qB \sim \frac{2\hbar}{\lambda^2} \quad \Rightarrow \quad n_\phi \equiv \frac{qB}{h} \sim \frac{2}{\lambda^2}$

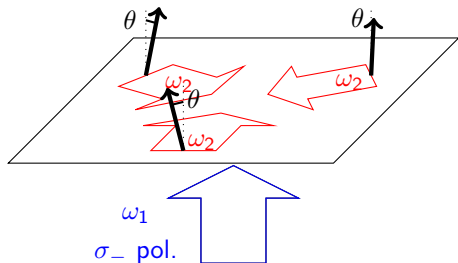
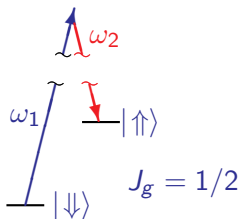
Example: Implementation for Hyperfine Levels

[NRC & Jean Dalibard, EPL **95**, 66004 (2011)]



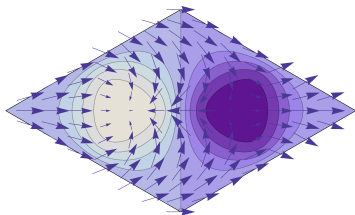
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$$\hat{V} \propto \begin{pmatrix} |\mathcal{E}_2^+|^2 - |\mathcal{E}_2^-|^2 & \mathcal{E}_2^z \mathcal{E}_1^{-*} \\ \mathcal{E}_2^{z*} \mathcal{E}_1^- & -|\mathcal{E}_2^+|^2 + |\mathcal{E}_2^-|^2 \end{pmatrix}$$

$N_\phi = 1$ (two level system)

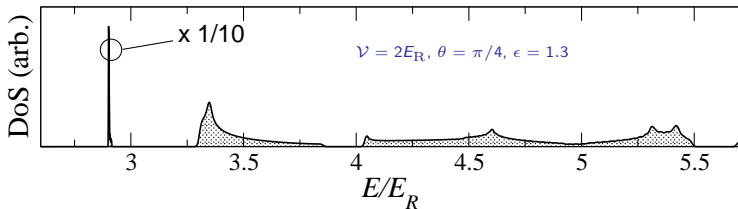


Bandstructure: $\frac{\mathbf{p}^2}{2M} \hat{\mathbb{I}} + \hat{V}(\mathbf{r})$

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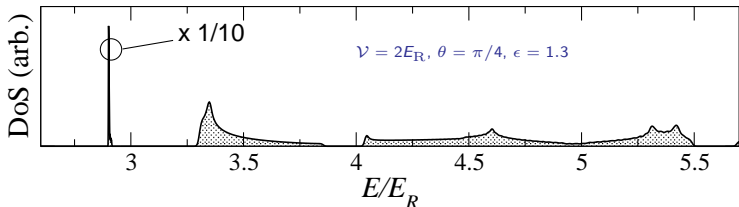
$J_g = 1/2$ (e.g. ^{171}Yb , ^{199}Hg , ^6Li)

$[E_R = \frac{\hbar^2}{2M\lambda^2}]$

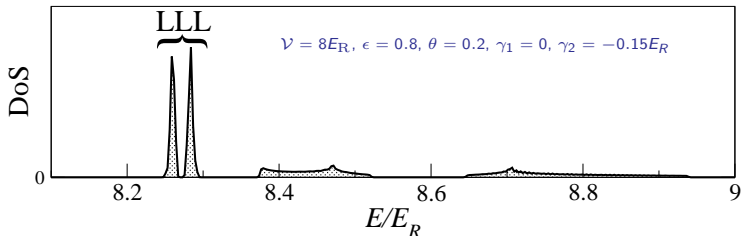


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$$[E_R = \frac{\hbar^2}{2M\lambda^2}]$$



$$J_g = 1 \text{ (e.g. } ^{23}\text{Na, } ^{39}\text{K, } ^{87}\text{Rb)}$$



- Narrow topological bands: analogous to lowest Landau level

Designing Optical Flux Lattices

[NRC & Roderich Moessner, PRL **109** 215302 (2012)]

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$$\hat{H} = \frac{\mathbf{p}^2}{2M} \hat{\mathbb{1}} + \hat{V}(\mathbf{r})$$

Optical lattices are conveniently defined in reciprocal space

Couplings $V_{\mathbf{k}'-\mathbf{k}}^{\alpha'\alpha} \equiv \langle \alpha', \mathbf{k}' | \hat{V} | \alpha, \mathbf{k} \rangle$ of internal states $\alpha = 1, \dots, N$

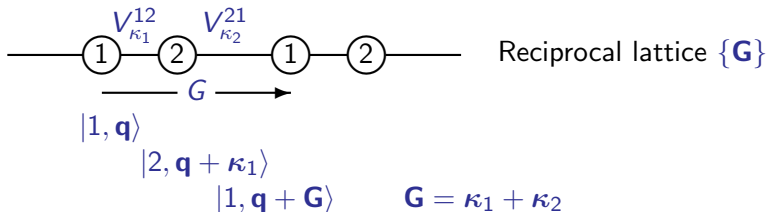
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Bloch's theorem $|\psi^{n\mathbf{q}}\rangle = \sum_{\alpha, \mathbf{G}} c_{\alpha\mathbf{G}}^{n\mathbf{q}} |\alpha, \mathbf{q} + \mathbf{g}_{\alpha} + \mathbf{G}\rangle$

$$E_{n\mathbf{q}} c_{\alpha\mathbf{G}}^{n\mathbf{q}} = \frac{\hbar^2 |\mathbf{q} + \mathbf{G} + \mathbf{g}_{\alpha}|^2}{2M} c_{\alpha\mathbf{G}}^{n\mathbf{q}} + \sum_{\alpha', \mathbf{G}'} V_{\mathbf{G} + \mathbf{g}_{\alpha} - \mathbf{G}' - \mathbf{g}_{\alpha'}}^{\alpha\alpha'} c_{\alpha'\mathbf{G}'}^{n\mathbf{q}}$$

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Adiabatic limit (K.E. $\rightarrow 0$)

$$E_{n\mathbf{q}} c_{\alpha\mathbf{G}}^{n\mathbf{q}} = \sum_{\alpha', \mathbf{G}'} V_{\mathbf{G} + \mathbf{g}_{\alpha} - \mathbf{G}' - \mathbf{g}_{\alpha'}}^{\alpha\alpha'} c_{\alpha'\mathbf{G}'}^{n\mathbf{q}}$$

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Bandstructure determines the dressed states in *real space*

$$[\hat{H} = \hat{V}(\mathbf{r})]$$

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conserved “momentum” \Leftrightarrow real space position, \mathbf{r}

Brillouin zone \Leftrightarrow real space unit cell

Bloch wavefunction \Leftrightarrow dressed state, $|n_{\mathbf{r}}\rangle$

band energies \Leftrightarrow local dressed state energies, $E_n(\mathbf{r})$

Berry curvature \Leftrightarrow local flux density, n_ϕ

Chern number, \mathcal{C} \Leftrightarrow flux through unit cell, N_ϕ

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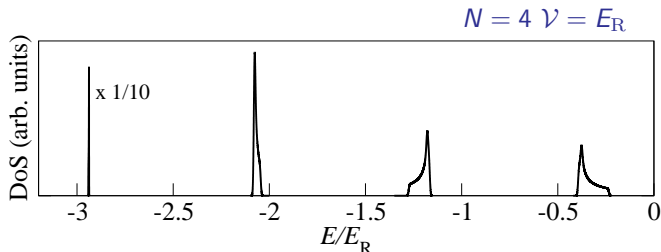
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For an optical flux lattice, the lowest energy band of the reciprocal-space tight-binding model has non-zero Chern number

OFLs with uniform Magnetic Field and Scalar Potential



- Low energy spectrum closely analogous to Landau levels
- Narrow bands \Rightarrow strongly correlated phases
- $N = 3$ scheme for ^{87}Rb shows robust Laughlin, CF/hierarchy and Moore-Read phases of bosons, even for weak two-body repulsion

[NRC & Jean Dalibard, PRL **110**, 185301 (2013)]

Outlook

- Strong magnetic field, $n_\phi \sim \frac{1}{\lambda^2}$
- Novel FQH states of 2D bosons [NRC & J. Dalibard, PRL (2013)]
- Correlated bosonic phases in 3D? [NRC, van Lankvelt, Reijnders & Schoutens, PRA '05]
- Strong-coupling superconductivity vs. FQH, $\xi_{\text{pair}} \sim \bar{a} \sim n_\phi^{-1/2}$
[for a cuprate superconductor, would need $B \gtrsim 10^5 \text{T!}$]

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 - Other Topological Bandstructures
 - “Chern insulators” with $\mathcal{C} > 1$
 - \mathbb{Z}_2 Topological Insulators in 2D and 3D
- ⇒ Exploration of “fractional topological insulators”

Summary

- ▶ Coherent optical coupling of internal states provides a powerful way to create topological bands for cold atoms.

$$\hat{H} = \frac{\mathbf{p}^2}{2M} \hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

- ▶ Simple laser set-ups lead to “optical flux lattices”: periodic magnetic flux with very high mean density, $n_\phi \sim 1/\lambda^2$.
- ▶ The bandstructure can be designed with significant control.
- ▶ These lattices offer a practical route to the study of novel strongly correlated topological phases in ultracold gases.