Climate Change, Directed Innovation, and Energy Transition: Should We Escape From Coal Through Gas?

Daron Acemoglu, Philippe Aghion, David Hemous

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1 Introduction

The growing concern about global warming has motivated academics and policy makers to think about the most effective way to move the world economy out of its current path of intensive use of coal and oil energy, a path which is widely considered to be responsible for the observed increase in CO2 concentration and consequently for the increase in temperatures. However, while there seems to be a fairly broad consensus on the need to eventually move away from carbon energy, opinions differ as to the optimal dynamic strategy to be followed. In particular, there is a hot debate between those who believe that intermediate sources of energy such as nuclear or gas energy should be immediately explored and developed in order to escape from coal, and those who fear that allowing for research and production around these intermediate sources of energy, would delay the desirable transition to fully clean and safe energy sources such as Eolian.

In this paper we develop a simple model to analyze the conditions under which allowing for research and innovation in intermediate sources of energy (e.g shale gas energy) can improve environmental quality in the short and/or medium term.

2 Basic model

Time is discrete and the economy comprises a continuum of researchers, and also a continuum of identical individuals whose utility depends positively on consumption and negatively on aggregate pollution. The final (consumption) good is produced according to:

$$Y_{ft} = A_{ft} L_{ft}^{1-\beta} E_t^{\beta},$$

where A_{ft} is TFP, L_{ft} is labor used in final good production and E_t is an energy composite. Later on (for calibration purposes) we may want to add capital, damages in the production function, energy-saving innovations, etc...

The energy composite is produced according to

$$E_t = \left(E_{c,t}^{\frac{\varepsilon-1}{\varepsilon}} + E_{s,t}^{\frac{\varepsilon-1}{\varepsilon}} + E_{g,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $E_{c,t}, E_{c,t}$, and $E_{c,t}$ denote energy from coal, shale gas and green (wind for example) energy respectively. The production of energy i is given by

$$E_{it} = \min \left(Y_{it}, R_{it} \right),$$

where Y_{it} represents an extraction input and R_{it} a resource use corresponding to that particular source of energy (coal, shale gas, and "wind"). Each resource i at date t involves a pollution intensity ξ_{it} so that: $P_{i,t} = \xi_i R_{i,t}$ with $\xi_c > \xi_s > 0 = \xi_g$. In other words, using the clean resource does not pollute the atmosphere, and the use of shale gas pollutes the atmosphere but less than that of the coal resource.

Aggregate pollution is then given by

$$P_t = \xi_g R_{i,t} + \xi_s R_{s,t} + \xi_c R_{c,t} = \xi_s R_{s,t} + \xi_c R_{c,t}.$$

We take the ξ's to remain fixed over time. Changes in ξ could be introduced to represent technological improvements in the capture of emissions ("clean coal" for instance or reducing methane leakage when shale gas is extracted). In addition one could add technological progress in front of the R variable to model resource saving innovation in energy production. At the moment though we shall concentrate on the effects of technological improvements in the extraction technology. Finally, we shall assume that R_g is in infinite supply, and to also assume that R_c and R_s are also in infinite supply (so that the price of all resources is 0).

The extraction input i is produced at time t according to

$$Y_{it} = L_{it}^{1-\alpha} \int_0^1 A_{ijt}^{1-\alpha} x_{ijt}^{\alpha} dj$$

where L_{it} is labor, x_{ijt} are intermediate inputs produced by local monopolists using the final good as an input: it costs ψ units of final food to produce one unit of intermediate input i. Due to the Leontief assumption and the infinite supply of resources we immediately get: $E_{it} = Y_{it} = R_{it}$ in equilibrium. Let

$$A_{it} = \int_0^1 A_{ijt} dj$$

In the next section on static equilibrium we derive conditions under which an increase in shale gas extraction productivity A_{st} increases or decreases aggregate pollution P_t at time t. In the subsequent section on dynamics we endogeneize the productivities on the three types of extraction activity (coal, shale gas, clean energy) by making them result from endogenous innovation activities, and we look for conditions under which a ban on research in gas extraction increases or decreases aggregate pollution in the short run and the long run.

3 Static equilibrium

Here we solve for the static equilibrium for given productivity vector $(A_{ijt})_{i,j}$. This equilibrium consists in labor demands L_{it} in the production of extraction inputs Y_{it}, and intermediate input production x_{ijt} by firm j in the extraction input sector i, such that (profit maximization by input producer (i,j)):

$$p_{it} L_{it}^{1-\alpha} A_{ijt}^{1-\alpha} x_{ijt}^{\alpha-1} = \frac{\psi}{\alpha^2}$$

hence

$$x_{ijt} = \left(\frac{\alpha^2}{\psi} p_{it}\right)^{\frac{1}{1-\alpha}} L_{it} A_{ijt}$$

with corresponding equilibrium profits:

$$\pi_{ijt} = (1-\alpha)\frac{\alpha^{\frac{1+\alpha}{1-\alpha}}}{\psi^{\frac{\alpha}{1-\alpha}}} p_{it}^{\frac{1}{1-\alpha}} L_{it} A_{ijt}$$

The corresponding equilibrium extraction input Y_{it} is given by:

$$Y_{it} = \left(\frac{\alpha^2}{\psi} p_{it}\right)^{\frac{\alpha}{1-\alpha}} L_{it} A_{it}$$

In the Appendix we compute the equilibrium wage and prices for c, g, s and the energy E. And the Appendix we also derive the equilibrium values of energy use on all three types of energy, and from there the equilibrium level of pollution. There we show:

Lemma 1 *Let*

$$A_{Et} = \left(A_{gt}^{-\varphi} + A_{st}^{-\varphi} + A_{ct}^{-\varphi}\right)^{-\frac{1}{\varphi}}$$

with $\varphi = (1-\alpha)(1-\varepsilon)$. *Then we have:*

$$P_t = \chi_2 A_{ft}^{\frac{\alpha}{1-\alpha\beta}} \left(\xi_c \left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon} + \xi_s \left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}\right) A_{Et}^{\frac{1-\alpha}{1-\alpha\beta}}$$

where χ_2 *is a constant.*

Now the question is how aggregate pollution P_t depends upon the productivity levels on the three types of extraction technologies: A_{st}, A_{ct} and A_{gt}. We have

$$\begin{aligned}
\frac{\partial \ln P_t}{\partial \ln A_{gt}} &= \underbrace{-(1-\alpha)\varepsilon \left(\frac{\xi_c\left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}}{\xi_c\left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon} + \xi_s\left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}} + \frac{\xi_s\left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}}{\xi_c\left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon} + \xi_s\left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}}\right)\frac{\partial \ln A_{Et}}{\partial \ln A_{gt}}}_{\text{substitution effect away from coal and shale}} + \underbrace{\frac{1-\alpha}{1-\alpha\beta}}_{\text{scale effect}}\frac{\partial \ln A_{Et}}{\partial \ln A_{gt}} \\
&= -(1-\alpha)\left(\varepsilon - \frac{1}{1-\alpha\beta}\right)\frac{\partial \ln A_{Et}}{\partial \ln A_{gt}}
\end{aligned}$$

Hence the substitution effect from coal and shale dominates if $\varepsilon > \frac{1}{1-\alpha\beta}$ (and we shall concentrate on this case from here on). In other words we focus on parameter values such that aggregate pollution is reduced by increasing productivity on the clean extraction technology.

What about the effect on aggregate pollution of increasing productivity on the shale gas technology? We have:

$$\frac{\partial \ln P_t}{\partial \ln A_{st}} = \underbrace{(1-\alpha)\varepsilon \left(\begin{array}{c} -\frac{\xi_c \left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}}{\xi_c \left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon} + \xi_s \left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}} \frac{\partial \ln A_{Et}}{\partial \ln A_{st}} \\ + \frac{\xi_s \left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}}{\xi_c \left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon} + \xi_s \left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}} \left(1 - \frac{\partial \ln A_{Et}}{\partial \ln A_{st}}\right) \end{array} \right)}_{\text{substitution effect away from coal and green!}} + \underbrace{\frac{1-\alpha}{1-\alpha\beta}}_{\text{scale effect}} \frac{\partial \ln A_{Et}}{\partial \ln A_{gt}}$$

$$= (1-\alpha)\varepsilon \frac{\xi_s \left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}}{\xi_c \left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon} + \xi_s \left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}} - (1-\alpha)\left(\varepsilon - \frac{1}{1-\alpha\beta}\right) \frac{\partial \ln A_{Et}}{\partial \ln A_{gt}},$$

where:

$$\frac{\partial \ln A_{Et}}{\partial \ln A_{st}} = \frac{A_{st}^{-\varphi}}{A_{Et}^{-\varphi}} = \frac{A_{st}^{(1-\alpha)(\varepsilon-1)}}{A_{Et}^{(1-\alpha)(\varepsilon-1)}}.$$

Therefore:

$$\frac{\partial \ln P_t}{\partial \ln A_{st}} = (1-\alpha)\varepsilon \frac{\xi_s \left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}}{\xi_c \left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon} + \xi_s \left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}} - (1-\alpha)\left(\varepsilon - \frac{1}{1-\alpha\beta}\right) \frac{A_{st}^{(1-\alpha)(\varepsilon-1)}}{A_{Et}^{(1-\alpha)(\varepsilon-1)}}$$

which can be positive or negative.

To gain intuition on the problem, consider first the simple case where $A_{gt} = 0$. In this case we have:

$$\frac{\partial \ln P_t}{\partial \ln A_{st}} = (1-\alpha)\left(-\varepsilon \frac{\left(\xi_c A_{ct}^{1-\alpha} - \xi_s A_{st}^{1-\alpha}\right) A_{ct}^{(1-\alpha)(\varepsilon-1)}}{\xi_c A_{ct}^{(1-\alpha)\varepsilon} + \xi_s A_{st}^{(1-\alpha)\varepsilon}} + \frac{1}{1-\alpha\beta} \right) \frac{A_{st}^{(1-\alpha)(\varepsilon-1)}}{A_{ct}^{(1-\alpha)(\varepsilon-1)} + A_{st}^{(1-\alpha)(\varepsilon-1)}}$$

which shows very clearly that whether substitution away from coal and towards shale gas, reduces or increases aggregate pollution, hinges a lot upon whether $\xi_s A_{st}^{1-\alpha} > \xi_c A_{ct}^{1-\alpha}$ or not!!!

We can rewrite

$$\frac{\left(\xi_c A_{ct}^{1-\alpha} - \xi_s A_{st}^{1-\alpha}\right) A_{ct}^{(1-\alpha)(\varepsilon-1)}}{\xi_c A_{ct}^{(1-\alpha)\varepsilon} + \xi_s A_{st}^{(1-\alpha)\varepsilon}} = \frac{1 - \frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{1-\alpha}}{1 + \frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{(1-\alpha)\varepsilon}}$$

which is decreasing in ξ_s and A_{st} and increasing in ξ_c and A_c.

Now conditional upon an increase in A_{st} decreasing pollution, this effect may increase or decrease with A_{st}. In fact we can prove:

Proposition 1 *Consider a world without green technologies, then: (i) an increase in the productivity of shale extraction decreases pollution whenever:* $\varepsilon \frac{1 - \frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{1-\alpha}}{1 + \frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{(1-\alpha)\varepsilon}} > \frac{1}{1-\alpha\beta}$. *This condition requires that* $\frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{1-\alpha} < 1$ *and is more likely to hold if* ξ_s *and* A_{st} *are low, and*

ξ_c *and* A_{ct} *are high; (ii) conditional upon* $\varepsilon \frac{1-\frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{1-\alpha}}{1+\frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{(1-\alpha)\varepsilon}} > \frac{1}{1-\alpha\beta}$*, the marginal effect of an increase in* A_{st} *on pollution may increase or decrease in* A_{st}.

Now let us introduce green technologies. Motivated by the previous observation, it makes sense to write:

$$\frac{\partial \ln P_t}{\partial \ln A_{st}} = (1-\alpha)\,\varepsilon \frac{\xi_s\left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}}{\xi_c\left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}+\xi_s\left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}} - (1-\alpha)\left(\varepsilon - \frac{1}{1-\alpha\beta}\right)\frac{A_{st}^{(1-\alpha)(\varepsilon-1)}}{A_{Et}^{(1-\alpha)(\varepsilon-1)}}$$

or

$$\begin{aligned}
\frac{\partial \ln P_t}{\partial \ln A_{st}} &= (1-\alpha)\left(\varepsilon \frac{\xi_s\left(\frac{A_{st}}{A_{Et}}\right)^{1-\alpha} - \left(\xi_c\left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}+\xi_s\left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}\right)}{\xi_c\left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}+\xi_s\left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}} + \frac{1}{1-\alpha\beta}\right)\frac{A_{st}^{(1-\alpha)(\varepsilon-1)}}{A_{Et}^{(1-\alpha)(\varepsilon-1)}} \\
&= (1-\alpha)\left(-\varepsilon \frac{1+\frac{\xi_s}{\xi_c}\left(\left(\frac{A_{st}}{A_{ct}}\right)^{(1-\alpha)\varepsilon} - \left(\frac{A_{Et}}{A_{ct}}\right)^{(1-\alpha)\varepsilon}\left(\frac{A_{st}}{A_{Et}}\right)^{1-\alpha}\right)}{\left(1+\frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{(1-\alpha)\varepsilon}\right)} + \frac{1}{1-\alpha\beta}\right)\frac{A_{st}^{(1-\alpha)(\varepsilon-1)}}{A_{Et}^{(1-\alpha)(\varepsilon-1)}} \\
&= (1-\alpha)\left(-\varepsilon \frac{1-\frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{1-\alpha}\left(1+\frac{(A_{gt})^{-\varphi}}{(A_{ct})^{-\varphi}}\right)}{\left(1+\frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{(1-\alpha)\varepsilon}\right)} + \frac{1}{1-\alpha\beta}\right)\frac{A_{st}^{(1-\alpha)(\varepsilon-1)}}{A_{Et}^{(1-\alpha)(\varepsilon-1)}}
\end{aligned}$$

Therefore, with green technologies, the overall substitution effect is still to decrease pollution if and only if

$$\frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{1-\alpha}\left(1+\frac{(A_{gt})^{-\varphi}}{(A_{ct})^{-\varphi}}\right) < 1$$

This expression is very close to the expression without green technology (we just have a correction factor in $1+\frac{(A_{gt})^{-\varphi}}{(A_{ct})^{-\varphi}}$). Thus, the higher the level of green technologies the less likely it is that shale gas is good for the environment. This establishes:

Proposition 2 *An increase in the productivity of the shale gas technology decreases pollution whenever* $\varepsilon \frac{1-\frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{1-\alpha}\left(1+\frac{(A_{gt})^{-\varphi}}{(A_{ct})^{-\varphi}}\right)}{1+\frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{(1-\alpha)\varepsilon}} > \frac{1}{1-\alpha\beta}$*. This condition requires that* $\frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{1-\alpha}\left(1+\frac{(A_{gt})^{-\varphi}}{(A_{ct})^{-\varphi}}\right) < 1$ *and is more likely to hold if* ξ_s, A_{st}, A_{gt} *are low, and/or if* ξ_c *and* A_{ct} *are high. Moreover, conditional upon* $\varepsilon \frac{1-\frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{1-\alpha}\left(1+\frac{(A_{gt})^{-\varphi}}{(A_{ct})^{-\varphi}}\right)}{1+\frac{\xi_s}{\xi_c}\left(\frac{A_{st}}{A_{ct}}\right)^{(1-\alpha)\varepsilon}} > \frac{1}{1-\alpha\beta}$*, the marginal effect of an increase in* A_{st} *on pollution can be increasing or decreasing in* A_{st}.

So far we have identified situations in which an increase in the productivity of shale is good for the environment: this is typically more likely to be the case when green technologies and shale technologies are not very advanced. One cannot, however, conclude that as shale becomes more developed the impact on pollution necessarily becomes less negative (as conditional upon being negative, the magnitude of the impact of an increased A_{st} on aggregate pollution may be increasing in A_{st}). This leads us to analyze the dynamics of energy transition, after endogeneizing technological progress on the three extraction technologies.

4 Dynamics

There are, of course, many different ways one can introduce innovation in this framework. In particular we can decide to have: an exogenous or endogenous amount of innovation, innovation in the three technologies or only in A_{gt} and A_{st}, innovation in A_{ft}, linear or concave innovation technologies,...We choose to start with the very simple case where: $A_{ft} = cte$. Moreover, as in AABH, we assume a fixed resource of innovation, namely a mass one population of scientists. we also assume the existence of an upper bound on the $A_i's$ which is reached for coal: $A_{ct} = \overline{A}$. This latter hypothesis is quite crucial of course (and a significant departure from AABH). We will have to consider alternatives but to some extent, it is not fully unrealistic: there is little progress in coal extraction per se...

We assume the same innovation structure as in AABH, with intermediate input innovators enjoying monopoly power for one period only. Intermediate input producers choose their sector of innovation (c, s, g) but they cannot choose which intermediate they target.

4.1 Solving the dynamic model

We then get that what drives the allocation of innovation between the two sectors is the ratio of expected profits, with expected profits satisfying

$$\Pi_{jt} = \eta_j \left(1+\gamma\right)\left(1-\alpha\right) \frac{\alpha^{\frac{1+\alpha}{1-\alpha}}}{\psi^{\frac{\alpha}{1-\alpha}}} p_{it}^{\frac{1}{1-\alpha}} L_{it} A_{i(t-1)}.$$

Therefore

$$\frac{\Pi_{st}}{\Pi_{gt}} = \frac{\eta_s}{\eta_g} \frac{p_{st}^{\frac{1}{1-\alpha}} L_{st} A_{s(t-1)}}{p_{gt}^{\frac{1}{1-\alpha}} L_{gt} A_{g(t-1)}} = \frac{\eta_s}{\eta_g} \frac{\left(1+\gamma\eta_s s_{st}\right)^{-\varphi-1}}{\left(1+\gamma\eta_g s_{gt}\right)^{-\varphi-1}} \left(\frac{A_{s(t-1)}}{A_{g(t-1)}}\right)^{-\varphi}$$

This is all exactly as in AABH. We then have that if A_{s0} is large enough relative to A_{g0}. In laissez-faire innovation will happen first in shale (until we reach the bound), and then later only will it be in coal.

Therefore we get that:

$$\begin{aligned} A_{st} &= \left(1+\gamma\eta_s\right)^t A_{s0} \text{ for } t \leq T_1 = \frac{1}{1+\gamma\eta_s} \ln \frac{\overline{A}}{A_{s0}} \\ A_{st} &= \overline{A} \text{ for } t > T_1 \end{aligned}$$

and

$$\begin{aligned} A_{gt} &= A_{g0} \text{ for } t \leq T_1 \\ A_{gt} &= \left(1+\gamma\eta_g\right)^{t-T_1} A_{g0} \text{ for } T_1 \leq t \leq T_2 + T_1 \text{ with } T_2 = \frac{1}{1+\gamma\eta_g} \ln \frac{\overline{A}}{A_{g0}} \end{aligned}$$

(we consider parameters such that T_1 and T_2 are integers - with obviously no loss of generality there).

This yields the equilibrium level of aggregate pollution:

$$P_t = \chi_2 A_f^{\frac{\alpha}{1-\alpha\beta}} \left(\xi_c \left(\frac{\overline{A}_c}{A_{Et}} \right)^{(1-\alpha)\varepsilon} + \xi_s \left(\frac{A_{st}}{A_{Et}} \right)^{(1-\alpha)\varepsilon} \right) A_{Et}^{\frac{1-\alpha}{1-\alpha\beta}} = \left(\xi_c \left(\frac{\overline{A}_c}{A_{Et}} \right)^{(1-\alpha)\varepsilon} + \xi_s \left(\frac{A_{st}}{A_{Et}} \right)^{(1-\alpha)\varepsilon} \right) A_{Et}^{\frac{1-\alpha}{1-\alpha\beta}}$$

after normalizing parameters such that $\chi_2 A_f^{\frac{\alpha}{1-\alpha\beta}} = 1$.

4.2 Should we ban research on shale gas extraction?

Now we consider the following policy experiment at time $t = 0$, which effectively bans research in gas. Under this policy experiment (for which all variables will have $\widetilde{\ }$) we then get that $\widetilde{A}_{st} = A_{s0}$ and

$$\widetilde{A}_{gt} = (1+\gamma\eta_g)^t A_{g0} \text{ for } 1 \le t \le T_2 = \frac{1}{1+\gamma\eta_g} \ln \frac{\overline{A}}{A_{g0}}$$

$$\widetilde{A}_{gt} = \overline{A} \text{ for } t \ge T_2$$

Now the question is how does P_t compares with $\widetilde{P}_t$? Assume that $\eta_g = \eta_c = \eta$ for simplicity. Assume also that $A_{g0} < A_{s0}$, so that $T_2 > T_1$.

4.2.1 Initial impact

Consider that $\gamma\eta$ is small enough that we can use linear approximations at the first period, then we get

$$\ln P_1 = \ln P_0 + \frac{\partial \ln P}{\partial \ln A_s} \gamma\eta$$

while

$$\ln \widetilde{P}_1 = \ln P_0 + \frac{\partial \ln P}{\partial \ln A_g} \gamma\eta$$

therefore $\widetilde{P}_1 > P_1$, if and only if

$$\frac{\partial \ln P}{\partial \ln A_s} < \frac{\partial \ln P}{\partial \ln A_g}.$$

Use the expressions from above to get

$$\begin{aligned} & \frac{\partial \ln P}{\partial \ln A_s} - \frac{\partial \ln P}{\partial \ln A_g} \\ = & (1-\alpha) \left(-\varepsilon \frac{1 - \frac{\xi_s}{\xi_c} \left(\frac{A_{st}}{A_{ct}} \right)^{1-\alpha} \left(1 + \frac{(A_{gt})^{-\varphi}}{(A_{ct})^{-\varphi}} \right)}{\left(1 + \frac{\xi_s}{\xi_c} \left(\frac{A_{st}}{A_{ct}} \right)^{(1-\alpha)\varepsilon} \right)} + \frac{1}{1-\alpha\beta} \right) \frac{A_{st}^{-\varphi}}{A_{Et}^{-\varphi}} + (1-\alpha) \left(\varepsilon - \frac{1}{1-\alpha\beta} \right) \frac{A_{gt}^{-\varphi}}{A_{Et}^{-\varphi}} \\ = & -(1-\alpha) \frac{A_{s0}^{-\varphi}}{A_{E0}^{-\varphi}} \left(\left(\varepsilon - \frac{1}{1-\alpha\beta} \right) \left(1 - \frac{A_{g0}^{-\varphi}}{A_{s0}^{-\varphi}} \right) - \varepsilon \frac{\xi_s}{\xi_c} \frac{\left(\frac{A_{s0}}{A_c} \right)^{1-\alpha} \left(1 + 2\frac{(A_{g0})^{-\varphi}}{(A_c)^{-\varphi}} \right)}{1 + \frac{\xi_s}{\xi_c} \left(\frac{A_{s0}}{A_c} \right)^{(1-\alpha)\varepsilon}} \right) \end{aligned}$$

The first term represents the gain in pollution reduction if $\xi_s = 0$: in this case it is obviously a bad idea to ban research on shale gas if productivity is initially higher on shale gas than on clean energy: allowing research and production on shale gas will help to steal market share from coal. This is all the more true the bigger is the initial gap in A_{s0}/A_{g0} (and $A_{s0} > A_{g0}$ is a necessary condition here to get a negative sign!).

The second term represents the fact that in reality shale does generate emissions... That term is greater when A_{g0} is large and when $\frac{\xi_s}{\xi_c}$ is large. The impact of A_{s0} and A_{c0} is ambiguous though... Now we can introduce the following notations to make things simpler

$$a_{it} = \left(\frac{A_{it}}{\overline{A}}\right)^{1-\alpha}$$

and we denote $\xi = \xi_s/\xi_c$. Therefore we have

$$\frac{\partial \ln P}{\partial \ln A_s} - \frac{\partial \ln P}{\partial \ln A_g} = -\varepsilon(1-\alpha)\left(\frac{a_{g0}}{a_{E0}}\right)^{\varepsilon-1}\left(\left(1-\frac{1}{(1-\alpha\beta)\,\varepsilon}\right)\left(1-\left(\frac{a_{g0}}{a_{s0}}\right)^{\varepsilon-1}\right) - \xi\frac{a_{s0}\left(1+2a_{g0}^{\varepsilon-1}\right)}{1+\xi a_{s0}^{\varepsilon}}\right)$$

Whether this is likely to be negative depends on how large the ratio

$$f(a_s, a_g) = \frac{1-\left(\frac{a_{g0}}{a_{s0}}\right)^{\varepsilon-1}}{a_{s0}\left(1+2a_{g0}^{\varepsilon-1}\right)}\left(1+\xi a_{s0}^{\varepsilon}\right)$$

is. This ratio is clearly decreasing in a_g and by looking at the function $\frac{1+\xi a_{s0}^{\varepsilon}}{a_{s0}}$ we could figure out whether or not it is unambiguous in a_g.

Lemma 2 *$\widetilde{P}_1 > P_1$ if $\left(1-\frac{1}{(1-\alpha\beta)\varepsilon}\right)\left(1-\left(\frac{a_{g0}}{a_{s0}}\right)^{\varepsilon-1}\right) > \xi\frac{a_{s0}\left(1+2a_{g0}^{\varepsilon-1}\right)}{1+\xi a_{s0}^{\varepsilon}}$: in this case banning shale gas initially increases pollution.*

4.3 Long-term impact

Consider now what happens in the longer term, namely at $T = T_2$. We have:

$$P_{T_2} = \left(\xi_c\left(\frac{\overline{A}}{A_{E T_2}}\right)^{(1-\alpha)\varepsilon} + \xi_s\left(\frac{\overline{A}}{A_{E T_2}}\right)^{(1-\alpha)\varepsilon}\right) A_{Et}^{\frac{1-\alpha}{1-\alpha\beta}}$$

where

$$\begin{aligned} {A_{E T_2}}^{-\varphi} &= 2\overline{A}^{-\varphi} + \left(A_{g0}\,(1+\gamma\eta)^{T_2-T_1}\right)^{-\varphi} \\ &= 2\overline{A}^{-\varphi} + A_{s0}^{-\varphi} \end{aligned}$$

Now remember that at time T_2, the green technology catches up with the frontier. Therefore at that time:

$$A_{E T_2} = \widetilde{A}_{E T_2}.$$

This in turn implies that

$$
\begin{aligned}
\widetilde{P}_{T_2} &= \left(\xi_c\left(\frac{\overline{A}}{\widetilde{A}_{E_{T_2}}}\right)^{(1-\alpha)\varepsilon} + \xi_s\left(\frac{A_{s0}}{\widetilde{A}_{E_{T_2}}}\right)^{(1-\alpha)\varepsilon}\right)\widetilde{A}_{E_{T_2}}^{\frac{1-\alpha}{1-\alpha\beta}} \\
&= \left(\xi_c\left(\frac{\overline{A}}{A_{E_{T_2}}}\right)^{(1-\alpha)\varepsilon} + \xi_s\left(\frac{A_{s0}}{A_{E_{T_2}}}\right)^{(1-\alpha)\varepsilon}\right)A_{E_{T_2}}^{\frac{1-\alpha}{1-\alpha\beta}}
\end{aligned}
$$

It then immediately follows that at $T = T_2$, we have $P_{T_2} > \widetilde{P}_{T_2}$: banning shale gas always decreases pollution at $T = T_2$! Question remains open as to what happens after time T_2.

For $t \in (T_2, T_1 + T_2]$, we have:

$$
\widetilde{P}_t \equiv \widetilde{P}_{T_2} = \left(\xi_c\left(\frac{\overline{A}}{\widetilde{A}_{E_{T_2}}}\right)^{(1-\alpha)\varepsilon} + \xi_s\left(\frac{A_{s0}}{\widetilde{A}_{E_{T_2}}}\right)^{(1-\alpha)\varepsilon}\right)\widetilde{A}_{E_{T_2}}^{\frac{1-\alpha}{1-\alpha\beta}}
$$

whereas

$$
P_t = \left(\xi_c\left(\frac{\overline{A}}{A_{Et}}\right)^{(1-\alpha)\varepsilon} + \xi_s\left(\frac{\overline{A}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}\right)A_{Et}^{\frac{1-\alpha}{1-\alpha\beta}}
$$

with

$$
\begin{aligned}
A_{Et}^{-\varphi} &= 2\overline{A}^{-\varphi} + \left(A_{g0}\left(1+\gamma\eta\right)^{t-T_1}\right)^{-\varphi} \\
&= 2\overline{A}^{-\varphi} + \left(A_{s0}\left(1+\gamma\eta\right)^{t-T_2}\right)^{-\varphi}
\end{aligned}
$$

so that

$$
A_{Et} > A_{E_{T_2}}.
$$

The comparison between $\widetilde{P}_t$ and P_t is thus a priori ambiguous: on the one hand $A_{Et}^{\frac{1-\alpha}{1-\alpha\beta}} > A_{E_{T_2}}^{\frac{1-\alpha}{1-\alpha\beta}}$ pushes towards $P_t > \widetilde{P}_t$: this reflects the fact that the economy operates on a larger scale when shale gas extraction is allowed; on the other hand $\left(\frac{\overline{A}}{A_{Et}}\right)^{(1-\alpha)\varepsilon} < \left(\frac{\overline{A}}{A_{E_{T_2}}}\right)^{(1-\alpha)\varepsilon}$ which pushes towards $P_t < \widetilde{P}_t$: this reflects the fact that pollution is (partly) mitigated by the substitution of coal for gas when shale gas extraction is allowed. [1] In fact we can show that for $\xi_s = 0$, i.e when shale gas extraction is as clean as the clean extraction technology, the latter effect dominates:

$$
\begin{aligned}
\widetilde{P}_t &= \xi_c\overline{A}^{(1-\alpha)\varepsilon}(2\overline{A}^{-\varphi} + A_{s0}^{-\varphi})^{(1-\alpha)(\frac{1}{1-\alpha\beta}-\varepsilon)(-1/\varphi)} \\
&> \\
P_t &= \xi_c\overline{A}^{(1-\alpha)\varepsilon}(2\overline{A}^{-\varphi} + [A_{s0}(1+\gamma\eta)^{t-T_2}]^{-\varphi})^{(1-\alpha)(\frac{1}{1-\alpha\beta}-\varepsilon)(-1/\varphi)}
\end{aligned}
$$

---

[1] Note however that $\frac{\overline{A}}{A_{Et}} > \frac{A_{s0}}{A_{E_{T_2}}}$, which results from the fact that

$$
\overline{A} > A_{s0}(1+\gamma\eta)^{t-T_2} > A_{s0}.
$$

This implies that the latter effect is strongest when $\xi_s = 0$, i.e when shale gas extraction is fully clean, which is not surprising.

since we assumed

$$\frac{1}{1-\alpha\beta} < \varepsilon.$$

Now what should we expect when $\xi_s = \xi_c$ and $t \in (T_2, T_1 + T_2]$? We know that $P_{T_2} > \widetilde{P}_{T_2}$ in that case (whereas $P_{T_2} > \widetilde{P}_{T_2}$ if $\xi_s = 0$). By continuity we still have $P_t > \widetilde{P}_t$ for $t > T_2$ and t close to T_2. Now to see what happens in that case for t sufficiently close to (and still less than) $T_1 + T_2$, we need to look at the case where $\xi_s = \xi_c = \xi$ and $t \geq T_1 + T_2$.

In that latter case we have:

$$P_t = 2\xi\left(\frac{1}{3^{-1/\varphi}}\right)^{(1-\alpha)\varepsilon})(3^{-1/\varphi}\overline{A})^{\frac{1-\alpha}{1-\alpha\beta}}$$

whereas

$$\widetilde{P}_t \equiv \xi\left(\left(\frac{\overline{A}}{(2\overline{A}^{-\varphi}+A_{s0}^{-\varphi})^{-1/\varphi}}\right)^{(1-\alpha)\varepsilon} + \left(\frac{A_{s0}}{(2\overline{A}^{-\varphi}+A_{s0}^{-\varphi})^{-1/\varphi}}\right)^{(1-\alpha)\varepsilon}\right)(2\overline{A}^{-\varphi}+A_{s0}^{-\varphi})^{-1/\varphi})^{\frac{1-\alpha}{1-\alpha\beta}}.$$

For $A_{s0} = 0$, the inequality

$$\begin{aligned} \widetilde{P}_t &= \xi\left(\frac{1}{2^{-1/\varphi}}\right)^{(1-\alpha)\varepsilon})(2^{-1/\varphi}\overline{A})^{\frac{1-\alpha}{1-\alpha\beta}} \\ &< \\ P_t &= 2\xi\left(\frac{1}{3^{-1/\varphi}}\right)^{(1-\alpha)\varepsilon})(3^{-1/\varphi}\overline{A})^{\frac{1-\alpha}{1-\alpha\beta}} \end{aligned}$$

boils down to

$$\frac{1}{2} < \frac{2}{3}(\frac{3}{2})^{\frac{\alpha\beta}{(\varepsilon-1)(1-\alpha\beta)}},$$

which itself follows from the fact that $2/3 > 1/2$ and $\frac{\alpha\beta}{(\varepsilon-1)(1-\alpha\beta)} > 0$ so that $(\frac{3}{2})^{\frac{\alpha\beta}{(\varepsilon-1)(1-\alpha\beta)}} > 1$.

For $A_{s0} = \overline{A}$, we simply have

$$\widetilde{P}_t = P_t,$$

so that by monotonicity

$$\widetilde{P}_t < P_t$$

when $A_{s0} \in (0, \overline{A})$ and $\xi_s = \xi_c = \xi$ and $t \geq T_1 + T_2$. Thus by continuity, this remains true for $t < T_1 + T_2$ but t close to $T_1 + T_2$.

5 Appendix: Proof of Lemma 1

We have in equilibrium:

$$w_t = (1-\alpha)\left(\frac{\alpha^2}{\psi}\right)^{\frac{\alpha}{1-\alpha}} p_{it}^{\frac{1}{1-\alpha}} A_{it}$$

for all i's, which gives a relationship between all p_i's and all A_i's, such that (for instance):

$$\frac{p_{ct}}{p_{st}} = \left(\frac{A_{ct}}{A_{st}}\right)^{-(1-\alpha)}$$

Further:

$$p_{Et}^{1-\varepsilon} = p_{ct}^{1-\varepsilon} + p_{gt}^{1-\varepsilon} + p_{st}^{1-\varepsilon}.$$

which translates into

$$\frac{p_{ct}}{p_{Et}} = \left(\frac{A_{ct}}{A_{Et}}\right)^{-(1-\alpha)}$$

with

$$A_{Et} = \left(A_{gt}^{-\varphi} + A_{st}^{-\varphi} + A_{ct}^{-\varphi}\right)^{-\frac{1}{\varphi}}$$

with $\varphi = (1-\alpha)(1-\varepsilon)$.

Next, demand for each energy input implies

$$\begin{aligned} p_{it} E_{i,t}^{\frac{1}{\varepsilon}} &= p_{Et} E_t^{\frac{1}{\varepsilon}} \\ \left(\frac{\alpha^2}{\psi}\right)^{\frac{\alpha}{1-\alpha}} p_{it}^{\frac{\alpha}{1-\alpha}+\varepsilon} L_{it} A_{it} &= p_{Et}^{\varepsilon} E_t \end{aligned}$$

Hence:

$$\frac{L_{ct}}{L_{st}} = \left(\frac{A_{ct}}{A_{st}}\right)^{-\varphi}$$

Let

$$L_{Et} = L_{ct} + L_{st} + L_{gt}$$

denote the aggregate labor demand. We have:

$$\frac{L_{ct}}{L_{Et}} = \left(\frac{A_{ct}}{A_{Et}}\right)^{-\varphi}$$

We can then write

$$E_t = \left(\frac{\alpha^2}{\psi}\right)^{\frac{\alpha}{1-\alpha}} p_{Et}^{\frac{\alpha}{1-\alpha}} L_{Et} A_{Et},$$

which implies that

$$\frac{Y_{it}}{E_t} = \left(\frac{A_{it}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}$$

Moreover, we have

$$w_t = (1-\alpha)\left(\frac{\alpha^2}{\psi}\right)^{\frac{\alpha}{1-\alpha}} p_{Et}^{\frac{1}{1-\alpha}} A_{Et},$$

which, together with

$$1 = \frac{1}{A_{ft}}\left(\frac{w}{1-\beta}\right)^{1-\beta}\left(\frac{p_{Et}}{\beta}\right)^{\beta},$$

implies:

$$p_{Et} = \left(\frac{\beta^{\beta}(1-\beta)^{1-\beta}}{(1-\alpha)^{1-\beta}}\right)^{\frac{1-\alpha}{1-\beta\alpha}} \left(\frac{\psi}{\alpha^2}\right)^{\frac{\alpha(1-\beta)}{1-\alpha\beta}} \left(\frac{A_{ft}}{A_{Et}^{1-\beta}}\right)^{\frac{1-\alpha}{1-\beta\alpha}}.$$

Now using the first order conditions for profit maximization in the final good sector, we obtain:

$$\frac{w_t L_{ft}}{\left(\frac{\alpha^2}{\psi}^{\frac{\alpha}{1-\alpha}}\right) p_{Et}^{\frac{1}{1-\alpha}} L_{Et} A_{Et}} = \frac{1-\beta}{\beta}$$

hence

$$\frac{L_{ft}}{L_{Et}} = \frac{1-\beta}{\beta(1-\alpha)}.$$

This in turn implies that L_{ft} and L_{Et} remain constant over time. In particular if $L_t = 1$, we have

$$L_{ft} = \frac{1-\beta}{1-\beta\alpha} \text{ and } L_{Et} = \frac{\beta(1-\alpha)}{1-\beta\alpha}$$

Therefore, we can express final output as

$$Y_t = \frac{1}{\beta} p_{Et} E_t = \frac{1}{\beta}\left(\frac{\alpha^2}{\psi}\right)^{\frac{\alpha}{1-\alpha}} p_{Et}^{\frac{1}{1-\alpha}} L_{Et} A_{Et} = \chi_1 \left(A_{ft} A_{Et}^{\beta(1-\alpha)}\right)^{\frac{1}{1-\beta\alpha}}$$

where $\chi_1 = \frac{\left((1-\alpha)^{\beta(1-\alpha)}\left(\frac{\alpha^2}{\psi}\right)^{\alpha\beta}\left(\beta^\beta(1-\beta)^{1-\beta}\right)\right)^{\frac{1}{1-\beta\alpha}}}{1-\beta\alpha}$.

The equilibrium aggregate production of energy E_t is then given by

$$E_t = \left(\frac{Y_t}{A_{ft} L_{ft}^{1-\beta}}\right)^{\frac{1}{\beta}} = \chi_2 \left(A_{ft}^{\alpha} A_{Et}^{1-\alpha}\right)^{\frac{1}{1-\alpha\beta}}$$

where

$$\chi_2 = \left(\left(\frac{1-\beta\alpha}{1-\beta}\right)^{1-\beta}\chi_1\right)^{\frac{1}{\beta}} = \frac{\left((1-\alpha)^{(1-\alpha)}\left(\frac{\alpha^2}{\psi}\right)^{\alpha}\beta(1-\beta)^{(1-\beta)\alpha}\right)^{\frac{1}{1-\beta\alpha}}}{(1-\beta\alpha)}.$$

Aggregate emissions are then given by:

$$P_t = \xi_c Y_{ct} + \xi_s Y_{st} = \left(\xi_c\left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon} + \xi_s\left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}\right) E_t$$

or equivalently

$$P_t = \chi_2 A_{ft}^{\frac{\alpha}{1-\alpha\beta}}\left(\xi_c\left(\frac{A_{ct}}{A_{Et}}\right)^{(1-\alpha)\varepsilon} + \xi_s\left(\frac{A_{st}}{A_{Et}}\right)^{(1-\alpha)\varepsilon}\right) A_{Et}^{\frac{1-\alpha}{1-\alpha\beta}}.$$

This establishes the Lemma.

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Cc: hemous@fas.harvard.edu forwards to gmail.com
Subject: Re: iteration

Thanks very much guys.

This is very interesting, but way too algebra intensive.

Let me see if I understand the basic point it's most interesting and novel here.

1. In a static or exogenous technology world, shale gas will reduce pollution. (Countering this is the Jevons effect I mentioned in the previous e-mail, that with cheaper shale gas the overall energy consumption increases, but I guess we care about the case in which this effect is dominated).

2. In endogenous technology, dynamic world, cheaper shale gas will discourage Green innovations, and can thus be bad.

Is this it? Do we care about anything else in the document? If so, there must be a way of getting this result with much less algebra, no?

In addition, I'm not sure whether I like the formulation with the "extraction input" being used as an aggregator with intermediates etc. Almost equivalent, but much easier to motivate, is to assume that some of the final good has to be used for extraction. This only changes the algebra marginally, but is much more palatable. But in any case, this still keeps the algebra quite complicated, which I think is not optimal.

Best

Daron

On 5/30/2014 4:38 AM, Aghion, Philippe wrote:
> Cher David,
> Ce fut un plaisir de travailler avec vous hier. Voici l'iteration qui integre nos progres. I will start looking at the comparative costs of reducing pollution by time T by the amount X respectively when we authorize shale gas extraction and when we ban it. Then we can look at North-South and the conditions under which allowing for shale gas helps counteract the pollution heaven effect.
> A tres bientot,
> Amities, philippe