Lignes Géodésiques et Segmentation d'images

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Some joint works with G. Peyré, S. Bougleux,
and PhD students R. Ardon, S. Bonneau and F. Benmansour.

Collège de France, 16 Janvier 2009

Overview

- Minimal Paths, Fast Marching and Front Propagation
- Anistropic Fast Marching and Perceptual Grouping
- Anistropic Fast Marching and Vessel Segmentation
- Closed Contour segmentation as a set of minimal paths in 2D
- Geodesic meshing for 3D surface segmentation
- Fast Marching on surfaces: geodesic lines and Remeshing – Isotropic, Adaptive, Anisotropic

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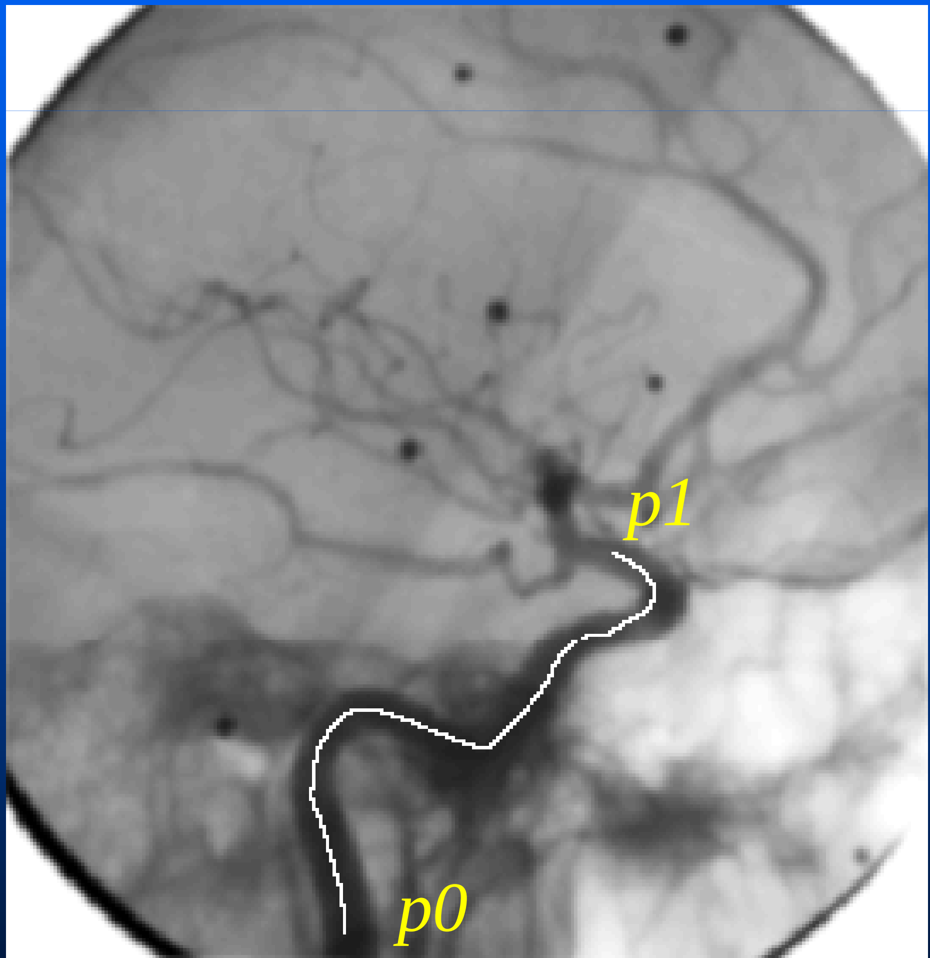
Paths of minimal energy



Looking for a path along which a feature Potential P(x,y) is minimal

example: a vessel
dark structure
P =gray level

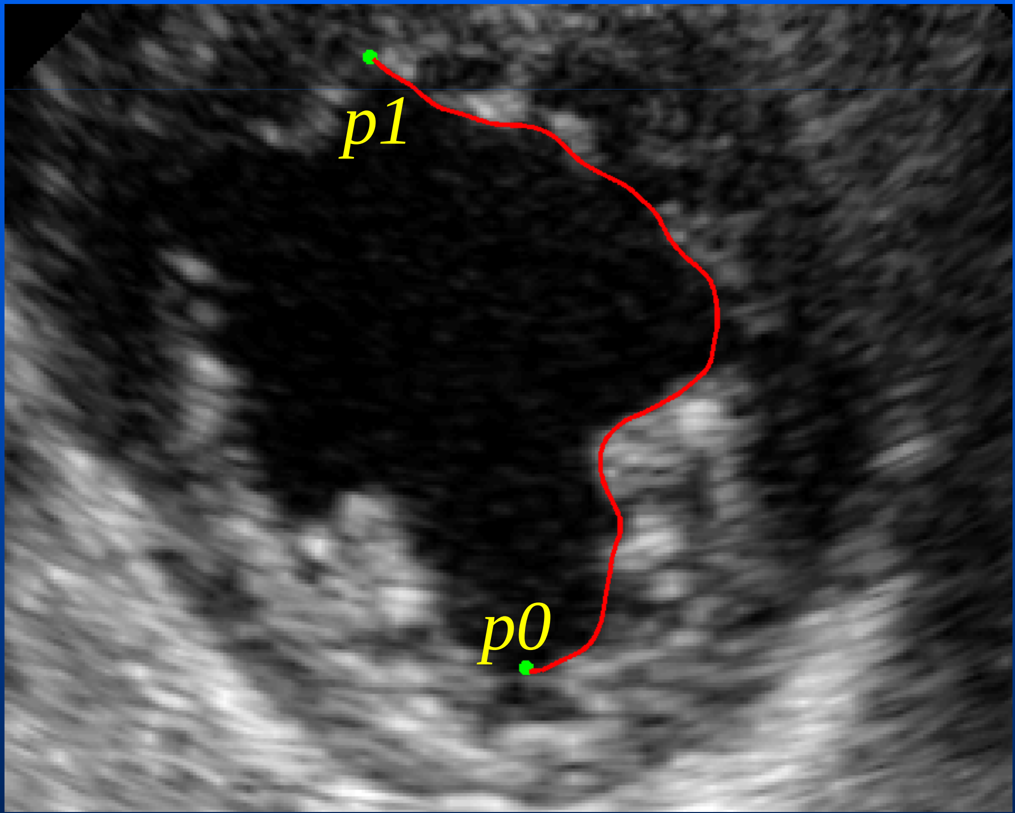
Input : Start point *p0=(x0,y0)*

End point *p1 =(x,y)*

Image

Output: Minimal Path

Paths of minimal energy



Looking for a path along which a feature Potential P(x,y) is minimal

example: cardiac ventricle contour
P =gradient based

Input : Start point *p0=(x0,y0)*

End point *p1 =(x,y)*

Image

Output: Minimal Path

Minimal Paths: Eikonal Equation

$$E(C)=\int_0^L P(C(s))ds$$

Potential P>0 takes lower values near interesting features :
on contours, dark structures, ...

STEP 1 : search for the surface of minimal action *U* of *p0* as the minimal energy integrated along a path between start point *p0* and any point *p* in the image

$$\text{Start point } C(0)=p0;$$

$$U_{p0}(p)=\inf_{C(0)=p0;C(L)=p} E(C)=\inf_{C(0)=p0;C(L)=p}\int_0^L P(C(s))ds$$

STEP 2: Back-propagation from the end point *p1* to the start point *p0*:

Simple Gradient Descent along U_{p0}

Minimal Paths: Eikonal Equation

STEP 1 : minimal action *U* of *p0* as the minimal energy integrated along a path between start point *p0* and any point *p* in the image

$$\text{Start point } C(0) = p0;$$

$$U_{p0}(p) = \inf_{C(0)=p0;C(L)=p} E(C) = \inf_{C(0)=p0;C(L)=p} \int_0^L P(C(s))ds$$

Solution of Eikonal equation:

$$\left\|\nabla U_{p0}(x)\right\| = P(x) \text{ and } U_{p0}(p0) = 0$$

Example P=1, U Euclidean distance to p0

Minimal Paths: Eikonal Equation

$$E(C) = \int_0^L P(C(s))ds$$

STEP 2: Back-propagation from the end point *p2* to the start point *p1*:

Simple Gradient Descent along U_{p1}

$$\frac{dC}{ds}(s) = -\nabla U_{p_1}(C(s)) \text{ with } C(0) = p_2.$$

Theorem 1: (Euler Lagrange of E) Any curve C which is a local minimum of energy E is a solution of

$$\nabla \mathcal{P}(C) \cdot \vec{n} = \mathcal{P}(C)\kappa$$

Definition 2 (Critical curves) *We say that C is a critical curve of the energy E if C is a solution of the Euler-Lagrange equation (5).*

Minimal Paths: Eikonal Equation

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$$\nabla \mathcal{P}(C) \cdot \vec{n} = \mathcal{P}(C)\kappa$$

Definition 3 (field lines) *We will say that $\mathcal{C}$ is a field line of ∇U_{p_1} if it is the solution of the ordinary differential equation*

$$\begin{cases} \frac{d\mathcal{C}(t)}{dt} = -\nabla U_{p_1}(\mathcal{C}(t)) \\ \mathcal{C}(0) = \mathbf{p}. \end{cases} \quad (11)$$

where $\mathbf{p}$ is a point of the image domain.

And we have the following property:

Theorem 4 (Field Lines and Euler-Lagrange equation) *If $U_{\mathbf{p}_1}$ is solution to the problem $\|\nabla U_{\mathbf{p}_1}\| = \mathcal{P}$ with $U_{\mathbf{p}_1}(\mathbf{p}_1) = 0$, every line field of $\nabla U_{\mathbf{p}_1}$ is a critical curve of the geodesic energy E.*

FAST MARCHING in 2D:

very efficient algorithm O(NlogN) for Eikonal Equation

Introduced by Sethian / Tsistsiklis

Numerical approximation of U(xij) as the solution to the discretized problem with upwind finite difference scheme

$$\|\nabla U\| = \tilde{P}$$

$$\max\left(u - U(x_{i-1,j}), u - U(x_{i+1,j}), 0\right)^2 + \max\left(u - U(x_{i,j-1}), u - U(x_{i,j+1}), 0\right)^2 = h^2 \tilde{P}(x_{i,j})^2$$

This 2nd order equation induces that :
action U at {i,j} depends only of the neighbors that have lower actions.

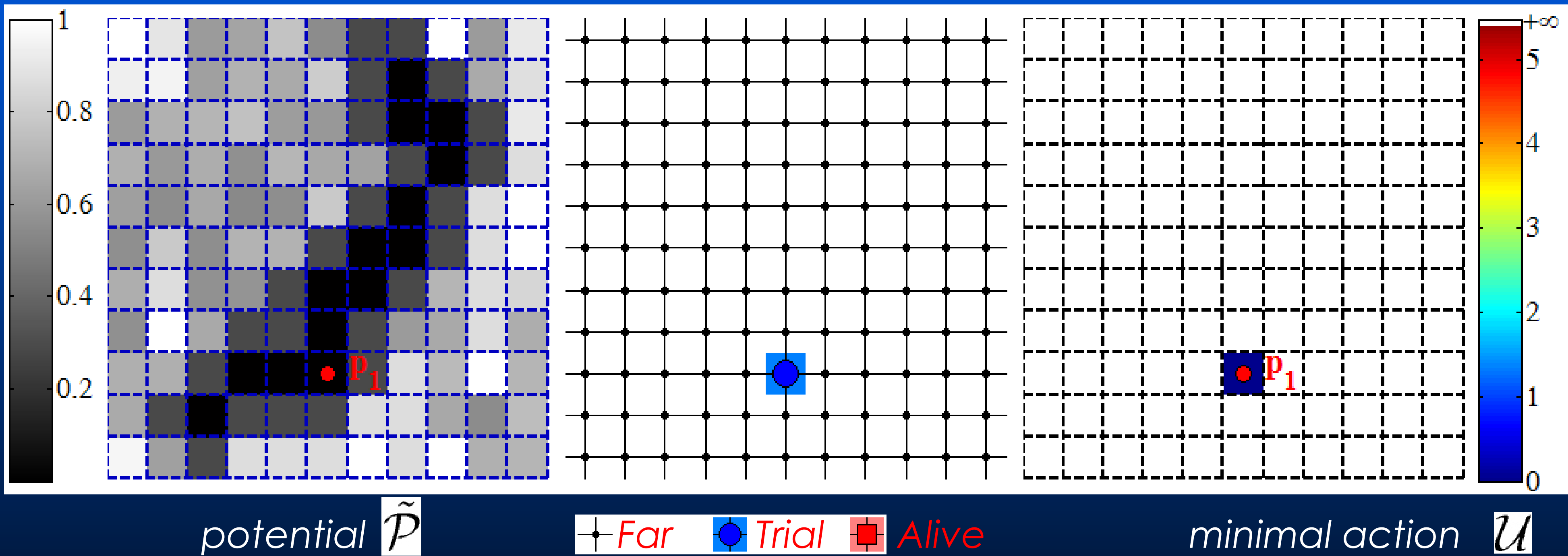
Fast marching introduces order in the selection of the grid points for solving this numerical scheme.

Starting from the initial point p0 with U = 0,
the action computed at each point visited can only grow.

Level sets of U can be seen as a Front propagation outwards.

Fast Marching Algorithm

Initialization

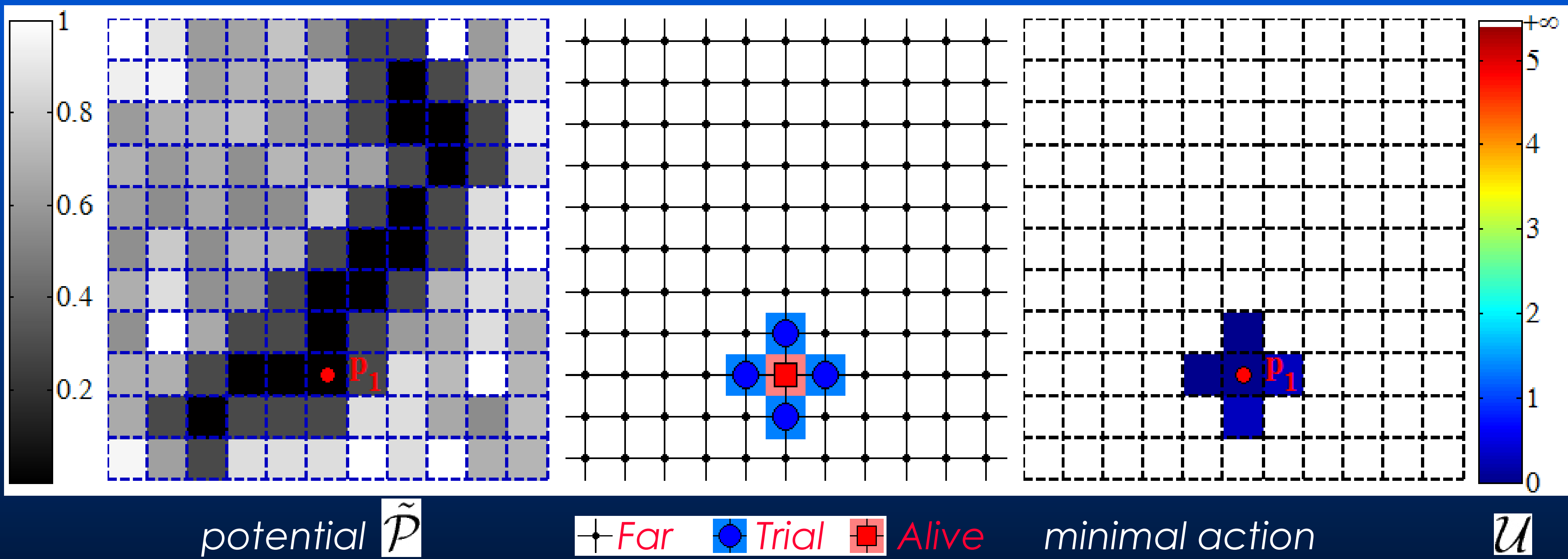


J. A. Sethian
A fast marching level set method for monotonically advancing fronts.
P.N.A.S., **93**:1591-1595, 1996.

Fast Marching Algorithm

Itération #1

- Find point $\mathbf{x}_{min}$ (*Trial* point with smallest value of $\mathcal{U}$).
- $\mathbf{x}_{min}$ becomes *Alive*.
- For each of 4 neighbors $\mathbf{x}$ of point $\mathbf{x}_{min}$:
  - If $\mathbf{x}$ is not *Alive*,
    - Estimate $\mathcal{U}(\mathbf{x})$ with upwind scheme.
    - $\mathbf{x}$ becomes *Trial*.



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Fast Marching Algorithm

Itération #2

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- $\mathbf{x}_{min}$ becomes *Alive*.
- For each of 4 neighbors $\mathbf{x}$ of point $\mathbf{x}_{min}$:

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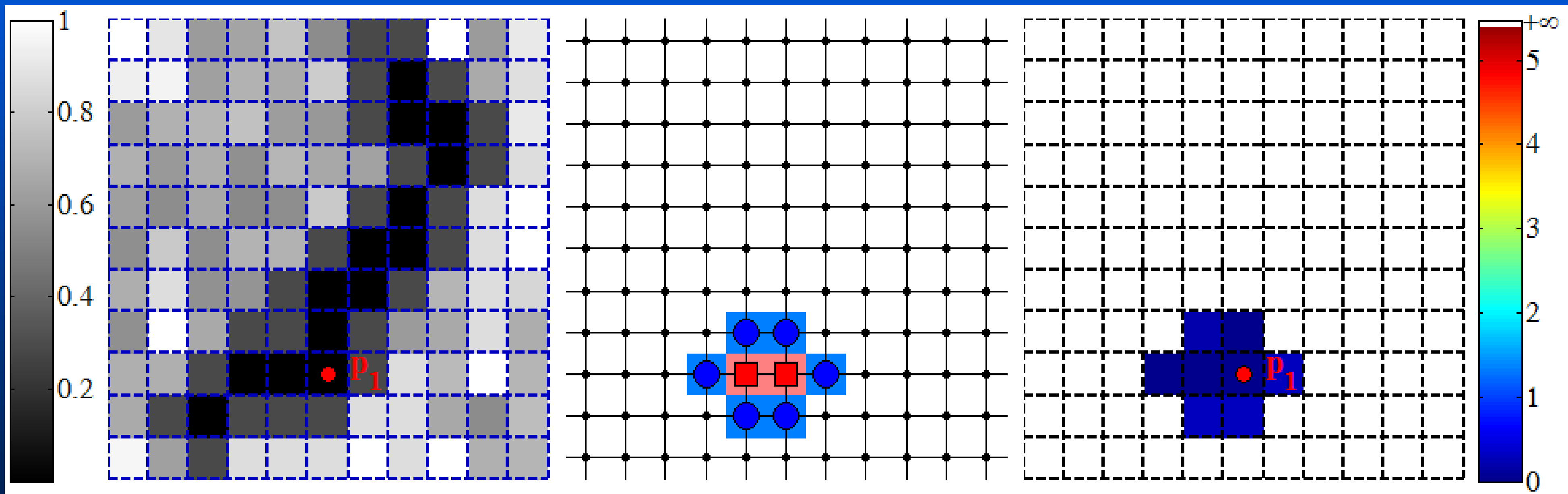
  $\mathbf{x}$ becomes *Trial*.



potential $\tilde{\mathcal{P}}$ — *Far* — *Trial* — *Alive* — *minimal action* $\mathcal{U}$

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Fast Marching Algorithm

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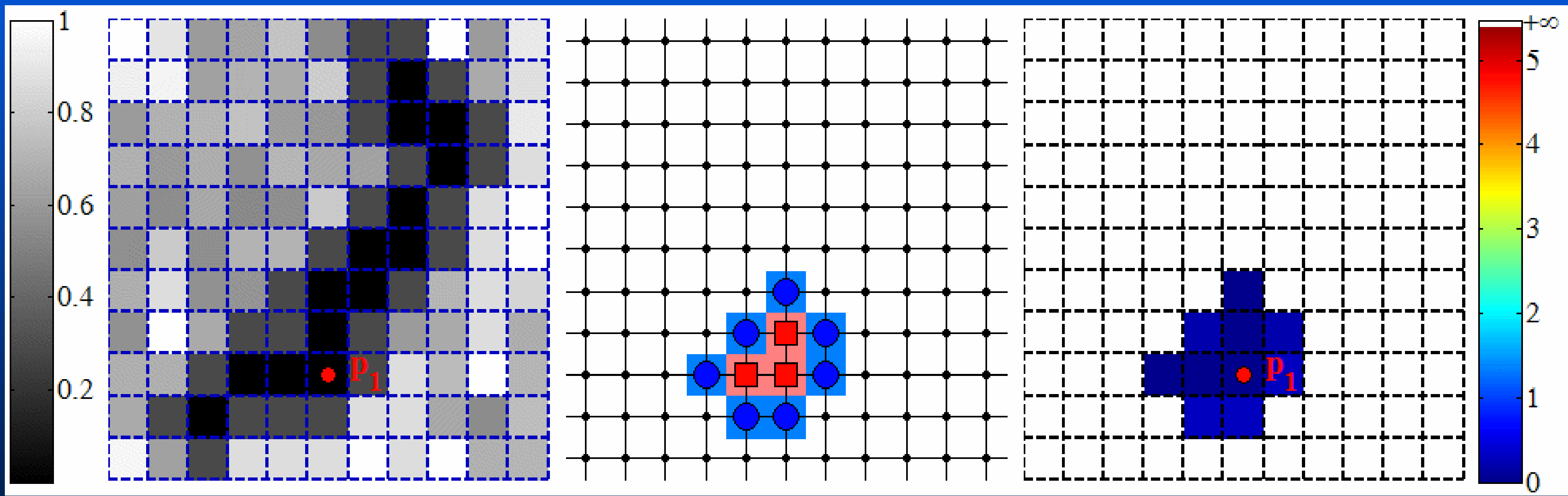

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Minimal Path between p1 and p2

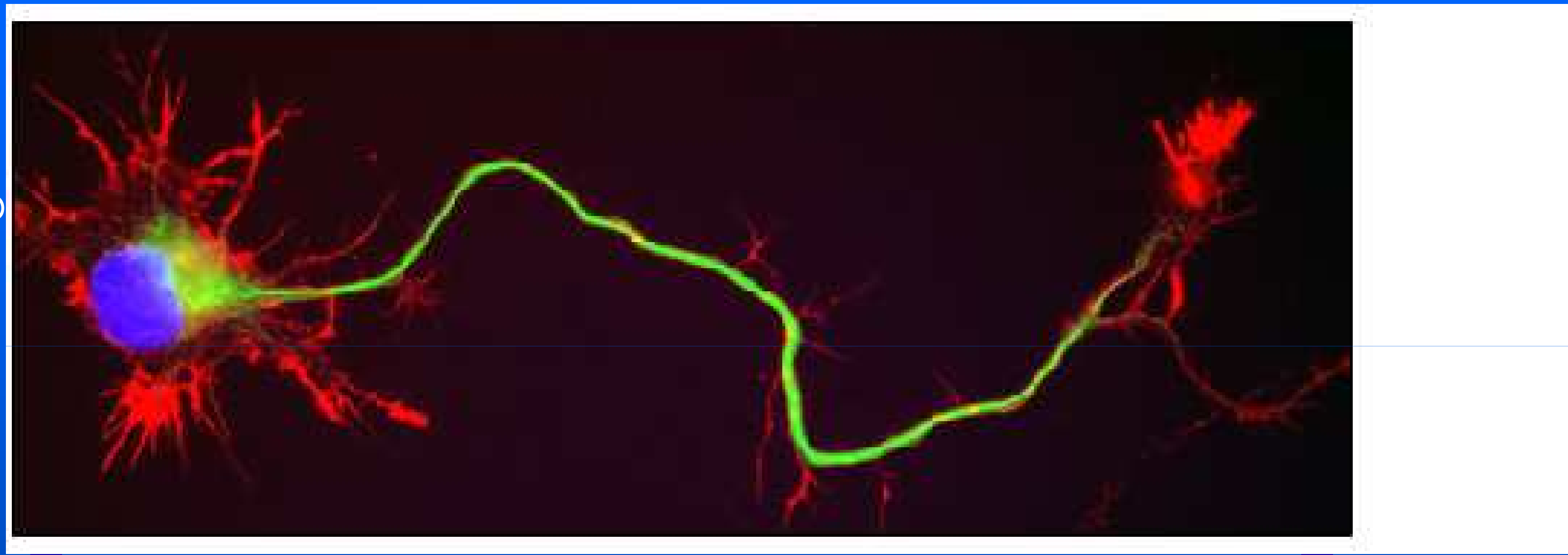
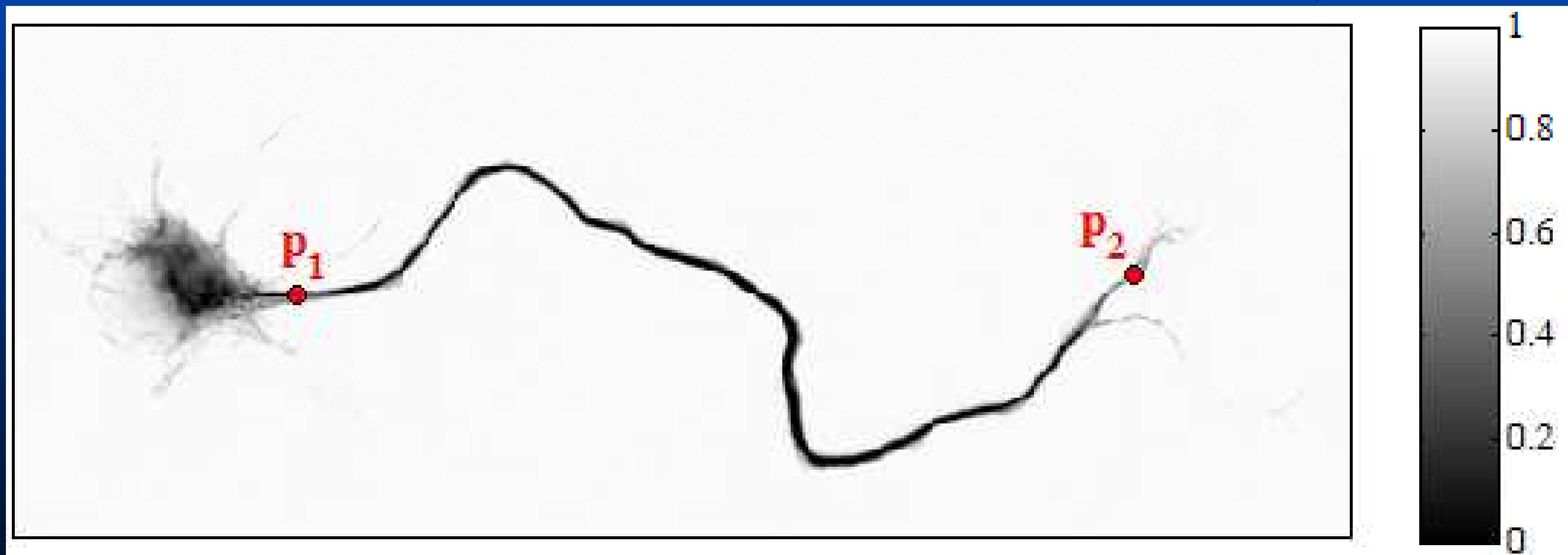
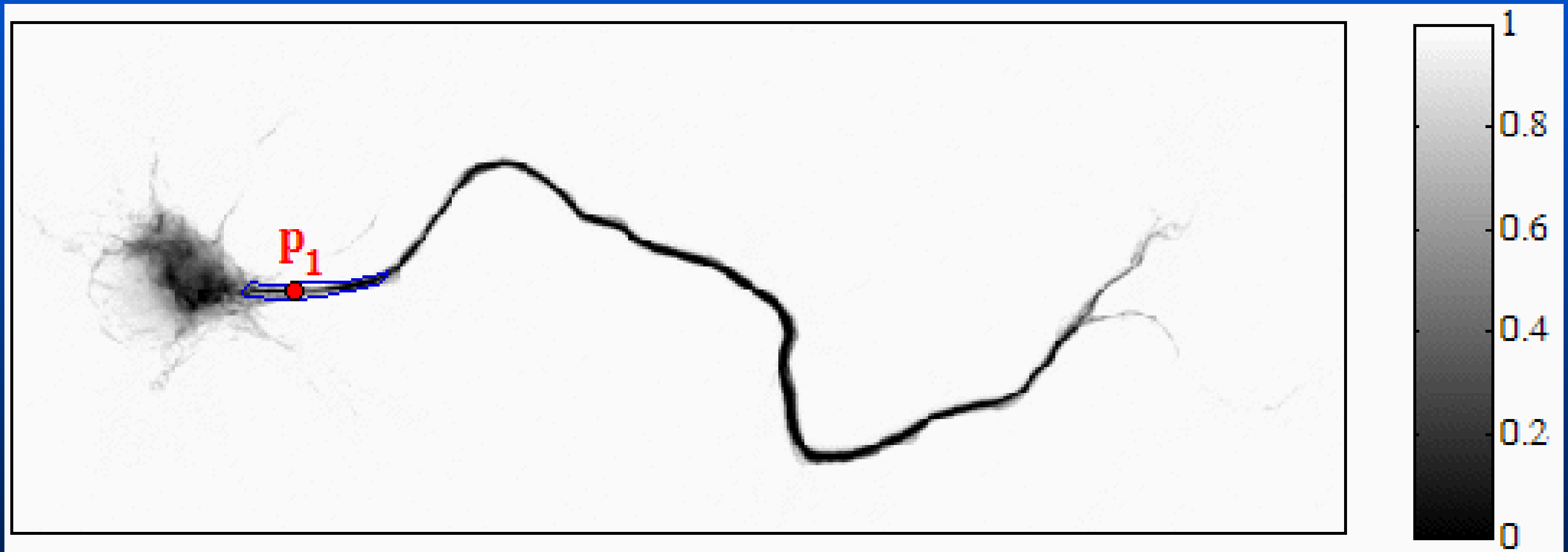


image I



potentiel $\mathcal{P} : \Omega \rightarrow \mathbb{R}^{+*}$

Minimal Path between p1 and p2



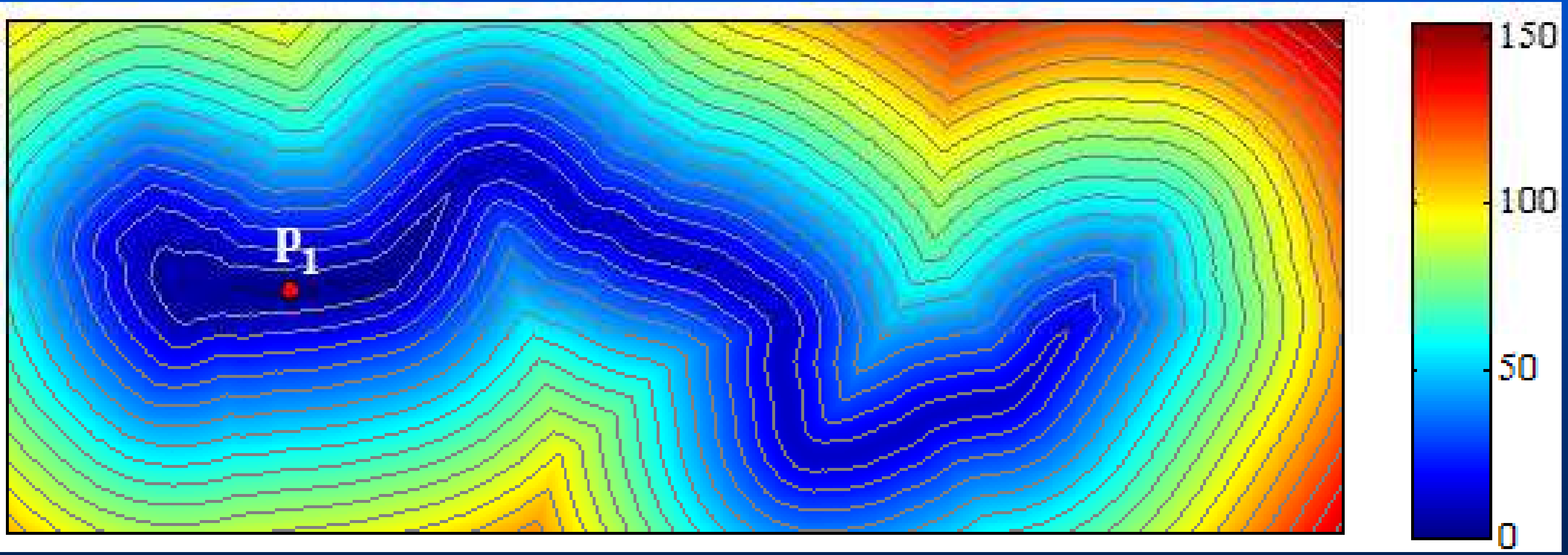
potentiel $\mathcal{P}$

L. D. Cohen, R. Kimmel
Global minimum for active contour models : a minimal path approach.
International Journal of Computer Vision, **25**:57-78, 1997.

Minimal Path between p1 and p2

Minimal action $\mathcal{U}_1 : \Omega \to \mathbb{R}^+$ solution of Eikonal equation :

$$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) \text{ pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(\mathbf{p}_1) = 0 \end{cases}$$



minimal action $\mathcal{U}_1$

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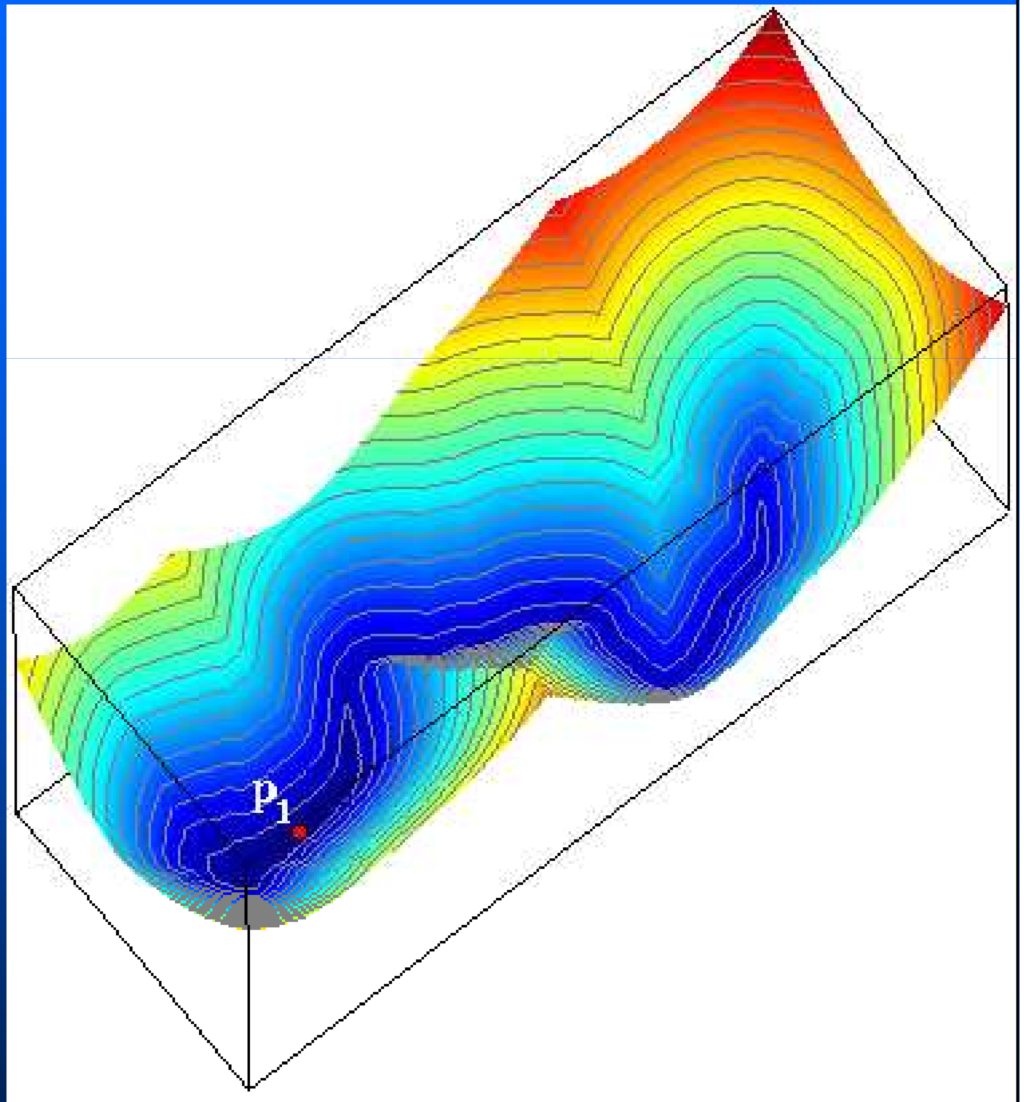

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Minimal Path between p1 and p2

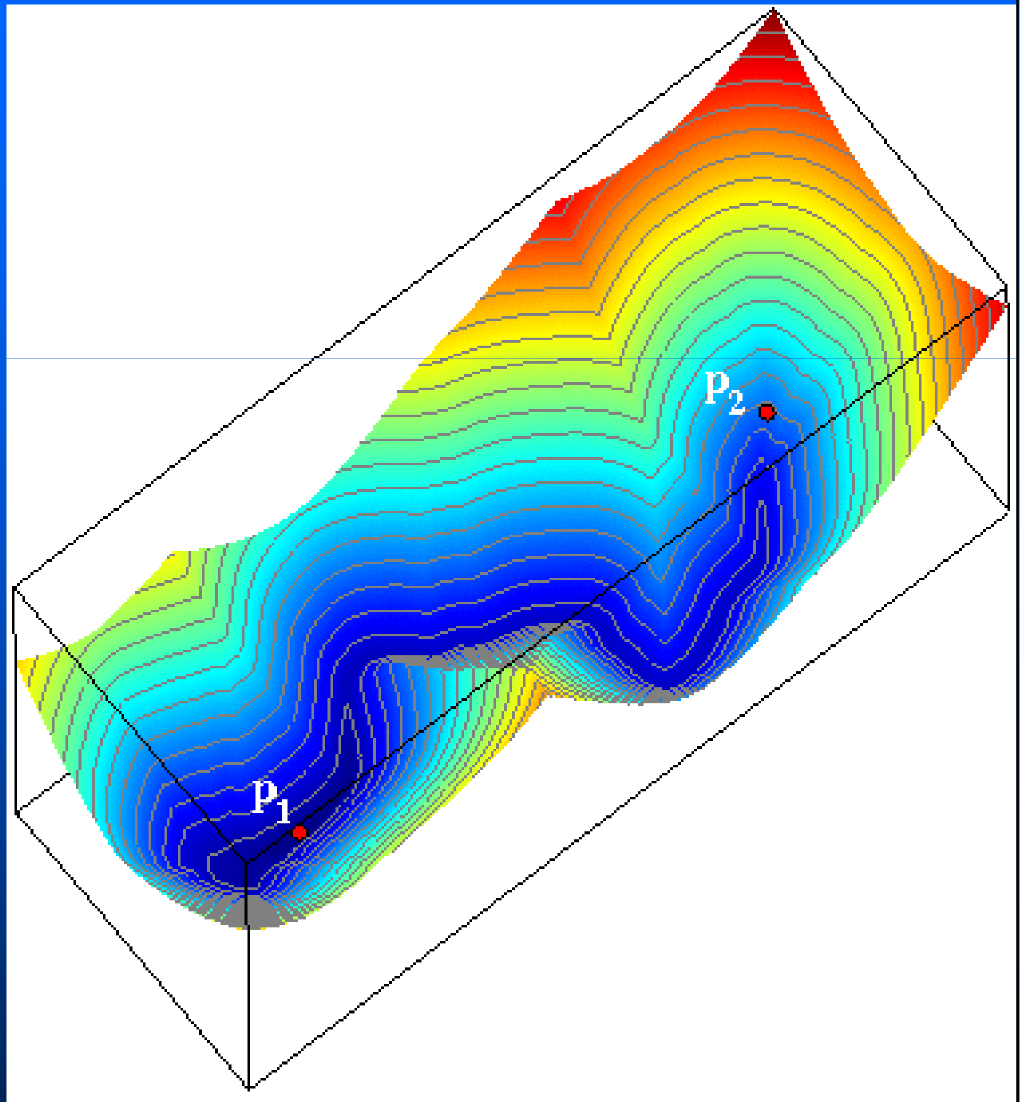
minimal path

$$\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2} = \min_{\gamma \in \mathcal{A}_{\mathbf{p}_1,\mathbf{p}_2}} \int_\gamma \tilde{\mathcal{P}}(\gamma(s))\mathrm{d}s$$

Is obtained by solving ODE:

$$\begin{cases} \dfrac{\partial \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)}{\partial s} = -\nabla \mathcal{U}_1(\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)) \\ \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(0) = \mathbf{p}_2 \end{cases}$$

⇨ simple gradient descent on $\mathcal{U}_1$ from $\mathbf{p}_2$ to $\mathbf{p}_1$



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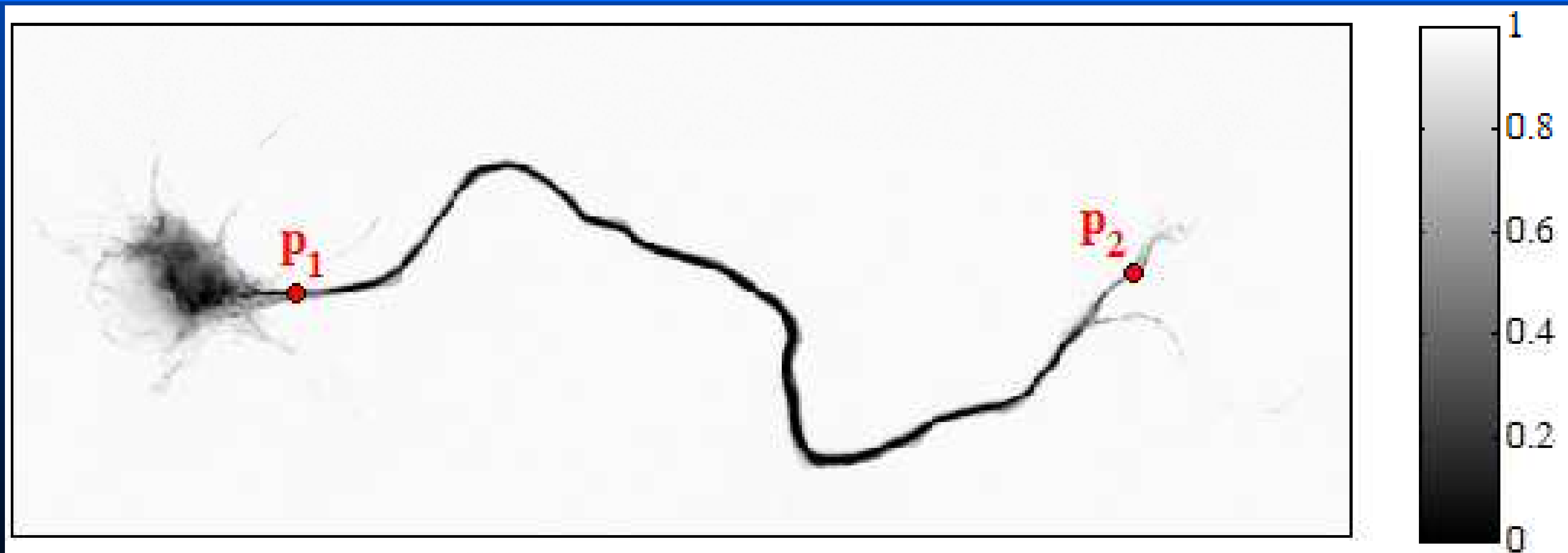
Minimal Path between p1 and p2

Step #1

$$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) \ \text{pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(\mathbf{p}_1) = 0 \end{cases}$$



potential $\mathcal{P}$

Minimal Path between p1 and p2

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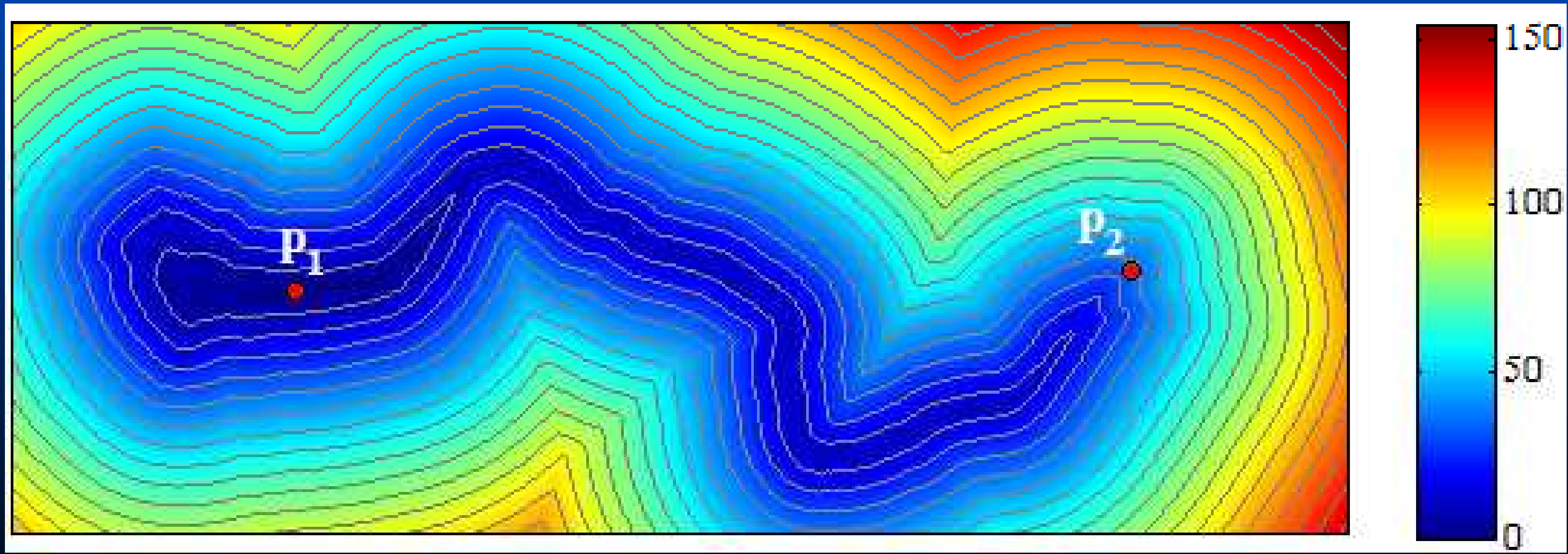

minimal action: $\mathcal{U}_1$

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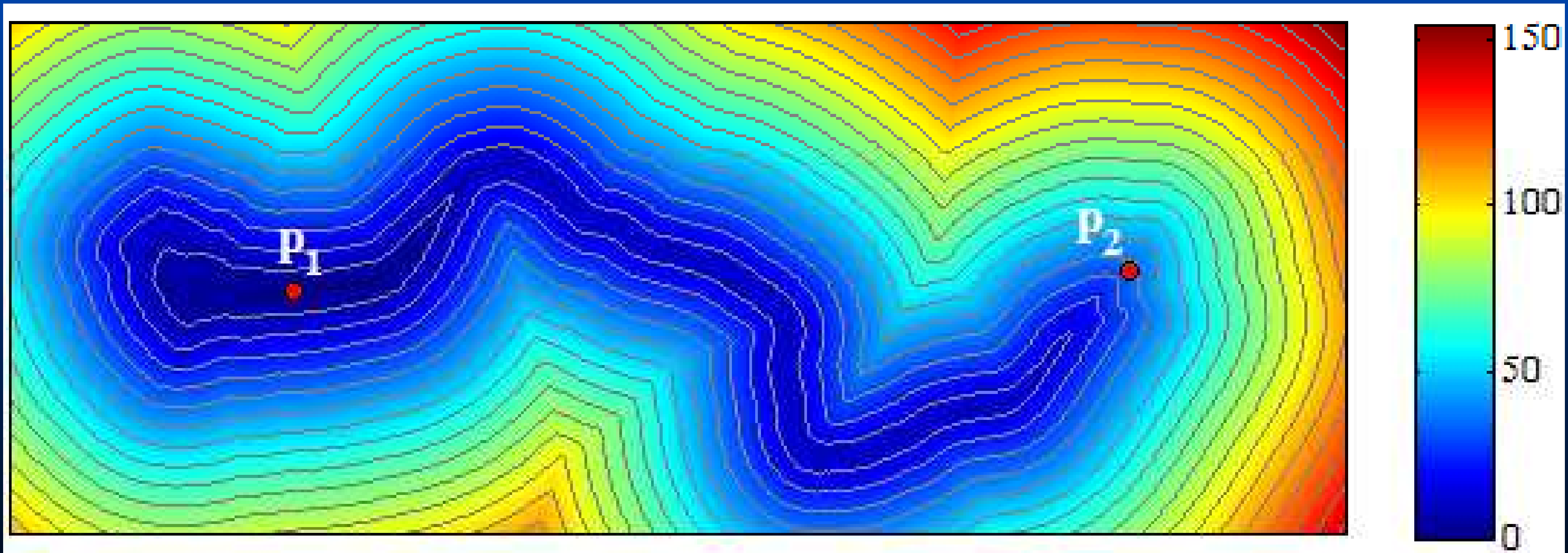
Step #2

gradient descent on $\mathcal{U}_1$ for extraction of minimal path $\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}$

$$\begin{cases} \frac{\partial \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)}{\partial s} = -\nabla \mathcal{U}_1(\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)) \\ \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(0) = \mathbf{p}_2 \end{cases}$$



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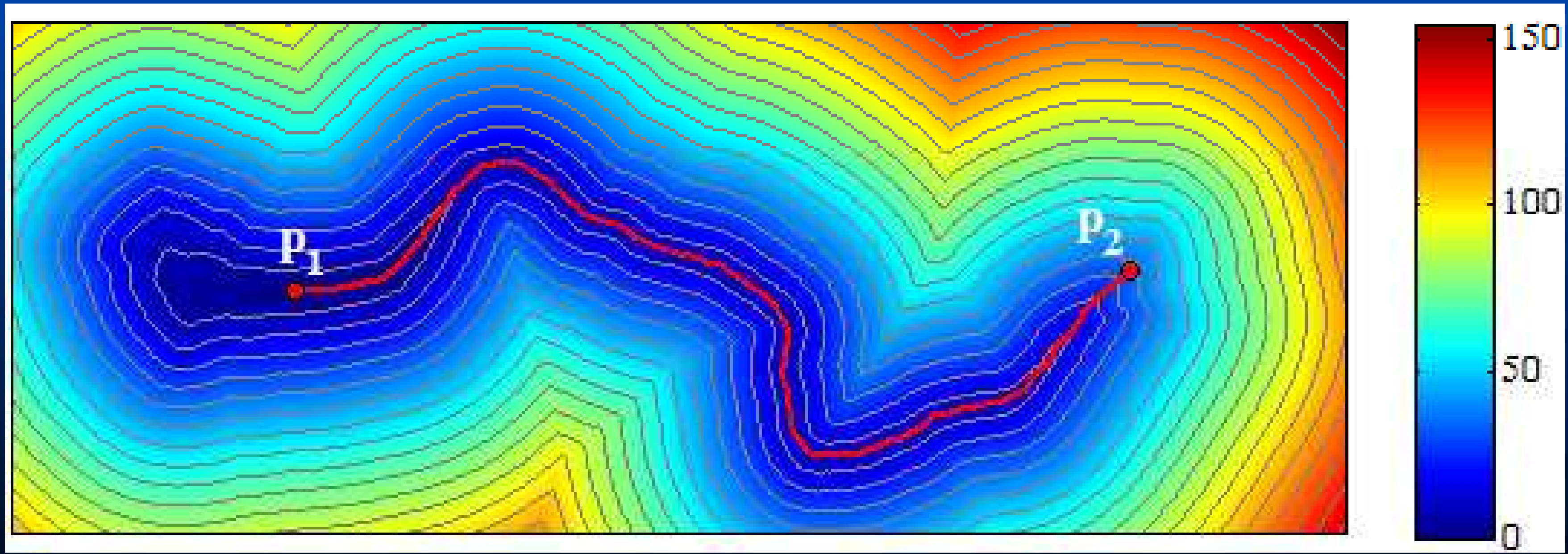

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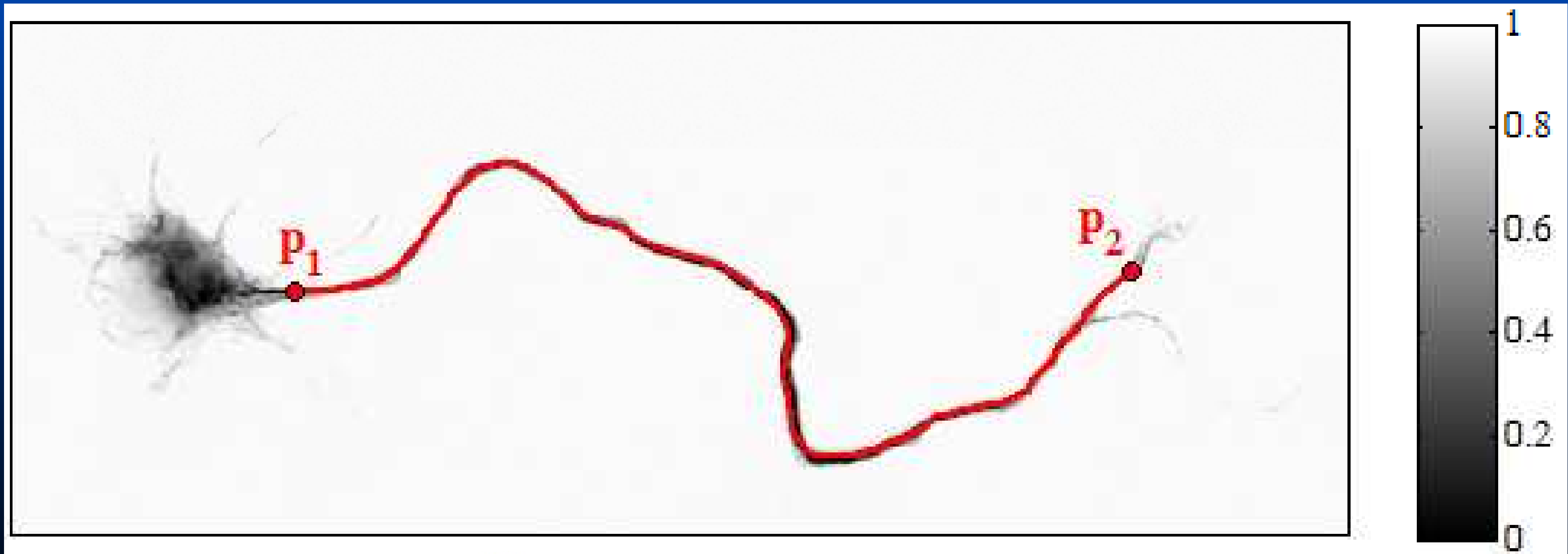


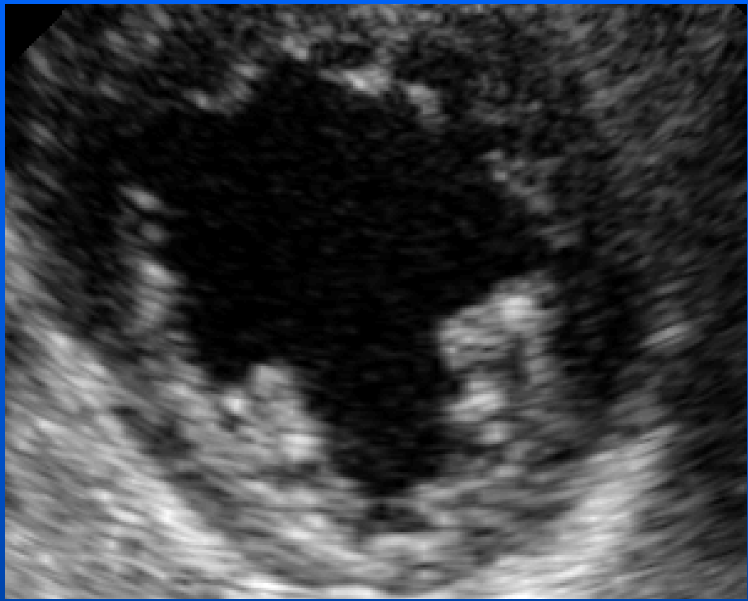
potential $\mathcal{P}$

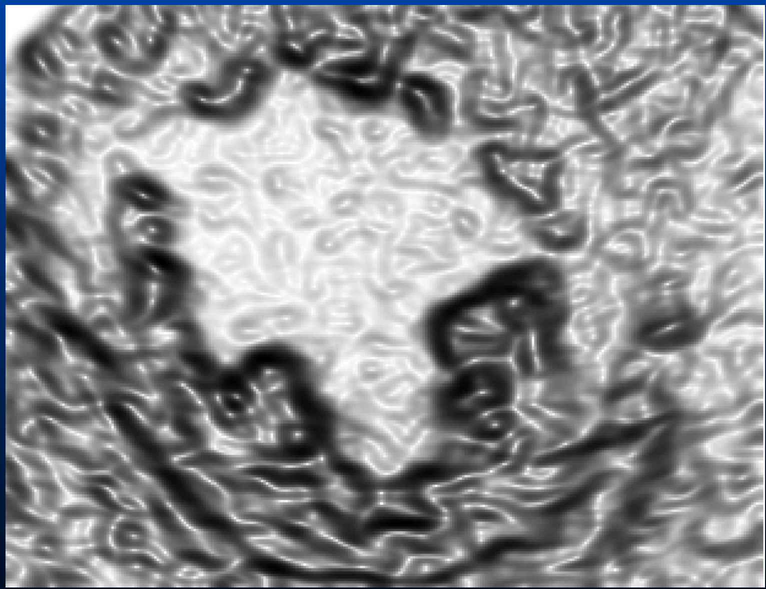
Minimal paths for 2D segmentation

Energy to minimize

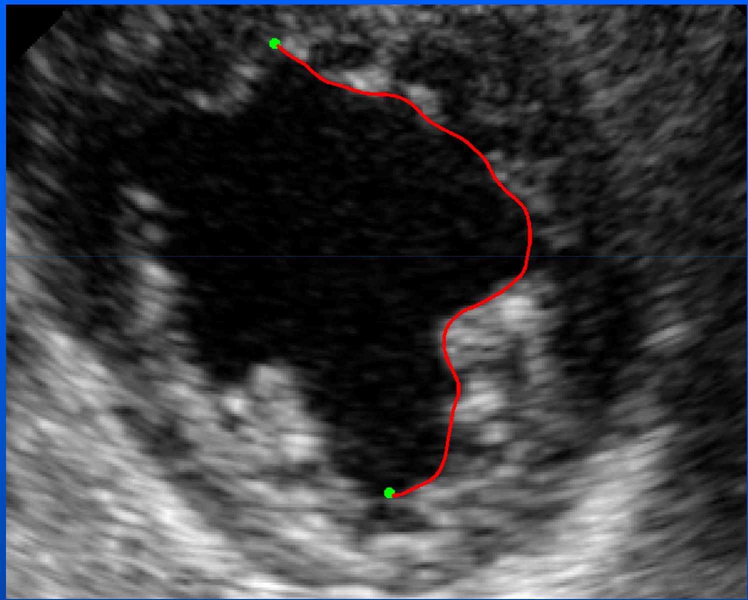
$$E(\gamma) = \int_0^L P\big(\gamma(t)\big)dt$$

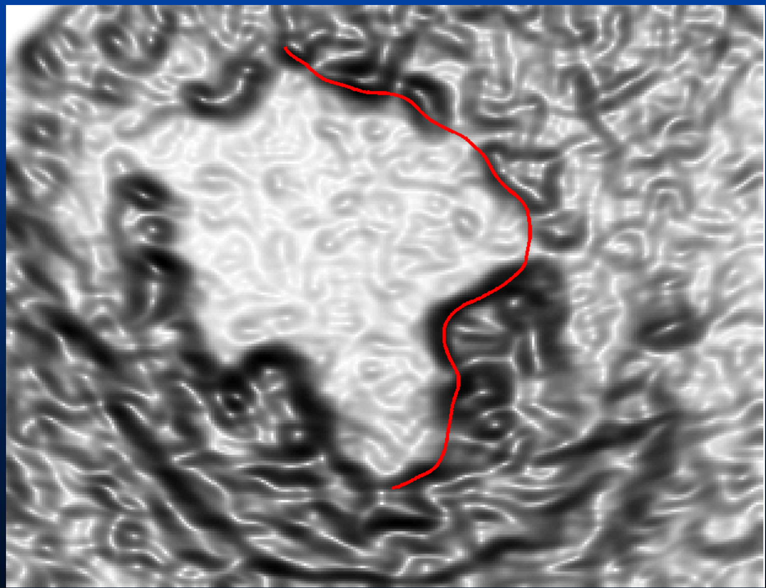
$$P : X \in \Omega \rightarrow \frac{1}{1+\alpha.\left|\nabla I_\sigma(X)\right|^2}$$





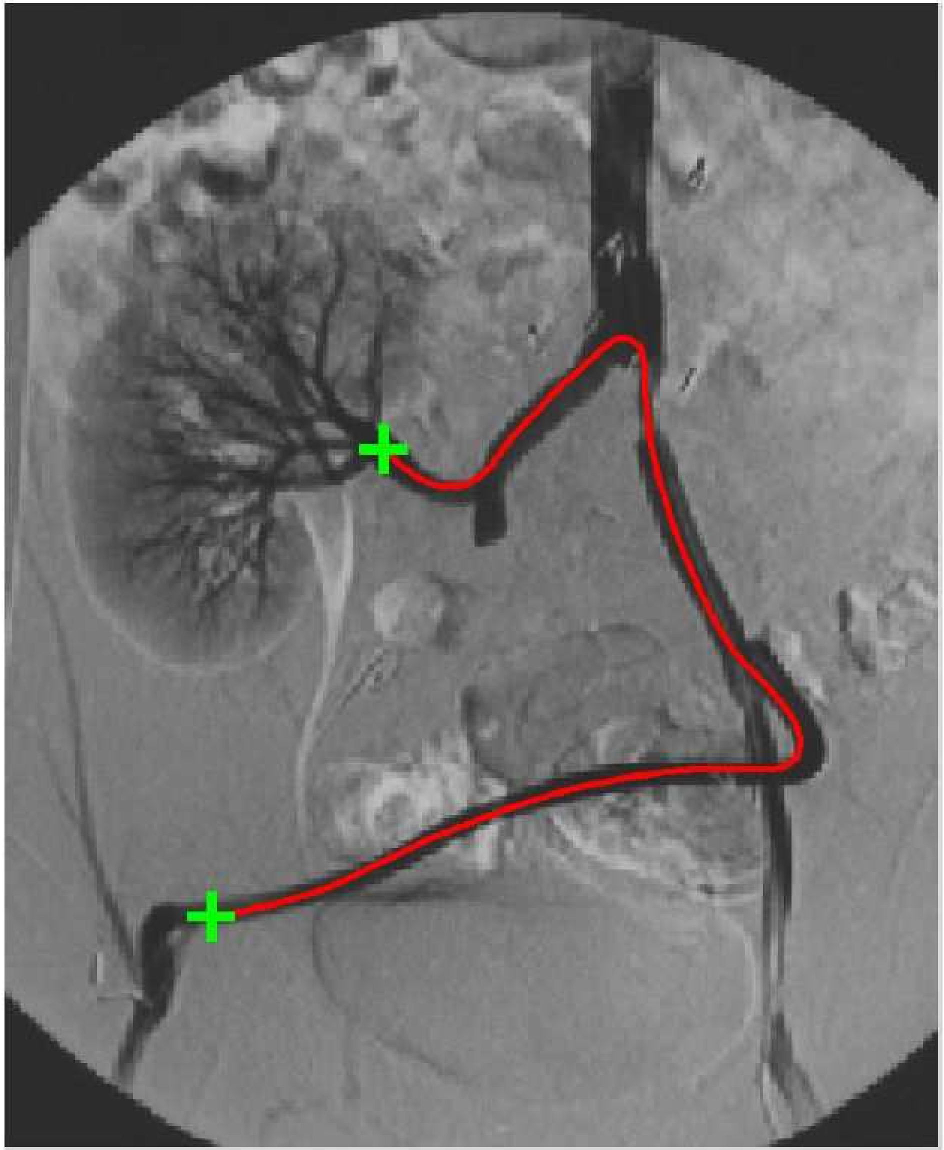
Minimal paths for 2D segmentation



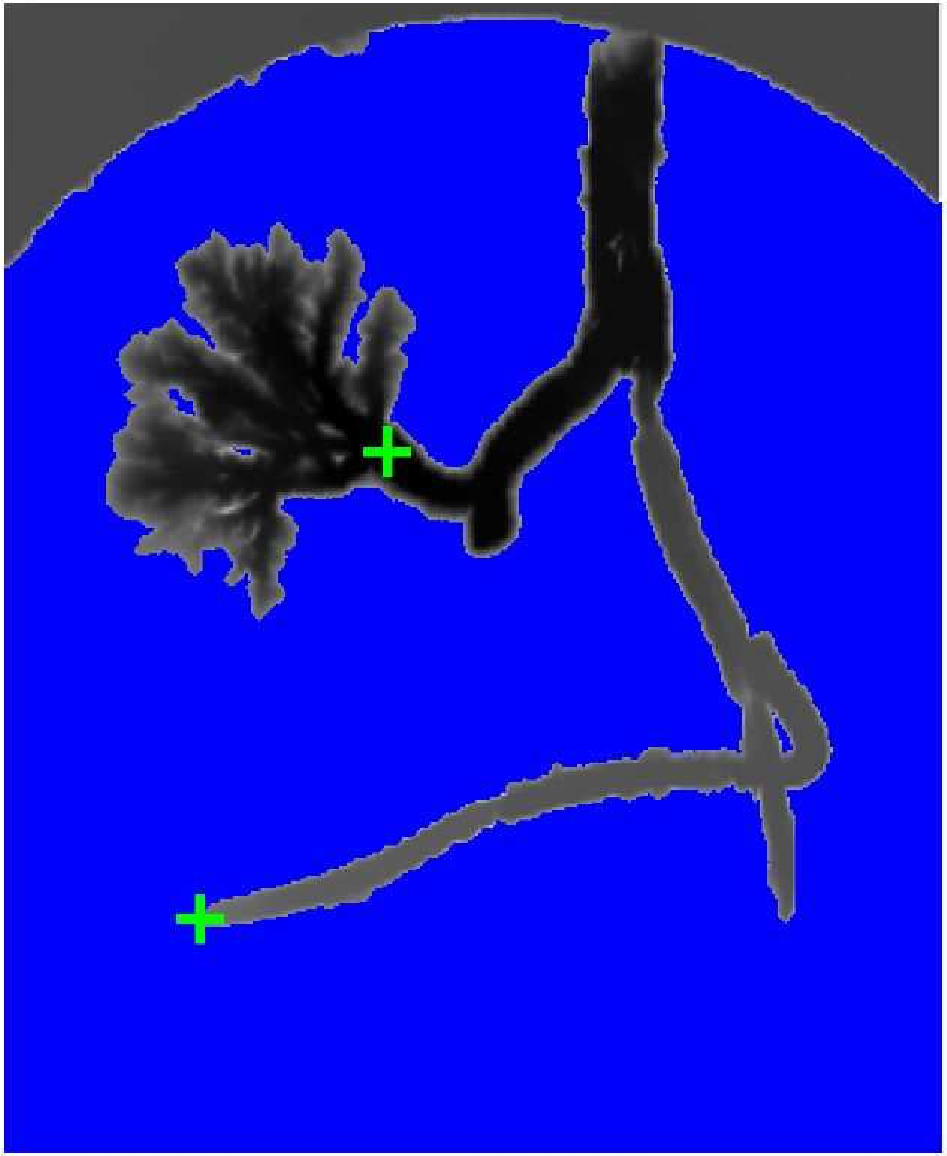


Minimal paths for 2D segmentation

- $P(\mathbf{x}) = w + (I(\mathbf{x}) - I(\mathbf{x}_0))^2 \Longrightarrow$ chemin d'intensité homogène

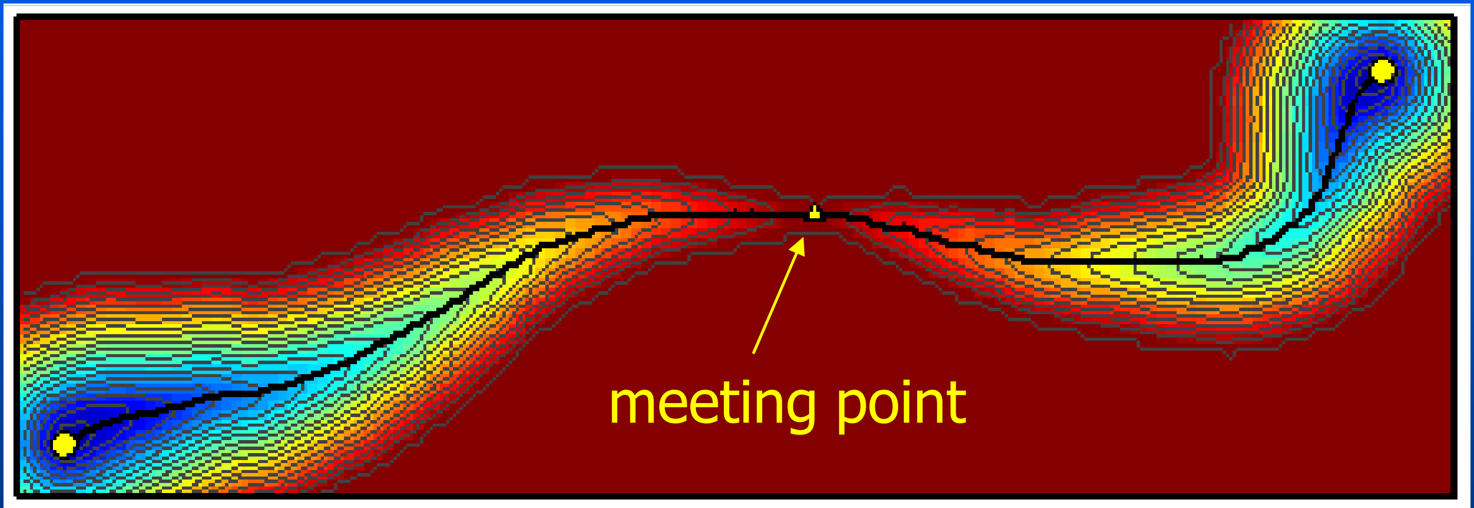


Chemin



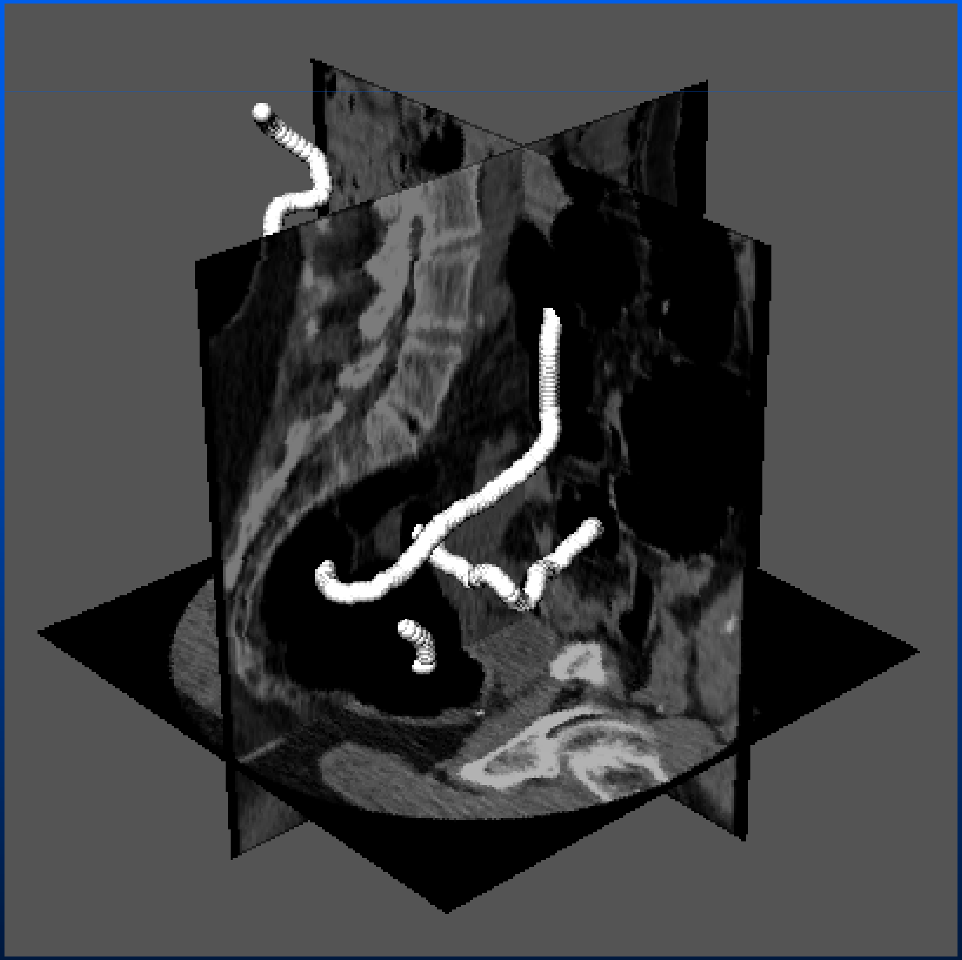
Carte de distance

Simultaneous propagation of two fronts until a shock occurs.

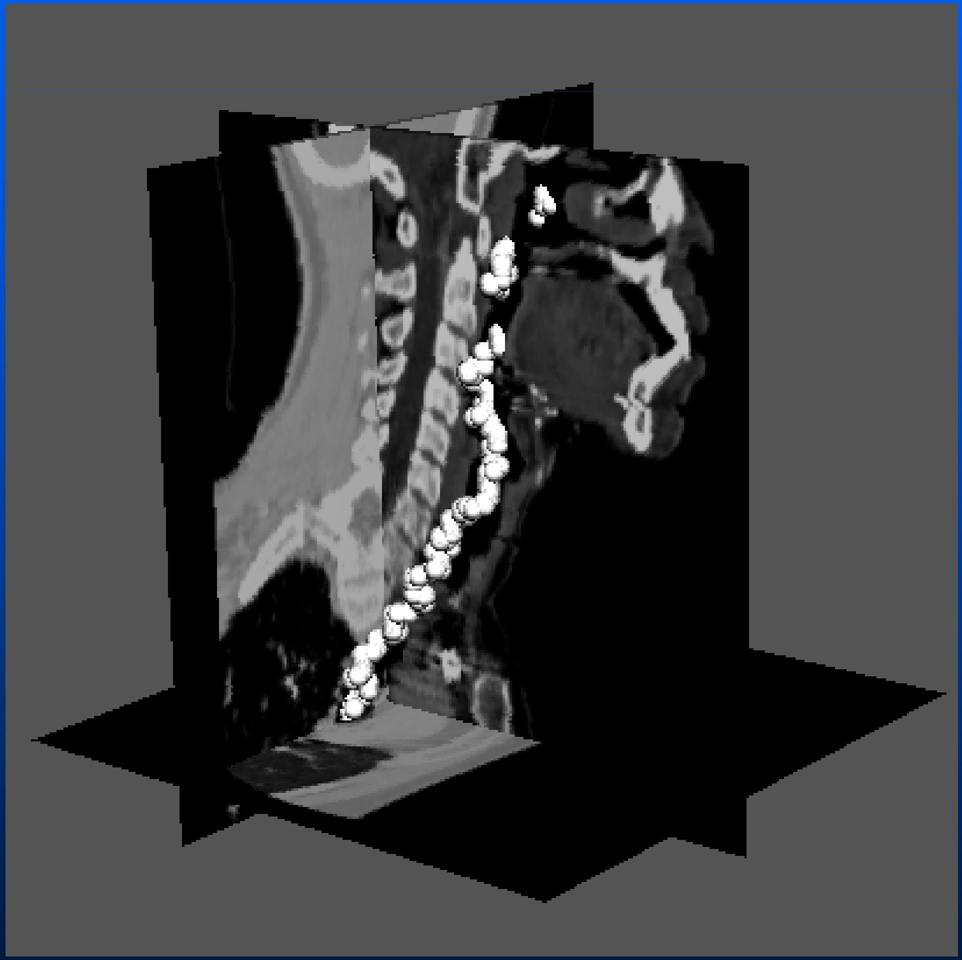


Reference:
T. Deschamps and L. D. Cohen
Minimal paths in 3D images and application to virtual endoscopy.
Proceedings ECCV'00, Dublin, Ireland, 2000.

Examples of 3D Minimal Paths



Colon 3D CT



Trachea 3D CT

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Riemannian Manifolds, Anisotropy and Geodesic Distances

- 2D Riemannian manifolds defined over a compact planar domain $\Omega \subset \mathbb{R}^2$
- Length of a curve $[0,1] \rightarrow \Omega$

$$L(\gamma) \stackrel{\text{def.}}{=} \int_0^1 \sqrt{\gamma'(t)^{\mathrm{T}} H(\gamma(t)) \gamma'(t)} \mathrm{d}t.$$

with $H:\Omega \rightarrow \mathbb{R}^{2\times 2}$ a metric tensor field of anisotropy $\alpha:\Omega \rightarrow [0,1]$

- Geodesic distance

$$d(x,y) = \min_{\gamma \in P(x,y)} L(\gamma), \quad \forall (x,y) \in \mathbb{R}^2$$

- Distance map $U_S:\Omega \rightarrow \mathbb{R}$ of a point set $S=\{x_k\}_k$

$$U_S(x) = \min_{x_k \in S} d(x, x_k), \quad \forall x \in \Omega$$

Anisotropy and Eikonal Equation

Theorem: U_{x_0} is the unique viscosity solution of the Hamilton-Jacobi equation

$$\|\nabla U_{x_0}\|_{H(x)^{-1}} = 1 \qquad \text{with} \qquad U_{x_0}(x_0) = 0,$$

where $\|v\|_A = \sqrt{v^{\mathrm{T}} A v}$.

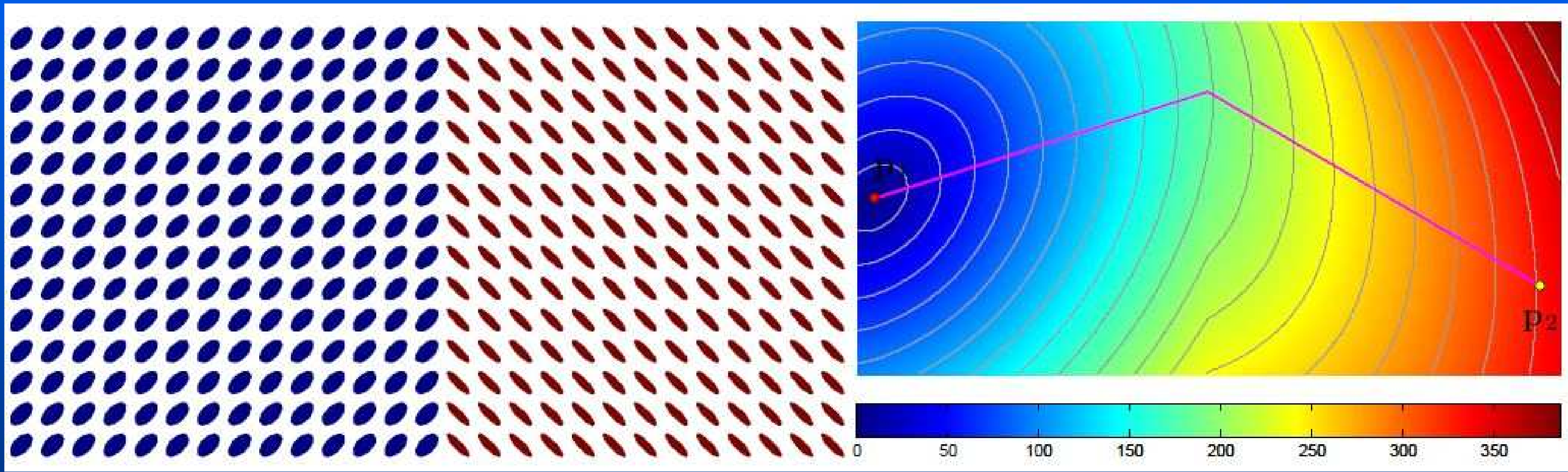
Geodesic curve γ between x_1 and x_0 solves

$$\gamma'(t) = -\frac{H(\gamma(t))^{-1}\nabla U_{x_0}}{\|H(\gamma(t))^{-1}\nabla U_{x_0}\|} \qquad \text{with} \qquad \gamma(0) = x_1.$$

Example: isotropic metric $H(x) = W(x)\mathrm{Id}_x$,

$$\|\nabla U_{x_0}\| = W(x) \qquad \text{and} \qquad \gamma'(t) = -\frac{\nabla U_{x_0}}{\|\nabla U_{x_0}\|}.$$

Anisotropy and Geodesics



Anisotropy and Geodesics

Tensor eigen-decomposition:

$$H(x) = \lambda_1(x) e_1(x) e_1(x)^{\mathrm{T}} + \lambda_2(x) e_2(x) e_2(x)^{\mathrm{T}} \quad \text{with} \quad 0 < \lambda_1 \leqslant \lambda_2,$$

Local anisotropy of the metric:

$$\alpha(x) = \frac{\lambda_2 - \lambda_1}{\lambda_2 + \lambda_1} = \frac{\sqrt{(a-b)^2 + 4c^2}}{a+b} \in [0,1] \quad \text{for} \quad H(x) = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$$

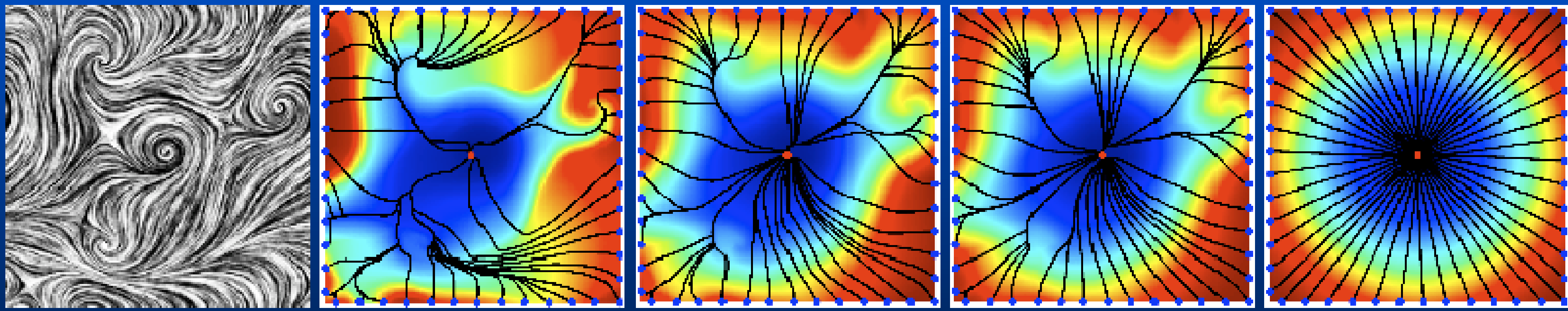
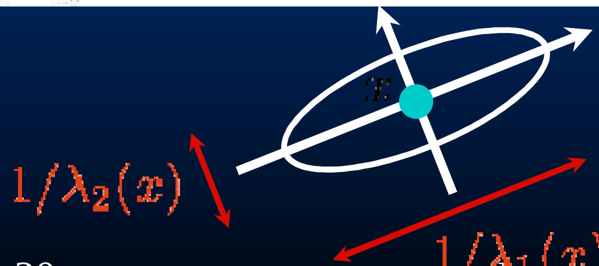
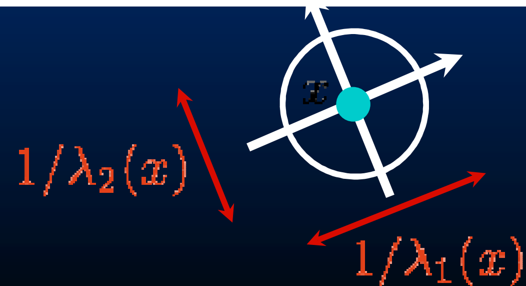


Image f $\alpha = .95$ $\alpha = .7$ $\alpha = .5$ $\alpha = 0$



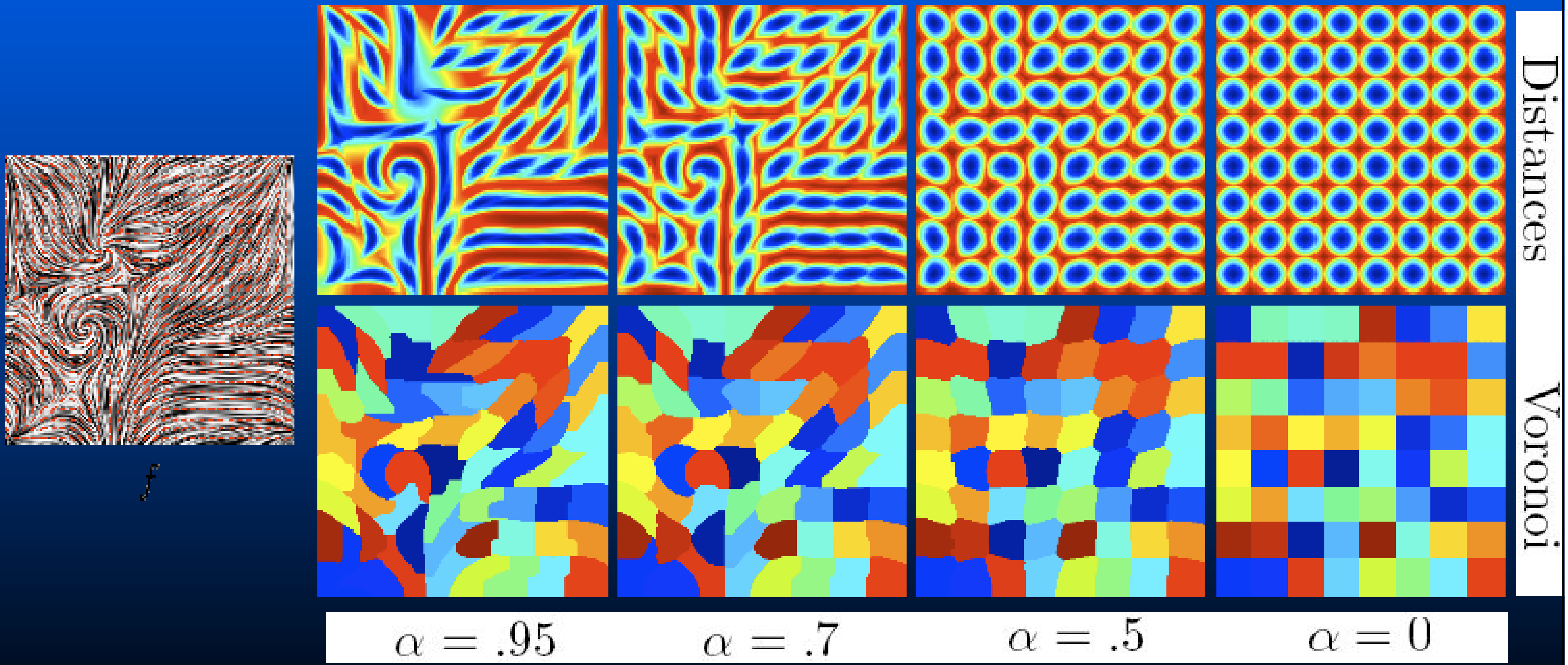


Anisotropic Voronoi Segmentation

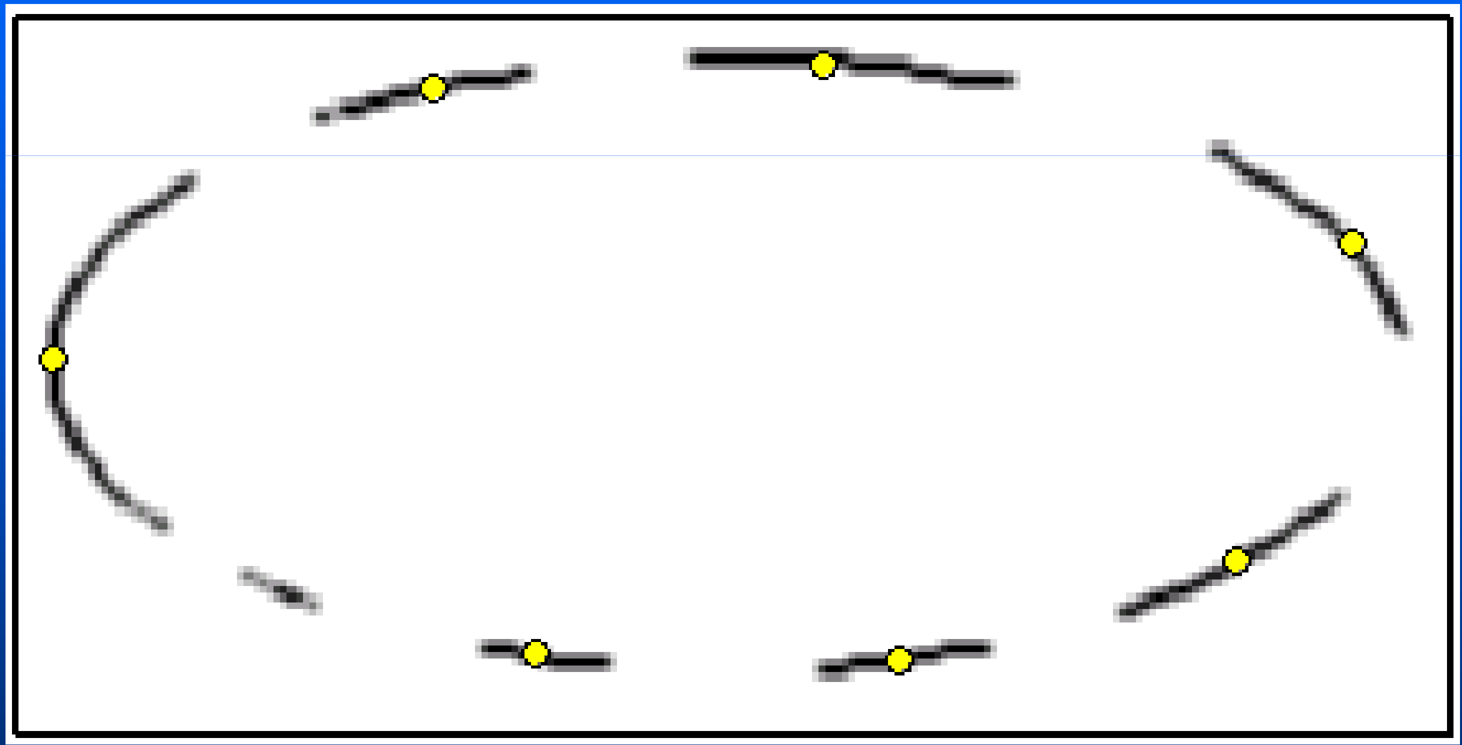
Voronoi segmentation:

$$\Omega = C_0 \bigcup_{x_i \in \mathcal{S}} \mathcal{C}_i \quad \text{where} \quad \mathcal{C}_i = \{x \in \Omega \setminus \forall j \neq i, \quad d(x_i, x) \leqslant d(x_j, x)\}$$

Outer cell: $\mathcal{C}_0 = \text{Closure}(\Omega^c)$.



The potential is an incomplete ellipse and 7 points are given.

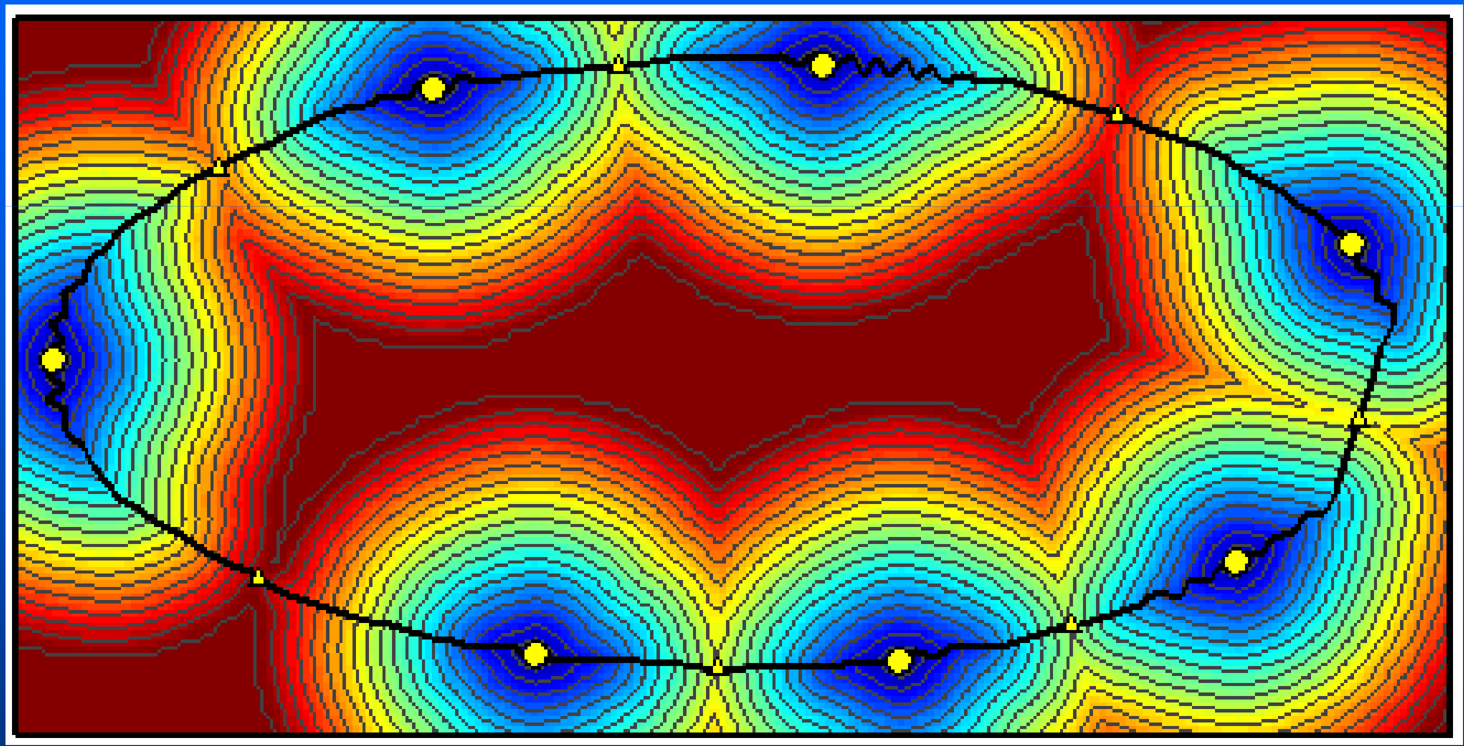


Reference:

L. D. Cohen

Multiple Contour Finding and Perceptual Grouping using Minimal Paths.

Journal of Mathematical Imaging and Vision, **14**:225-236, 2001.



Reference:
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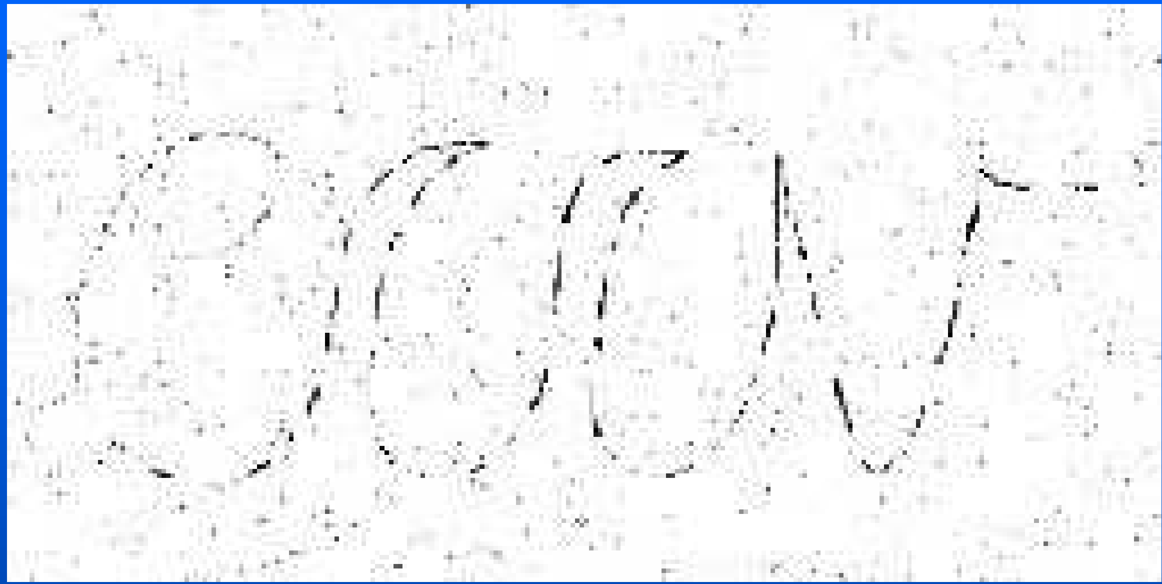
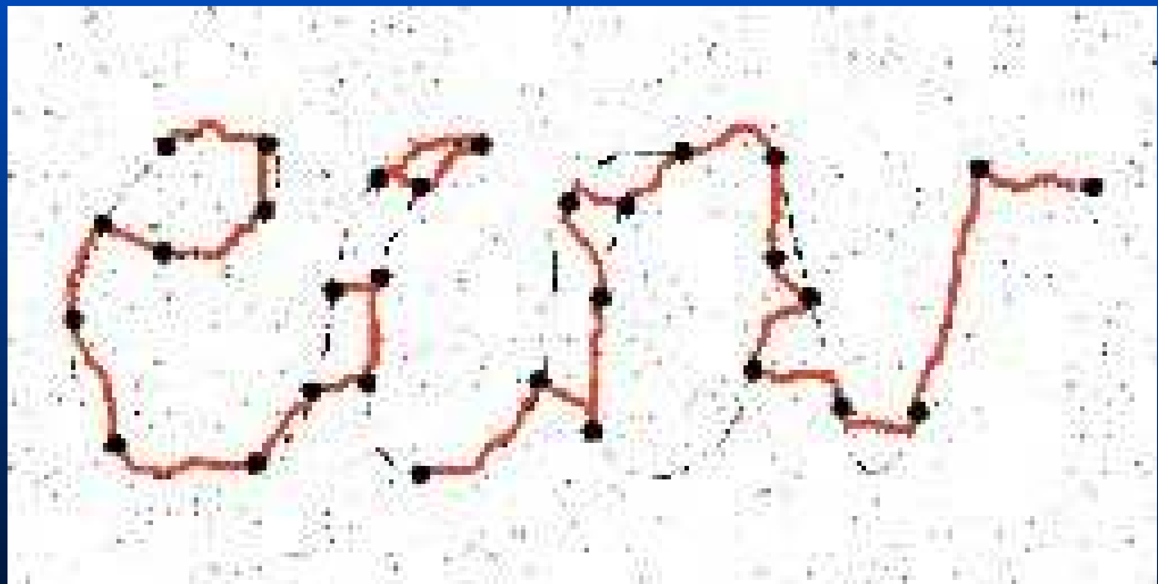
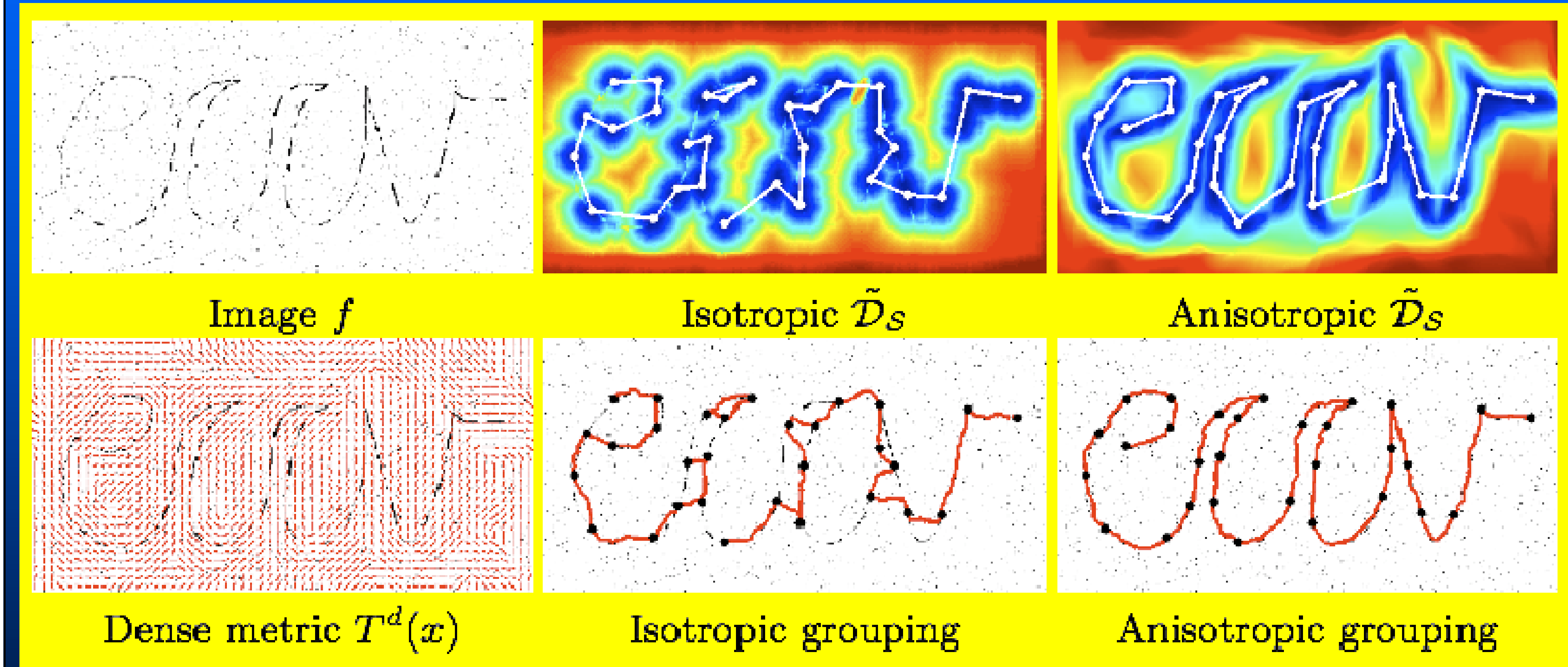
Multiple Contour Finding and Perceptual Grouping using Minimal Paths.
Journal of Mathematical Imaging and Vision, **14**:225-236, 2001.

Perceptual Grouping using Minimal Paths





Using the orientation with anisotropic geodesics



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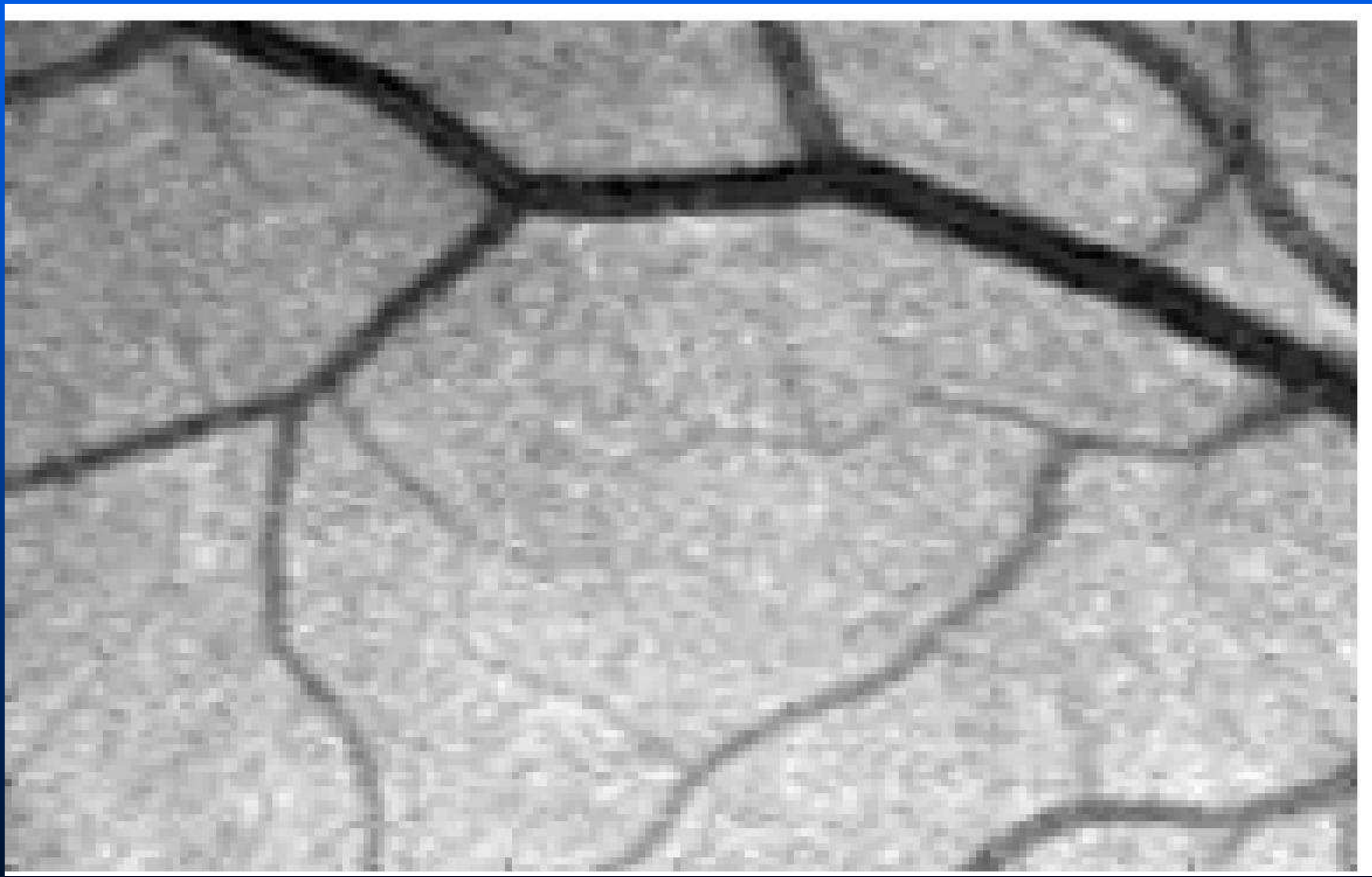
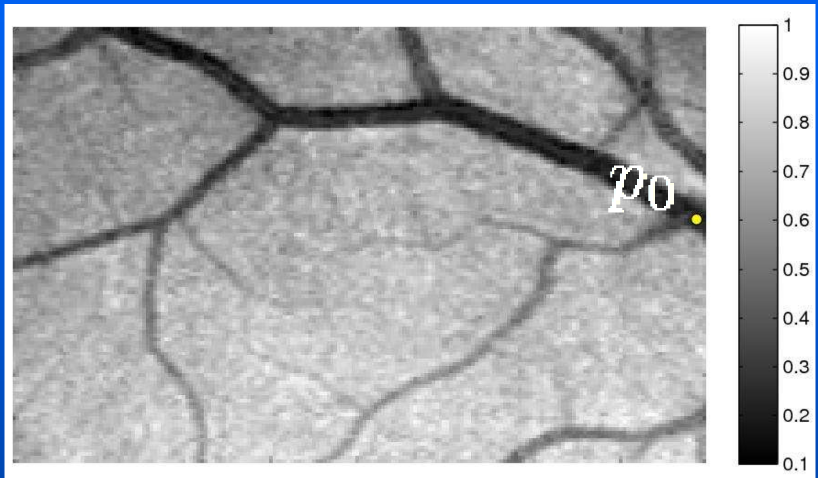
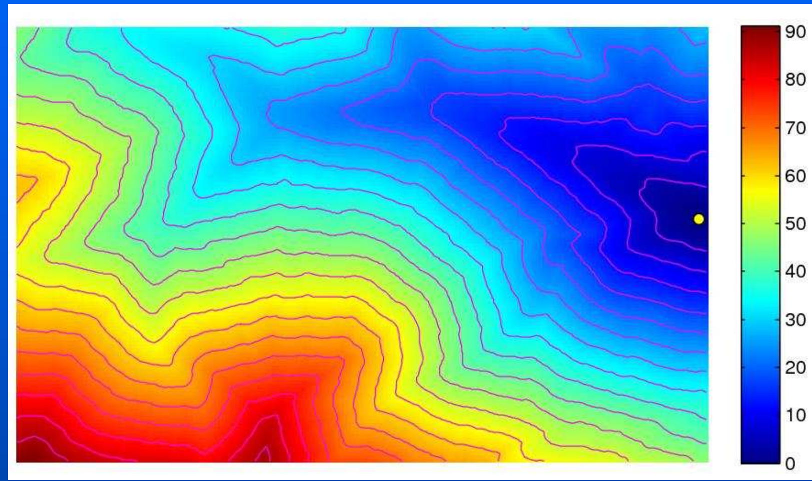
3D Minimal Paths for tubular shapes in 2D

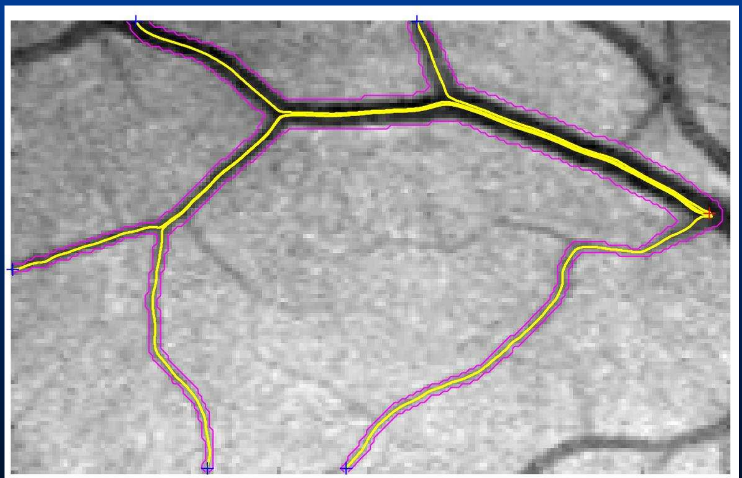
2D in space , 1D for radius of vessel

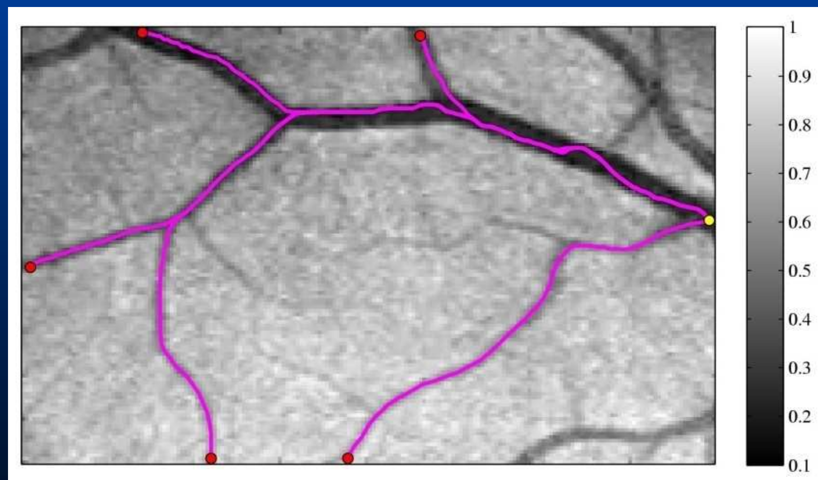


3D Minimal Paths for tubular shapes in 2D
Motivation









Orientation dependent Energy

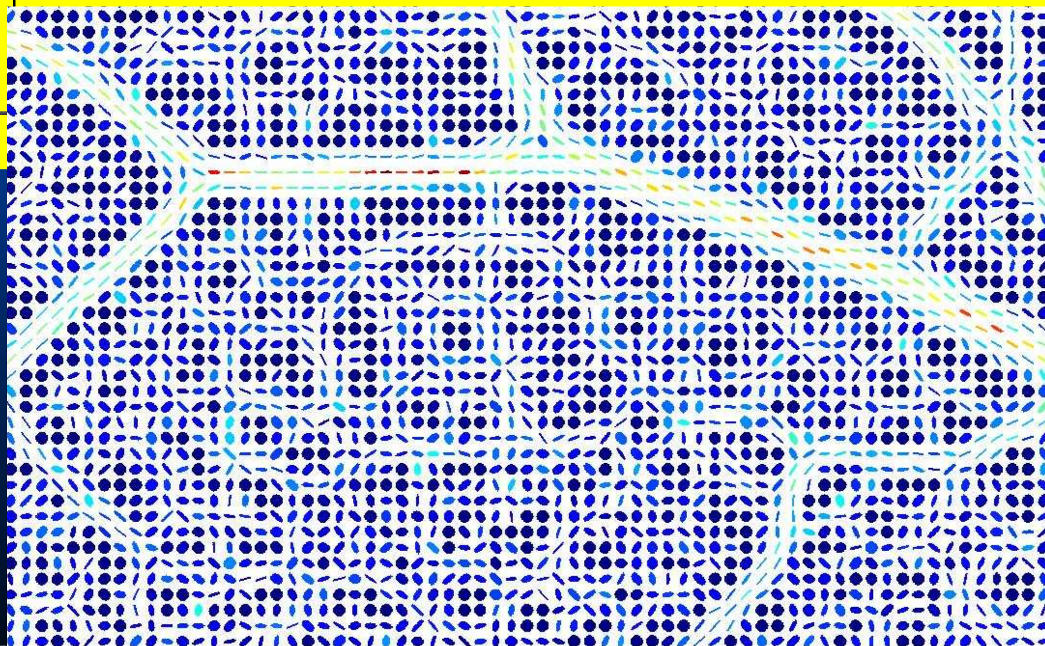
Minimal paths method : looking for a path minimizing the energy

$$E(\mathcal{C}) = \int_0^L P(\mathcal{C}(s))ds$$

Since the tubular structures have directions, we should consider the orientation:

$$E(\mathcal{C}) = \int_0^L P(\mathcal{C}(s), \mathcal{C}'(s))ds$$

where $P(\mathcal{C}, \mathcal{C}') = \sqrt{\mathcal{C}'^T \mathcal{M}(\mathcal{C})\mathcal{C}'}$ [illegible] th-way $\mathcal{C}$, relative to a metric $\mathcal{M}$.



3D Minimal Path for tubular shapes in 2D

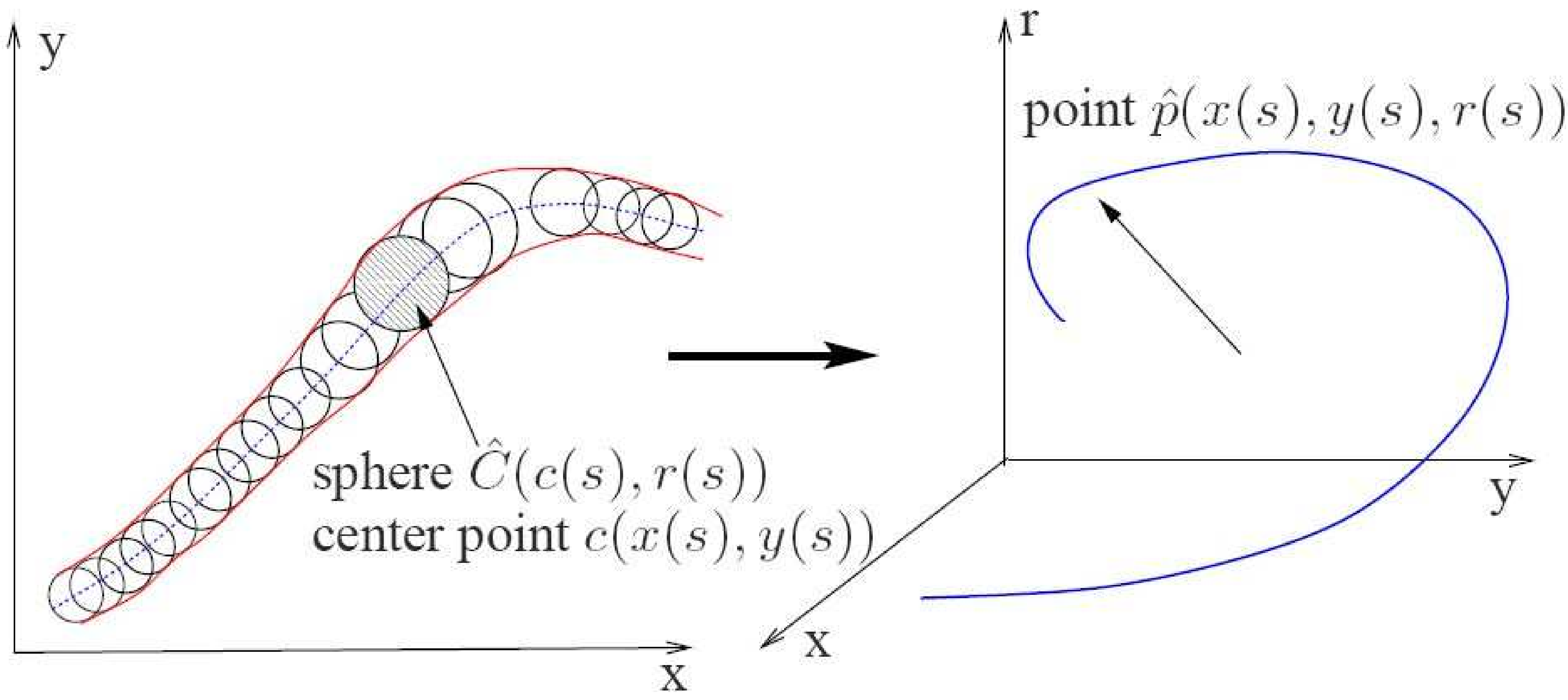


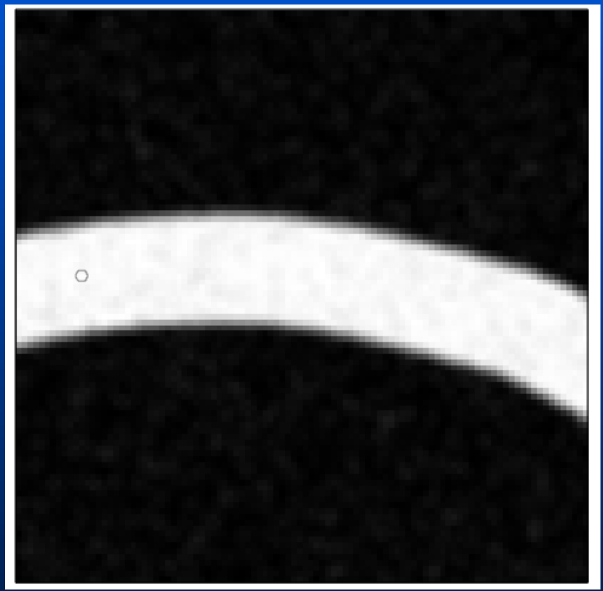
Figure 1. A tubular surface is presented as the envelope of a family of spheres with continuously changing center points and radii.

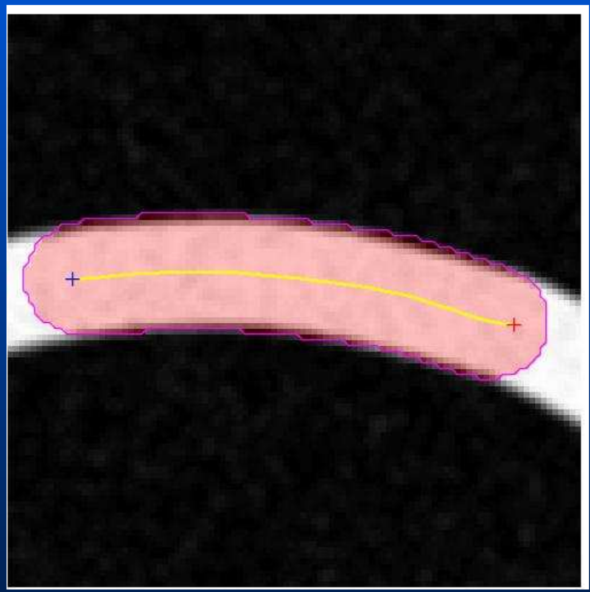
Examples of 3D Minimal Paths for tubular shapes in 2D

Anisotropic Fast Marching algorithm to solve

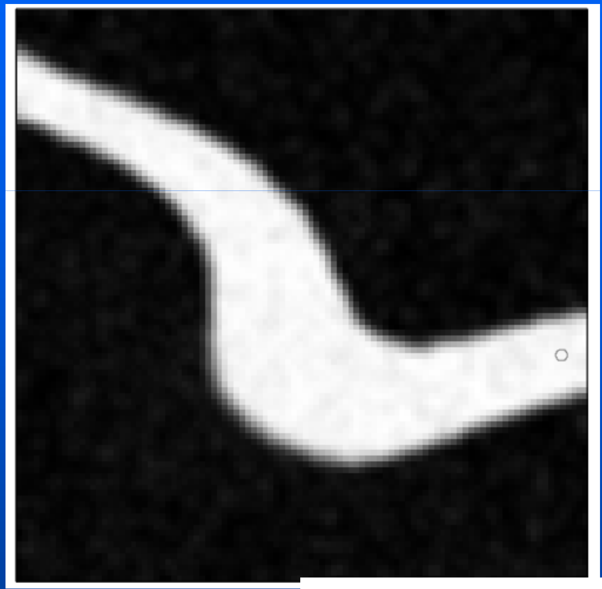
$$\|\nabla\mathcal{U}(x)\|_{\mathcal{M}^{-1}} = \sqrt{\nabla\mathcal{U}(x)^T\mathcal{M}^{-1}(x)\nabla\mathcal{U}(x)} = 1 \text{ and } \mathcal{U}_{p_0}(p_0) = 0$$

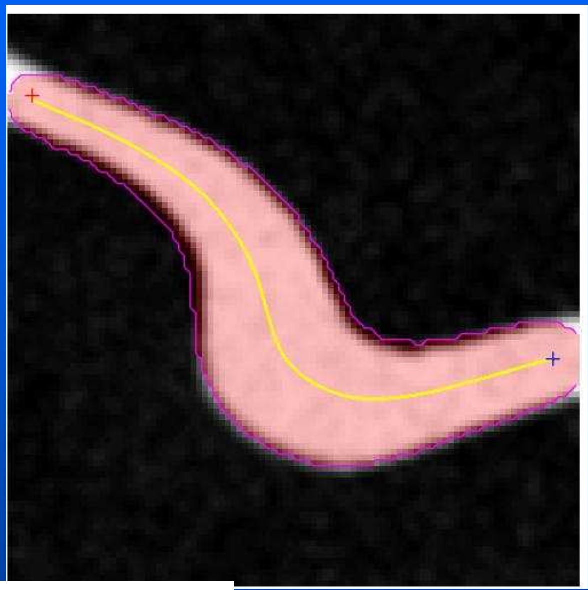
and back-propagation $\mathcal{C}' \propto \mathcal{M}^{-1}\nabla\mathcal{U}$

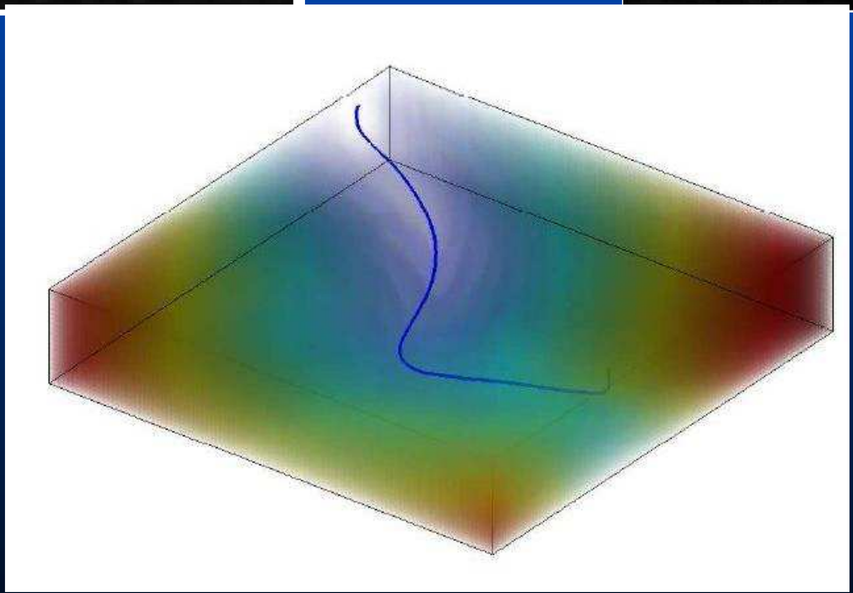




Examples of 3D Minimal Paths for tubular shapes in 2D

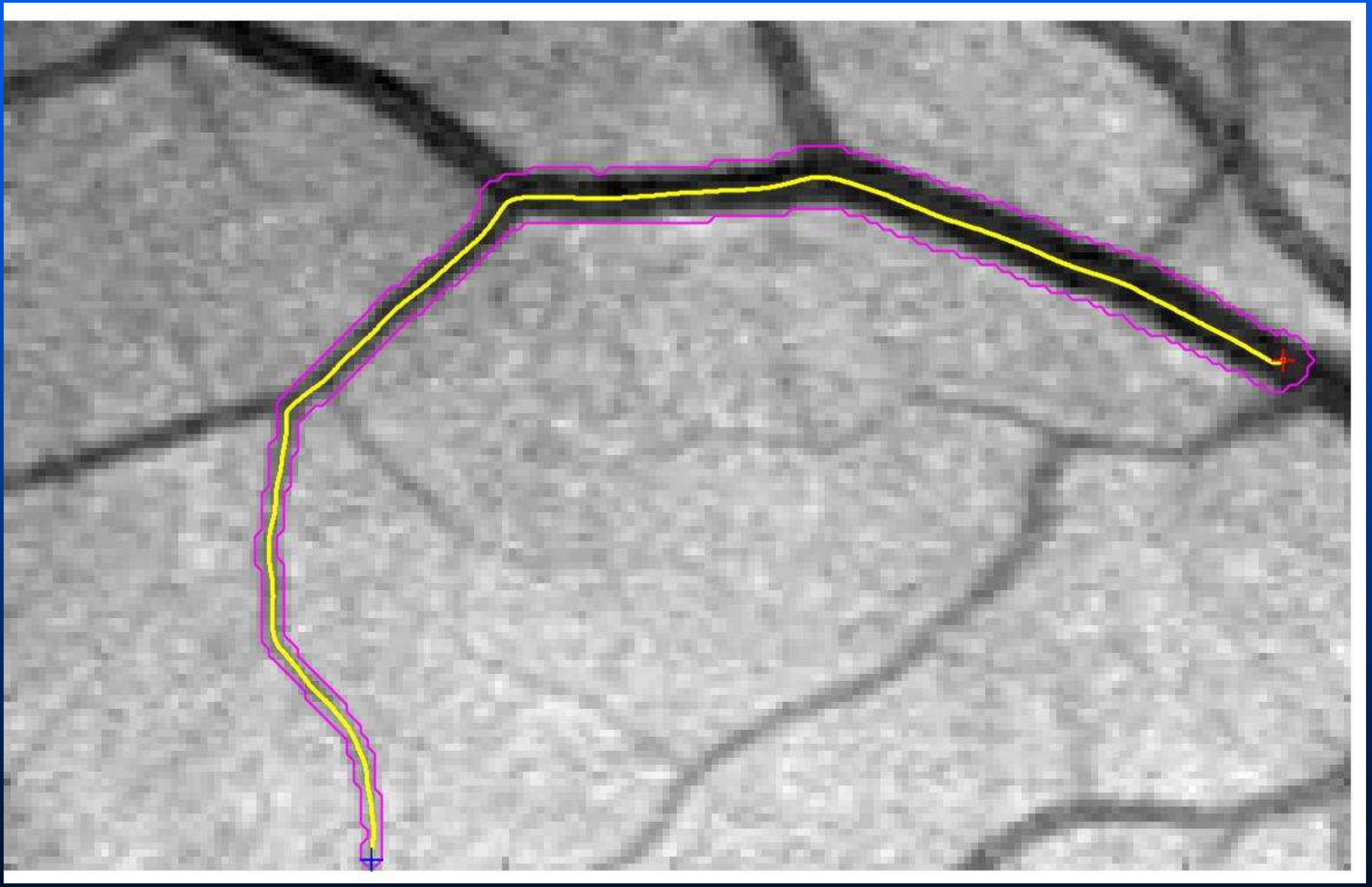






Examples of 3D Minimal Paths for tubular shapes in 2D

2D in space , 1D for radius of vessel



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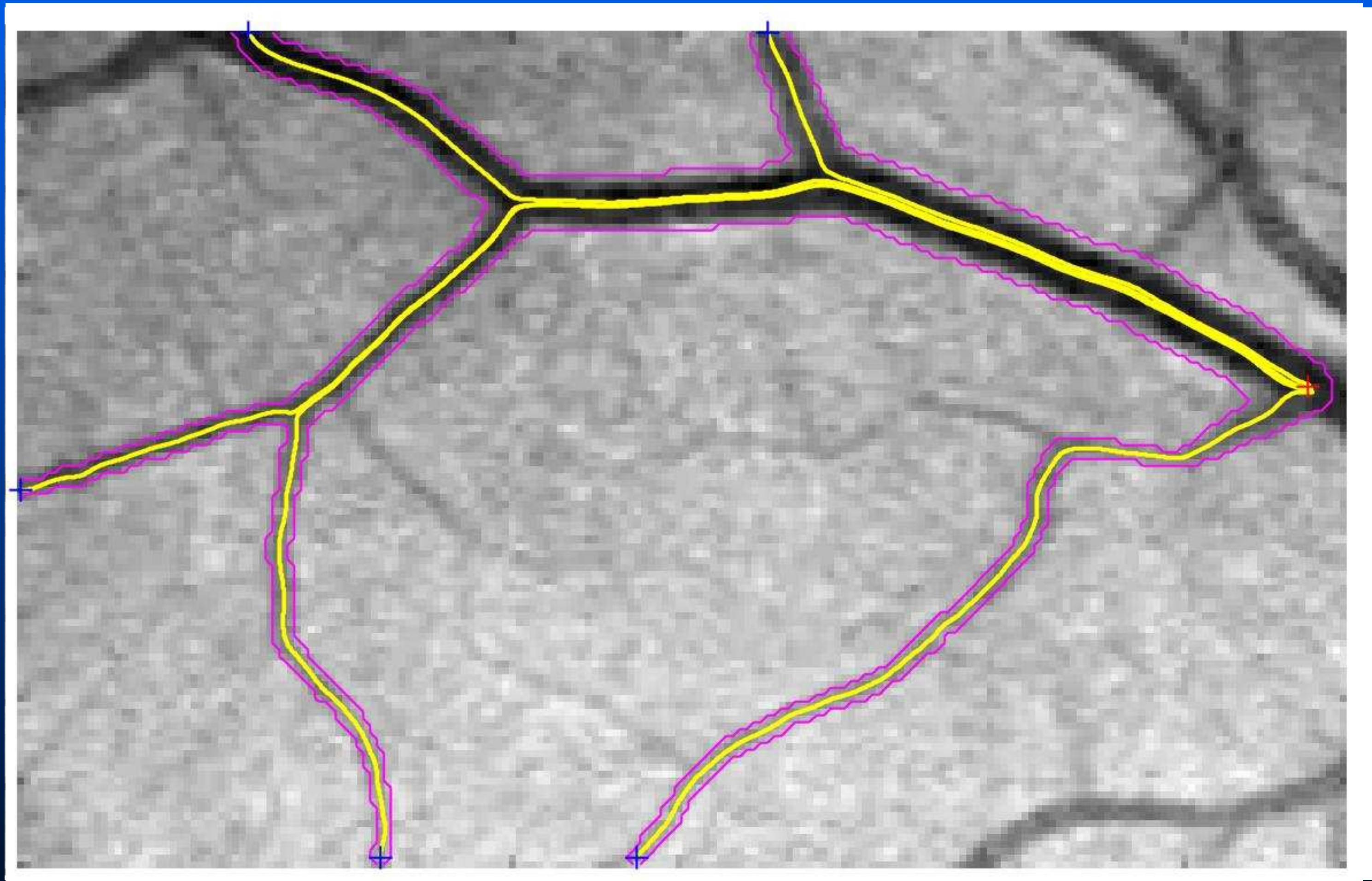
2D in space , 1D for radius of vessel



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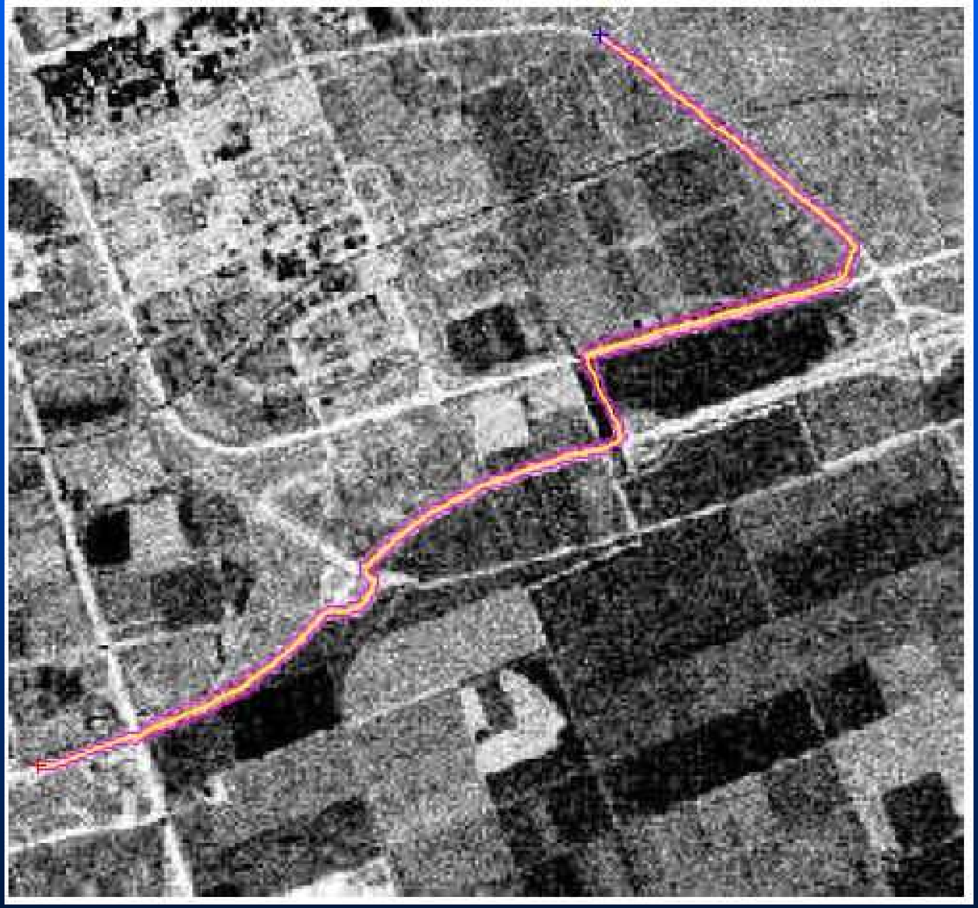
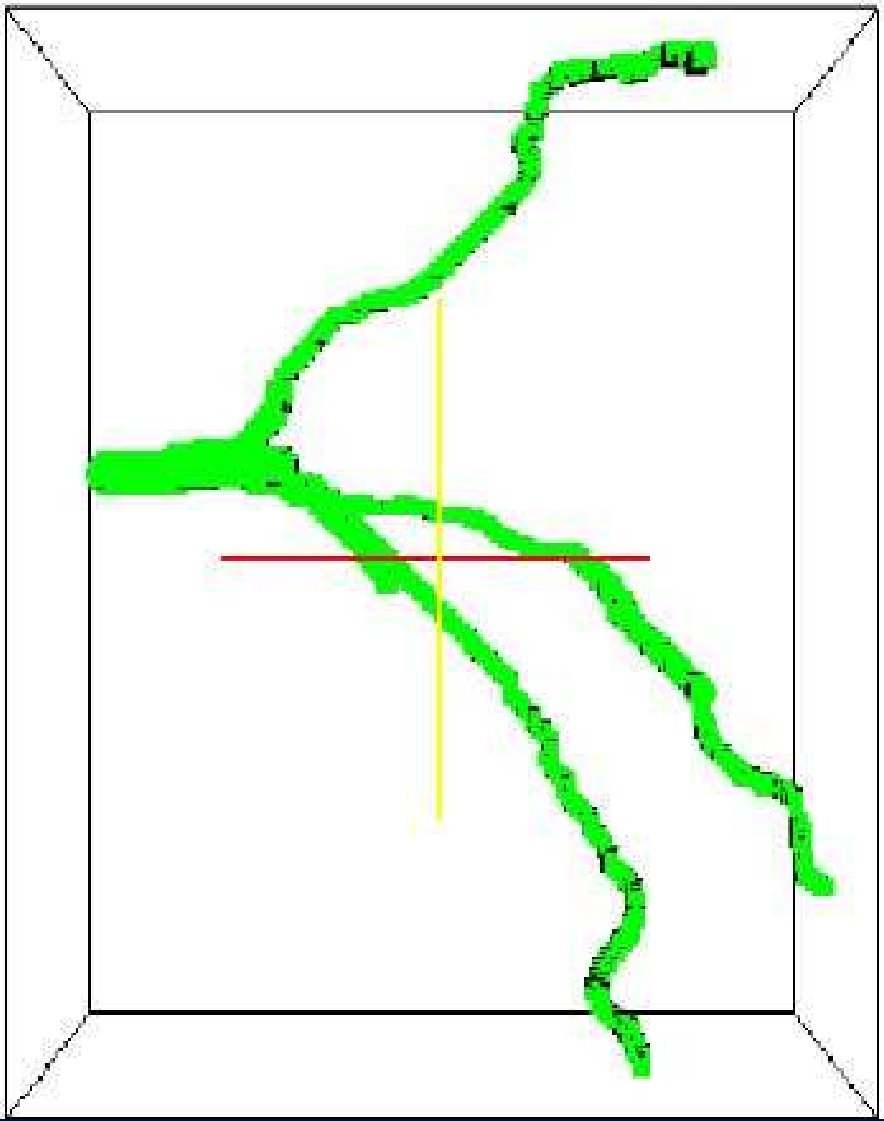
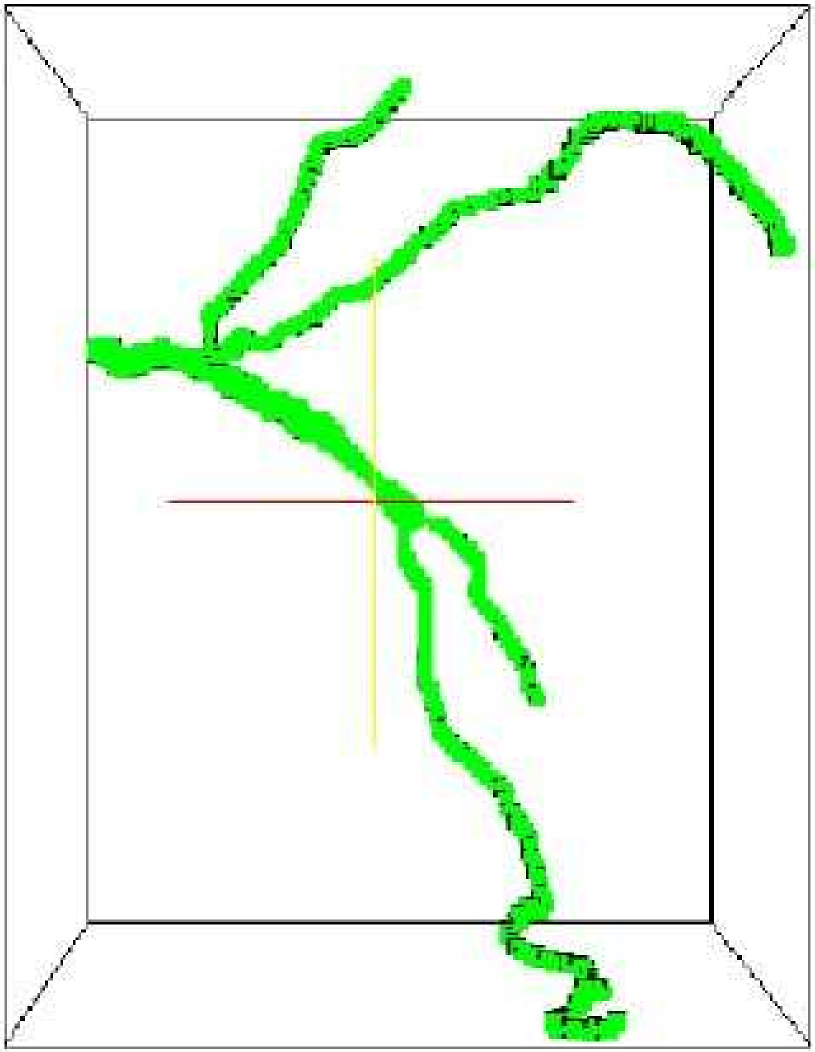
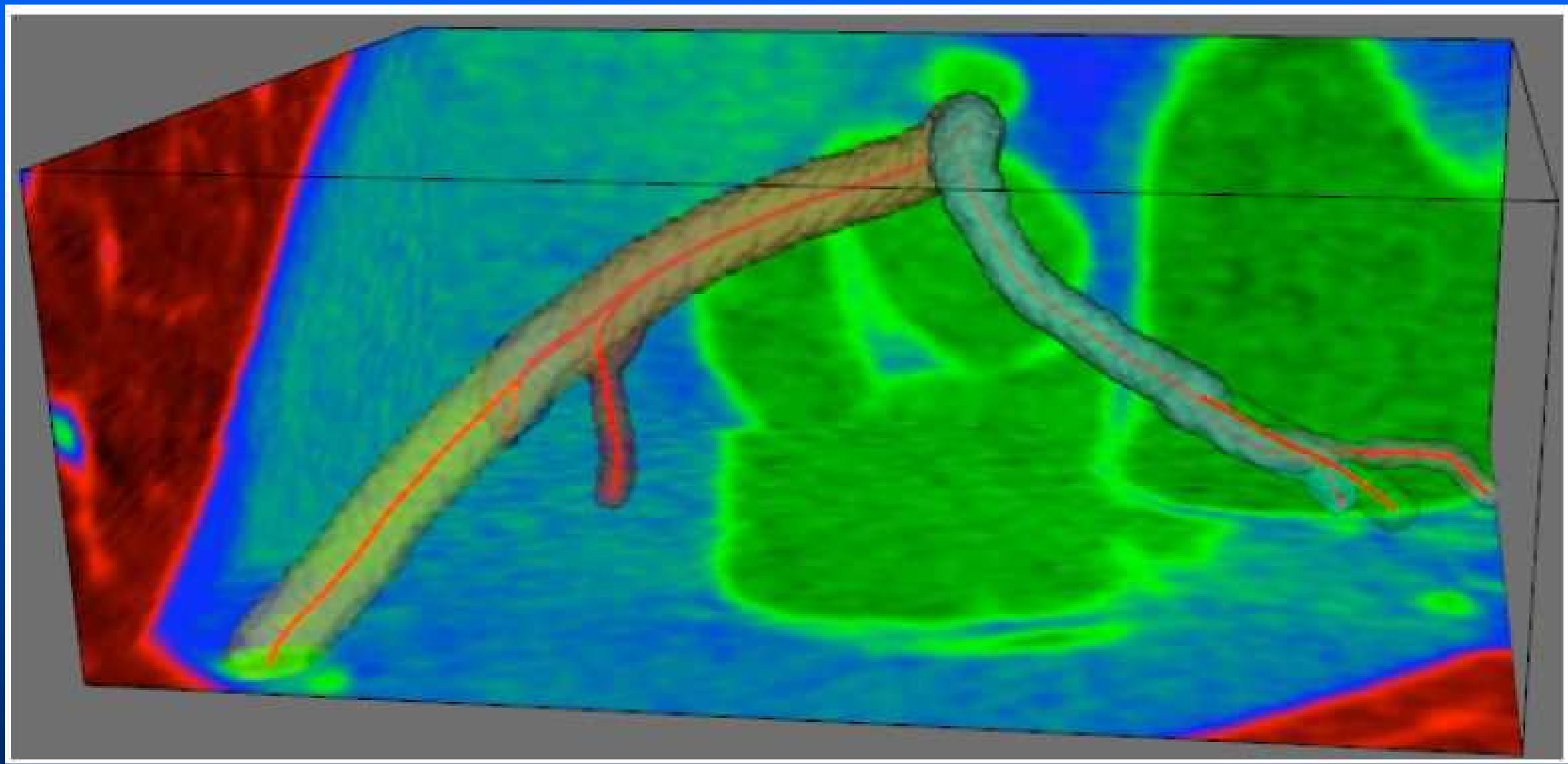


Examples of 4D Minimal Paths for tubular shapes in 3D



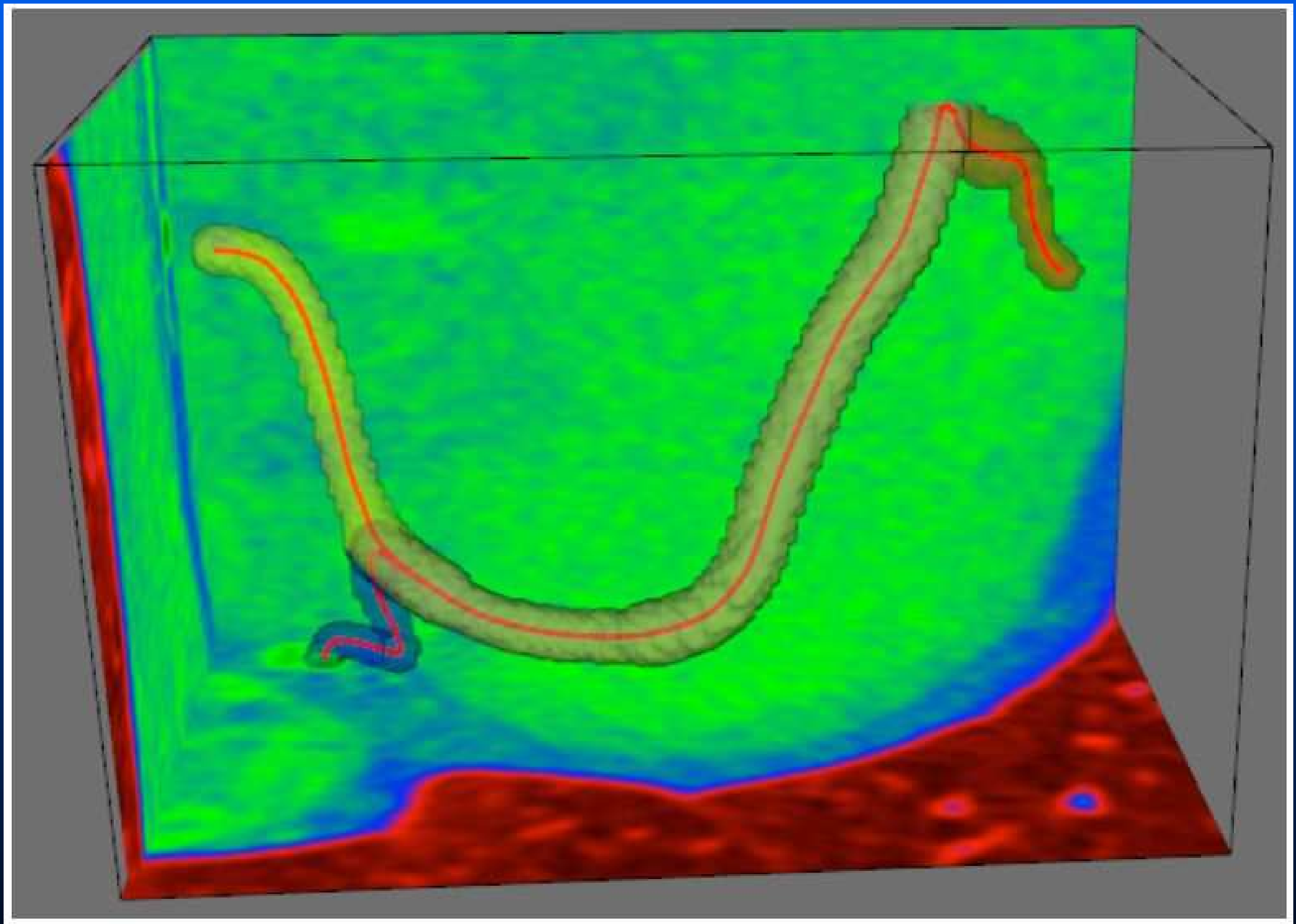


Examples of 4D Minimal Paths for tubular shapes in 3D



Examples of 4D Minimal Paths for tubular shapes in 3D

3D in space , 1D for radius of vessel



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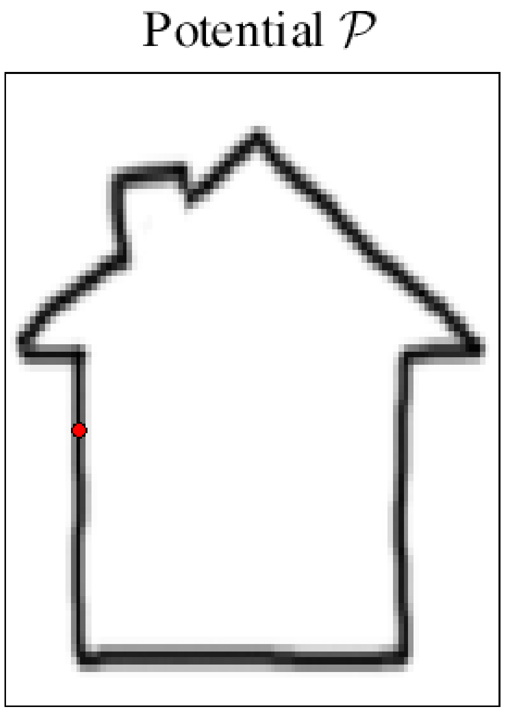
Finding a closed contour by growing minimal paths and adding keypoints



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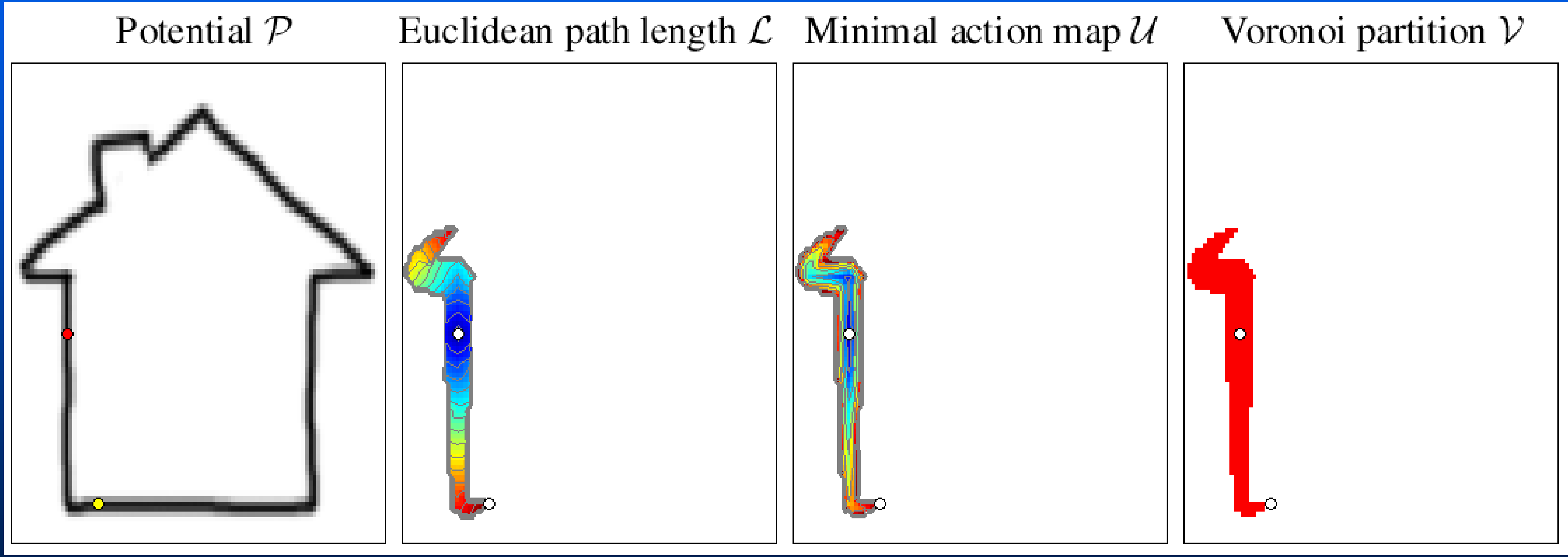

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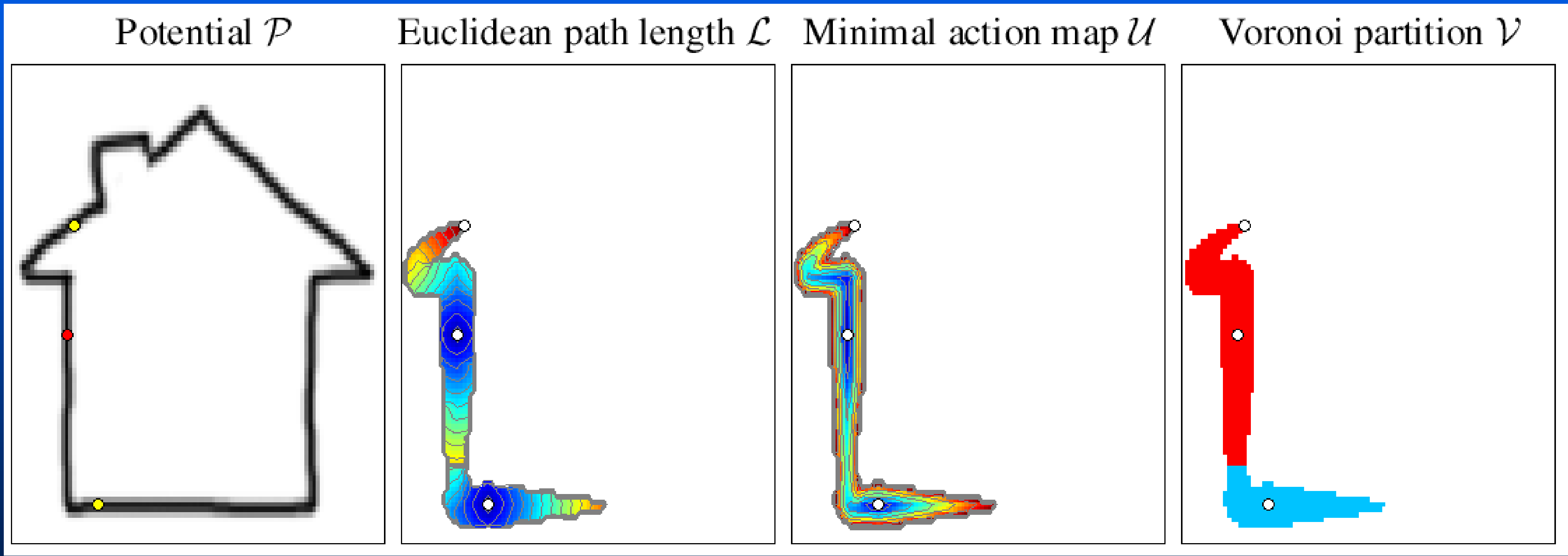

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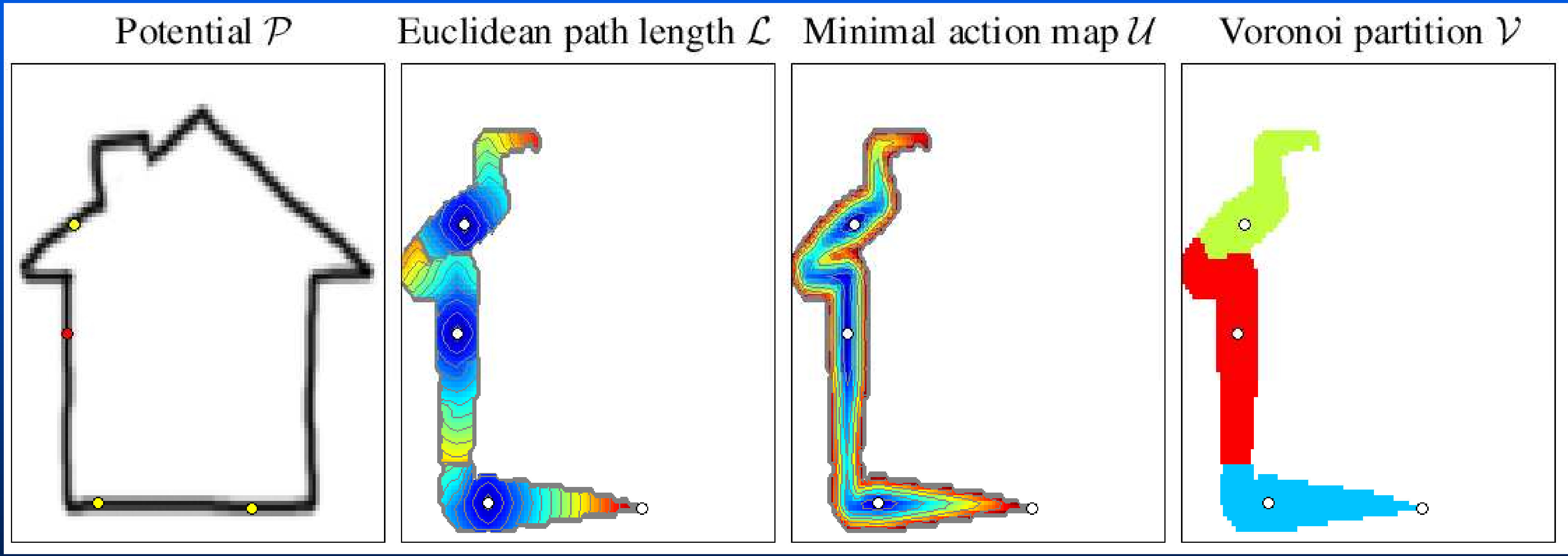

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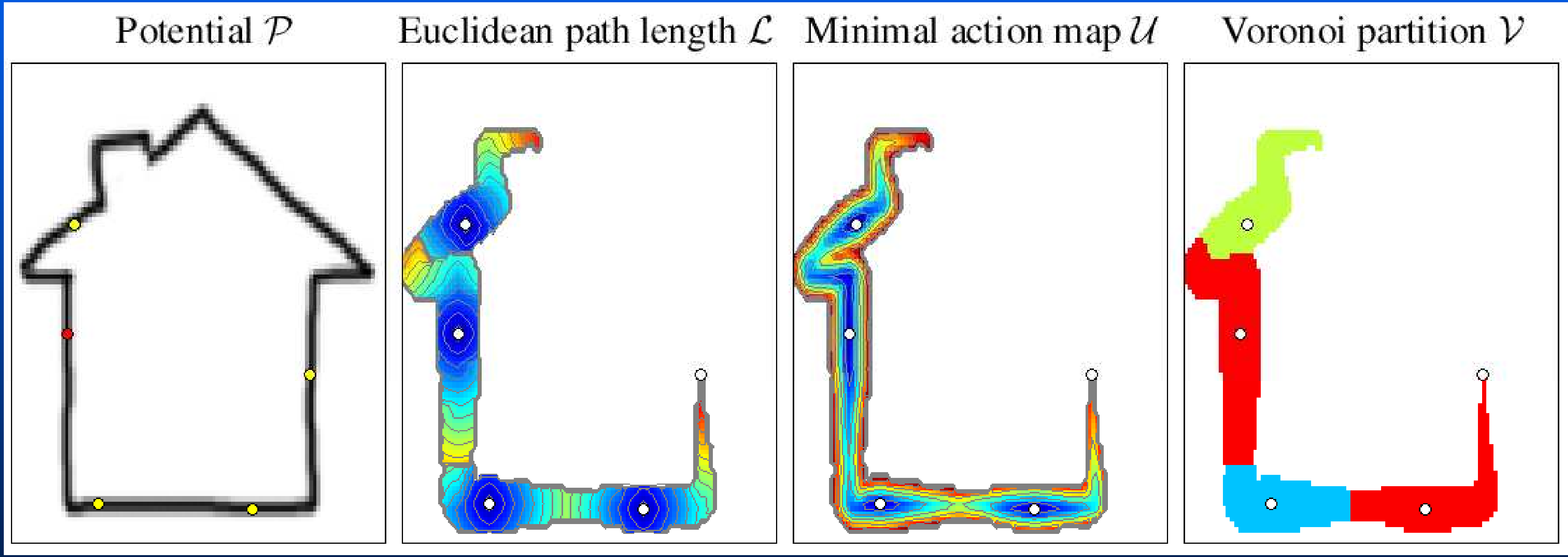

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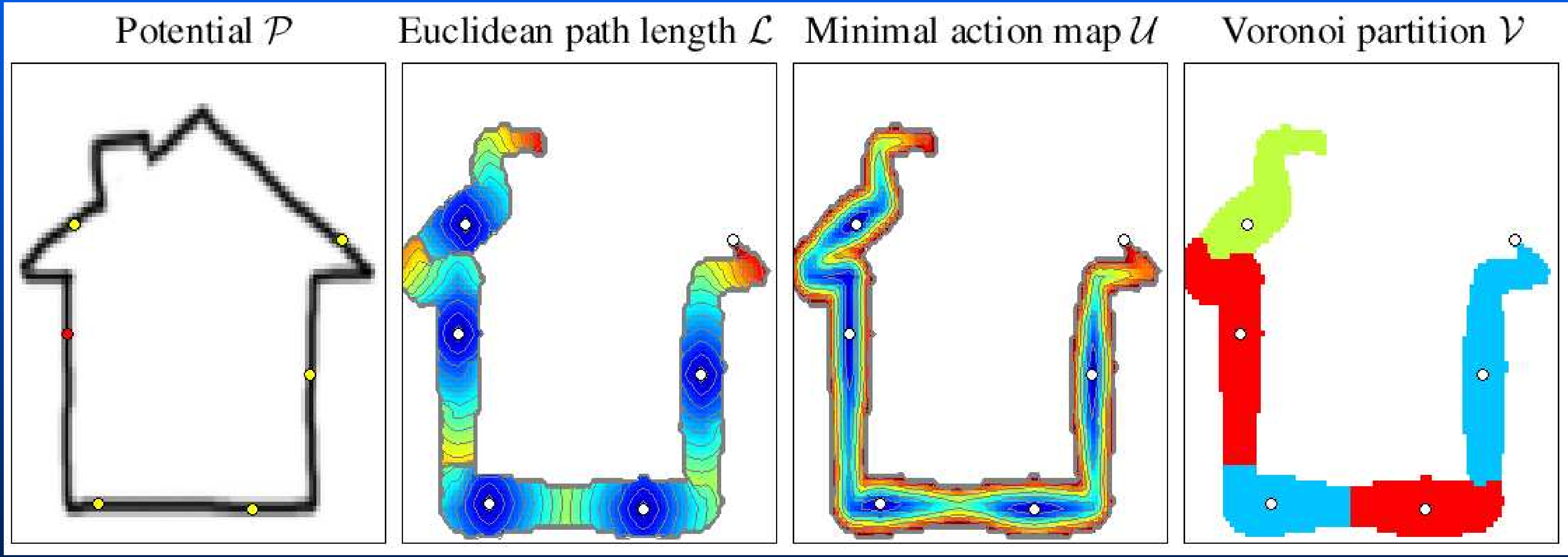

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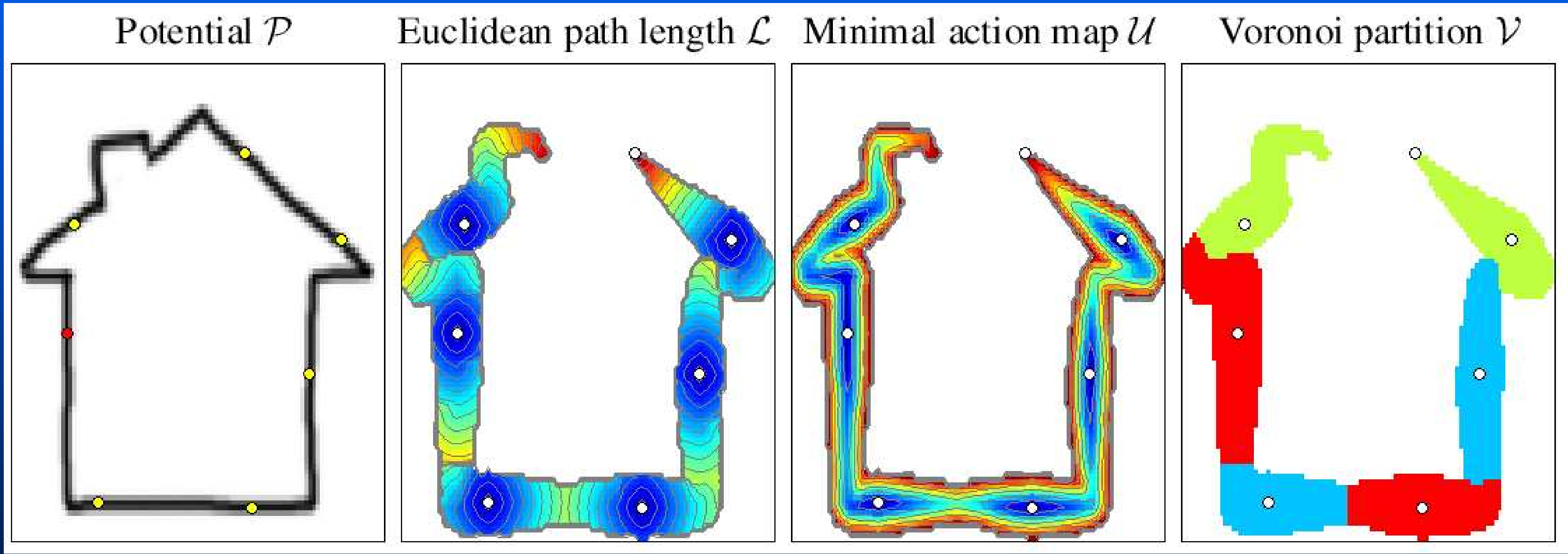

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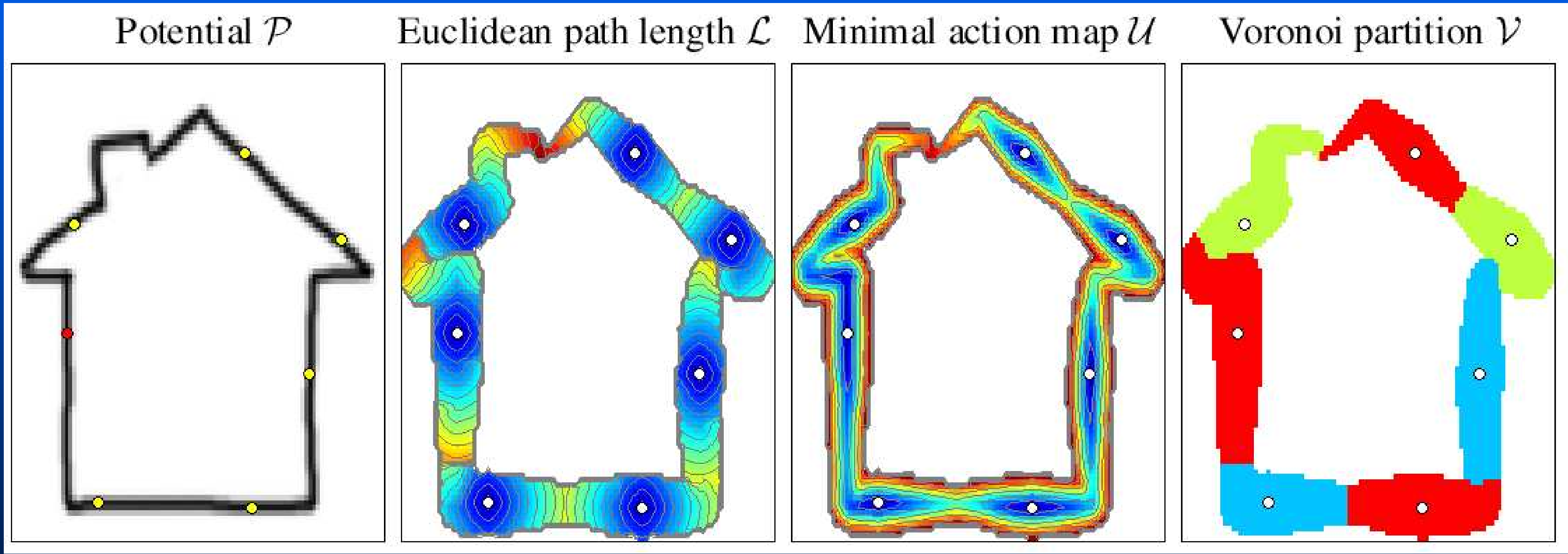
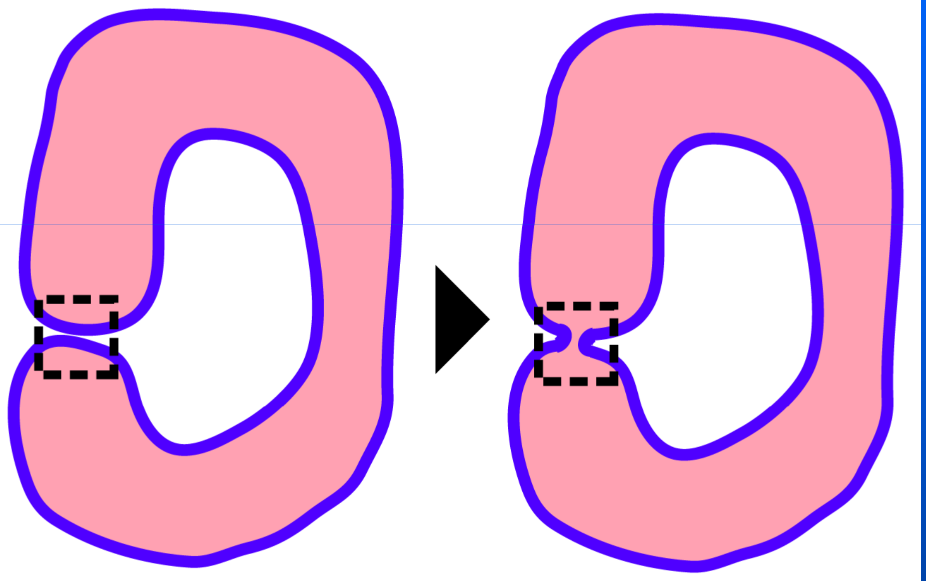
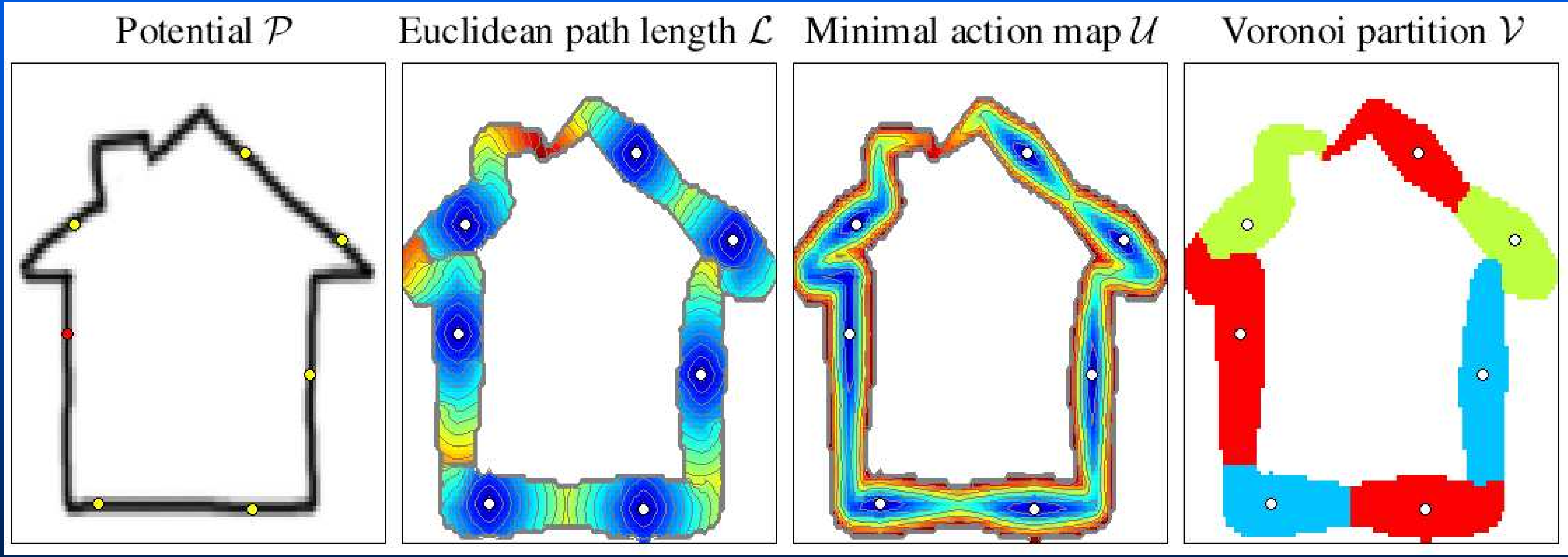


Adding keypoints: Stopping criterion

The propagation must be stopped as soon as the domain visited by the fronts has the same topology as a ring.



Finding a closed contour by growing minimal paths and adding keypoints



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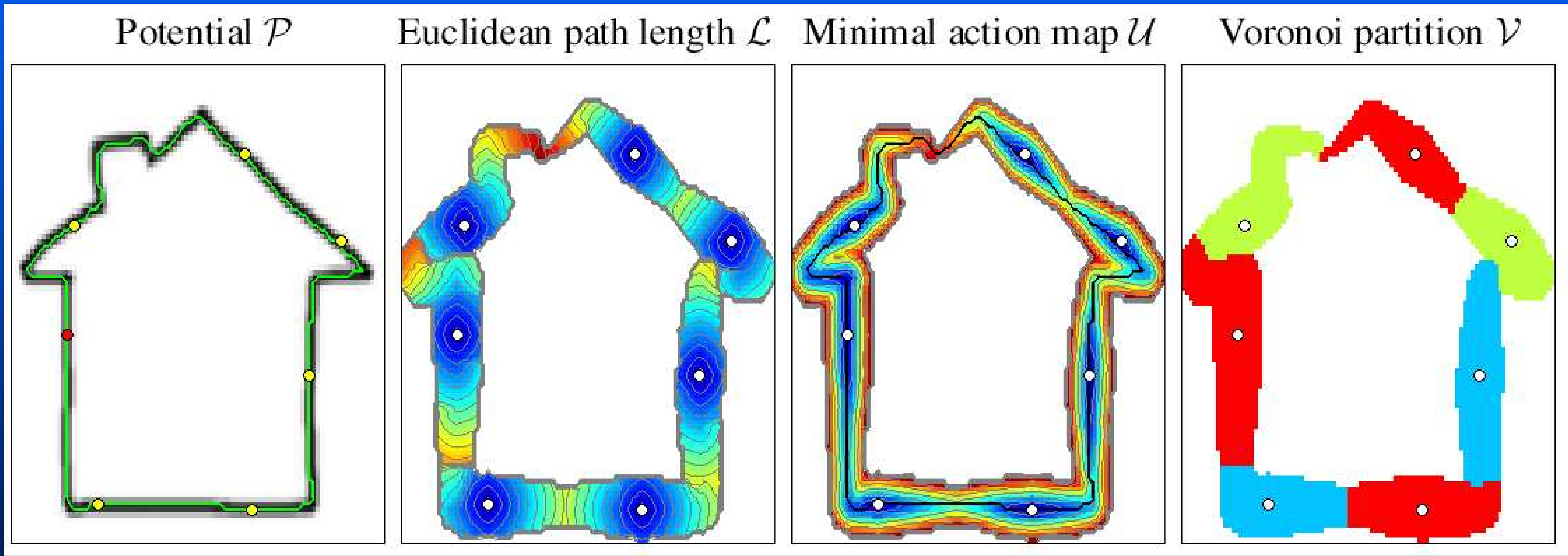

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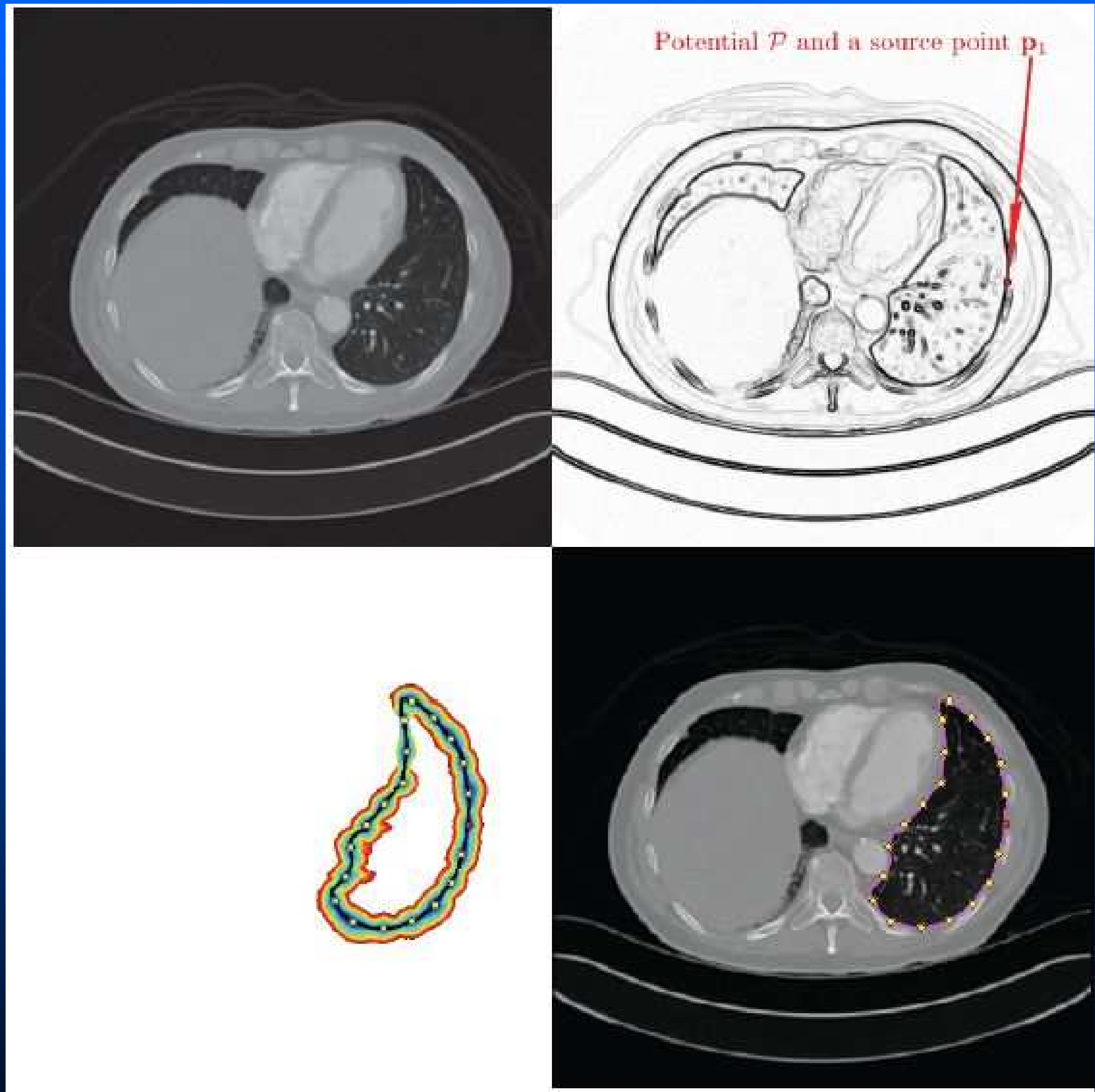
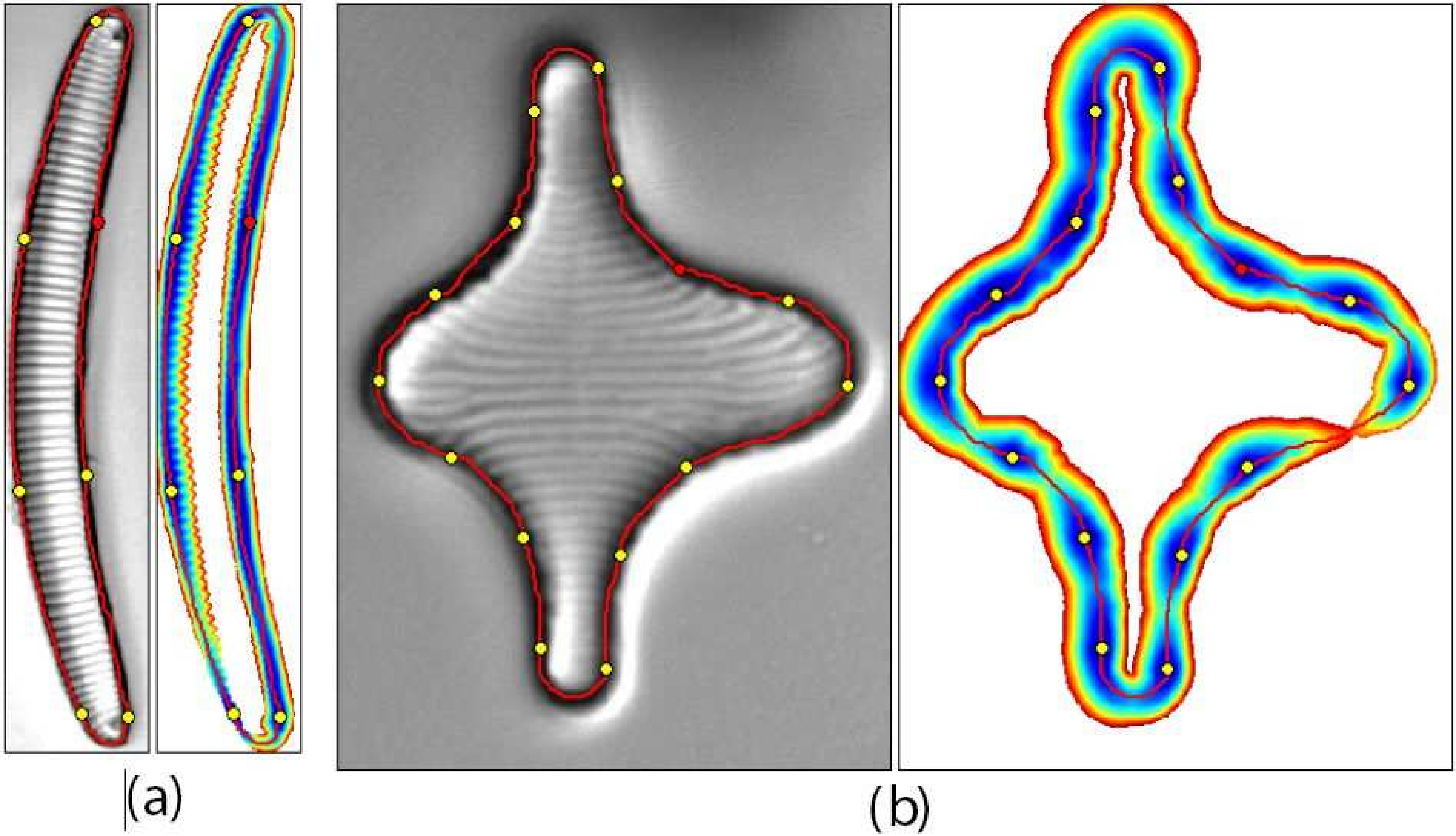
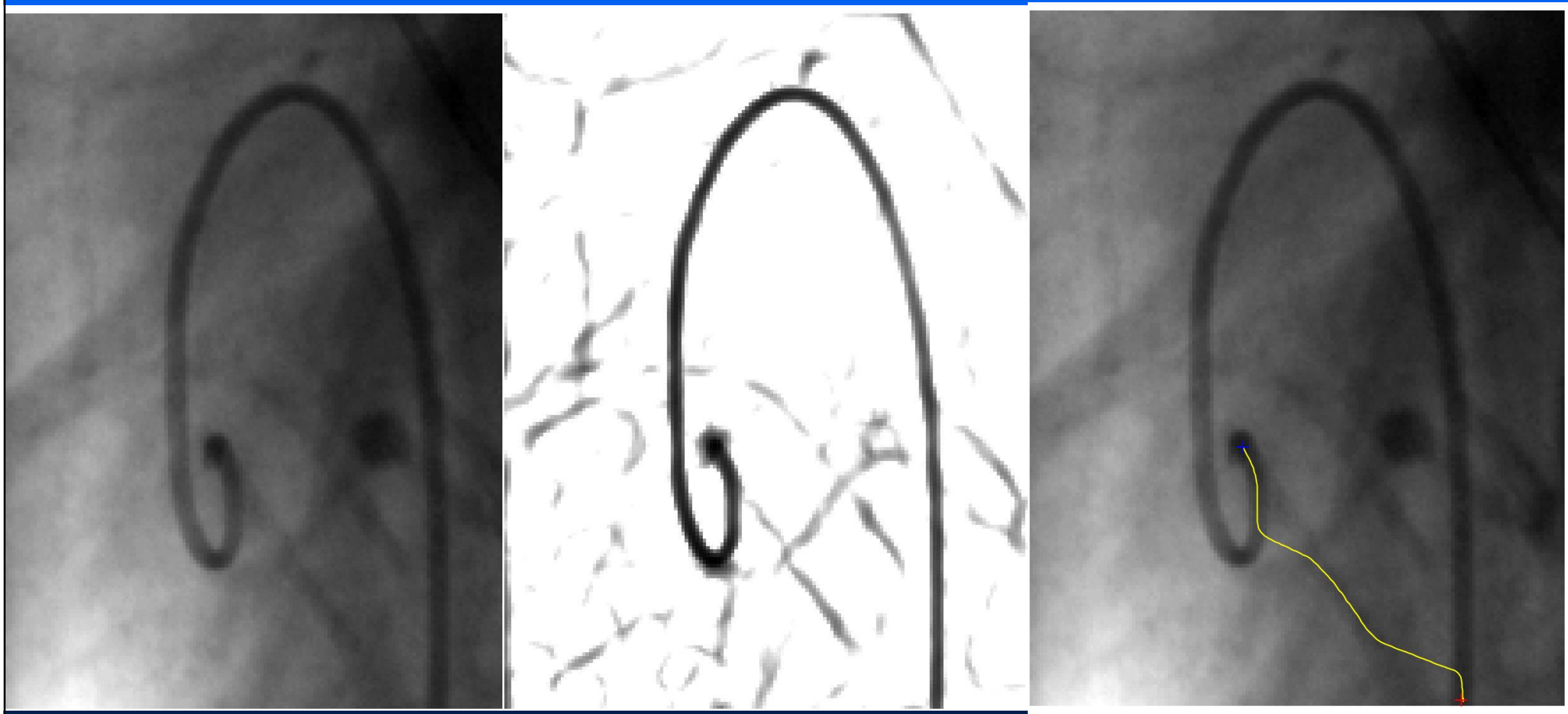


Finding a closed contour by growing minimal paths



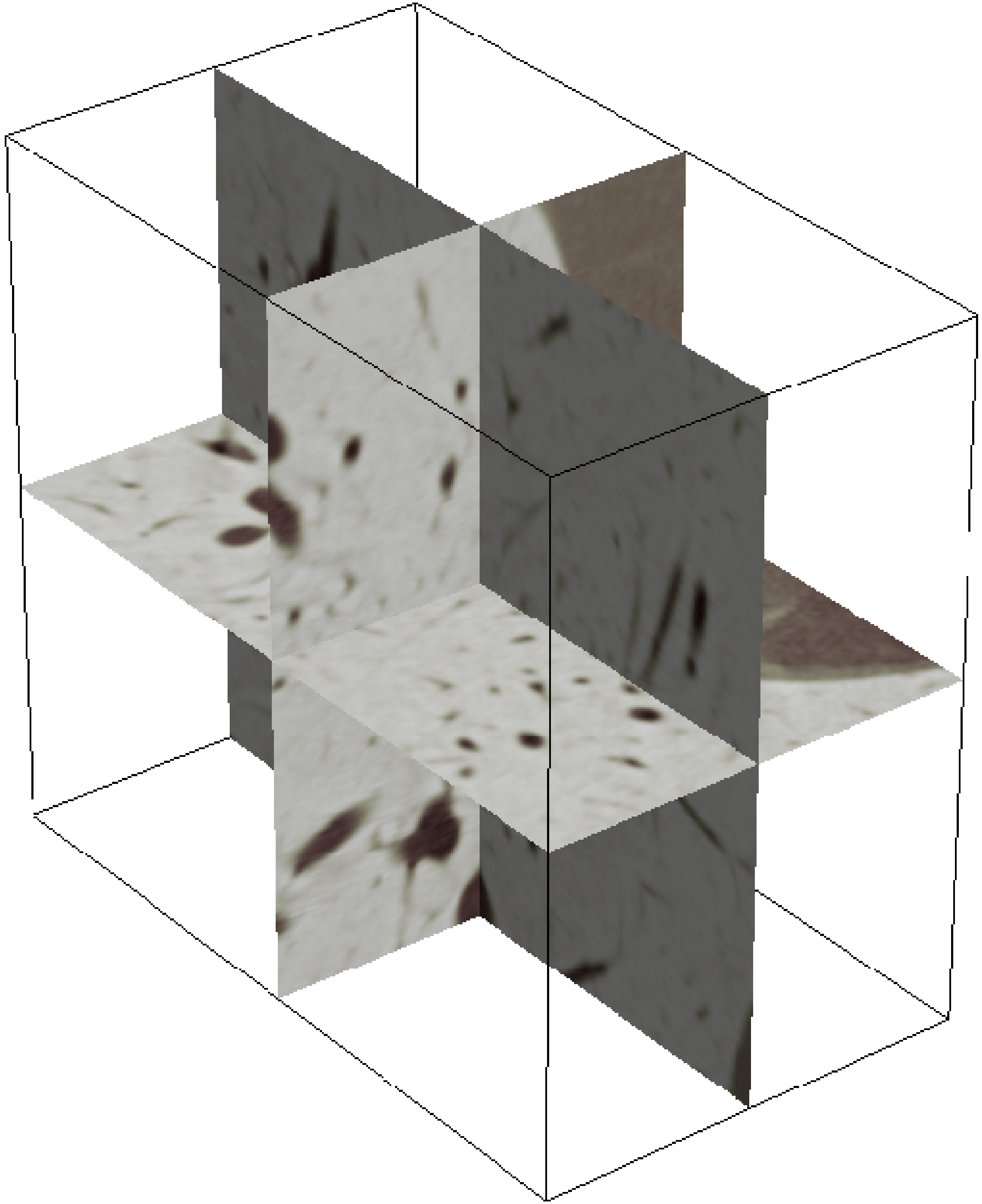
Finding a contour between two points by growing minimal paths

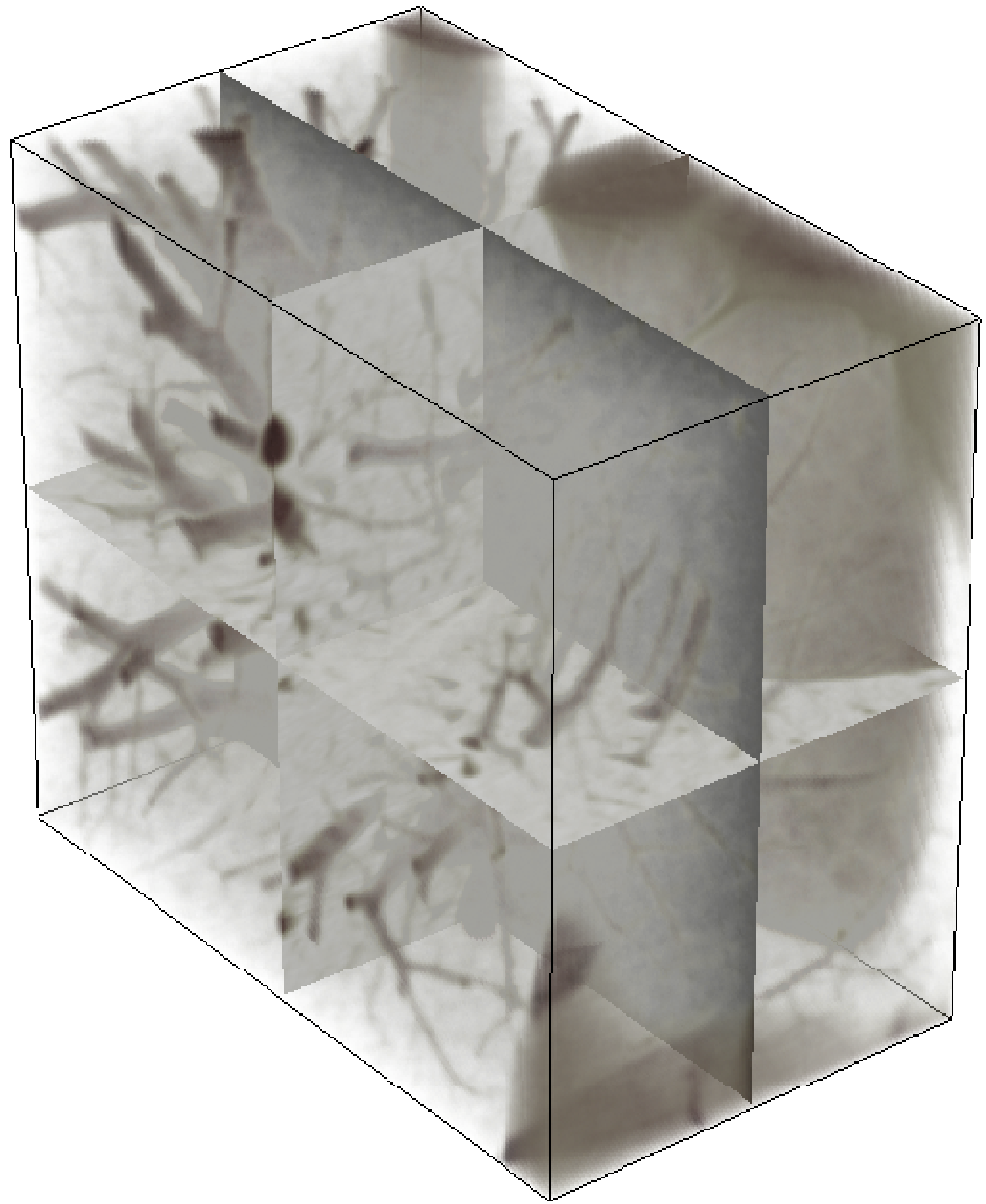


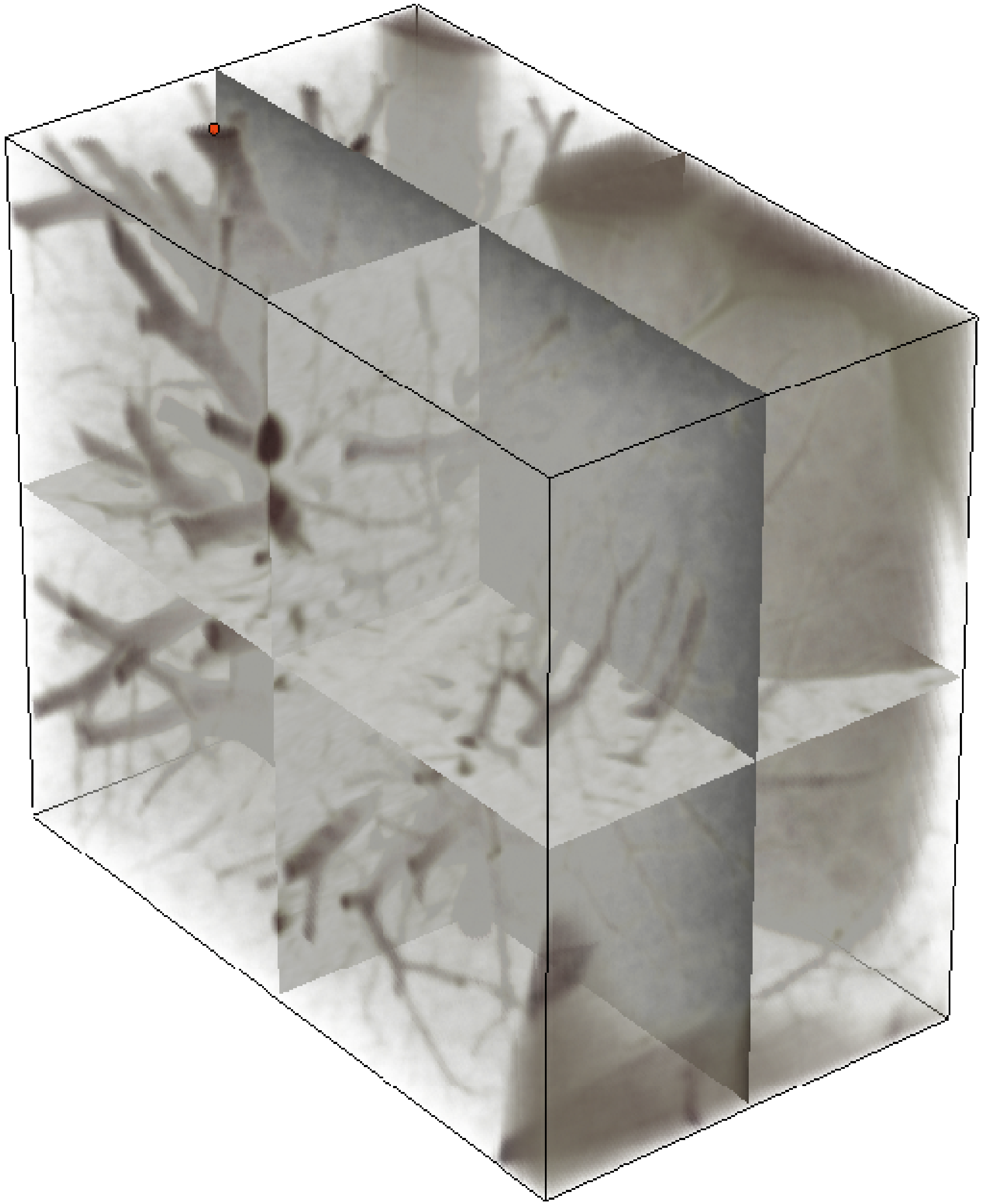
Finding a contour between two points by growing minimal paths

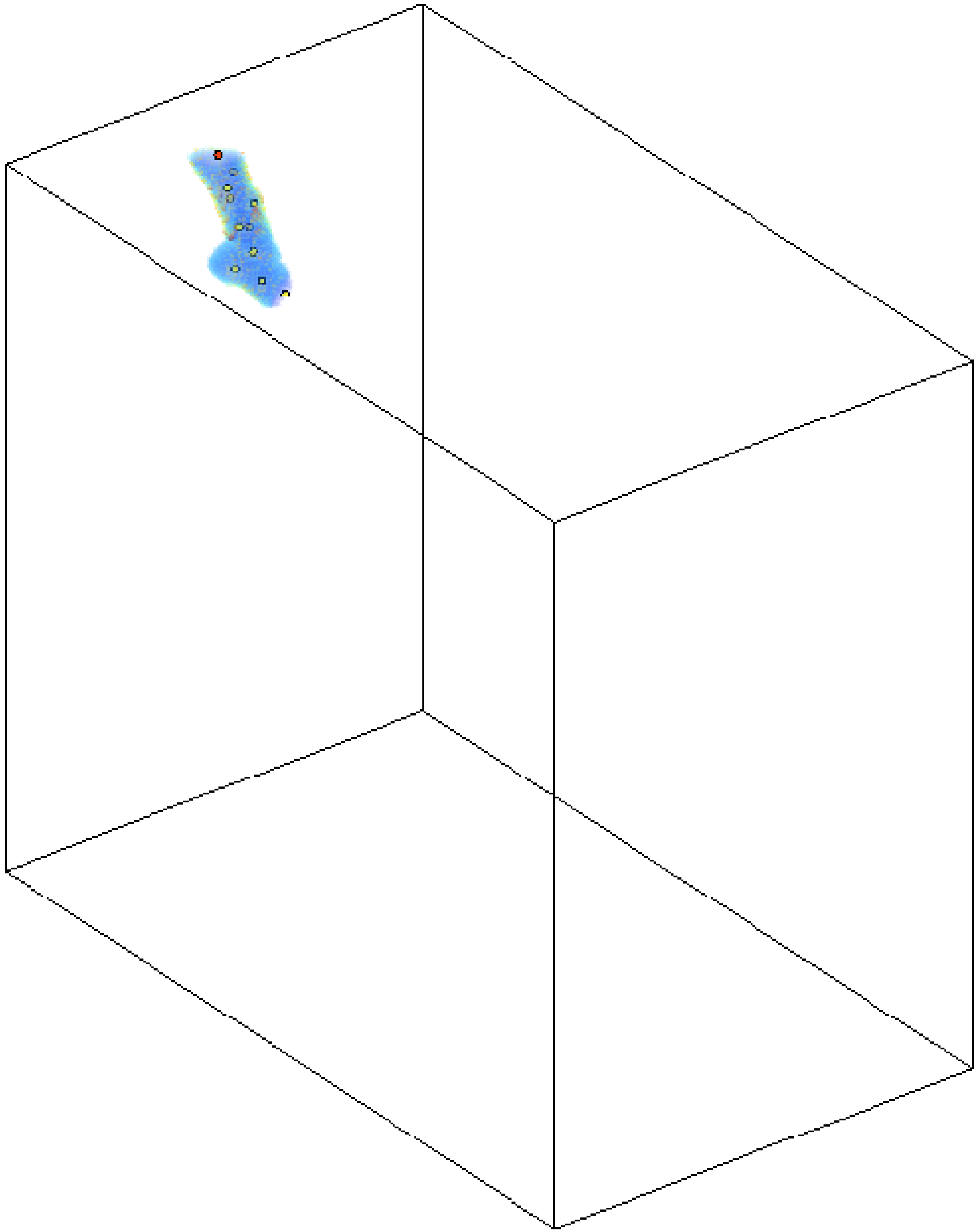
Extension to 3D vessel segmentation

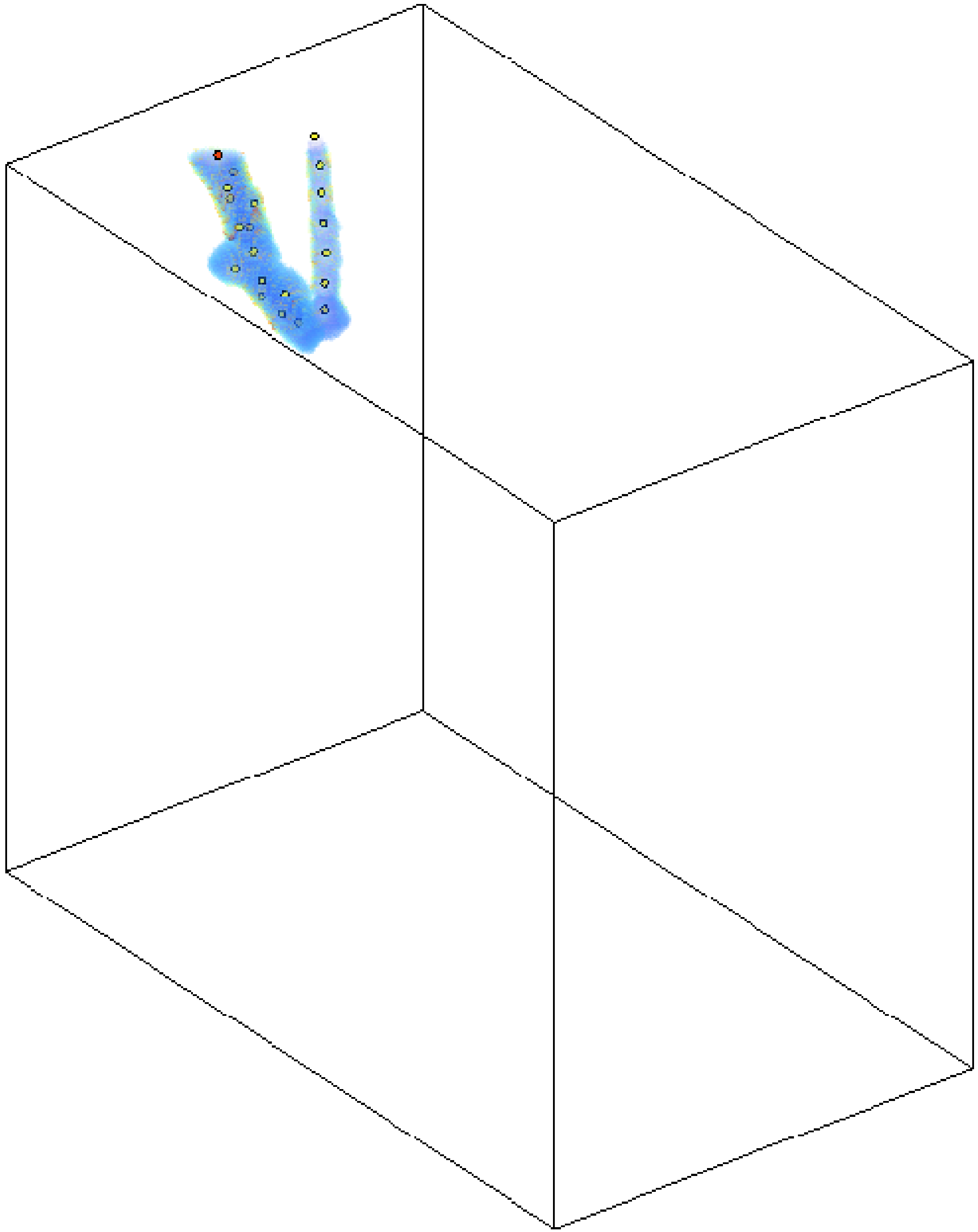
Example of results of the keypoints method in a 3D image of Pulmonary Arteries

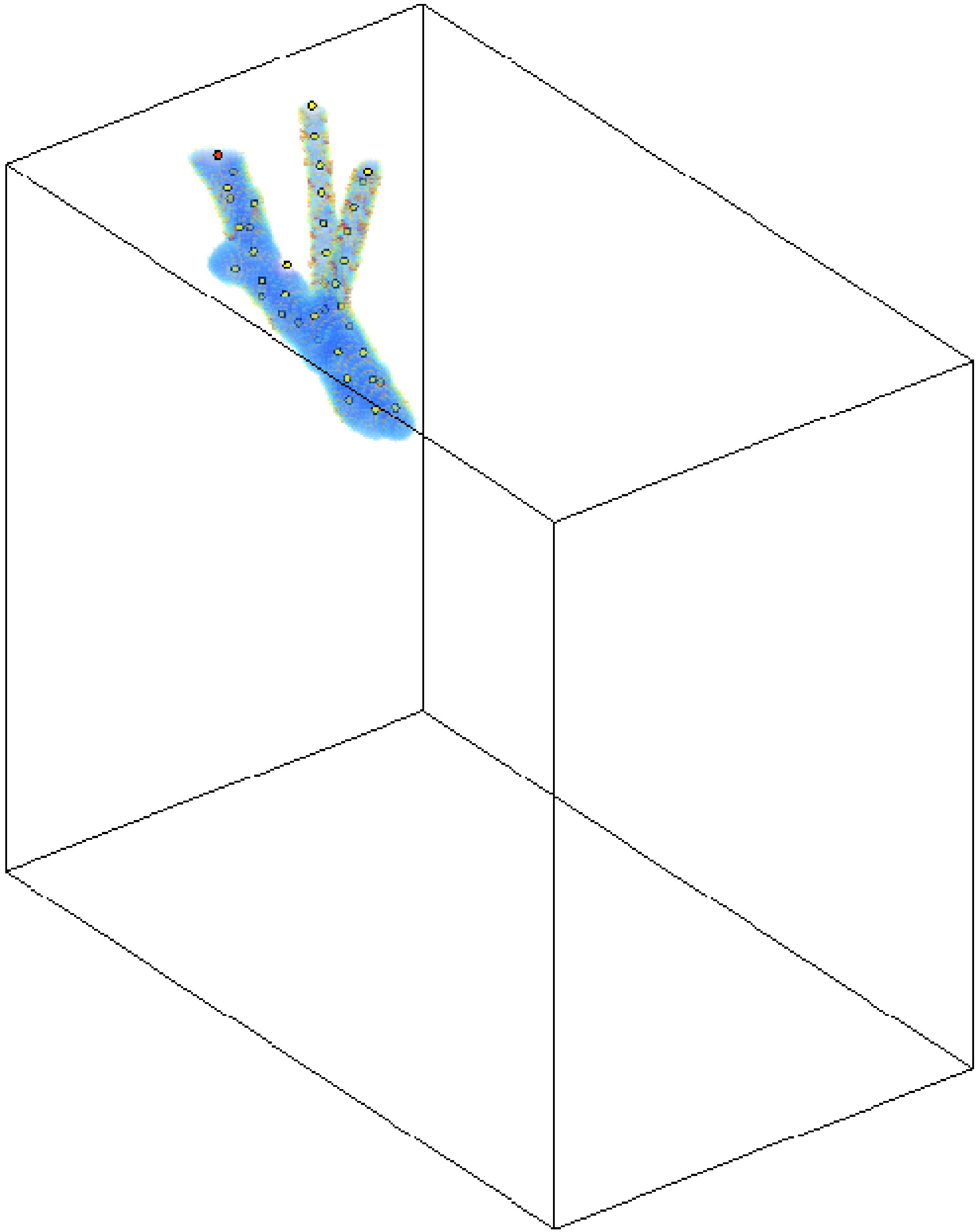


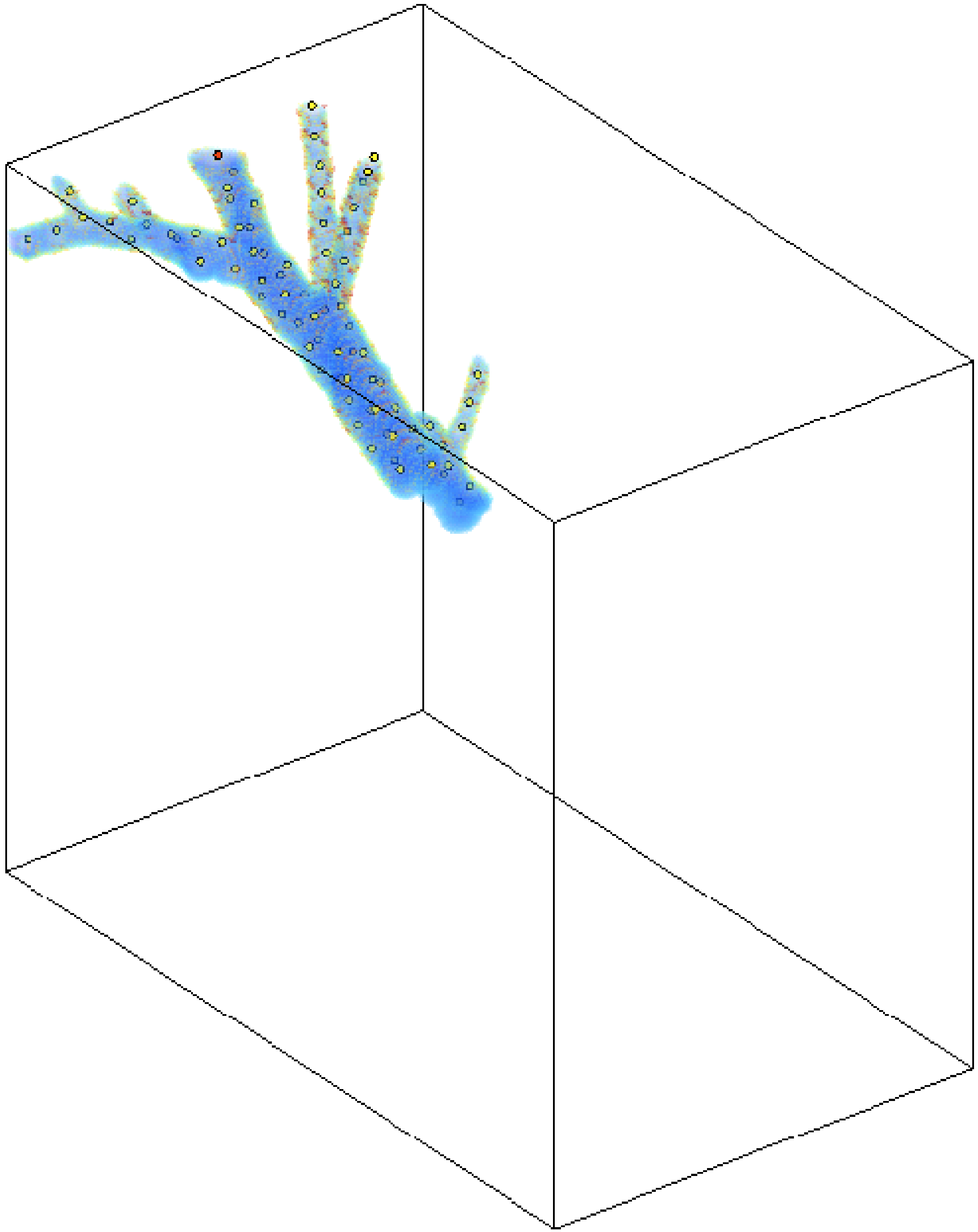


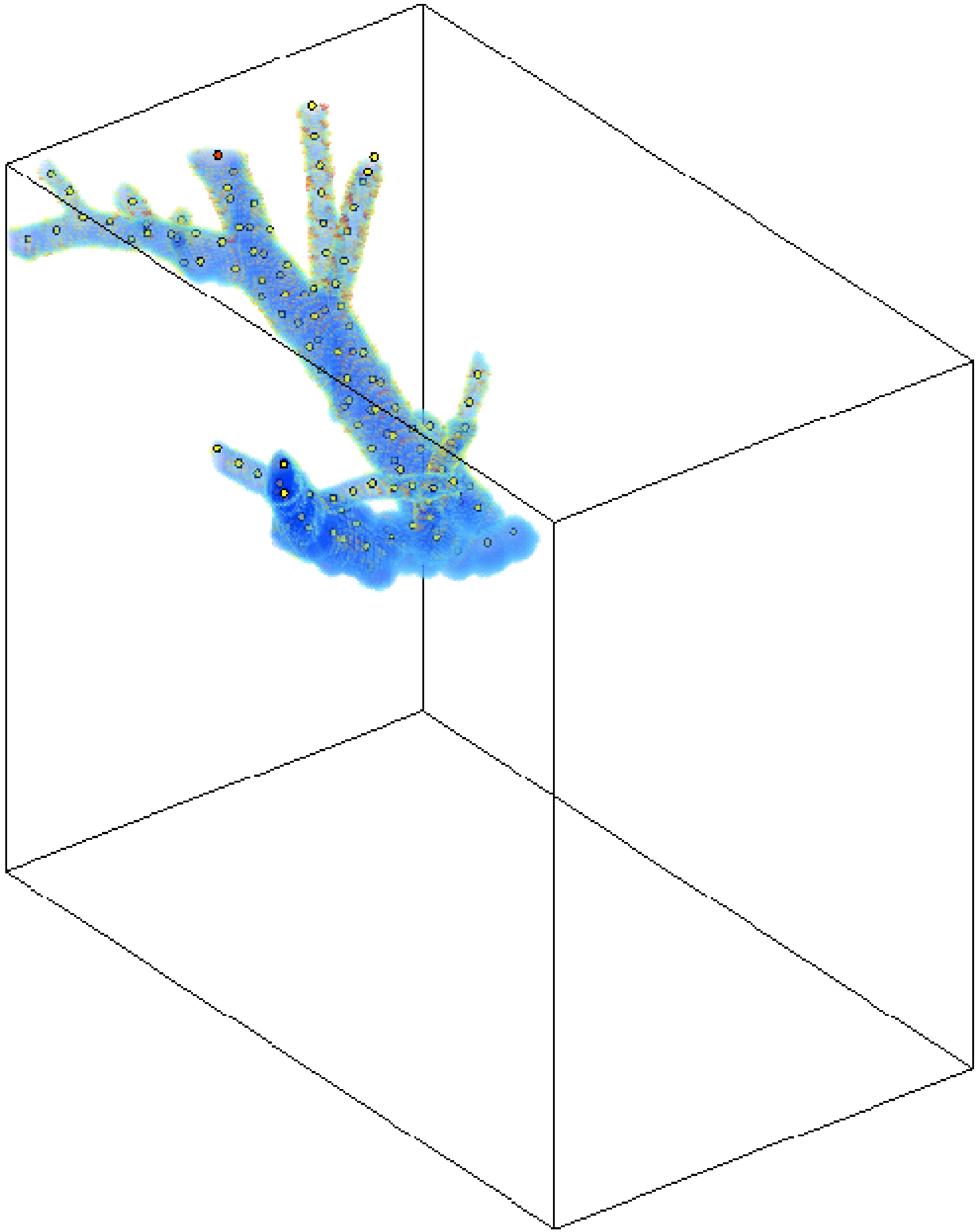


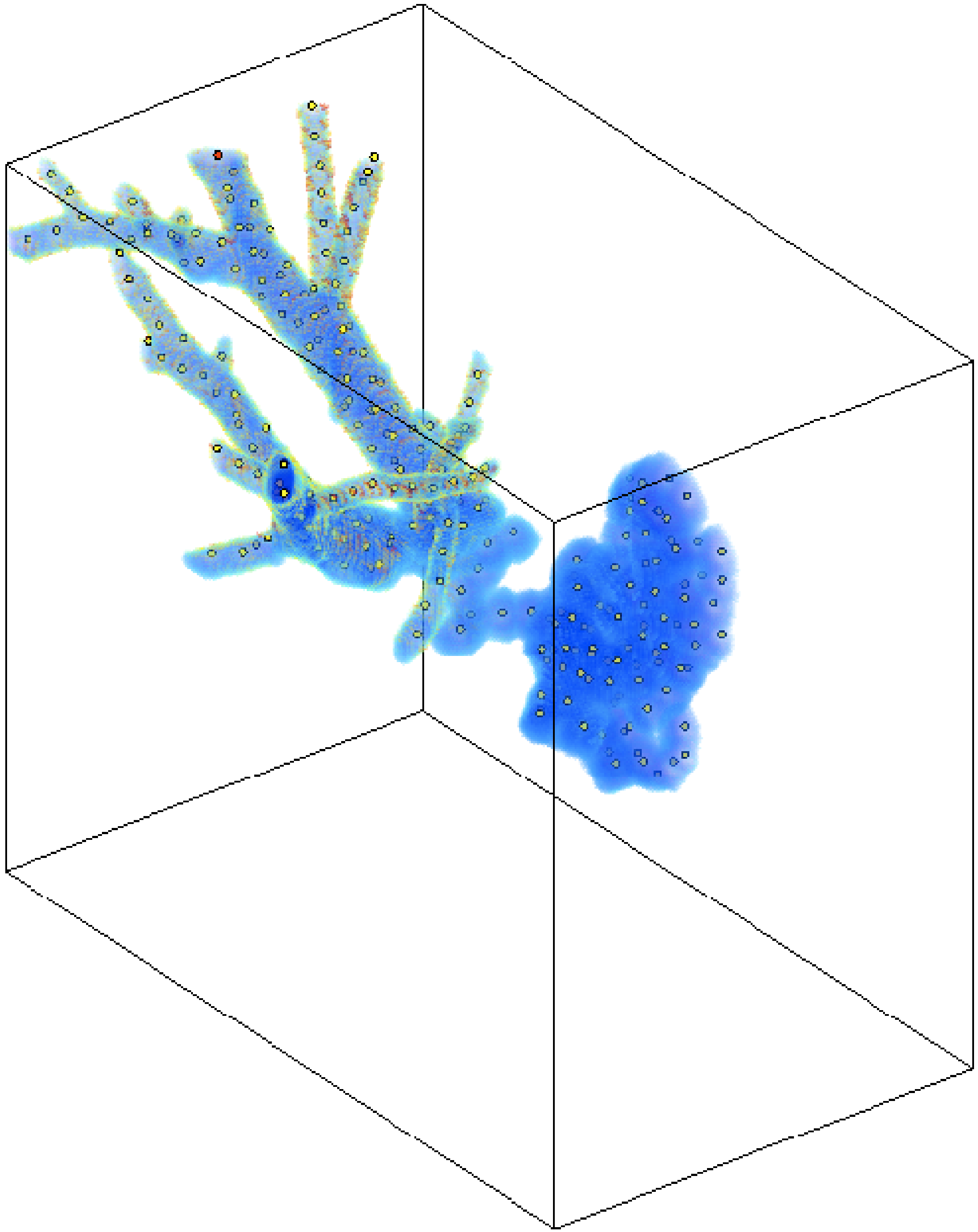


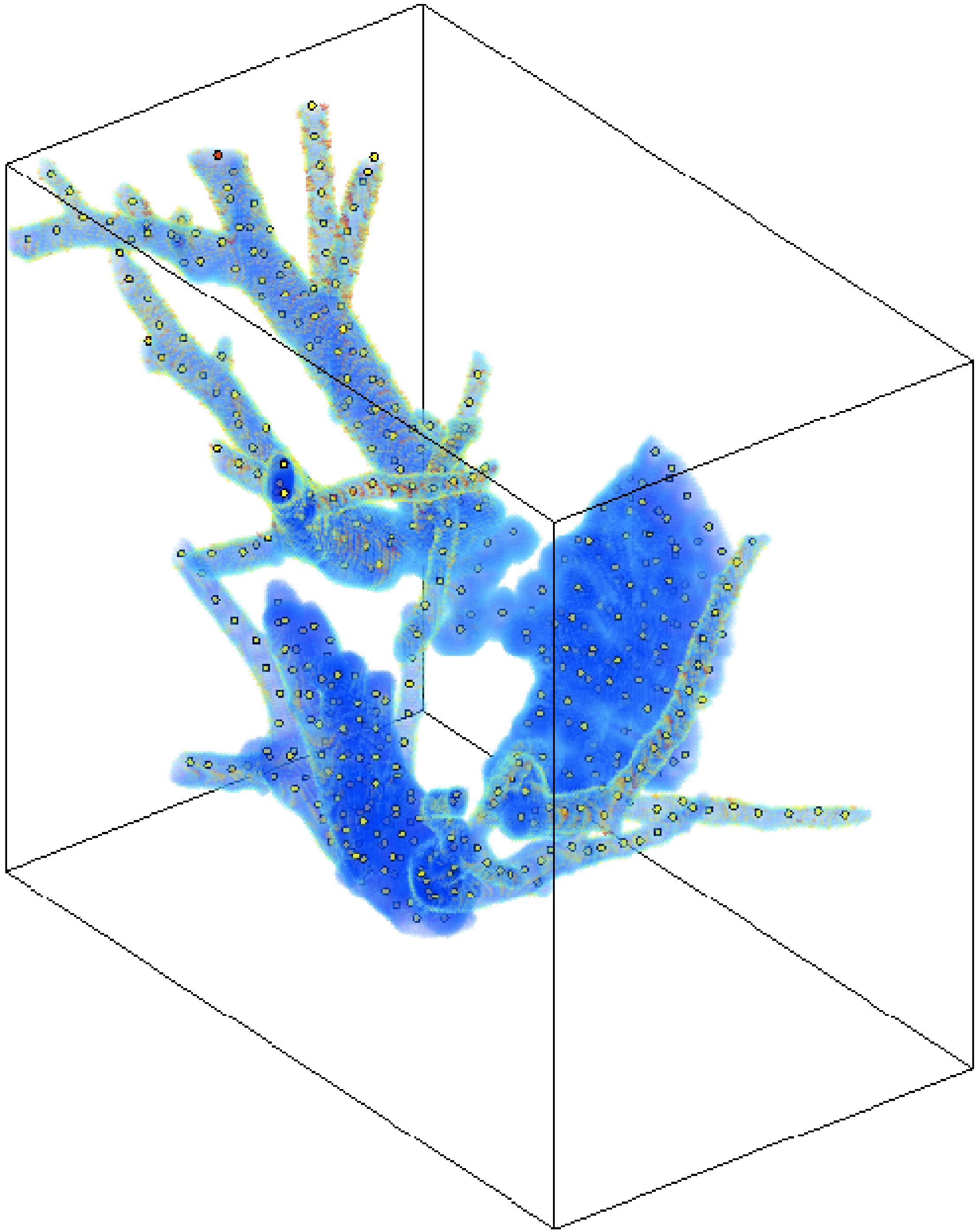












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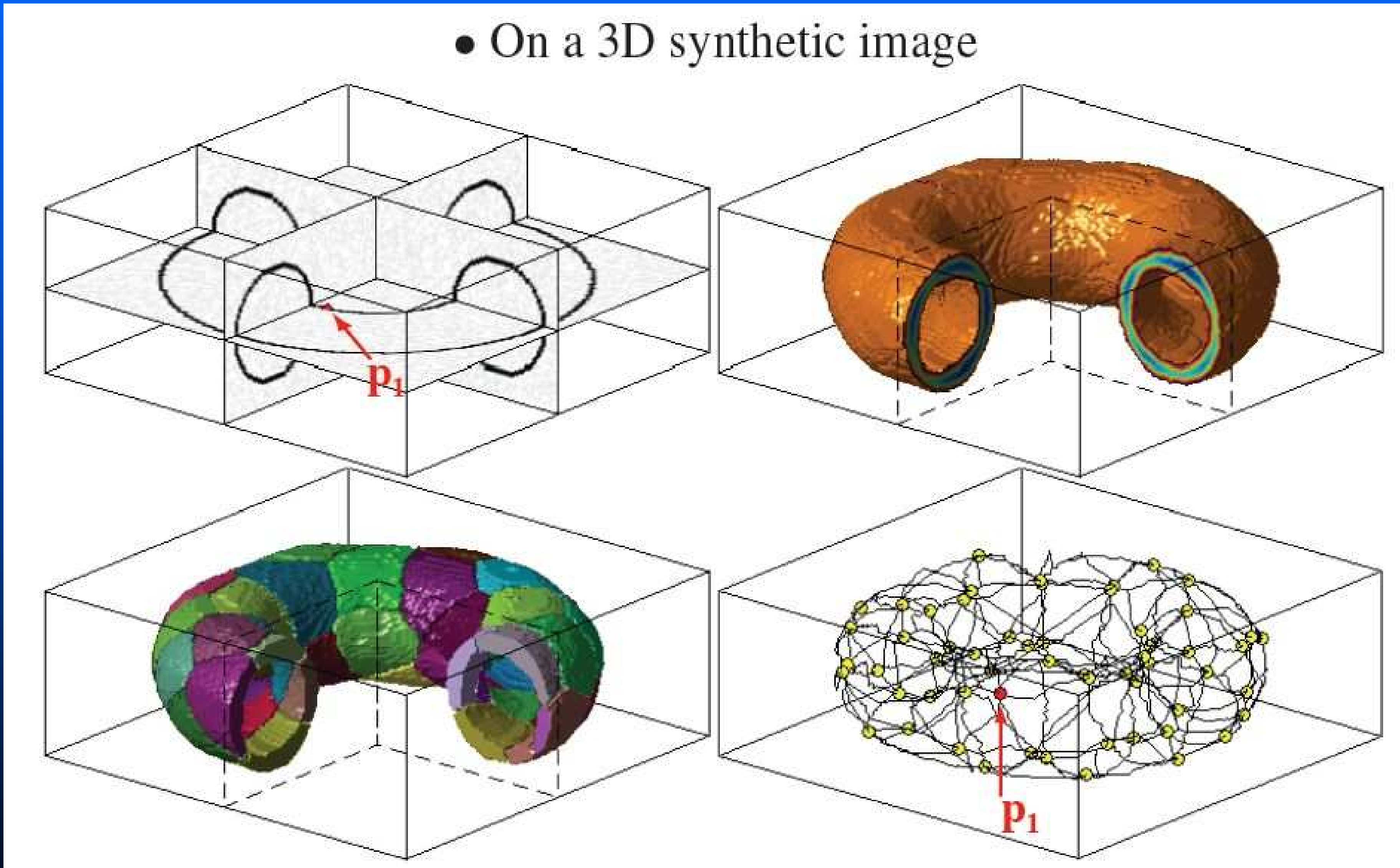
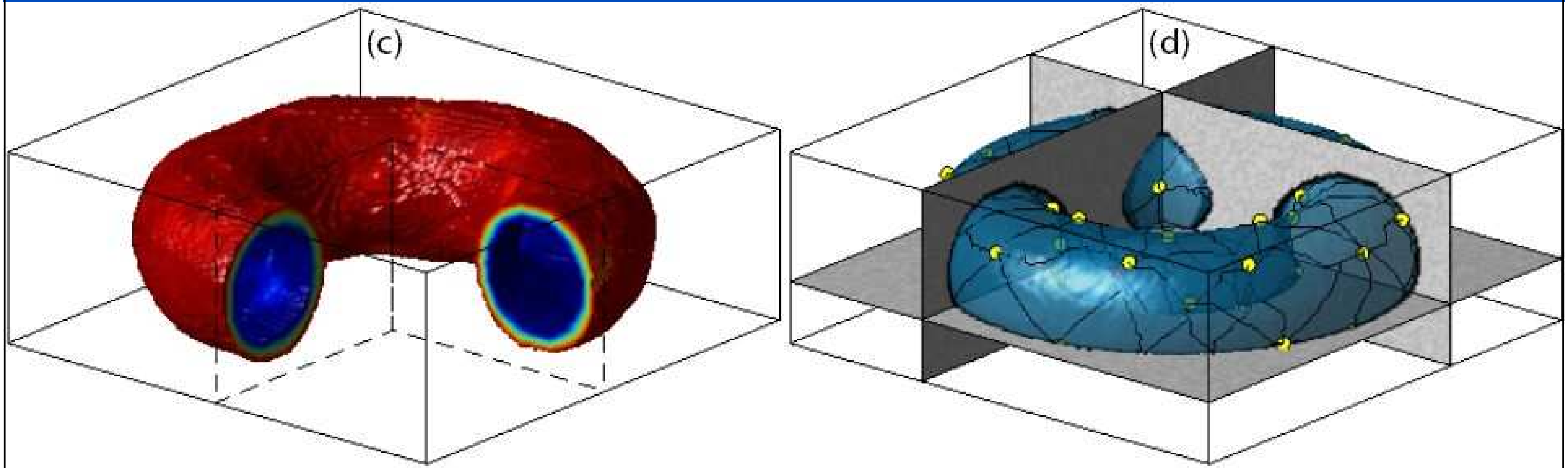
3D extension: Finding a closed surface by growing minimal paths. Result is a Geodesic Mesh

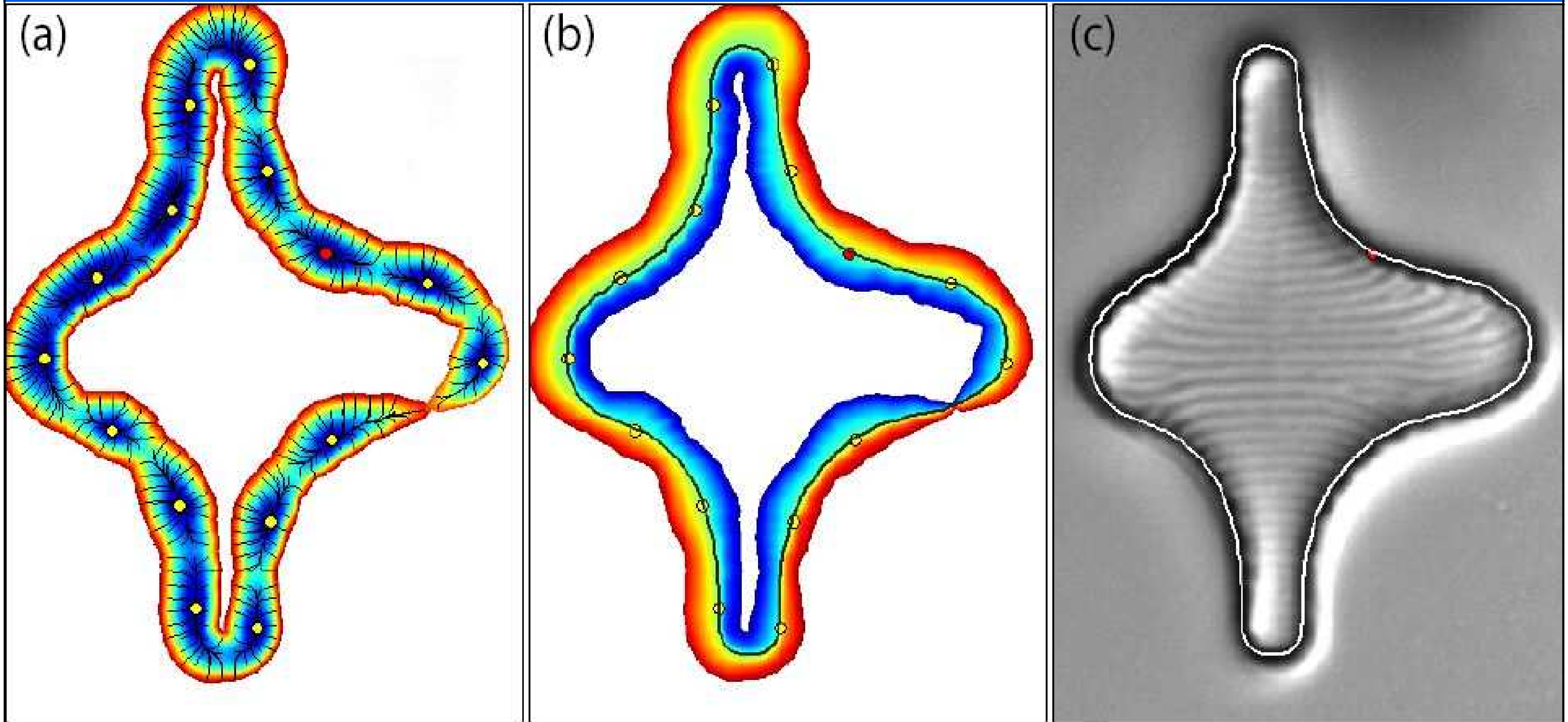


3D extension: Finding a closed surface by growing minimal paths.
Result is a Geodesic Mesh

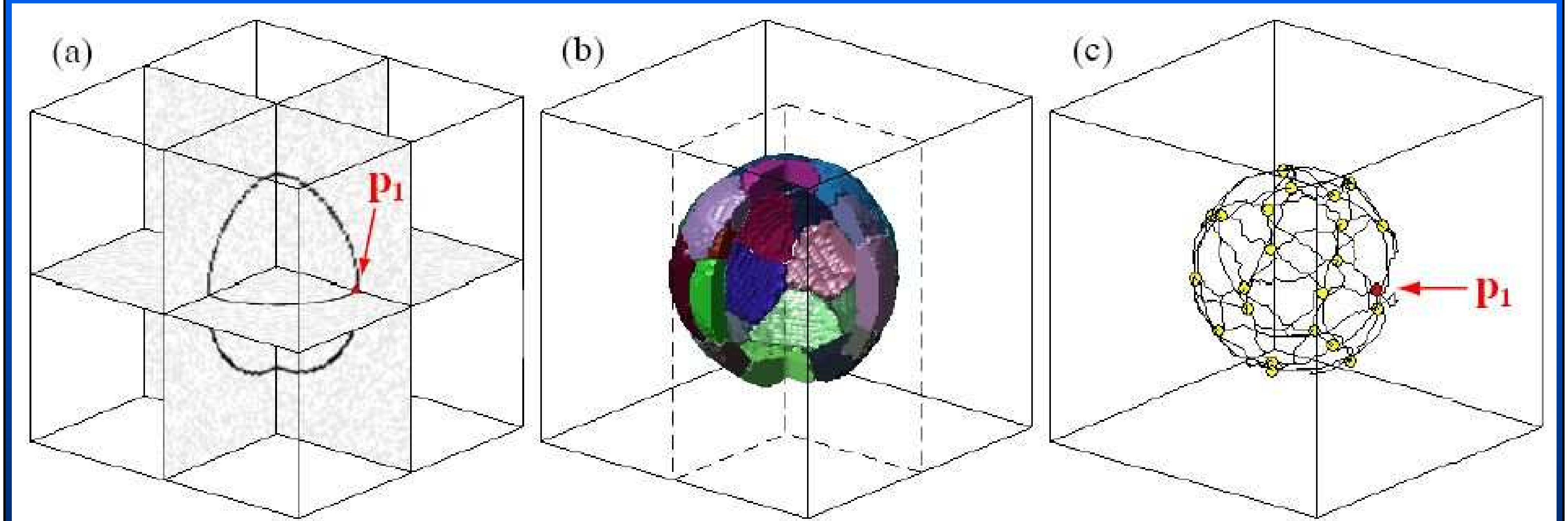
- Mesh is completed to a surface using a Transport equation



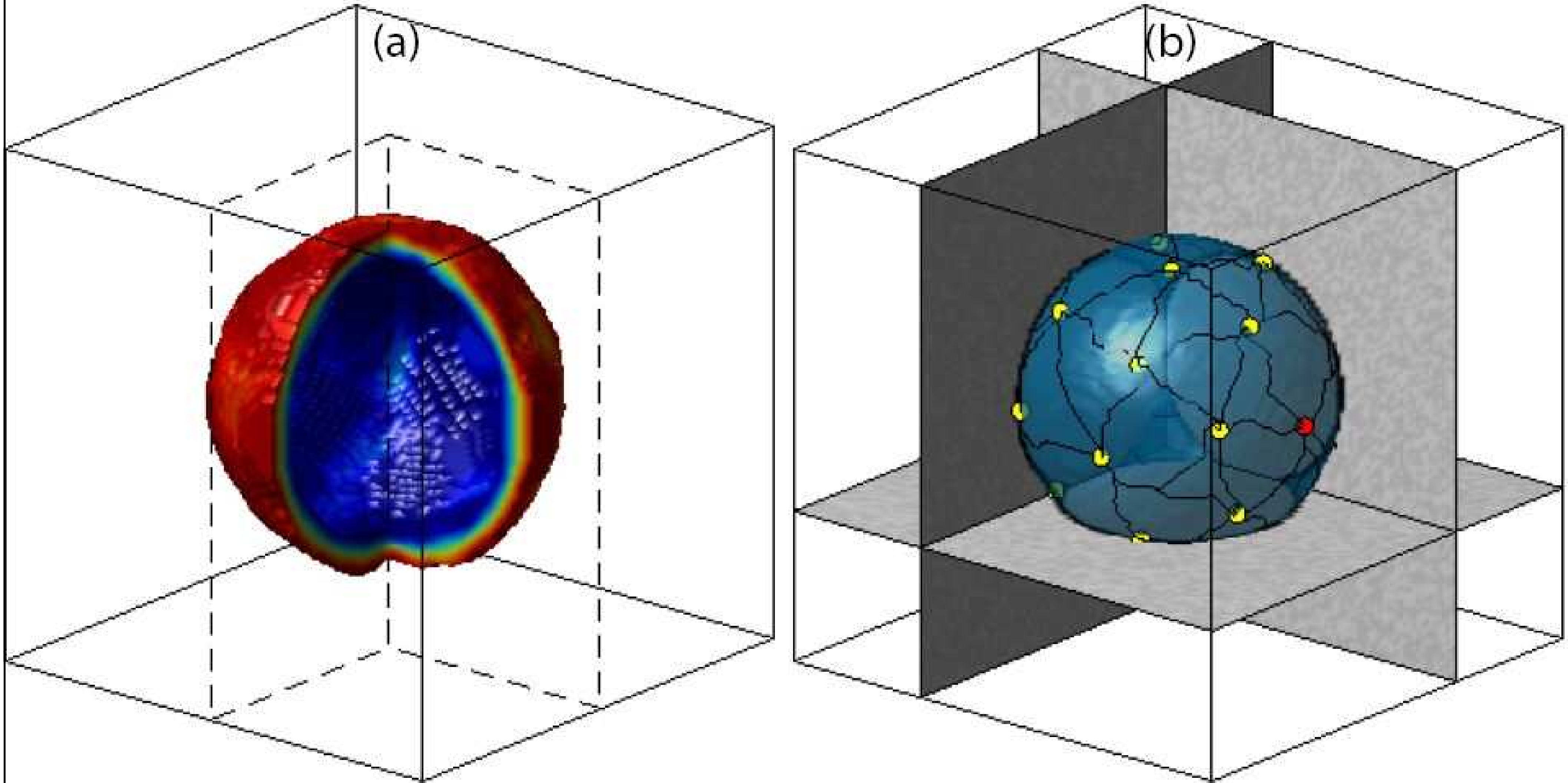
- Mesh is completed to a surface using a Transport equation
- Example for a 2D image.



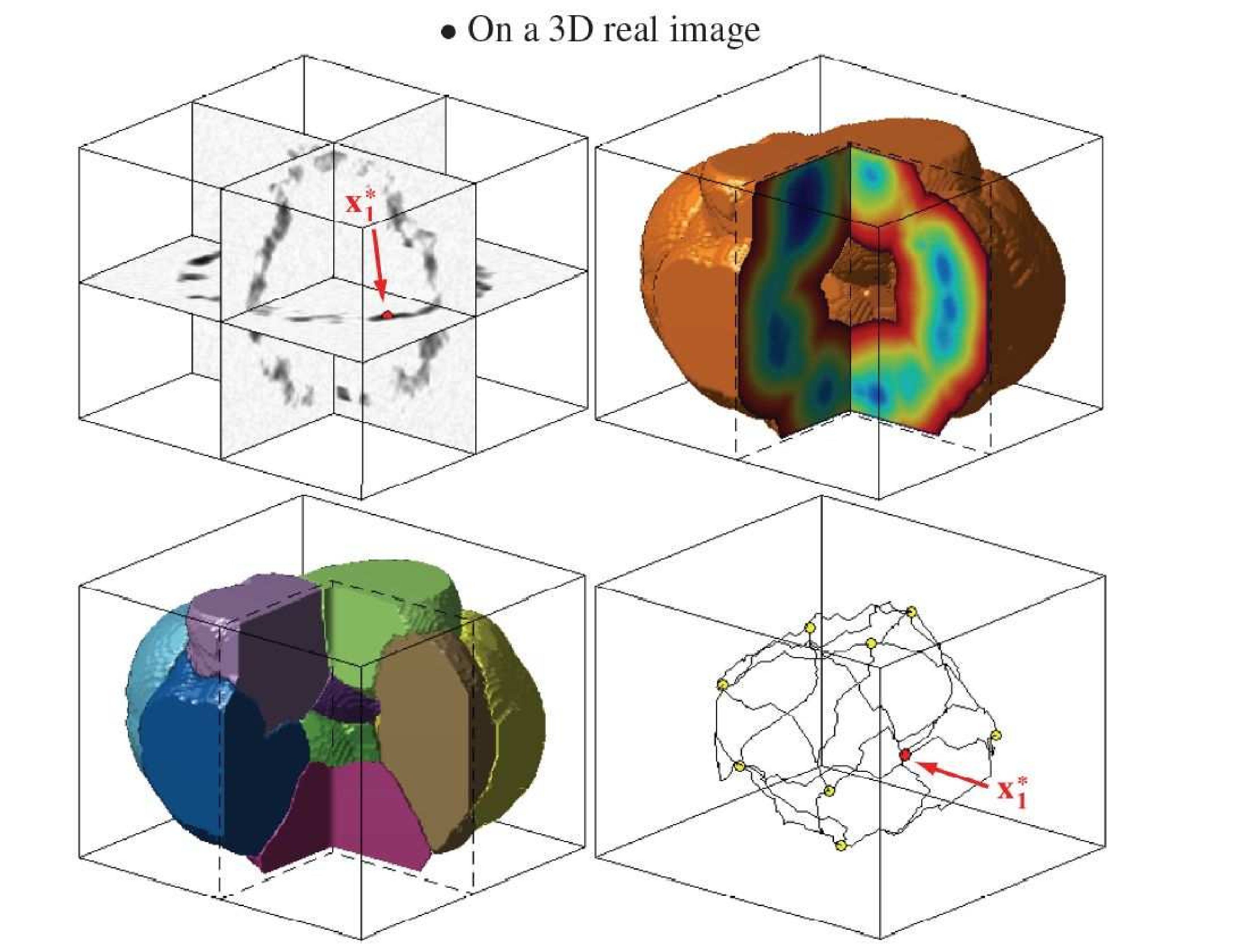
- Example for a 3D sphere: geodesic mesh



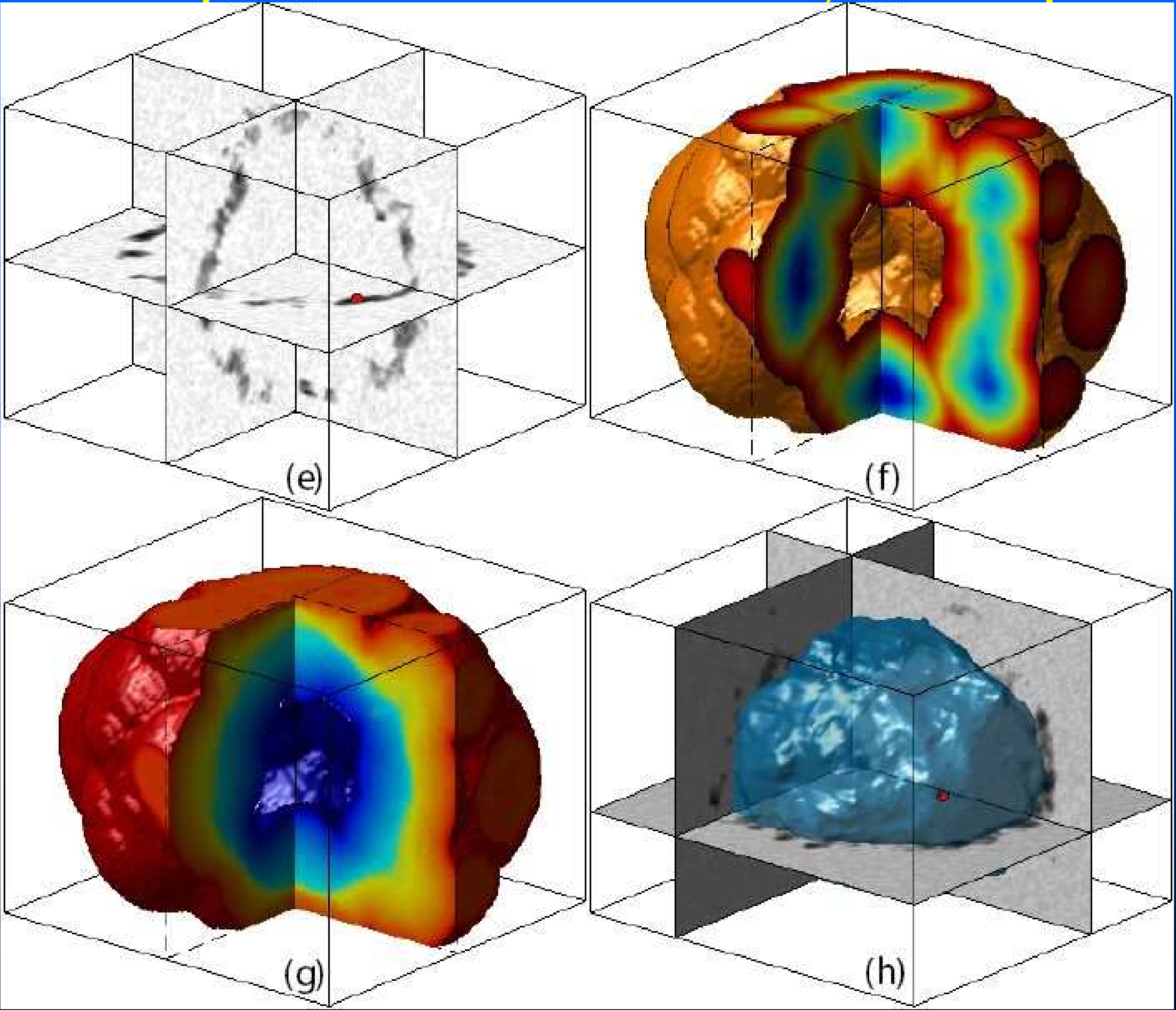
- Example for a 3D sphere: geodesic mesh
- Mesh completed to a surface by Transport



■ Example for a 3D real image: geodesic mesh



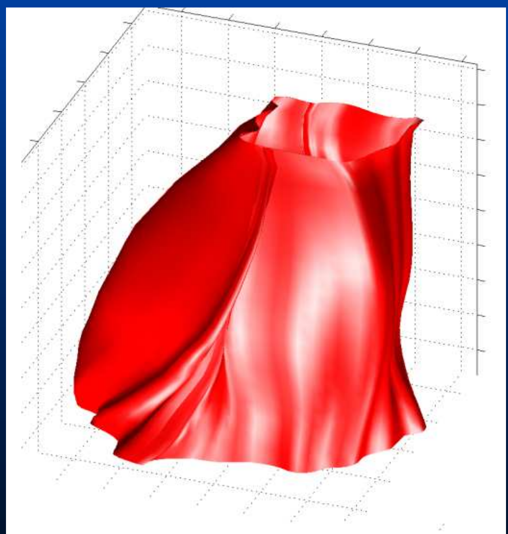
- Example for a 3D real image: geodesic mesh
- Mesh completed to a surface by Transport

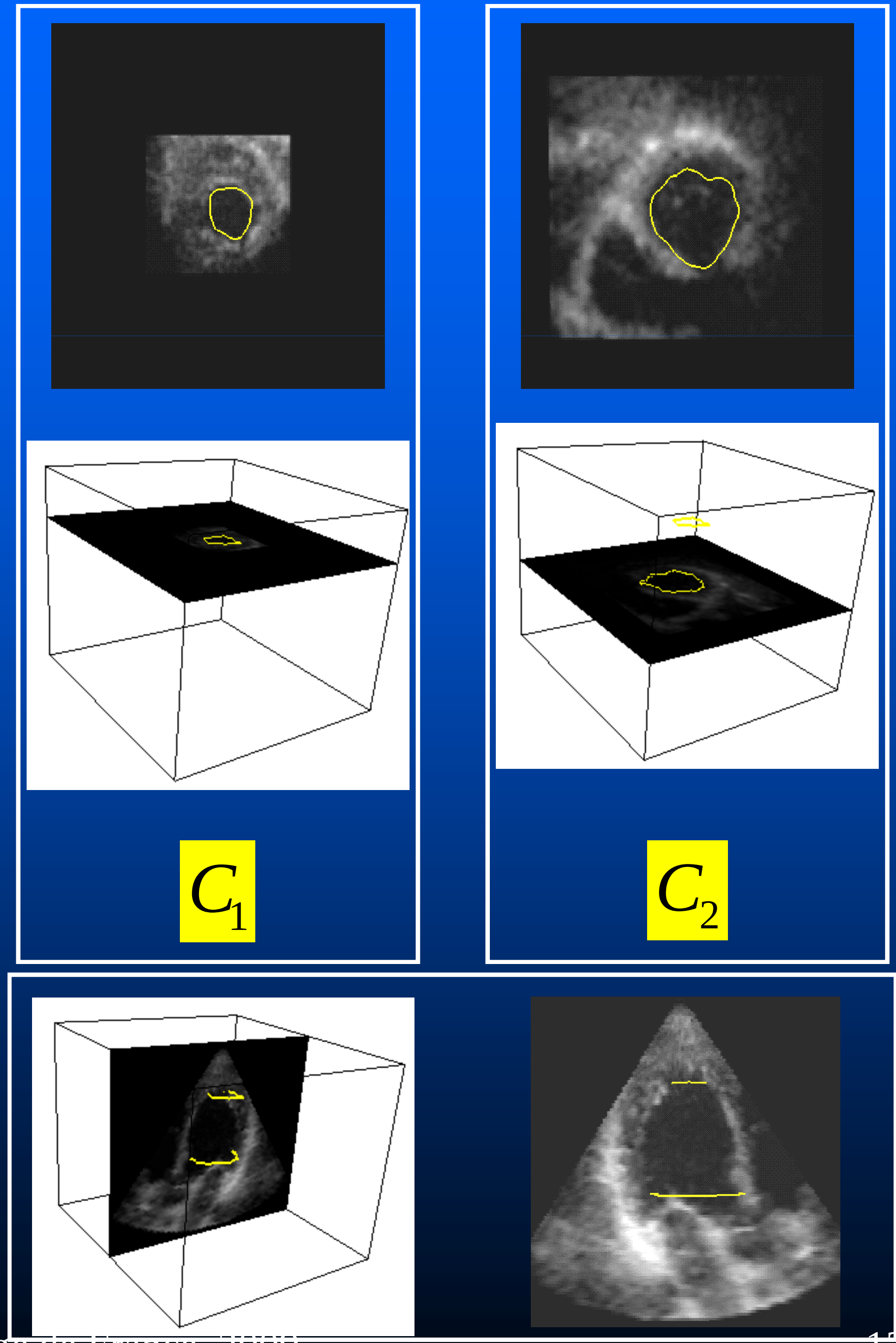


Fast Constrained Surface Extraction by Minimal Paths

- Input:
  1. 3D image.
  2. Two closed curves (C1,C2) drawn by expert on two slices.

- Goal:
  - Fast algorithm to obtain a surface lying on the two curves and segmenting the object of interest.





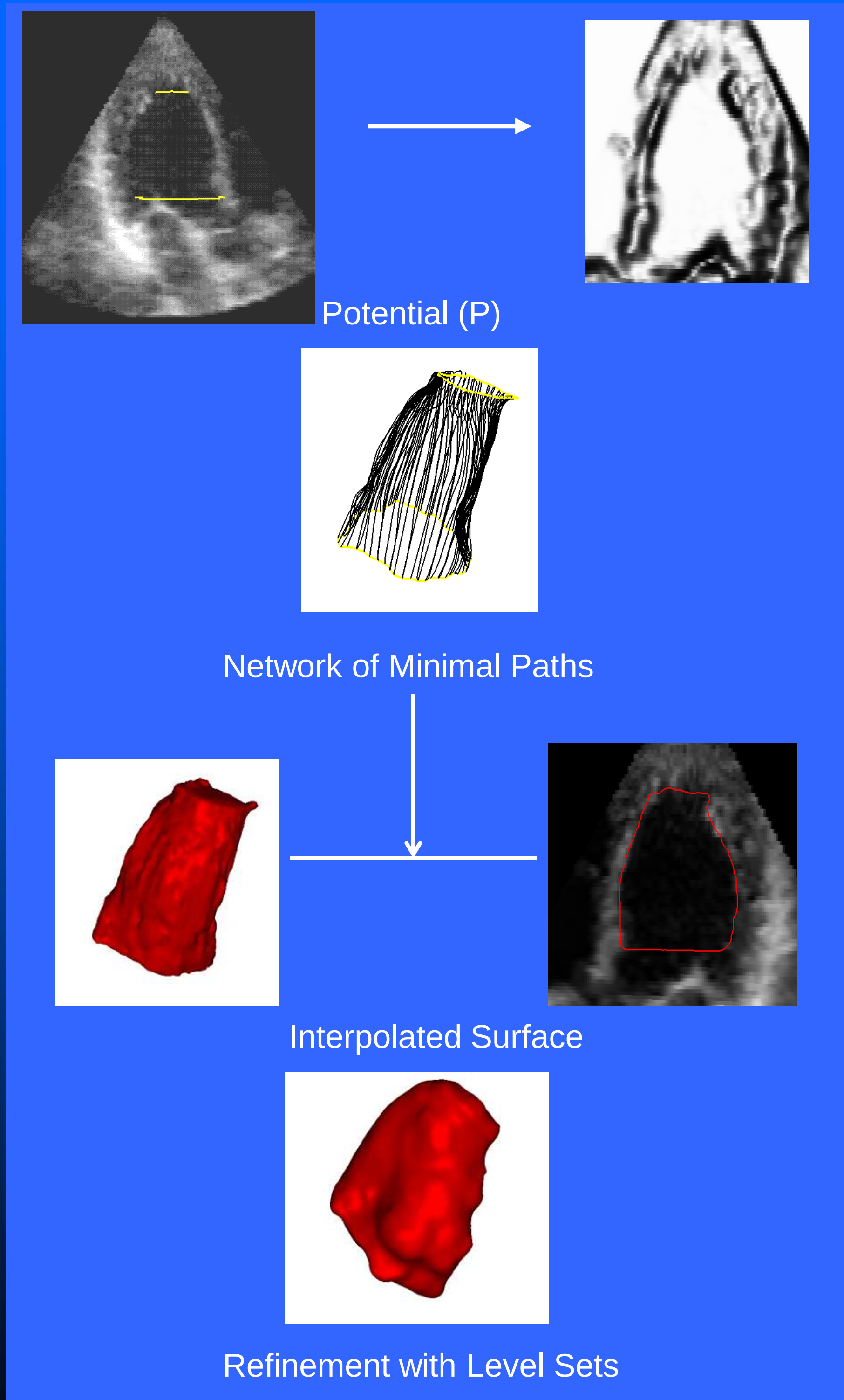
Solution proposed

From a potential (P) describing the image features

- We create a network of paths $S_{C_1}^{C_2}$ *linking* the given curves C1 and C2 and *globally* minimizing

$$E(C) = \int_C P(C)ds$$

- We interpolate them in order to generate the segmenting surface.
- If further precision is needed an active model can be used to refine the segmentation.

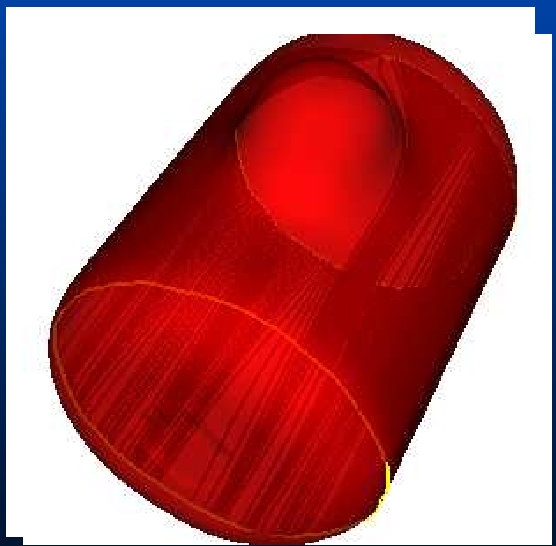


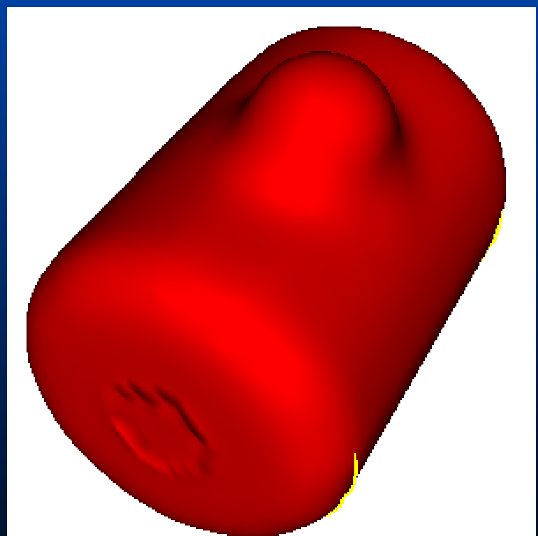
Hypothesis : Ψ satisfies on image domain

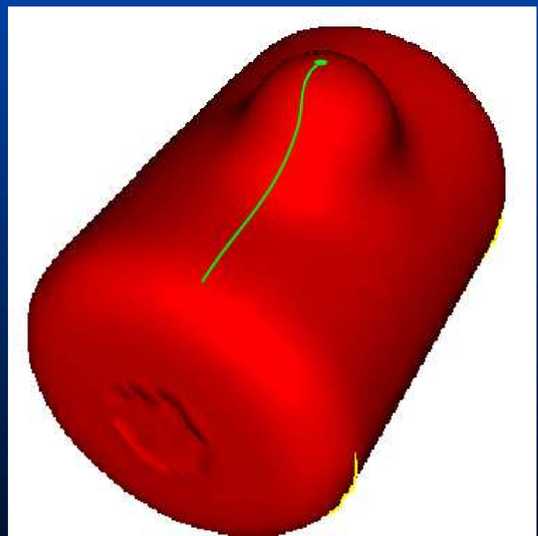
$$\forall p \in \Omega, \left\langle \nabla\Psi(p), \nabla U_{\Gamma_1}(p) \right\rangle = 0 \qquad \Gamma_2 \subset \Psi^{-1}(0)$$

$$\forall p \in \Omega,\ p \in \Psi^{-1}(0) \Rightarrow C_{\Gamma_1}^{p} \subset \Psi^{-1}(0)$$

$\Psi^{-1}(0)$ is composed only of minimal paths leading to Γ_1







Construction of Ψ when Γ_1 and Γ_2 are planar (usual case for applications).

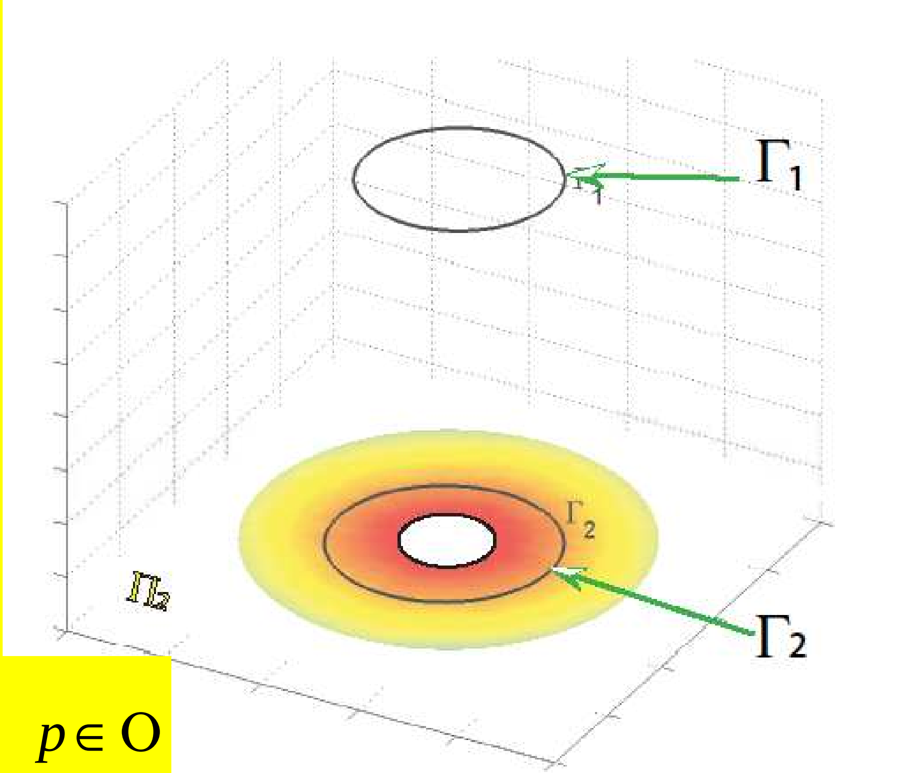
$$\begin{cases} \forall p \in \Omega, \langle \nabla\Psi(p), \nabla U_{\Gamma_1}(p)\rangle + \mathrm{H}(\Psi) = 0 \\ \Gamma_2 \subset \Psi^{-1}(0) \end{cases}$$

⇓

$$\begin{cases} p \in \Psi^{-1}(0) \Rightarrow \mathrm{C}_{\Gamma_1}^{p} \subset \Psi^{-1}(0) \\ S_{\Gamma_1}^{\Gamma_2} \subset \Psi^{-1}(0) \end{cases}$$

$$V_\eta^2 = \{p \in \Pi_2 \text{ such that } |d_2(p)| \le \eta\}$$

$$\mathrm{O} = \mathrm{int}(\Omega) - V_\eta^2$$



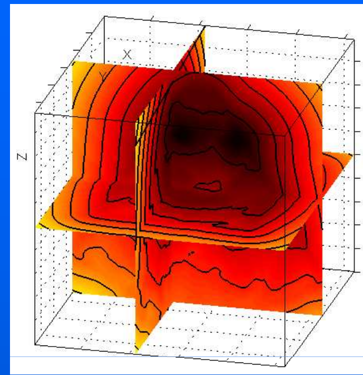
By choosing H(Ψ) = α.Ψ, we have to solve this problem:

$$\begin{cases} \langle \nabla\Psi(p), \nabla U_{\Gamma_1}(p)\rangle + \alpha.\Psi = 0 & \text{if} \quad p \in \mathrm{O} \\ \Psi(p) = d_2(p) & \text{if} \quad p \in V_\eta^2 \\ \Psi(p) = \min\limits_{p \in V_\eta^2}(d_2(p)) & \text{if} \quad p \in \partial\Omega \end{cases}$$

Step 1: numerical Resolution of eikonal equation by :

Fast Marching, Group Marching, Fast Sweeping

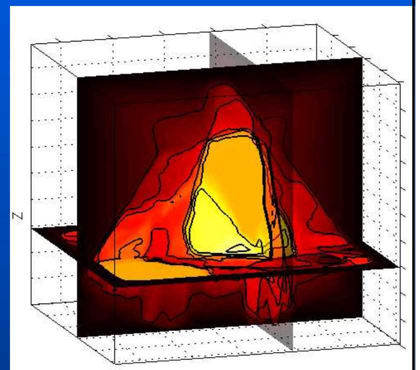
$$\left\| \nabla U_{\Gamma_1} \right\| = P$$



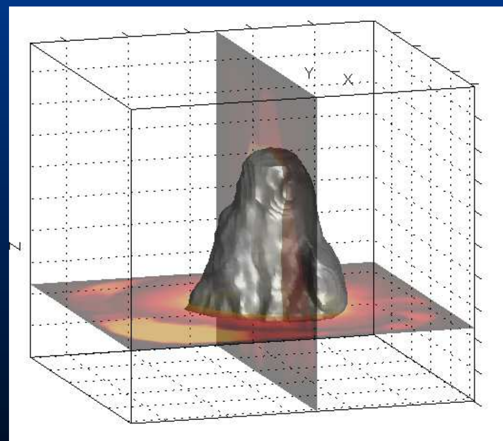
- **Etape 2:** Resolution of transport equation

$$\begin{cases} \left\langle \nabla \Psi(p), \nabla U_{\Gamma_1}(p) \right\rangle + \alpha . \Psi = 0 \\ \Psi = 0 \quad \text{on} \quad \Gamma_2 \end{cases}$$

  - By iterative approach
  - By *Fast Marching* approach.
  - By *Fast Sweeping* approach.



- **Step 3:** Detection of zero level set
  - by *Marching Cube, Marching Tetrahedra…*

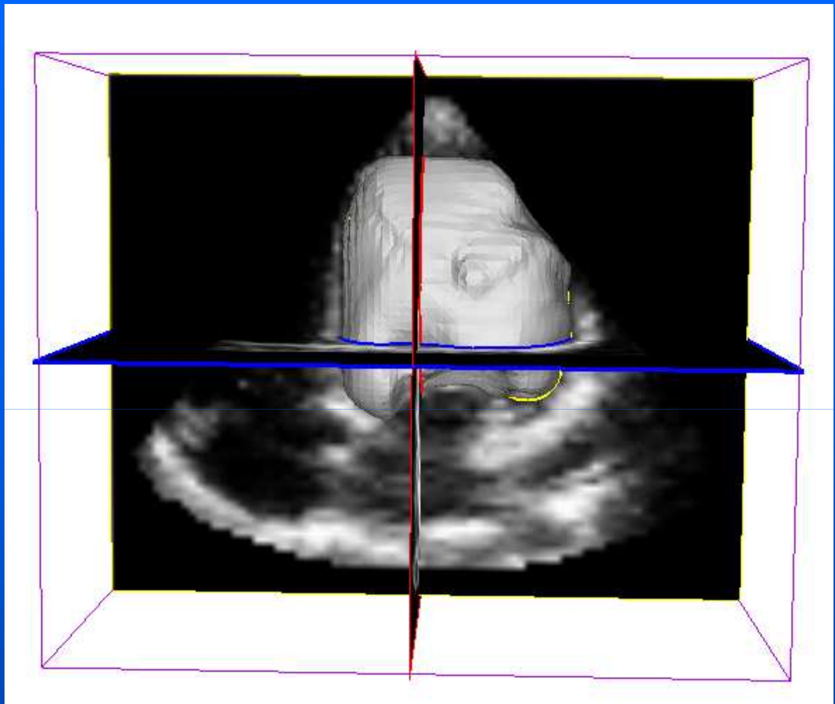


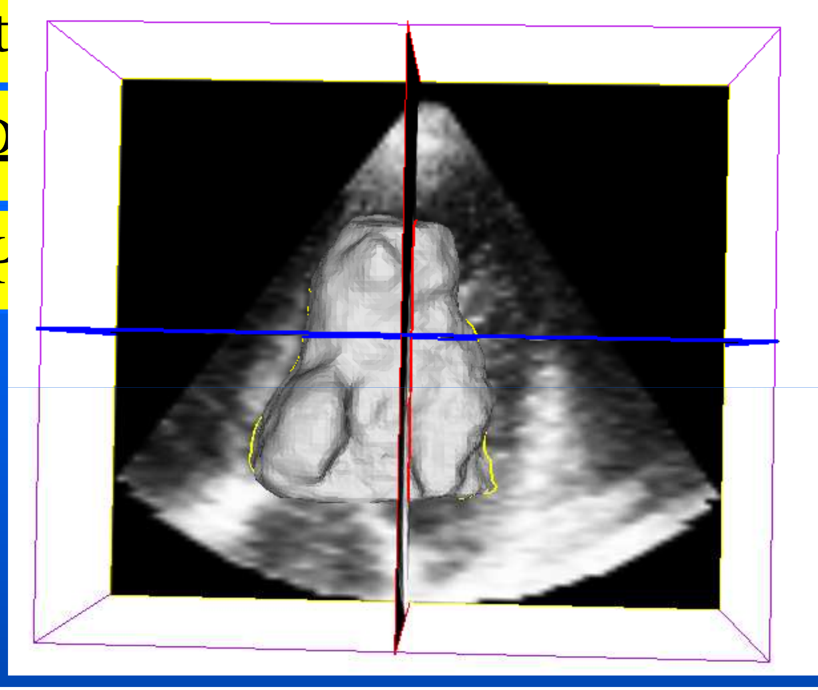
Examples of path network : implicit approach as zero level set of a transport equation

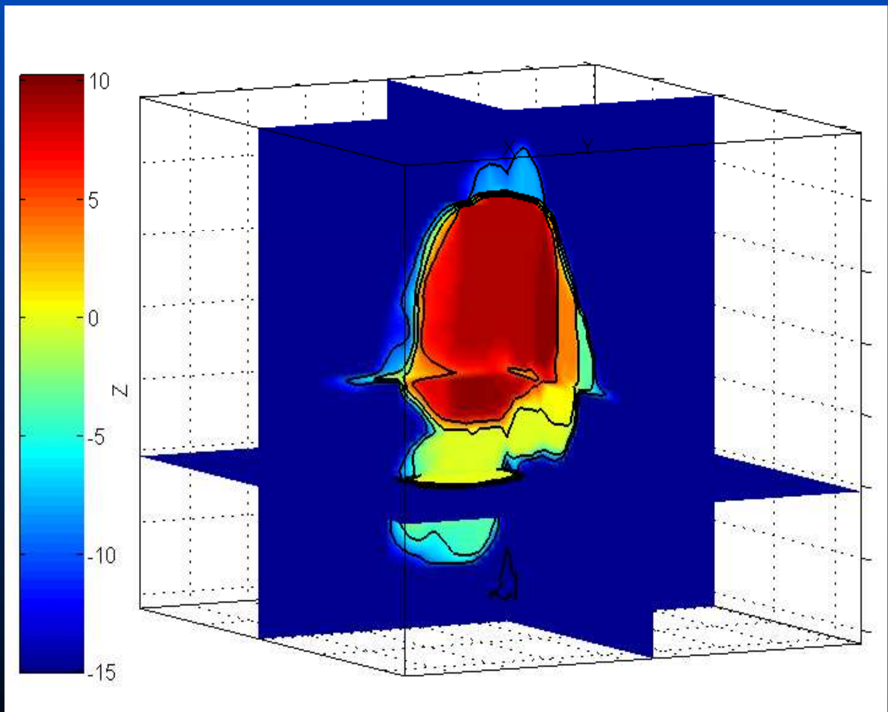
Hypot… …main

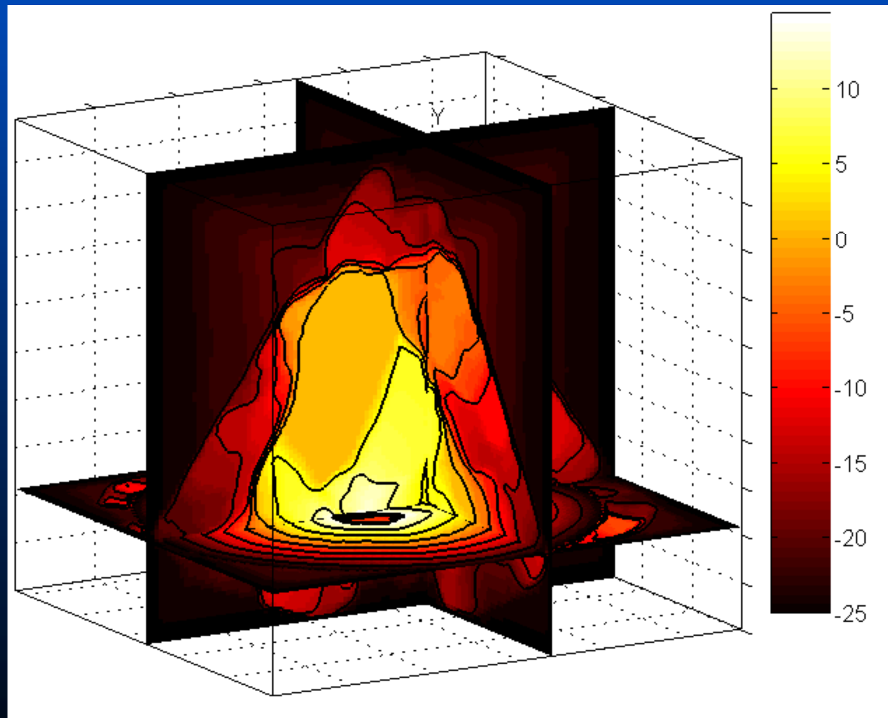
$\forall p \in \Omega$…

$\Gamma_2 \subset \Psi$…







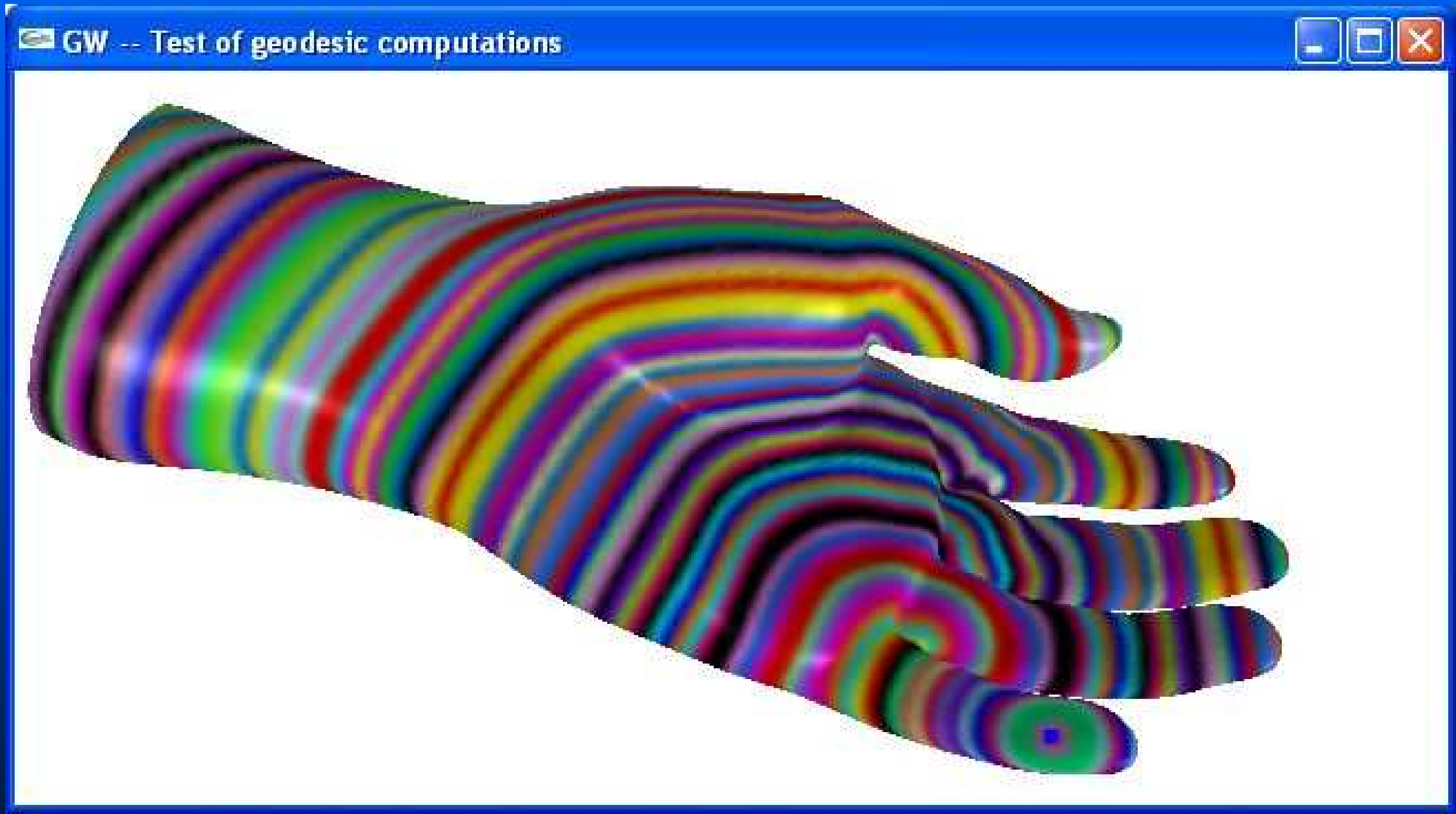


Overview

- Minimal Paths, Fast Marching and Front Propagation
- Anistropic Fast Marching and Perceptual Grouping
- Anistropic Fast Marching and Vessel Segmentation
- Closed Contour segmentation as a set of minimal paths in 2D
- Geodesic meshing for 3D surface segmentation
- Fast Marching on surfaces: geodesic lines and Remeshing – Isotropic, Adaptive, Anisotropic

Fast Marching on a surface and Remeshing

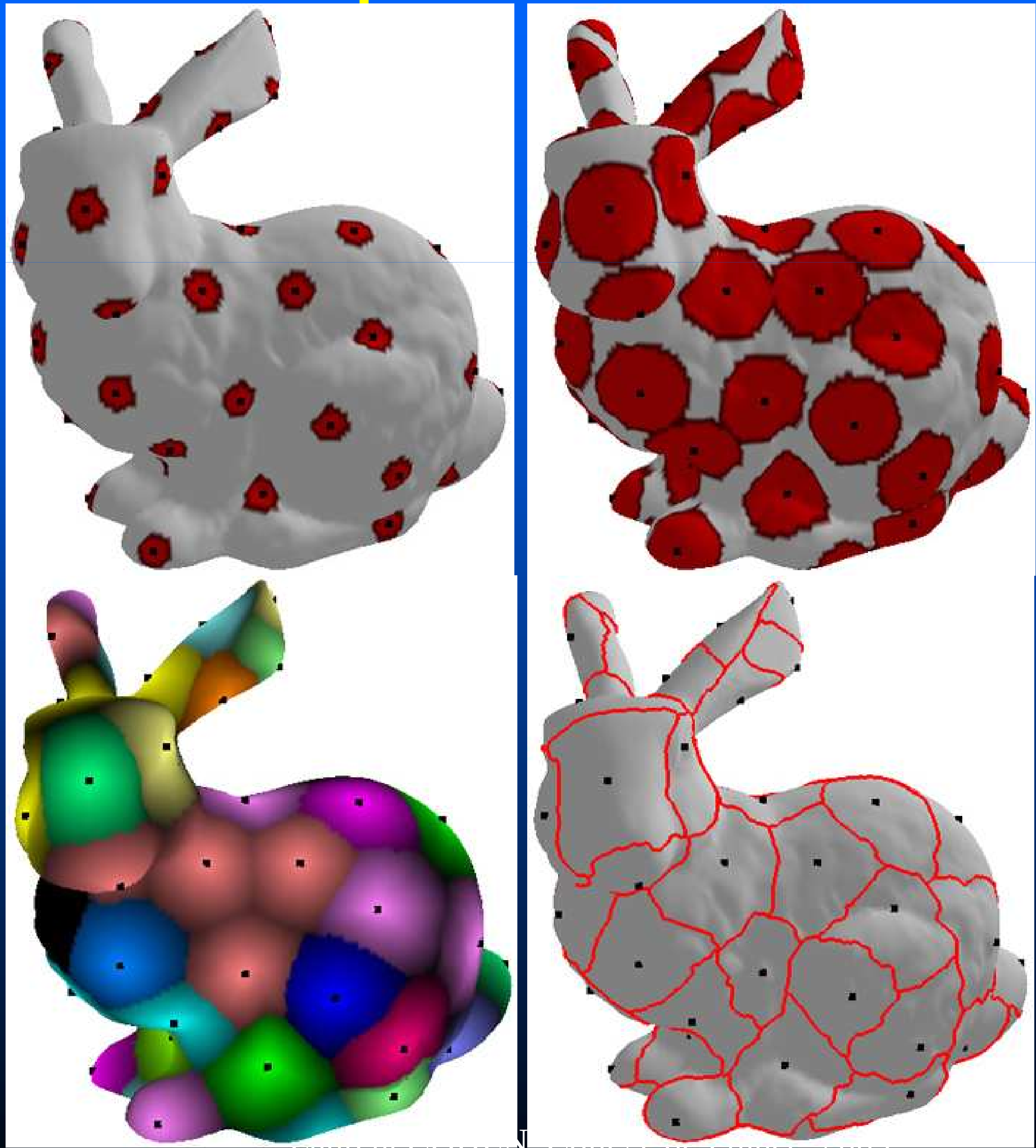
Front Propagation on a surface from one point.



Fast Marching on a surface

Geodesic lines on a surface

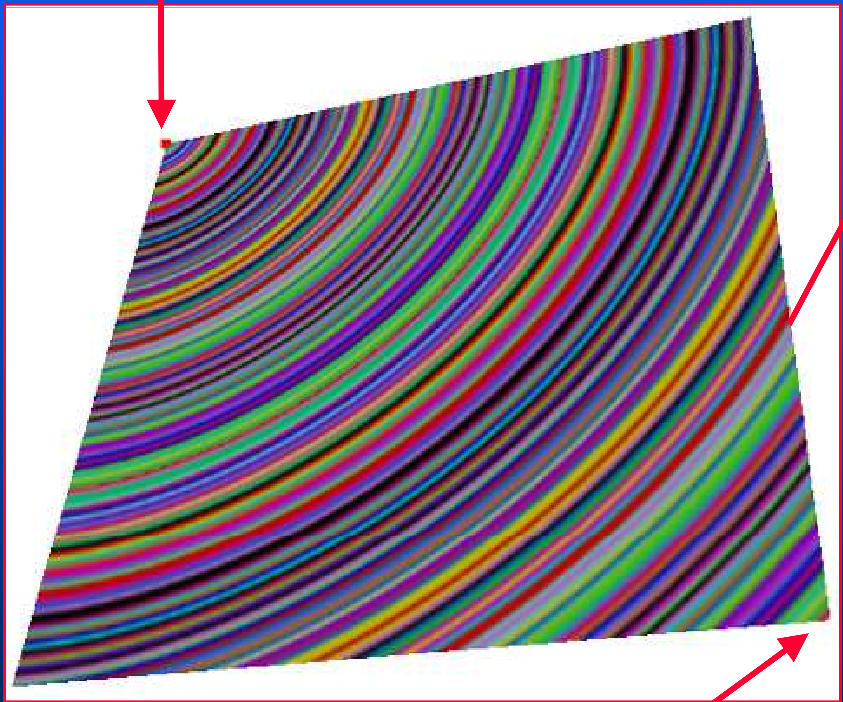
Example of Voronoi

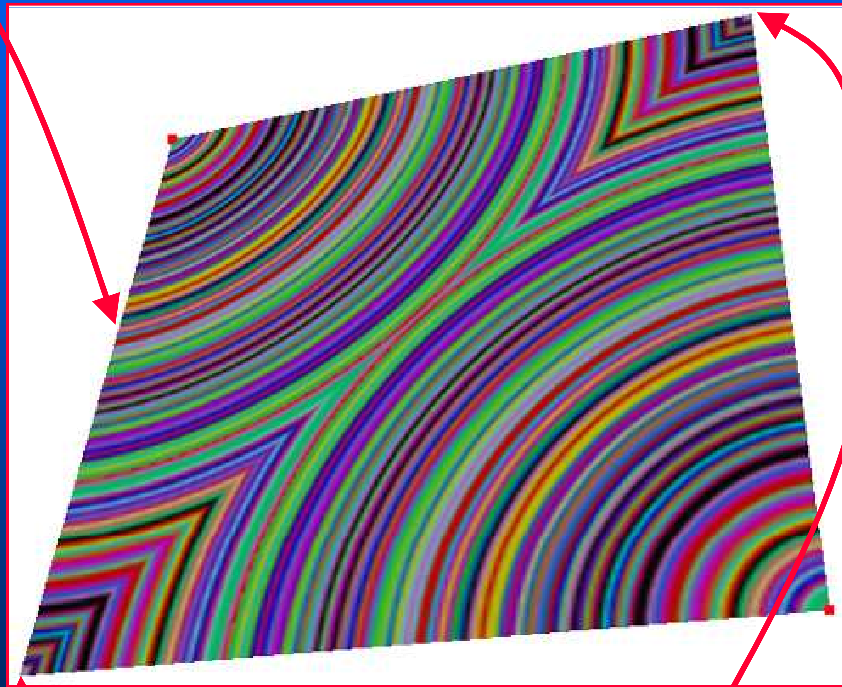


Sampling with uniform distribution

Choose first point anywhere

update the geodesic distance

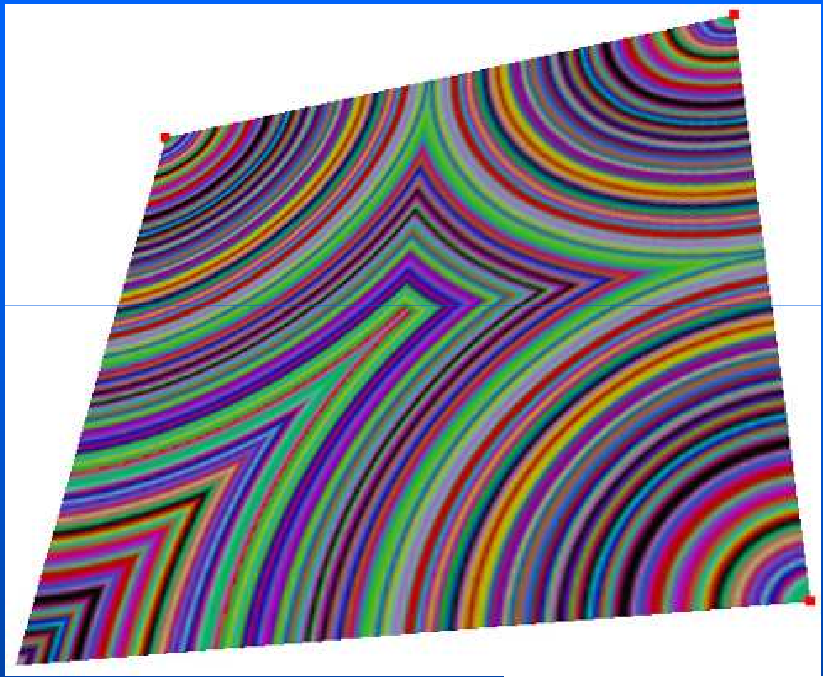


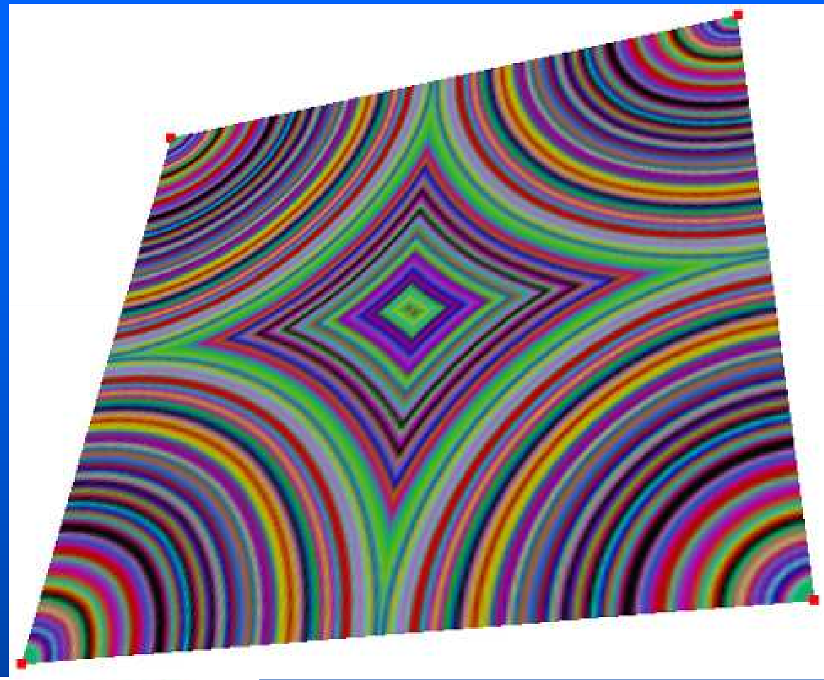


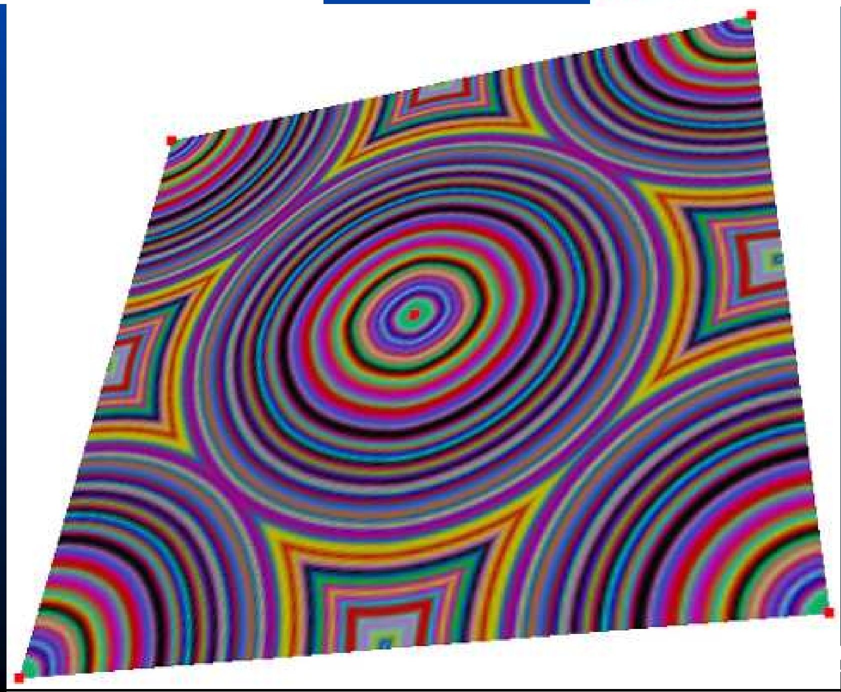
choose the furthest point

The two new furthest points

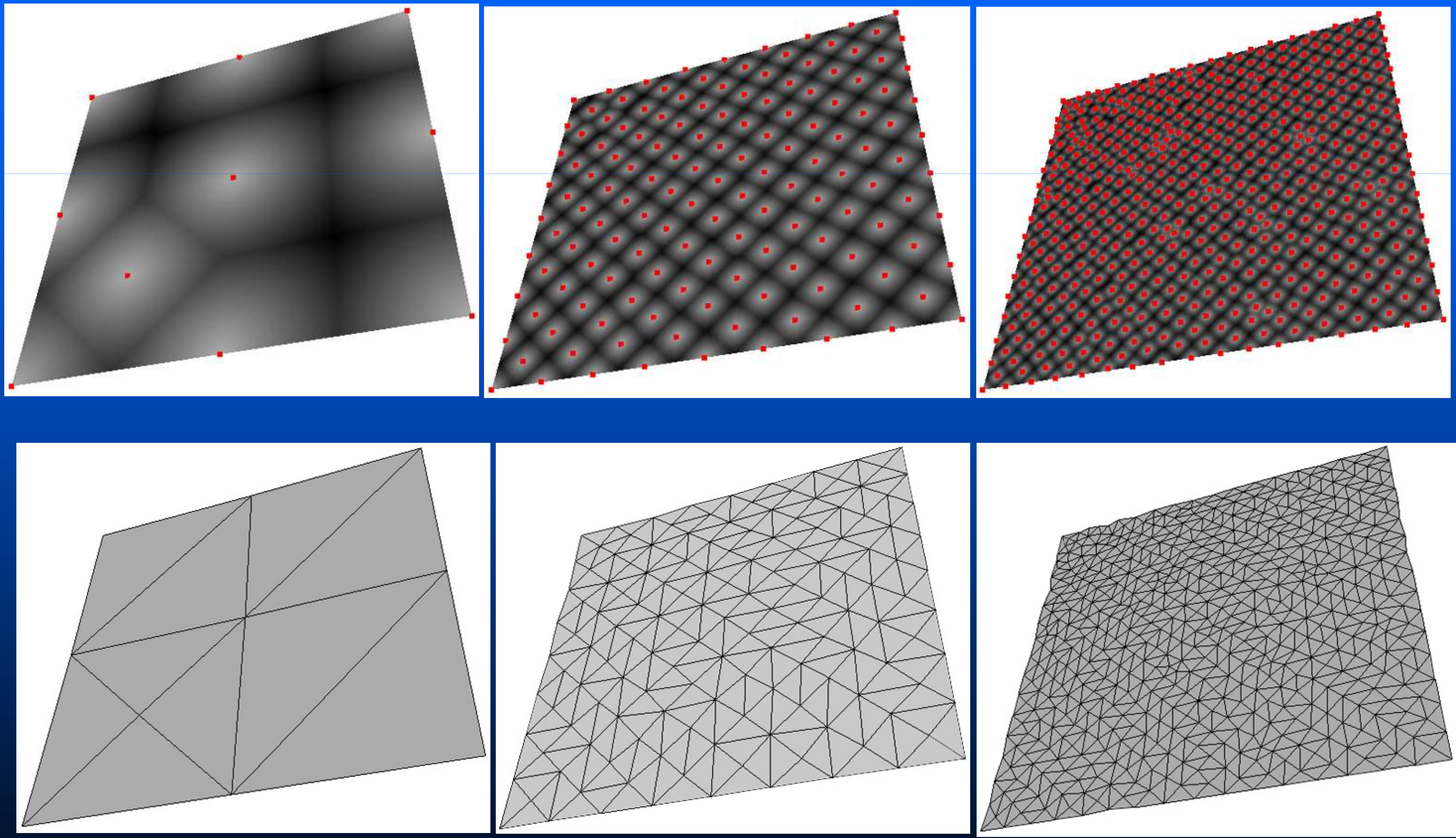
Sampling with uniform distribution



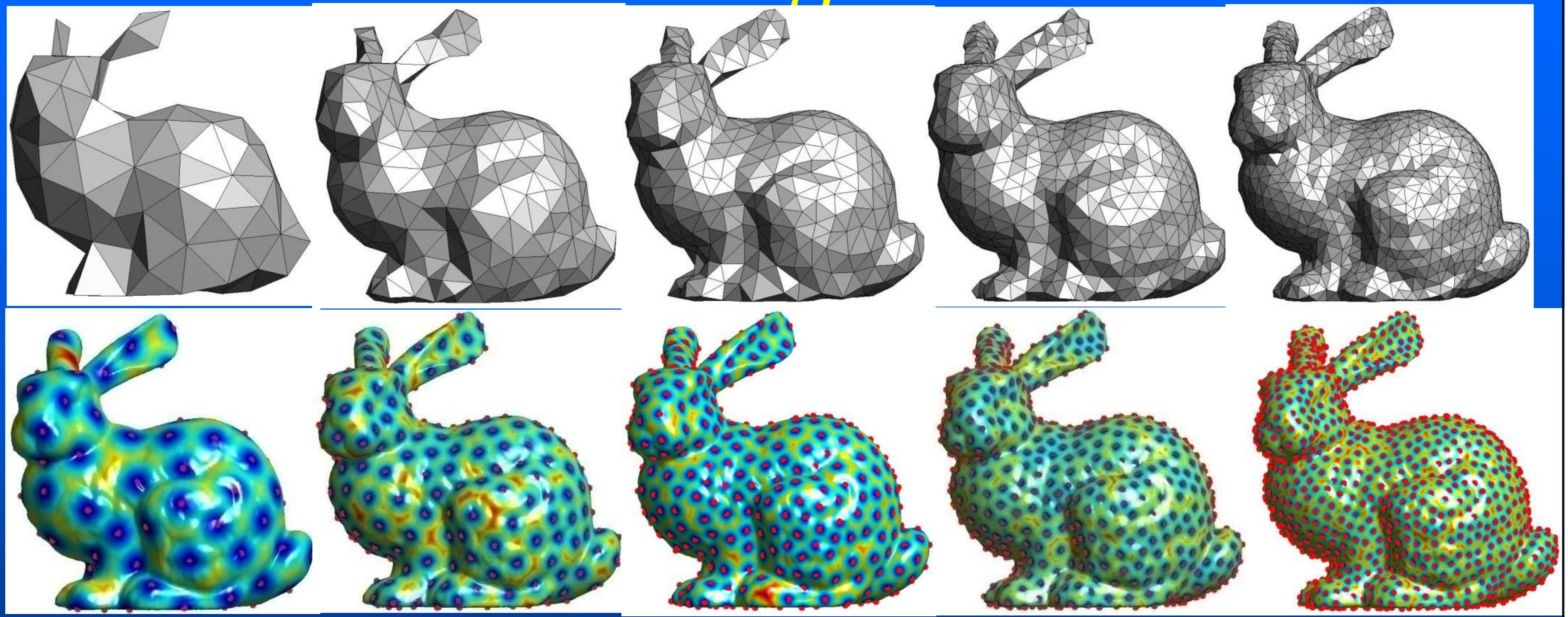


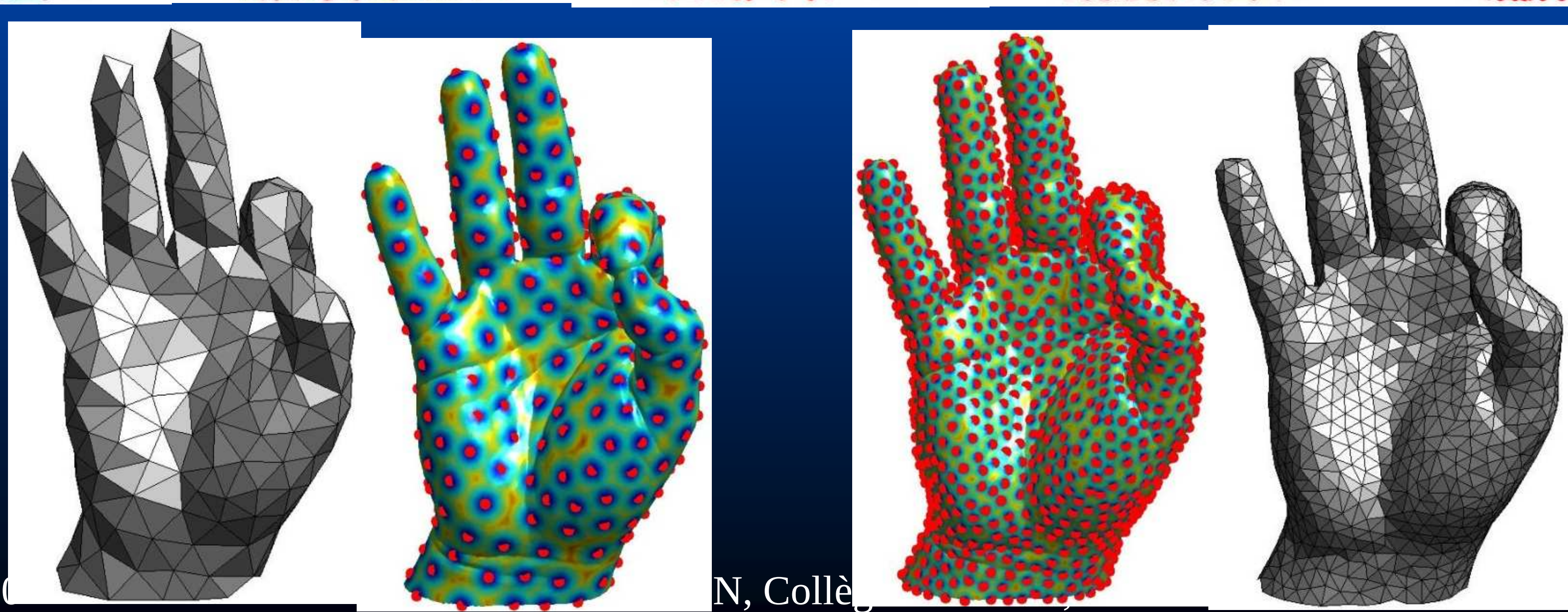


Sampling on a plane



Uniform Remeshing



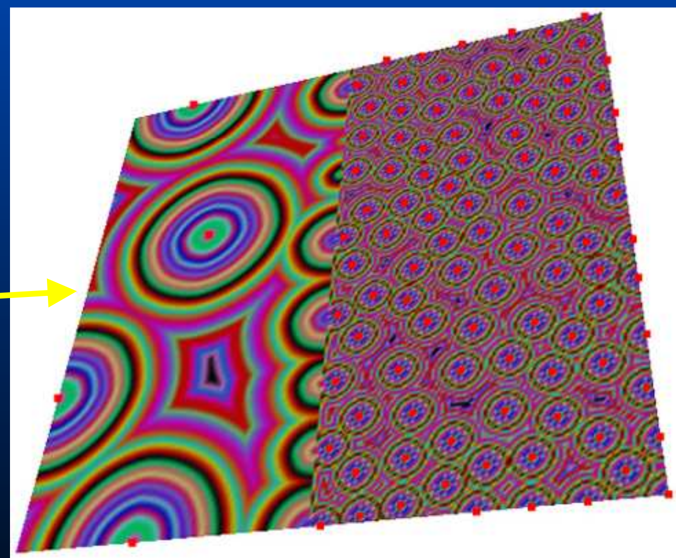


Non constant speed function

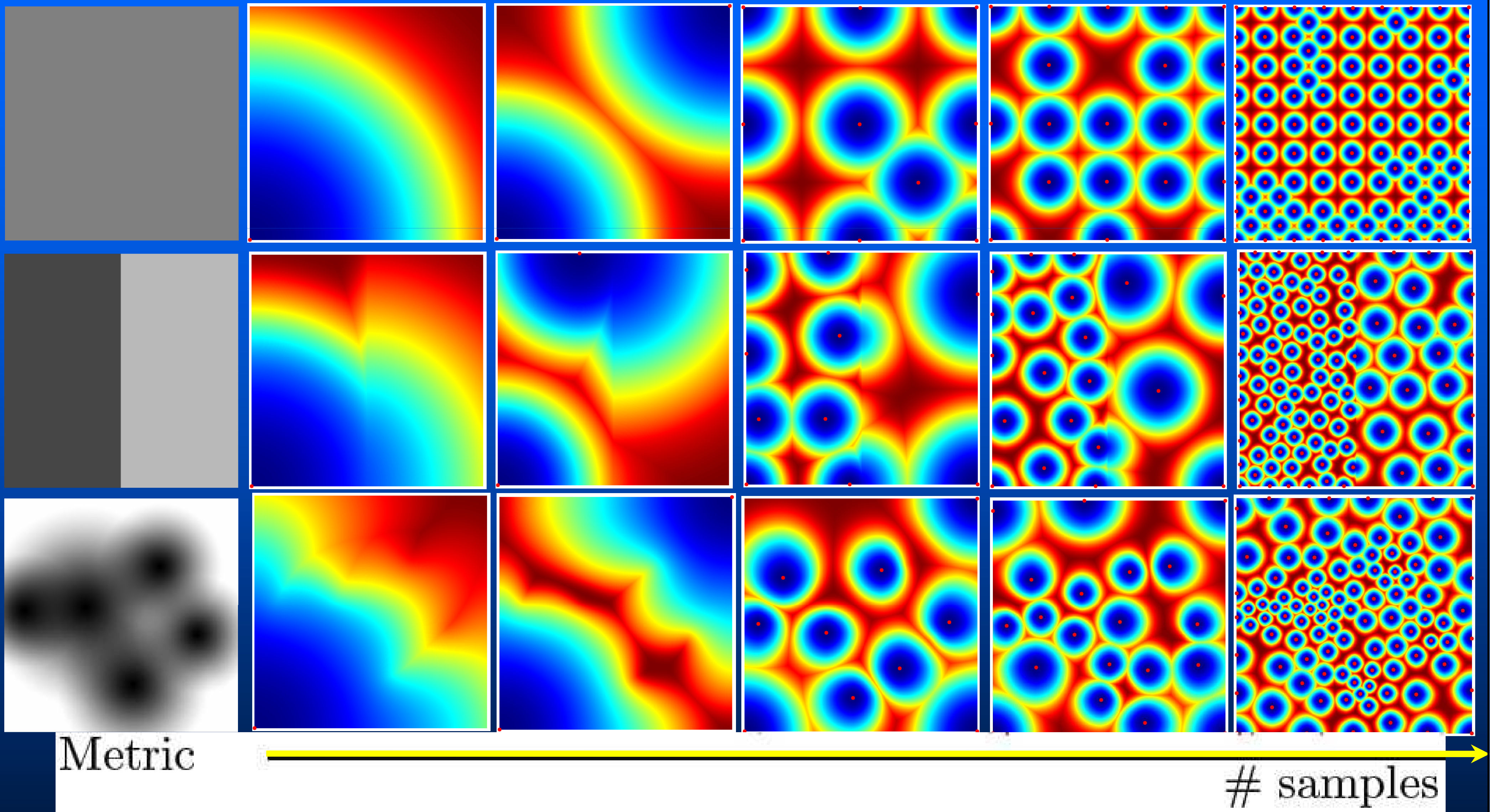
High Speed

Low speed

A little later …



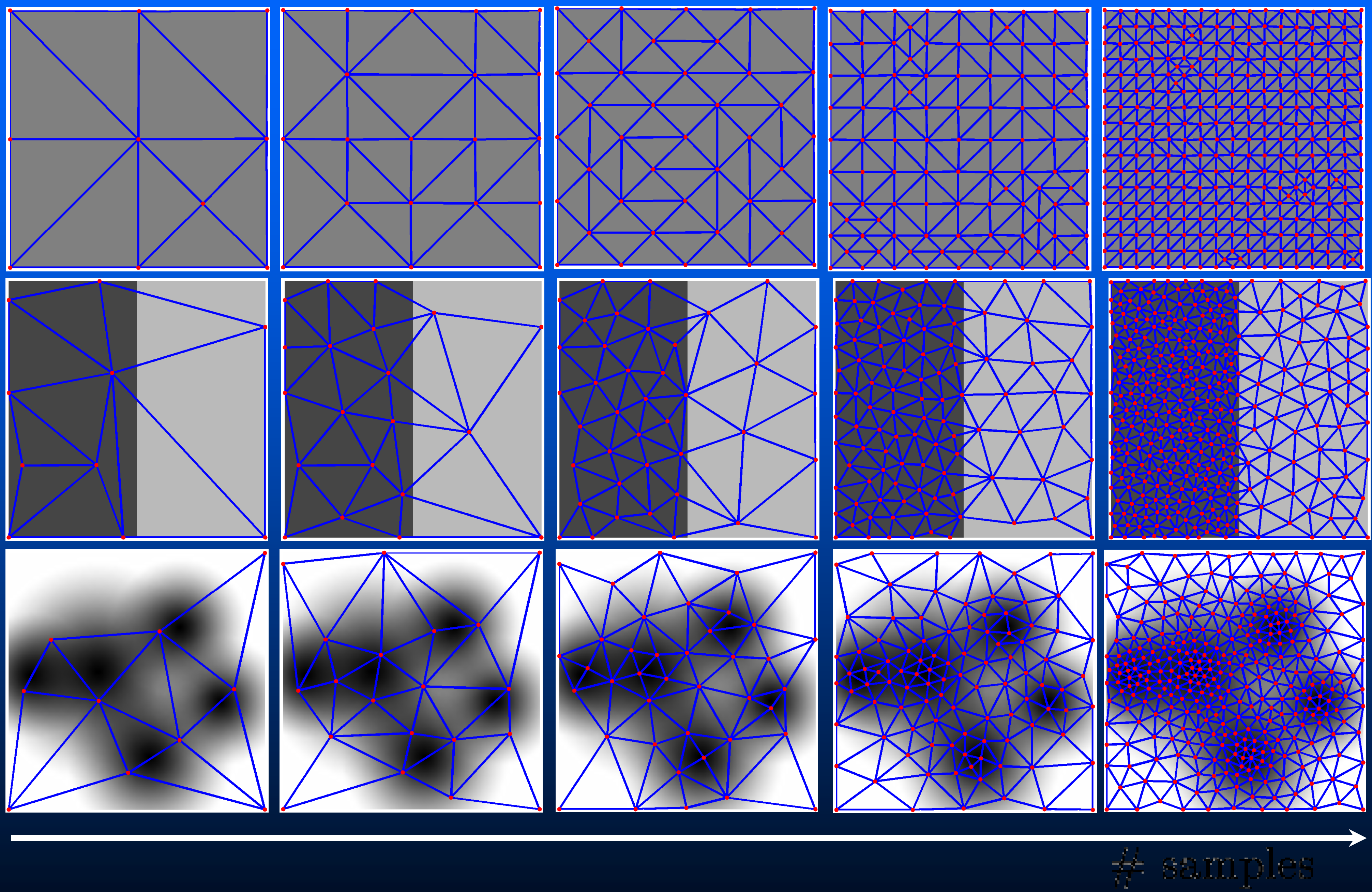
Farthest Point Sampling



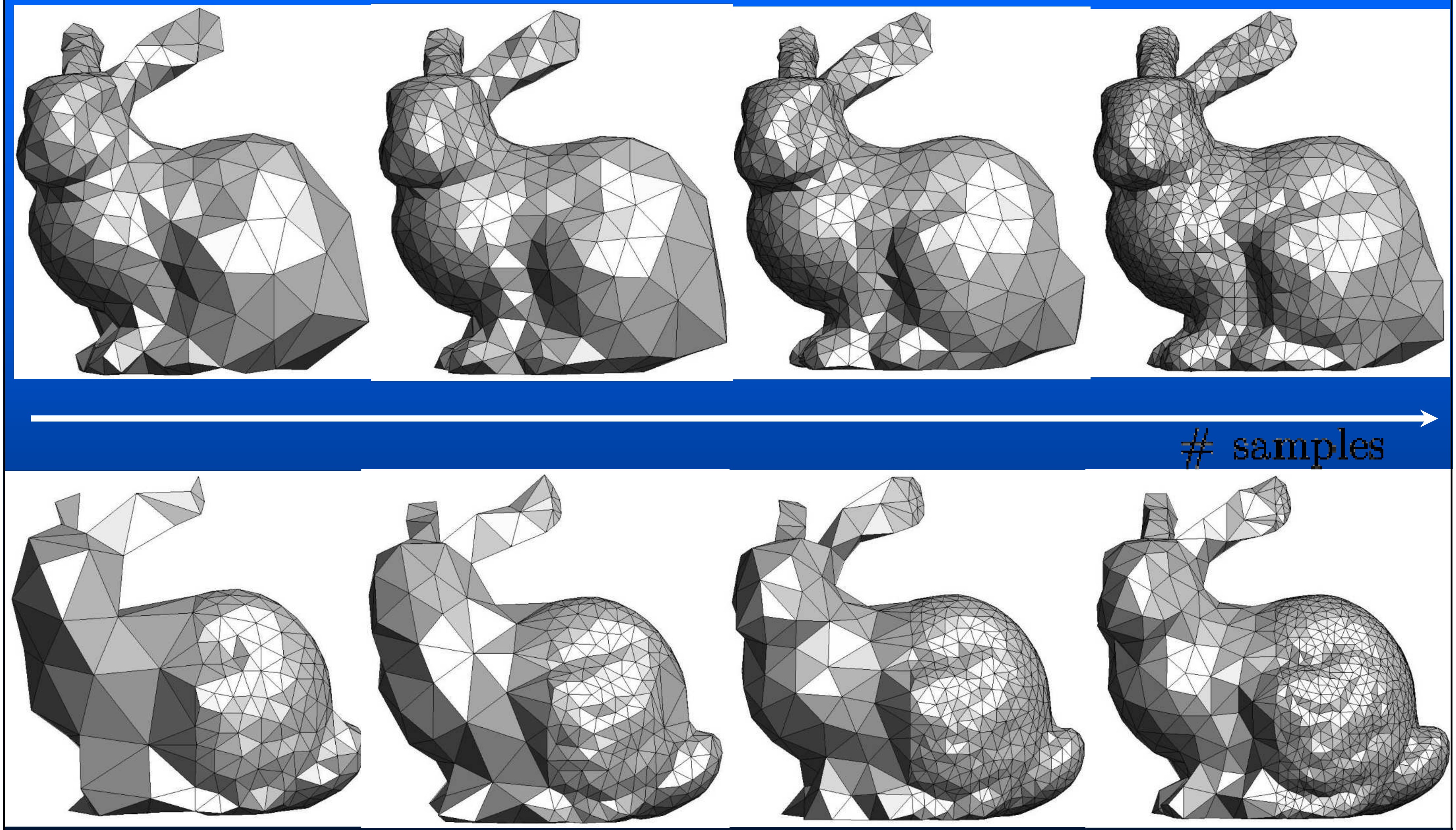
$W(x)$ small $\Longrightarrow$ front moves slowly,

$\Longrightarrow$ denser sampling.

Farthest Point Triangulation

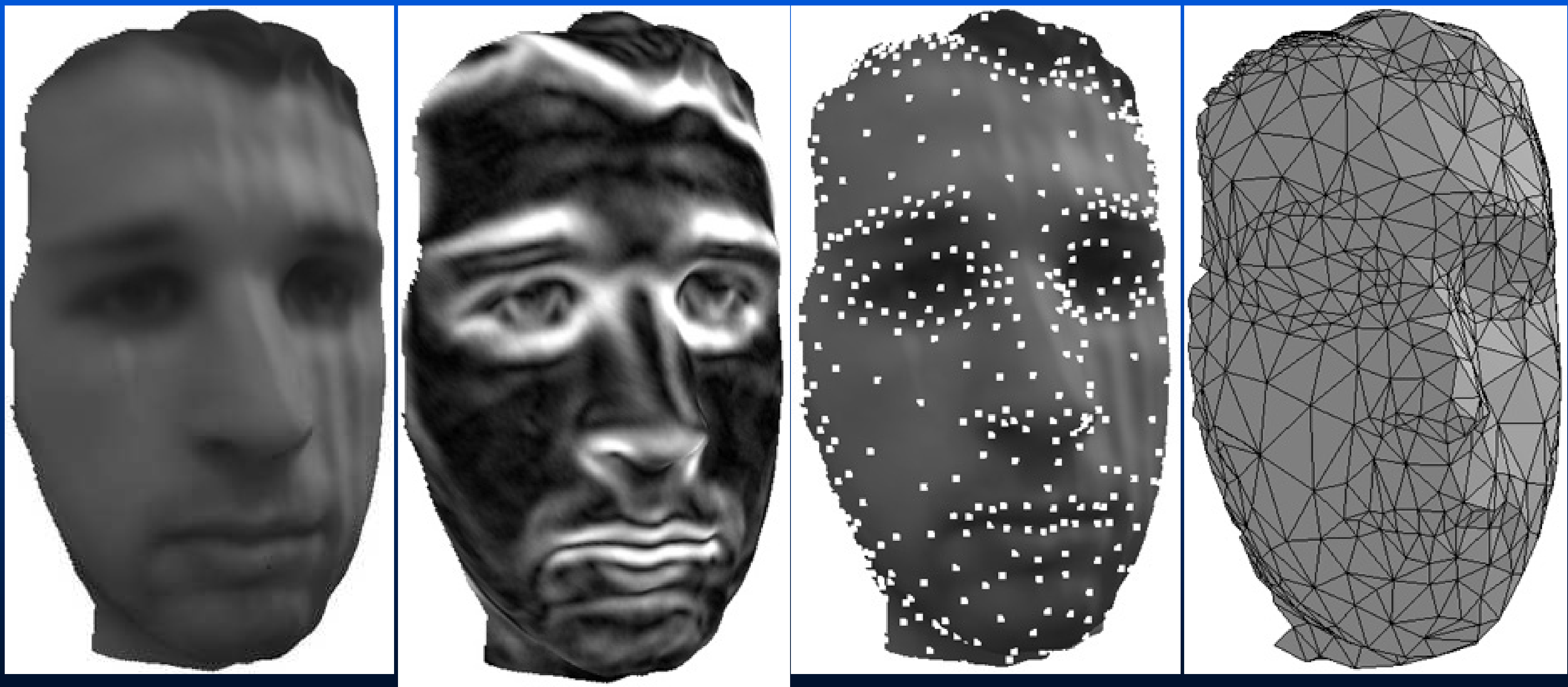


Adaptive Remeshing

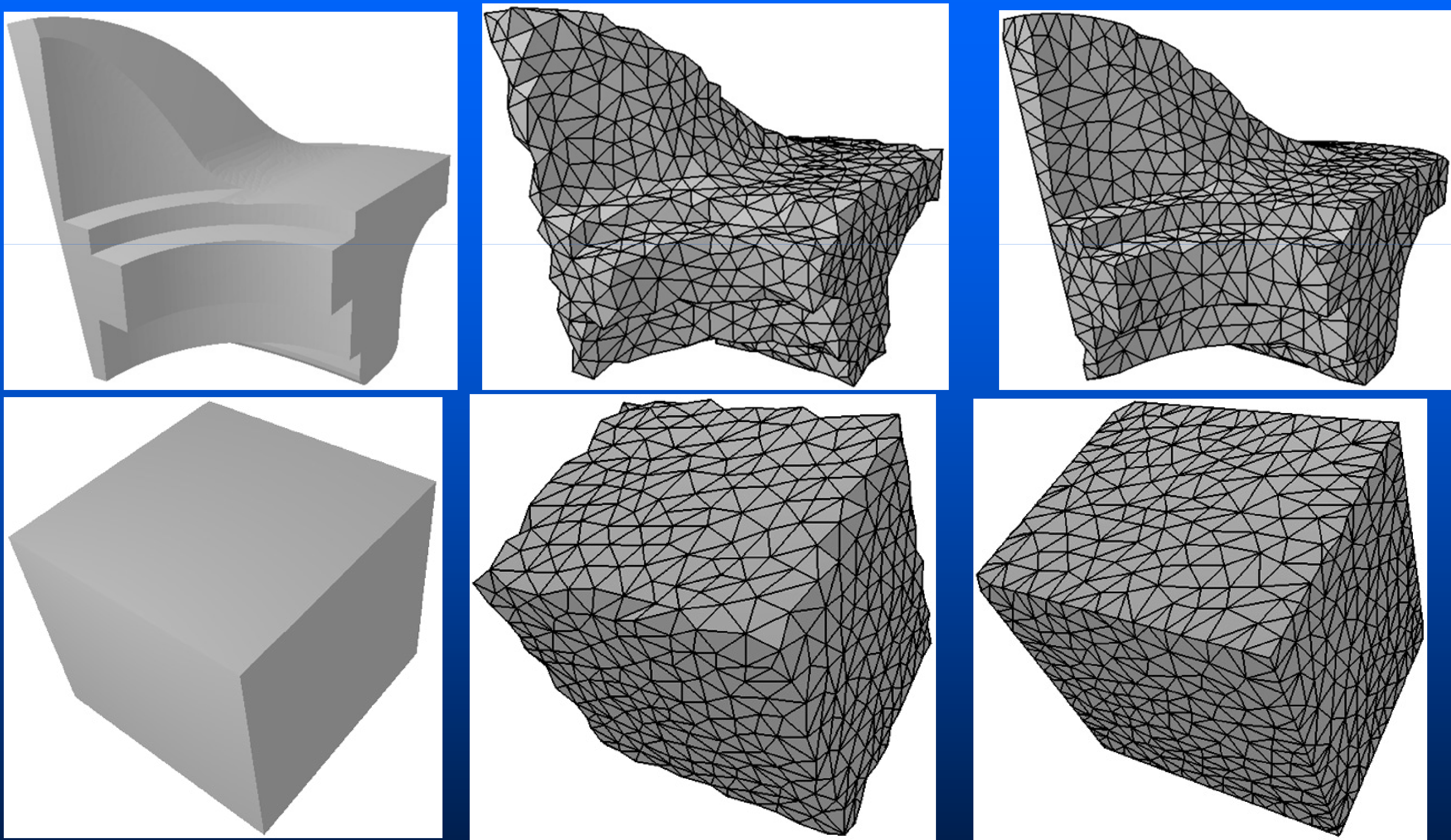


Density Given by a Texture

- A texture: $T : S \xrightarrow{\varphi} [0,1]^2 \xrightarrow{I} \mathrm{IR}$
- Adaptive speed : $F = 1/P(v) = 1/\left(\varepsilon + \left|\overrightarrow{\mathrm{grad}}(I)(\varphi(v))\right|\right)$

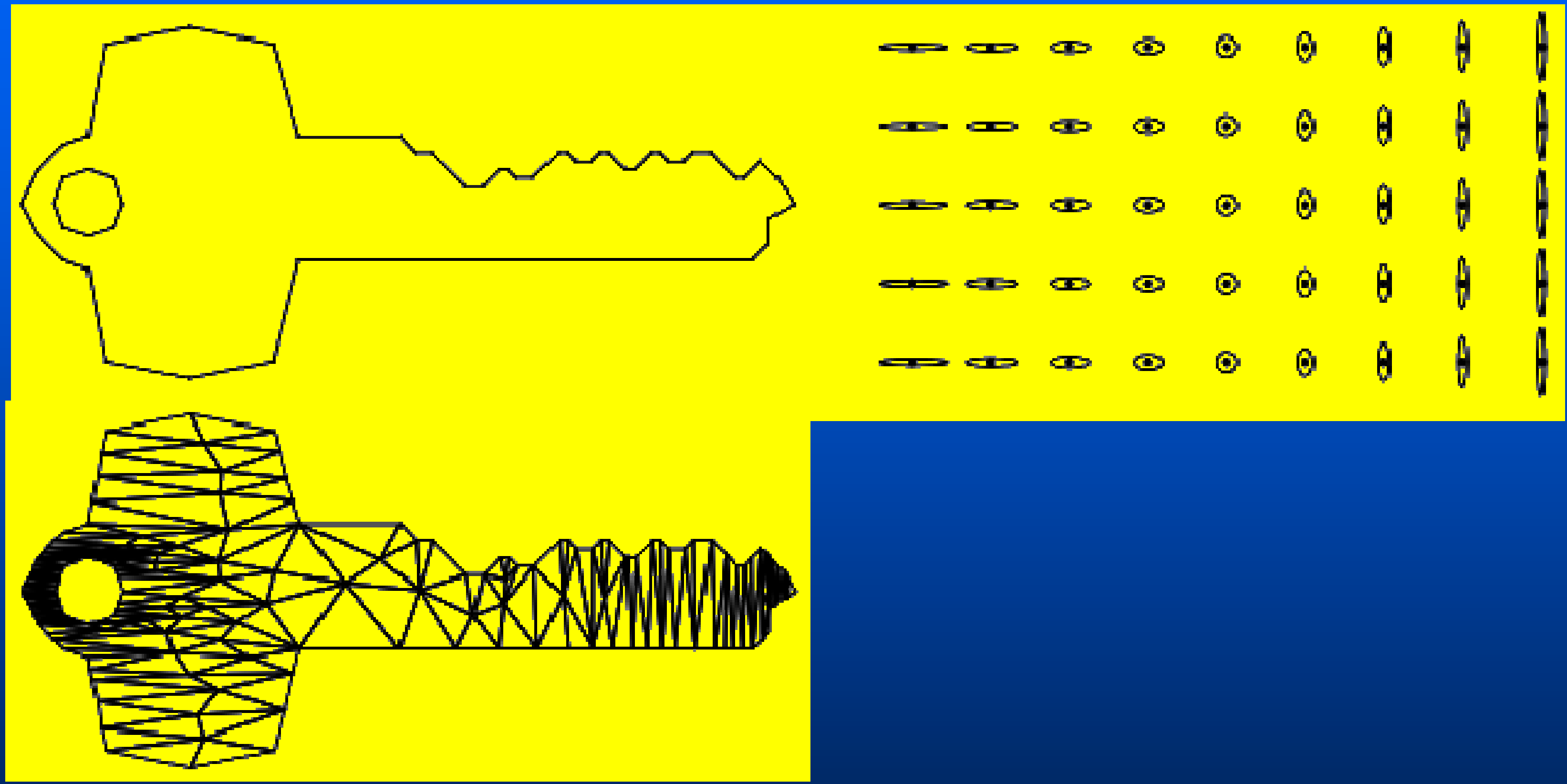


Examples of Remeshing

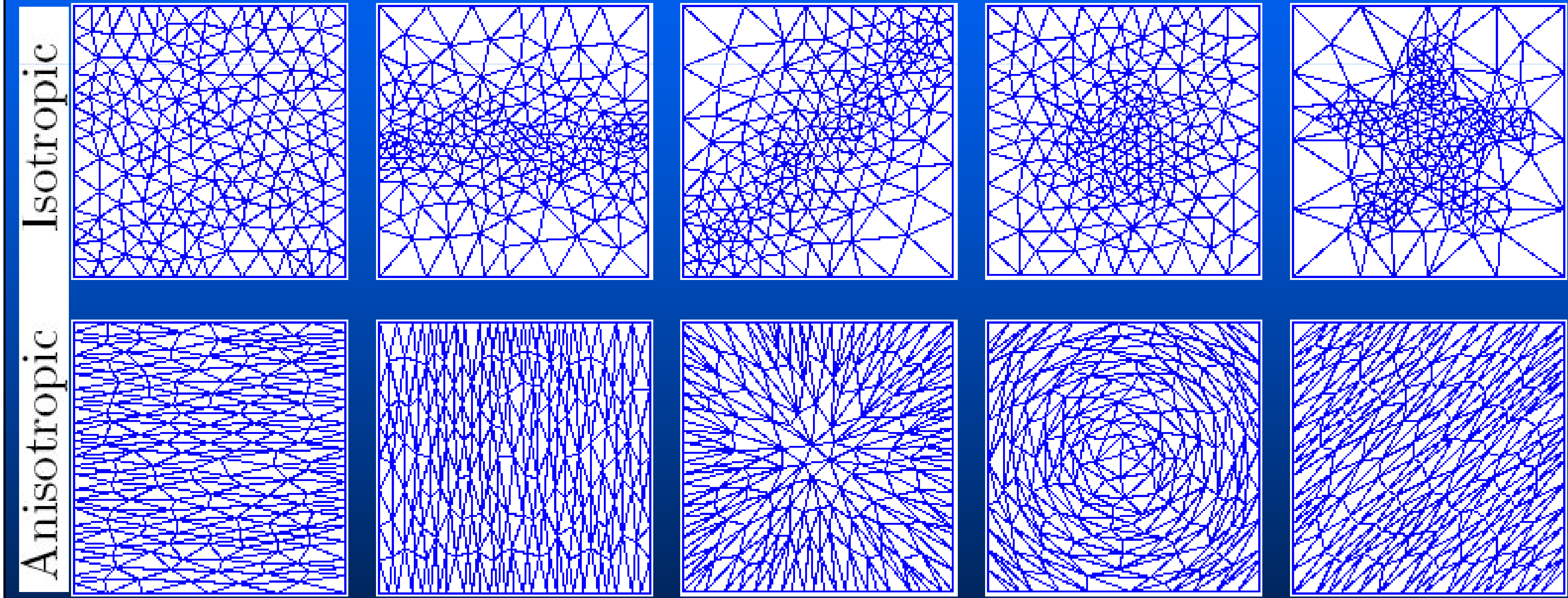


Original mesh **Uniform** **Curvature adapted**

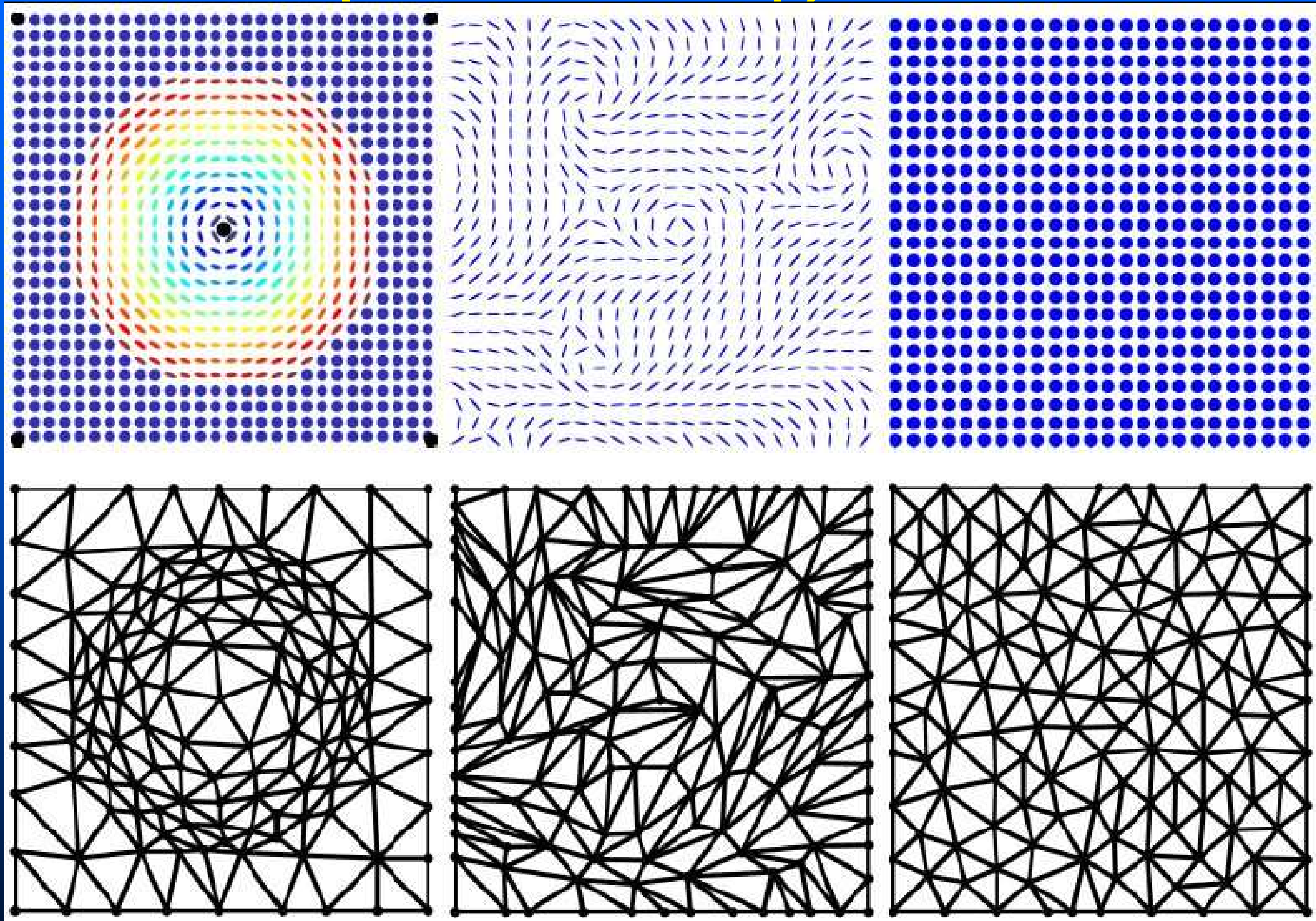
Examples of Anisotropic Meshing



Isotropic vs. Anisotropic Meshing

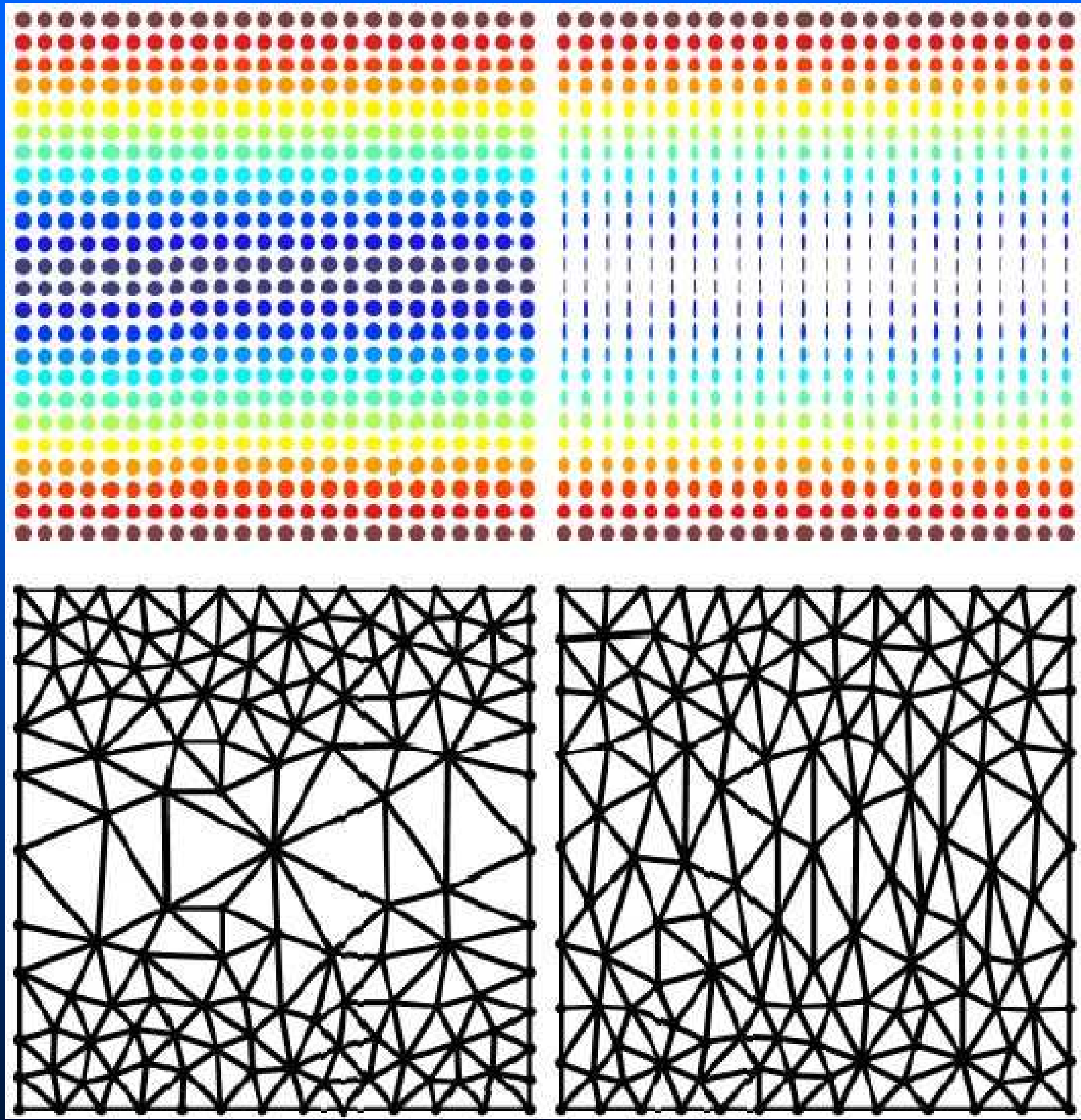


Anisotropic Meshing



farthest point strategy

Anisotropic Meshing



farthest point strategy

Thank you !

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Global minimum for active contour models: A minimal path approach Laurent D. Cohen and R.~Kimmel. in International Journal of Computer Vision, August 1997.

Minimal Paths and Fast Marching Methods for Image Analysis. , Laurent~D. Cohen, In Mathematical Models in Computer Vision: The Handbook, Nikos Paragios and Yunmei Chen and Olivier Faugeras Editors, Springer 2005.

Fast Constrained Surface Extraction by Minimal Paths. , Roberto Ardon and Laurent D. Cohen. International Journal on Computer Vision, Special Issue on Variational and Level Set Methods in Computer Vision (VLSM 2003), 69(1):127--136, August 2006.

Geodesic Remeshing Using Front Propagation. , Gabriel Peyre and Laurent D. Cohen. International Journal on Computer Vision, Special Issue on Variational and Level Set Methods in Computer Vision (VLSM 2003), 69(1):145--156, August 2006.

A new implicit method for surface segmentation by minimal paths in {3D} images. , Roberto Ardon, Laurent D. Cohen and Anthony Yezzi. Applied Mathematics and Optimization, 55(2):127-144, March 2007.

Anisotropic Geodesics for Perceptual Grouping and Domain Meshing. Sebastien Bougleux and Gabriel Peyr\'e and Laurent D. Cohen. Proc. tenth European Conference on Computer Vision (ECCV'08)}, Marseille, France, October 12-18, 2008.

Finding a Closed Boundary by Growing Minimal Paths from a Single Point on 2D or 3D Images. Fethallah Benmansour and Laurent D. Cohen. Journal of Mathematical Imaging and Vision. To appear, 2009.

Geodesic Methods for Shape and Surface Processing, Gabriel Peyre and Laurent D. Cohen in Advances in Computational Vision and Medical Image Processing: Methods and Applications, Springer, 2009.

Tubular anisotropy for 3D vessels segmentation. Fethallah Benmansour and Laurent D. Cohen. Preprint, 2009.

Lignes Géodésiques et Segmentation d'images

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Some joint works with G. Peyré, S. Bougleux,

and PhD students R. Ardon, S. Bonneau and F. Benmansour.

Collège de France, 16 Janvier 2009