The continuous spectrum in peak-shaped elastic bodies (Vibrating Black Holes)

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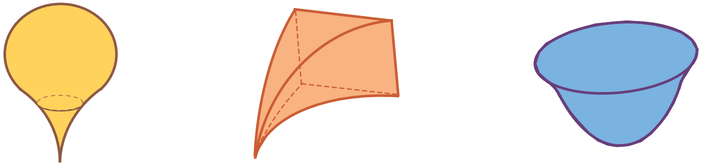
A peak-shaped body.

A peak-shaped (non-Lipschitz) domain Ω:

- In the vicinity of the coordinate origin the domain Ω coincides with the peak
  $\Pi = \{x = (y, z) = (y_1, \dots, y_{n-1}, z) : z \in (0, d), z^{-1-\gamma} y \in \omega\}$
- ω is the cross-section, $\gamma > 0$ the sharpness exponent of the peak Π (Π is a cone for $\gamma = 0$, a Lipschitz domain)

The peak– and beak– and basin–shaped domains:



The Neumann problem for the Laplace operator.

The spectral problem

$$-\Delta u = \lambda u \quad \text{in} \quad \Omega, \qquad \partial_\nu u = 0 \quad \text{on} \quad \partial\Omega \setminus \mathcal{O},$$

The spectrum is discrete.

- If $\partial\Pi$ is the graph $z = f(y)$, the compactness of the embedding $H^1(\Omega) \subset L^2(\Omega)$ is evident.
- If not (e.g., the peak cross-section ω is an annulus), one may apply the Hardy–type inequality
$$\|r^{-1}u; L^2(\Omega)\| \leq c_\Omega \|u; H^1(\Omega)\|$$
and make use of the big weight r^{-1} where r is the distance from $\mathcal{O}$, namely the embedding operator becomes the sum of two operator, small (in the ε-neighborhood V_δ of $\mathcal{O}$) and compact (in the Lipschitz domain $\Omega \setminus \overline{V_\delta}$).

The elasticity equations in three dimensions.

- $u = (u_1, u_2, u_3)$ is a displacement vector ($n = 3$).
- $\varepsilon_{jk}(u) = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j}\right)$ and $\sigma_{jk}(u) = \sum_{p,q=1}^{3} a_{jk}^{pq}\varepsilon_{jk}(u)$

  are Cartesian components of the strain (deformation) and stress tensors, respectively.
- The differential equations and the boundary conditions:

$$-\sum_{k=1}^{3} \frac{\partial}{\partial x_k}\sigma_{jk}(u) = \lambda\rho u_j \quad \text{in} \quad \Omega,$$

$$\sum_{k=1}^{3} \nu_k \sigma_{jk}(u) = 0 \quad \text{on} \quad \partial\Omega \setminus \mathcal{O},$$

  where $\nu = (\nu_1, \nu_2, \nu_3)$ is the outward normal and $\rho > 0$ the material density.

The variational formulation of the elasticity spectral problem.

The elastic energy $\frac{1}{2}a(u,u;\Omega)$:

$$a(u,u;\Omega) = \int\limits_{\Omega} \sum_{j,k=1}^{3} \sigma_{jk}(u)\varepsilon_{jk}(u)\,dx \geq$$

$$\geq c_a \int\limits_{\Omega} \sum_{j,k=1}^{3} \varepsilon_{jk}(u)\varepsilon_{jk}(u)\,dx = \frac{c_a}{4} \int\limits_{\Omega} \sum_{j,k=1}^{3} \left| \frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right|^2 dx$$

The integral identity:

$$\int\limits_{\Omega} \sum_{j,k=1}^{3} \sigma_{jk}(u)\varepsilon_{jk}(v)\,dx = \lambda \int\limits_{\Omega} \rho \sum_{j=1}^{3} u_j v_j\,dx, \quad \forall v \in C_c^{\infty}(\overline{\Omega} \setminus \mathcal{O})^3$$

The Korn inequality.

In a Lipschitz domain Ω:

$$\int\limits_\Omega \left|\nabla_x u_j\right|^2 dx \le c_\Omega \left(\int\limits_\Omega \left| \frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right|^2 dx + \int\limits_\Omega \left|u_j\right|^2 dx \right)$$

(summation over $j, k = 1, 2, 3$)

Some names:

Korn A. (1908), Friedrichs K.O. (1947), Gobert J. (1962), Nečas J. (1967), Kondratiev V.A., Oleinik O.A. (1988)

The spectrum of the elasticity system with the traction-free boundary conditions is discrete:

$$0 = \lambda_1 = \ldots = \lambda_6 < \lambda_7 \le \ldots \le \lambda_m \le \ldots \to +\infty$$

The absence of Korn's inequality in a peak-shaped domain.

The family of trial vector functions:

Displacements

$$\psi_1^m(x) = \varphi_m(z), \quad \psi_2^m(x) = 0, \quad \psi_3^m(x) = -y_1 \partial_z \varphi_m(z),$$

where $\varphi_m(z) = \varphi(mz)$, $\varphi \in C_c^\infty(1,2)$, $m \to +\infty$, and strains

$$\varepsilon_{pq}(\psi^m) = 0,\ p,q = 1,2, \quad \varepsilon_{23}(\psi^m) = 0,$$
$$\varepsilon_{13}(\psi^m) = \frac{1}{2}\left(\frac{\partial \psi_1^m}{\partial z} + \frac{\partial \psi_3^m}{\partial y_1}\right) = 0, \quad \varepsilon_{33}(\psi^m) = -y_1 \frac{\partial^2 \varphi_m}{\partial z^2}.$$

Norms:

$$\int\limits_{\Pi} |\psi^m(x)|^2 \, dx \sim \left(m^{-1-\gamma}\right)^2 \times m^{-1} = m^{-3-2\gamma},$$

(area of the cross-section) $\times$ (length of the support)

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Norms:

$$\int\limits_{\Pi} |\varepsilon_{jk}(\psi^m; x)|^2 \, dx \sim m^{-3-2\gamma} \times \left(m^{-1-\gamma}\right)^2 \times \left(m^2\right)^2 = m^{-1-4\gamma},$$

(area $\times$ length) $\times$ (y_1) $\times$ (φ_m'')

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Norms:

$\|\psi^m; L_2(\Pi)\|^2 \sim m^{-3-2\gamma}, \qquad \|\nabla_x \psi^m; L_2(\Pi)\|^2 \sim m^{-1-2\gamma},$

$$\|\varepsilon(\psi^m); L_2(\Pi)\|^2 \sim m^{-1-4\gamma}.$$

The appearance of the essential spectrum.

Recall the calculated norms:

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- The energy space $\mathcal{E}$ is the completion of $C_c^\infty(\overline{\Omega} \setminus \mathcal{O})^3$ in the norm $(a(u,u;\Omega) + \|\rho^{1/2}u; L_2(\Omega)\|^2)^{1/2}$.
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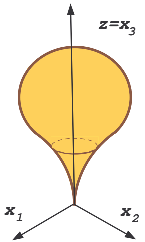
- If $\gamma > 0$, the embedding $\mathcal{E} \subset H^1(\Omega)^3$ is wrong!
- If $\gamma \geq 1$, the embedding $\mathcal{E} \subset L^2(\Omega)^3$ is not compact, thus, the essential spectrum is not empty!

The anisotropic weighted Korn inequality in peak-shaped domains (an incomplete version).

Nazarov, 1998, 2008:

$$\int\limits_{\Omega} \Big(r^{-2}|u_3|^2 + r^{-2+2\gamma}\big(|u_1|^2 + |u_2|^2\big)\Big)\, dx \le$$

$$\le c_\Omega \Big(\int\limits_{\Omega} \left| \frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right|^2 dx + \int\limits_{\Omega} \left| u_j \right|^2 dx \Big).$$



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The inferences:

- If $\gamma < 1$ the embedding $\mathcal{E} \subset L^2(\Omega)^3$ is compact and the spectrum of the elasticity problem for a body with an insufficient sharp peak is discrete.
- The essential spectrum cannot appear due to longitudinal vibrations.

The continuous spectrum.

The essential and continuous spectra coincide:

Due to a series of results by V. Maz'ya and B. Plamenevskii, the kernel of the problem operator in weighted Sobolev spaces has a finite dimension.
Thus an eigenvalue of infinite multiplicity is impossible.

The physical conclusions:

- The continuous spectrum provokes for wave processes. Since the continuous spectrum is caused by the peak top, the wave processes must be located in the peak.
- A wave process in a finite volume may produce a black hole for vibration.

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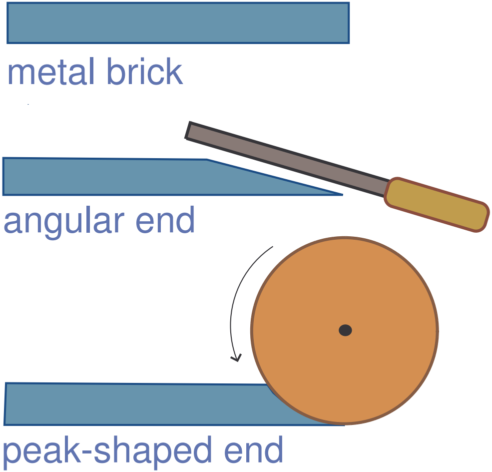
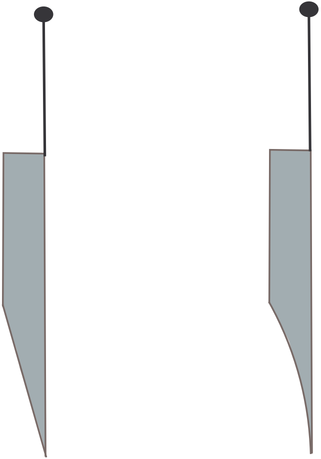
Preparing a thought experiment.



A thought experiment.



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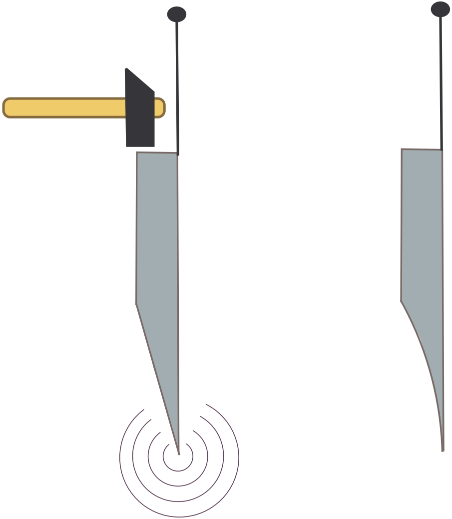

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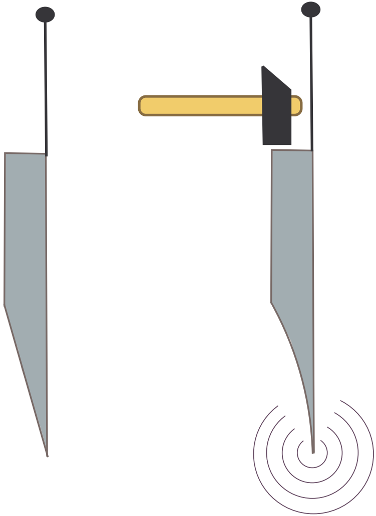

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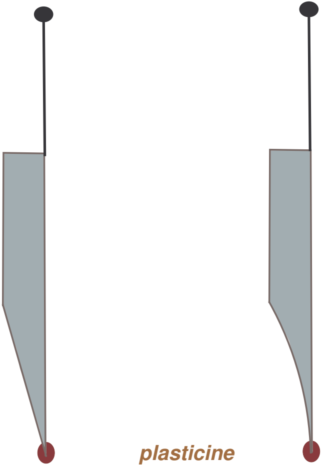

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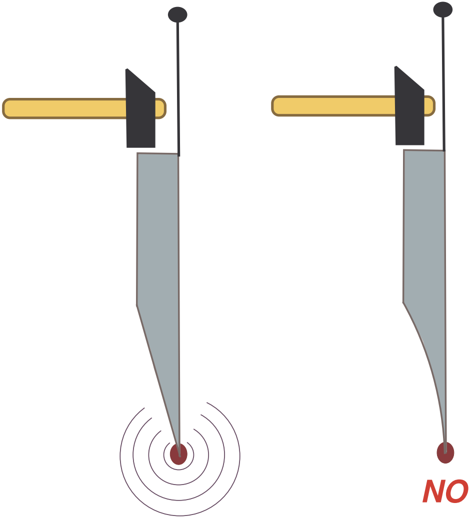


The real experiment.

The engineering theory:

Mironov M.A. (1988, 1992, ...), Krylov V.V. (2002, 2004...)

Wave processes. An engineering model.

The one-dimensional theory of beams (Mironov M.A.;1988):

$$\frac{d^2}{dz^2}B(z)\frac{d^2}{dz^2}w(z) = \lambda\rho M(z)w(z), \quad z \in (0,1),$$

.

where $B(z) = b\left(z^2\right)^4$ is the inertia moment and $M(z) = m\left(z^2\right)^2$ is the area of the cross-section.

The differential equation of the Euler type (indeed, $8-2\times2=4$):

$$\frac{d^2}{dz^2}z^8\frac{d^2}{dz^2}w(z) = \mu z^4 w(z), \quad z \in (0,1),$$

$$w(z) = z^{\beta-5/2} \quad \Rightarrow \quad \left(\beta^2 - \frac{49}{4}\right)^2\left(\beta^2 - \frac{25}{4}\right)^2 = \mu,$$

$$\beta_\pm^2 = \frac{49+25}{8} \pm \sqrt{\frac{(49+25)^2}{64} - \frac{49\times25}{16} + \mu}$$

Waves :

$$\beta_{\pm}^2 = 8^{-1}\big(49 + 25 \pm \sqrt{(49-25)^2 + 64\,\mu}\big)$$

The right-hand side with minus is positive if and only if

$\mu < \mu_* = 4^{-2}5^2 7^2.$

- If $\mu < \mu_*$, four solutions have the form $z^{\pm\beta_\pm - 5/2}$, two solutions with $+\beta_\pm$ possess finite energy $\int z^8 \big|\partial_z^2 w(z)\big|^2 dz$.
- If $\mu \geq \mu_*$, two solutions have the form $z^{\pm i\beta_- - 5/2}$ and imply oscillating waves while the energy is infinite for three solutions with the exponents $-\beta_+ - 5/2$ and $\pm i\beta_- - 5/2$ and the problem cannot have a solution in H^2.

Wave processes. An engineering model (Mironov ;1988).

Harmonic oscillations:

$$\exp(it\kappa)z^{\pm i\beta-5/2} = z^{-5/2}\exp(i(t\kappa \pm \beta \ln z))$$

- $z^{-5/2}$ is the normalization factor such that the integral $\int z^8 |\partial_z^2 w(z)|^2\, dy$ over the cross-section ($z^{-1}dz$ is taken off) stays constant.
- The function above does not change if $t\kappa \pm \beta \ln z =$const. Hence, during the time interval of length Δt the relative distance of the wave propagation is equal to $\exp(\mp\beta^{-1}\kappa\Delta t)$ and it takes an infinite time either to go to the top (minus), or to come from the top (plus).

The mathematical theory is not completed yet.

Some results:

- If $\gamma = 1$, the continuous spectrum includes $[\lambda_\dagger, +\infty)$.
- If $\gamma > 1$, the continuous spectrum implies $[0, +\infty)$.
- Under a symmetry assumption, an infinite series of eigenvalues are embedded into the continuous spectrum.
- Accumulation effect for eigenvalues of elastic bodies with blunted peaks and peak-shaped inclusions.

Some publications:

- *Nazarov S.A.* The spectrum of the elasticity problem for a spiked body. Siberian Math. J. **49**, 5 (2008)
- *Bakharev F. L., Nazarov S.A.* On the structure of the spectrum of the elasticity problem for a body with a super-sharp spike. Siberian Math. J. **50**, 4 (2009)
- *Nazarov S.A.* Oscillations of an elastic body with a heavy rigid spike-shaped inclusion J. Appl. Math. Mech. **72**, 5 (2008)

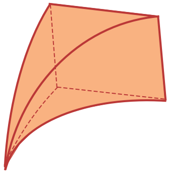
Beak-shaped solids.

Some results:

- If $\gamma = 1$, the essential spectrum includes the ray $[\lambda_\dagger, +\infty)$.
- If $\gamma > 1$, the essential spectrum includes the point $\lambda = 0$.

Publications:

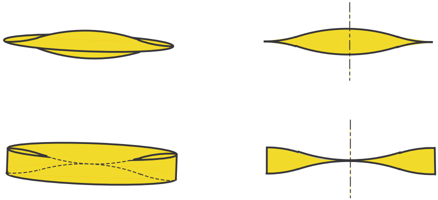
- *Nazarov S.A., Polyakova O.R.* Asymptotic behavior of the stress-strain state near a spatial singularity of the boundary of the beak tip type J. Appl. Math. Mech. **57**, 5 (1993)
- *Cardone G., Nazarov S.A., Taskinen J.* A criterion for the existence of the essential spectrum for beak-shaped elastic bodies J. Math. Pures Appl. **92**, 6 (2009)



Plates with sharp edges (Mikhlin S.G.; 1970)

Degenerate forth-order equation:

- The criterium of the continuous spectrum is proved in
- *Campbell A., Nazarov S.A., Sweers G. H.* Spectra of two-dimensional models for thin plates with sharp edges SIAM J. Math. Anal. **42**, 6 (2010)



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The common weak point:

- RADIATION CONDITIONS ARE NOT CREATED YET!
- For a scalar problem in
- *Nazarov S.A., Taskinen J.* Radiation conditions at the top of a rotational cusp in the theory of water-waves M2AN **45** (2011)

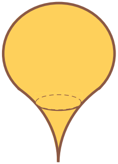
Vibrating Black Holes.

The Neumann problem for elliptic system of second-order equations.

The spectral problem.

$$\overline{\mathcal{D}(-\nabla_x)}^{\top}\mathcal{A}(x)\mathcal{D}(\nabla_x)u(x) = \lambda\mathcal{B}(x)u(x), \quad x \in \Omega,$$
$$\overline{\mathcal{D}(\nu(x))}^{\top}\mathcal{A}(x)\mathcal{D}(\nabla_x)u(x) = 0, \quad x \in \partial\Omega \setminus \mathcal{O}.$$

- the peak $\Pi = \{x = (y, z) : z \in (0, d), z^{-1-\gamma}y \in \omega\}$.
- $u = (u_1, \ldots, u_k)^{\top}$ is the column vector function.
- λ is the spectral parameter.



The algebraic completeness.

The coefficient matrices.

- $\mathcal{A}$ and $\mathcal{B}$ are matrices of sizes $K \times K$ and $k \times k$, respectively, Hermitian, positive definite, measurable, uniformly bounded.

The matric differential operator.

- $\mathcal{D}(\nabla_x)$ is a $K \times k$ matrix of first-order differential operators with constant (complex) coefficients.
- $\mathcal{D}(\xi)$ is algebraically complete (Nečas J., 1967), that is, with a certain $\rho_{\mathcal{D}} \in \mathbb{N} = \{1, 2, \dots\}$, for any row $p = (p_1, \dots, p_k)$ of homogeneous polynomials in ξ of degree $\rho \geq \rho_{\mathcal{D}}$, one finds a polynomial row $q = (q_1, \dots, q_K)$ such that

$$p(\xi) = q(\xi)\mathcal{D}(\xi), \quad \xi \in \mathbb{R}^n$$

The elasticity system is a particular case with $k = 3$ and $K = 6$.

The variational formulation.

The integral identity.

$$(\mathcal{A}\mathcal{D}(\nabla_x)u, \mathcal{D}(\nabla_x)v)_\Omega = \lambda(\mathcal{B}u, v)_\Omega, \qquad \forall v \in H^1(\Omega)^k.$$

The generalized Korn inequality in a Lipschitz domain.

The algebraic completeness provides (Nečas J., 1967)

$$\|u; H^1(\Theta)\| \le c_\Theta^{\mathcal{D}}\big(\|\mathcal{D}(\nabla_x)u; L^2(\Theta)\| + \|u; L^2(\Theta)\|\big),\ u \in H^1(\Theta)^k.$$

The polynomial property (Nazarov S., 1997, 1999).

- For any point x^0 and any domain $\Xi \subset \mathbb{R}^n$ and with some finite-dimensional subspace $\mathcal{P}$ of vector polynomials, $(\mathcal{A}(x^0)\mathcal{D}(\nabla_x)u, \mathcal{D}(\nabla_x)v)_\Xi = 0,\ u \in H^1(\Xi)^k \ \Leftrightarrow\ u \in \mathcal{P}\big|_\Xi$.
- $\Rightarrow$ the ellipticity and the Shapiro–Lopatinskii conditions.
- $p = (p_1, \ldots, p_k)^\top \in \mathcal{P} \quad \Rightarrow \quad \mathrm{deg} p_j < \rho_{\mathcal{D}}$
- In elasticity $\mathcal{P}$ is the space of rigid motions (translations+rotations)

The criterion of the discrete spectrum.

Theorem.

- The spectrum of the problem in the peak-shaped domain Ω is discrete for any $\gamma > 0$ if and only if
  the polynomial subspace $\mathcal{P}$ does not contain a polynomial dependent on z, i.e. $\partial_z p = 0$ for any $p \in \mathcal{P}$.
- If the polynomial subspace $\mathcal{P}$ includes a polynomial
  $p(y, z) = p^m z^m + p^{m-1} z^{m-1} + \cdots + p^1 z^1 + p^0$
  with $m \geq 1$ and $p^j \in \mathbb{C}^k$, $p^m \neq 0$,
  then in the case $\gamma \geq 1/m$ the continuous spectrum σ_c of the problem is not empty. Moreover, $0 \in \sigma_c$ for $\gamma > 1/m$.

For the elasticity problem a rotation vector depends on z.

The operator formulation of the boundary value problem.

The Hilbert space.

Let $\mathcal{H}$ be the completion of $C_c^\infty(\overline{\Omega} \setminus \mathcal{O})^k$ with respect to the norm generated by the scalar product

$$\langle u, v \rangle = (\mathcal{A}\mathcal{D}(\nabla_x)u, \mathcal{D}(\nabla_x)v)_\Omega + (\mathcal{B}u, v)_\Omega.$$

The continuous positive self-adjoint operator $\mathcal{T}$ is defined by

$$\langle \mathcal{T}u, v \rangle = (\mathcal{B}u, v)_\Omega, \qquad u, v \in \mathcal{H}.$$

The new spectral parameter $\mu = (1+\lambda)^{-1}$.

The variational formulation of the boundary value problem

$$(\mathcal{A}\mathcal{D}(\nabla_x)u, \mathcal{D}(\nabla_x)v)_\Omega = \lambda(\mathcal{B}u, v)_\Omega, \qquad \forall v \in H^1(\Omega)^k.$$

is equivalent to the abstract equation

$$\mathcal{T}u = \mu u, \qquad u \in \mathcal{H}.$$

Steps of the proof.

Step 1.

If $\partial_z p = 0$ for any $p \in \mathcal{P}$, then the following weighted Korn inequality is valid:

$$\|r^{-1}u; L^2(\Omega)\| \le c\big(\|\mathcal{D}(\nabla_x)u; L^2(\Omega)\| + \|u; L^2(\Omega)\|\big)$$

This inequality is similar to the Hardy-type inequality in the case of the Laplace operator but the proof needs some matric algebra.

Step 2.

A sequence $\{u^m\}$ in $\mathcal{H}$ is constructed and the properties

$$\|u^m; \mathcal{H}\| \ge c > 0; \quad u^m \to 0 \text{ weakly in } \mathcal{H};$$

$$\|\mathcal{T}u^m - u^m; \mathcal{H}\| \to 0$$

of the singular Weyl sequence for the operator $\mathcal{T}$ at the point $\mu = 1$ are verified.

The structure of the test functions is approximately the same as in elasticity.

The paper.

Nazarov S.A. On the essential spectrum of boundary value problems for systems of differential equations in a bounded peak-shaped domain. Funct. Anal. Appl. **43**,1 (2009).

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Nazarov S.A. On the essential spectrum of boundary value problems for systems of differential equations in a bounded peak-shaped domain. Funct. Anal. Appl. **43**,1 (2009).

THANK YOU

FOR ATTENTION !