The hypothesis of a language of thought in infancy

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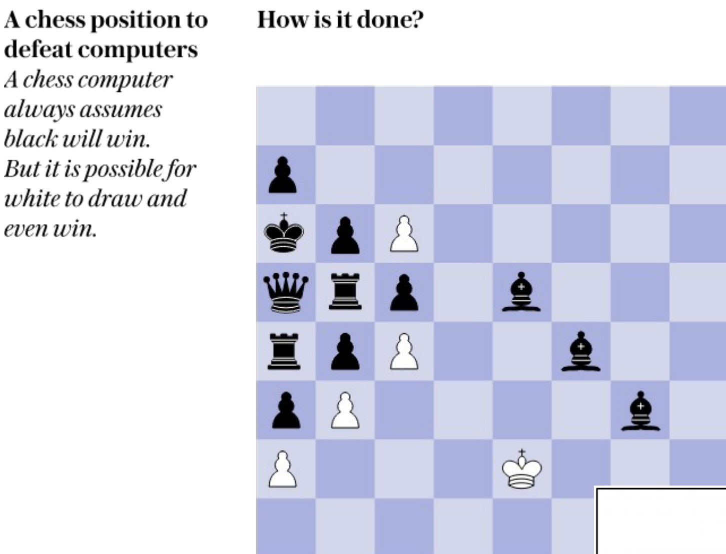
Outline

- **Motivation**
- **Central hypothesis: a Language of Thought**
- **Experimental domains:**
  - Evidence in adults from logical learning
  - Logic and compositionality in children and infants
  - Hierarchical biases in children, adults, primates
- **Cognitive change by learning isomorphisms**

Humans have broad algorithmic knowledge



$$*54{\cdot}43.\ \vdash :.\ \alpha, \beta \in 1 \,.\supset :\, \alpha \cap \beta = \Lambda \,.\equiv .\, \alpha \cup \beta \in 2$$

Dem.

$$\vdash .\, *54{\cdot}26 \,.\supset \vdash :.\, \alpha = \iota`x \,.\, \beta = \iota`y \,.\supset :\, \alpha \cup \beta \in 2 \,.\equiv .\, x \neq y \,.$$

$$[*51{\cdot}231] \qquad \equiv .\, \iota`x \cap \iota`y$$

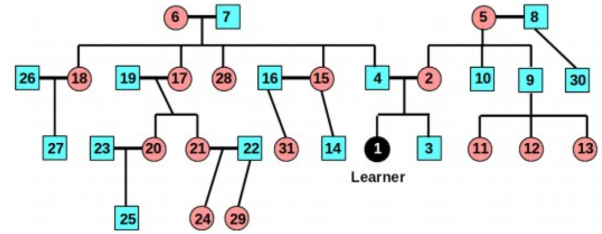
$$[*13{\cdot}12] \qquad \equiv .\, \alpha \cap \beta = \Lambda \qquad (1)$$

$$\vdash .\, (1) \,.\, *11{\cdot}11{\cdot}35 \,.\supset$$

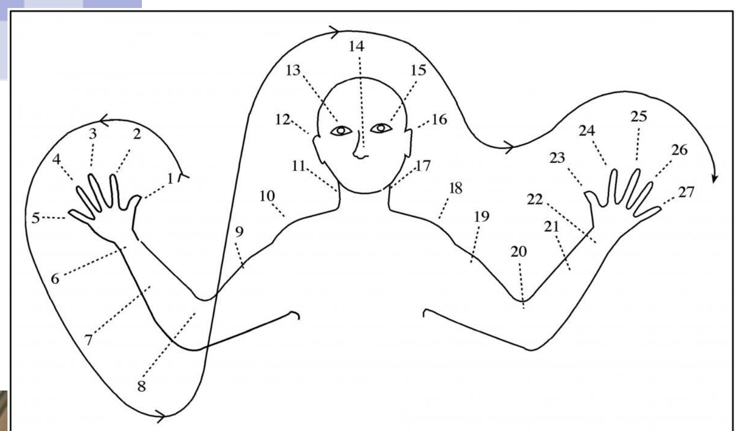
$$\vdash :.\, (\exists x, y) \,.\, \alpha = \iota`x \,.\, \beta = \iota`y \,.\supset :\, \alpha \cup \beta \in 2 \,.\equiv .\, \alpha \cap \beta = \Lambda \qquad (2)$$

$$\vdash .\, (2) \,.\, *11{\cdot}54 \,.\, *52{\cdot}1 \,.\supset \vdash .\, \text{Prop}$$

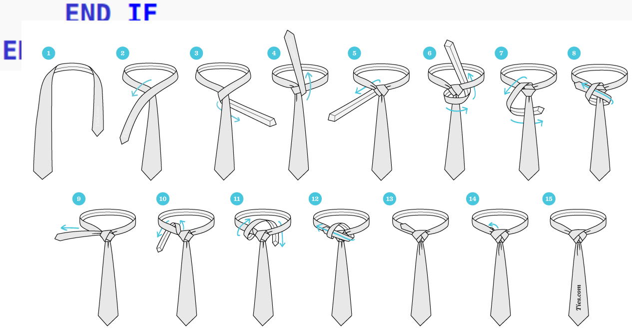
From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.



```
FUNCTION itFib (n)
    n1 = 0
    n2 = 1
    FOR k = 1 TO ABS(n)
        sum = n1 + n2
        n1 = n2
        n2 = sum
    NEXT k
    IF n < 0 THEN
        itFib = n1 * ((-1) ^ ((-n) + 1))
    ELSE
        itFib = n1
    END IF
EI
```



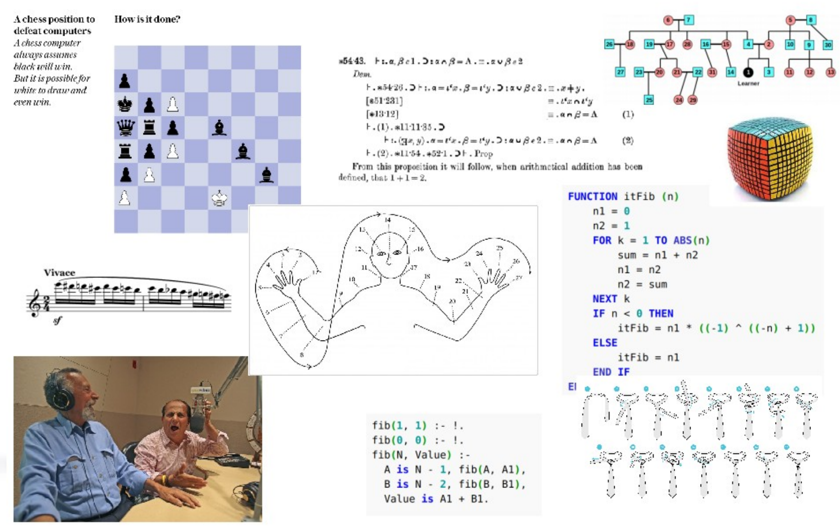
```
fib(1, 1) :- !.
fib(0, 0) :- !.
fib(N, Value) :-
  A is N - 1, fib(A, A1),
  B is N - 2, fib(B, B1),
  Value is A1 + B1.
```



The “Central Problem” for cognitive development

- How can we start with what a baby knows and end up with what an ordinary adult knows?

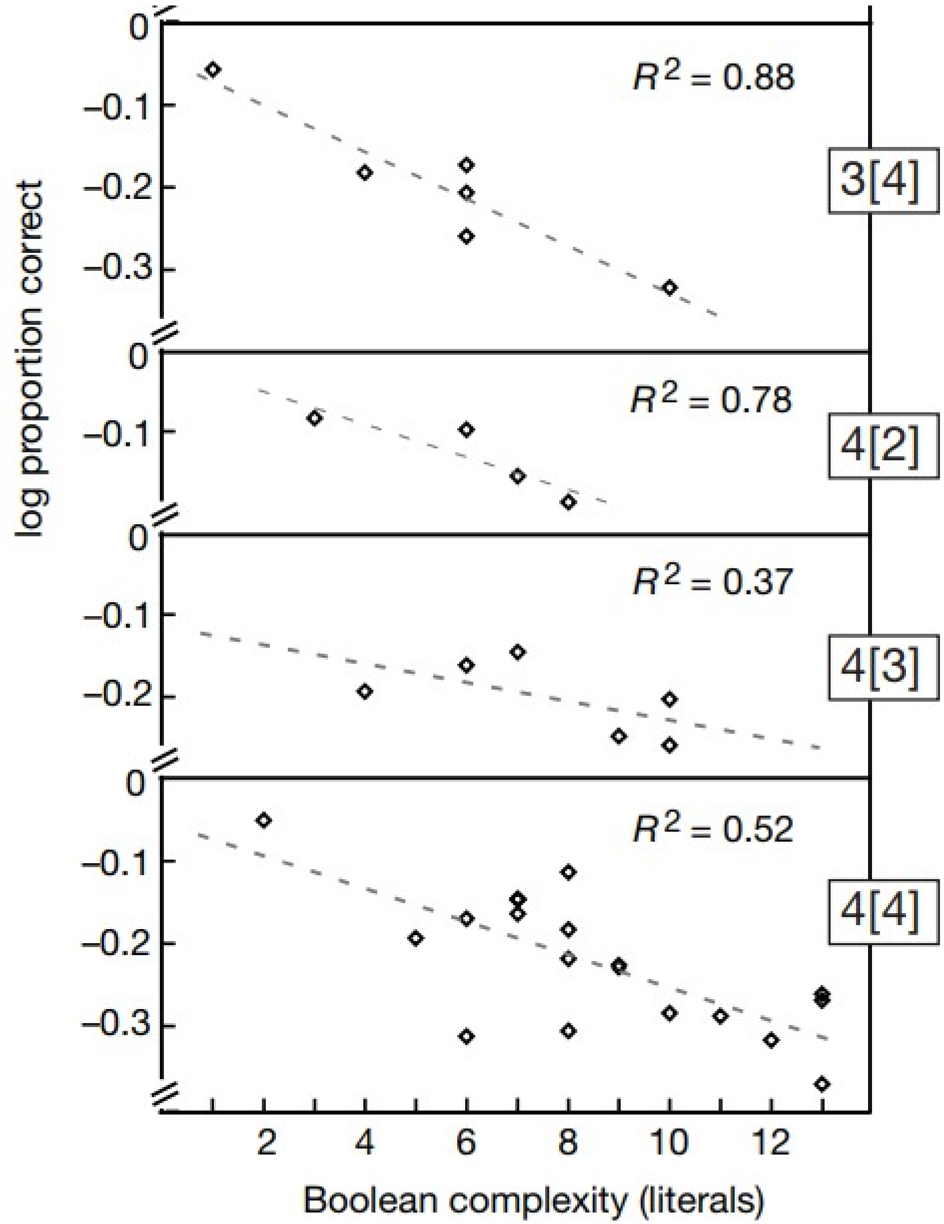
Humans have broad algorithmic knowledge



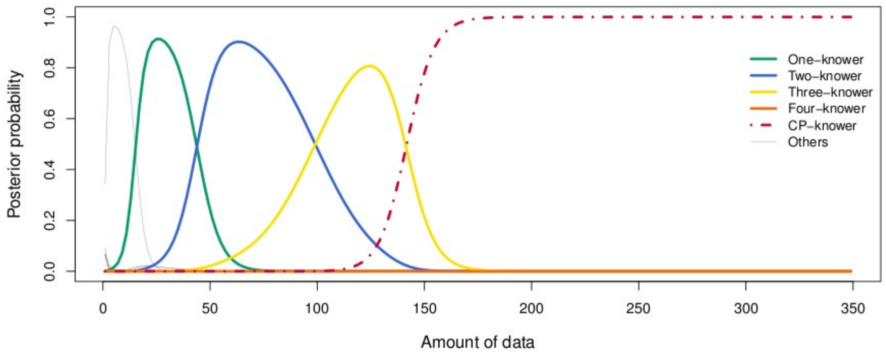
Working hypothesis of my work

- **Learning is *like* a statistician programming** – learners compose operations in new ways to form generative models (explanatory theories) of observed data.
    - Programs written in the LOT (Fodor 1975)
    - Nature provides the primitive + inference mechanisms
    - Input drives the creation of specific representations

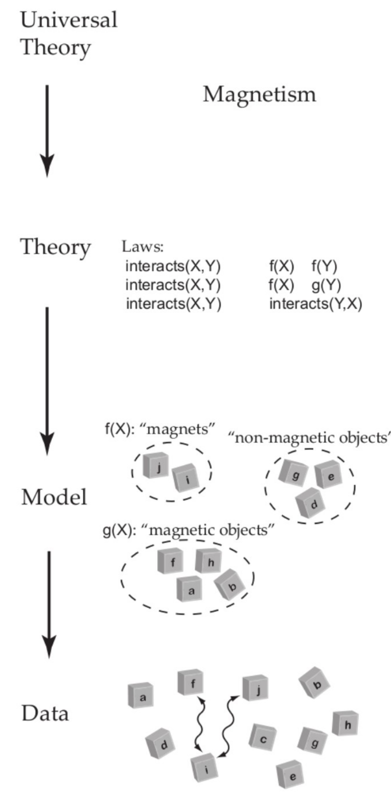
Feldman (2000)

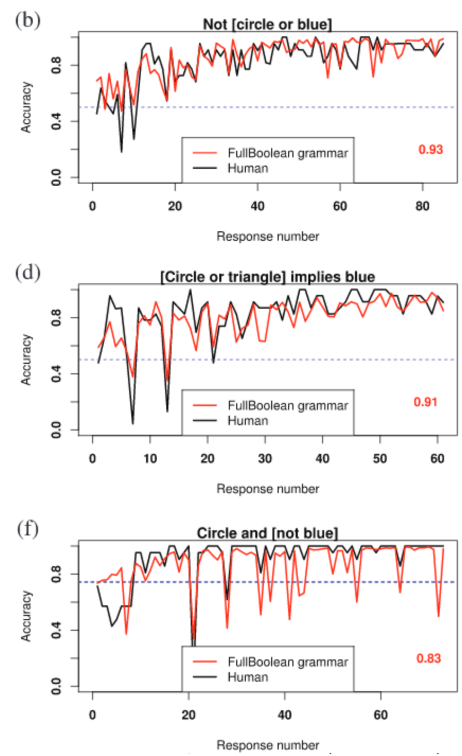


Simple: red(x)
. and(red(x), circle(x))
. or(not(red(x)), circle(x))
Complex: or(and(red(x), circle(x)), square(x))

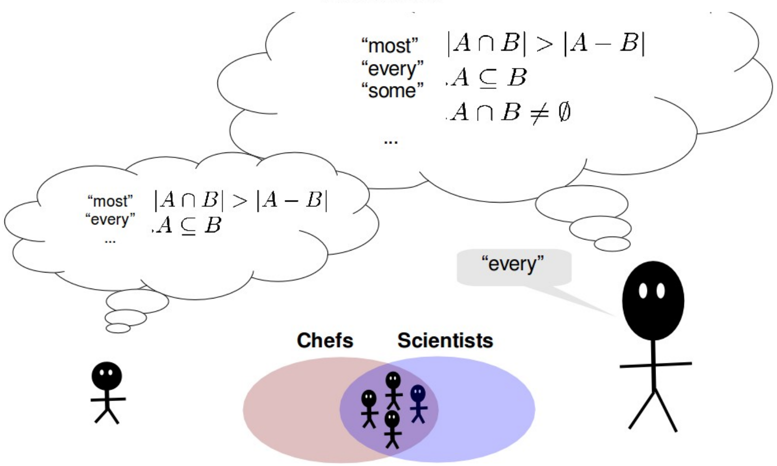



```
define F(S):
    if singleton(S):
        return "one"
    else:
        x = select(S)
    return wordAfter(F(S \ x))
```





Ullman et al. 2010, 2012



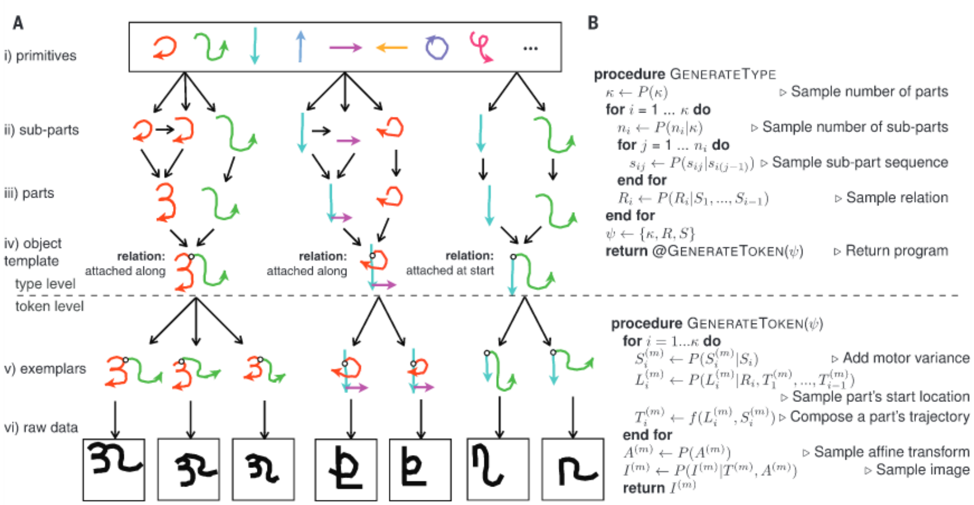
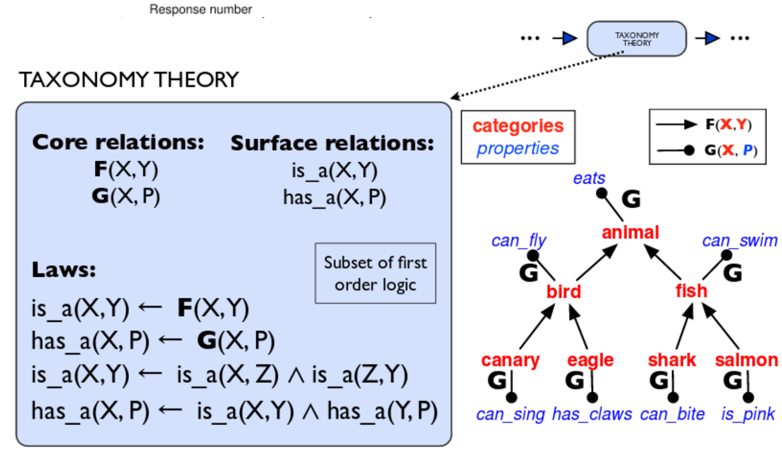


Fig. 3. A generative model of handwritten characters. (**A**) New types are generated by choosing primitive actions (color coded) from a library (i), combining these subparts (ii) to make parts (iii), and combining parts with relations to define simple programs (iv). New tokens are generated by running these programs (v), which are then rendered as raw data (vi). (**B**) Pseudocode for generating new types ψ and new token images $I^{(m)}$ for $m = 1, \ldots, M$. The function $f(\cdot, \cdot)$ transforms a subpart sequence and start location into a trajectory.

Lake et al 2015



Katz & Tenenbaum 2008

Central hypothesis of my work

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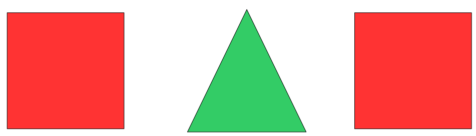
Predictions of Bayesian-LOT

- Learning curves should follow LOT predictions.

Compositional learning in adults

(Piantadosi 2011, Piantadosi, Goodman, Tenenbaum 2010, 2016)



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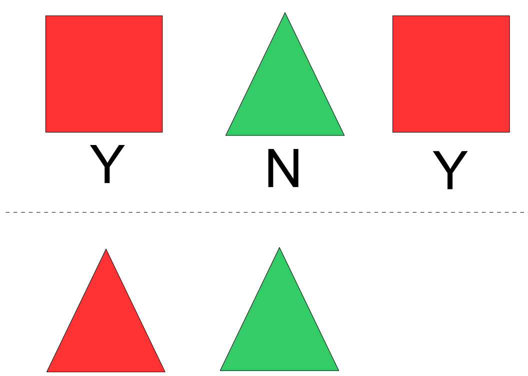

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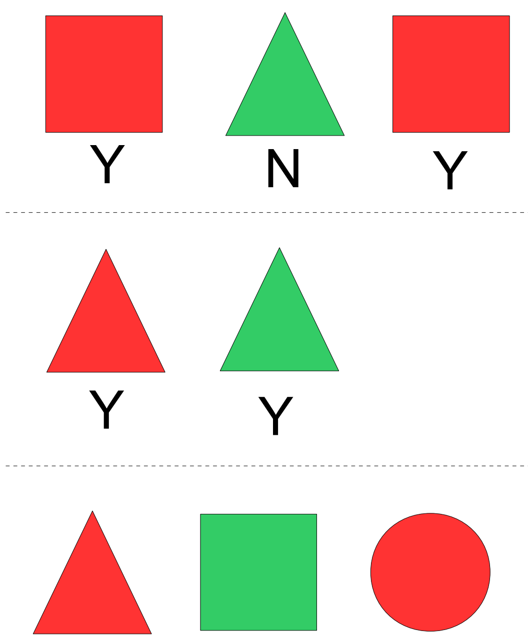

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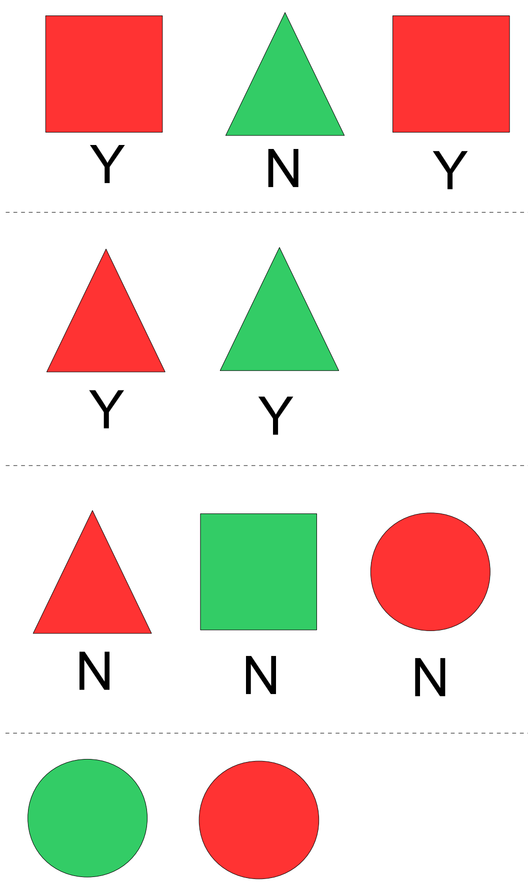

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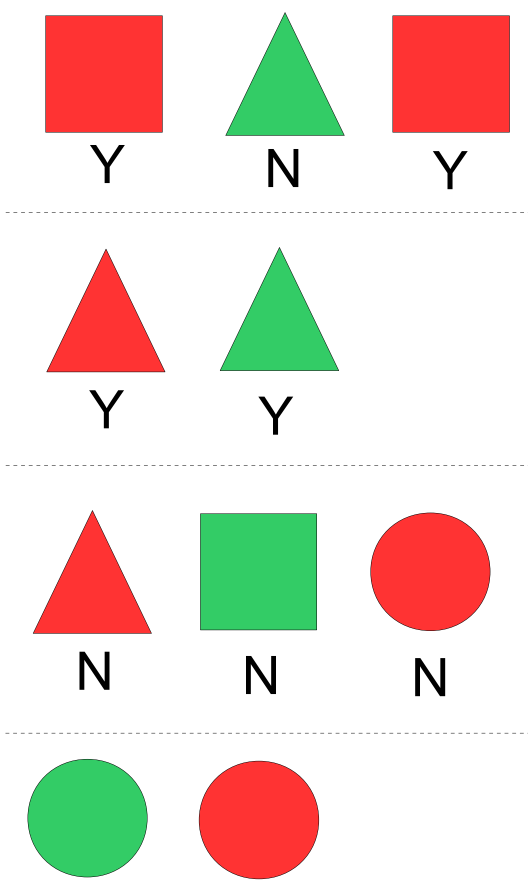
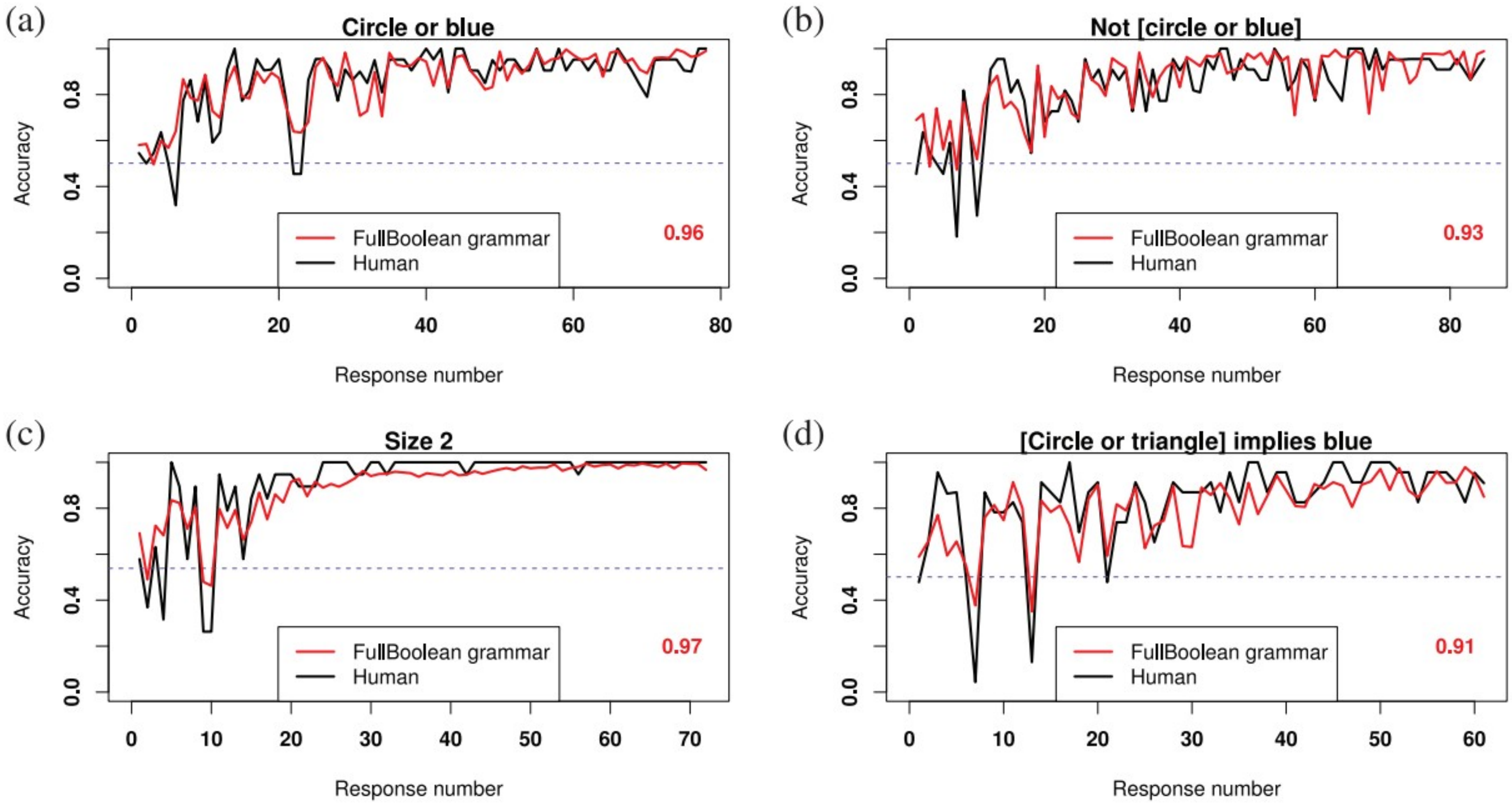


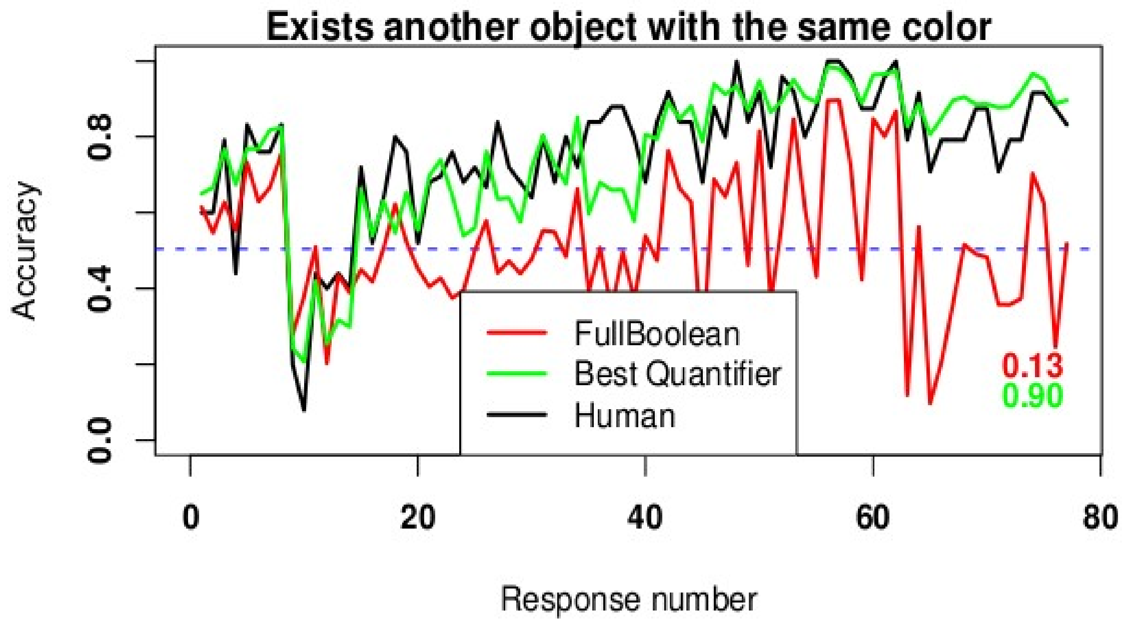
$$\lambda A\ .\ \exists x.\ \mathrm{shape}(x) = \mathrm{shape}(A)$$

- "True"
- "False"
- "Blue"
- "Circle"
- "Not circle"
- "Circle or blue"
- "Circle or triangle"
- "Blue or green"
- "Circle and blue"
- "Circle and not blue"
- "Not [circle and blue]"
- "Not [circle or blue]"
- "Not [[Not cirle] or blue]"
- "Circle xor blue"
- "Not [circle xor blue]"
- "Circle xor [not blue]"
- "Everything iff triangle"
- "Size 3"
- "Size 2"
- "Size 1"
- "Size 3 or size 1"
- "Size 3 or size 2"
- "Size 2 or size 1"
- "Size 1 and blue"
- "Size 1 or blue"
- "One of largest or smallest"
- "Not one of largest or smallest"
- "One of the largest of its shape"
- "Unique largest"
- "Unique largest and blue"
- "Unique largest or blue"
- "One of the largest"
- "One of the largest and blue"
- "One of the largest or blue"
- "There exists a smaller object"
- "There exists a smaller blue object"
- "Same shape as a blue object"
- "Same shape as a [blue object or circle]"
- "Same shape as a [blue object or green object]"
- "[Same shape as a blue object] and not blue"
- "[Same shape as a blue object] or green"
- "[Same shape as a blue object] and green"
- "[Same shape as a blue object] and circle"
- "Same shape as the unique largest"
- "Same shape as one of the largest"
- "Same shape as one of the largest and blue"
- "Same shape as one of the largest or blue"
- "Same shape as the unique largest but not the largest"
- "Same shape as one of the largest but not one of the largest"
- "Unique blue object"
- "Unique circle"
- "The unique element and is [blue or green]"
- "The unique element and is [blue and circle]"
- "The unique element and is [blue or circle]"
- "Unique largest blue object"
- "Unique largest [blue or green] object"
- "Same shape as the unique largest blue object"
- "Same shape as one of the largest blue objects"
- "Exists another object with the same shape"
- "Exists another object with the same size"
- "[Exists another object with the same shape] or blue"
- "[Exists another object with the same shape] and blue"
- "Exists another object with the same color"
- "Does not exist another object with same shape"
- "Does not exist another object with same shape and color"
- "Every other object with the same shape is the same color"
- "[Every other object with the same shape is same color] or blue"
- "[Every other object with the same shape is same color] or circle"
- "Every other object with the same shape is not the same color"
- "There exists another blue object with the same shape"
- "There exists another object with the same shape, and one with the same color"
- "There exists another object with the same shape, and a different one with the same color "
- "There exists another object with the same shape that has another with the same color"
- "Shares a feature with every object"
- "Circle implies blue"
- "Blue implies circle"
- "[Not blue] implies circle"
- "[Not blue] implies [not circle]"
- "Blue implies size=1"
- "[Circle or triangle] implies blue"
- "[Circle and blue] or [triangle and green]"
- "Circle or blue or [triangle and green]"
- "Circle or [blue and triangle]"
- "Circle or [blue implies triangle]"
- "There exists a blue object of the same shape"
- "Same size as a circle"
- "Same shape as another object which is blue"
- "Same shape as another object which is [blue or green]"
- "Same shape as a [blue or green] object (potentially itself)"
- "The unique object that is [blue or circle]"
- "The unique object that is [blue or green]"
- "The unique object that is [blue and circle]"
- "The unique object"
- "Same size as the unique blue object"
- "Unique smallest"
- "One of the smallest"
- "One of the smallest of its shape"
- "The unique smallest of its shape"
- "Exactly one other element is blue"
- "Exactly one element is blue"
- "Exactly one other element is the same color"
- "Same shape as exactly one blue object"
- "Same shape as exactly one other blue object"
- "Every other object with the same shape is blue"
- "Every object with the same shape is blue"
- "Every-other-atleastone object with the same shape is blue"
- "Every-atleastone object with the same shape is the same color"
- "Every-other-atleastone object with the same shape is not the same color"


(a)
Circle or blue
Accuracy
Response number
FullBoolean grammar
Human
0.96
(b)
Not [circle or blue]
0.93
(c)
Size 2
0.97
(d)
[Circle or triangle] implies blue
0.91

Summary of language comparison

(Piantadosi 2011, Piantadosi, Goodman, Tenenbaum 2010, in prep)

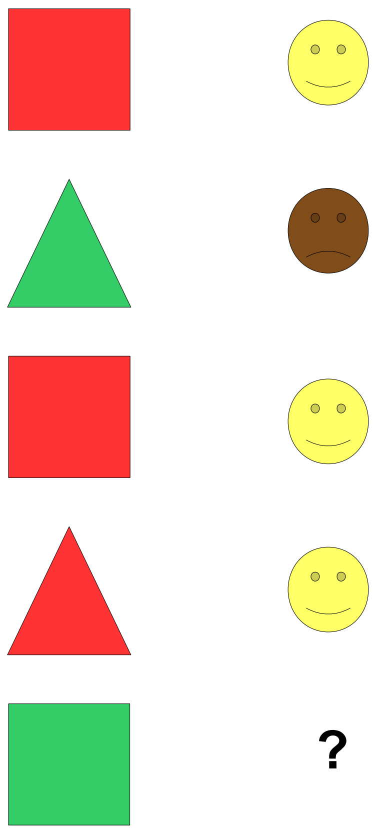


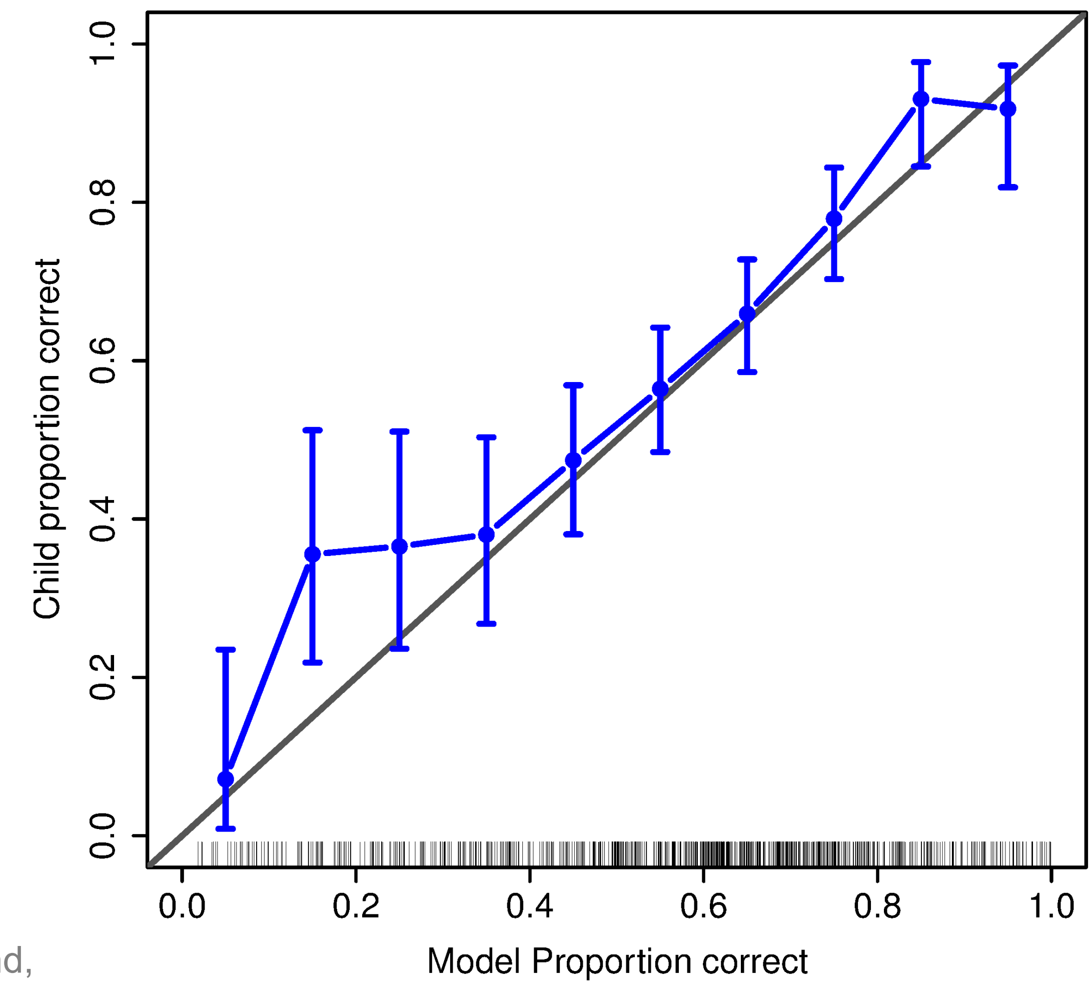
Grammar	H.O. LL
FULLBOOLEAN	-16315.27
CNF	-16333.59
DNF	-16368.31
BICONDITIONAL	-16385.01
IMPLIES	-16442.40
HORNCLAUSE	-16487.25
SIMPLEBOOLEAN	-16490.51
NAND	-16902.68
NOR	-16917.49
UNIFORM	-19482.72
EXEMPLAR	-23645.13
ONLYFEATURES	-31662.08
RESPONSE-BIASED	-37906.77
LOGISTIC	–

FOL	One-Or-Fewer	Small-Cardinalities	2nd-Ord.-Quan.	H.O. LL
✓	·	·	·	-79279.95
✓	✓	·	·	-79560.90
·	✓	·	·	-79642.46
·	✓	✓	·	-79972.75
✓	✓	·	✓	-80198.75
✓	·	·	✓	-80267.46
✓	·	✓	·	-80285.38
·	✓	·	✓	-80300.00
·	·	✓	·	-80614.35
✓	✓	✓	·	-80942.77
✓	✓	✓	✓	-81138.27
·	✓	✓	✓	-81289.85
✓	·	✓	✓	-81596.68
·	·	✓	✓	-81651.36
FULLBOOLEAN				-81773.43
BICONDITIONAL				-81967.68
SIMPLEBOOLEAN				-82144.71
·	·	·	·	-82219.08
CNF				-82685.21
DNF				-82752.82
·	·	·	✓	-82853.59

3~4 year olds' inferences follow model predictions

(Piantadosi & Aslin in prep)





(Bruner, Goodnow, & Austin, 1956; Shepard, Hovland, & Jenkins, 1961; Nosofsky, Palmeri, & McKinley, 1994; Feldman, 2000, 2003a, 2003b; Goodman, Tenenbaum, Feldman, & Griffiths, 2008, ...)

Summary

- Learner's inferences in controlled laboratory tasks follow what you would expect from LOT theories.
- In fact, we can reverse engineer their LOT's components from behavior.

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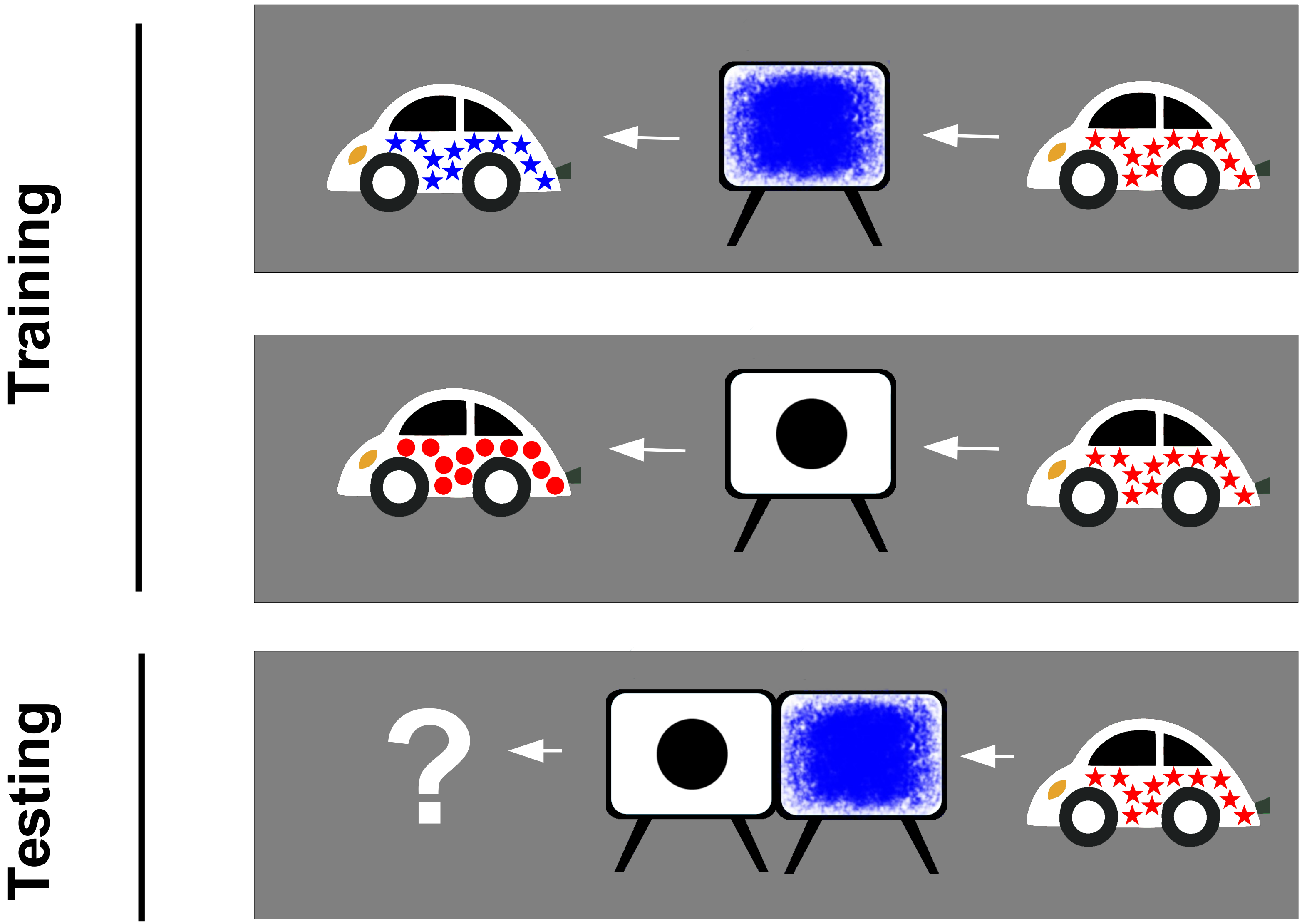
Compositionality in 3-4 year olds

(Piantadosi & Aslin, in prep)



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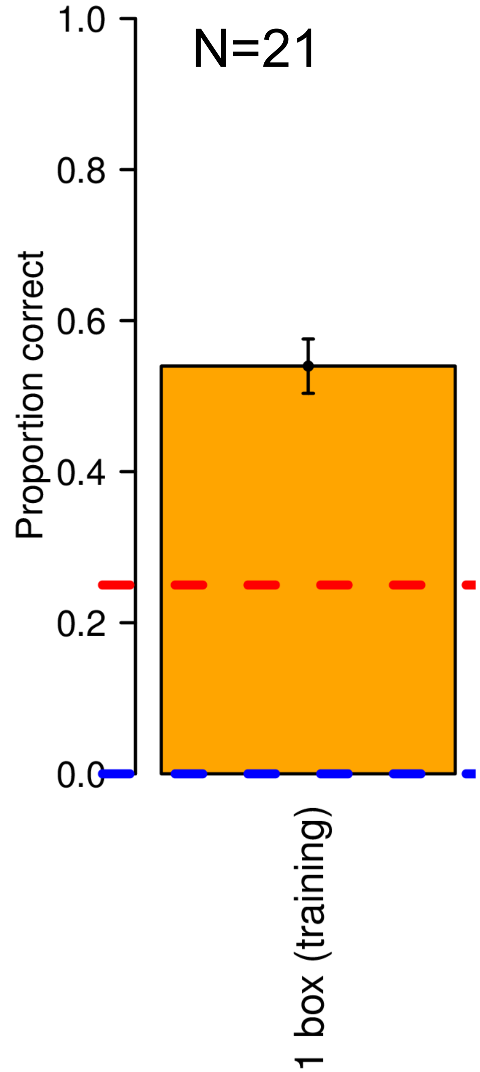

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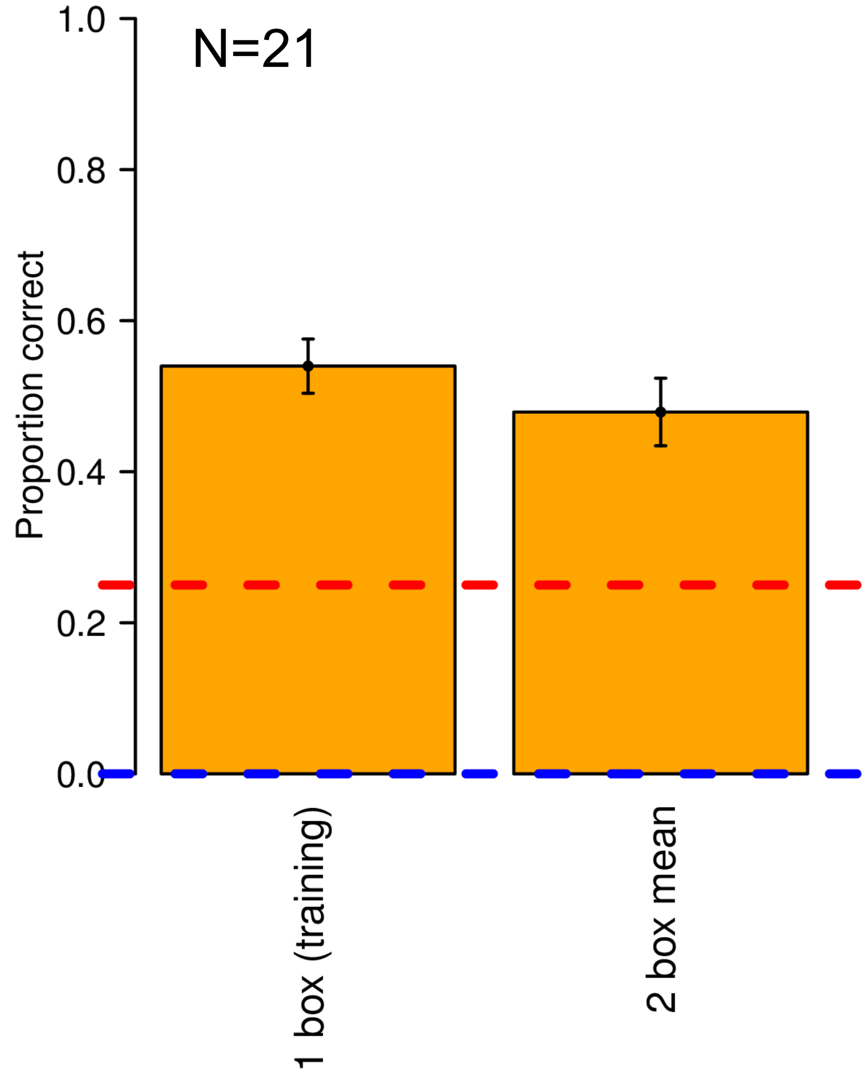


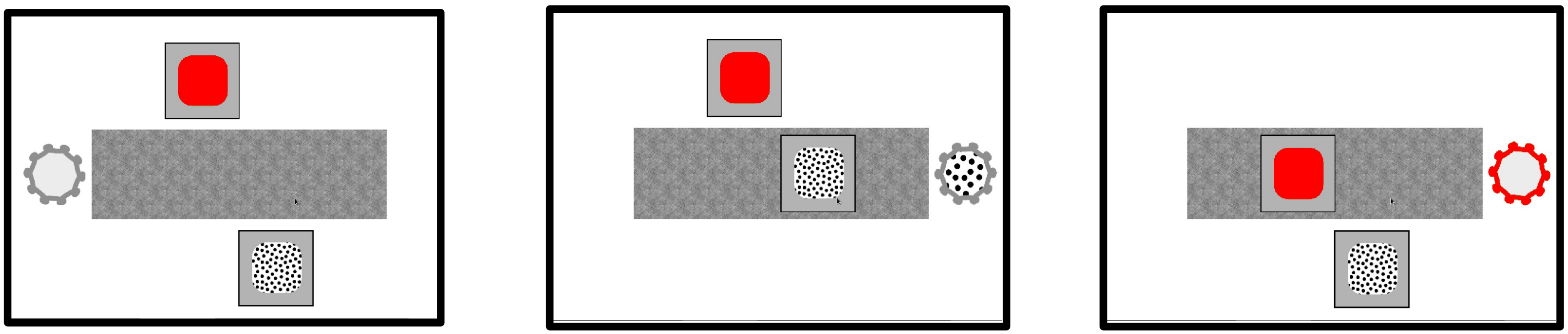
Compositionality in 3-4 year olds

- **Conceptual combination through composition essentially for “free” by this age.**

Infant test of compositionality

(Piantadosi & Aslin, in prep)

- **Familiarization:**



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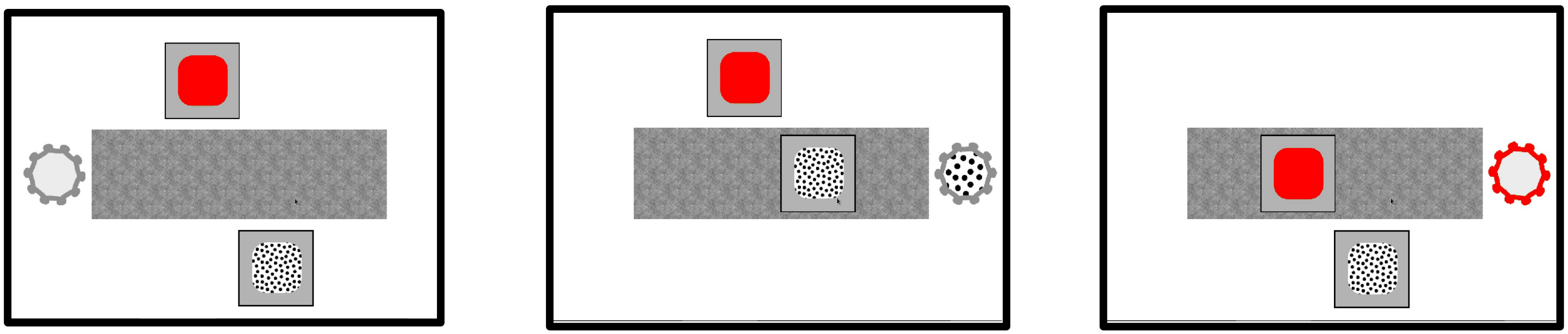
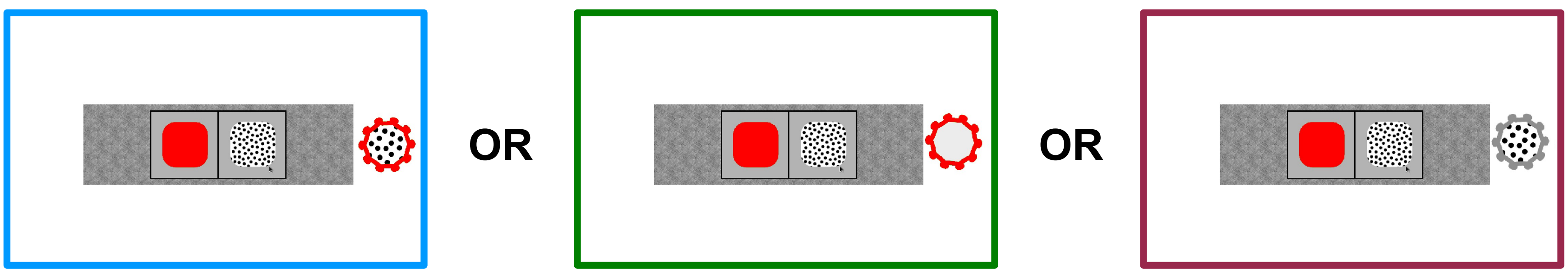
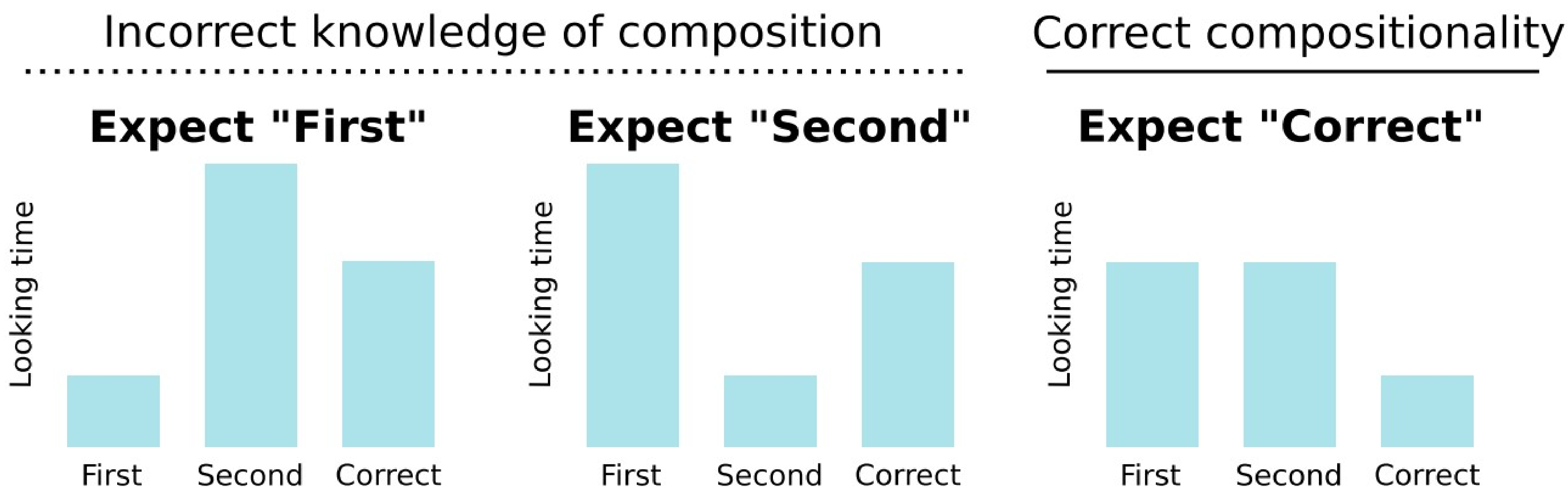


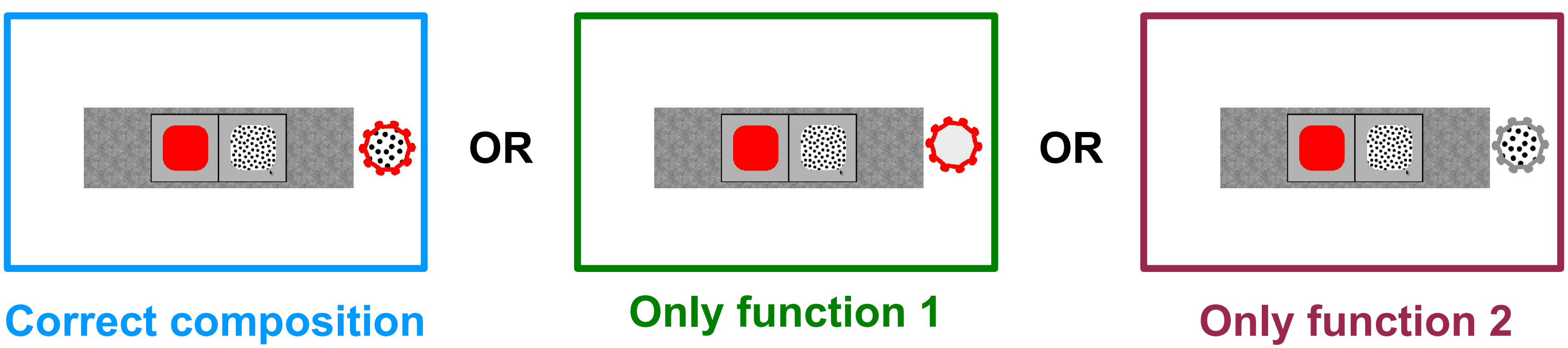
- **Test:**

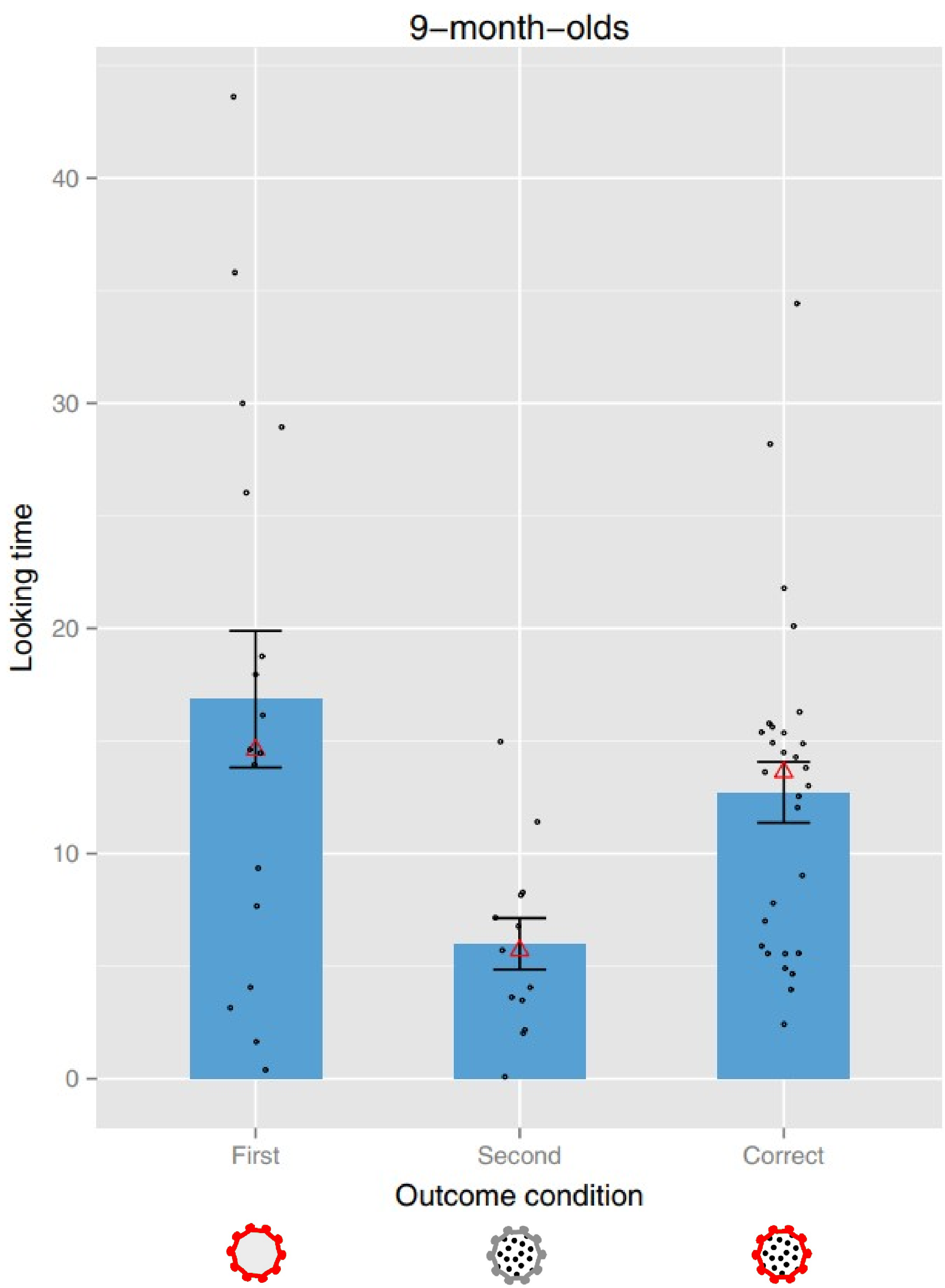


Correct composition | Only function 1 | Only function 2

Predictions






9–month–olds
Looking time
40
30
20
10
0
First
Second
Correct
Outcome condition

Summary

- **It is as though 9mo infants expect only the *second* function to have applied.**
  - Consistent with some working memory limitations, e.g.,
    - 1+1+1 fails, but 2+1 and 2-1 succeed (Moher, Tuerk, & Feigenson, 2012, Baillargeon, Miller, & Constantino, 1994)
    - ABA baiting fails but AAB baiting succeeds (Feigenson & Yamaguchi 2009)

- **A possible generalization:** infants can't multiply update a mental model

Compositionality: summary

- **This kind of composition not apparent in early development.**
  - Plausibly due to memory limitations, tracking and representing multiple operations.

- **Ability emerges in at least some capacity by 3~4 years.**
  - May not make sense to think of the very *earliest* learning in terms of compositional hypothesis testing
  - But it explains generalization patterns well in 3-4 year olds.

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- **Cognitive change by learning isomorphisms**

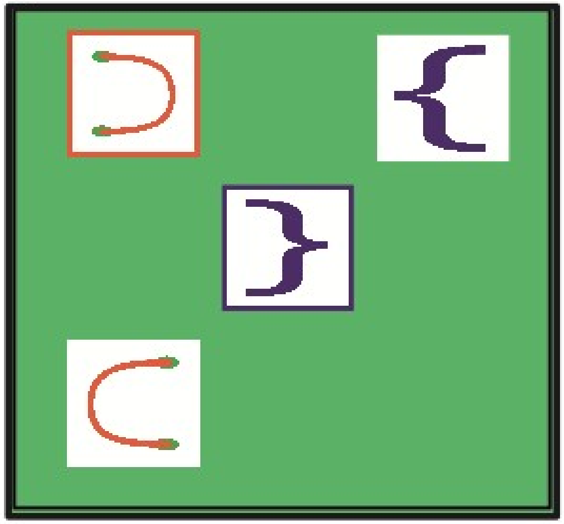
Predictions of Bayesian-LOT

- **Learning curves should follow LOT predictions.**
- **Children should be able to compose mental operations.**
- **Humans should automatically structure thought hierarchically.**

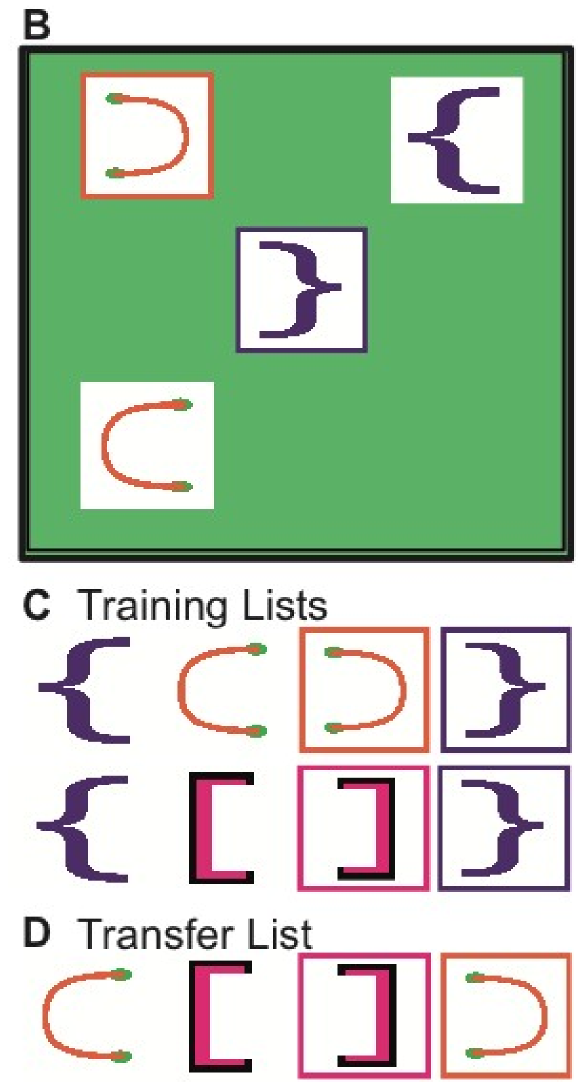
Simple:	red(x)
.	and(red(x), circle(x))
.	or(not(red(x)), circle(x))
Complex:	or(and(red(x), circle(x)), square(x))

Ferrigno, Cheyette, Cantlon, & Piantadosi

B



Ferrigno, Cheyette, Cantlon, & Piantadosi



Ferrigno, Cheyette, Cantlon, & Piantadosi

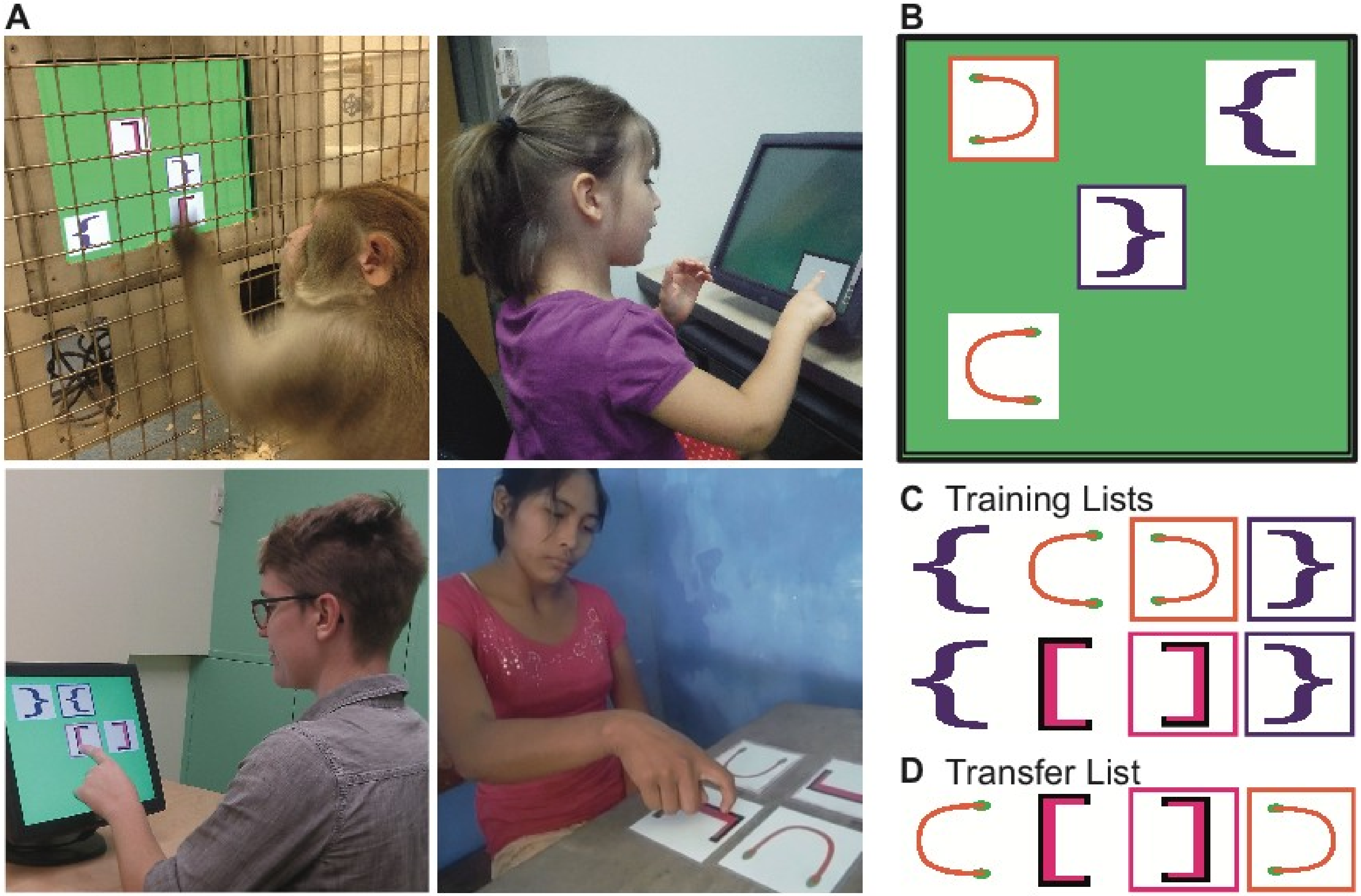
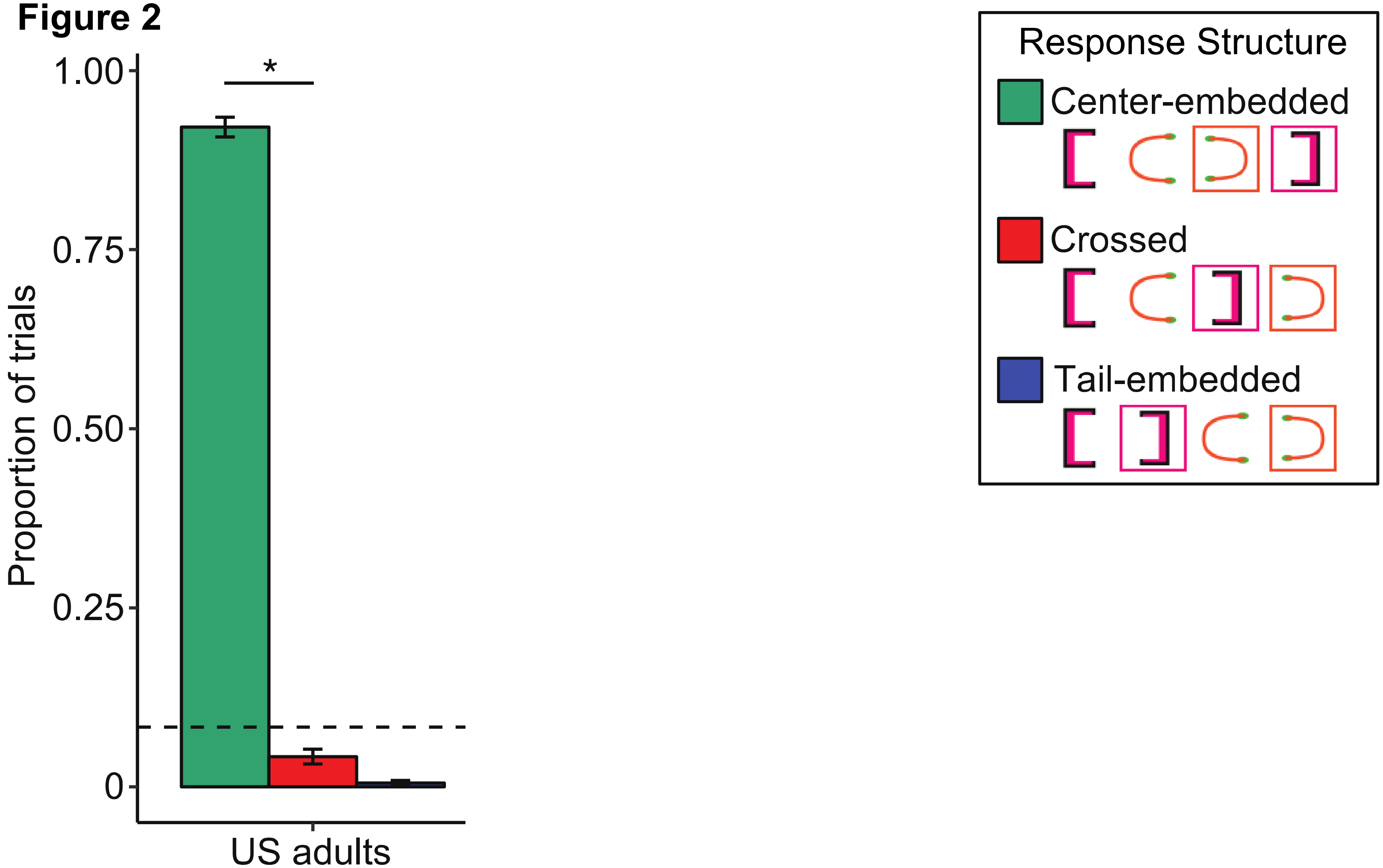
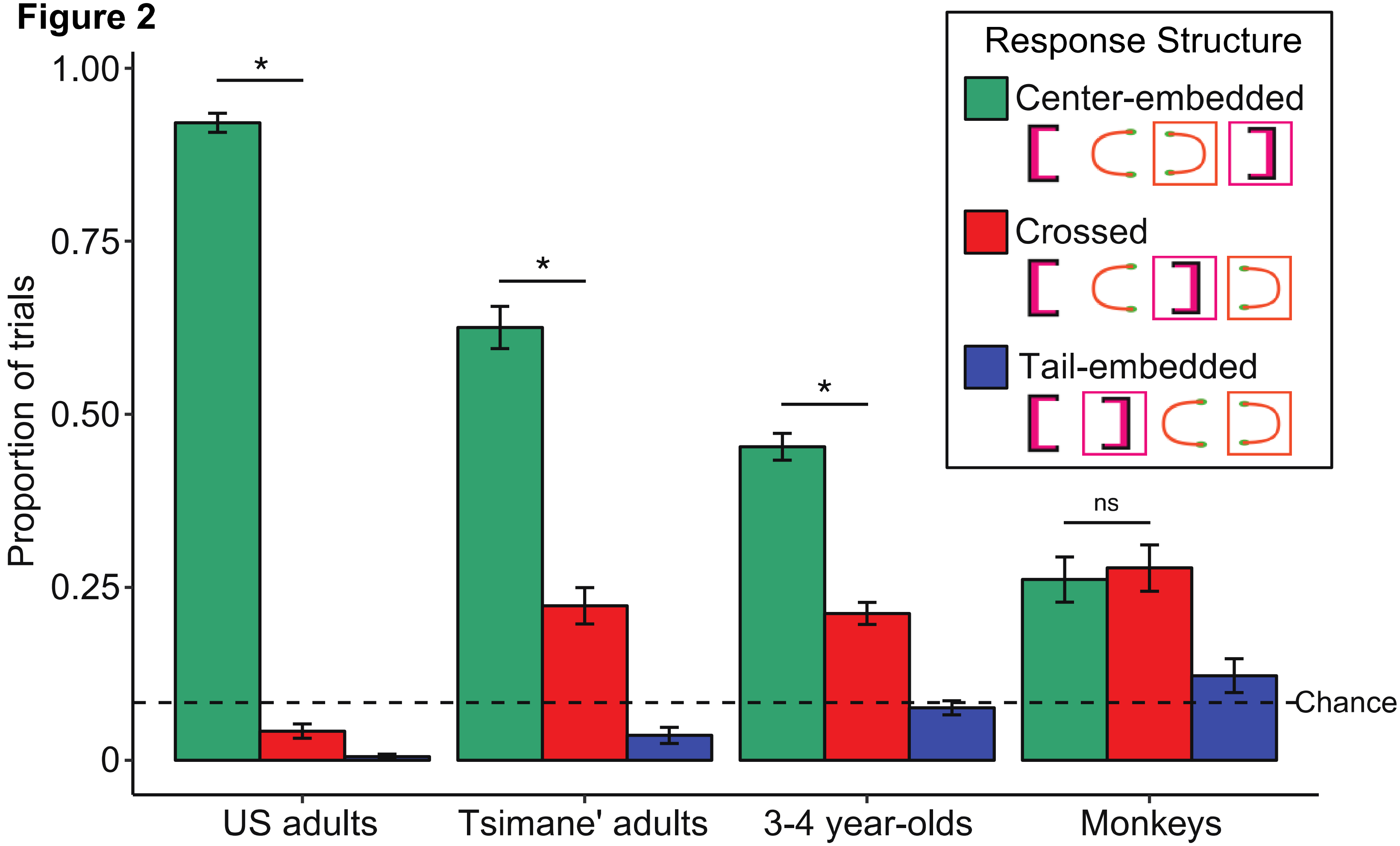


Figure 2



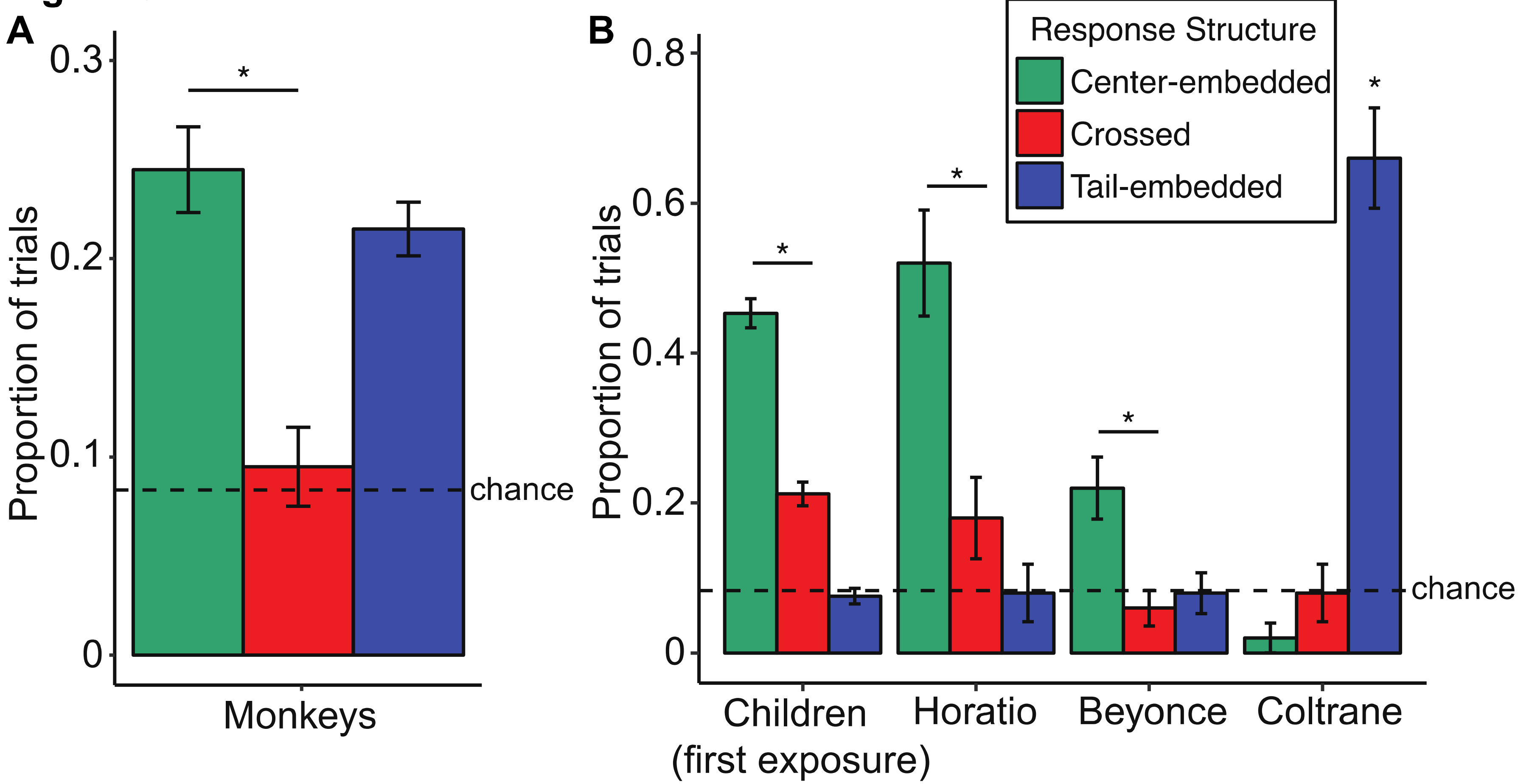
1.00
0.75
0.50
0.25
0
Proportion of trials
*
US adults
Response Structure
Center-embedded
Crossed
Tail-embedded

Figure 2



1.00
0.75
0.50
0.25
0
Proportion of trials
*
*
*
ns
Chance
US adults
Tsimane' adults
3-4 year-olds
Monkeys
Response Structure
Center-embedded
Crossed
Tail-embedded

Figure 3


A
0.3
0.2
0.1
0
*
chance
Proportion of trials
Monkeys
B
0.8
0.6
0.4
0.2
0
Proportion of trials
Response Structure
Center-embedded
Crossed
Tail-embedded
*
*
*
*
chance
Children
(first exposure)
Horatio
Beyonce
Coltrane

Summary

- Recursive/hierarchical inferences are automatic for humans, across culture and ages.
- These inferences are possible for monkeys, but not as natural.

Outline

- **Motivation: broad algorithmic abilities**
- **Central hypothesis: a Language of Thought**
- **Experimental domains:**
  - Evidence in adults from logical learning
  - Logic and compositionality in children and infants
  - Hierarchical biases in children, adults, primates
- **Cognitive change by learning isomorphisms**

Predictions of Bayesian-LOT

- Learning curves should follow LOT predictions.
- Children should be able to compose mental operations.
- Humans should automatically structure thought hierarchically.

Predictions of Bayesian-LOT

- **Learning curves should follow LOT predictions.**
- **Children should be able to compose mental operations.**
- **Humans should automatically structure thought hierarchically.**

Some challenges

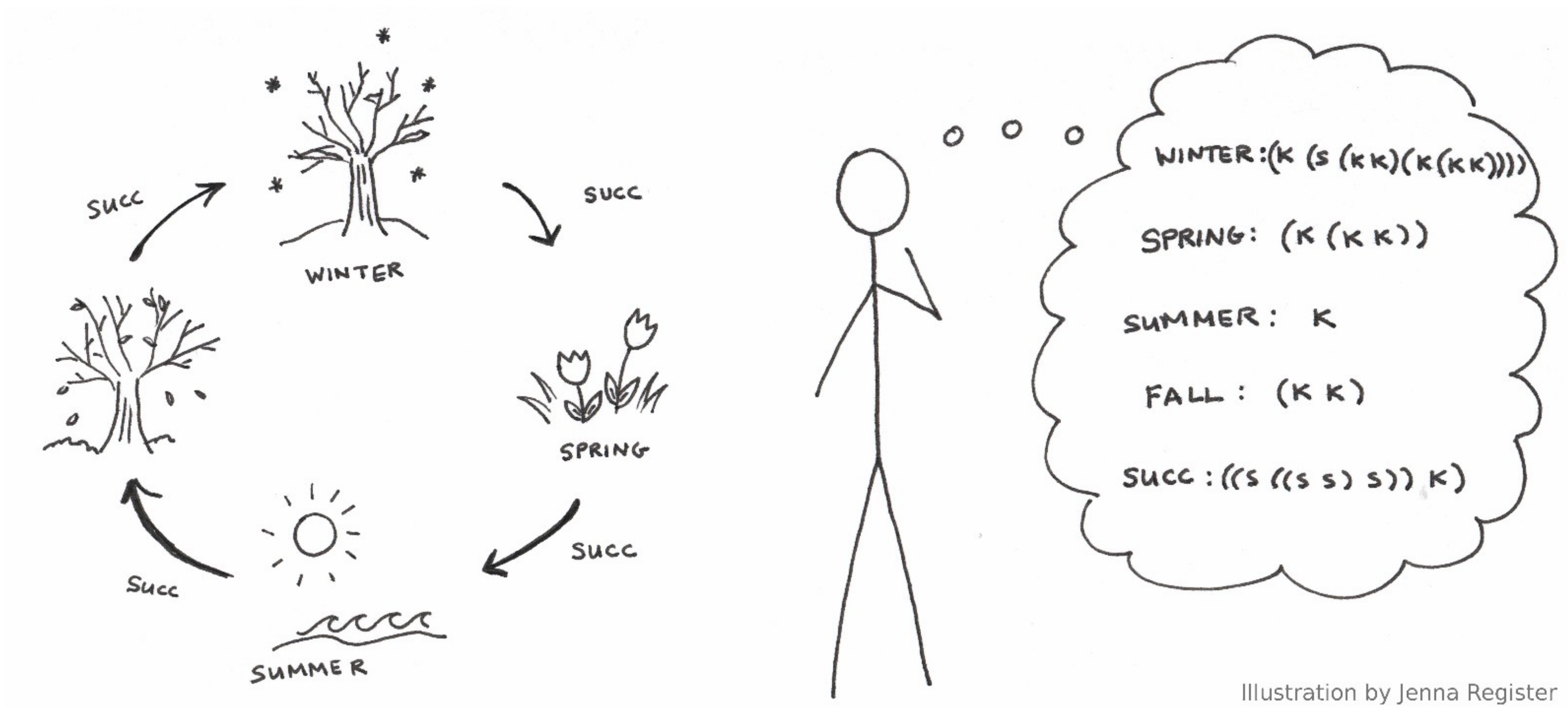
- **What if infants don't show abilities with primitives we hypothesize are there?**
  - From what could you learn something so basic as logic?
- **How do children learn so many different kinds of systems?**

Human learn many formal systems

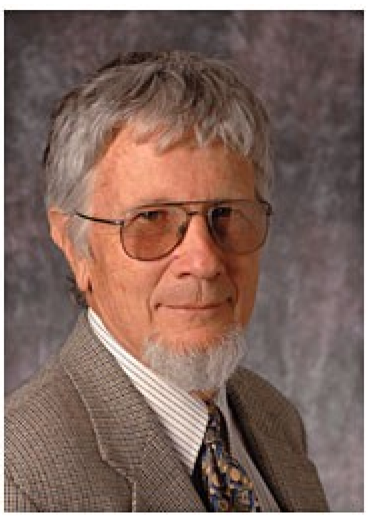
- **Basic logic** (e.g., and, or, not, iff)
- **Natural language logic** (e.g. “and”, “or”)
- **First-order logic quantifiers** (e.g. ∀,∃)
- **Second-order quantification** (e.g. there exists a property P …)
- **Generalized quantifiers** (e.g. natural language “most”)
- **Grammars** (e.g. context-free grammars)
- **Programming languages** (e.g. Python, Haskell, Prolog)
- **Tree structures and relations** (e.g. kinship systems)
- **Dominance hierarchies/relations** (e.g. Putin > Trump)
- **Physics** (e.g. block stacking)
- **Arbitrary graphs** (e.g. Boston subway map)
- **Games** (e.g. tic-tac-toe, nim, battleship)
- **Simulations** (e.g. hypotheticals)

ChurIso

- **What is needed:** a mental language in which we can build any kind of system we need to learn about.
- **How can we do this?** Learn isomorphisms.



Isomorphism as the heart of representation



"A mental representation is a functioning isomorphism between a set of processes in the brain and a behaviorally important aspect of the world. This way of defining a representation is taken directly from the mathematical definition of a representation. To establish a representation in mathematics is to establish an isomorphism (formal correspondence) between two systems of mathematical investigation (for example, between geometry and algebra) that permits one to use one system to establish truths about the other (as in analytic geometry, where algebraic methods are used to prove geometric theorems)."

Randy Gallistel

Combinatory logic

Moses Schönfinkel

John von Neumann

Haskell Curry

A candidate kind of representation system

- Combinatory logic permits arbitrary computations with only TWO primitives, both tree manipulations:
  - (**K** x y) $\rightarrow$ x   other notation: K(x,y) $\rightarrow$ x
  - (**S** x y z) $\rightarrow$ ((x z) (y z))   other notation: S(x,y,z) $\rightarrow$ x(z,y(z))
  - Currying – a function without enough arguments can take the next in line

    e.g. ((K x) y) $\rightarrow$ (K x y) $\rightarrow$ x

An example

```
true  := (K K)
false := K
and   := ((S (S (S S))) (K (K K)))
or    := ((S S) (K (K K)))
not   := ((S ((S K) S)) (K K))
```

```
(or true false) = (((S S) (K (K K)))  (K K)  K)
```

An example

```
true  := (K K)
false := K
and   := ((S (S (S S))) (K (K K)))
or    := ((S S) (K (K K)))
not   := ((S ((S K) S)) (K K))
```

```
(or true false) = (((S S) (K (K K))) (K K) K)
→ (((S S) (K (K K)) (K K)) K)       ; Currying rule
```

An example

```
true   := (K K)
false  := K
and    := ((S (S (S S))) (K (K K)))
or     := ((S S) (K (K K)))
not    := ((S ((S K) S)) (K K))
```

```
(or true false) = (((S S) (K (K K))) (K K) K)
→ (((S S) (K (K K)) (K K)) K)      ; Currying rule
→ ((S S (K (K K)) (K K)) K)        ; Currying rule twice
```

An example

```
true   :=  (K K)
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and    :=  ((S (S (S S))) (K (K K)))
or     :=  ((S S) (K (K K)))
not    :=  ((S ((S K) S)) (K K))
```

```
(or true false) = (((S S) (K (K K)))  (K K)  K)
→ (((S S) (K (K K)) (K K))  K)          ; Currying rule
→ ((S S (K (K K)) (K K)) K)             ; Currying rule twice
→ ((S (K K) ((K (K K)) (K K))) K)       ; Definition of S
```

An example

```
true   := (K K)
false  := K
and    := ((S (S (S S))) (K (K K)))
or     := ((S S) (K (K K)))
not    := ((S ((S K) S)) (K K))
```

```
(or true false) = (((S S) (K (K K)))  (K K)  K)
→ (((S S) (K (K K)) (K K))  K)          ; Currying rule
→ ((S S (K (K K)) (K K)) K)             ; Currying rule twice
→ ((S (K K) ((K (K K)) (K K))) K)       ; Definition of S
→ ((S (K K) (K (K K) (K K))) K)         ; Currying
→ ((S (K K) (K K)) K)                   ; Definition of K
→ (S (K K) (K K) K)                     ; Currying
→ ((K K) K ((K K) K))                   ; Definition of S
→ ((K K K) ((K K) K))                   ; Currying
→ (K ((K K) K))                         ; Definition of K
→ (K (K K K))                           ; Currying
→ (K K)                                 ; Definition of K
```

Church encoding

- **This technique, from mathematical logic, is known as <u>Church Encoding</u>**
  - Use one logical system to mimic the behavior of another.


HOT HEISS
7753

Churlso

- **My lab has been working on a library to infer church encodings from simple relational information.**

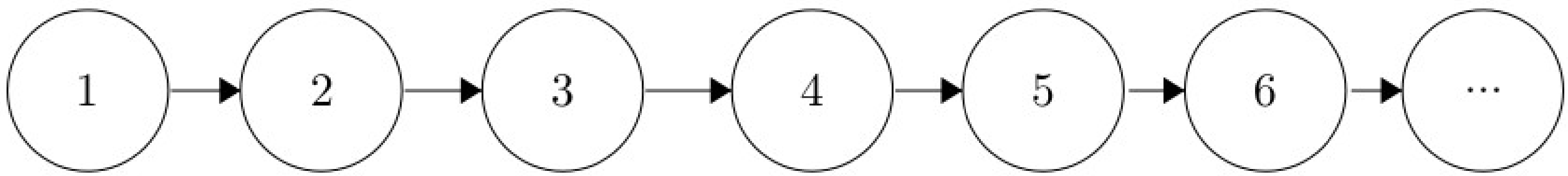
	Observed relations	Mental Representation
Seasons	(succ winter) → spring (succ spring) → summer (succ summer) → fall (succ fall) → winter	spring := (**K** (**K** **K**)) winter := (**K** (**S** (**K** **K**) (**K** (**K** **K**)))) fall := (**K** **K**) summer := **K** succ := ((**S** ((**S** **S**) **S**)) **K**)
1, 2, Many	(succ one) → two (succ two) → many (succ many) → many	many := (**S** **K** **K**) two := (**K** (**S** **K** **K**)) one := **K** succ := ((**S** (**S** **K** **K**)) (**K** (**S** **K** **K**)))

- **How we infer:** use ideas from the inductive LOT – prefer encodings with short running time, simple structure.

Number
($\mathbb{Z}$)

```
(succ one) → two
(succ two) → three
(succ three) → four
```

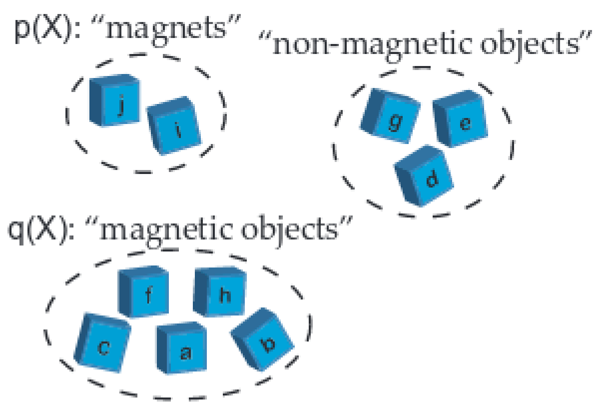
```
succ := K
one := S
two := (K S)
three := (K (K S))
four := (K (K (K S)))
```

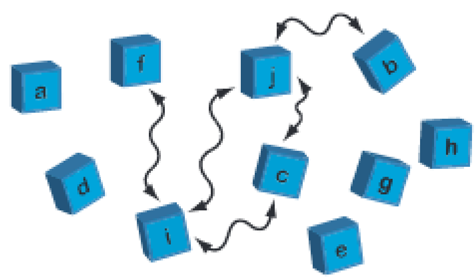


Magnetism

Core Predicates: p(X), q(X)
Surface Predicates: interacts(X,Y)
Laws:

$\text{interacts}(X,Y) \longleftarrow p(X) \wedge p(Y)$
$\text{interacts}(X,Y) \longleftarrow p(X) \wedge q(Y)$
$\text{interacts}(X,Y) \longleftarrow \text{interacts}(Y,X)$





Ullman, Goodman, & Tenenbaum (2012)

Magnetism

```
(attract p1 p2) ↛ True
(attract p2 p1) ↛ True
(attract p1 n1) → True
(attract p1 n2) → True
(attract p2 n1) → True
(attract p2 n2) → True
(attract n1 n2) ↛ True
(attract n2 n1) ↛ True
(attract n1 p1) → True
(attract n1 p2) → True
(attract n2 p1) → True
(attract n2 p2) → True
; and one single example
(attract n1 x) → True
```

```
attract  := ((S S) (K I))
n1       := K
n2       := K
p2       := (K K)
p1       := (K K)
```

Magnetism

```
(attract p1 p2) ↛ True
(attract p2 p1) ↛ True
(attract p1 n1) → True
(attract p1 n2) → True
(attract p2 n1) → True
(attract p2 n2) → True
(attract n1 n2) ↛ True
(attract n2 n1) ↛ True
(attract n1 p1) → True
(attract n1 p2) → True
(attract n2 p1) → True
(attract n2 p2) → True
; and one single example
(attract n1 x) → True
```

```
attract := ((S S) (K I))
n1      := K
n2      := K
p2      := (K K)
p1      := (K K)
x       := (K K)
```

A
|
B
|
C
|
D

Dominance
$(a \succ b \succ c \succ d)$

```
True := K
(dom a b) → True
(dom a c) → True
; No information a,d relation
(dom b c) → True
(dom b d) → True
(dom c d) → True
(dom b a) ↛ True
(dom c a) ↛ True
(dom c b) ↛ True
(dom d b) ↛ True
(dom d c) ↛ True
(dom b a) ↛ True
(dom c b) ↛ True
(dom d c) ↛ True
```

```
a := (K (K K))
b := (S (S K))
c := (S K K)
d := (K K)
dom := (((S (K (S (K (S S (K K))) K)) S
    ↪) (S (K (S (K (S S (K K))) K)) S)
    ↪) K)
```

Domain	Facts	Representation
Reversal	`(reverse x y) → (y x)`	`reverse := ((S (K (S (S K K)))) K)`

If-else	`True  := (K K)`
	`False := K`
	`(ifelse True x y) → x`
	`(ifelse False x y) → y`
		`ifelse := ((S ((S K) S)) (S K)))`

Identity	`(identity x) → x`	`identity := (S K K)`
Repetition	`(repeat f x) → (f (f x))`	`repeat := ((S (S (K S) K)) (S K K)` `)`

Recursion	`(Y f) --→ (f (Y f))`	`Y := (((S (K S) K) ((S ((S (K (S (`
		`K (S S (K K))) K)) S) (S (S`
		`(S K K))))) S)) (S (K S) K))`

Mutual recursion	`(f (g (Y* f g))) --→ (Y* f`
	`g)`
		`Y* := (((S (K S) K) (((S (K S) K)`
		`((S (S (K S) K)) ((S (K (S (`
		`K (S S (K K))) K)) S) (S (K`
		`S) K)))) (S ((S (K (S (K (S`
		`S (K K))) K)) S) (S K K)))))`
		`(S (K S) K))`

Apply	`(apply f x ) → (f x)`	`apply = (S K K)`

Tree, List	`(first (pair x y)) → x`
	`(rest (pair x y)) → y`
		`pair := (((S (K S) K) (S (K (S (K`
		`(S S (K K))) K)) S)) ((S (K`
		`(S (K (S S (K K))) K)) S) (S`
		`(K (S (K (S S (K K))) K)) S`
		`)))`
		`first := ((S (S K K)) (K (S K)))`
		`rest := ((S (S K K)) (K K))`

Towards neural implementation

- **The rules of combinatory logic are simple tree manipulations**

  (**K** x y) → x

  (**S** x y z) → ((x z) (y z))

  - There exist neural implementations of these operations

    (e.g. tensor product coding, Boltzcons, etc.)

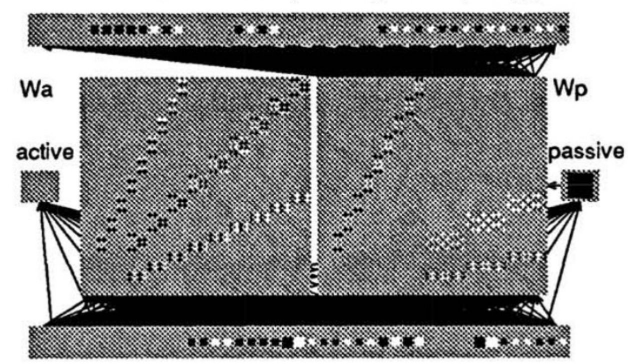


Figure 1: Recursive tensor product network processing a passive sentence

Legendre, Miyata, Smolensky (1990)

Lessons from Combinatory Logic

- **There is a real sense in which theories need not assume cognitive content (cf Fodor) –** not even basic logic and computation.

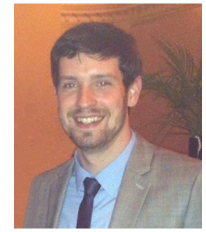
- **A productive metaphor for the development of a LOT:** a simple, Turing-complete dynamical system in which you can construct a “model” (church encoding) of any other.

General Summary

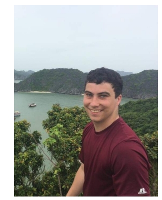
- **I have charted out domains of current experimental and computational work on the LOT, from infancy and beyond**:
  - Tight LOT-predictions in adult learning experiments.
  - “Free” compositionality in toddlers
  - Interesting limitations in infancy
  - Biases towards hierarchical/recursive structures across humans
- **Churlso** is a neurally-implementable model capable of inferring any logical structures and generalizing+deducing.
- **This work generally** pushes the LOT hypothesis out of the domain of philosophy and into experimental psychology and machine learning.

Acknowledgements

- **Stephen Ferrigno & Jessica Cantlon**
  (Tsimane'/kid/adult/primate work)




- **Sam Cheyette & Frank Mollica**
  (compositional learning models)




- **Jenna Register and Yuan Yang**
  (Churiso, learning models)


- **Josh Tenenbaum** & **Noah Goodman**
  (compositional learning models)


- **Dick Aslin & Holly Palmeri**
  (Kid compositional experiments)


Members of Kidd Lab, Computation and Language Lab

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