

# The hunting of the SNARC:

The discovery of number-space interactions  
and their cerebral correlates

Stanislas Dehaene, Collège de France

and INSERM-CEA Cognitive Neuroimaging Unit, NeuroSpin, Saclay, France



# The serendipitous discovery of the SNARC effect (*Spatial-Numerical Association of Response Codes*)

Stanislas Dehaene, Sergini Bossini and Pascal Giraux 1993

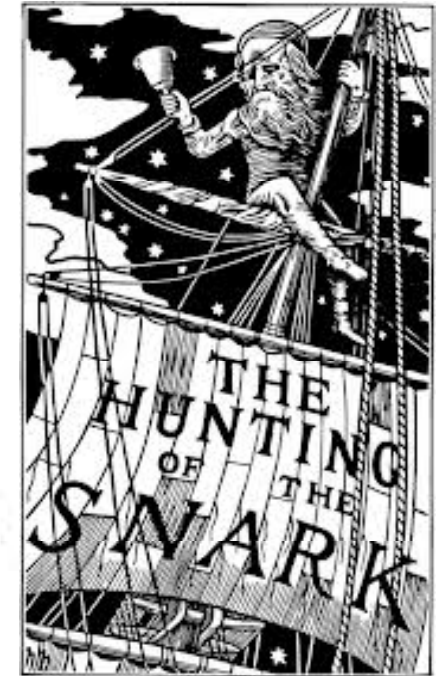
A study initially designed to investigate the semantics of numbers:

How do we know that 4 is even and 5 is odd?

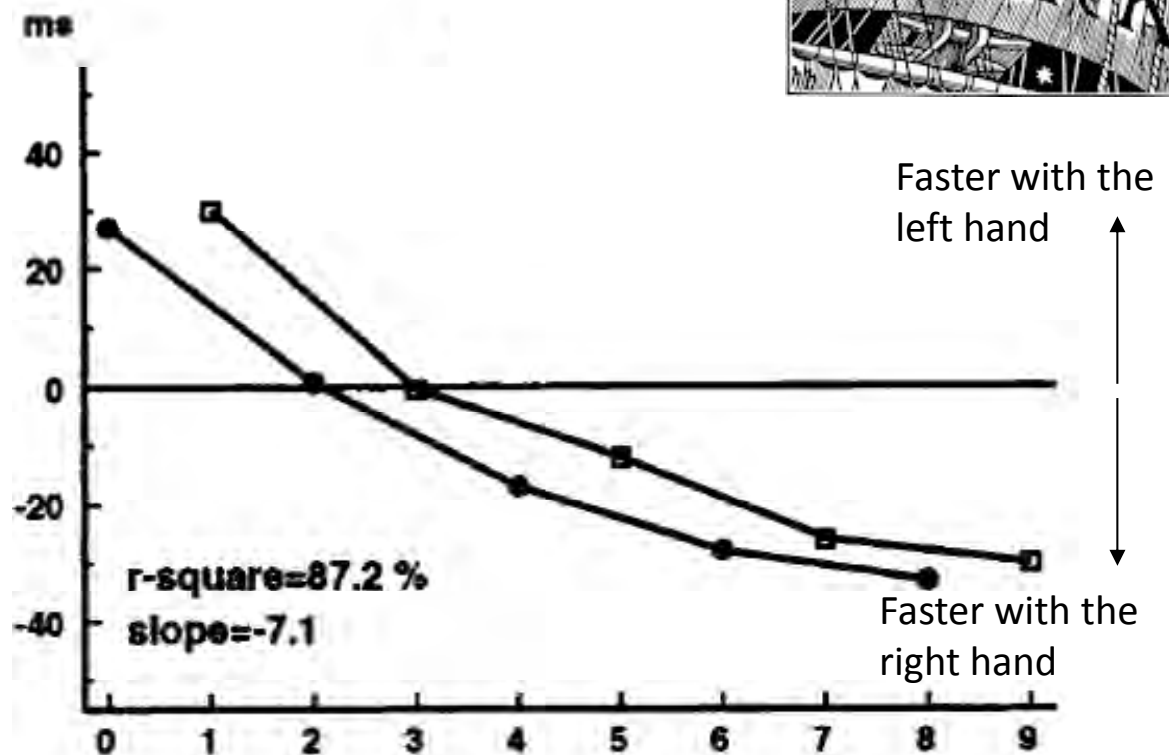
Task = click a button for even numbers, another for odd digits.

Balanced assignment of the « odd » and « even » responses to the left and right hands in different blocks.

A triple interaction of block, parity, and magnitude...

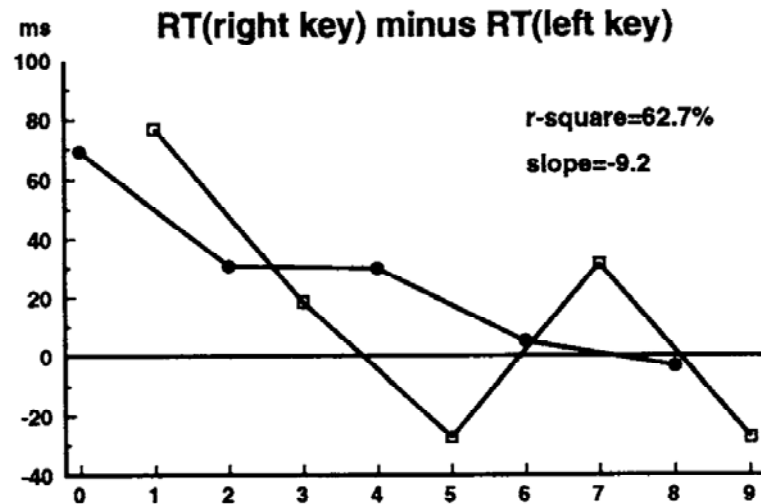


RT(right key) minus RT(left key)

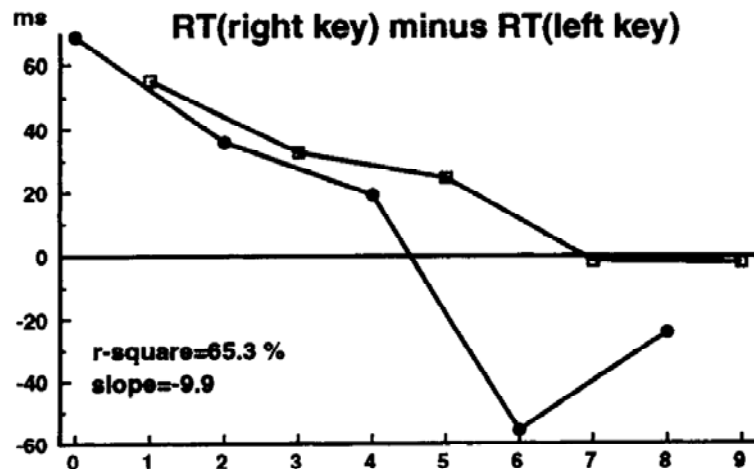


# “What I tell you three times is true”

The effect is identical in left-handers.

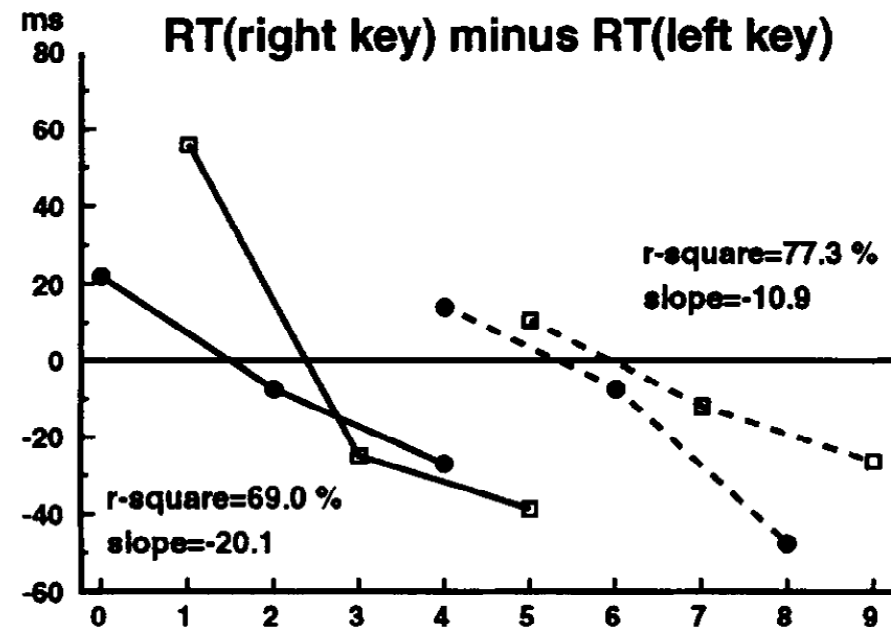


It depends on the side of space, not the hand (crossed-hands)



the Bellman

It depends on the *relative* size of numbers, not their absolute value.





## The cultural origins of the SNARC effect: the importance of reading direction

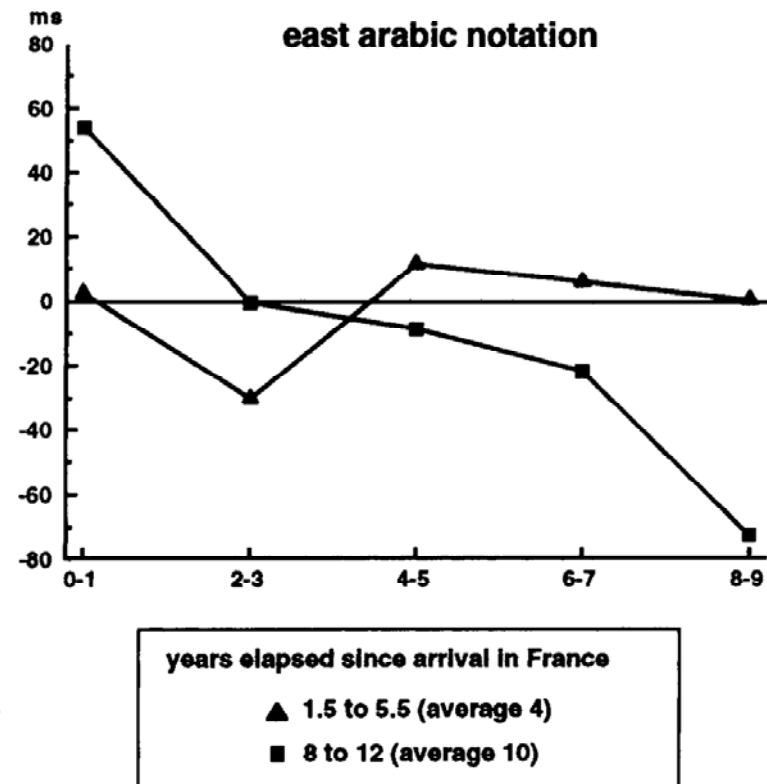


In **right-to-left readers**, the effect tends to disappear (Dehaene et al., 1993) or even reverse (Zebian, 2005; Hung et al., 2008; Shaki et al, 2008, 2009, 2012).

We now know that the effect is flexible, and that reading direction is just one of the biases that determines it.

**In children**, with parity judgment, the effect appears around 3<sup>rd</sup> grade (Berch et al., 1999), around the time that children automatize the links between numbers and space (Girelli, JECP 2000).

But, links between number, time and space are present even in infancy (De Hevia, 2010 ; Lourenco & Longo, 2010).



# The fate of the SNARC

Journal of Experimental Psychology: General  
1993, Vol. 122, No. 3, 371–396

Copyright 1993 by the American Psychological Association, Inc.  
0096-3445/93/\$3.00

## The Mental Representation of Parity and Number Magnitude

Stanislas Dehaene, Serge Bossini, and Pascal Giraux

Nine experiments of timed odd–even judgments examined how parity and number magnitude are accessed from Arabic and verbal numerals. With Arabic numerals, Ss used the rightmost digit to access a store of semantic number knowledge. Verbal numerals went through an additional stage

Nombre total de citations

Cité 967 fois

Nombre de citations par an



Articles Google Scholar

The mental representation of parity and number magnitude.

S Dehaene, S Bossini, P Giraux - Journal of Experimental Psychology: General, 1993

# Some remarkable extensions of the SNARC effect

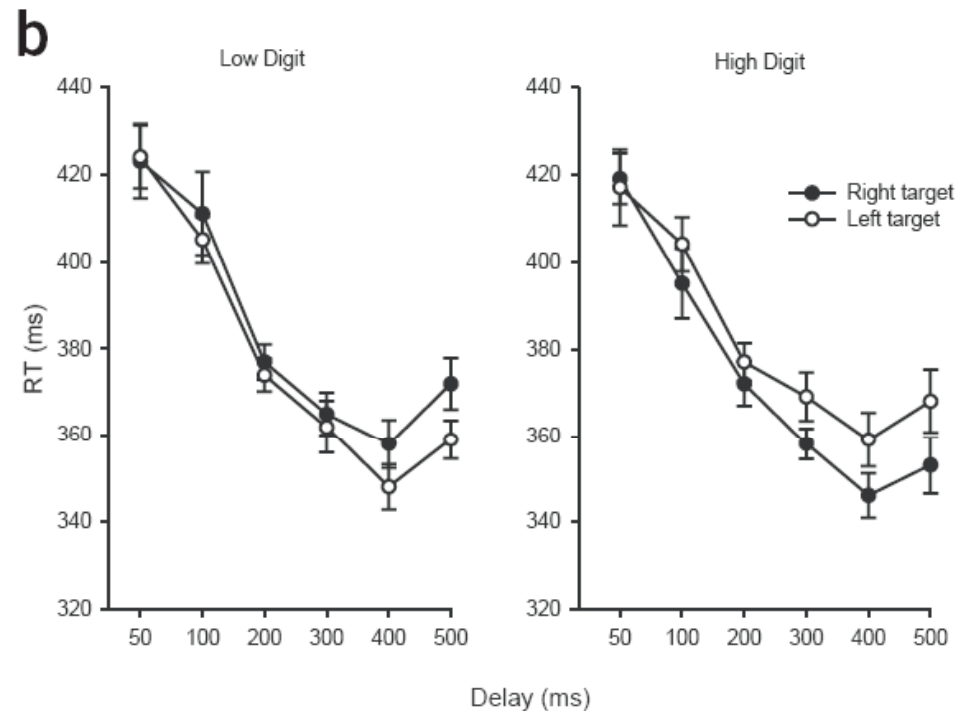
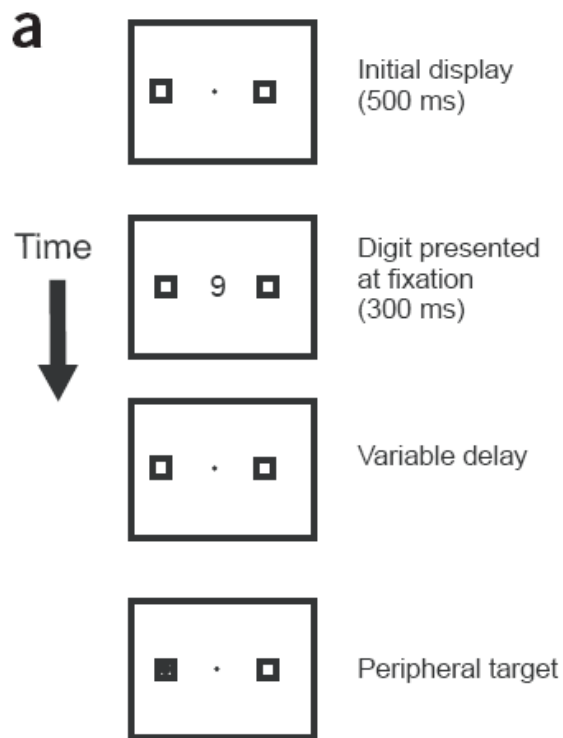
**Zorzi, Priftis, & Umiltà, *Nature*, 2002:**

Hemineglect patients fail in bisecting two numbers: they may place 17 in the middle of 11 and 19!



**Fischer et al., *Nature Neuroscience*, 2003:**

The detection of a flash is faster in the right hemifield after a large central digit, and in the left hemifield after a small central digit



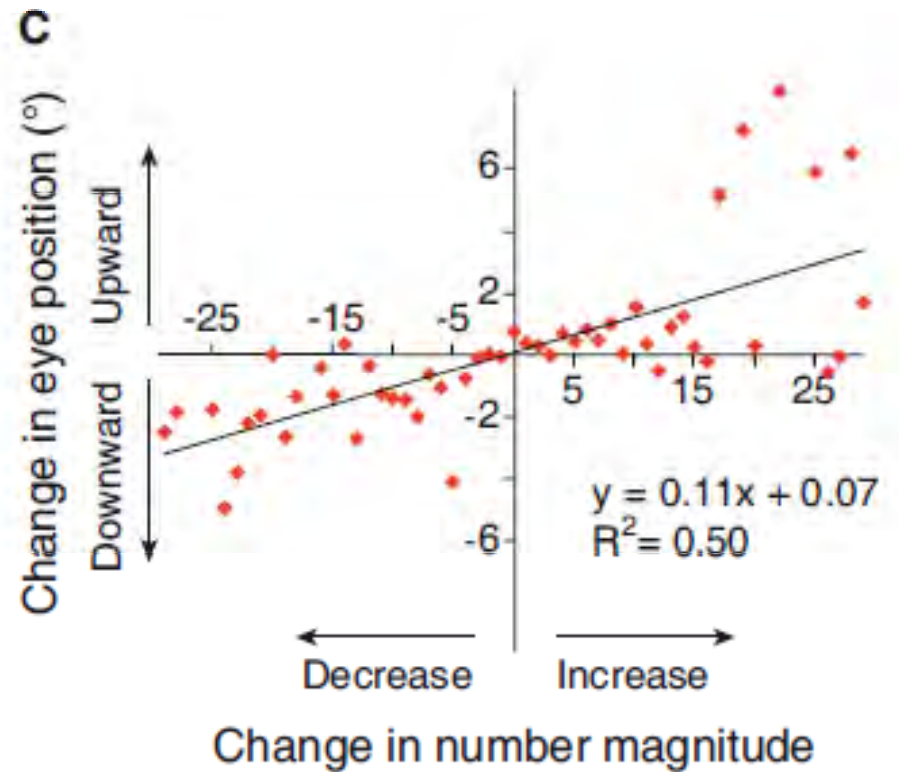
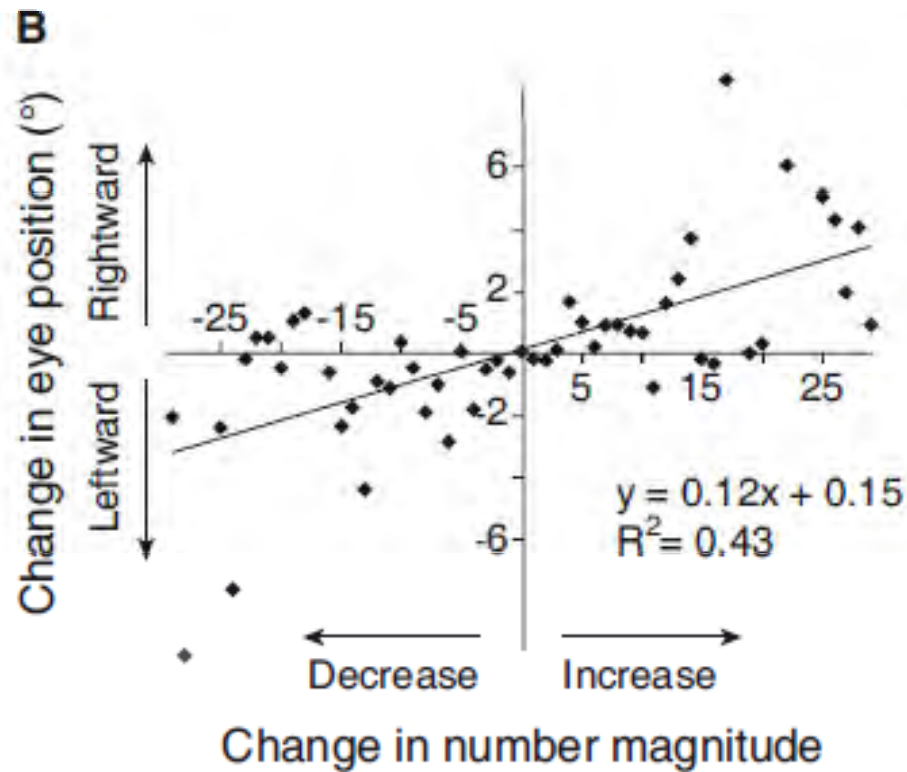
RNG

# Some remarkable extensions of the SNARC effect



Loetscher et al., *Current Biology*, 2010:

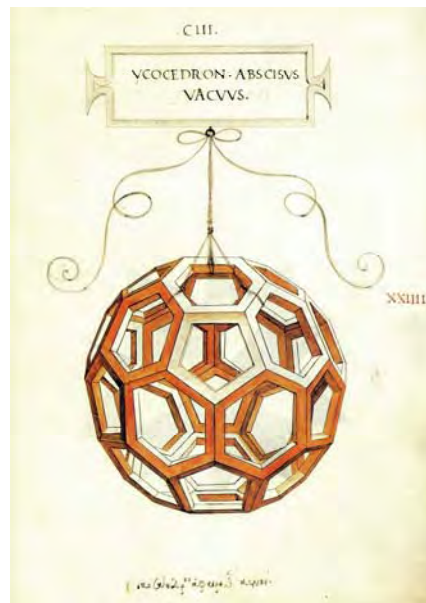
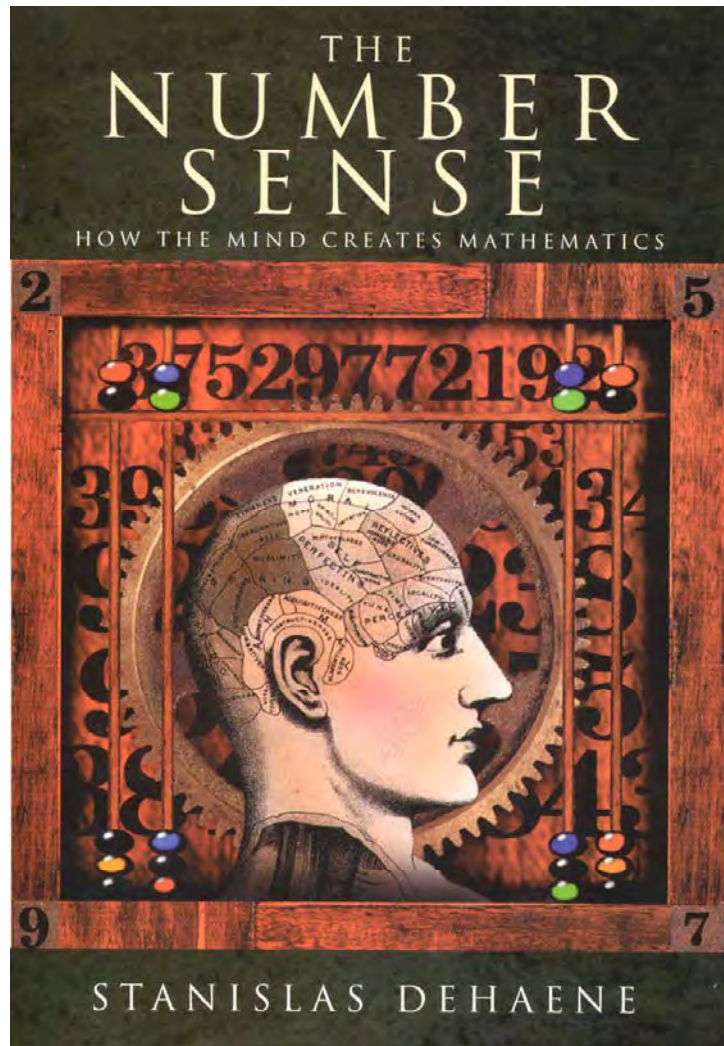
During the generation of random numbers in darkness, human adults move their eyes rightwards and upwards for larger numbers.



# Space, Time And Number:

three fundamentally intertwined foundations of mathematics

The representation of numbers as a « line », and the many other metaphors of space, play an essential role in the construction of mathematics.





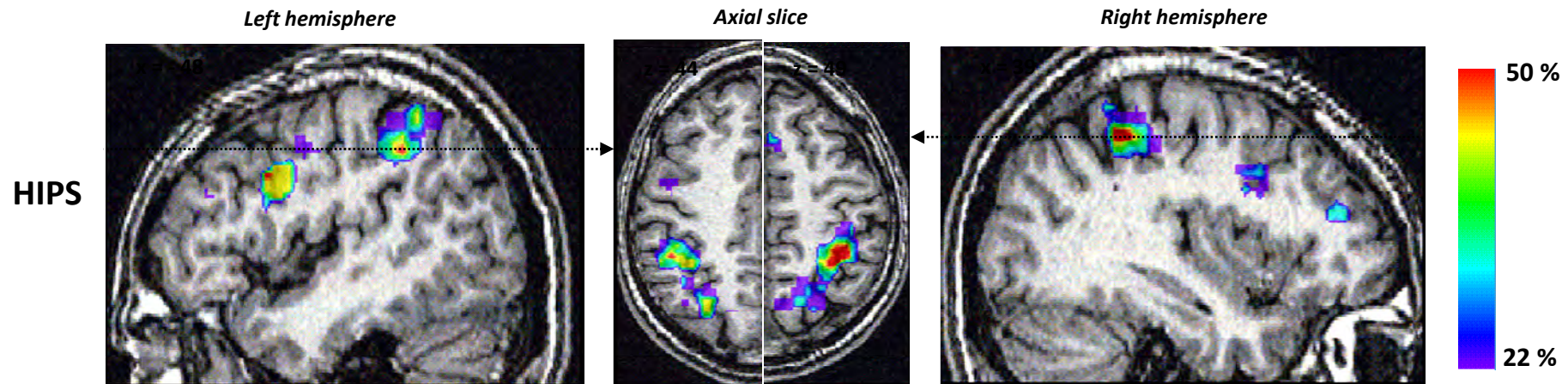
## The place for the SNARC?

"Just the place for a Snark!" the Bellman cried, As he landed his crew with care;  
Supporting each man on the top of the tide By a finger entwined in his hair.  
"Just the place for a Snark! I have said it twice: That alone should encourage the crew.  
Just the place for a Snark! I have said it thrice: What i tell you three times is true."



the Bellman


## Number sense and the horizontal segment of the intraparietal sulcus (HIPS)



- All numerical tasks activate this region  
(e.g. addition, subtraction, comparison, approximation, digit detection...)
- This region fulfils two criteria for a semantic-level representation:
  - It responds to number in various formats (Arabic digits, written or spoken words), more than to other categories of objects (e.g. letters, colors, animals...)
  - Its activation varies according to a semantic metric (numerical distance, number size)

## An array of numerical and spatial areas around the intraparietal sulcus

 Calculation only

 Calculation and Language

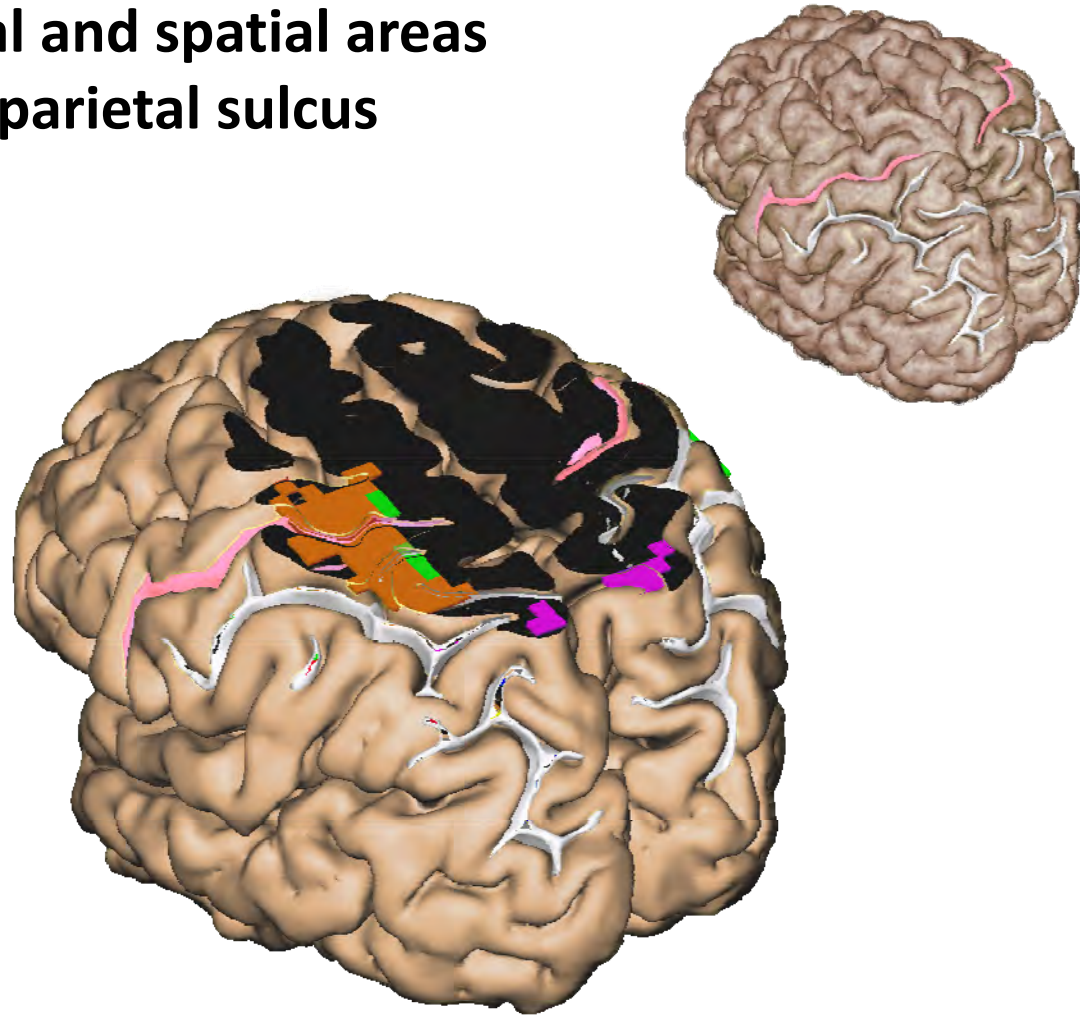
 Grasping only

 Manual tasks

 Visuo-spatial tasks

 Attention only

 Saccades only

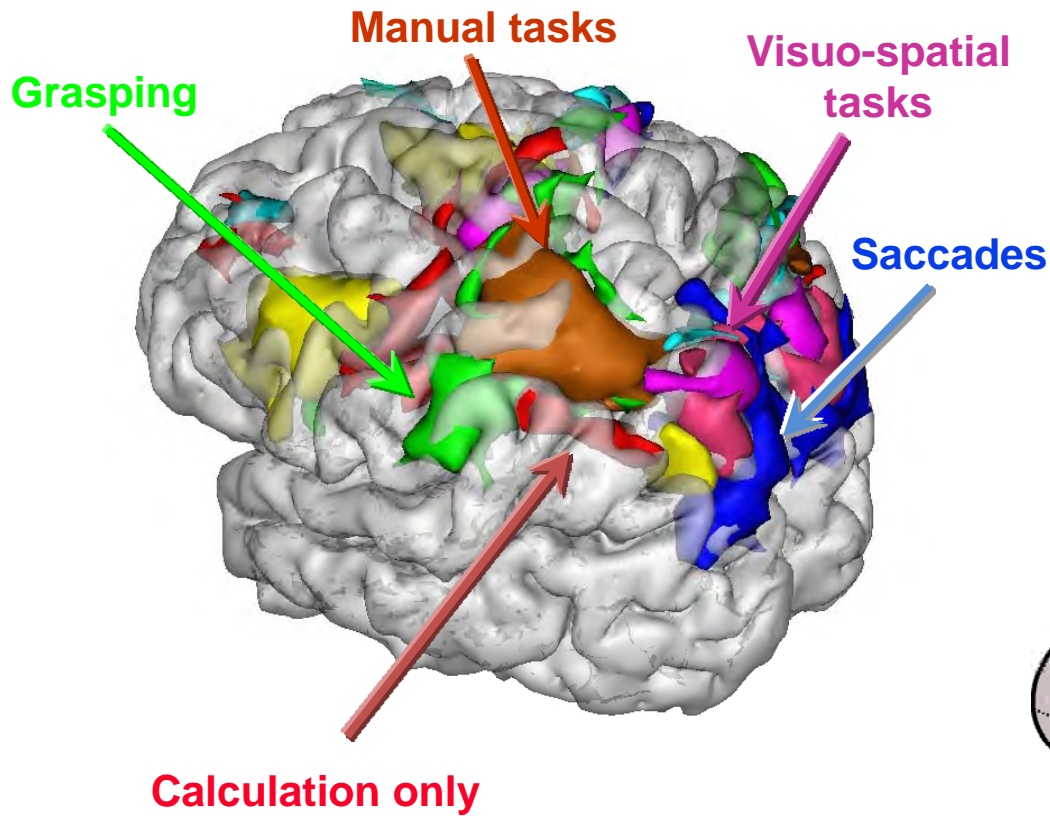


Simon, O., Mangin, J. F., Cohen, L., Le Bihan, D., & Dehaene, S. (2002). *Neuron*, 33(3), 475-487.

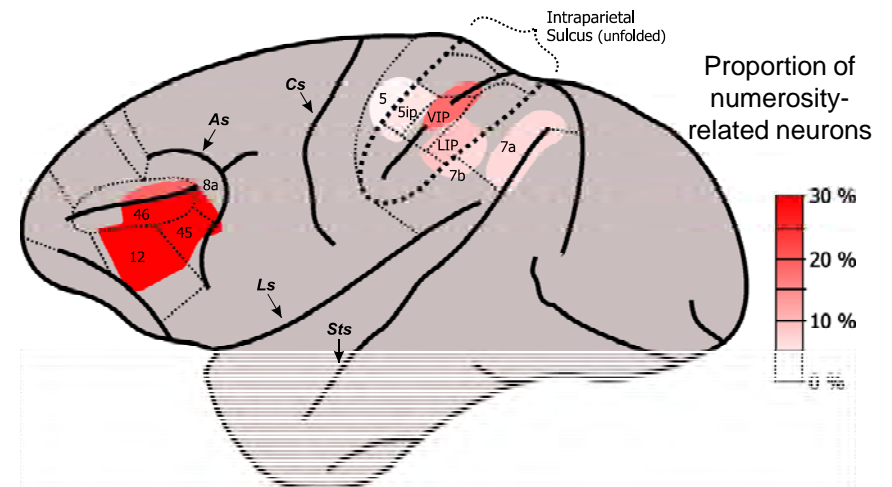
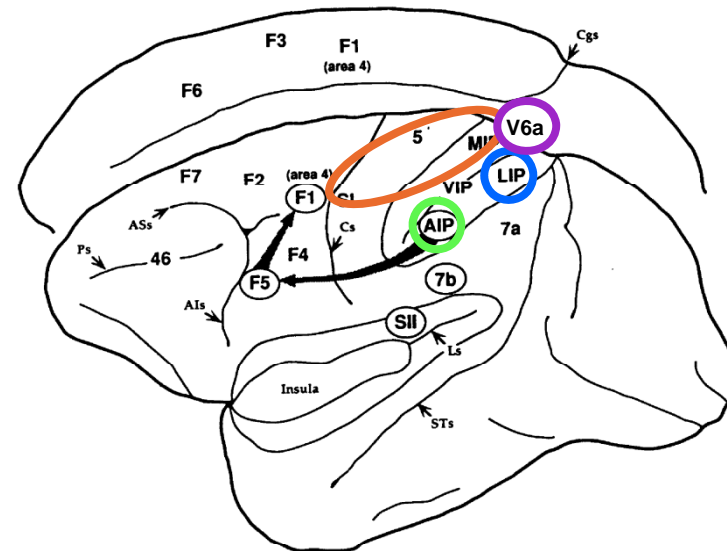
# The intraparietal region: a crucial site for number and space in the human and monkey brain

Simon et al, *Neuron* 2002; Nieder et al., *Science* 2002, *PNAS* 2004

## Human brain



## Macaque monkey





The beaver: Ed Hubbard

# Hunting the SNARC with fMRI

“They sought it with thimbles, they sought it with care;  
They pursued it with forks and hope...”  
Hubbard, Pinel & Dehaene, 2006; never published...



The beaver's lesson

## 1. Parity judgement task

Normal  
Odd - Right



Normal  
Even - Right



Crossed  
Odd - Right



Crossed  
Even - Right



Three orthogonal factors: Hand (left or right); Side of space (left or right);  
Size of number (small or large)

## 2. Saccade Paradigm

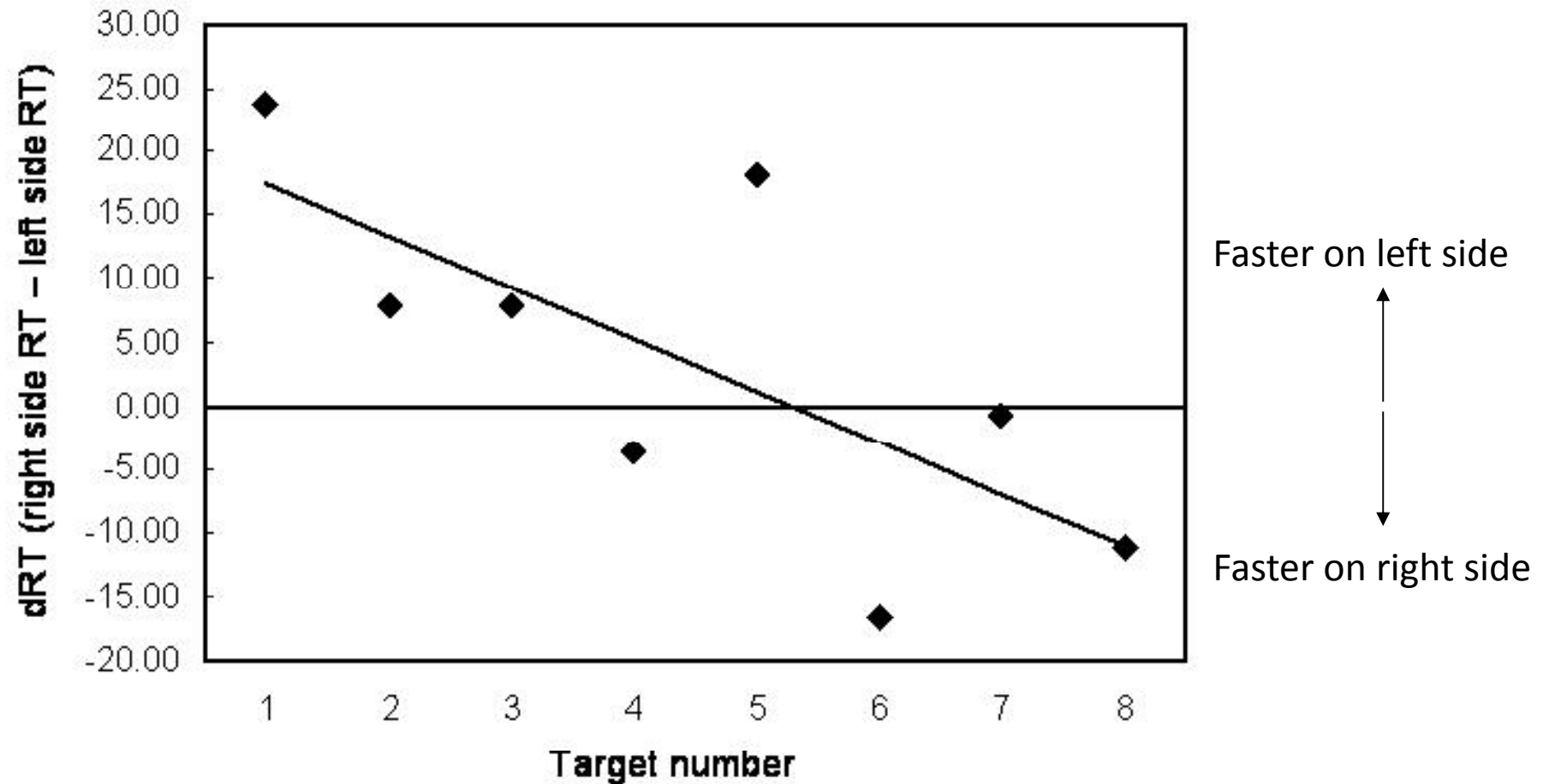
One factor: Saccade direction  
(left or right)

## 3. Arithmetic localizer

Activation to subtraction problem (spoken or  
written) relative to matched  
non-numerical sentences

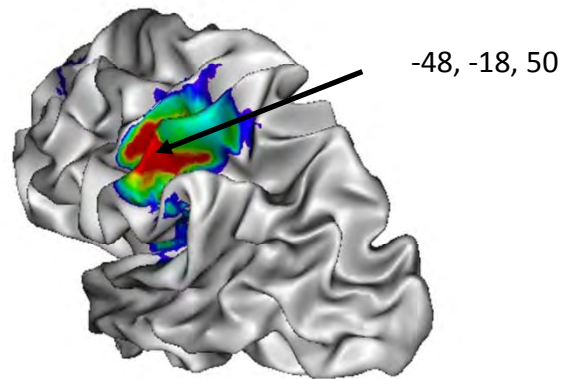
# Behavioral data collected during fMRI

- replication of the SNARC effect
- association with side of space
- no association with hand

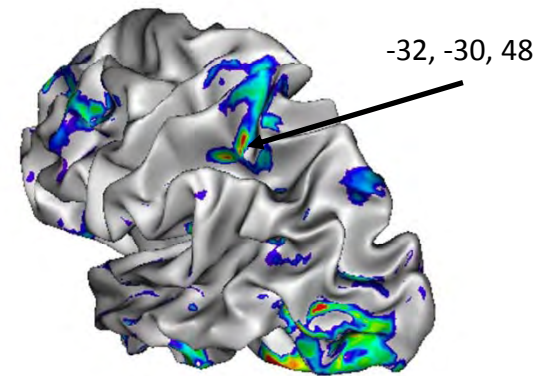


# Space and number:

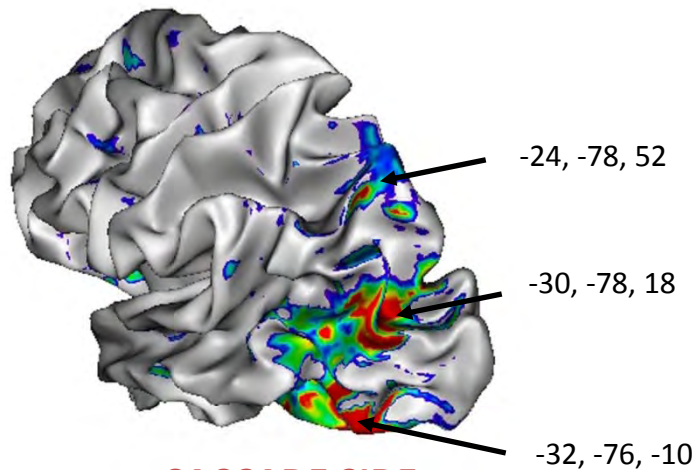
## Definition of four parietal regions of interest (ROI)



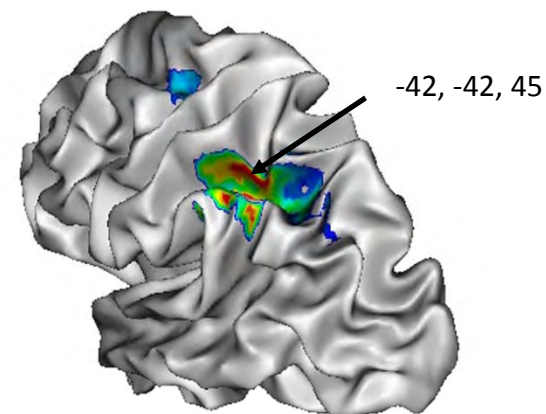
**RESPONSE HAND**  
Right > Left Hand



**RESPONSE SPACE**  
Right > Left Response Side



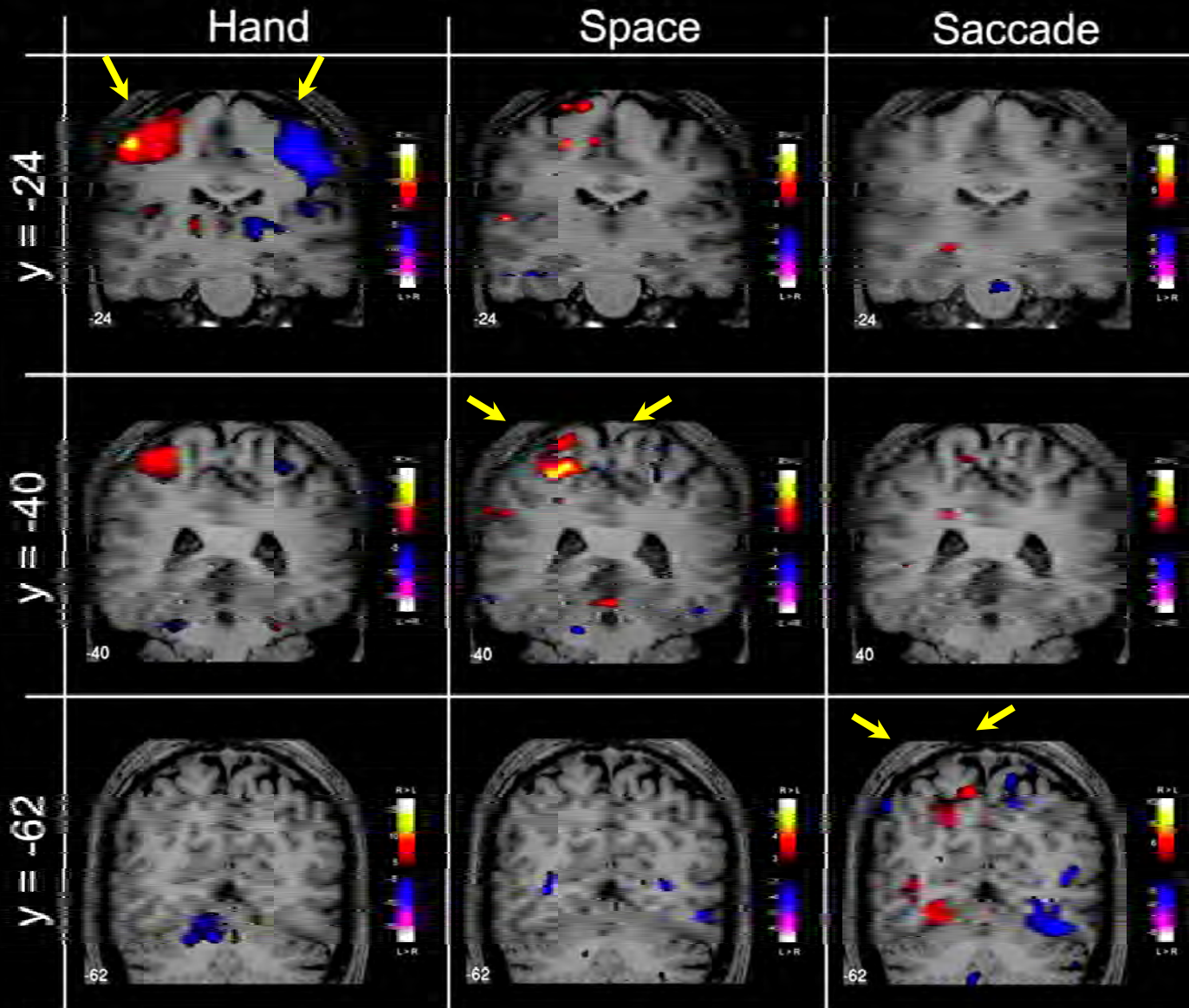
**SACCADE SIDE**  
Right > Left Saccades



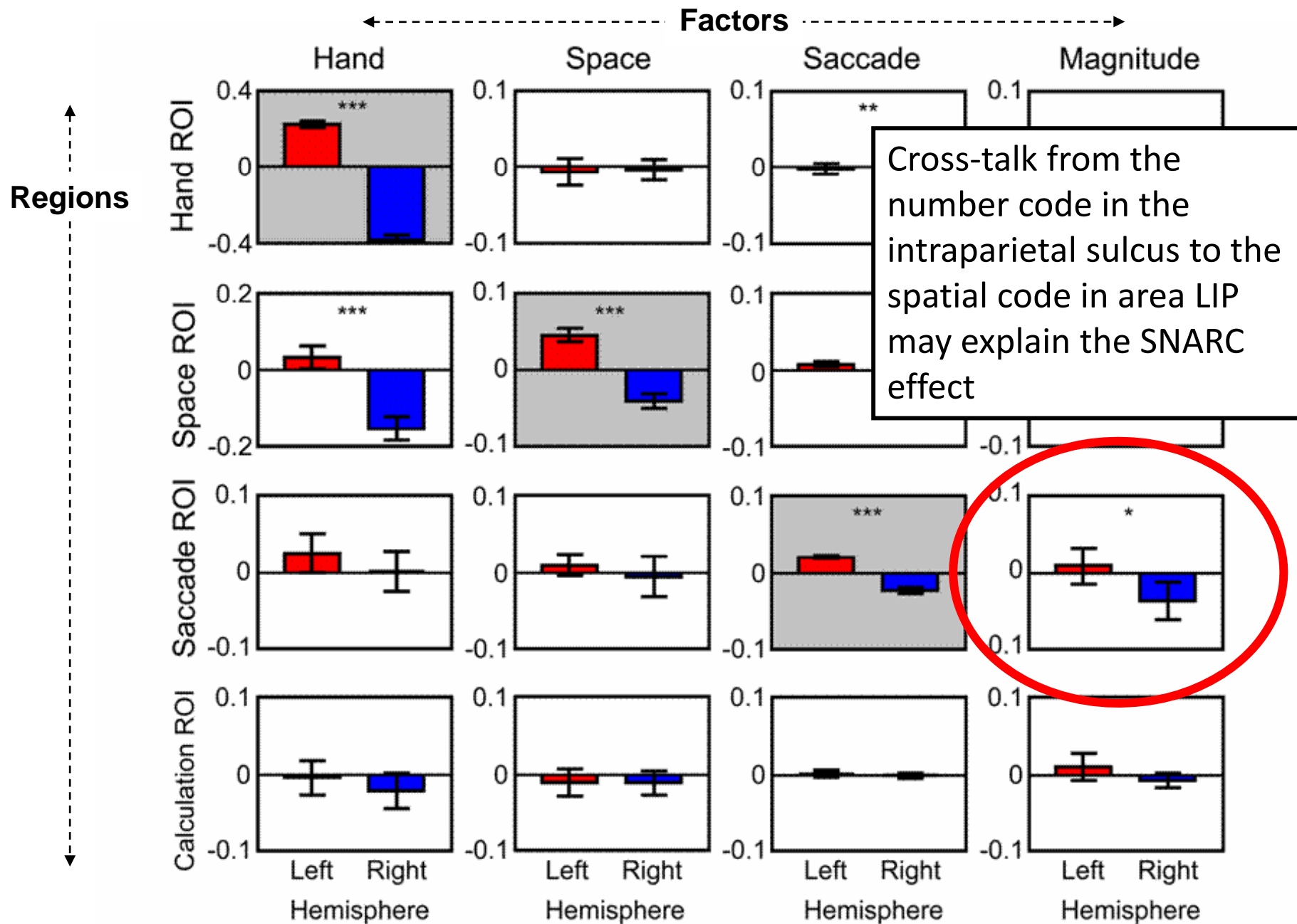
**CALCULATION**  
Calculation (A+V) – Phrases

# Contrasts

three coronal slices



# Effect of each factor in each region ('right' – 'left')



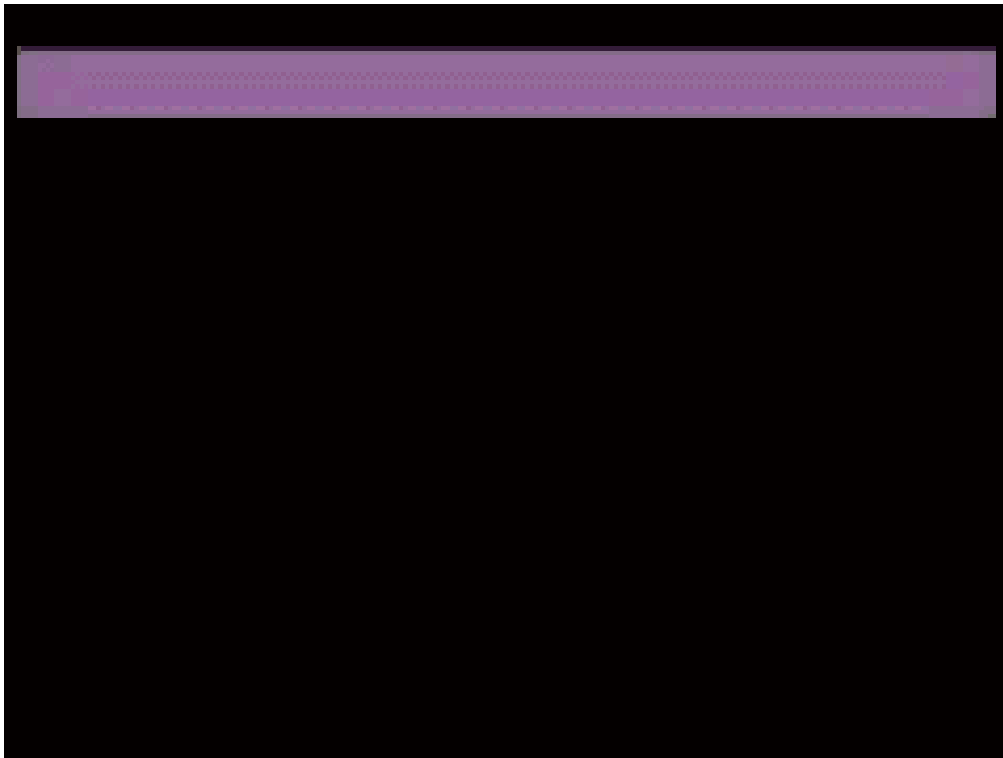


# Does the SNARC influence calculation?

Is an arithmetic operation comparable to a movement on the number line?

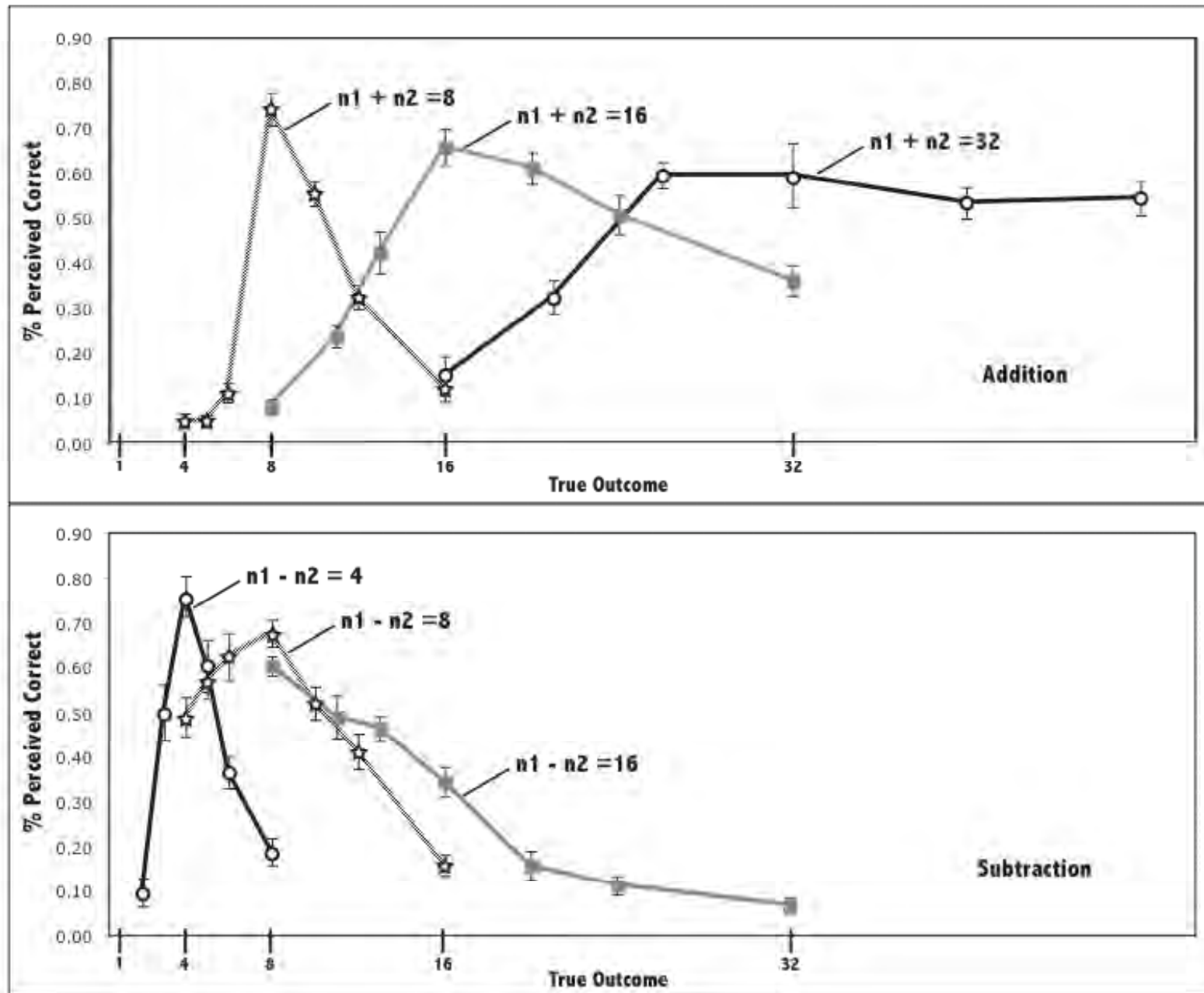
Approximate calculation can be tested with simple movies.

When  $5 + 5$  does not make 10... ..infants look longer at such impossible events



McCrink, K., & Wynn, K. (2004). Large-number addition and subtraction by 9-month-old infants. *Psychol Sci*, 15(11), 776-781.

# How do adults estimate additions and subtractions?



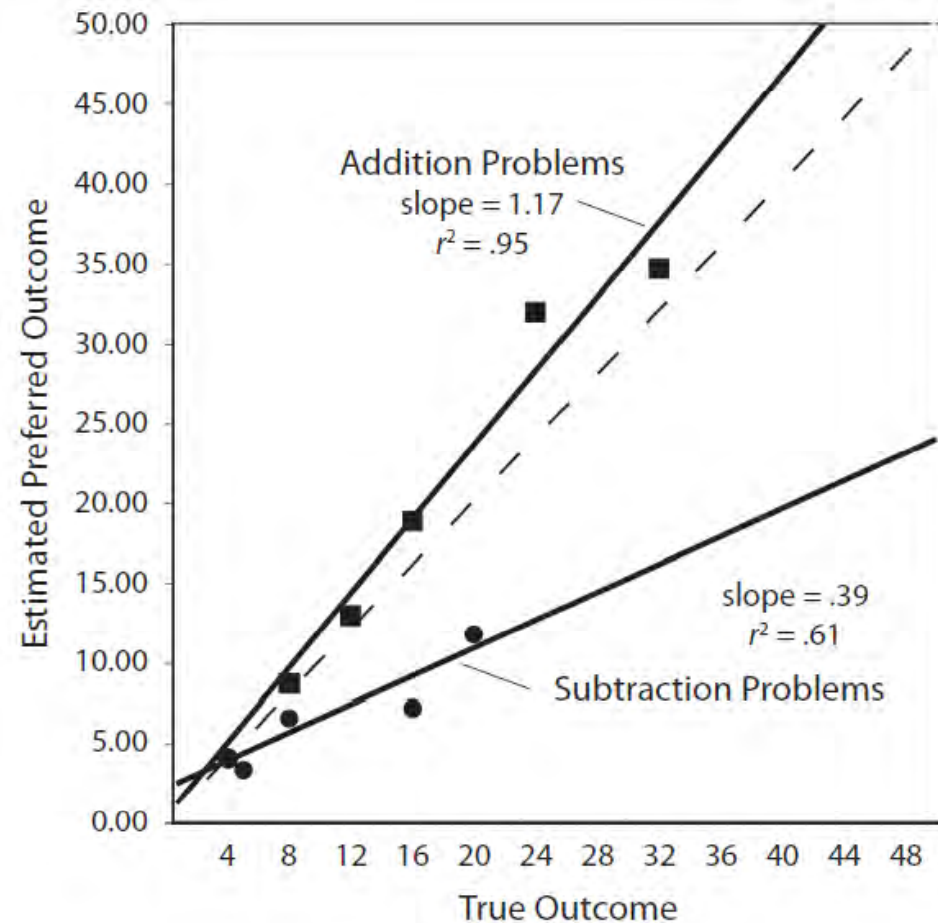
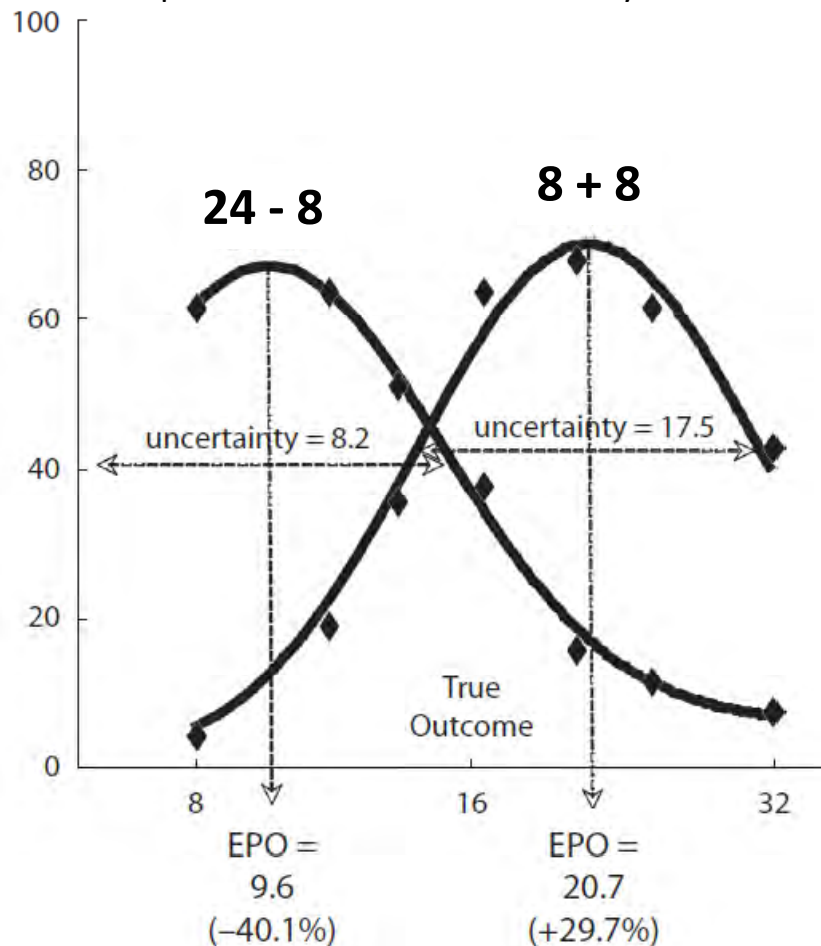
# An « operational momentum » effect

additions are biased towards larger numbers

subtractions are biased towards smaller numbers

Calculation resembles movement on the number line.

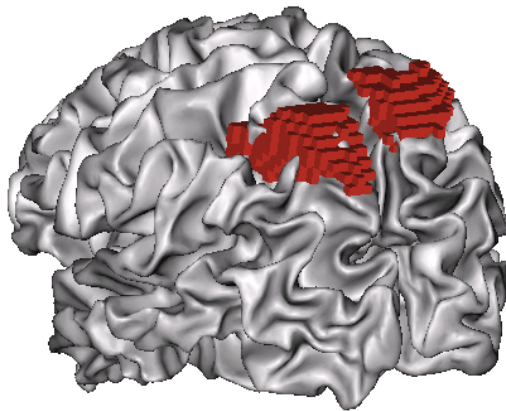
McCrink, K., Dehaene, S., & Dehaene-Lambertz, G. (2007). Moving along the number line: operational momentum in nonsymbolic arithmetic. *Percept Psychophys*, 69(8), 1324-1333.



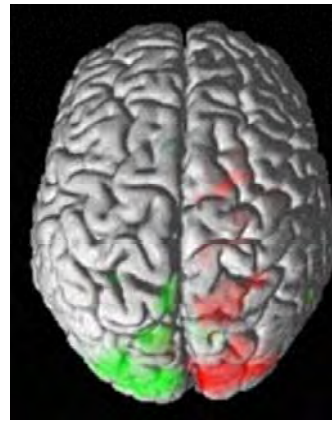
# Cross-talk between number and space during calculation

Knops, Thirion, Hubbard & Dehaene, *Science*, 2009

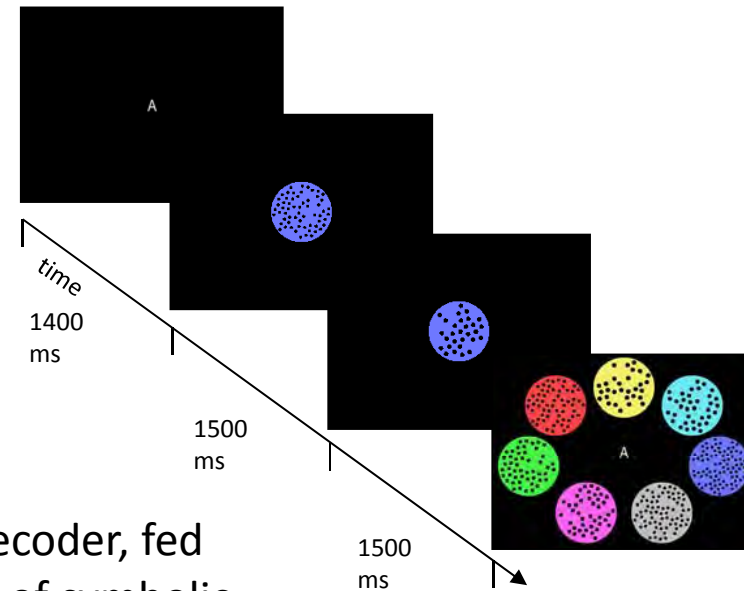
## Training block: eye movements



- Decoding eye movements to the left (red) vs. to the right (green)
- The decoder predicts novel left or right trials with 70% accuracy on average (range: 56%-85%). Classification is above chance ( $p < 0.05$ ) in 14/15 subjects.



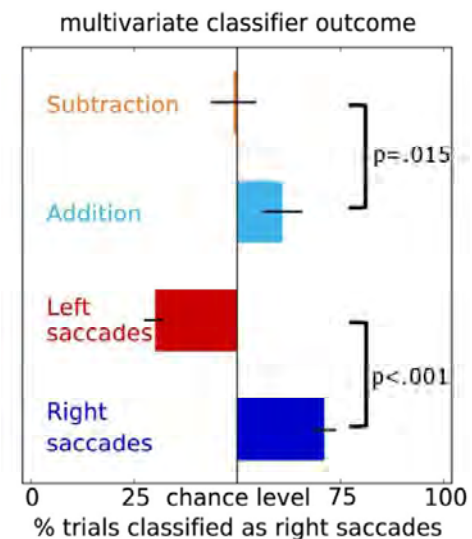
## Test block: Addition / Subtraction



- The same decoder, fed with images of symbolic or non-symbolic calculation, generalizes:

The distinction between left and right eye movements can also be used to distinguish subtraction from addition

(with Arabic or Dot notation)



# The shape of the SNARC: From logarithmic to linear

Siegler & Opfer, 2003; Siegler & Booth, 2004

Number-Space mapping task: « Please point to where number x should fall »



A major change occurs during mathematical education: a switch from a logarithmic to a linear understanding of number

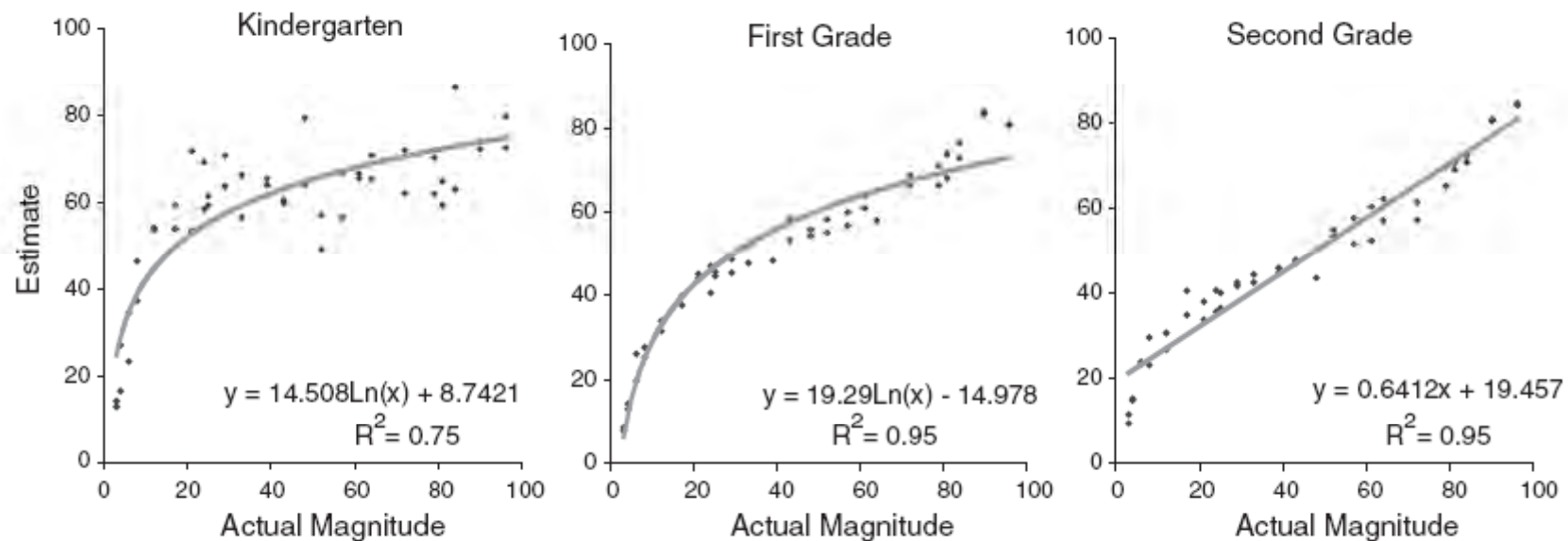


Figure 2. Progression from logarithmic pattern of median estimates among kindergartners (left panel) to linear pattern of estimates among second graders (right panel) in Experiment.



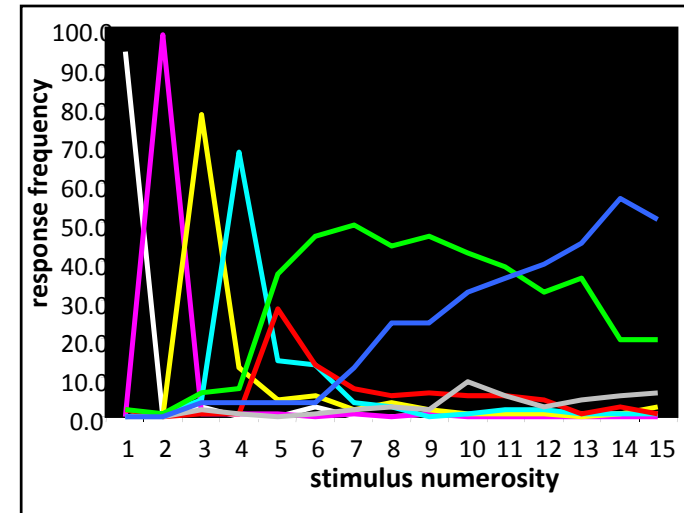
# Numerical cognition without words in the Mundurucu

Pica, Lemer, Izard, & Dehaene, Science, 2004

- pug ma = one
- xep xep = two
- ebapug = three
- ebadipdip = four
- pug pōgbi = one hand
- xep xep pōgbi = two hands
- adesu/ade gu = some, not many
- ade/ade ma = many, really many

A reduced lexicon of number words

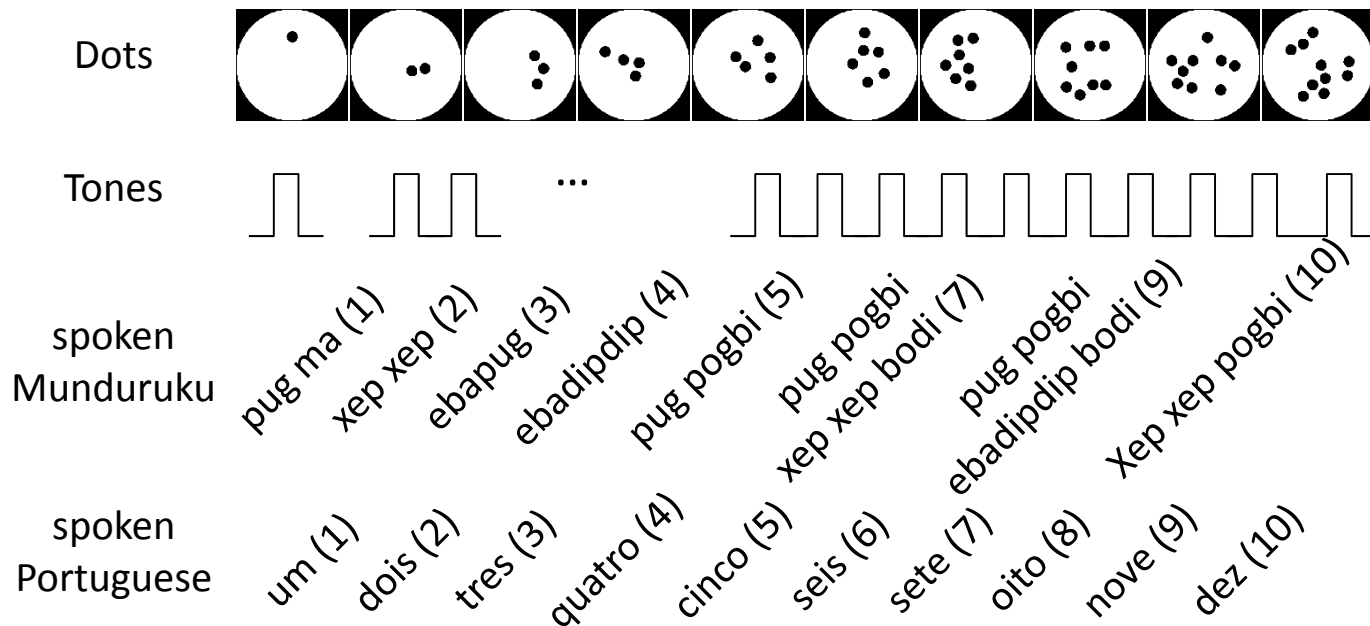
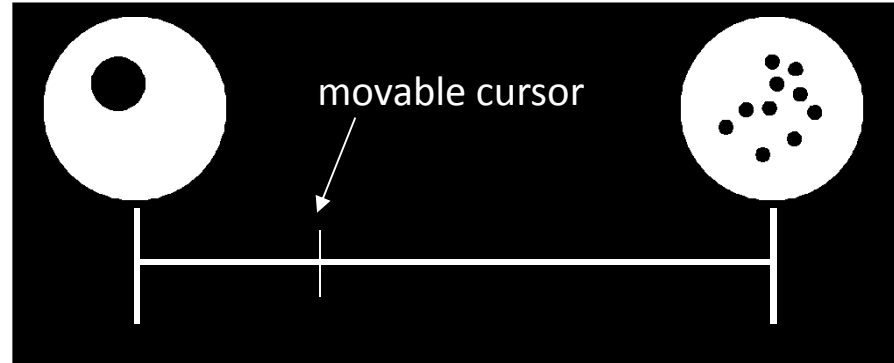
Mundurucu number words refer to approximate numerosity



Mundurucu adults and children can do approximate arithmetic with non-verbal numerosities (e.g. 40+30 is larger than 50) but not exact arithmetic (e.g. 7-6=1)

# Number-Space mapping in the Mundurucu

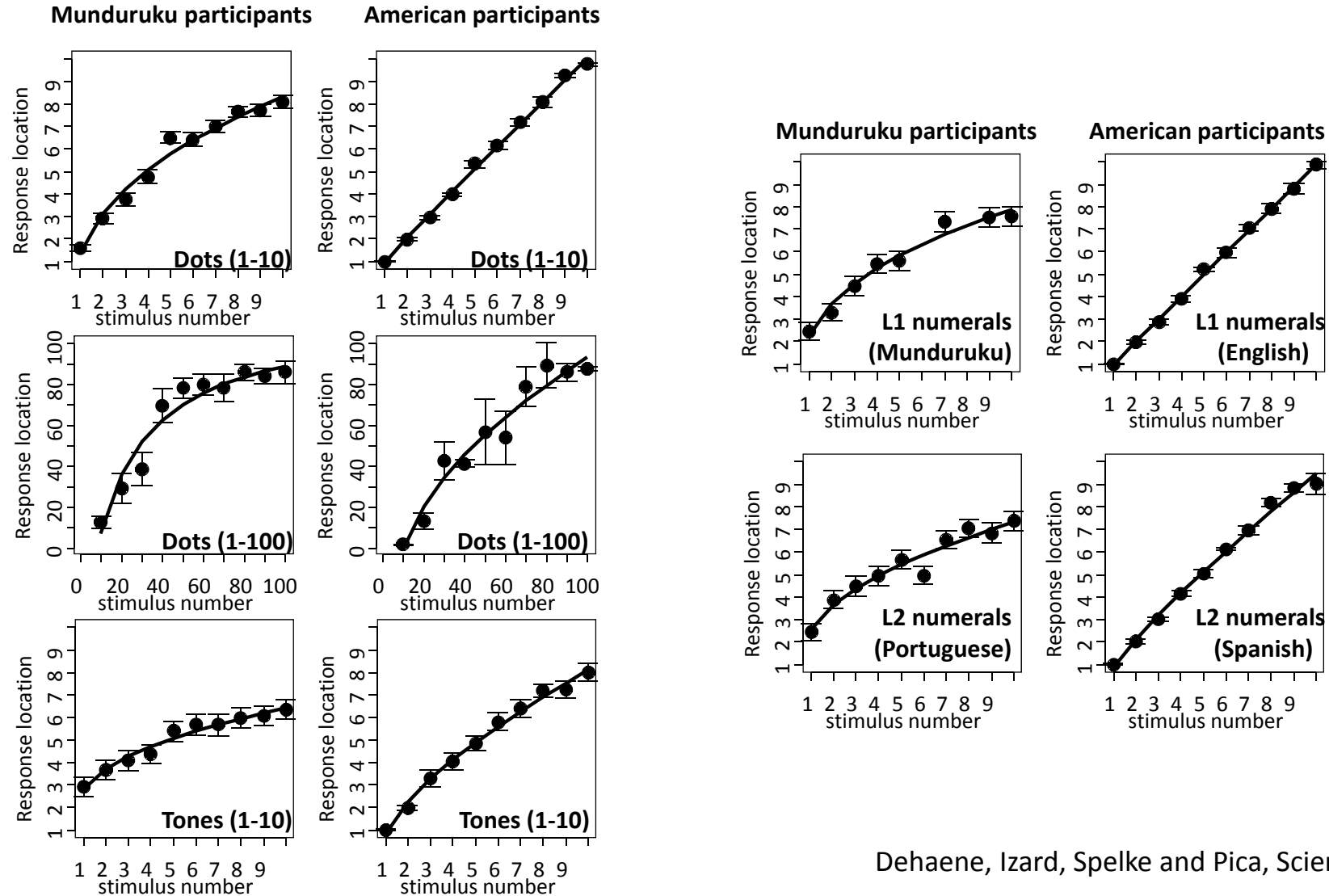
Mundurucu children and adults were asked to point to the location corresponding to a certain number.



# Logarithmic Number-Space mapping in Mundurucu adults

Mundurucu children and adults show a compressive mapping

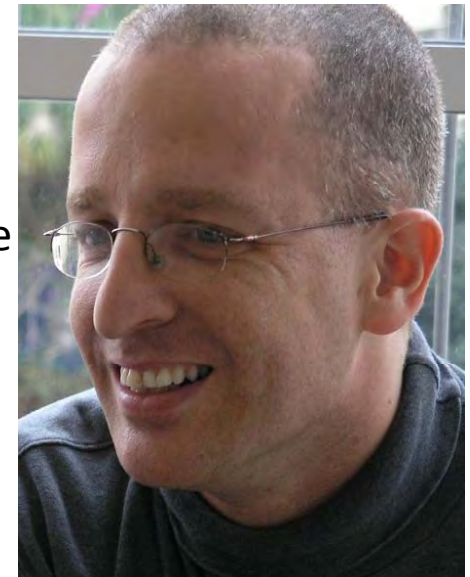
- For dot patterns or series of 1-10 tones
- For Mundurucu words and even for Portuguese numerals





# Decomposing the number-space task

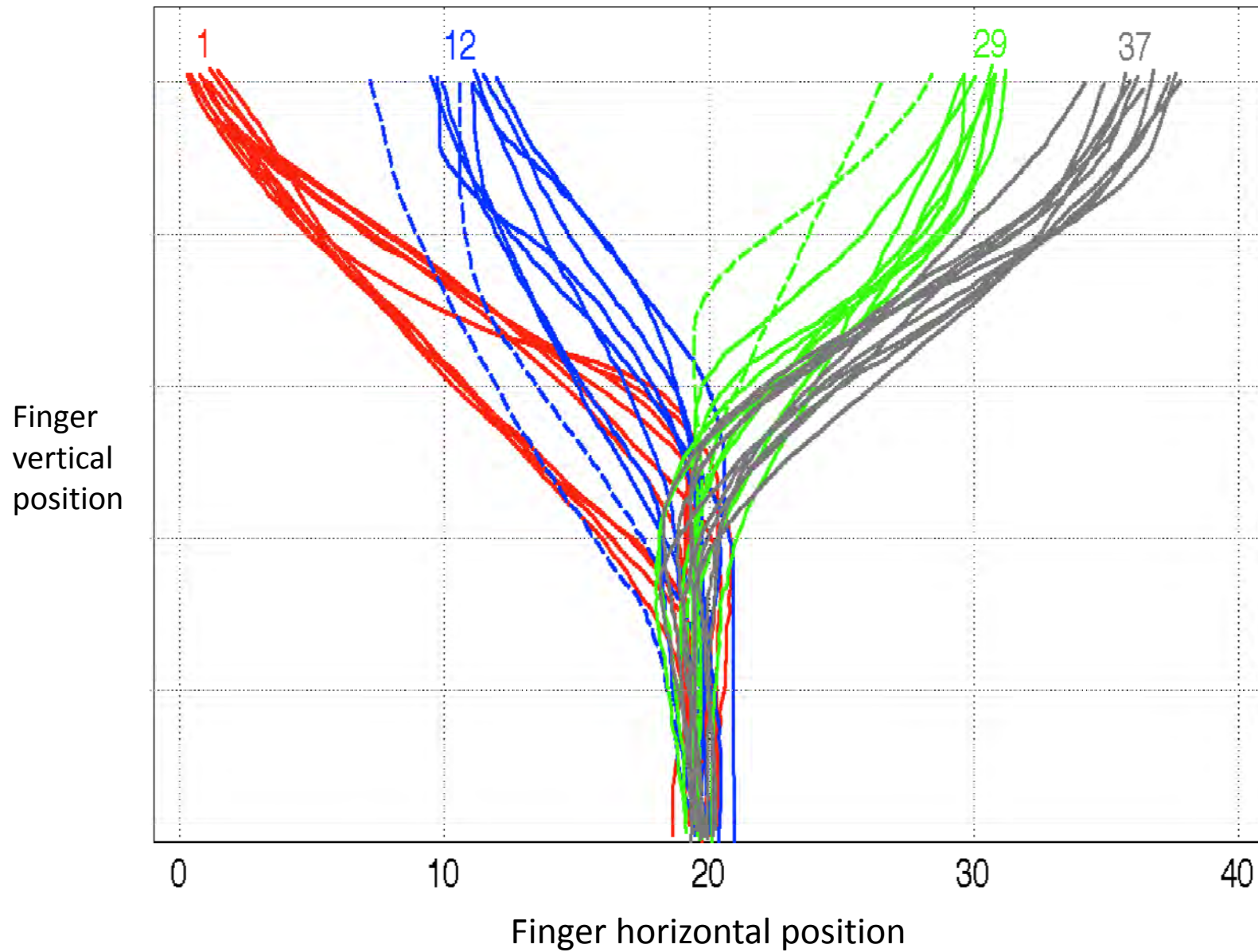
Dror Dotan programmed the iPad to digitize the finger trajectory while the number-to-line task (with two-digit numbers between 0 and 40)



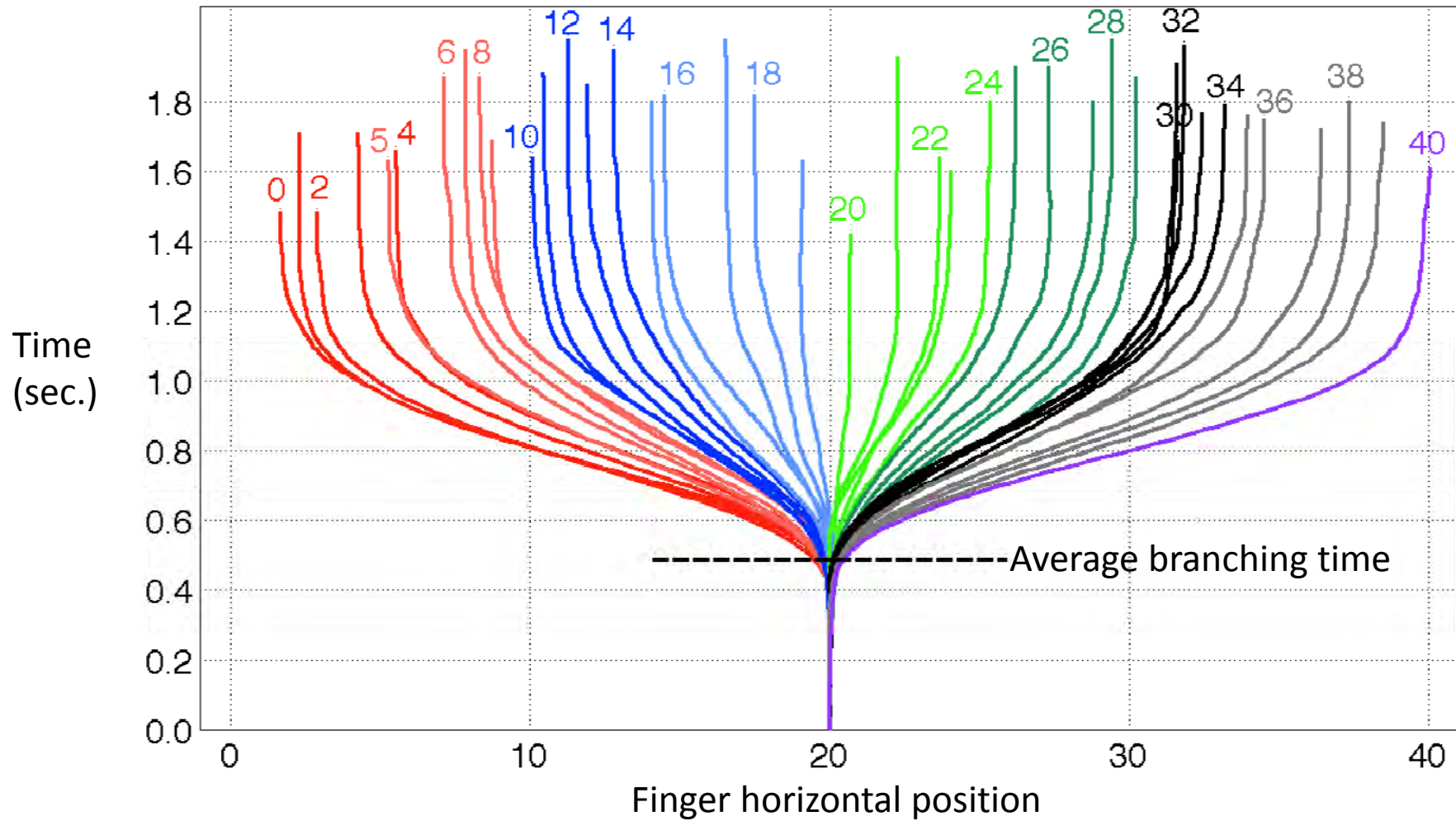
Dror Dotan



# Decomposing the number-space task



# Decomposing the number-space task



# Decomposing the number-space task

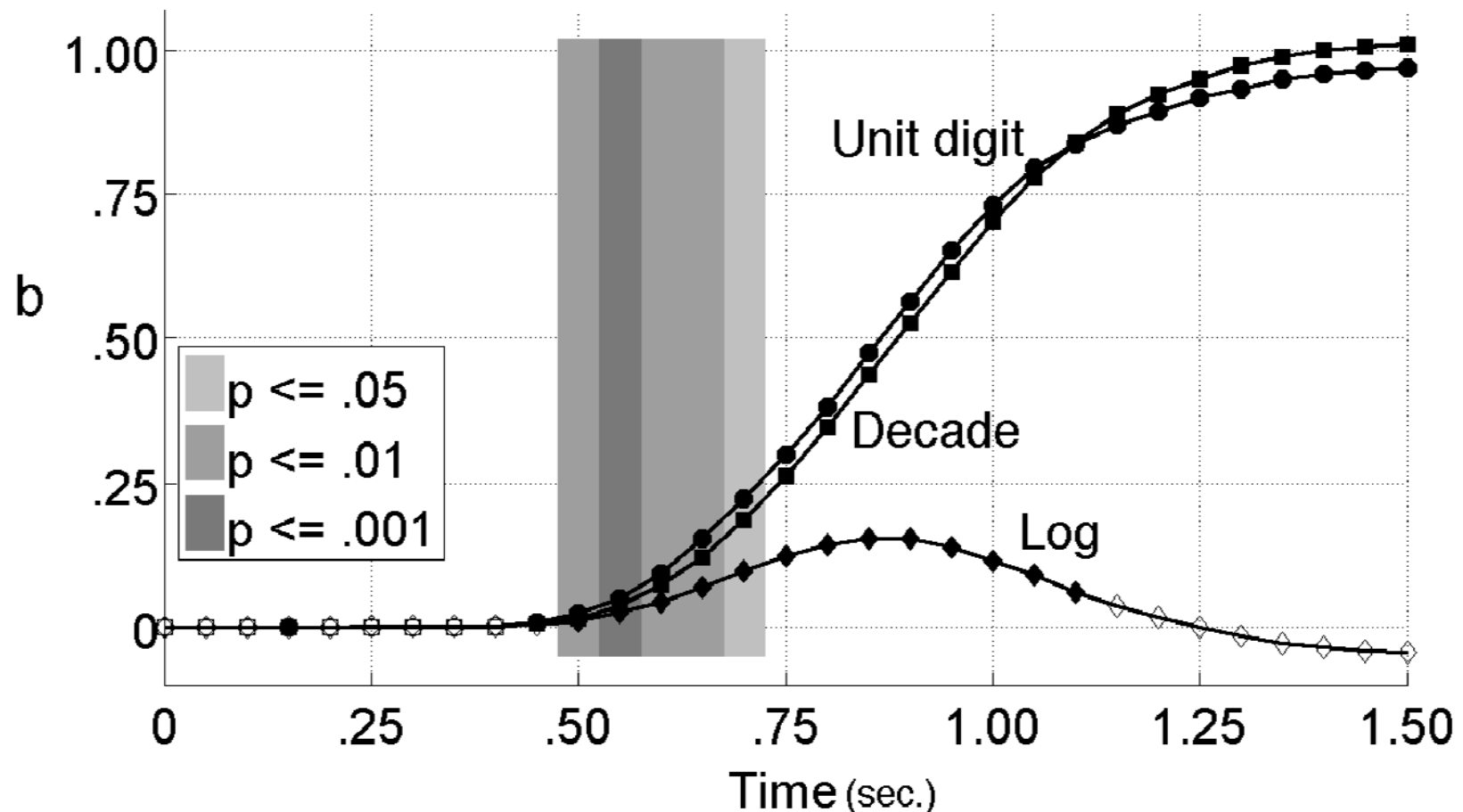
Units and decades are processed nearly simultaneously

→ holistic apprehension of the quantity – or even enhancement of units

There is a transient effect of the log quantity

→ a dormant compressive representation in educated adults.

See Dror Dotan's poster for other interesting points.



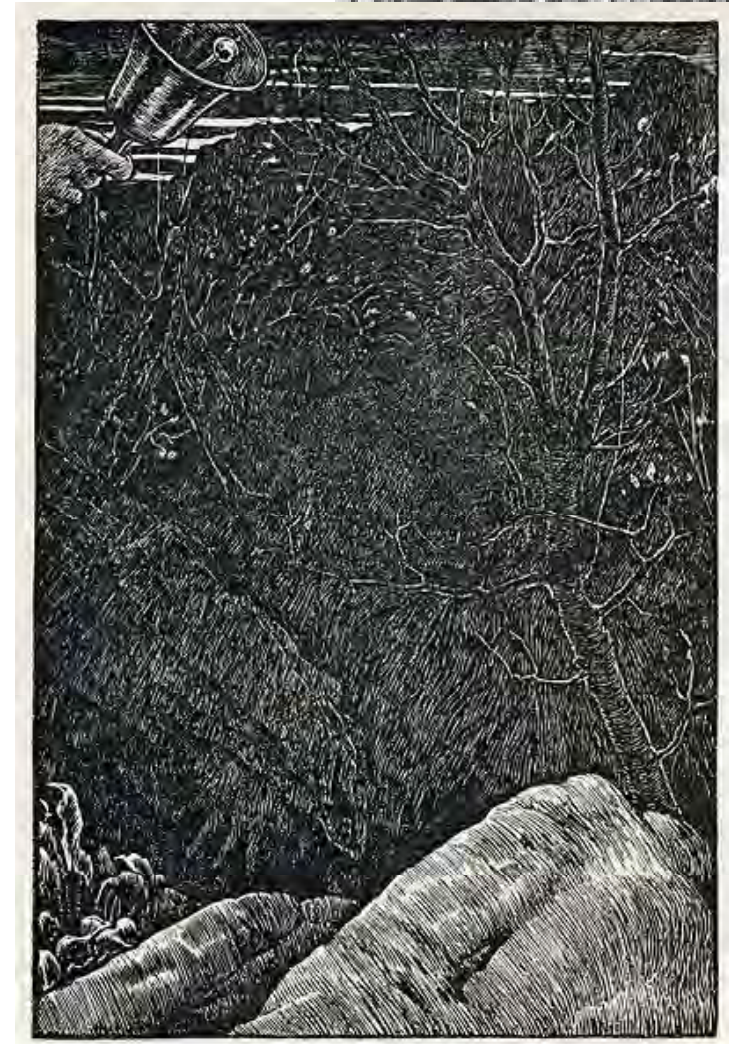
# The end of the hunting?

““It's a Snark!" was the sound that first came to their ears,  
And seemed almost too good to be true.  
Then followed a torrent of laughter and cheers  
Then the ominous words ``It's a Boo---"  
(...)

In the midst of the word he was trying to say  
In the midst of his laughter and glee  
He had softly and suddenly vanished away  
For the Snark *was* a Boojum, you see.”

We will hear today some very interesting challenges to the SNARC story:

- Do different types of spatial-numerical effects have different origins? (e.g. synesthesia)
- Does the SNARC depend just on working memory? (Van Dijck & Fias, 2011)
- Does the SNARC effect exist even in newborn chicks?! (see papers and poster by Rugani et al.)





Each thought he was thinking of nothing but "Snark"  
And the glorious work of the day!