Conversion thermoélectrique:

Nanofils désordonnés et Cavités quantiques chaotiques :

L'intérêt des bords de bandes

en régime élastique cohérent et en régime inélastique activé.

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❑ **I . <u>Elastic Coherent Regime (Low temperatures)</u>**

A - 1d localized systems:

- **Measure of the conductance** of disordered nanowires in the field effect transistor device configuration **(Sanquer et al)** and the Mott formula for the thermopower.

- **Theory using an Anderson model for a 1d disordered chain** Thermoelectric transport (i) the bulk; (ii) the edge and (iii) eventually the outside of the impurity band - Typical behavior and fluctuations of the thermopower

B - 0d chaotic systems

- **Measure of the thermopower** of a chaotic cavity (Molenkamp et al).

- **Random matrix theory** of the thermopower of a chaotic cavity:
Universal thermopower distribution in the bulk (Wigner-Dyson) and at the edge (Tracy-Widom) of the spectrum.

❑ **II. <u>Inelastic Activated Regime (Intermediate temperatures) in 1d.</u>**

Mott variable range hopping and Miller-Abrahams resistor network, Seebeck and Peltier coefficients in the bulk and near the edges of the impurity band.

❑ **III. <u>Thermoelectric transport at room temperature (Kim et al).</u>**
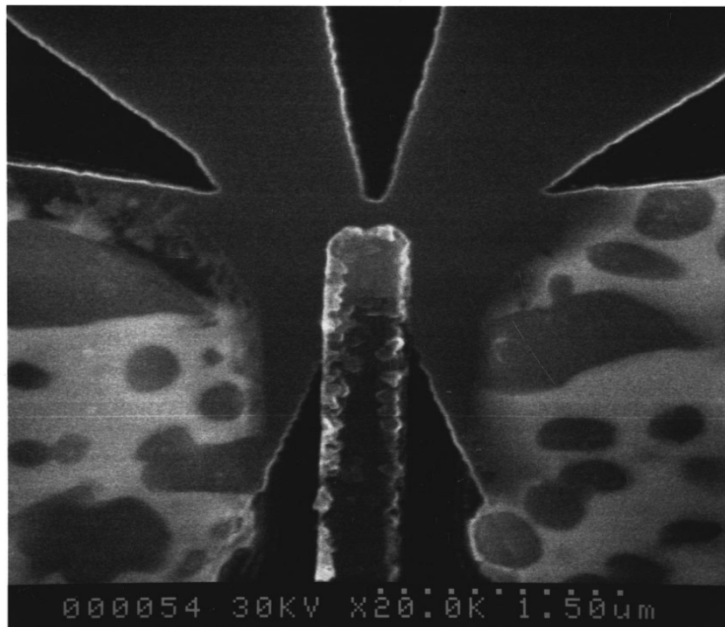
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Quantum coherent elastic transport in the field effect transistor device configuration at very low temperature

We study the transport through gated GaAs:Si wires of 0.5 μm length in the insulating regime and observe transport via tunneling at very low temperature. We describe the mean positive magnetoconductance and the mesoscopic fluctuations of the conductance ~versus energy or magnetic field! purely within one-electron interference model.

FIG. 1. SEM picture of the GaAs:Si **submicronic** MESFET. The 0.5-mm-thick aluminum Schottky gate is visible on the bottom. **The gate** does not cover the whole constriction width, but **covers entirely the conducting channel if one considers the depletion width**. The GaAs is doped at 10^{23} **Si m^{-3}**) 300-nm-thick layer is etched toform four large contact pads to the active region under the gate.AuxGe12xNi Ohmic contacts are visible on the right and the left. The volume of the active region is estimated to be **0.2 x 0.2 x 0.5 μm^3** (taking into account depletion layers for V_{gate}= 0 V, about 120 nm).

Anderson Insulator ➡️ ⬅️ **Conductor**
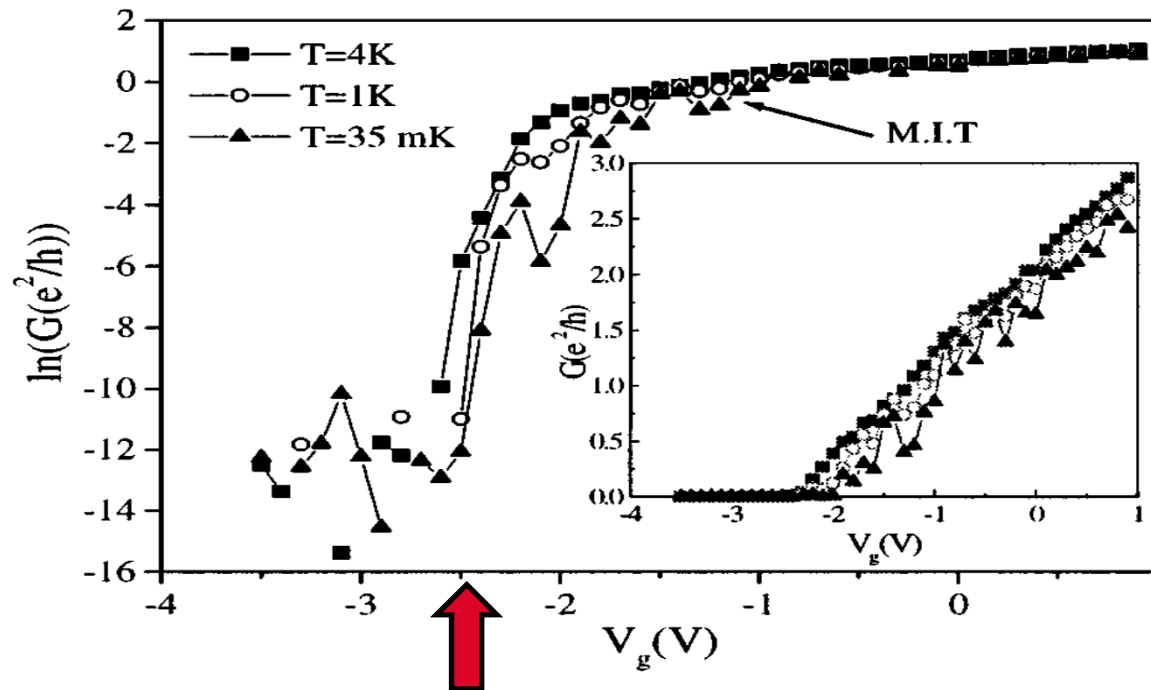
ln G(V gate) at three temperatures in a large gate voltage range
(details of the conductance pattern are not seen for this gate voltage sampling)
Inset: the same curve in a linear scale. Note the linearity at voltages above the transition.
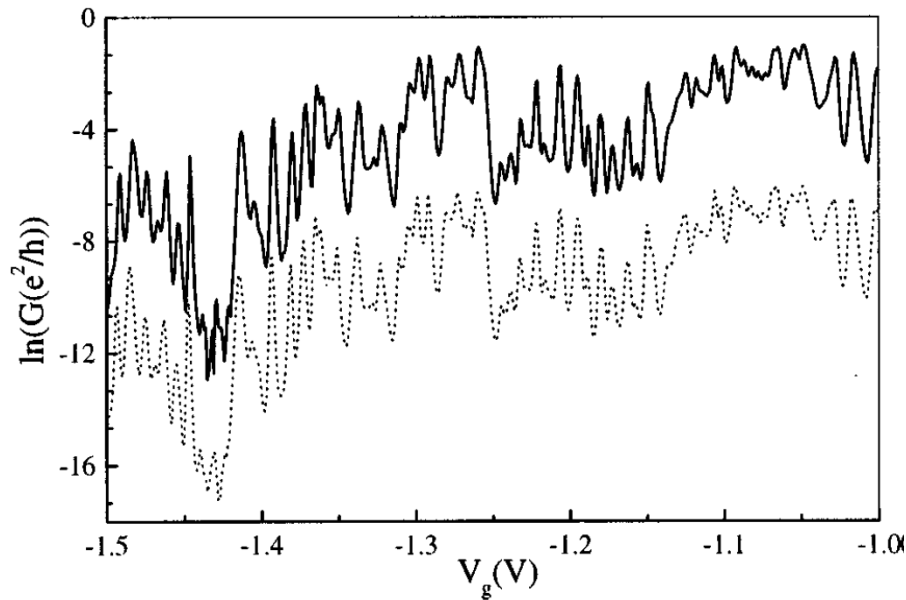
➡️ **Edge of the impurity band = -2,5 V**
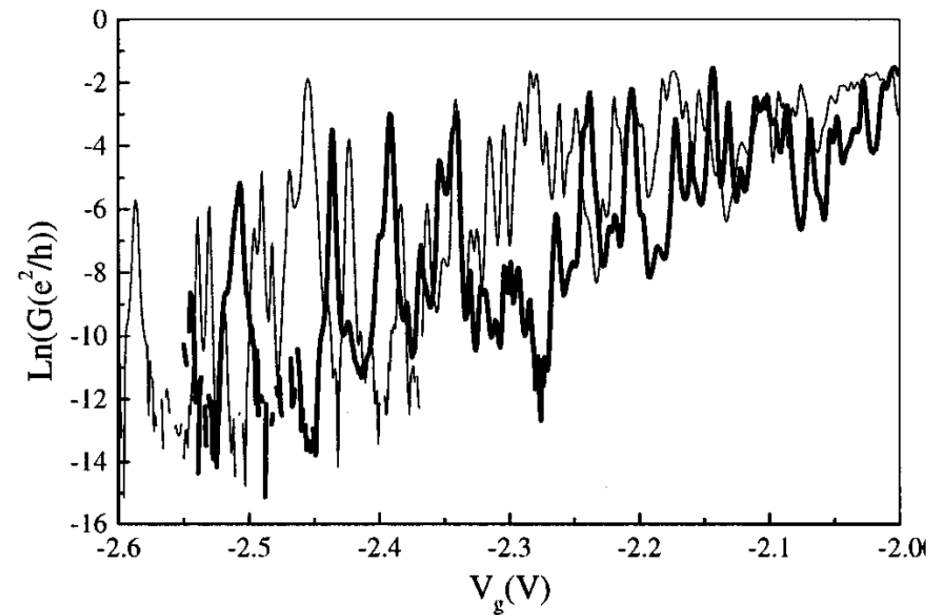**(Complete depletion of the disordered nanowire)**
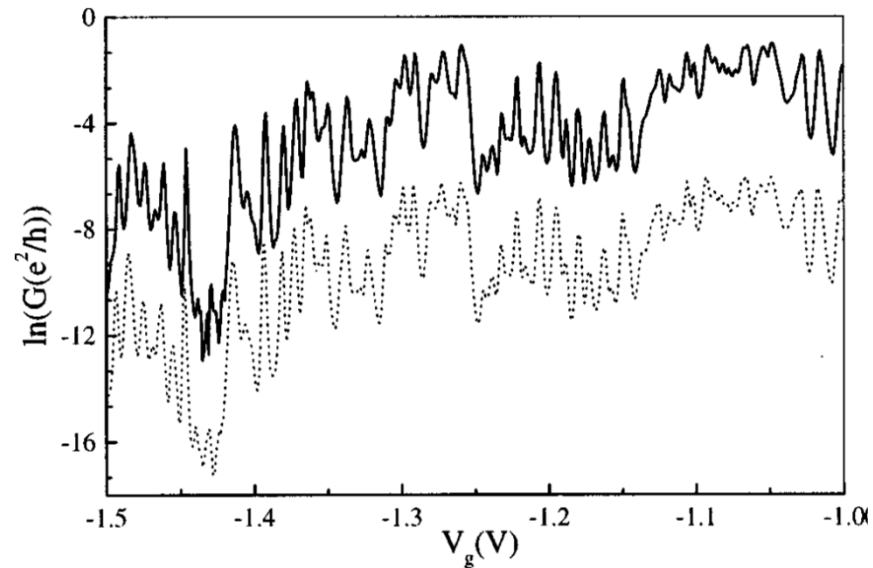
ln G(V gate) at $T = 100\ mK$ in the **0.5-μm-long** sample for two successive experiments without thermal cycling, showing the excellent reproducibility of the conductance pattern (curves are shifted for clarity).
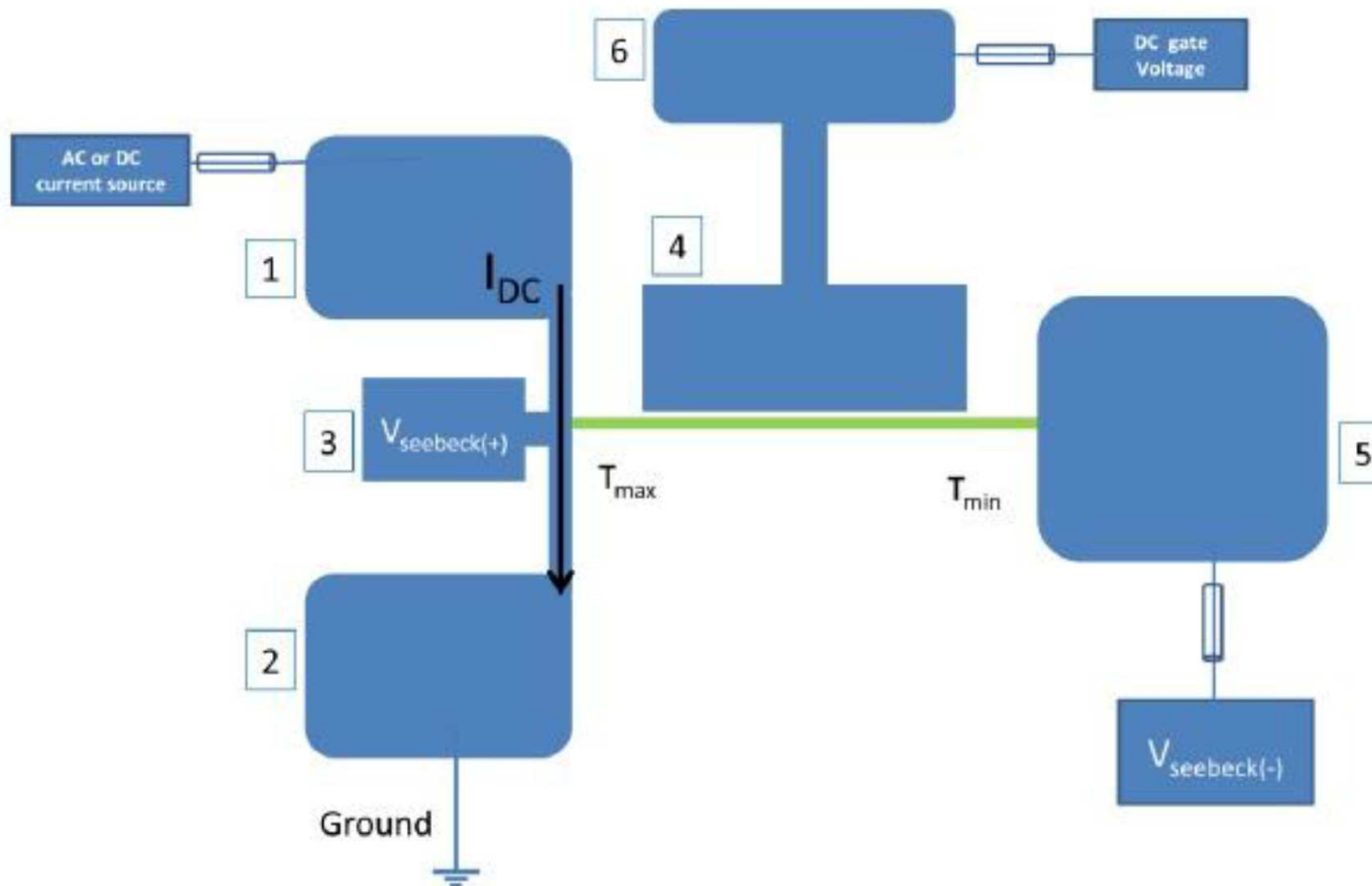
Bulk of the impurity band

Edge of the impurity band

Mott Formula :

$$S \approx \frac{\partial \ln(G)}{\partial V_g}$$

Larger thermopower at the band edges

Disordered nanowire
in the field effect transistor device configuration:

R. Bosisio, G. Fleury and JLP

1d lattice of length L (**N** sites) with nearest hopping terms t, random on-site potentials and gate potential V_G

Anderson Localization with **localization length ξ(E)**

Left lead
(self-energy Σ_L)

Right lead
(self-energy Σ_R)

V_G
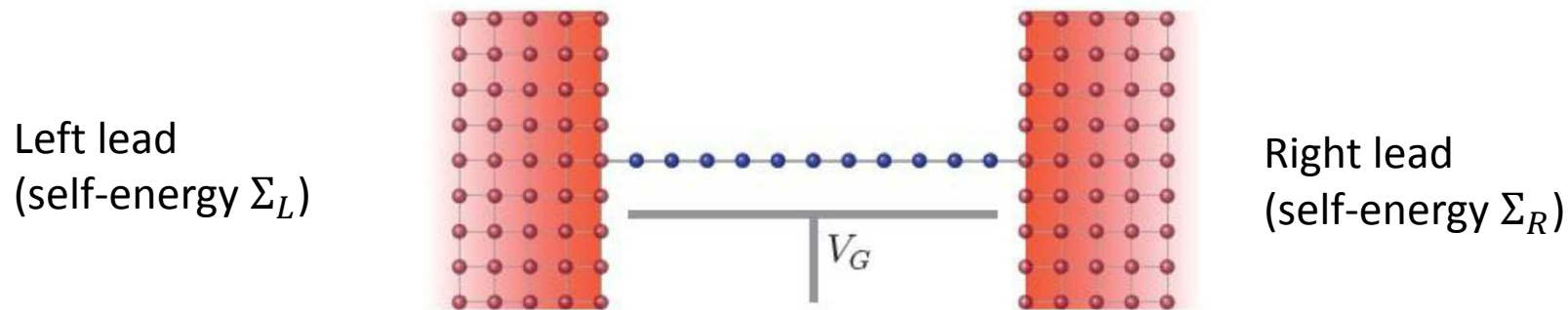
$$H_{1d} = -\boldsymbol{t}.\Sigma_{ij}\left(c_j^+ c_i + H.C \right) + \Sigma_i\, \boldsymbol{\varepsilon}_i n_i$$

$$H_{gate} = \Sigma_i \boldsymbol{V_G}\, n_i$$

$\boldsymbol{\varepsilon_i}$ Box distribution of **width W** and **center 0**

Study of the localized limit $N > \xi$

Elastic coherent transport,   *Linear Response,*   *Sommerfeld expansions*

"Mott   Formula "

Typical **thermopower**

$$S = t \left. \frac{\mathrm{d} \ln \mathcal{T}}{\mathrm{d}E} \right|_{E_F}$$
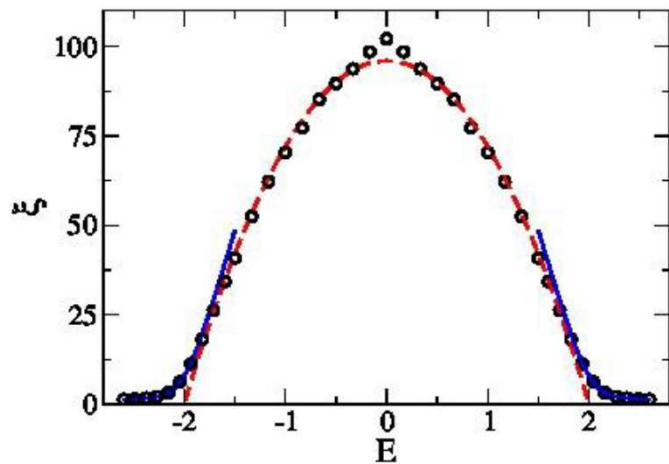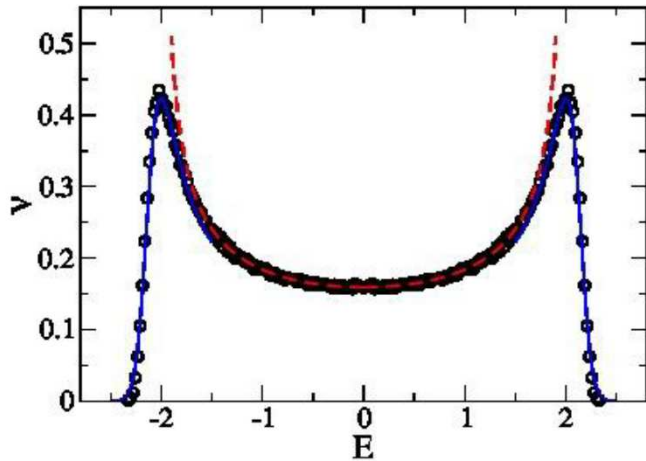
$$[\ln \mathcal{T}]_0(E) = -\frac{2N}{\xi(E)}$$

$$\mathcal{S} = \frac{\pi^2}{3} \left(\frac{k_B}{e} \right) \left(\frac{k_B T}{t} \right) S$$ In physical units

To predict the typical behavior of S, one just need to know how the localization length ξ depends on the energy E.

Weak Disorder expansions of the 1d density of states v=ρ/N and of the localization length ξ (assuming $V_G = 0$)

Numerical check with $W = 1$

BULK

$$\rho_b(E)/N = \frac{1}{2\pi t \sqrt{1 - (E/2t)^2}}$$

$$\xi_b(E) = \frac{24}{W^2}\left(4t^2 - E^2\right)$$

B. Derrida & E. Gardner, J. Physique 45, 1283 (1984)

EDGE

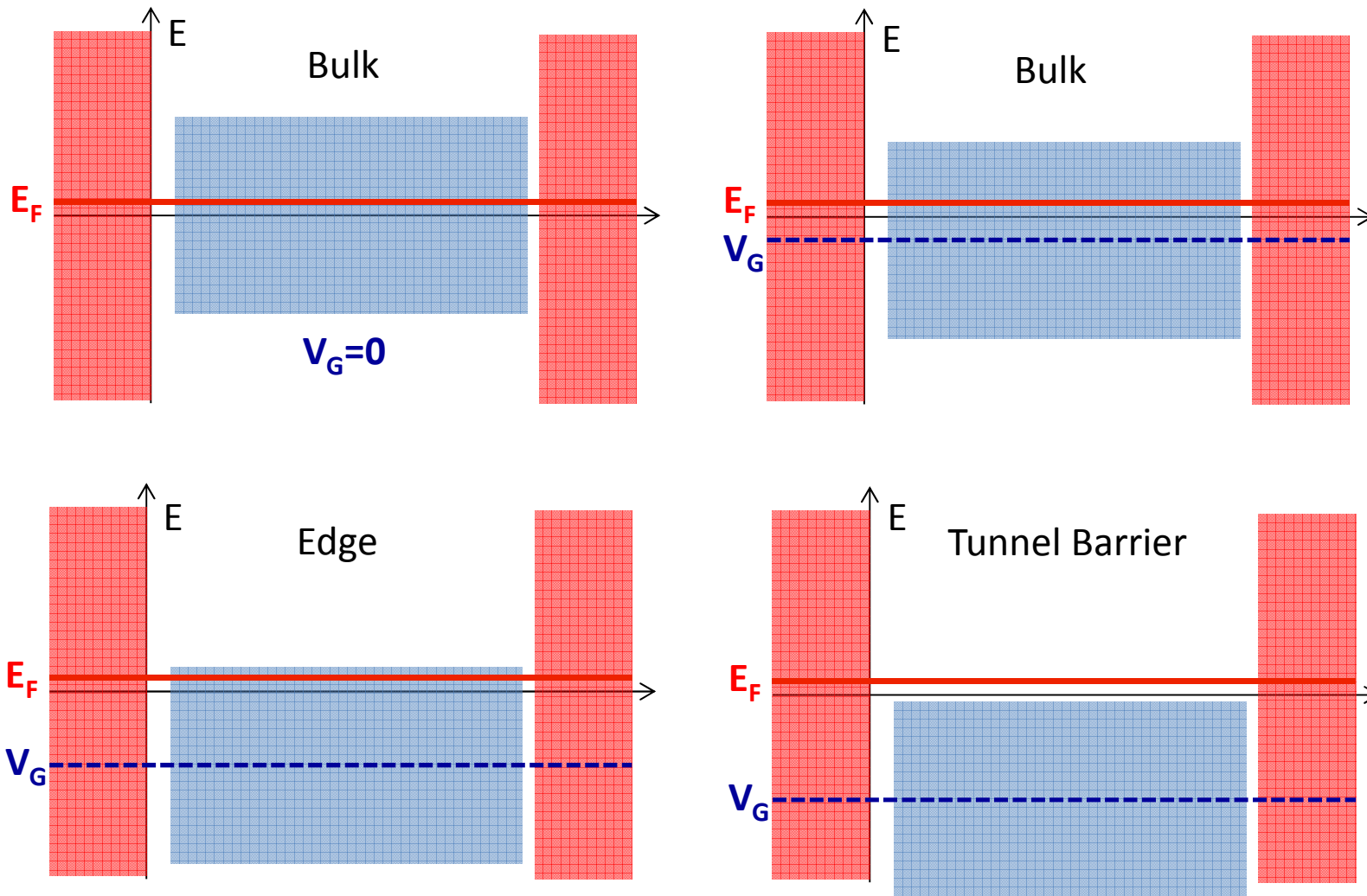$$\rho_e(E)/N = \sqrt{\frac{2}{\pi}}\left(\frac{12}{tW^2}\right)^{1/3}\frac{\mathcal{I}_1(X)}{[\mathcal{I}_{-1}(X)]^2}$$

$$\xi_e(E) = 2\left(\frac{12t^2}{W^2}\right)^{1/3}\frac{\mathcal{I}_{-1}(X)}{\mathcal{I}_1(X)}$$

$$X = (|E| - 2t)t^{1/3}(12/W^2)^{2/3}$$

$$\mathcal{I}_n(X) = \int_0^\infty y^{n/2} e^{-\frac{1}{6}y^3 + 2Xy}\, dy$$

EFFECT OF GATE VOLTAGE ON THE IMPURITY BAND

What matters is the <u>relative</u> position of E_F inside the impurity band

TYPICAL THERMOPOWER AT LOW T:
WEAK DISORDER THEORY & NUMERICAL CHECK WITH W=1

Using Sommerfeld expansions for having Mott formula

Bulk:

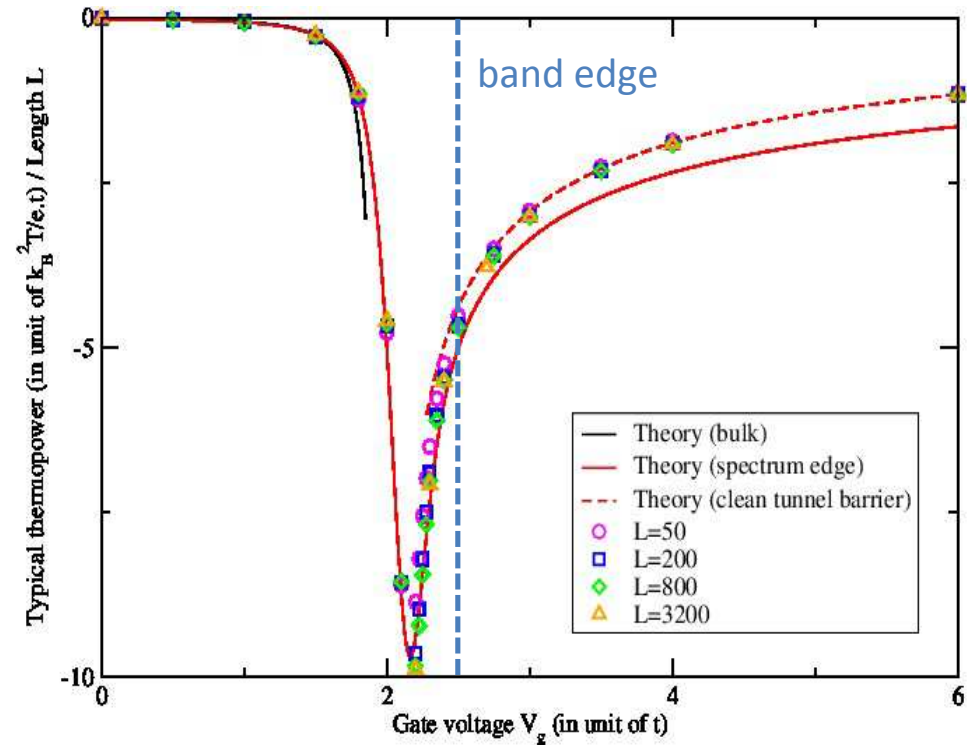$$S_0^b = -N \frac{(E_F - V_g) W^2}{96 t^3 [1 - ((E_F - V_g)/2t)^2]^2}$$

Edge:

$$S_0^e = -2N \left(\frac{12 t^2}{W^2} \right)^{1/3} \left\{ \frac{\mathcal{I}_3(X)}{\mathcal{I}_{-1}(X)} - \left[\frac{\mathcal{I}_1(X)}{\mathcal{I}_{-1}(X)} \right]^2 \right\}$$

$$X = (|E_F - V_g| - 2t) t^{1/3} (12/W^2)^{2/3}$$

Tunnel Barrier:

$$\frac{S_0^{TB}}{N} \underset{N \to \infty}{\approx} \frac{1}{N} \frac{2t}{\Gamma(E_F)} \left. \frac{d\Gamma}{dE} \right|_{E_F} \pm \frac{1}{\sqrt{\left(\frac{E_F - V_g}{2t} \right)^2 - 1}}$$

band edge

Typical thermopower (in unit of $k_B^2 T/e.t$) / Length L

Theory (bulk)
Theory (spectrum edge)
Theory (clean tunnel barrier)
○ L=50
□ L=200
◇ L=800
△ L=3200

Gate voltage V_g (in unit of t)

**Large Enhancement of the Thermopower
near the band edge of the nanowire**

R. Bosisio, G. Fleury and J-L. Pichard, (2013)

MESOSCOPIC FLUCTUATIONS: THERMOPOWER DISTRIBUTIONS

In the Bulk $\longrightarrow$ Cauchy distribution (demonstrated in [1] for the case $S_0=0$)

$$P(S) = \frac{1}{\pi}\frac{\Lambda}{\Lambda^2 + (S - S_0)^2}$$
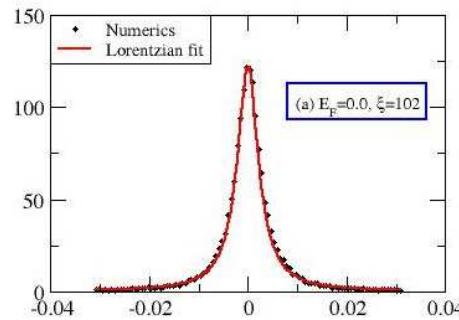
$$\Lambda = \frac{2\pi t}{\Delta_F}$$

S_0 = typical thermopower
Δ_F = mean level spacing near E_F

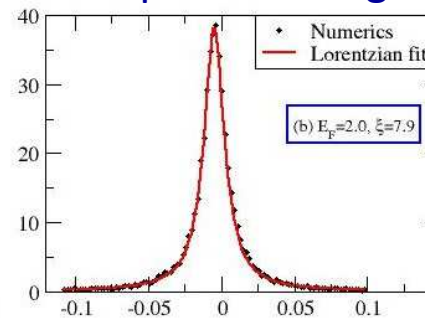Near the Edge $\longrightarrow$ Gauss distribution (characterized numerically)

$$P(S) = \frac{1}{\sqrt{2\pi}\lambda}\exp\left[-\frac{(S - S_0)^2}{2\lambda^2}\right]$$

$$\lambda \approx 0.6\frac{Wt\sqrt{N}}{(E_F - V_g)^2 - (2t + W/4)^2}$$

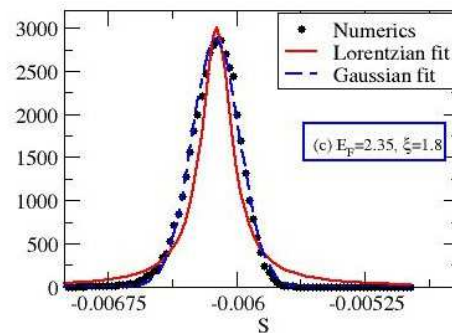1D nanowire with disorder W=1 → Spectrum edge at E=2.5t

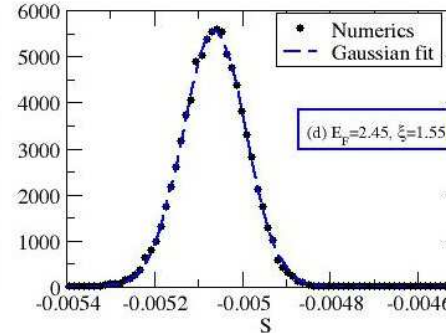$V_g = 0.0$ (a) $E_F=0.0$, $\xi=102$ (b) $E_F=2.0$, $\xi=7.9$ $V_g = 2.0$

$V_g = 2.35$ (c) $E_F=2.35$, $\xi=1.8$ (d) $E_F=2.45$, $\xi=1.55$ $V_g = 2.45$

[1] S. A. van Langen, P. G. Silvestrov, and C.W. J. Beenakker, Supperlattices Microstruct. 23, 691 (1998).
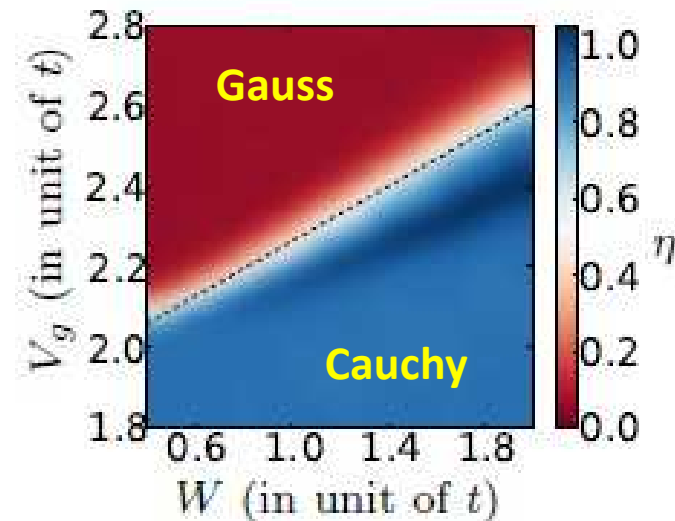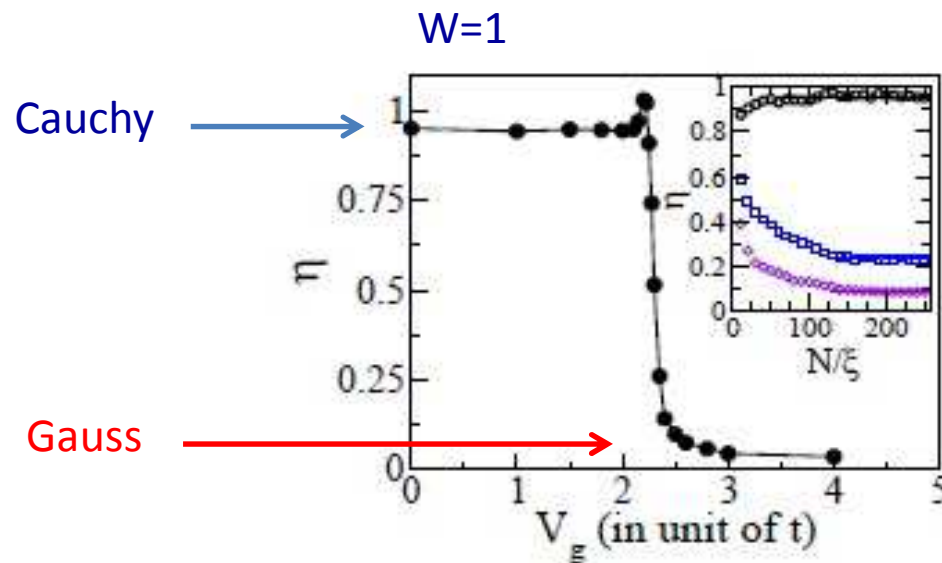[2] R.Bosisio, G. Fleury and J-L. Pichard, (2013)

$$\eta = \frac{\int dS\,|P(S) - P_G(S)|}{\int dS\,|P_L(S) - P_G(S)|}$$

Parameter which measures the "distance" between the observed numerical distribution and the best Lorentzian (P_L) and Gaussian (P_G) fits

- $\eta = 1$ if Cauchy distribution
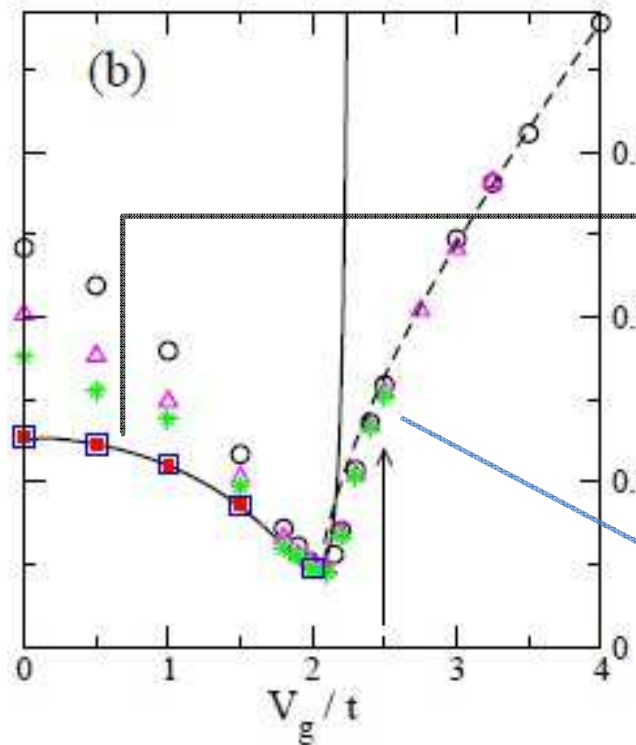- $\eta = 0$ if Gauss distribution

Edge: $V_G = 2{,}5$

[1] R.Bosisio, G. Fleury and J-L. Pichard,   arXiv:1310.4923v2 [cond-mat.mes-hall]

**Validity of the Sommerfeld expansion
leading to Mott formula for S**

Validity of Sommerfeld Expansion $\longrightarrow$ Wiedemann-Franz (WF) law, Mott formula

Range of validity of W-F law for $W = 1$ and $E_F = 0$ as a function of V_G

Sommerfeld temperature is
proportional to the mean energy
level spacing in the system:

$$k_B T_c \propto \Delta_F$$

Proportionality constant depends
on required precision

Result for the tunnel barrier:

$$N k_B T_c \propto t \sqrt{[(E_F - V_g)/(2t)]^2 - 1}$$

Estimation for Si nanowire: ~ 100 mK

TEP fluctuations at the spectrum edge of a chaotic scatterer

Low temperature elastic coherent thermoelectric transport

Delay-time and thermopower distributions at the spectrum edges of a chaotic scatterer

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Chaotic quantum dot
(Measure of the thermopower)

Thermopower of a Chaotic Quantum Dot

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(Received 10 November 1998)

The thermovoltage of a chaotic quantum dot is measured using a current heating technique. The fluctuations in the thermopower as a function of magnetic field and dot shape display a non-Gaussian distribution, in agreement with simulations using random matrix theory. We observe no contributions from weak localization or short trajectories in the thermopower. [S0031-9007(99)08804-3]

trical conductance of small—characteristic size ler than the electron mean free path—confined stems (usually denoted as quantum dots) shows ctuations. These fluctuations display correla- nction of an external parameter such as shape or ield, which can be described in a statistical man- electrons can, in fact, be viewed as billiard balls a classically chaotic system where many random at the system walls occur. Because of the wave- of the electrons, quantum mechanics is needed these systems fully. Chaos in quantum dots has tigated [1−3] in conductance measurements but s turns out to be difficult. So-called short tra-] and weak localization effects [1,5] add up to re of chaotic motion. Moreover, current heat- electrons in the dot appears to be unavoidable in e measurements. Electron heating effects in the

tion of parametric derivatives ($X = E, B$, shape,.. conductance of a QD is the subject of recent F vestigations [12,13]. The probability distributior thermopower is again expected to be non-Gaus chaotic conductors, exhibiting cusps at zero ampli nonexponential tails [13,14].

In this paper, we present magnetothermopow surements of a statistical ensemble of chaotic QI observed thermopower fluctuations show a non-C distribution. We present a numerical fit based c which describes the experimental data. We den that effects such as short trajectories, weak loca and dephasing are absent in thermopower measur

In Fig. 1a the measured device is shown schen A QD (lithographic size 800 nm × 700 nm) is statically defined (gates A, B, C, and D) in a s high-mobility 2-dimensional electron gas (2DE

Quantum dot with **2 opened channels for each QPCs**

Measure at 50 mK with a temperature difference of the same order
typical size 800 nm
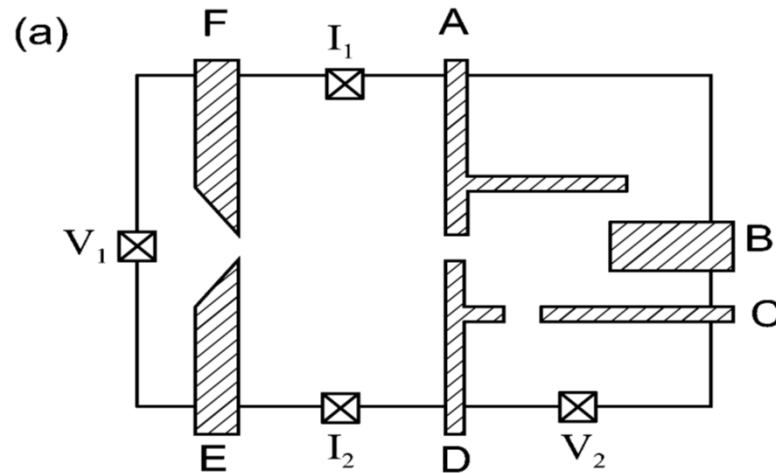
- (a) Schematic top view of the measured sample. The crosses denote the Ohmic contacts to the 2DEG; the hatched areas denote the gates. The heating current is applied between I1 and I2. The thermovoltage is measured between V1 and V2,

- The QD is defined by applying a negative voltage to gates A, B, C, and D.
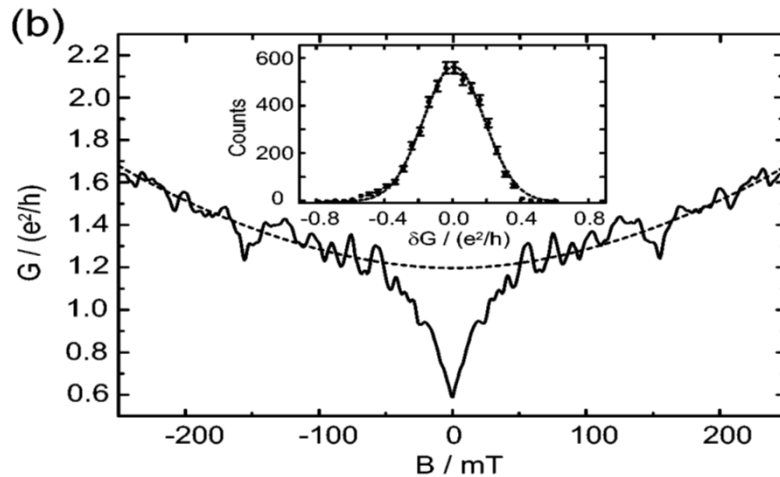
- (b) Magnetoconductance of the QD, averaged over
- a large number of different QD configurations. The influence
- of short trajectories is characterized by the dashed line. Inset:
- Conductance distribution for $|B| \$ 50$ mT

Thermovoltage
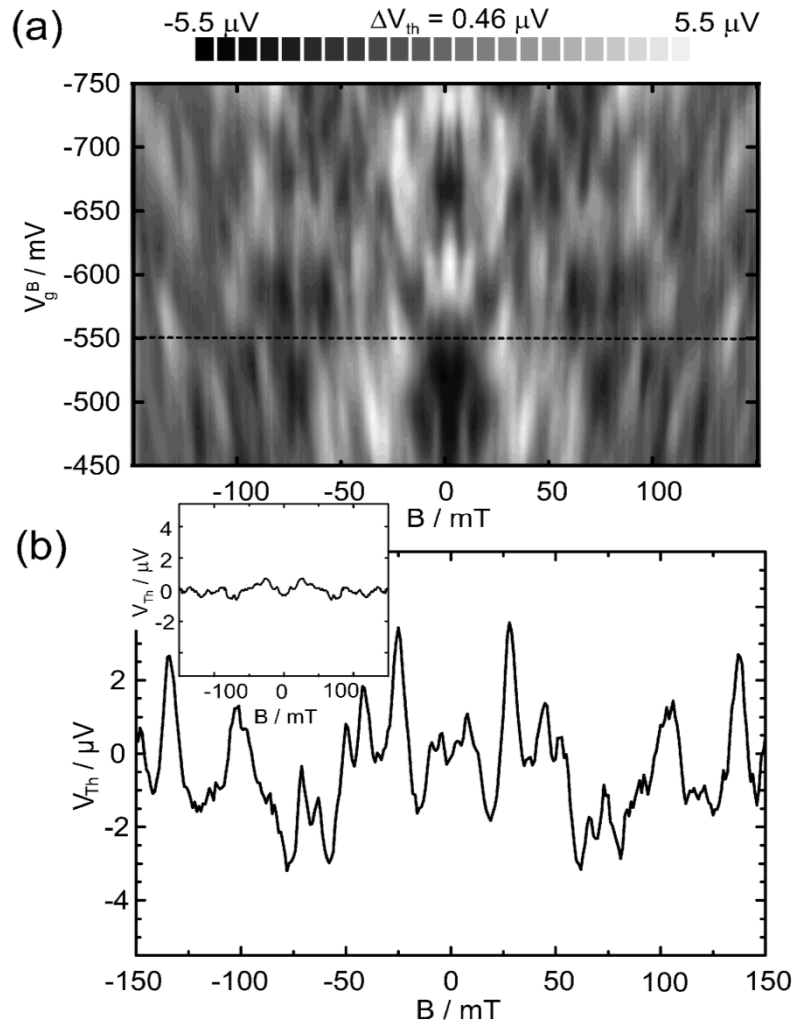
(a) The gate voltage is changed by a constant small amount of dV_B 10 mV for each magnetic field sweep. The light areas denote a large (maximum $5.5\,mV$) positive thermovoltage, and the dark areas a large (maximum $25.5\,mV$)negative thermovoltage.

(b) Individual thermovoltage trace
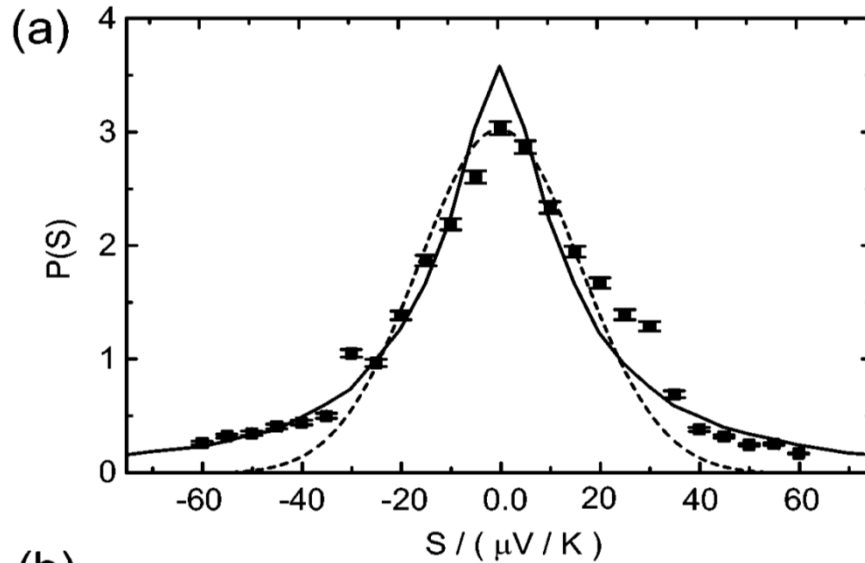
- **(a) Thermopower distribution in the presence of TRS ($B < 40$ mT).**
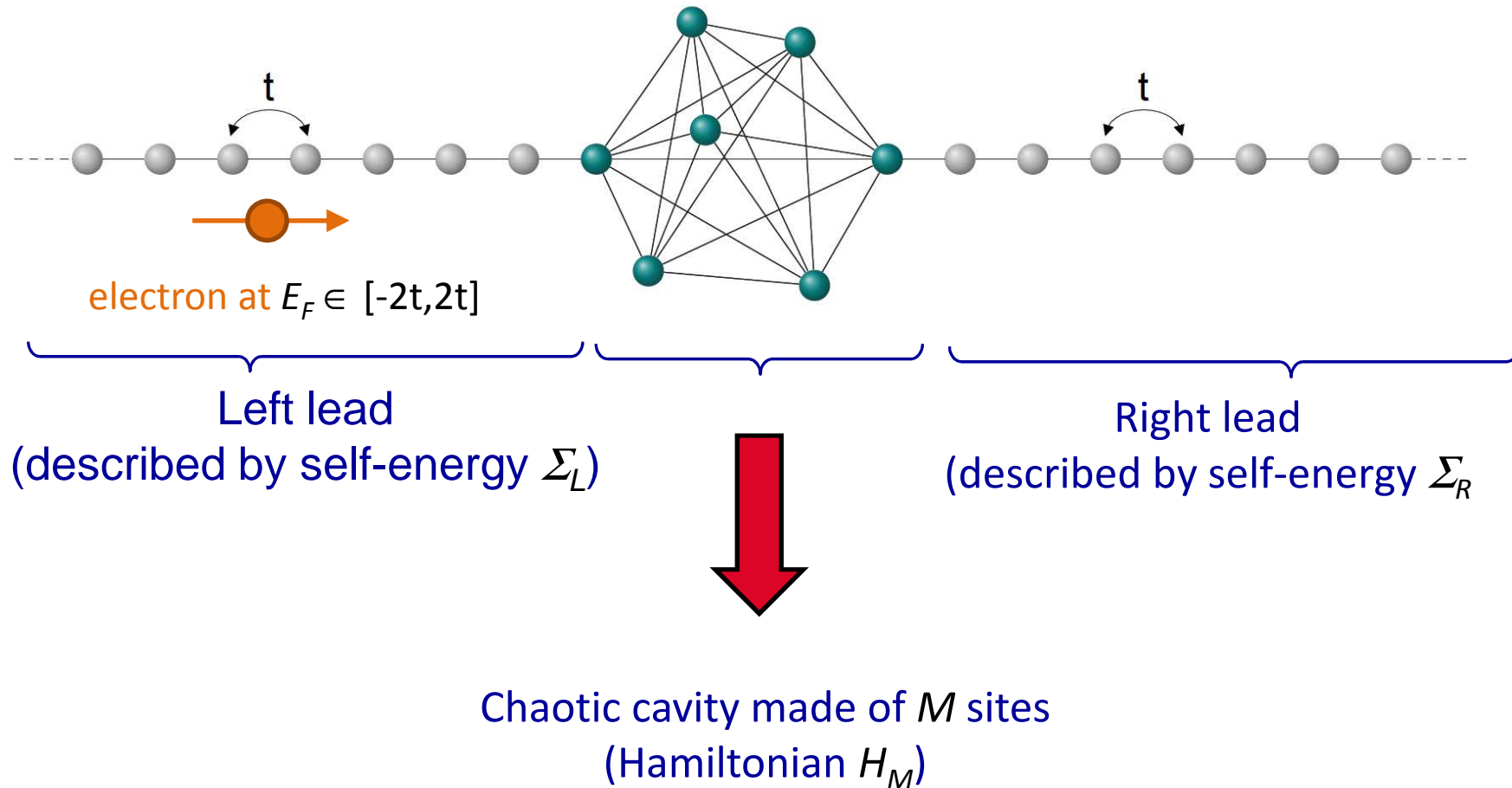 Experimental results (dots),
  simulation results (solid line),
  Gaussian fit (dashed line).
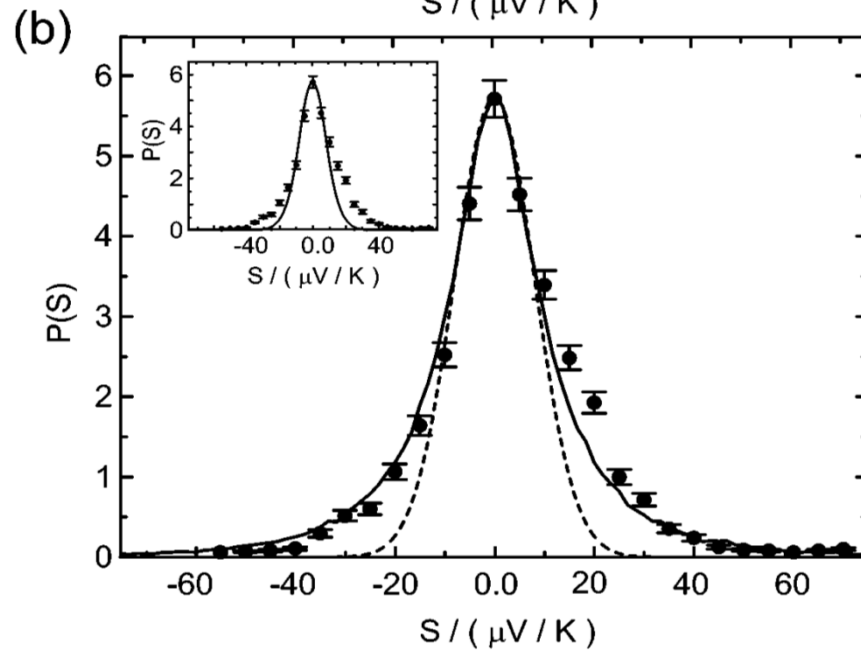
- **(b) Thermopower distribution for broken TRS ($B > 50$ mT).**

  Experimental results (dots),
  simulation results (solid line),
  and Gaussian fit (dashed line).

t

t

electron at $E_F \in$ [-2t,2t]

Left lead
(described by self-energy Σ_L)

Right lead
(described by self-energy Σ_R)

Chaotic cavity made of M sites
(Hamiltonian H_M)

Exact relation between scattering matrix _S_ and Hamiltonian H_M (see Datta) :

$$S(E) = -\mathbb{I}_2 + i\tau^\dagger\sqrt{\Gamma}\frac{1}{E-H_M-\Sigma_L-\Sigma_R}\sqrt{\Gamma}\,\tau \quad \text{with} \quad \Gamma = Im(\Sigma_L) = Im(\Sigma_R)$$

With energy-dependent self-energies $\Sigma_L(E)$ and $\Sigma_R(E)$

Chaotic Scattering: $\quad S \in COE \leftrightarrow \begin{cases} P(dS) = \rho(S).\mu(dS) \\ \rho(S) = constant \end{cases}$

Chaotic scattering at E_F if Hamiltonian $\quad H_M \in$ Cauchy ensemble
with adjusted parameters for the considered size M and Fermi energy E_F

Cauchy Ensemble

Notation $H_M \in C(M, \varepsilon, \Gamma)$ $\quad$ *Center ε and Width* $\quad \Gamma$

$$P(dH_M) = \rho(H_M)\mu(dH_M)$$

$$\rho(H_M) \propto \det\left([H_M - \epsilon \mathbf{1}_M]^2 - [\Gamma^2 \mathbf{1}_M]\right)^{-\frac{M+1}{2}}$$

- **For $M = 2$** $\qquad$ $S \in COE \leftrightarrow H_2 \in C(2, \epsilon = \frac{E_F}{2}, \Gamma = \Gamma_F)$

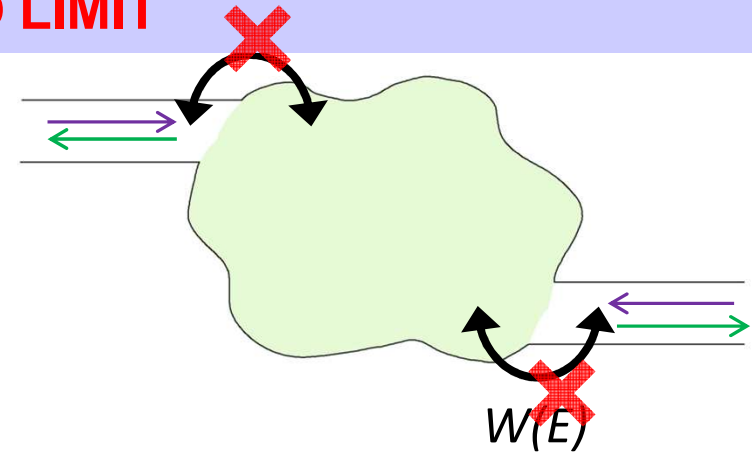- **For M > 2** $\qquad$ $H_M \in C\left(M, \varepsilon = \frac{E_F}{2}, \Gamma = \Gamma_F\right) \to S \in COE$

Two leads with N modes   **THE WIDE-BAND LIMIT**

$$S(E) = I - 2i\pi W^\dagger \frac{1}{E - H_M + i\pi WW^\dagger} W$$

Hypothesis : $Re[\Sigma]$ neglected and
$Im[\Sigma] = W$ is energy-independent
Correct in the wide-band limit (ie in the bulk)

W(E)

Scattering approach
$S(E_F)$ in the circular ensemble

$\longleftrightarrow$

Hamiltonian approach
H_M in the **Gaussian Ensemble**

$W(\times)$
$M \to \infty$

N modes

$N = 1$

Derivation of delay-time distrib. $P(\tau_1, \tau_2, ...)$
where τ_i = eigenvalues of $Q = -i\hbar S^\dagger \frac{dS}{dE}$
$\Rightarrow$ Laguerre ensemble for $1/\tau_i$

Thermopower
distribution
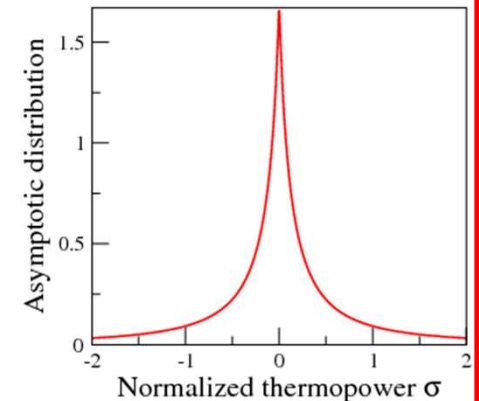
Time-delay distribution : Brouwer, Frahm and Beenakker, *PRL* **78** 4737 (1997)
Seebeck distrib. for N=1 : Van Langen, Silvestrov & Beenakker, *Superlatt. MicroStr.* **23** 691 (1998)

Thermopower of **Single-Channel** Disordered and **Chaotic** Conductors

S. A. van Langen, P. G. Silvestrov and C. W. J. Beenakker   (**S is a 2x2 COE matrix**)

$$\sigma_k = \frac{\Delta}{2\pi} \frac{\partial \log \tau(E)}{\partial E}\bigg]_{E=E_F} \quad (where\ \Delta\ \text{is the level spacing})$$

$$P(\sigma_k) = \int dc\, P(c) \iint d\tau_1 d\tau_2 P(\tau_1,\tau_2) \int dT p(T)\, (\tau_1 + \tau_2)\delta(\sigma_k - a(c,\tau_1,\tau_2,T)),$$

$$a(c,\tau_1,\tau_2,T) = \frac{\Delta}{2\pi\hbar} c(\tau_1 - \tau_2)\sqrt{1/T - 1}$$

- p (T)=1/(2√T) and $P(c) \propto \dfrac{1}{\sqrt{1-c^2}}$

- $P(\tau_1,\tau_2) \propto |\tau_1 - \tau_2| \dfrac{1}{\tau_1 \tau_2} e^{-\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)\frac{\pi\hbar}{\Delta}}$ **RMT Laguerre ensemble**

Hartree Correction Factor $\tau_1 + \tau_2$

**Quantum Mechanical Time-Delay Matrix in Chaotic Scattering**

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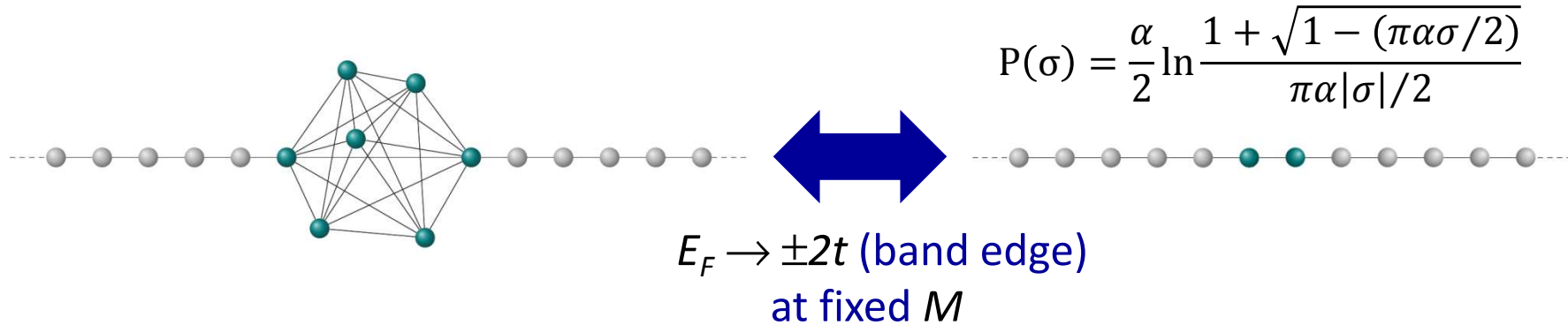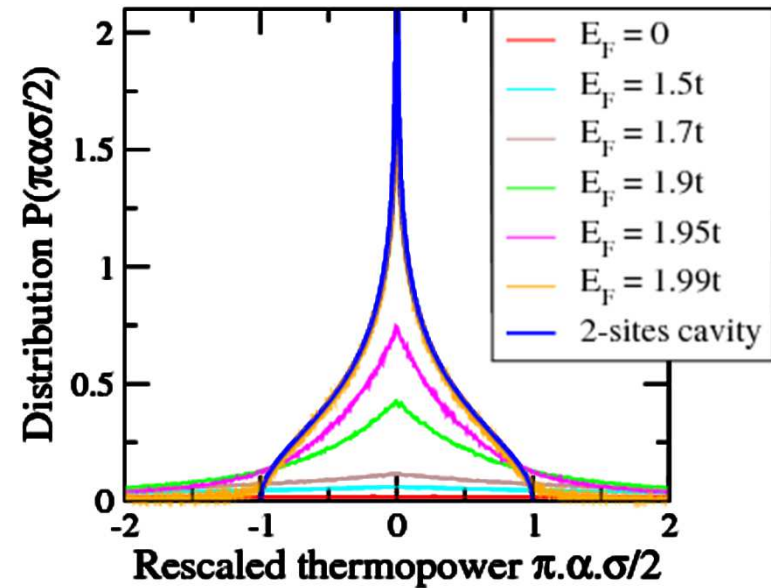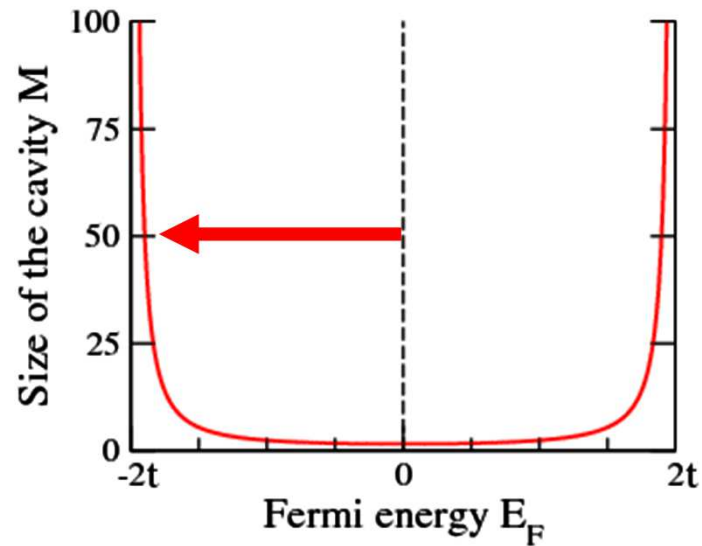
(Received 21 March 1997)

We calculate the probability distribution of the matrix Q 2i ¯ hS21SyE for a chaotic system with scattering matrix S at energy E. The eigenvalues tj of Q are the so-called proper delay times, introduced by Wigner and Smith to describe the time dependence of a scattering process. The distribution of the inverse delay times turns out to be given by the Laguerre ensemble from random matrix theory.
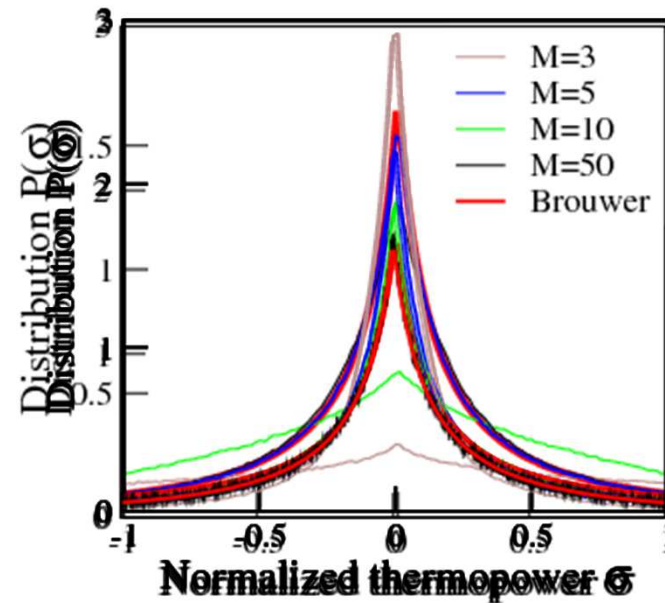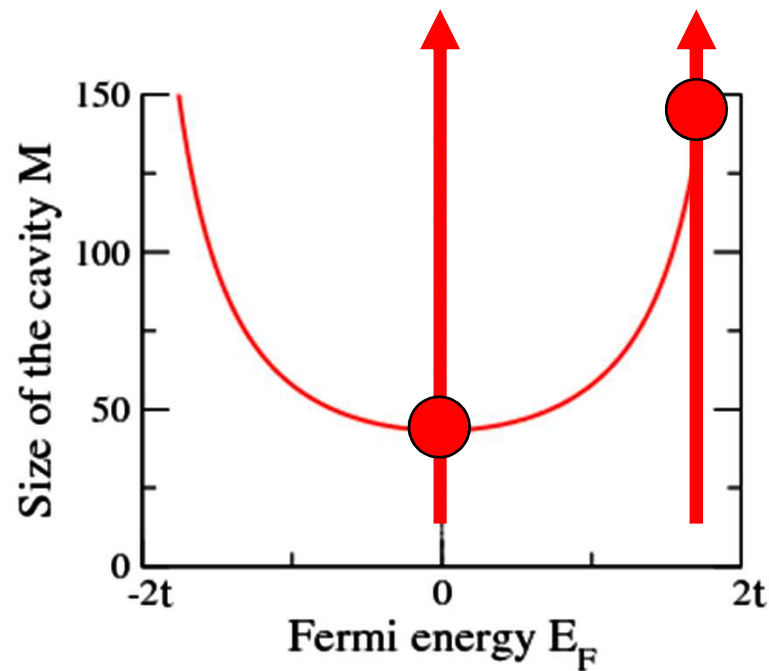
At fixed energy E_F and large size M, we recover previous result valid in the bulk

$$P(\sigma) = \frac{\alpha}{2} \ln \frac{1 + \sqrt{1 - (\pi\alpha\sigma/2)}}{\pi\alpha|\sigma|/2}$$

$E_F \rightarrow \pm 2t$ (band edge)
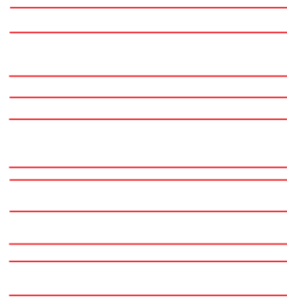at fixed M

Close to the band edge, the thermopower distribution is **very different** from the one in the bulk. It is given by the analytical result derived for a 2-sites cavity

BULK STATISTICS versus EDGE STATISTICS
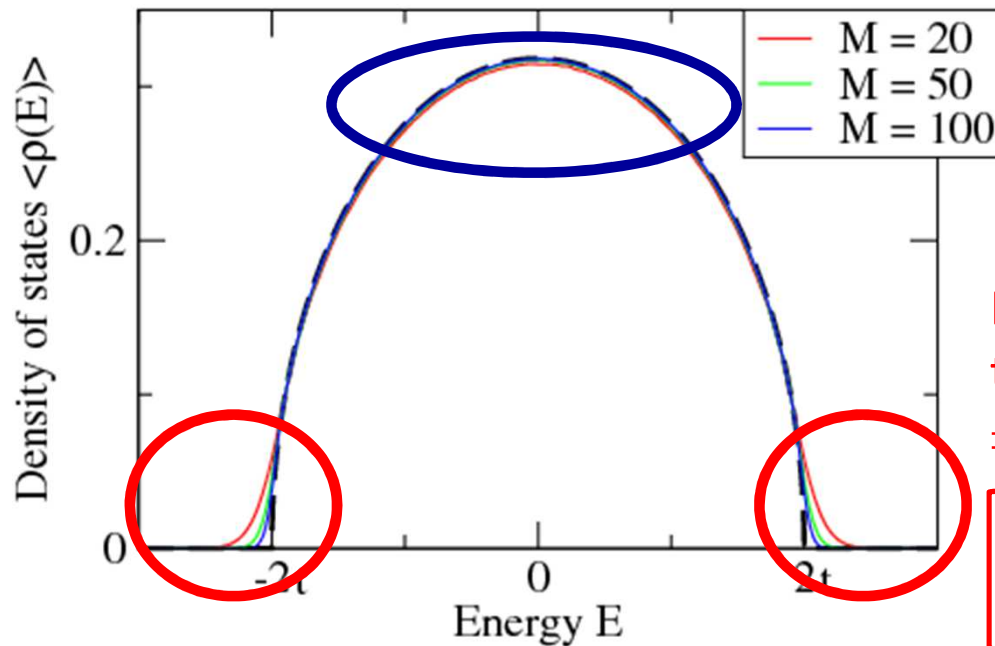
Closed chaotic cavity

Hamiltonian H_M in Gaussian ensemble :

$$P(H_M) \propto \exp(-tr\, \frac{MH_M^2}{4t^2})$$

Oriol Bohigas et al (1983)
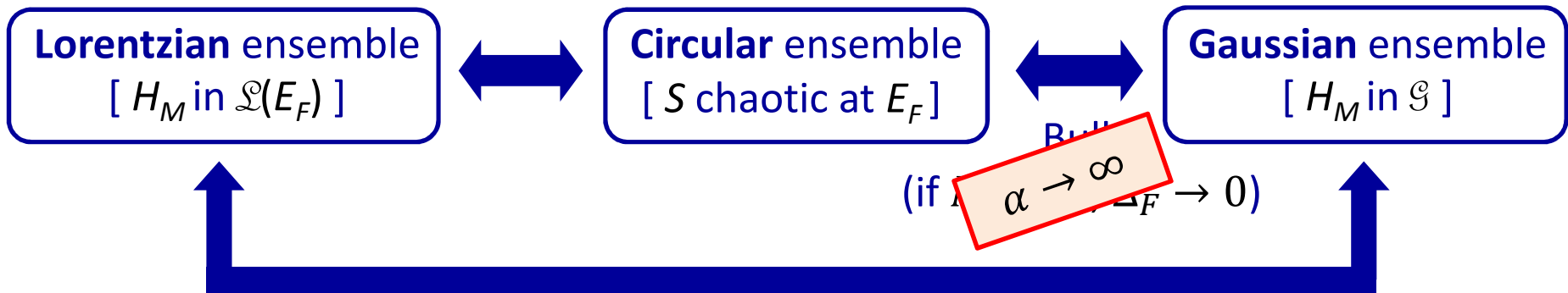
Universal statistics of eigenvalues in the **bulk**

[• Wigner nearest-neighbor spacing distrib.
• Sine-Kernel for the 2-points correlation fct]

M = 20
M = 50
M = 100

Different universality class at the **edges** of the spectrum
$\Rightarrow$ Tracy-Widom

[• TW level spacing distribution
• Airy-Kernel for the 2-points correlation fct]

Density of states $\langle \rho(E) \rangle$

Energy E

Tracy and Widom, *Comm. Math. Phys* **159**, 151 (1994); Forrester, *Nucl. Phys. B* **402**, 709 (1993)

CONNEXION TO TRACY-WIDOM PARAMETRIZATION α

Lorentzian ensemble
[H_M in $\mathcal{L}(E_F)$]

$\longleftrightarrow$

Circular ensemble
[S chaotic at E_F]

$\longleftrightarrow$

Gaussian ensemble
[H_M in $\mathcal{G}$]

(if $\alpha \to \infty$ $\Delta_F \to 0$)

$\alpha \to \infty$

If $M \to \infty$ with the same Δ_F
for t $\alpha \to \infty$ ensembles

$$\alpha = \frac{1}{\pi M}\left(\frac{E \pm 2t}{2t}\right)^{3/2}$$

Tracy-Widom rescaling
with the parameter α

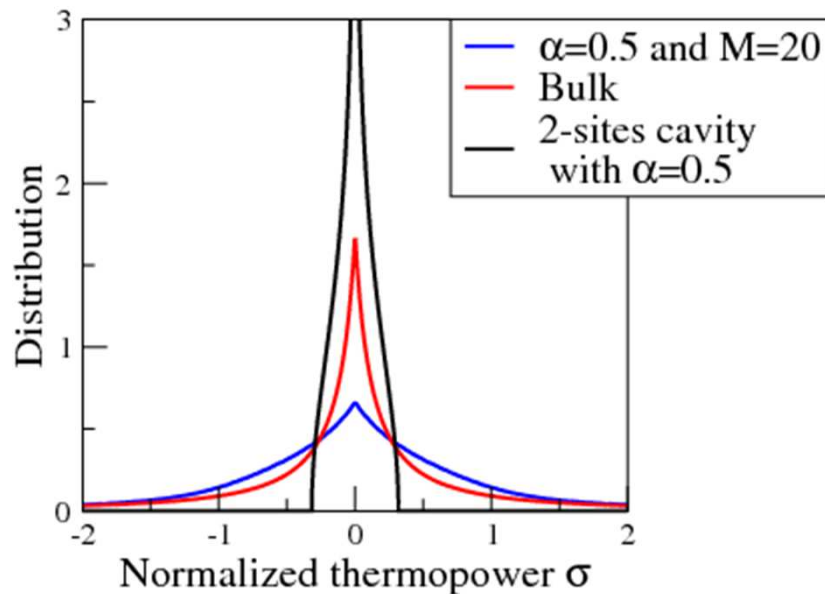$$\langle \rho(\alpha) \rangle \propto \frac{1}{\alpha^{1/6}}\exp(-K\alpha)$$

Lorentzian ensemble

Density of states at E_F $\langle\rho_F(E)\rangle$

0.4

0.2

0

$E_F = 0$
$E_F = 1$
$E_F = -1.5$

-2   0   2
Energy E (in unit of t)

Gaussian e...

Density of states $\langle\rho(E)\rangle$

0.4

0.2

0

-2   0   2
Energy E (in unit of t)

0.1

0

1.5   2   2.5

*Brouwer, PhD Thesis (1997); Vavilov et al, PRL **86**, 874 (2001)*
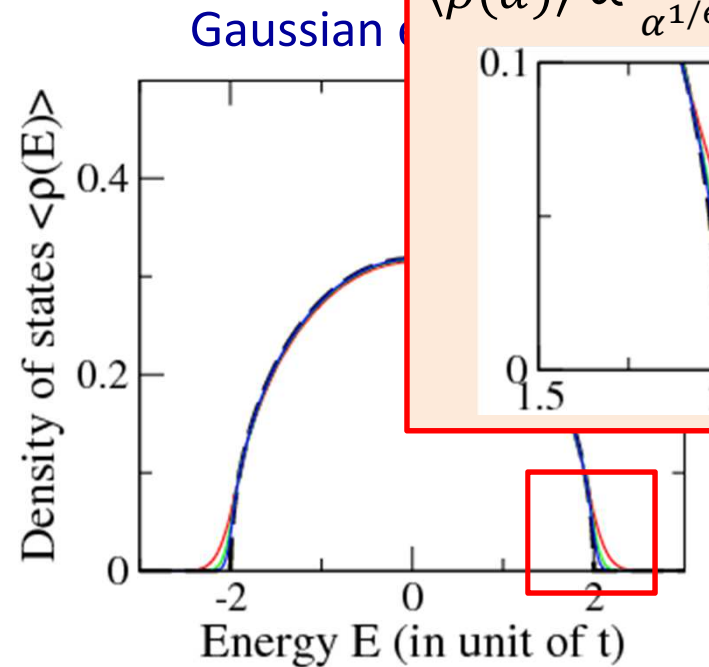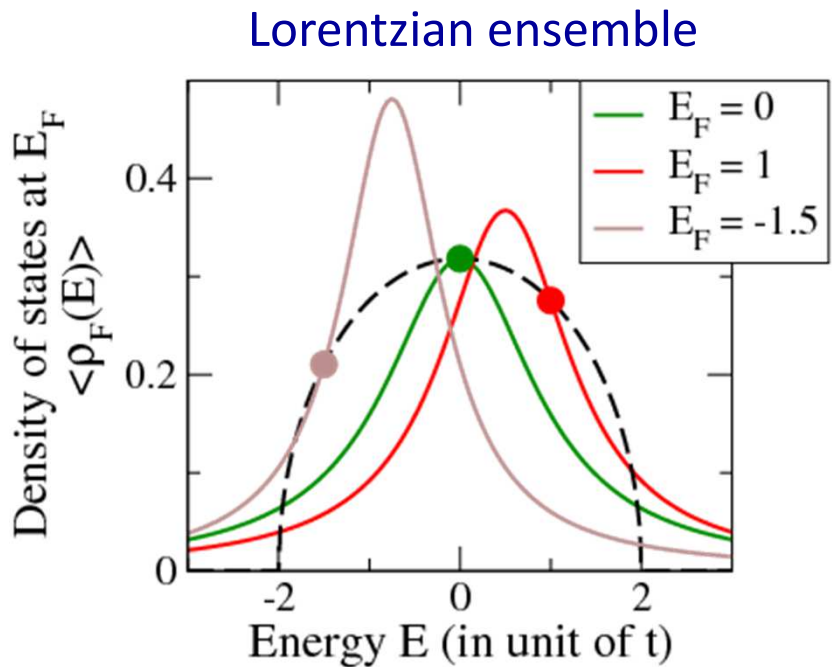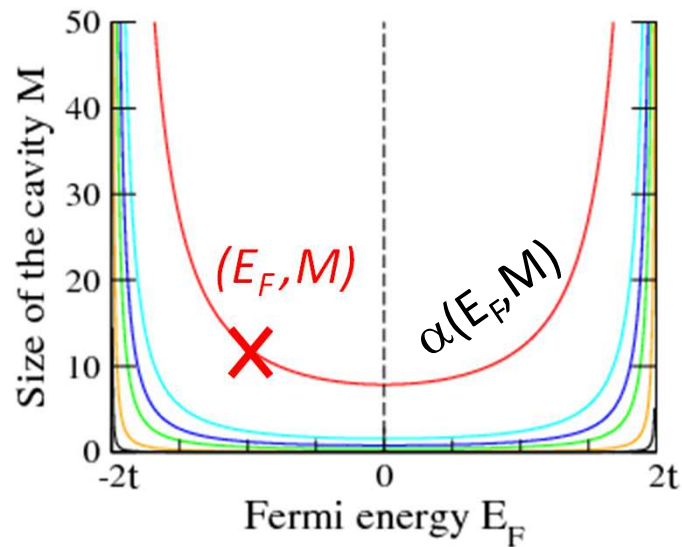
$$\alpha(E_F, M) = \frac{\Gamma_F^2}{\Delta_F t} = \frac{1}{8\pi} \left| \frac{E_F - 2t}{tM^{-1/3}} \right|^{2/3} \left| \frac{E_F + 2t}{tM^{-1/3}} \right|^{2/3}$$
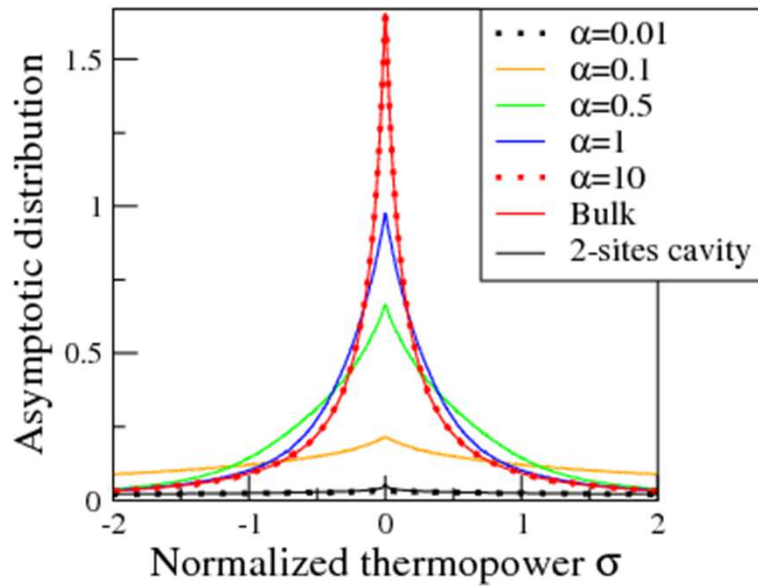
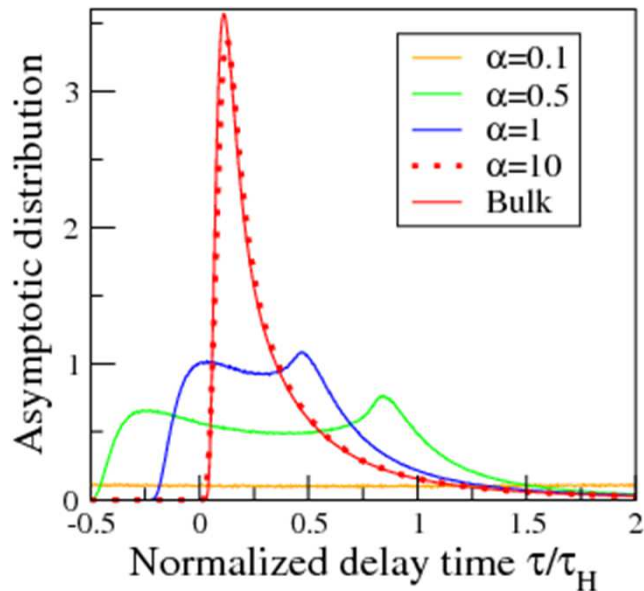Measures the energy distance to the band edge in unit of $tM^{-1/3}$

TRACY-WIDOM SCALING

$$\eta = \frac{\int d\sigma \, |P(\sigma) - P_{bulk}(\sigma)|}{\int d\sigma \, |P_{M=2}(\sigma, \alpha) - P_{bulk}(\sigma)|}$$

DESCRIPTION OF THE BULK-EDGE CROSSOVER

- Infinity of asympotic thermopower distributions, indexed by $\alpha \in [0,\infty]$

- The two extreme limits are analytically understood : $\alpha \to \infty$ (bulk)
  $\alpha \to 0$ (2-sites cavity)

- The parameter α appears to be the Tracy-Widom parameter

- Same conclusions for the delay-time distribution

Summary for the TEP fluctuations of a chaotic scatterer

- Mesoscopic TEP Fluctuations of Chaotic scatterers exhibit universal forms which can be described by the theory of the random matrices

- 1d model with chaotic scattering is characterized by a one parameter family of universal distributions

- **Bulk Limit $\alpha \rightarrow \infty$;**

- **Edge Limit $\alpha \rightarrow 0$**

- Same energy rescaling (**parameter α**) leading to the Tracy-Widom asymptotic universal form for the energy level distribution at the band edges of Gaussian ensembles of random Hermitian matrices.

Variable Range Hopping (VRH) Transport in gated disordered NWS

Hopping between pairs of localized states <u>mediated by phonons</u>
Conductance: competition between **<u>tunneling</u>** and <u>activated</u> processes

$$G_{ij} \sim e^{-2|x_i - x_j|/\xi} \, e^{-(|E_i - \mu| + |E_j - \mu| + |E_i - E_j|)/2k_B T}$$
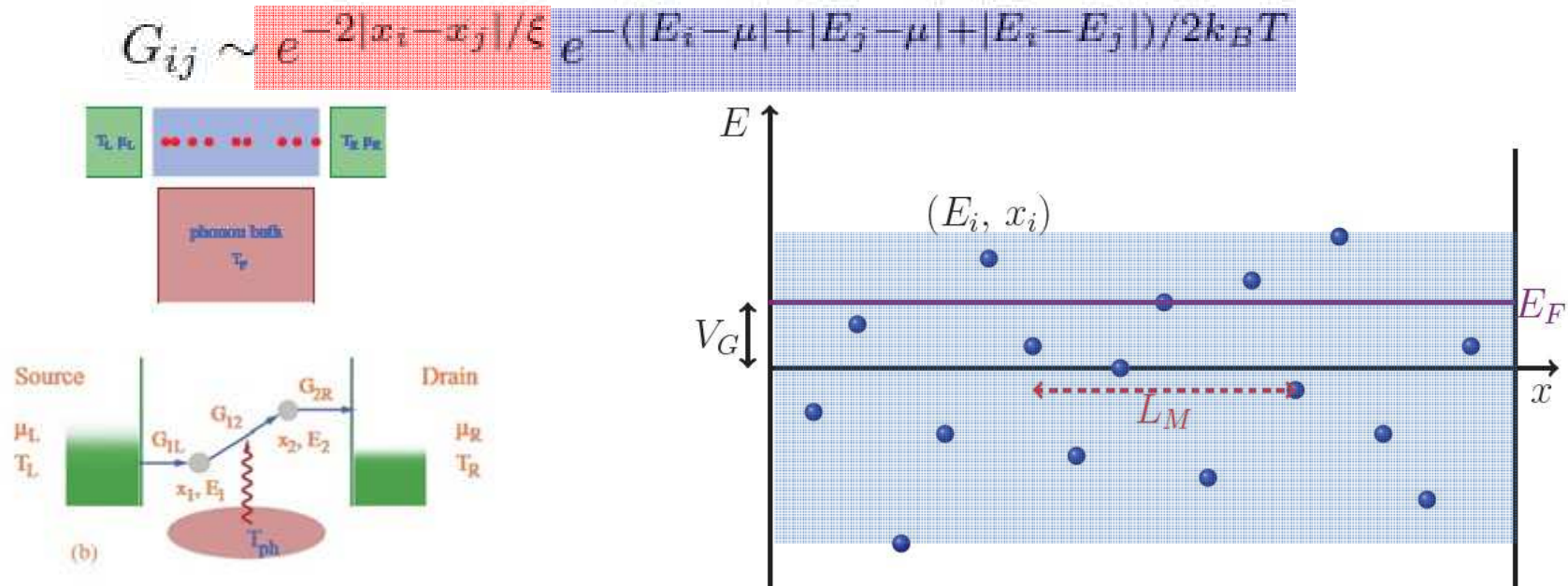
Maximization of the conductance yields the scale of typical hop:

$$L_M \simeq \left(\frac{\xi}{2\nu T}\right)^{1/2}$$ Mott's Hopping length

ξ = localization length
ν = density of states / volume

[1] J-H. Jiang, O. Entin-Wohlman and Y. Imry, Phys. Rev. B 87, 205420 (2013).
[2] R.Bosisio, G. Fleury and J-L. Pichard, (2013)

TEMPERATURE SCALES

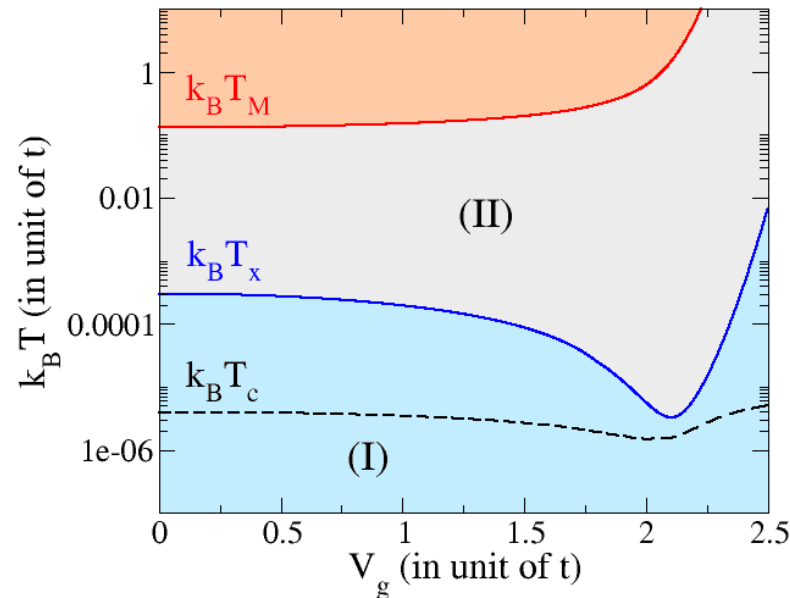$$L_M \simeq \left(\frac{\xi}{2\nu T}\right)^{1/2}$$

Low T: $L_M \gg L$ → elastic transport

Increasing T: $L_M \sim L$ → onset of inelastic processes $\longrightarrow$ $T_x \sim \dfrac{\xi}{2\nu L^2}$

T

Increasing T: $L_M \sim \xi$ → simple activated transport $\longrightarrow$ $T_M \simeq (\xi^d \nu)^{-1}$

VRH Typical Conductance :
(Mott's picture)

$$G(T) \sim exp\left\{-\left(\frac{T_M}{T}\right)^{1/(d+1)}\right\}$$

d=1 (dimensionality)

What about the thermopower?

Conductance: Kurkijärvi (1973), Lee (1984), Fogler (2005)
Thermopower: Zvyagin (~80's)

MILLER-ABRAHAMS RESISTOR NETWORK

1. Transition rates Between localized states

[Inelastic transition rates (Fermi Golden Rule)]

$$\Gamma_{ij} = \gamma_{ij}\, f_i(1 - f_j)\left[N_{BE}(\varepsilon_i - \varepsilon_j) + \theta(\epsilon_i - \epsilon_j)\right]$$

$$\gamma_{ij} = \alpha_{e-ph}.\, e^{-|x_i - x_j|/\xi}$$

Between lead and localized states

[Elastic tunneling rates]

$$\Gamma_{Li} = \gamma_{Li}\, f_i(1 - f_j) \qquad \gamma_{ij} = e^{-|x_i - x_j|/\xi}$$

2. Conductances
$$G_{ij} = \frac{e^2}{k_B T}\Gamma_{ij}$$

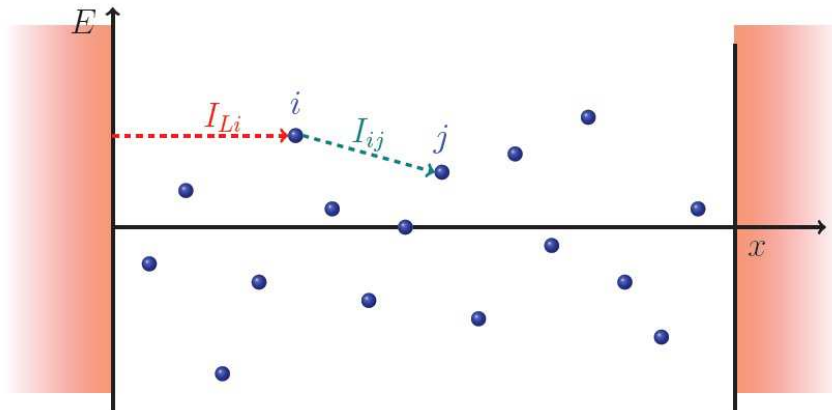**3. Local chemical potential (out of equilibrium transport)**

$$f_i(\mu) \rightarrow f_i(\mu + \delta\mu_i)$$

4. Current
$$I_{ij} = G_{ij}\frac{\delta\mu_i - \delta\mu_j}{e}$$

RANDOM RESISTOR NETWORK [1,2]

energy levels localized at (random) positions x_i

I_{ij}: hopping current between sites i and j

$I_{iL(R)}$: tunneling current between site i and leads

$f_i = f_i^0 + \delta f_i$ "local" FD distribution

Current conservation at node i :

$$\left(\sum_{j \neq i} I_{ij}\right) + I_{iL} + I_{iR} = 0$$

Electric current:

$$I_L^e = \sum_i I_{iL} = -\sum_i I_{iR}$$

Heat current:

$$I_{L(R)}^Q = \sum_i \left(\frac{E_i - \mu_{L(R)}}{e}\right) I_{iL(R)}$$

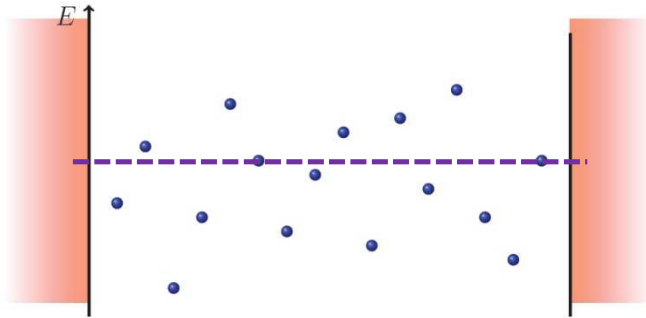Peltier: $\Pi = \dfrac{I_L^Q}{I_L^e}$

Thermopower:
(from Onsager relations) $S = \dfrac{\Pi}{T}$

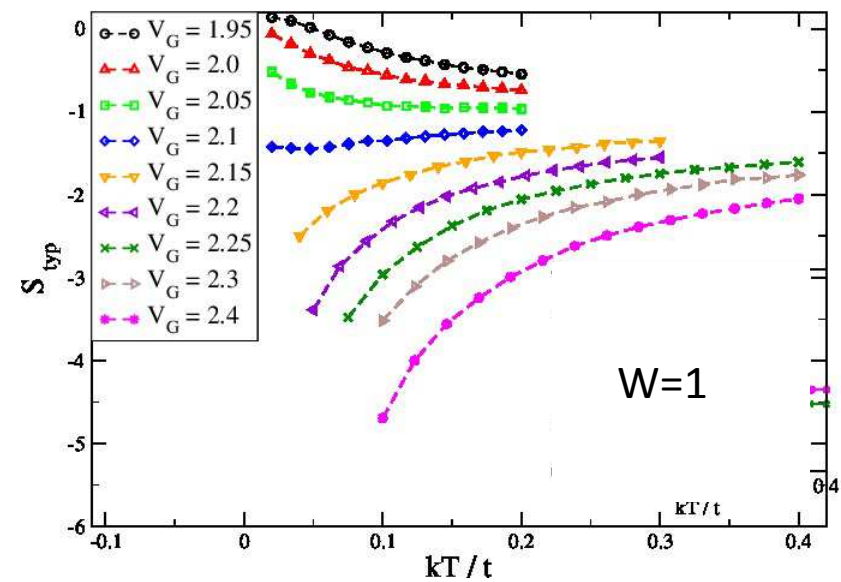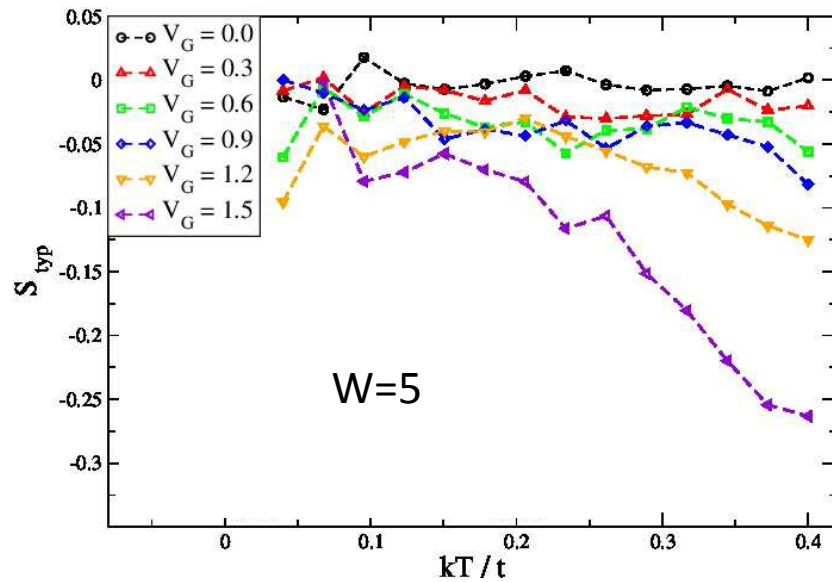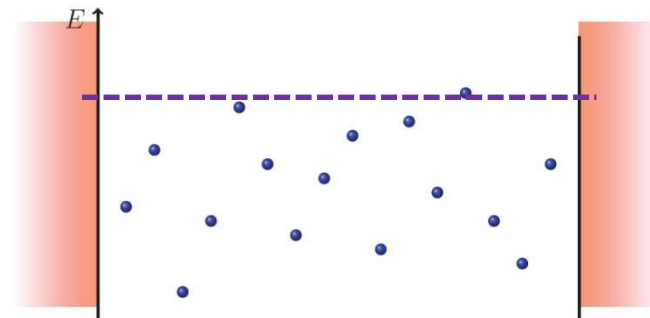[1] A. Miller and E. Abrahams, Phys. Rev. 120, 745 (1960)
[2] J-H. Jiang, O. Entin-Wohlman and Y. Imry, Phys. Rev. B 87, 205420 (2013).

EFFECT OF V_g ON TYPICAL THERMOPOWER IN VRH

$$\text{EFFECT OF } V_g \text{ ON TYPICAL THERMOPOWER IN VRH}$$

[1] R.Bosisio, G. Fleury and J-L. Pichard, (2013)

http://arxiv.org/pdf/1307.0249v1.pdf

Electric Field Effect Thermoelectric Transport in Individual Silicon and Germanium/Silicon Nanowires

Yuri M. Brovman1, Joshua P. Small1, Yongjie Hu2, Ying Fang2, Charles M. Lieber2, and Philip Kim1

1 Department of Applied Physics and Applied Mathematics and Department of Physics,

Columbia University, New York, New York, 10027, USA and

2 Department of Chemistry and Chemical Biology,

Harvard University, Cambridge, MA 02139, USA

We have simultaneously measured conductance and thermoelectric power (TEP) of individual silicon and germanium/silicon core/shell nanowires in the field effect transistor device configuration. As the applied gate voltage changes, the TEP shows distinctly different behaviors while the electrical conductance exhibits the turn-off, subthreshold, and saturation regimes respectively. At room temperature, peak TEP value of ~300μ V/K is observed in the subthreshold regime of the Si devices.

Substantially large peak TEP values are observed in the subthreshold regime of the Si and Ge/Si devices, **indicating largely enhanced TEP near the band edge of semiconducting NWs.**
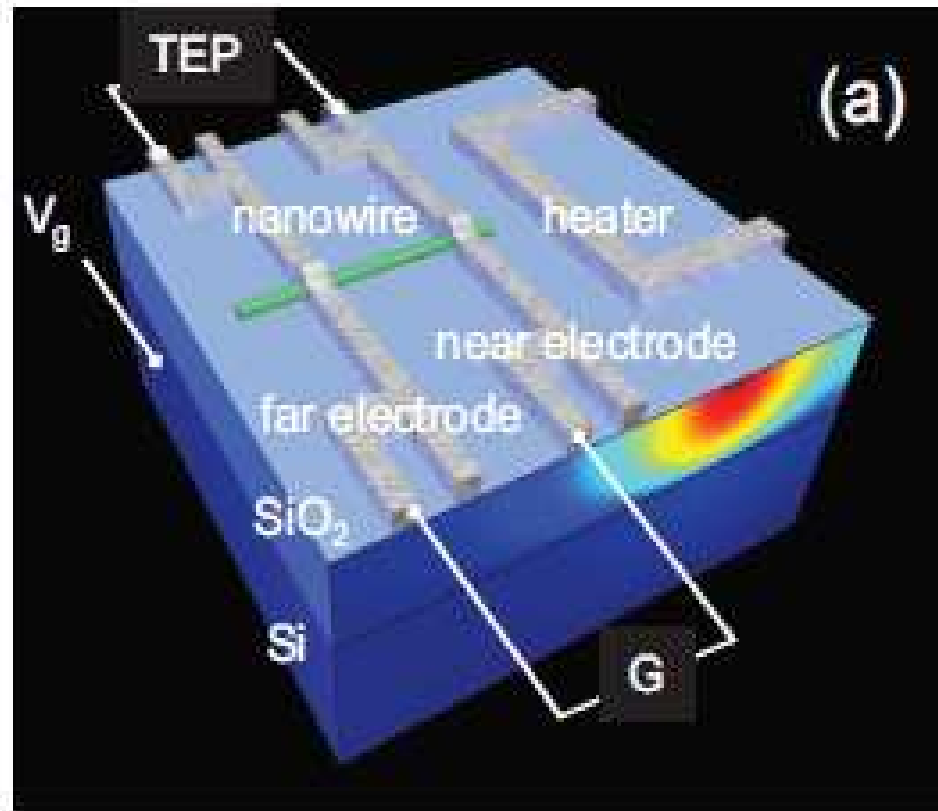
Schematic diagram of the simultaneous measurement technique of conductance and thermopower on individual nanowires. The finite element simulation shows a temperature profile, with red being the hottest and blue being the bath temperature, of the cross section of the substrate.
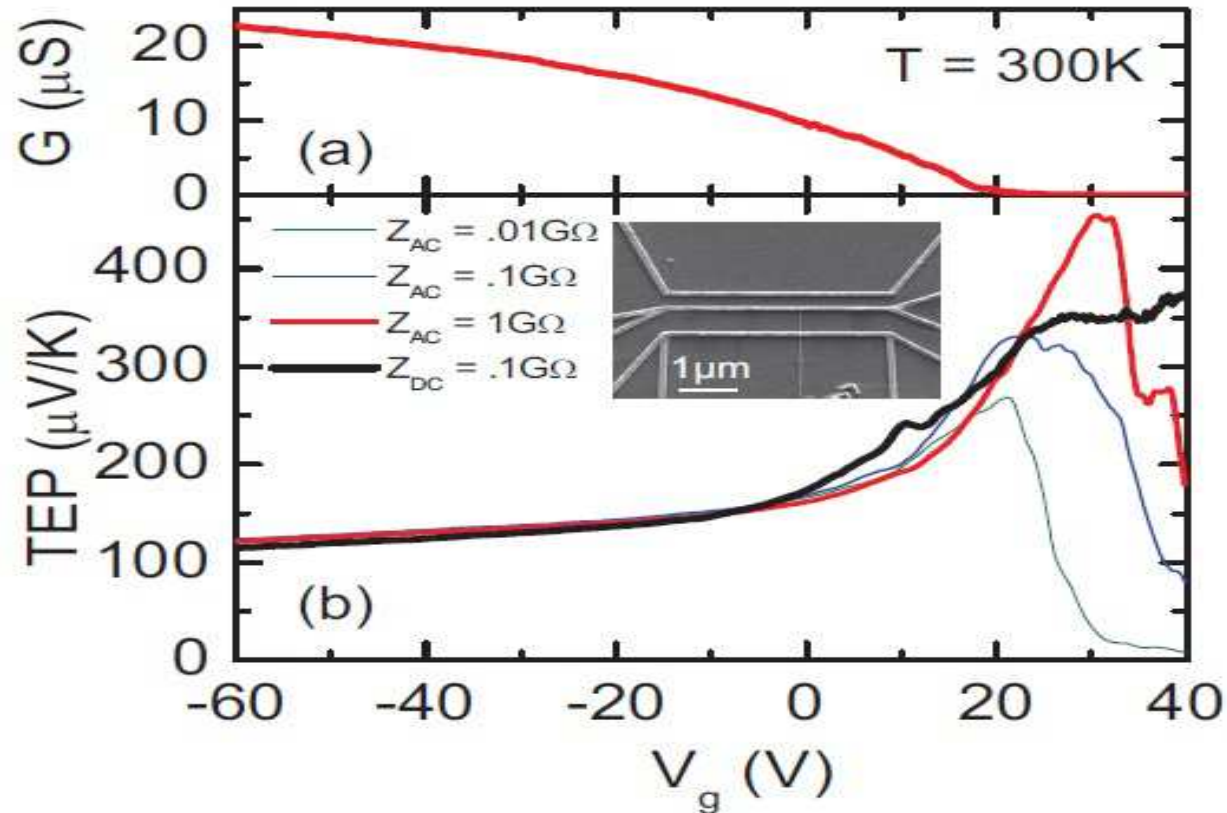
Conductance (a) and thermopower (b) of a Ge/Si nanowire as a function of gate voltage taken at T = 300 K. The inset in (b) shows a typical SEM image of a 12 nm Ge/Si device. Large input impedance becomes important when measuring TEP near the band edge of a semiconductor, as the FET device turns off.

THANK YOU

Thermometer for the 2D Electron Gas using 1D Thermopower

N. J. Appleyard, J. T. Nicholls, M. Y. Simmons, W. R. Tribe, and M. Pepper
Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, United Kingdom

$$S = \frac{\Delta V}{T_e - T_l}\Big]_{I=0} = \frac{(\pi k_B)^2}{3e}(T_e + T_l)\frac{\partial(\ln G)}{\partial \mu}$$
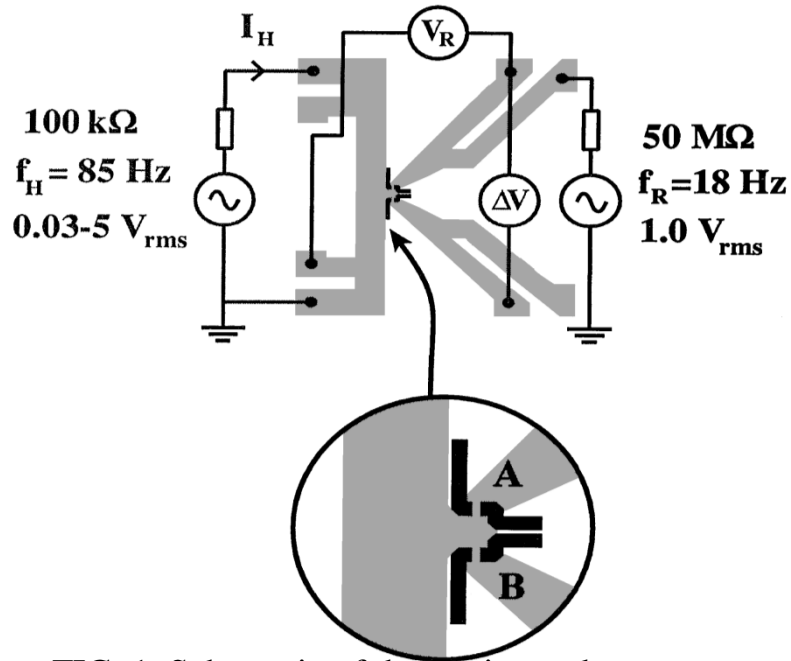
FIG. 1. Schematic of the device and measurement circuit. The etched mesa, shown in grey, consists of a heating channel and two voltage probes, where the two 1D constrictions are defined. The four-terminal resistance R is measured simultaneously with the thermopower S, but at a different frequency. Magnified view: The two pairs of split gatesdefining the constrictions A and B are shown in solid black.
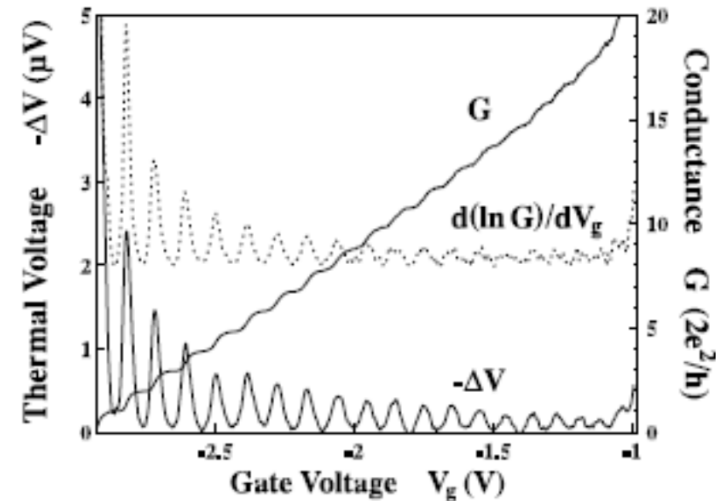
FIG. 2. Experimental traces of the conductance G and the thermopower voltage from constriction A, using a heating current of **1.5 mA** at a lattice temperature of **305 mK,** so that $T_e \sim$ **600 mK**. The dashed line shows the predicted thermopower signal from the Mott relation [Eq. (1)].