Strongly coupled phonon fluid and Goldstone modes in an anharmonic quantum solid: Transport and chaos

Evyatar Tulipman
Weizmann Institute of Science, Israel

Based on: ET and Erez Berg PRR 2020, PRB 2021

Motivation

Theoretical side

MB quantum chaos "dictates" **transport properties** of strongly correlated systems?

Planckian bound on transport

$$\tau_{\mathrm{tr}} \geq s \frac{\hbar}{k_B T}$$

(Sachdev, Zaanen)

?

$\longleftrightarrow$

Bound on chaos

$$\tau_L \geq \frac{\hbar}{2\pi k_B T}$$

(Maldacena-Shenker-Stanford)

Motivation

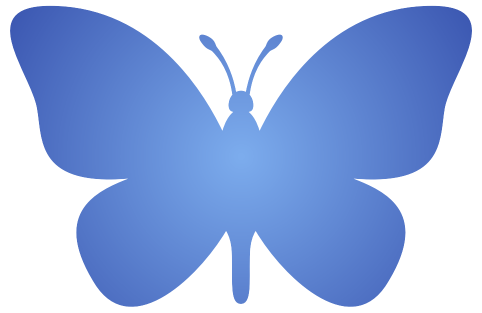
Theoretical side

Conjecture: thermal and chaos diffusivities are universally related (Hartnoll, Blake)

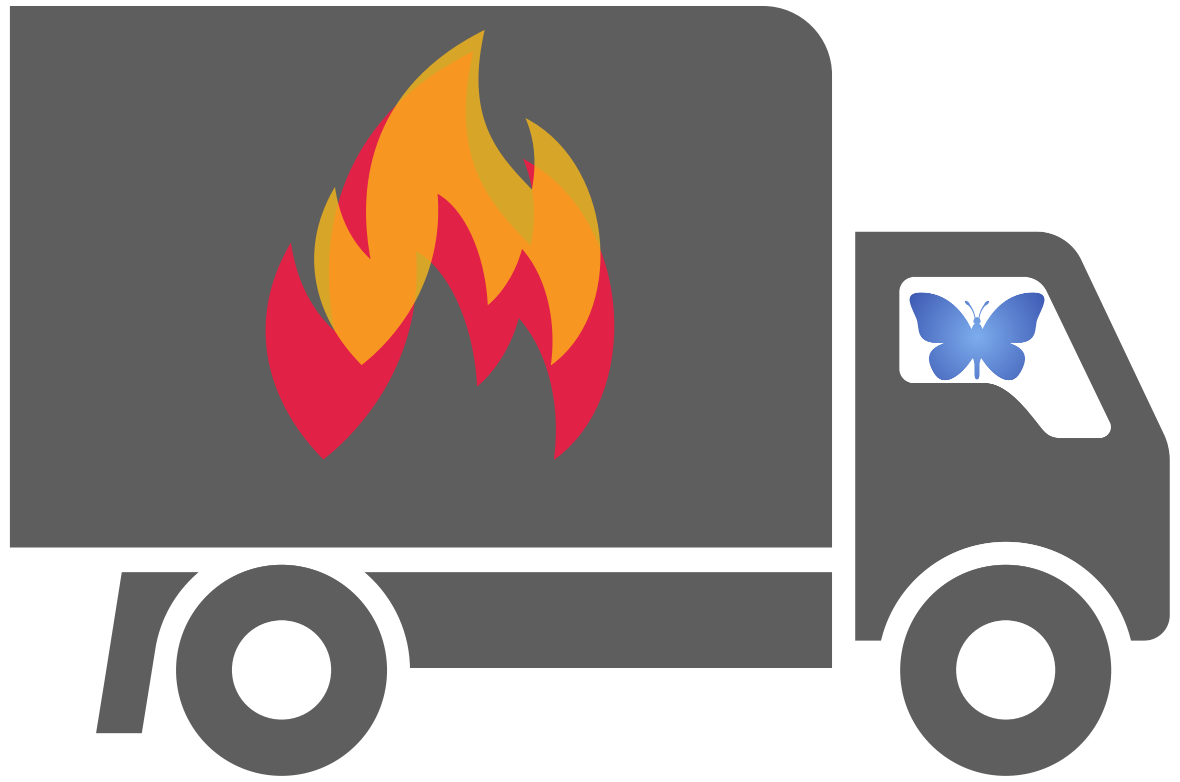
$$D_{th} \sim D_L$$

$$D_{th} = \frac{1}{d} v^2 \tau_{th}$$

$$D_L = \frac{1}{d} v_B^2 \tau_L$$



Planckian bound on diffusion $D_{th} \gtrsim s v_B^2 \frac{\hbar}{k_B T}$?



True in many generic cases (counterexample: inhomogeneous SYK chains, Gu et al. 17)

Motivation

Experimental side

- Planckian dissipation in “bad insulators” ?

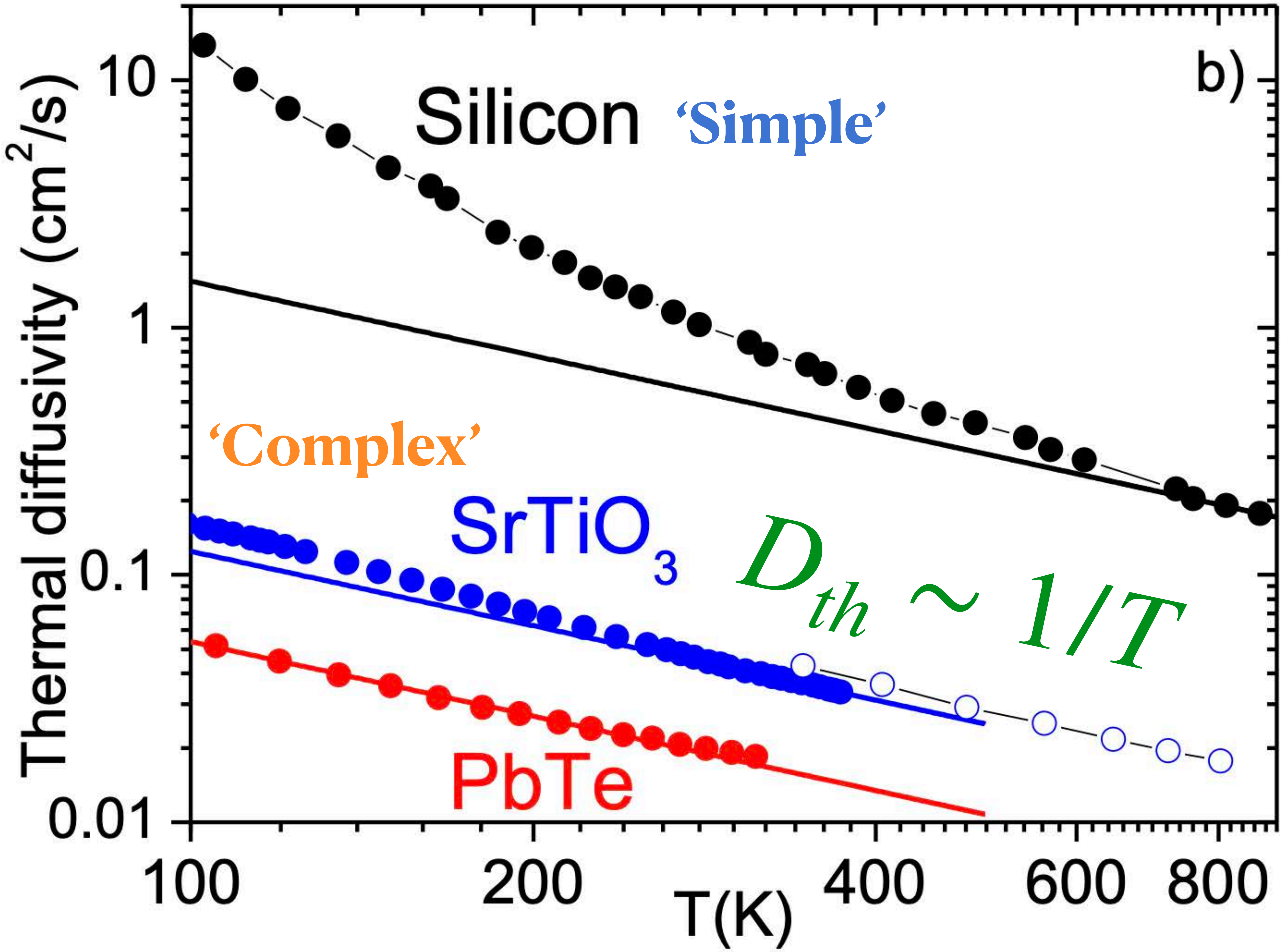
- Host of **complex insulating compounds** (many atoms in unit cell), e.g. complex oxides like $SrTiO_3$, with *anomalously small thermal diffusivities*

- “Bad insulators” = bad thermal conductors

Zhang, Martelli, Behnia, Kapitulnik et al. (PRL 18, PNAS 18, PRB 21, Journal of Physics: Condensed Matter 19)

Motivation

Experimental side



⇒ Analyze in terms of thermal relaxation times

$$(1) \quad \tau_{th} \equiv \frac{dD_{th}}{v^2}$$

averaged speed of sound v_{ph}

Parametrize in units of Planckian timescale

$$(2) \quad \tau_{th} = \alpha \frac{\hbar}{k_B T}$$

Zhang, Martelli, Behnia, Kapitulnik et al. (PRL 18, PNAS 18, PRB 21, Journal of Physics: Condensed Matter 19)

Motivation

Experimental side

$$\tau_{th} = \alpha \frac{\hbar}{k_B T}$$

Simple insulators: $\alpha \sim \mathcal{O}(50) \gg 1$

Classical, textbook ph-ph umklapp $D_{th} \sim 1/T$

Naive dimensional analysis $\alpha \sim \sqrt{M_{\text{ion}}/m_{\text{electron}}} \gg 1$

Sample	v_{ph}, 10^5cm/s	α	ℓ_{ph}(300 K), Å
$SrTiO_3$ (20)	7.87	2.7	5.1
$LaAlO_3$ (24, 32)	6.72	2.9	3.86
$KTaO_3$ (24, 33)	7.5	3.1	5.56
$KNbO_3$ (24, 34)	7.0	1.6	2.69
$NdGaO_3$ (24, 35)	6.5	1.65	2.61
$YAlO_3$ (24, 36)	8.25	1.8	3.54
$MgSiO_3$ (37, 38)	8.0	1.05	2.1
Disordered $SrTiO_3$ (24)	7.87	1.9	3.57
GGG (39, 40)	6.55	2.5	3.98
$PbWO_4$ (41, 42)	3.47	3.0	2.65
BeO (43, 44)	11.3	11	46
Silicon (20)	8.43	23	202
Natural diamond (45–47)	18.0	50	55,000

Complex insulators: $\alpha \sim 1$

No phonon quasiparticles? (Phonon fluid?)

Empirically bounded $\alpha \geq 1$

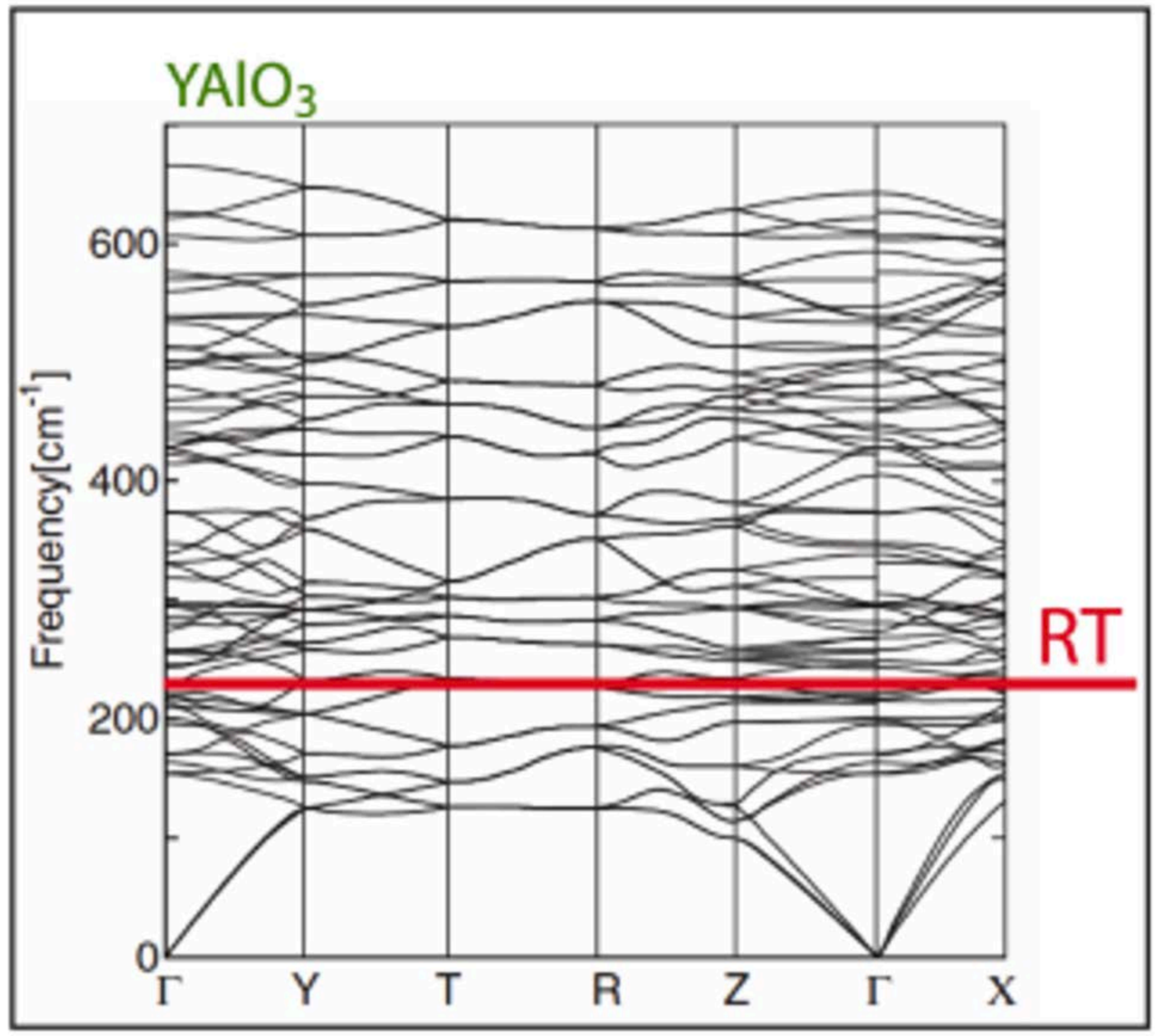
Zhang, Martelli, Behnia, Kapitulnik et al. (PRL 18, PNAS 18, PRB 21, Journal of Physics: Condensed Matter 19)

Motivation

Experimental side

Is $D_{th} \sim 1/T$ regime classical in complex insulators?

Host optical branches
well above RT

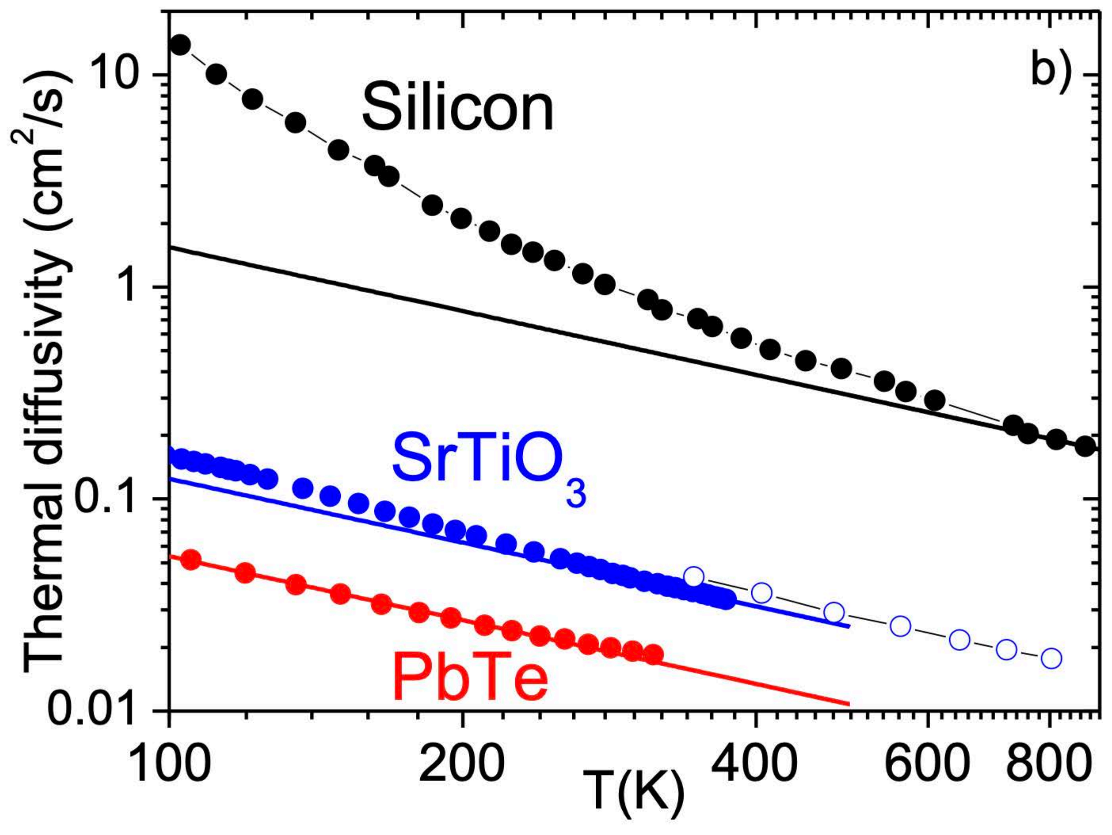


Structurally complex: pronounced anharmonicities

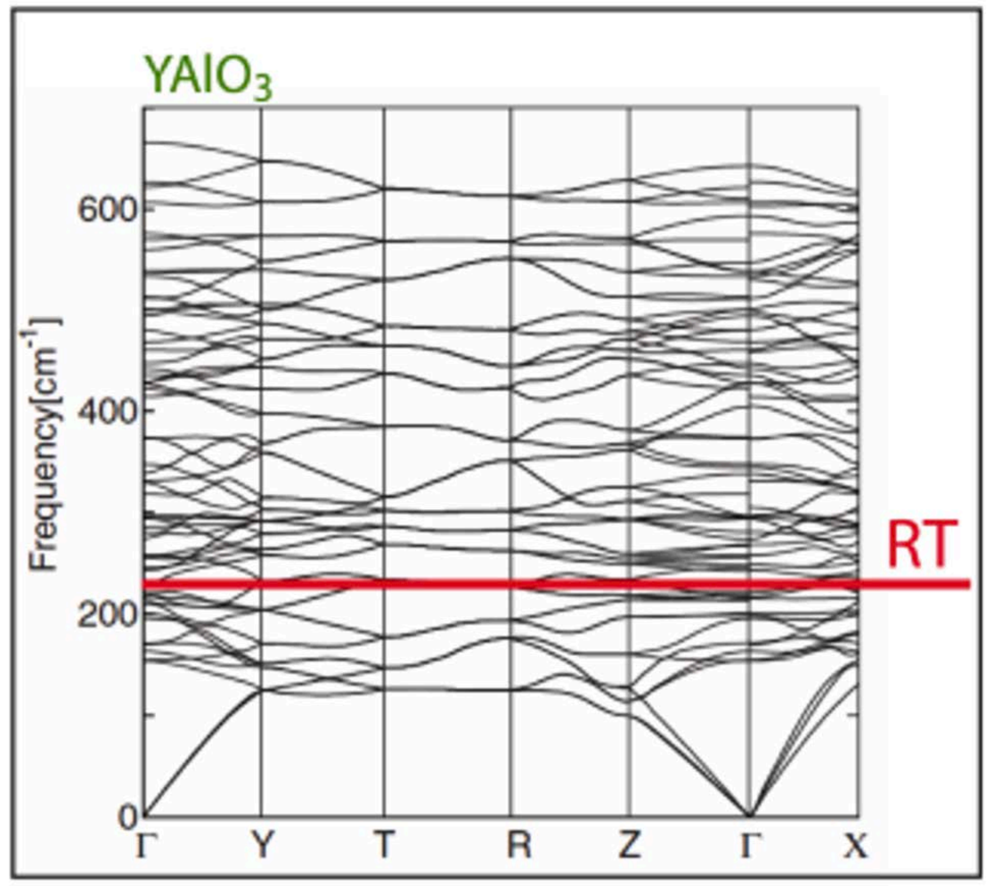
Zhang, Martelli, Behnia, Kapitulnik et al. (PRL 18, PNAS 18, PRB 21, Journal of Physics: Condensed Matter 19)

Motivation

Theoretical + experimental



Sample	v_{ph}, 10^5cm/s	s	ℓ_{ph}(300 K), Å
$SrTiO_3$ (20)	7.87	2.7	5.1
$LaAlO_3$ (24, 32)	6.72	2.9	3.86
$KTaO_3$ (24, 33)	7.5	3.1	5.56
$KNbO_3$ (24, 34)	7.0	1.6	2.69
$NdGaO_3$ (24, 35)	6.5	1.65	2.61
$YAlO_3$ (24, 36)	8.25	1.8	3.54
$MgSiO_3$ (37, 38)	8.0	1.05	2.1
Disordered $SrTiO_3$ (24)	7.87	1.9	3.57
GGG (39, 40)	6.55	2.5	3.98
$PbWO_4$ (41, 42)	3.47	3.0	2.65
BeO (43, 44)	11.3	11	46
Silicon (20)	8.43	23	202
Natural diamond (45–47)	18.0	50	55,000



Planckian diffusivity from a strongly coupled "phonon fluid"?

Bound on diffusion?

Relation to MB quantum chaos?

Zhang, Martelli, Behnia, Kapitulnik et al. (PRL 18, PNAS 18, PRB 21, Journal of Physics: Condensed Matter 19)

Outline

of talk

Model of strongly coupled phonons

Phase diagram and relation to SYK model

Dynamics, thermal transport and MB quantum chaos

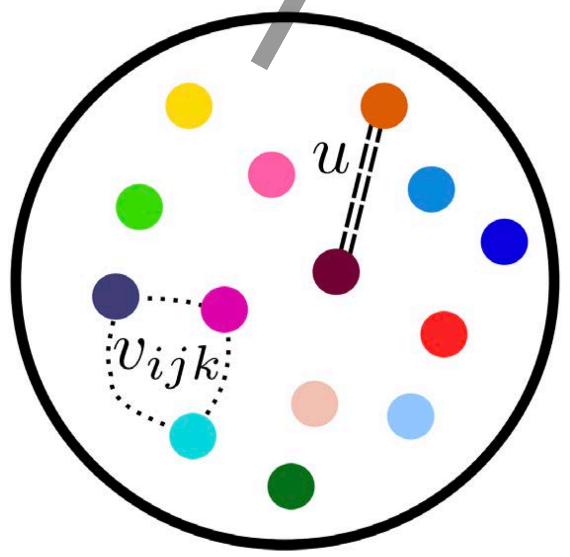
Summary

Model

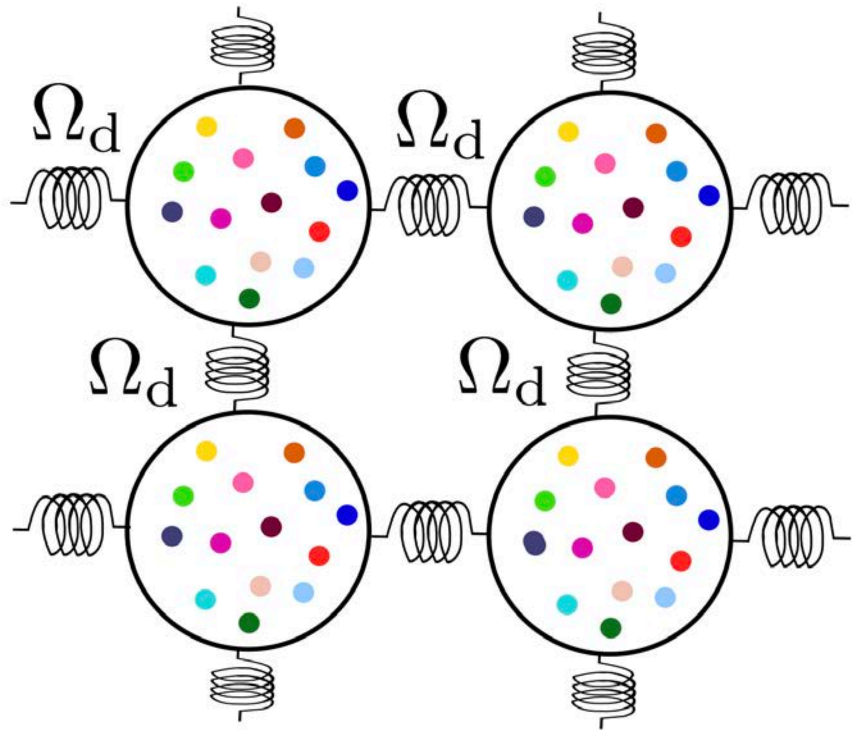
Hamiltonian and some preliminaries

$N \gg 1$ strongly coupled **optical phonons** (+ acoustic modes, later) in a d-dimensional lattice, $H = \sum_{\boldsymbol{r}} (H_{\boldsymbol{r},0} + H_{\boldsymbol{r},int})$

Unit cell



Lattice



Gaussian cubic couplings:

$$\overline{v_{ijk}} = 0, \quad \overline{v_{ijk}^2} = 2v^2$$

Strong coupling:

$$\Omega_i \sim \Omega_v \sim \Omega_u$$

$\Omega_{i,v,u}$ - energy scales

Solvable in 'weakly dispersive' limit:

$$\Omega_{i,v,u} \gg \Omega_{\rm d}$$

Convenient to consider $\Omega_i = \Omega_0$

$$H_{\boldsymbol{r},0} = \sum_{i=1}^{N} \frac{\pi_{i,\boldsymbol{r}}^2}{2} + \frac{\Omega_i^2}{2}\phi_{i,\boldsymbol{r}}^2 + \frac{\Omega_{{\rm d},i}^2}{2} \sum_{\boldsymbol{\delta}} \left(\phi_{i,\boldsymbol{r}+\boldsymbol{\delta}} - \phi_{i,\boldsymbol{r}}\right)^2$$

$$H_{\boldsymbol{r},\rm int} = \frac{1}{N} \sum_{ijk} v_{ijk} \phi_{i,\boldsymbol{r}} \phi_{j,\boldsymbol{r}} \phi_{k,\boldsymbol{r}} + \frac{u}{4N} \left(\sum_{i=1}^{N} \phi_{i,\boldsymbol{r}}^2 \right)^2$$

In this talk, 3 simple variants of H:

(1) single unit cell (0 + 1 dimensional model)

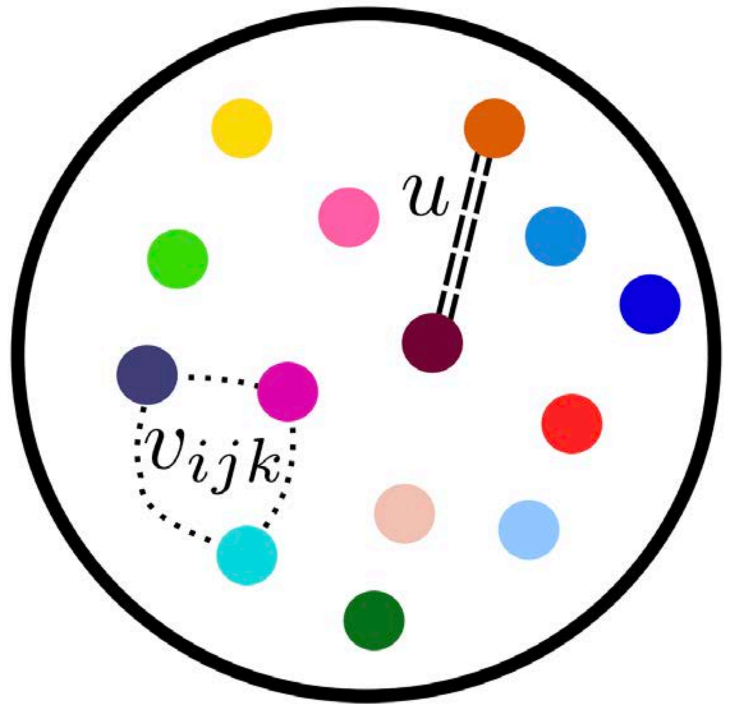
(2) lattice with a single optical branch

(3) lattice with optical and acoustic branches

0+1 model

Saddle-point equations

Unit cell with a single optical branch $\Omega_i \equiv \Omega_0$:

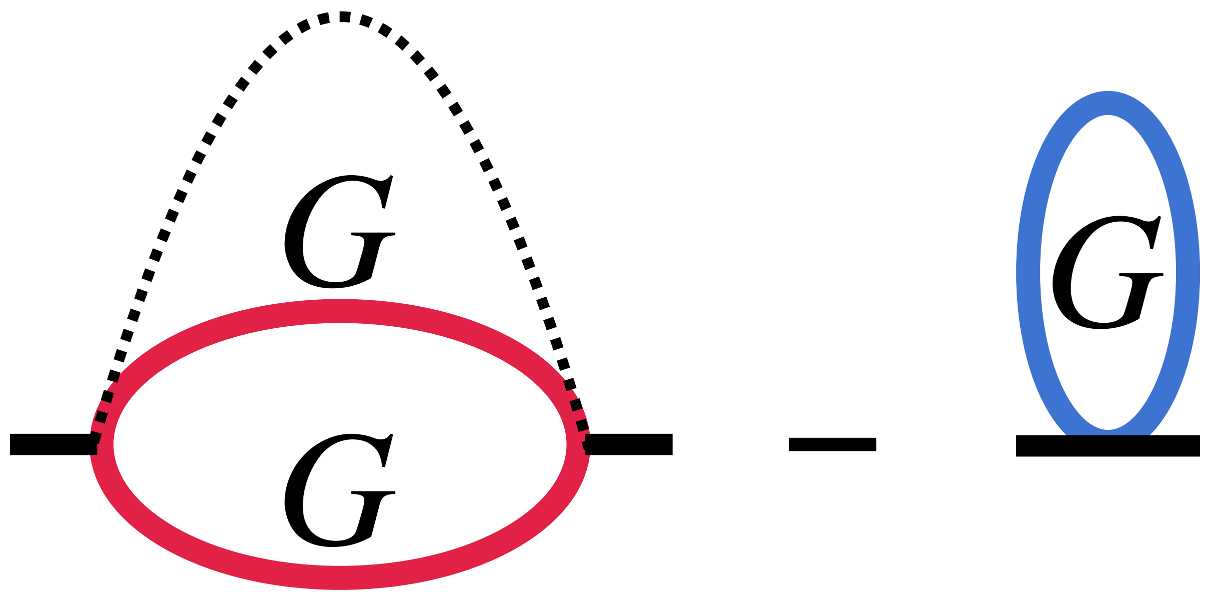


$$H = \sum_{i=1}^{N} \frac{\pi_i^2}{2} + \frac{\Omega_0^2}{2}\phi_i^2 + \frac{1}{N}\sum_{ijk} v_{ijk}\phi_i\phi_j\phi_k + \frac{u}{4N}\left(\sum_{i=1}^{N}\phi_i^2\right)^2$$

$N \to \infty$ saddle-point equations for replica diagonal solution:

$$G(i\omega) = \frac{1}{\omega^2 + \Omega_0^2 - \Pi(i\omega)} \qquad \Pi(\tau) = v^2 G^2(\tau) - u G(\tau)\delta(\tau)$$

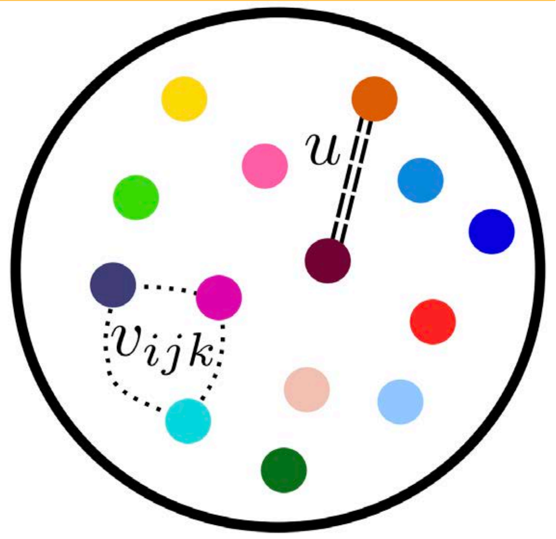
$\Pi =$



<u>Comment:</u> 0+1 model similar to quantum spherical p-spin glass model (Cugliandolo et al. '00)

0+1 model

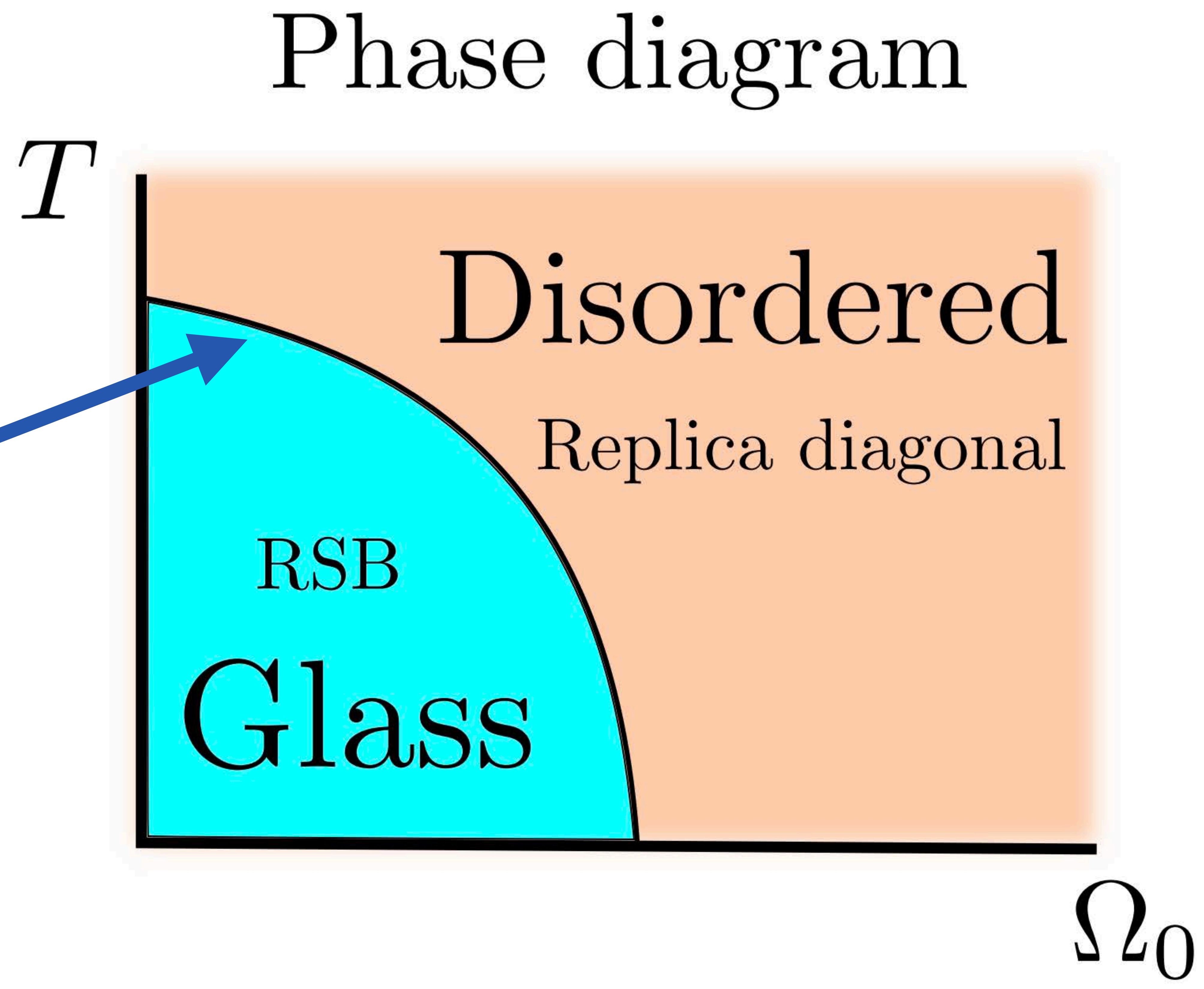
Phase diagram



- Phase diagram crucial to study dynamics in self-averaging phase
- Cranking up cubic anharmonicities v_{ijk} induces a 1^{st} order transition to a replica-symmetry breaking phase

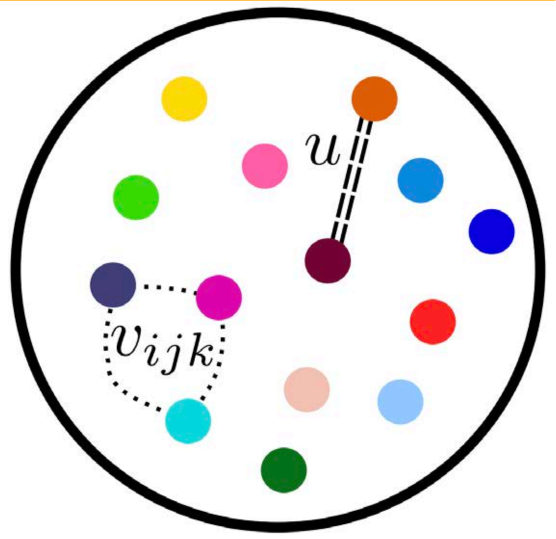
1^{st} order

- Phase diagram of lattice model is essentially identical



0+1 model

Relation to the SYK model



- Can we tune to an SYK-like critical point?

  Naively, $\Omega_0 \to \Omega_*, \quad u \to 0$:

$$G(i\omega) \approx \frac{1}{\Omega_*^2 - \Pi(i\omega)} \qquad \Pi(\tau) = v^2 G(\tau)^2$$

$$\Longrightarrow \qquad G(\tau) \sim |\tau|^{-2\Delta}$$

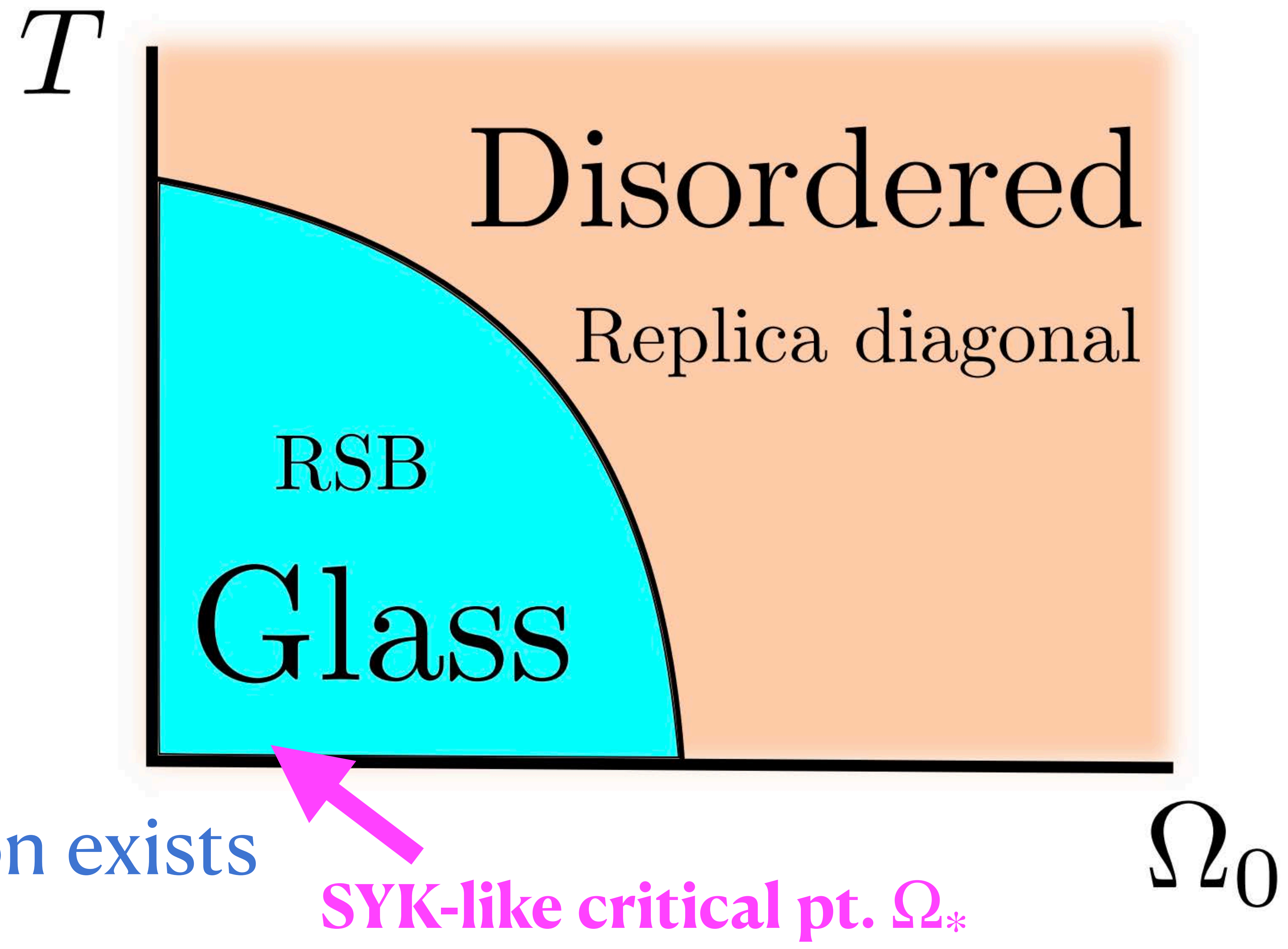
SYK-like solution is not realized:

(1) Saddle-point solution at Ω_* is RSB

(2) CFT has complex scaling dimensions

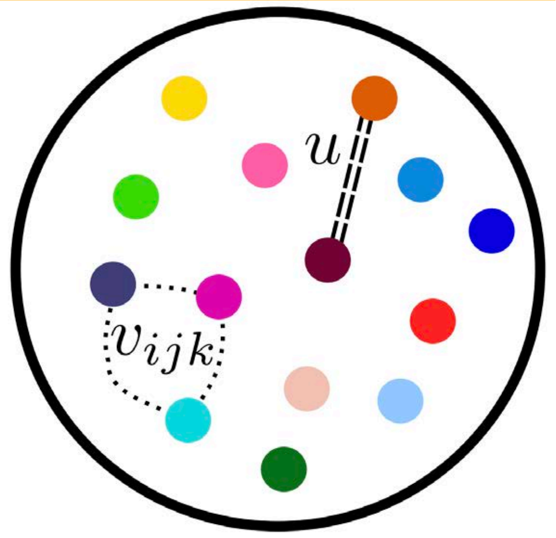
(3) Even without randomness: a thermodynamically-favorable, gapped solution exists

Phase diagram

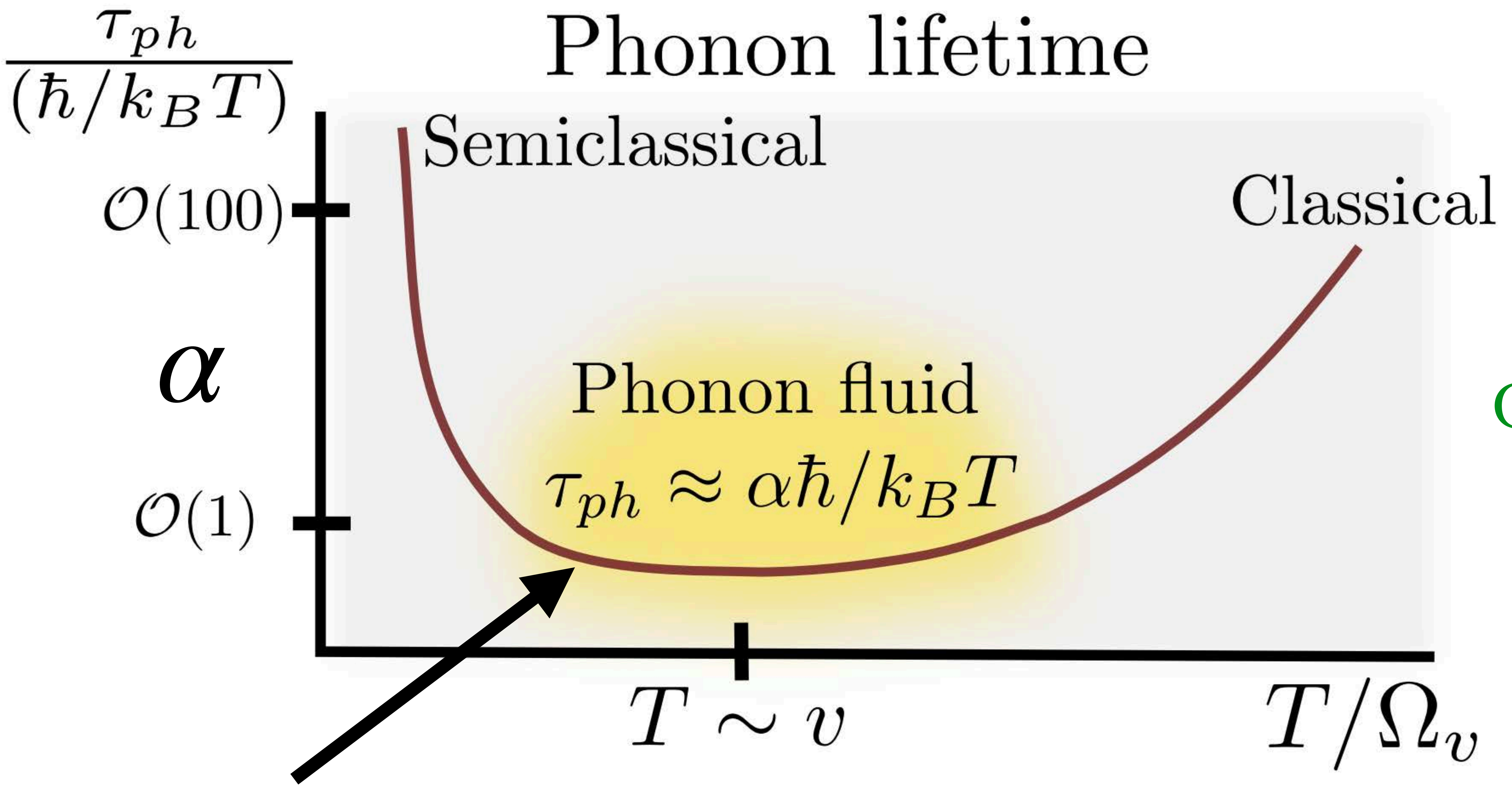


0+1 model

Dynamical regimes



- Dynamical regimes via phonon lifetime τ_{ph}: $\quad G_R(t) \sim \exp\left(-t/\tau_{ph}\right)$
- Generic behavior in strongly coupled regime:

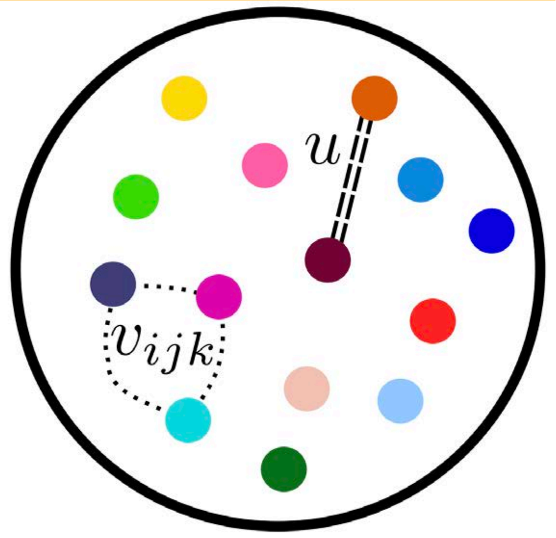


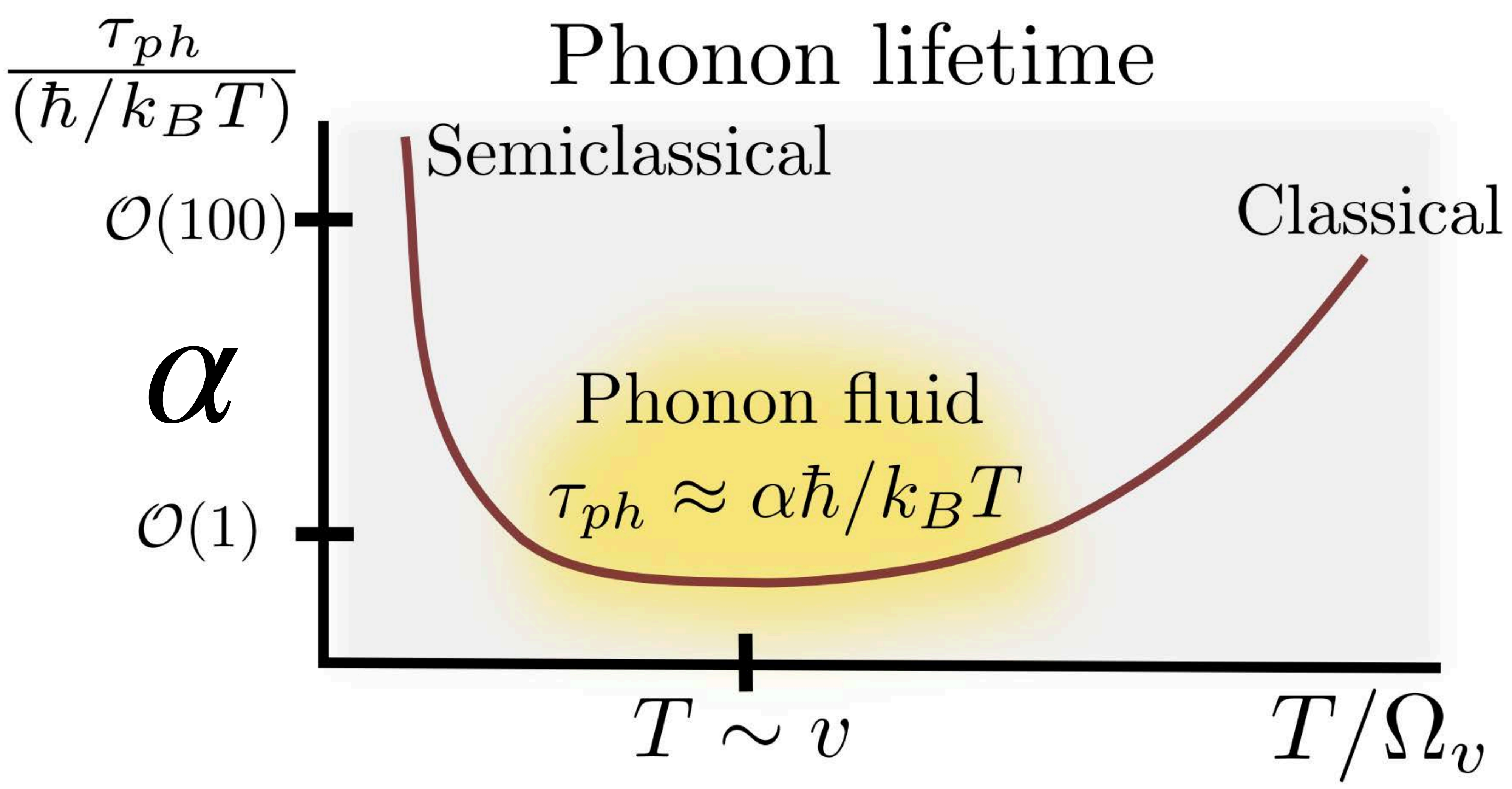
Generically, at strong coupling
$\alpha \approx 5 - 15$

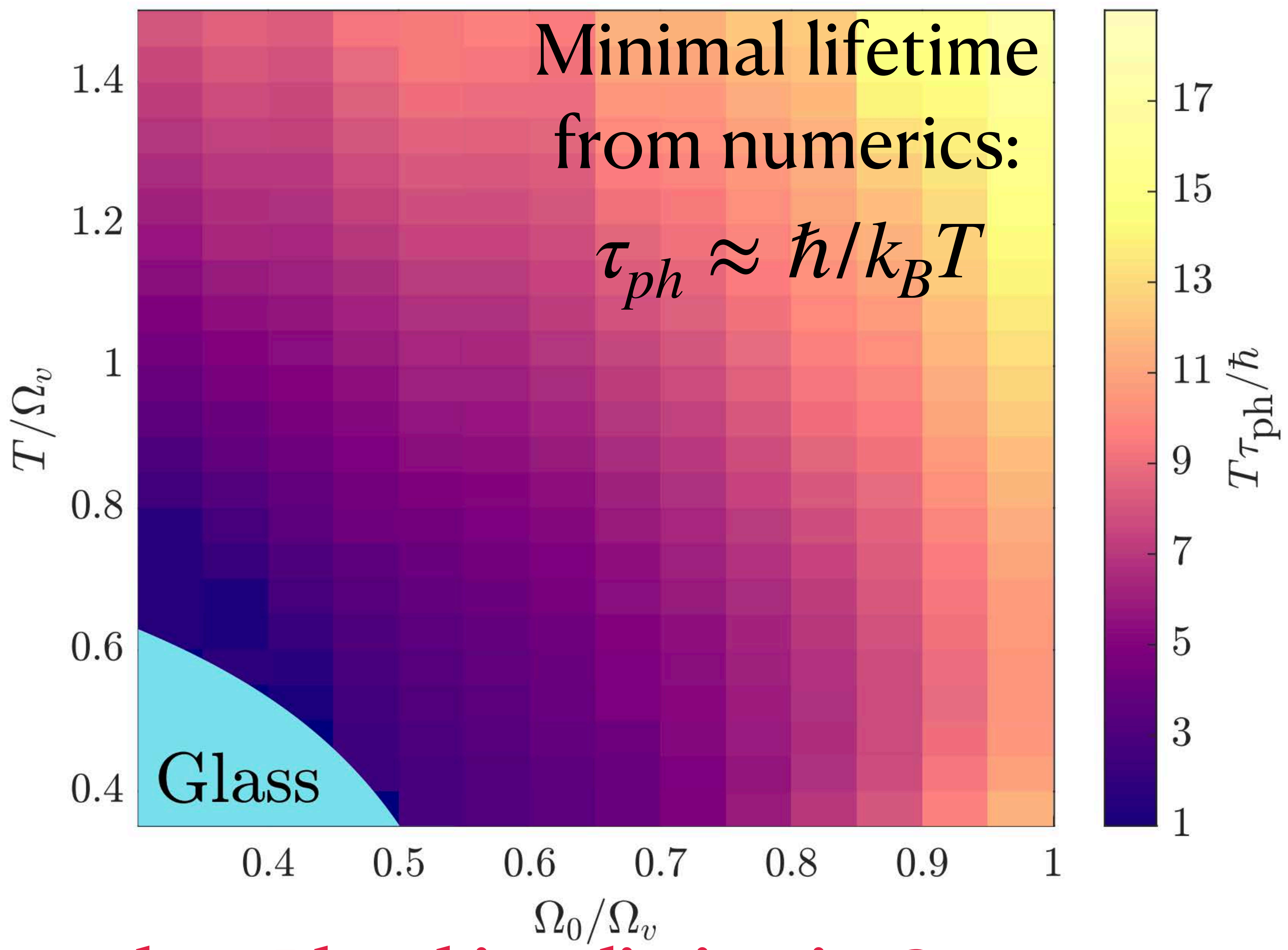
Wide intermediate-T Planckian regime

0+1 model

Minimal lifetime in line with Planckian bound





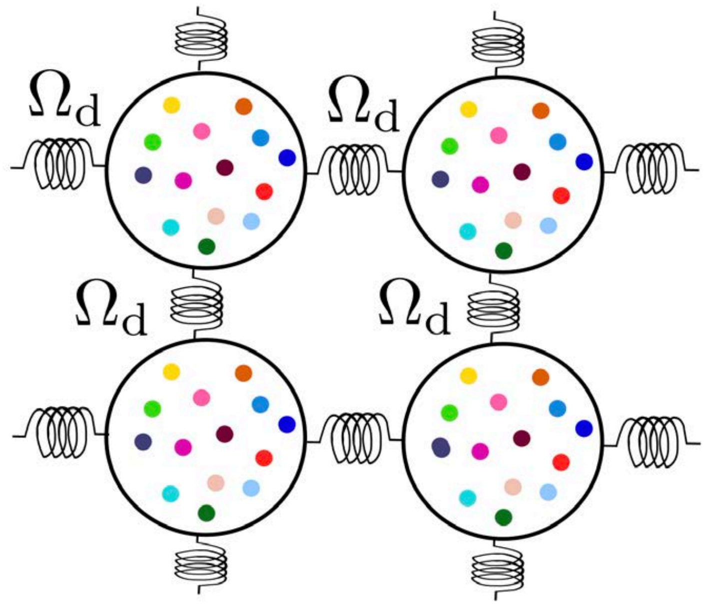


$\alpha \gtrsim 1$, minimal in vicinity of glass transition

Phonon fluid regime corresponds to Planckian dissipation?

Lattice model - single optical branch

Saddle point equation and reminder



- Saddle point equations for a single optical branch: (d=1)

$$G(i\omega, k) = \frac{1}{\omega^2 + \Omega_o^2 + 4\Omega_d^2 \sin^2(k/2) - \Pi(i\omega, k)}$$

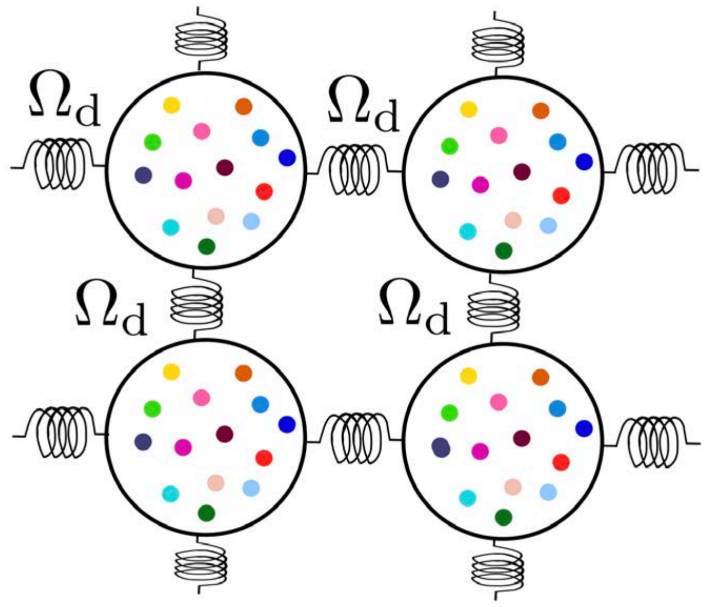
$$\Pi(\tau, r) = v^2 G(\tau, r)^2 - uG(\tau, r)\delta(\tau)\delta(r)$$

- Weakly dispersive limit $\Omega_d \ll \Omega_0$: $\Pi(i\omega, k) \approx \Pi(i\omega)$
  $\Rightarrow$ single particle dynamics inherited from 0+1 model

- A single velocity scale $v_o \equiv \Omega_d^2/\overline{\Omega}_o$, where $\overline{\Omega}_o^{\,2} = \Omega_o^2 - \Pi(0)$ is the renormalized frequency

Lattice model - single optical branch

Transport and chaos



- Consider **thermal** and **chaos** diffusivities

$$D_{\rm th} = \kappa_{\rm th}/c \qquad \mathrm{OTOC}(t,\boldsymbol{r}) \sim \frac{1}{N}\exp\left(\lambda_L t - \frac{r^2}{D_L t}\right)$$

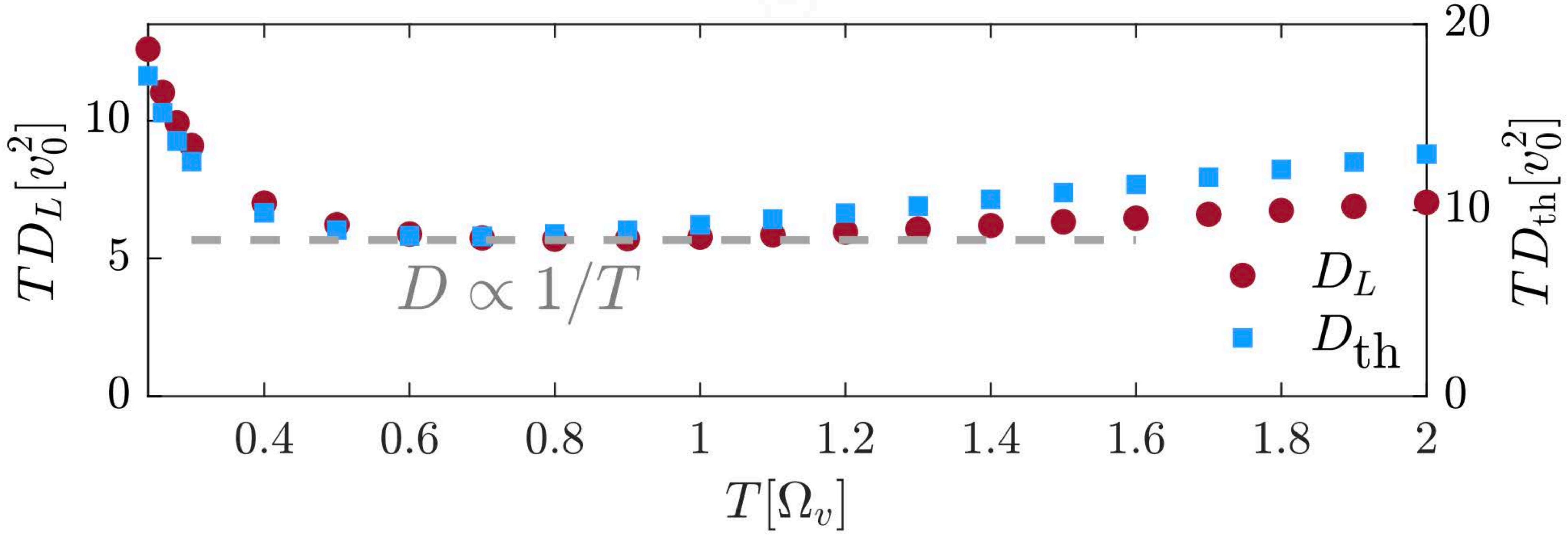
Diffusivities are related:

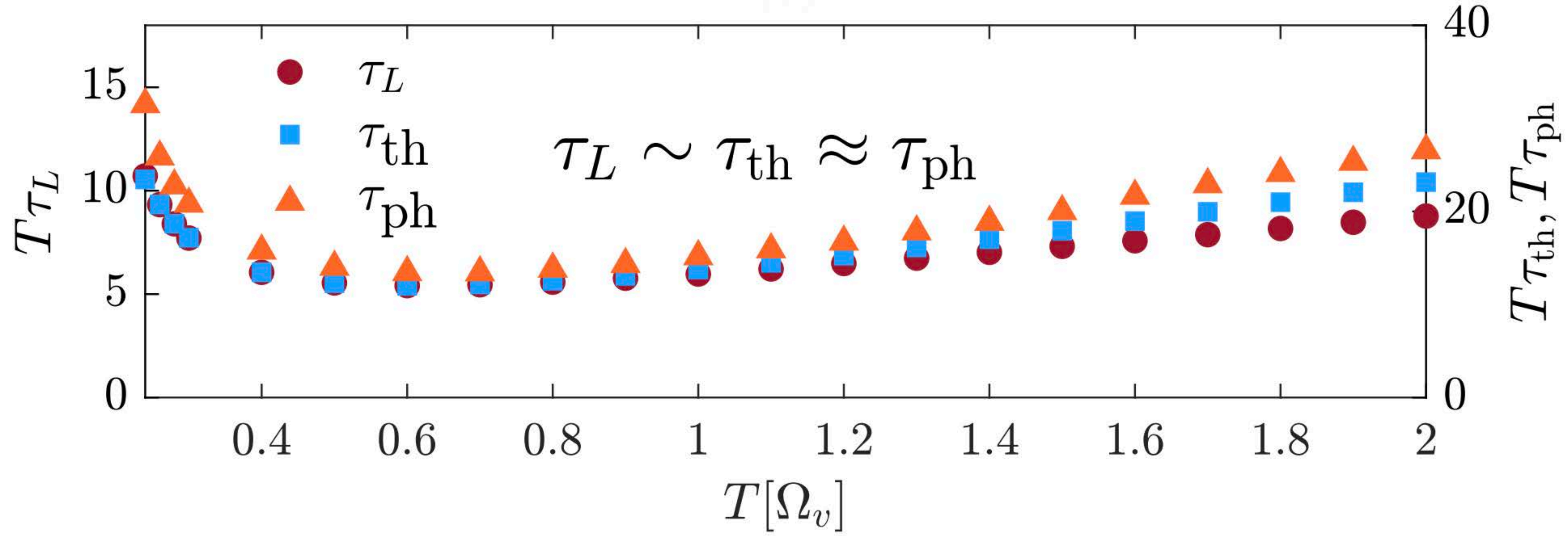
$$D_L \approx \gamma D_{\rm th}, \quad \gamma \sim 1-3$$

Planckian dissipation in phonon fluid regime:

$$\tau \approx \alpha \frac{\hbar}{k_B T}, \quad \alpha \sim 5-15$$

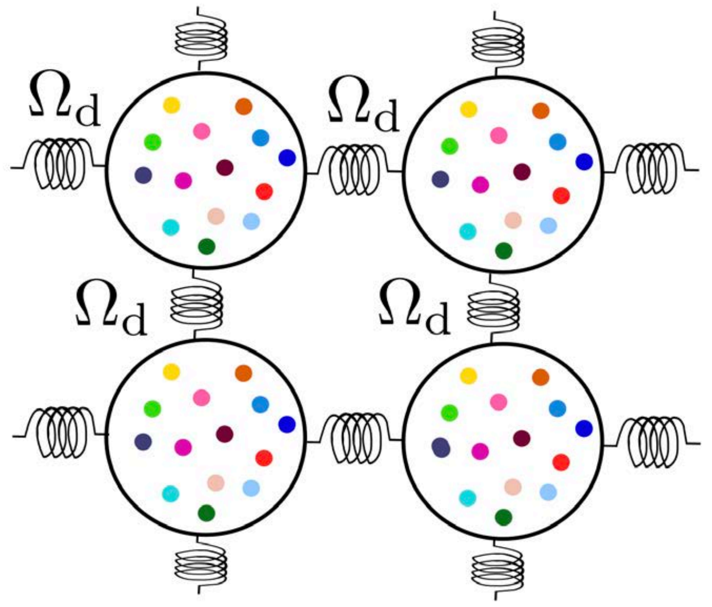
$v_B \approx v_{\rm optical}$ weakly $T-$dependent at intermediate temperatures





Adding acoustic (Goldstone) modes

Towards a more realistic model



- Acoustic phonons weakly coupled, fast and long lived at $k \to 0$

$$\tau_{\text{acoustic}}(k) \sim 1/k^2$$

- N_o optical modes + N_a acoustic modes

$$\phi_{i,r} \mapsto \phi_{i,r+1} - \phi_{i,r} \quad (\approx \partial_r \phi_{i,r})$$

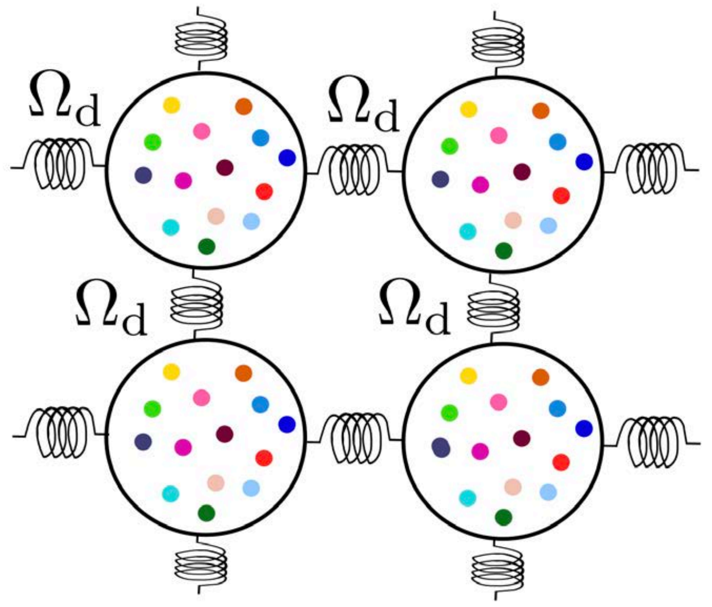
- Large N limit: fixed $n_o = N_o/N, \quad n_a = N_a/N$

  Consider small fractions of acoustic modes: $n_a \ll n_o$ (optical phonons = bath)

$$G_{ac}(i\omega, k) = \frac{1}{\omega^2 + 4\Omega_a^2 \sin^2(k/2) - \Pi_a(i\omega, k)}, \quad \Pi_{\text{acoustic}}(i\omega, k) = 4\sin^2\left(\frac{k}{2}\right)\Pi_{\text{optical}}(i\omega)$$

Adding acoustic (Goldstone) modes

Towards a more realistic model

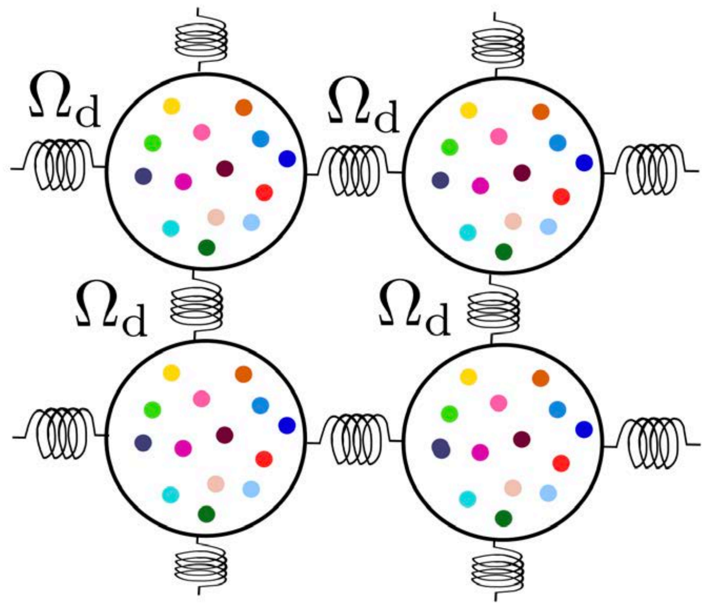


Coexistence of short-lived phonon fluid and long-lived Goldstone modes

What dominates thermal transport? MB quantum chaos?

Transport vs. Chaos take 2

Coexistence of acoustic and optical modes



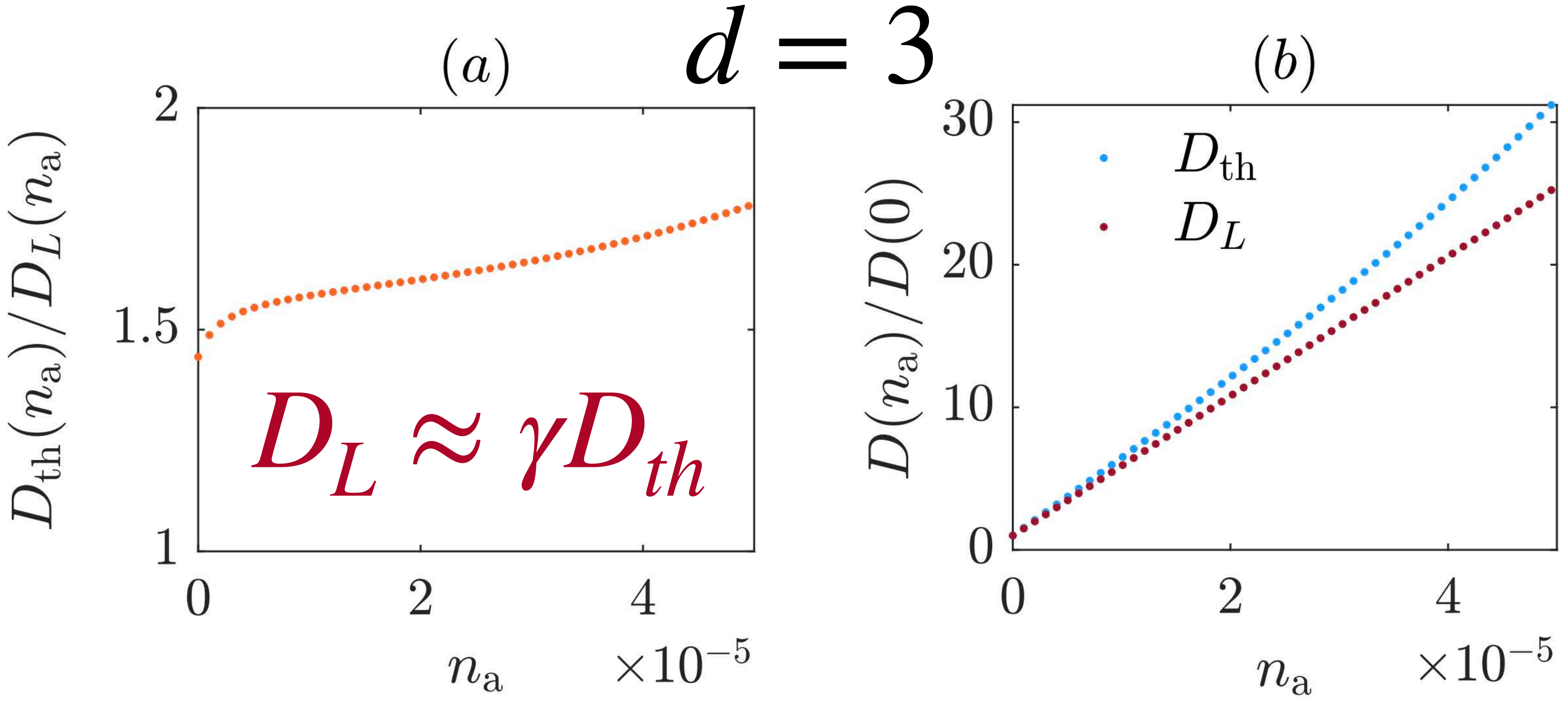
- Lower dimensions $d = 1,2$:
  Acoustic modes dominate thermal transport, but not chaos!
  For any $n_a > 0$,

$$D_{th} \to \infty, \quad D_L < \infty$$

- In $d = 3$, relation generically restored (but anisotropy can break off relation again)

$d = 3$

γ

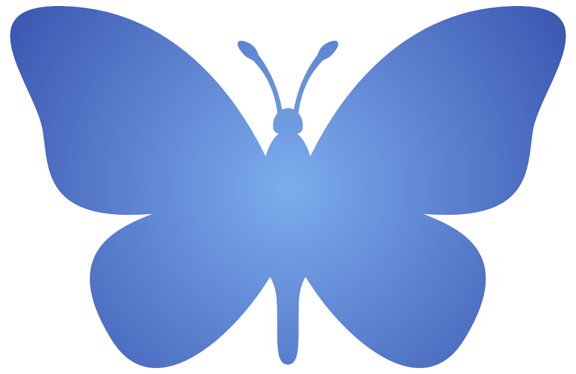


Transport vs. Chaos take 2

Coexistence of acoustic and optical modes

- Intuitively:

τ_{th} dominated by longest-lived op's



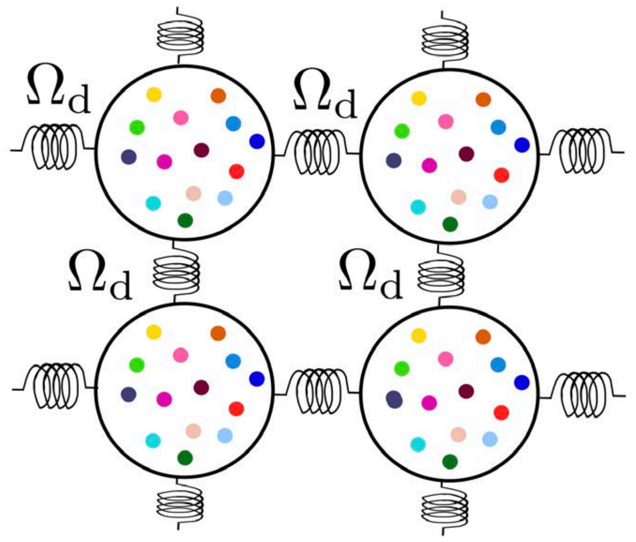
τ_L dominated by shortest-lived op's

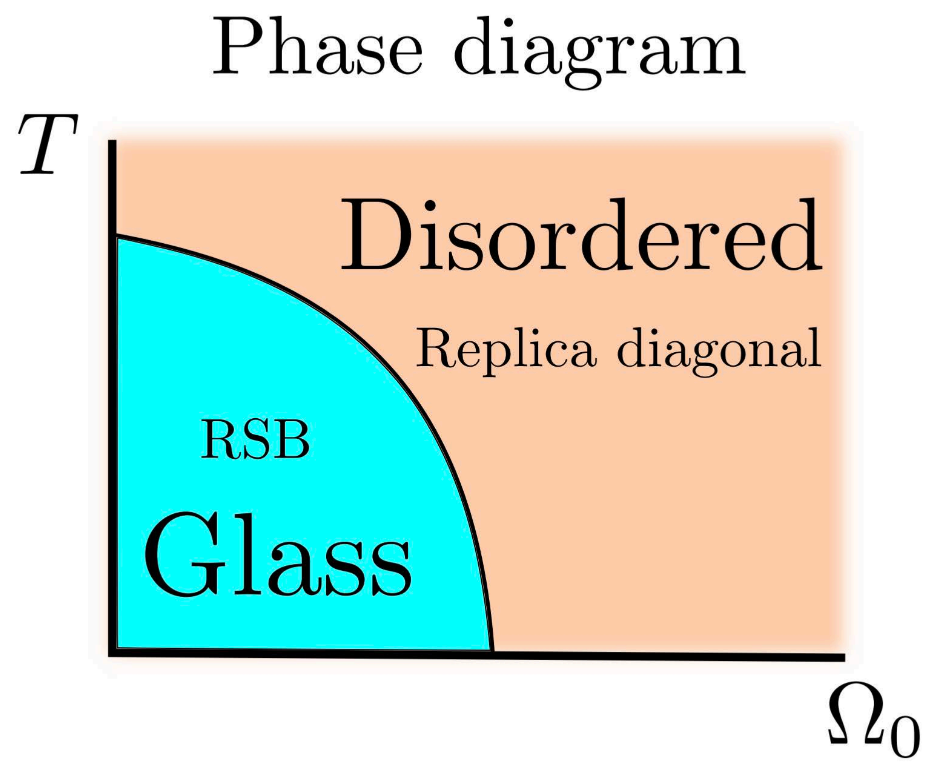
If short-lived op's = long-lived op's, expect $D_L \sim D_{th}$, otherwise, relation can be broken

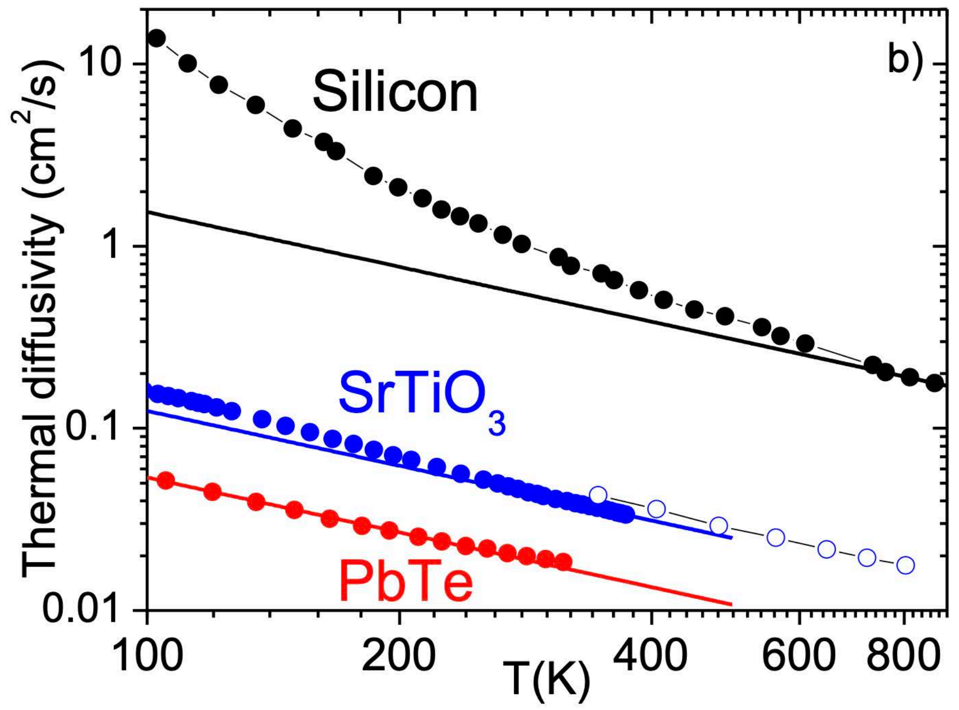
Summary

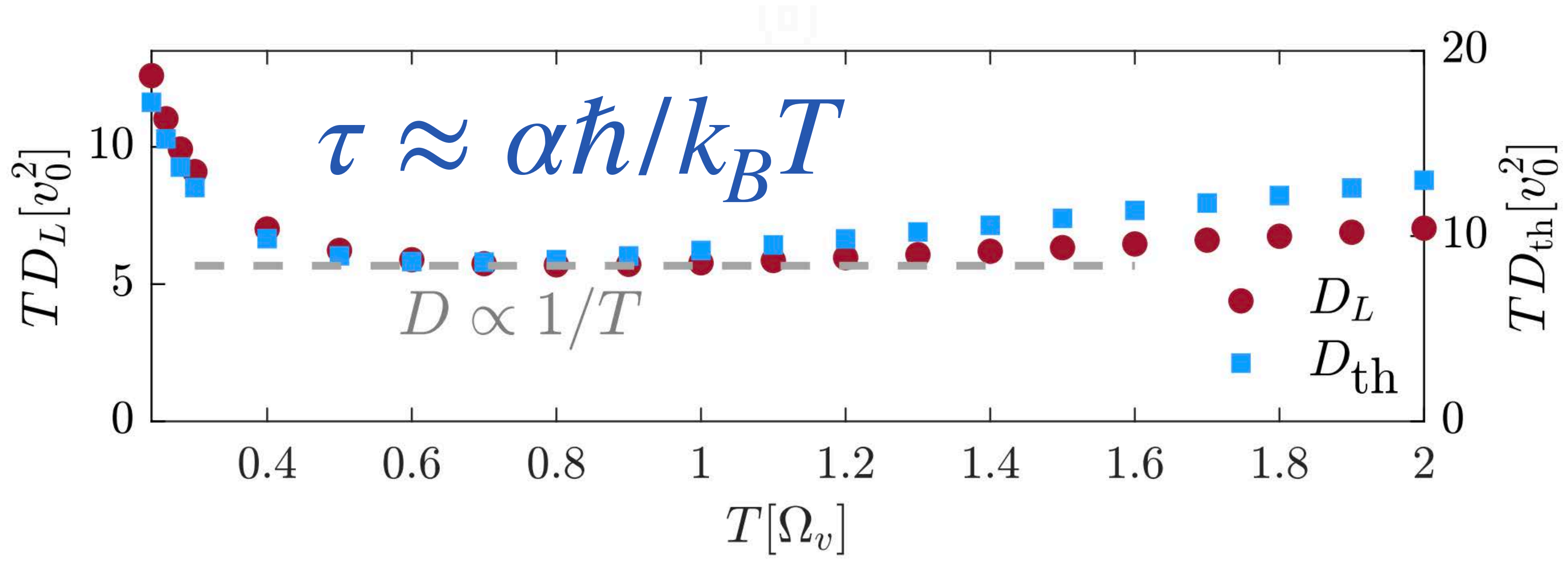
of talk

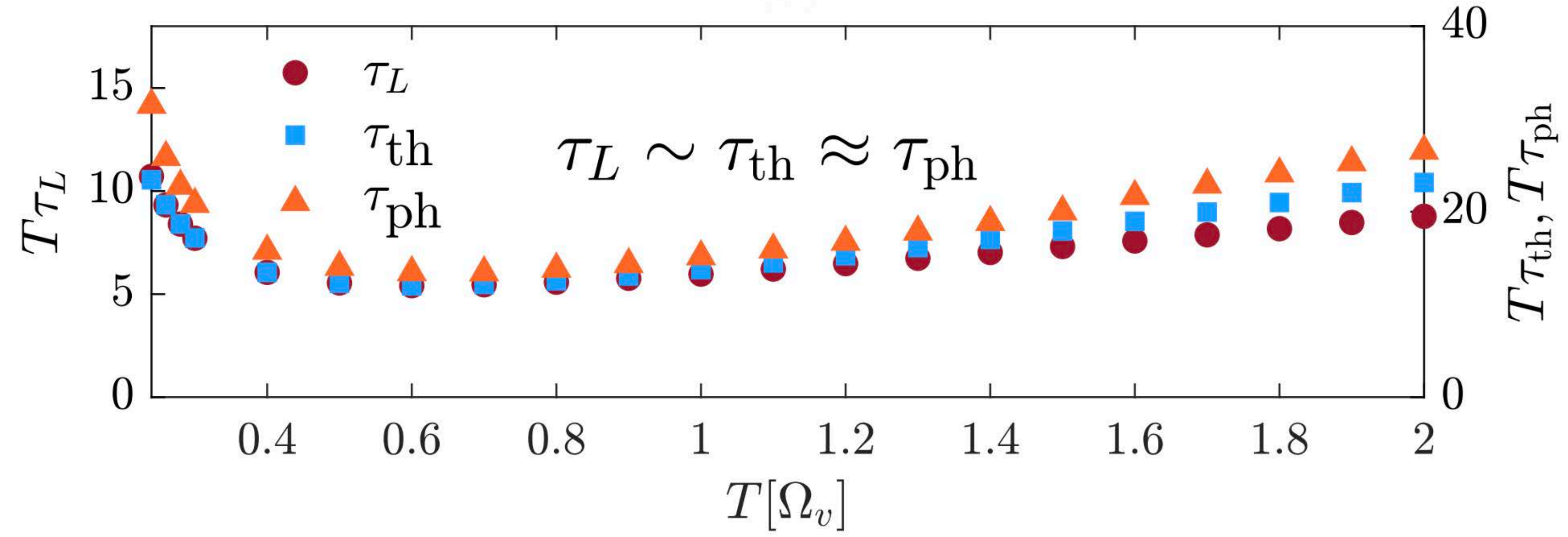
- Motivation: Planckian thermal diffusivities in complex insulators and possible relation to many-body quantum chaos
- Theoretical model shows emergent Planckian dissipation at intermediate-T
- Transport and chaos are related, but relation can be infinitely violated for coexisting phonon fluid and Goldstone modes
- Also in papers: Multiple optical branches, scrambling with multiple scales, and more…











Thank you for your attention!