

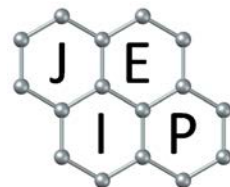
Dynamics, Transport and Chaos through a Strange Quantum Bath

Marco Schiro'
CNRS, JEIP College de France

Strange Metals, SYK Models and Beyond, June 2-3 2022



COLLÈGE
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Jeunes Equipes
de l'Institut de Physique
du Collège de France



In Collaboration with:

 Ancel Larzul, PhD Student @CdF



 Steven Thomson (CdF \rightarrow Free U Berlin)



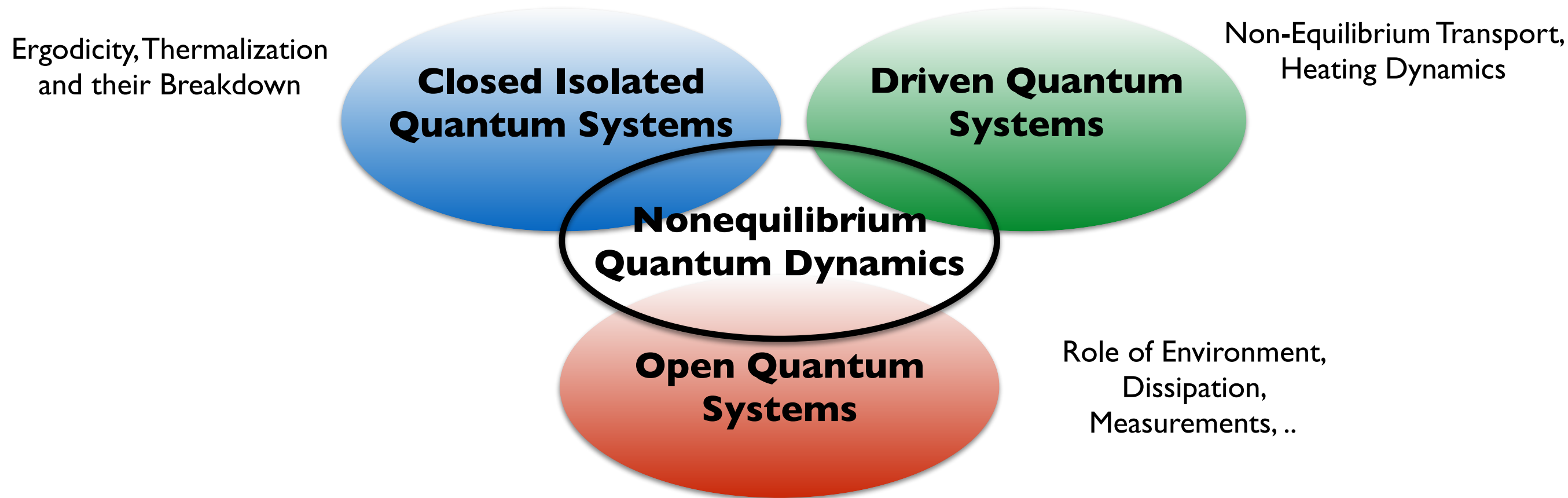
- A. Larzul, M. Schiro', *Phys. Rev. B* 105 045105 (2022)
- A. Larzul, S. J. Thomson, M. Schiro', *arXiv:2204.06434*
- A. Larzul, M. Schiro', *arXiv.2206.XXX* (to appear)

Funding:



Quantum Matter Out of Equilibrium

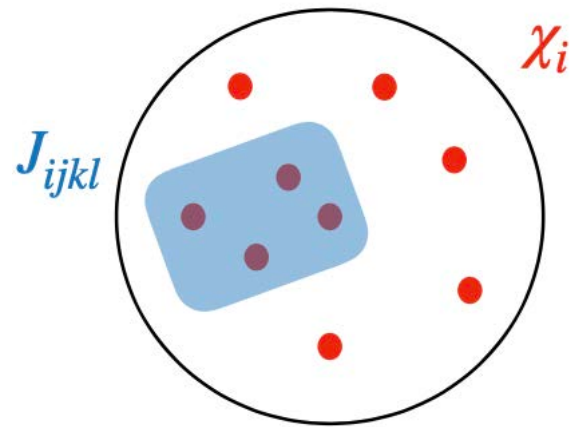
General principles to classify **dynamical** behaviour of quantum many-body systems?



Simple models that capture “generic” behavior?

Sachdev-Ye-Kitaev Model

Sachdev&Ye,
Georges, Parcollet&Sachdev,
Kitaev, Stanford&Maldacena,..



$$H_4 = -\frac{1}{4!} \sum_{ijkl} J_{ijkl} \psi_i \psi_j \psi_k \psi_l$$

$$\{\psi_i, \psi_j\} = \delta_{ij}$$

$$\langle J_{ijkl} \rangle = 0, \quad \langle J_{ijkl}^2 \rangle = 6 \frac{J^2}{N^3}$$

Exactly Solvable in Large N: Minimal Model for Non-Fermi-Liquid
(gapless, but no quasiparticles)

Fluctuations beyond Large N described by Schwarzian Action (gravity analogue)

Maximally Chaotic — Saturates the Bound on Chaos

Bound on chaos

$$\lambda_L \leq \frac{2\pi}{\beta}$$

Maldacena&Stanford

Can this model give us insights about the behaviour of quantum matter far from equilibrium?

Outline

- 📌 Thermalisation in closed, isolated SYK models
- 📌 SYK as a “Strange Quantum Bath”: Equilibration Dynamics
- 📌 SYK as a “Strange Quantum Bath”: Energy Transport & Chaos

Dynamics and Thermalization of Isolated SYK Models

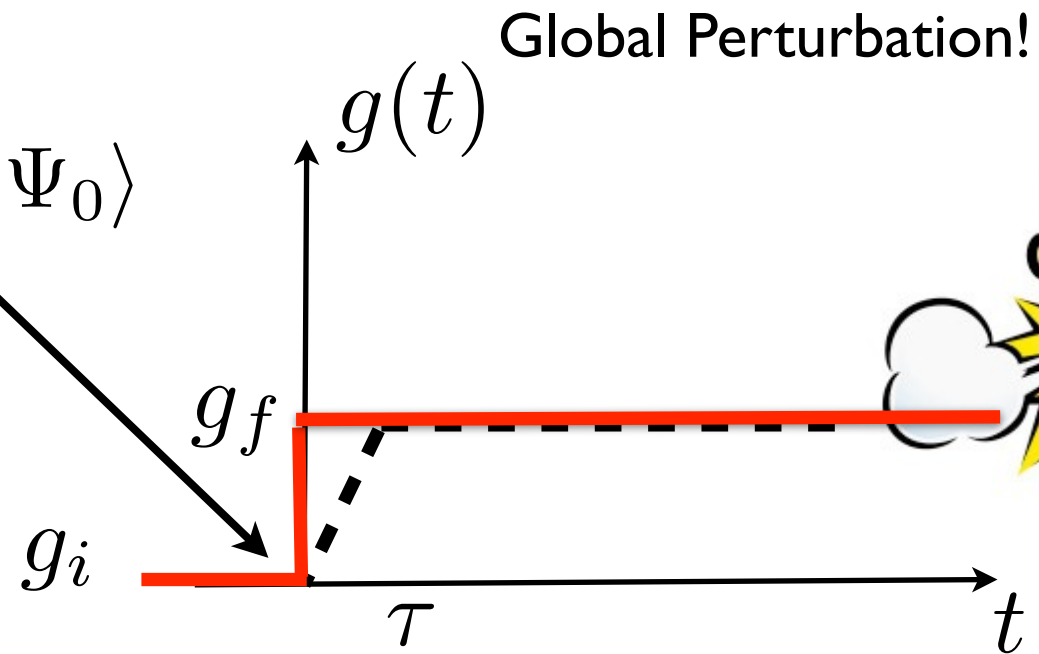
- *A. Larzul, M. Schiro', Phys. Rev. B 105 045105*

Quantum Thermalization

$$H[g(t)] = H_0 + g(t) H_1$$

$$|\Psi(t=0)\rangle = |\Psi_0\rangle$$

$$i\partial_t |\Psi(t)\rangle = H |\Psi(t)\rangle$$



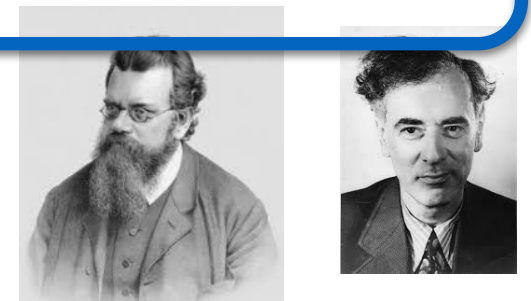
Unitary Quantum Dynamics (isolated system, no external bath)

- 📌 Pure states remain pure $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$ $\text{Tr}\rho^2(t) = \text{Tr}\rho_0 = 1$
- 📌 Quantum Mechanics is linear!

📌 How generic quantum systems approach Thermal Equilibrium after a perturbation?

Interactions induce scattering, loose memory initial condition...

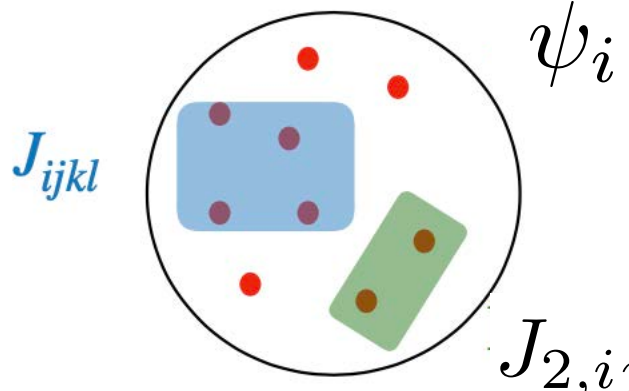
$$\frac{dn_{\mathbf{k}}}{dt} = I_{coll}[n_{\mathbf{k}}(t)]$$



📌 In modern terms: “Typical” (High-Energy) Excited States are “Thermal” (ETH)

Dynamics of SYK Models

📌 “Mixed” SYK:



$$H = \frac{i}{2} \sum_{ij} J_{2,ij} \psi_i \psi_j - \frac{1}{4!} \sum_{ijkl} J_{4,ijkl} \psi_i \psi_j \psi_k \psi_l$$

”SYK₂”
”SYK₄”

📌 Equilibrium: Crossover from NFL to FL scaling at $T^* \sim J_2^2 / J_4$

Parcollet&Georges('99),...,Song et al (2017),...

📌 Dynamics: Keldysh field theory is “exactly solvable” in Large-N

📌 Kadanoff-Baym Equations for Green's functions

$$G^{\alpha\beta}(t_1, t_2) = -\frac{i}{N} \sum_i \langle \psi^\alpha(t_1) \psi^\beta(t_2) \rangle$$

Large N Kadanoff-Baym Equations for Mixed SYK

Larzul&MS PRB(2022)

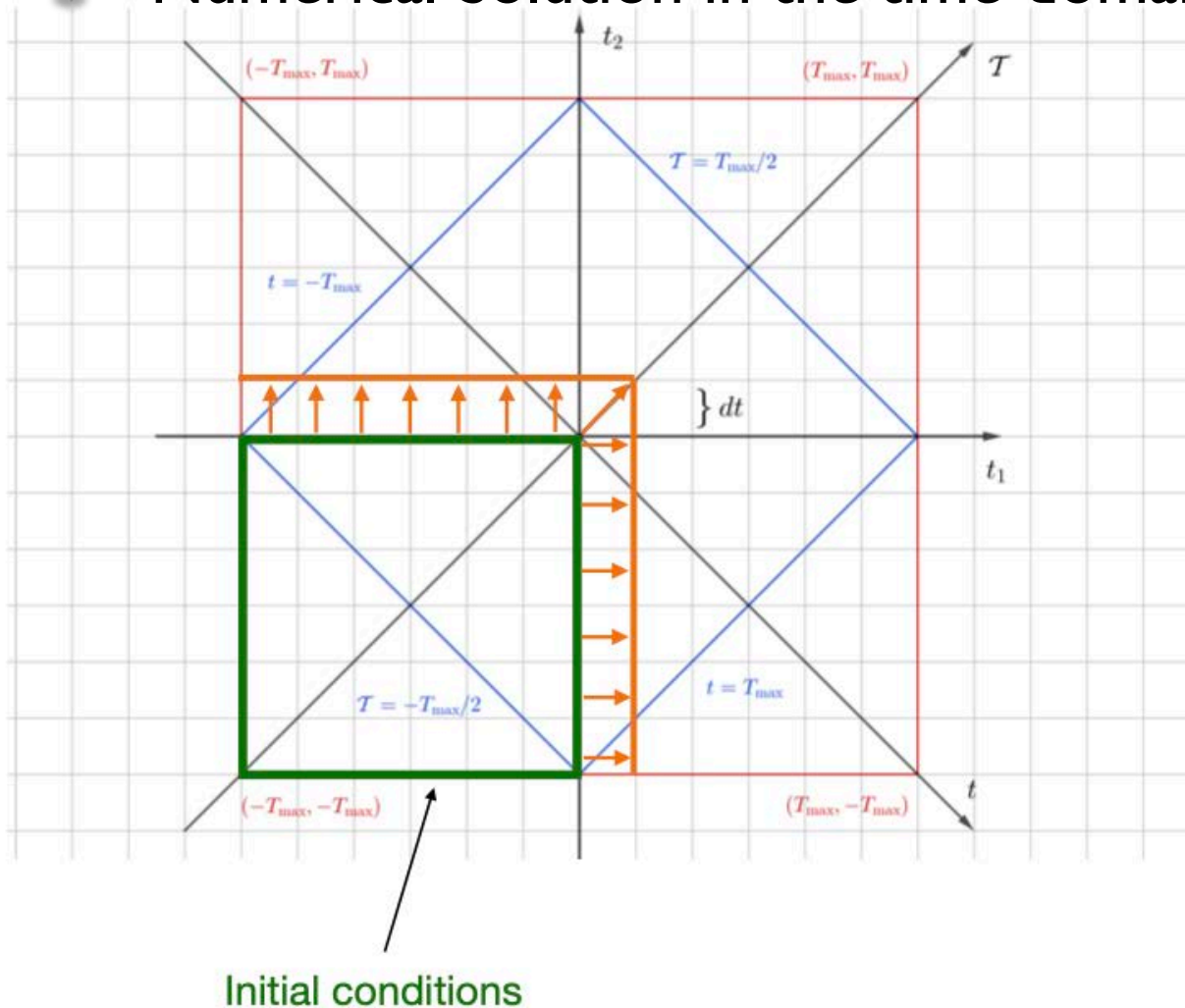
$$i\partial_{t_1} G^{>,<}(t_1, t_2) = \int_{-\infty}^{+\infty} dt \left[\Sigma^R(t_1, t) G^{>,<}(t, t_2) + \Sigma^{>,<}(t_1, t) G^A(t, t_2) \right]$$

$$-i\partial_{t_2} G^{>,<}(t_1, t_2) = \int_{-\infty}^{+\infty} dt \left[G^R(t_1, t) \Sigma^{>,<}(t, t_2) + G^{>,<}(t_1, t) \Sigma^A(t, t_2) \right]$$

$$\Sigma^{\alpha\beta}(t_1, t_2) = -\alpha\beta J_4(t_1) J_4(t_2) G^{\alpha\beta}(t_1, t_2)^3 + J_2(t_1) J_2(t_2) G^{\alpha\beta}(t_1, t_2)$$

Self-Energy term

Numerical Solution in the time-domain (similar to Nonequilibrium DMFT)



Initial Condition+Propagation(predictor/corrector)

Wigner Transform

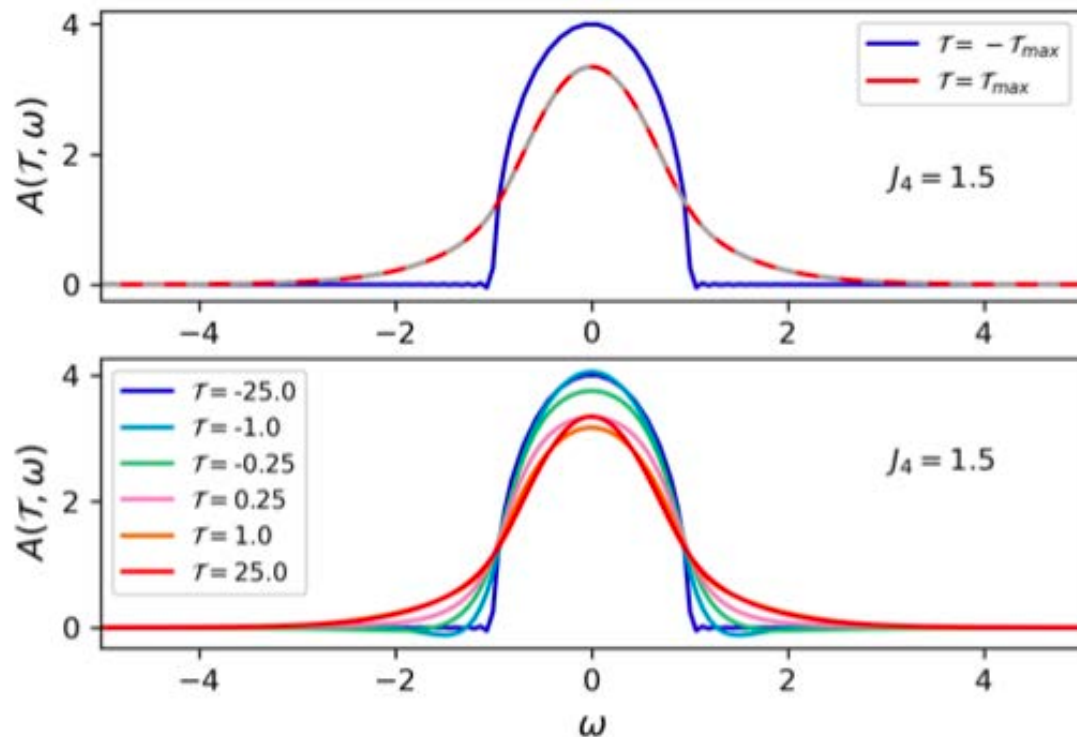
$$G(\mathcal{T}, \omega) = \int dt e^{i\omega t} G\left(t_1 = \mathcal{T} + \frac{t}{2}, t_2 = \mathcal{T} - \frac{t}{2}\right)$$

Thermalization of Mixed SYK Model

Larzul&MS PRB(2022)

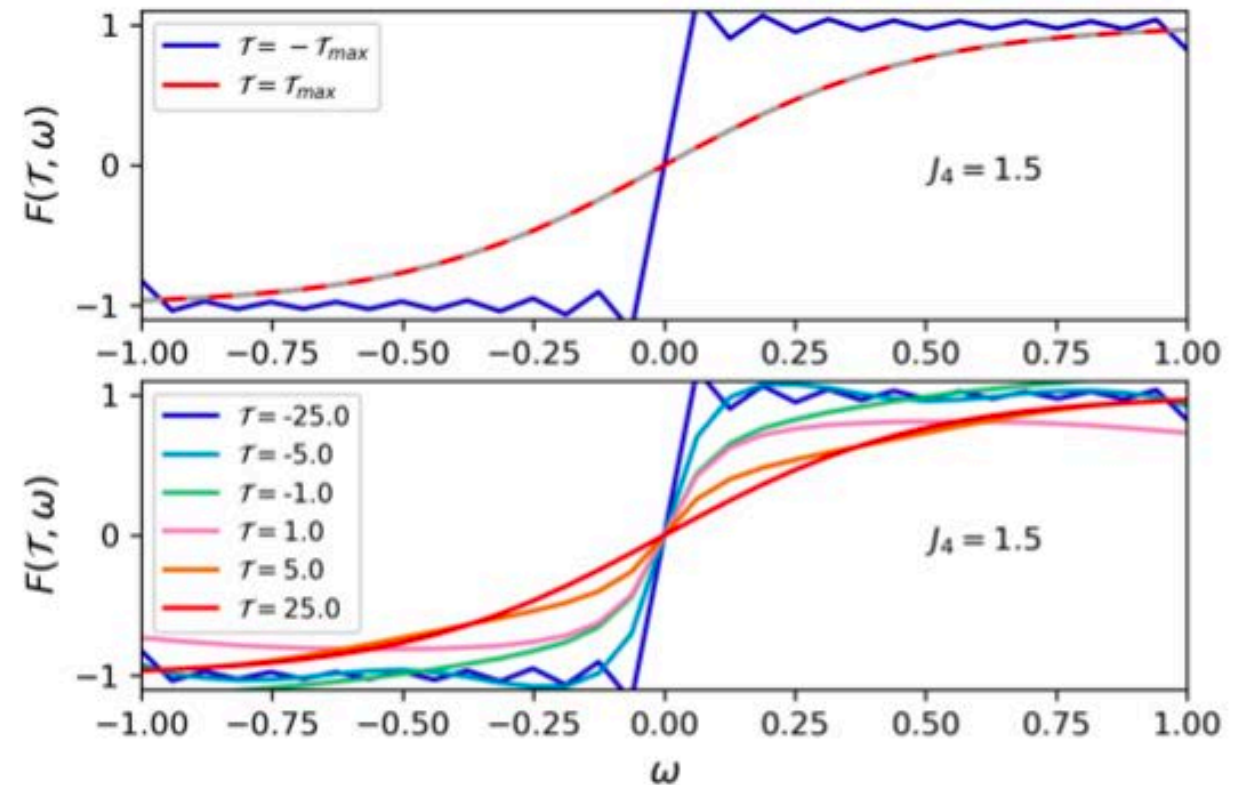
Spectral Function

$$A(\mathcal{T}, \omega) = -2\text{Im}G^R(\mathcal{T}, \omega)$$



Distribution Function

$$iG^K(\mathcal{T}, \omega) = F(\mathcal{T}, \omega)A(\mathcal{T}, \omega)$$



Exactly solvable Large N model that shows thermalisation !

See Also Numerical/Analytical Studies of ETH
(for example J.Sonner)

Allows to extract dynamics of Effective Temperature

$$F(\mathcal{T}, \omega) \equiv \tanh\left(\frac{\beta_{eff}(\mathcal{T})\omega}{2}\right)$$

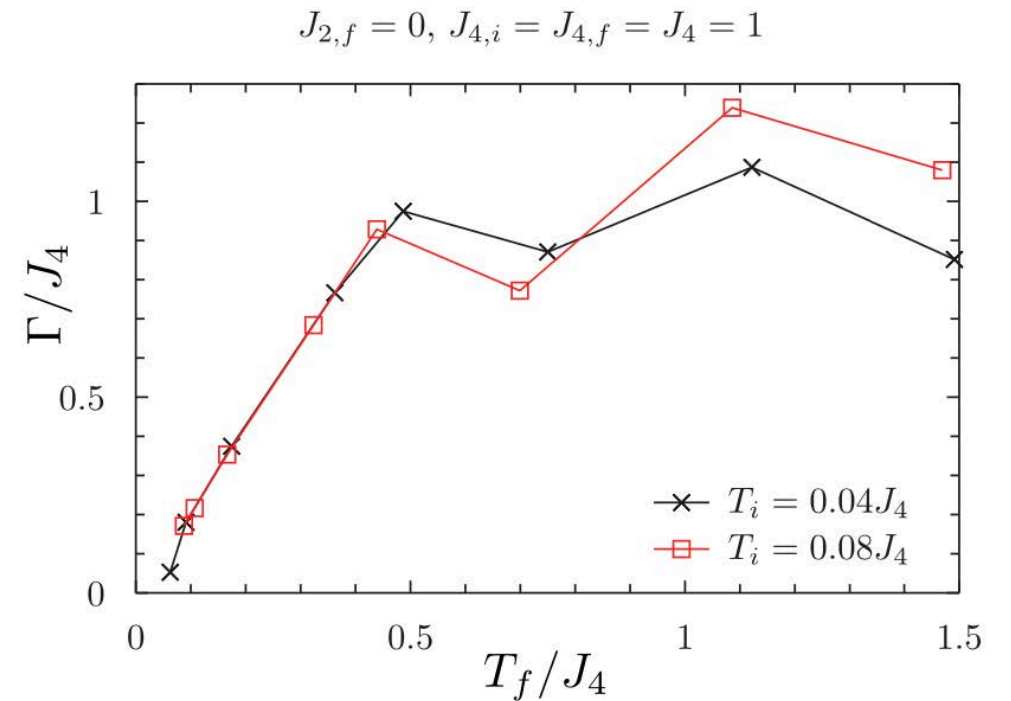
Effective temperature matches the energy injected

Thermalization time?

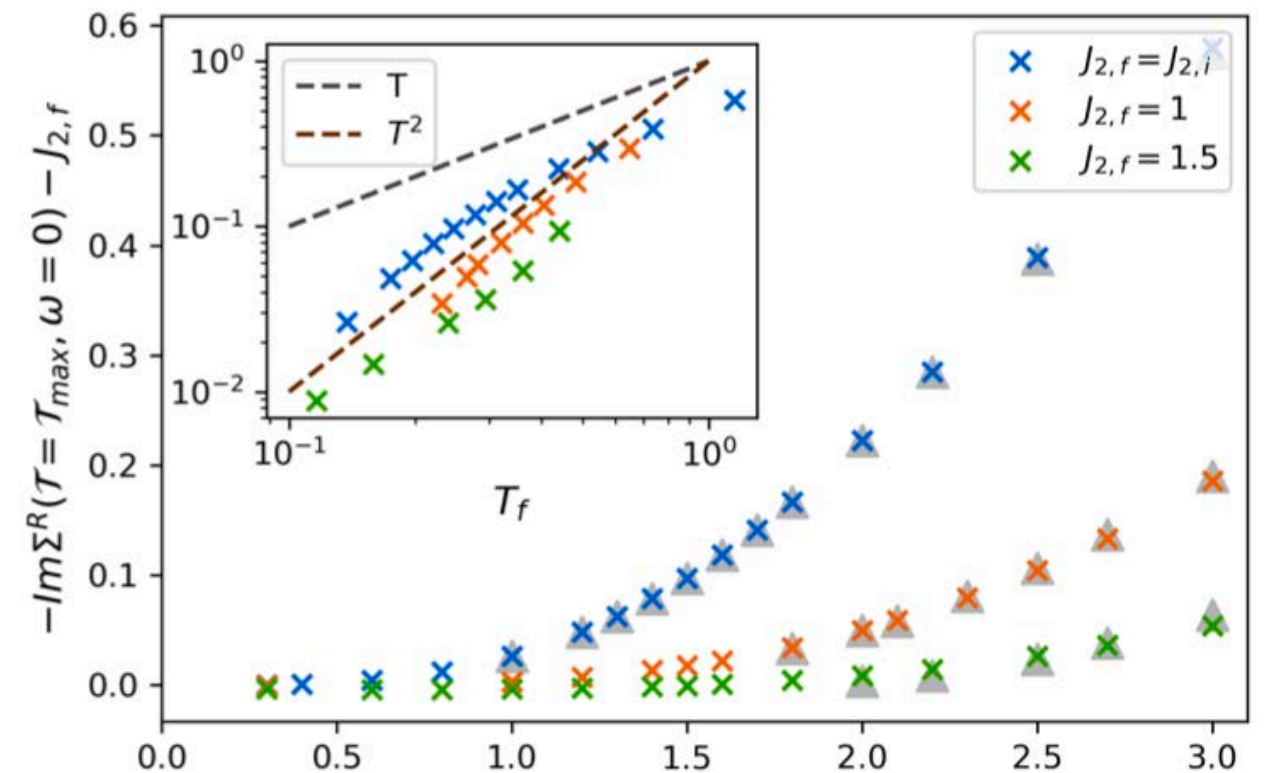
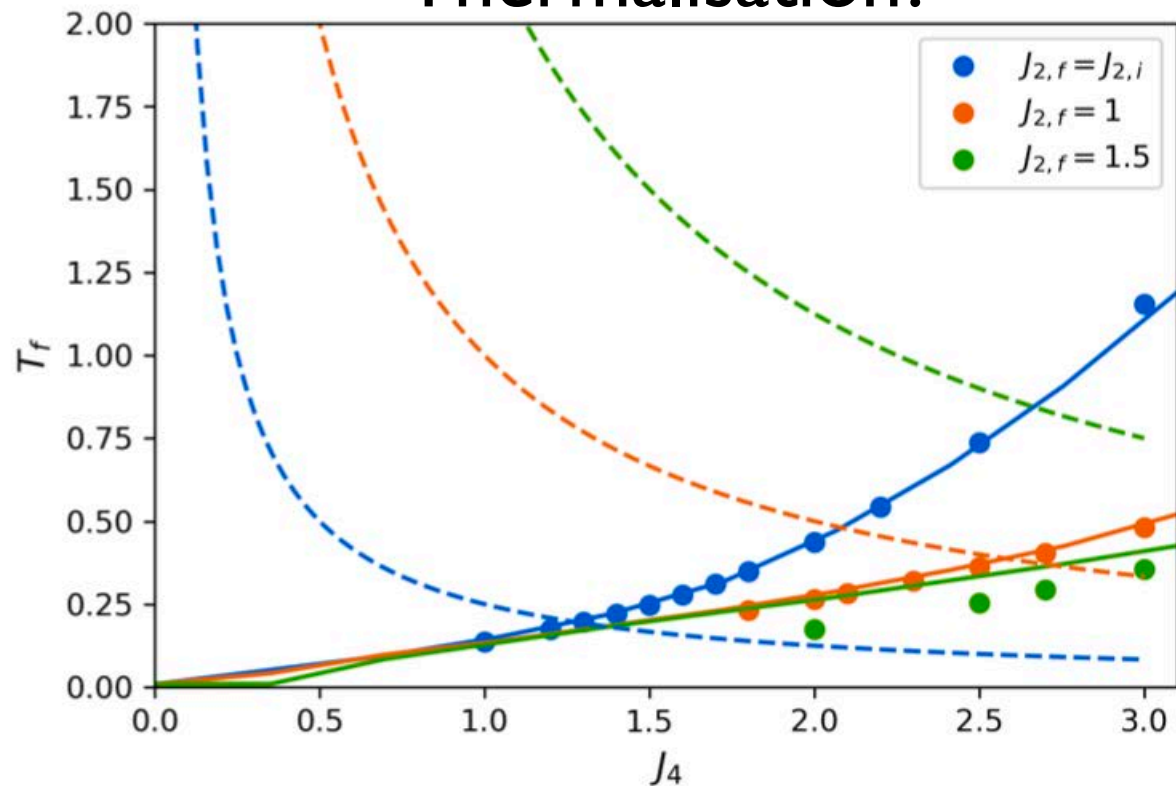
Quenches in pure SYK4:

“Planckian” Rate $\Gamma \sim T_f$

Eberlein et al PRB(2017)



Mixed SYK Role of Quasiparticles for Thermalisation?



Slow down of thermalisation rate

$$\Gamma \sim Im\Sigma^R(T) \sim T_f^2$$

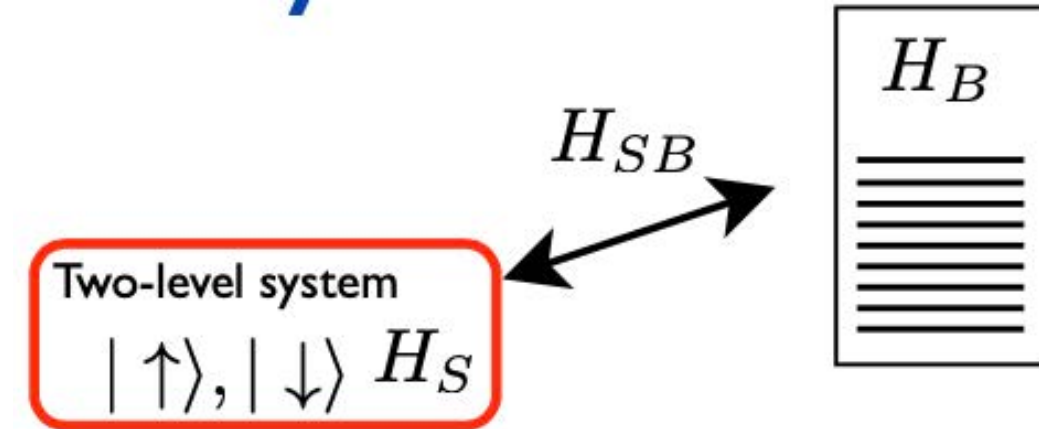
Larzul&MS PRB(2022)

SYK Model as a “Strange” Quantum Bath

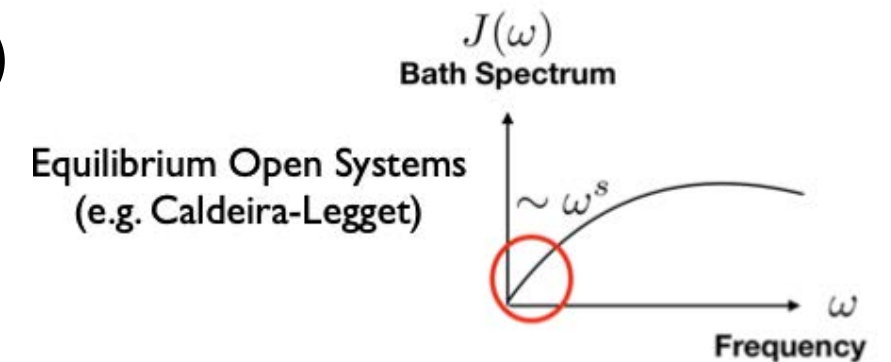
- *A. Larzul, S. J. Thomson, M. Schiro, arXiv:2204.06434*

Open Quantum Systems

📌 System+Bath Picture cft. Breuer&Petruccione



📌 Bath is usually treated as a set of non-interacting degrees of freedom (electrons, phonons, photons,..)



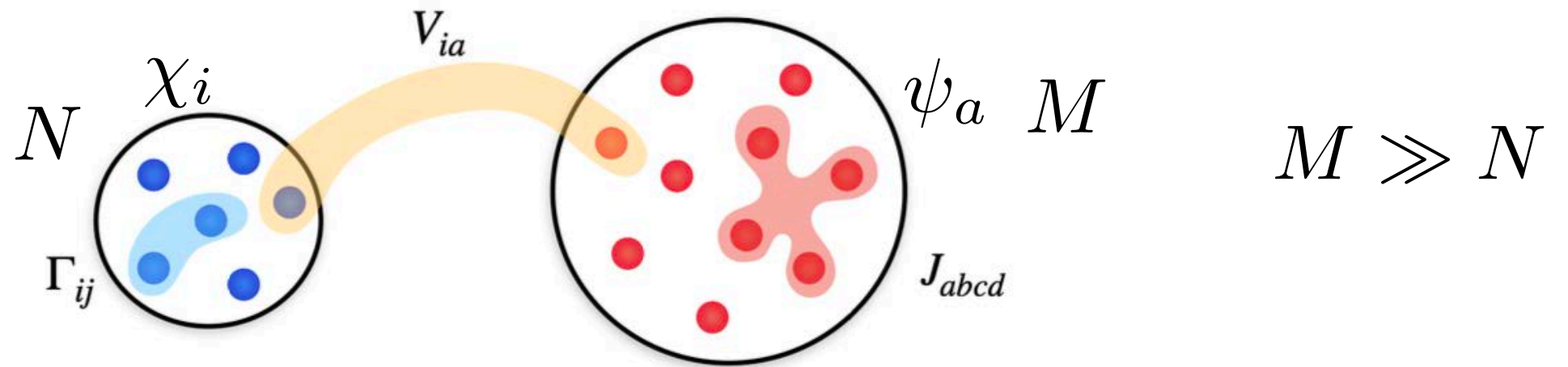
📌 Physics of “interacting” quantum baths?

📌 SYK4 provides an example of critical (gapless) bath without quasiparticles

📌 Is it SYK4 a good “thermalizer”?

Are Fast Scramblers Good Thermal Baths?

Couple a small SYK2 “System” to a large SYK4 “Bath”



$$H(t) = \frac{i}{2} \sum_{i,j=1}^N \Gamma_{i,j} \chi_i \chi_j + H_B[\psi] + \theta(t) H_{SB}[\chi, \psi]$$

Related Models (Banjeree&Altman'17, Zhang '19, Cheipesh'20, Can'20,..)

Bath fully interacting and described by SYK4

$$H_B^{(4)} = -\frac{1}{4!} \sum_{a,b,c,d=1}^M J_{abcd} \psi_a \psi_b \psi_c \psi_d$$

Sudden switching of linear system-bath coupling

$$H_{SB} = i \sum_{i=1}^N \sum_{a=1}^M V_{ia} \chi_i \psi_a$$

$$\overline{J_{abcd}^2} = \frac{3! J^2}{M^3}, \quad \overline{V_{ia}^2} = \frac{V^2}{M}$$

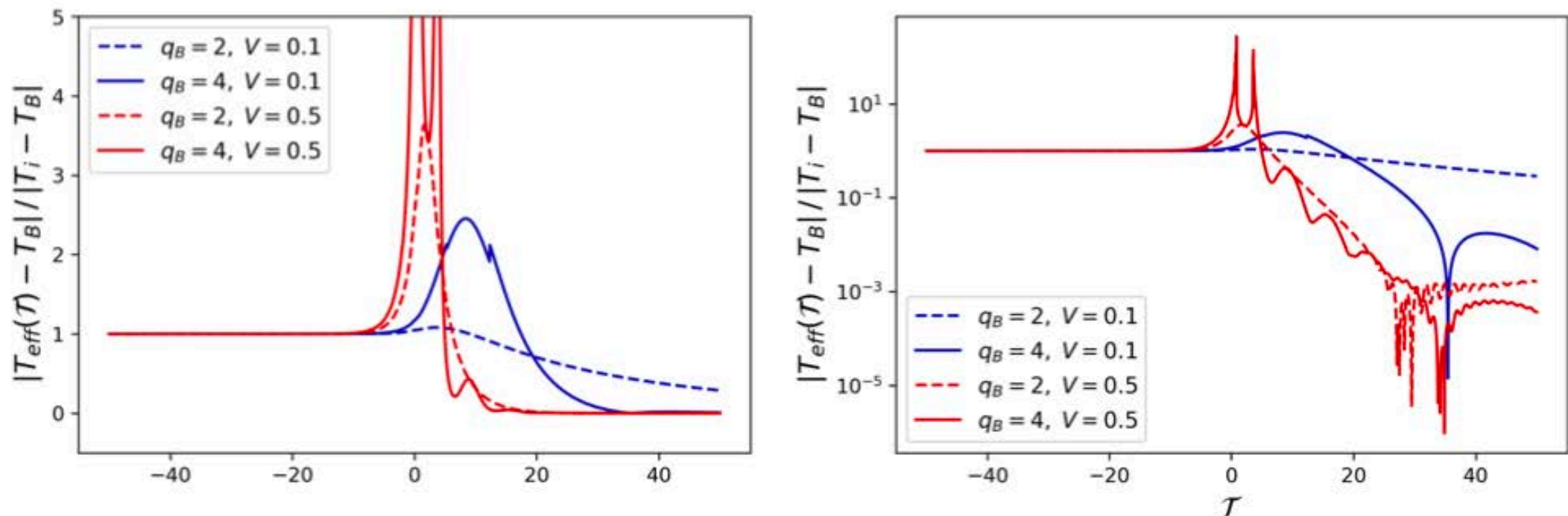
$$\overline{\Gamma_{ij}^2} = \frac{\Gamma^2}{N}$$

Compare with the case in which both system and bath are SYK2

Dynamics of Effective Temperature

Initial Condition: System hotter than the bath $T_S > T_B$

Compare Thermalization Rate for SYK4 and SYK2 baths



$$T_{eff}(\mathcal{T}) = T_B + (T_S - T_B)e^{-\lambda\mathcal{T}}$$

Thermalization to the bath temperature

Thermalization rate $\lambda(V)$ at weak and strong system-bath coupling V

SYK4 bath is a better thermalizer at small V than large V

Weak-Coupling: Fast thermalisation to a “Strange” Bath

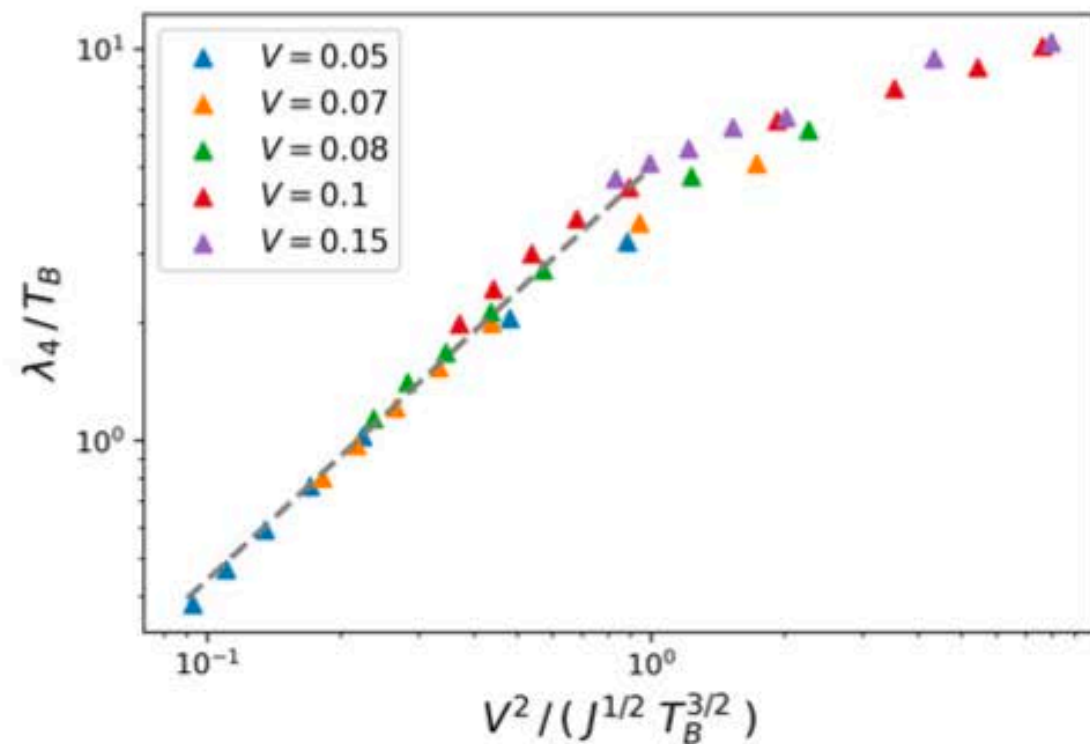
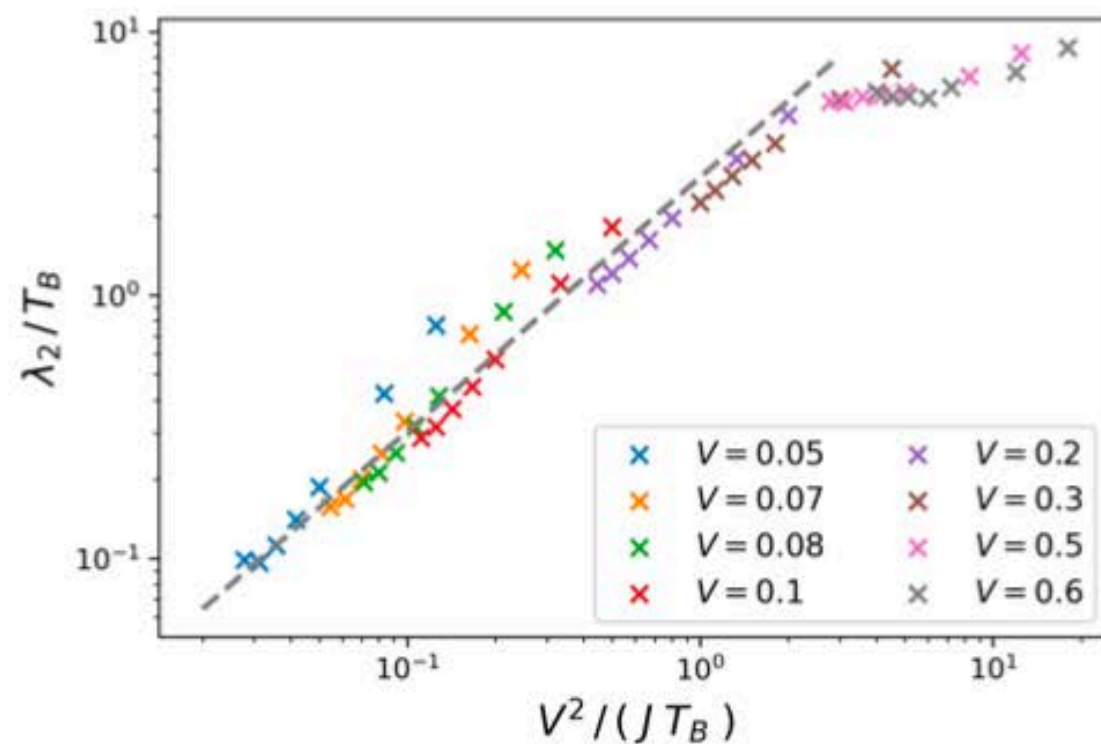
Kadanoff-Baym Equation
(Gradient expansion)

$$\partial_{\mathcal{T}} F_S(\mathcal{T}, \omega) = i\Sigma_S^K(\mathcal{T}, \omega) - iF_S(\mathcal{T}, \omega)(\Sigma_S^R(\mathcal{T}, \omega) - \Sigma^R(\mathcal{T}, \omega))$$

$$F_S(\mathcal{T}, \omega) = \tanh\left(\frac{\beta_f \omega}{2}\right) + \left[F_S(\mathcal{T}_0, \omega) - \tanh\left(\frac{\beta_f \omega}{2}\right)\right] e^{-\lambda(\omega)(\mathcal{T} - \mathcal{T}_0)}$$

📌 Thermalization time for SYK2 bath vs SYK4 bath

$$\lambda_2 \sim V^2 / J \quad \lambda_4 \sim V^2 / \sqrt{JT_B} \quad \lambda_4 \gg \lambda_2$$



📌 SYK4 bath is parametrically more effective than a FL bath as “thermaliser”

Energy Transport Through a Strange Quantum Bath

- *A. Larzul, M. Schiro', arXiv.2206.XXX (to appear)*

Thermal Transport of Quasiparticles

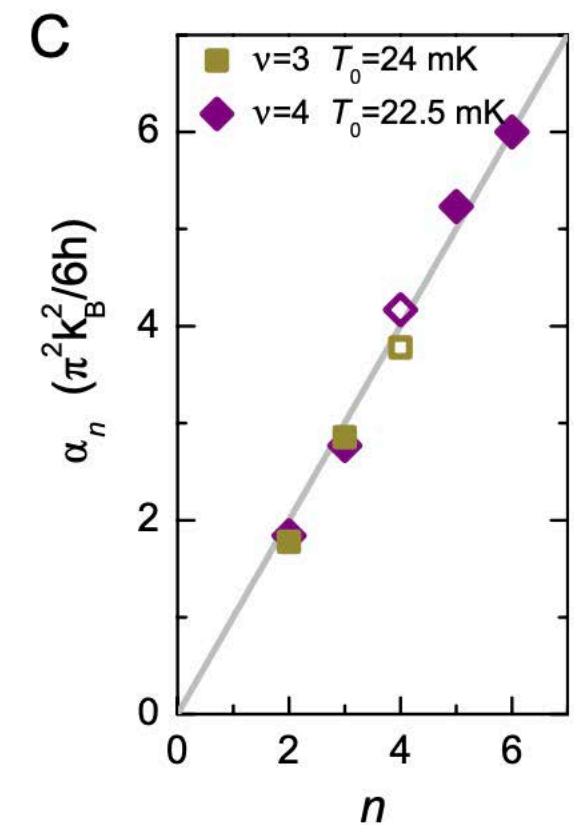
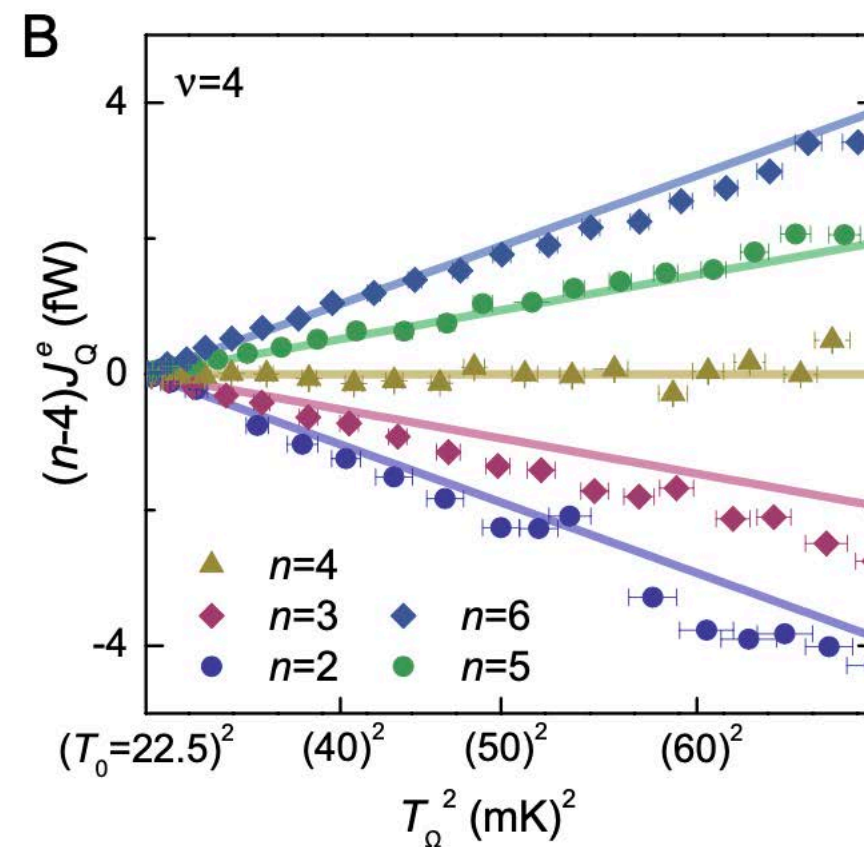
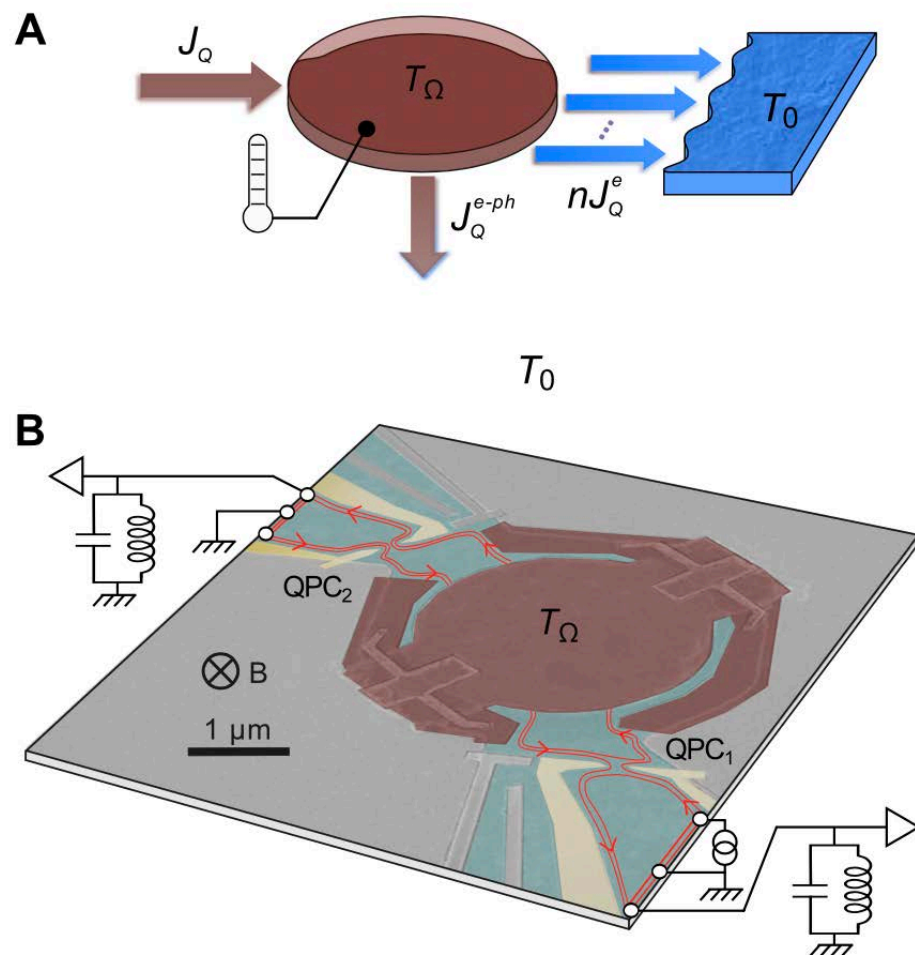
Example: Ballistic channel (Landauer-Buttiker)

J. Pekola & B. Karimi RMP(2021)

$$J_Q^e(T_\Omega, T_0) = \frac{\pi^2 k_B^2}{6h} (T_\Omega^2 - T_0^2).$$

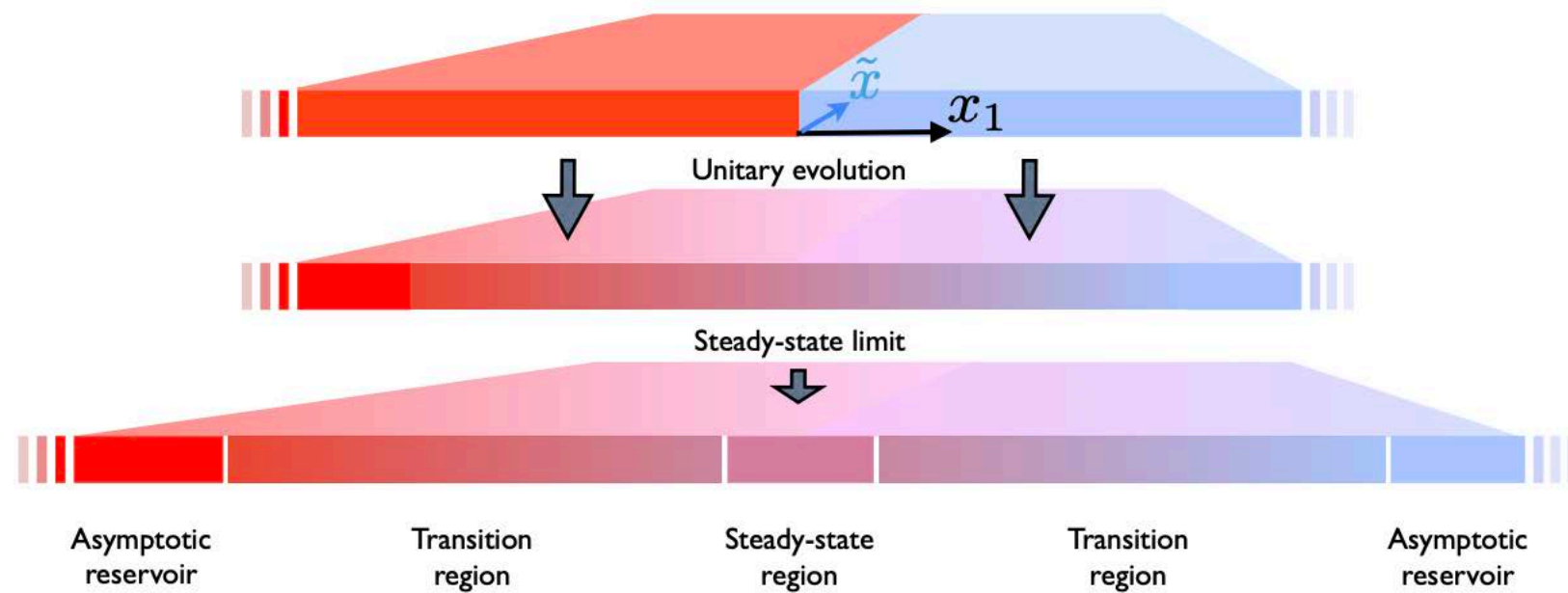
$$G(T) = \frac{\pi^2 k_B^2 T}{3h}$$

Quantum of Thermal Conductance



S. Jezouin, ..., F. Pierre, Science (2013)

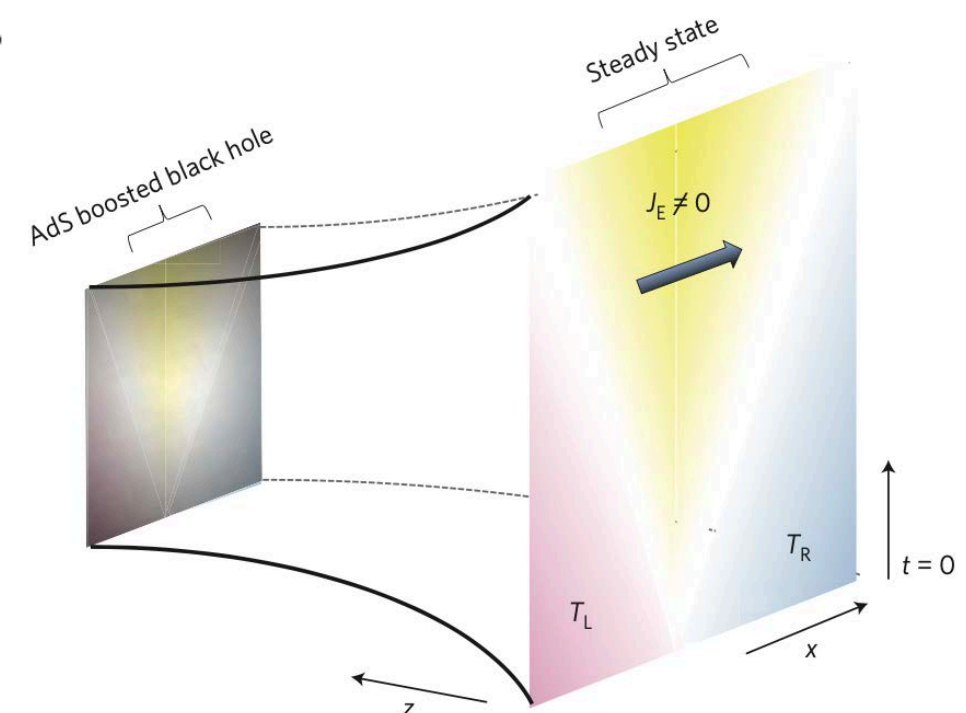
Thermal Transport from CFT



📌 **Steady-State Energy Current from merging CFTs ($d=1$)**

D. Bernard, B. Doyon (2012)

$$J = \frac{c\pi}{12\hbar} k_B^2 (T_l^2 - T_r^2).$$

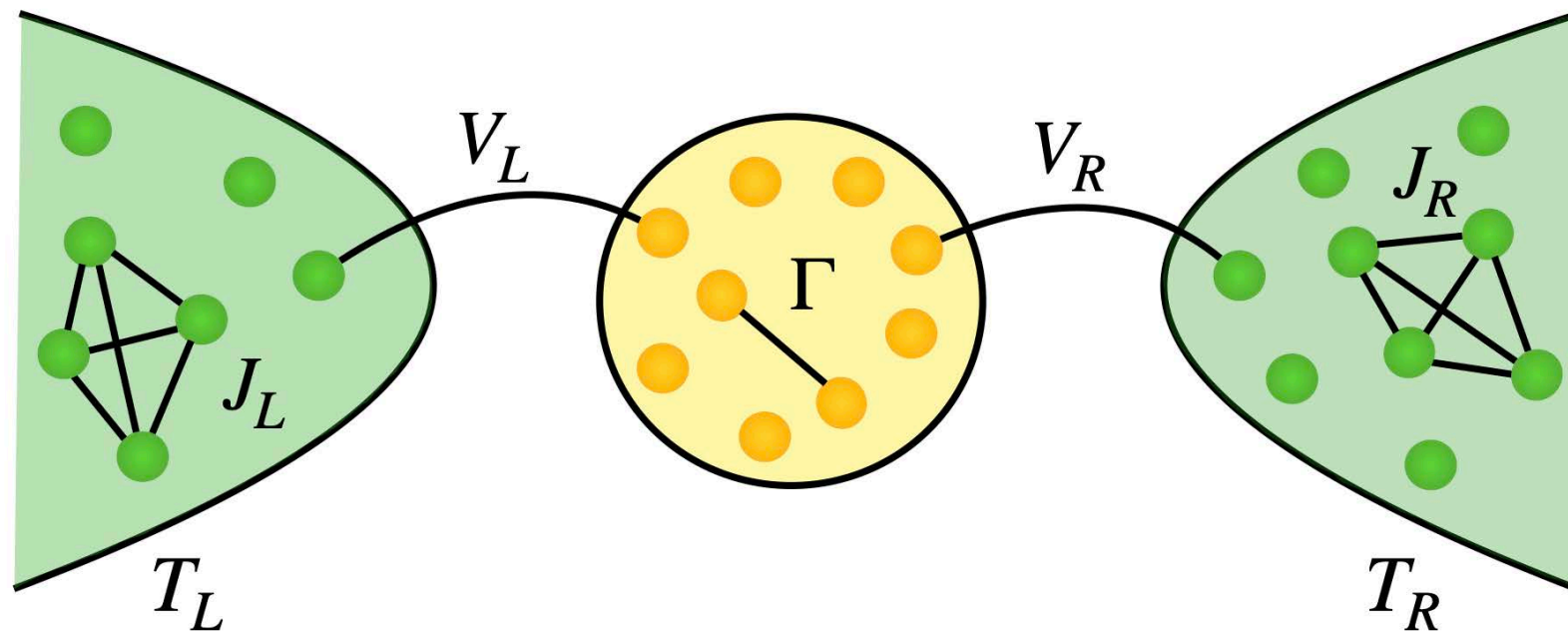


📌 **Insights from AdS for $d > 1$:
“boosted Black Hole”**

Bhaseen, ..., Schalm, Nature Phys 2015

SYK Energy Transport Setup

Two SYK4 bath at different temperatures coupled by a SYK2 island



$$H = \sum_{\alpha=L,R} H_4^\alpha + H^S + \sum_{\alpha=L,R} H^{S\alpha}$$

$$H_4^\alpha = -\frac{1}{4!} \sum_{a,b,c,d=1}^M J_{abcd} \psi_a^\alpha \psi_b^\alpha \psi_c^\alpha \psi_d^\alpha \quad , \quad \overline{J_{abcd}^2} = \frac{3!J^2}{M^3}, \quad \overline{V_{ia}^2} = \frac{V^2}{M}$$

$$H^S = \frac{i}{2} \sum_{i,j=1}^N \Gamma_{ij} \chi_i \chi_j \quad H^{S\alpha} = i \sum_{i=1}^N \sum_{a=1}^M V_{ia} \chi_i \psi_a^\alpha \quad \overline{\Gamma_{ij}^2} = \frac{\Gamma^2}{N}$$

Exactly solvable in large N,M at fixed $p = N/M$

Energy Current Between SYK

Focus here: Energy Current in the stationary state $\mathcal{J}_\alpha = \overline{\dot{E}_\alpha(t)} = i \overline{\langle [H, H_\alpha] \rangle}(t)$

Exact Formula in large N,M limit

$$\mathcal{J} = -\frac{NV^2}{2} \int \frac{d\omega}{2\pi} \omega (G_L^<(\omega) - G_R^<(\omega)) G_S^>(\omega)$$

Current depends on Green's functions of L/R Baths and System (coupled through Dyson Equation)

Analogous to Meir-Wingreen Formula for charge transport in quantum dots: here interacting reservoirs!

Here: assume $N \ll M$ (large bath, not renormalised by the system)

Linear Thermal Conductance

$$\mathcal{G}(T) = \frac{NV^2}{2} \int \frac{d\omega}{2\pi} \omega A_S^{eq}(\omega) f_{eq}(-\omega) \frac{\partial}{\partial T} (A_B^{eq}(\omega) f_{eq}(\omega))$$

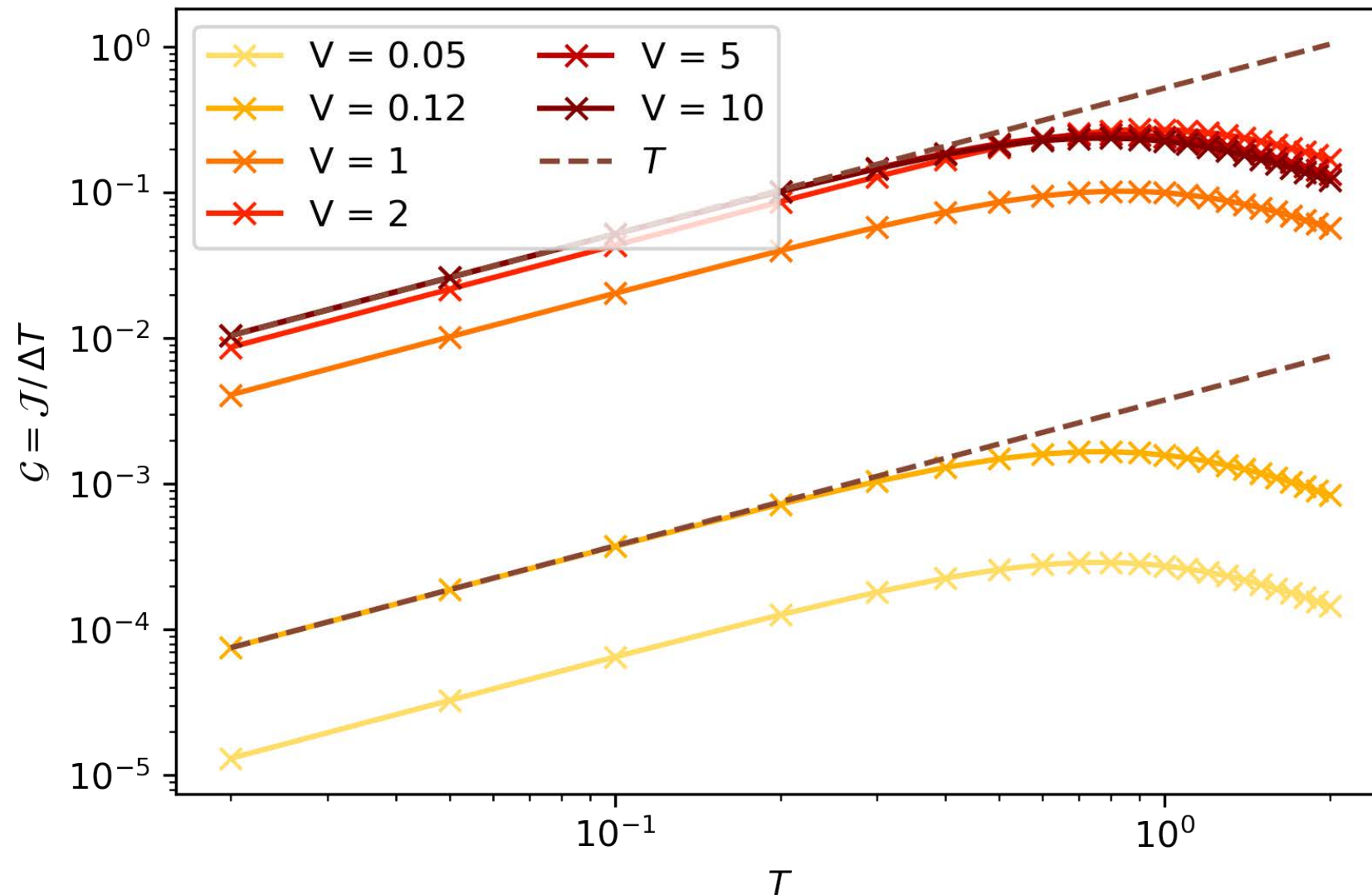
Warmup: Energy Transport by Quasiparticles

Consider SYK2 baths

$$T_{L/R} = T \pm \Delta T$$

Thermal Conductance

$$\mathcal{G} \sim \mathcal{J} / \Delta T$$



📌 Thermal Conductance is linear in T ,
for all couplings V

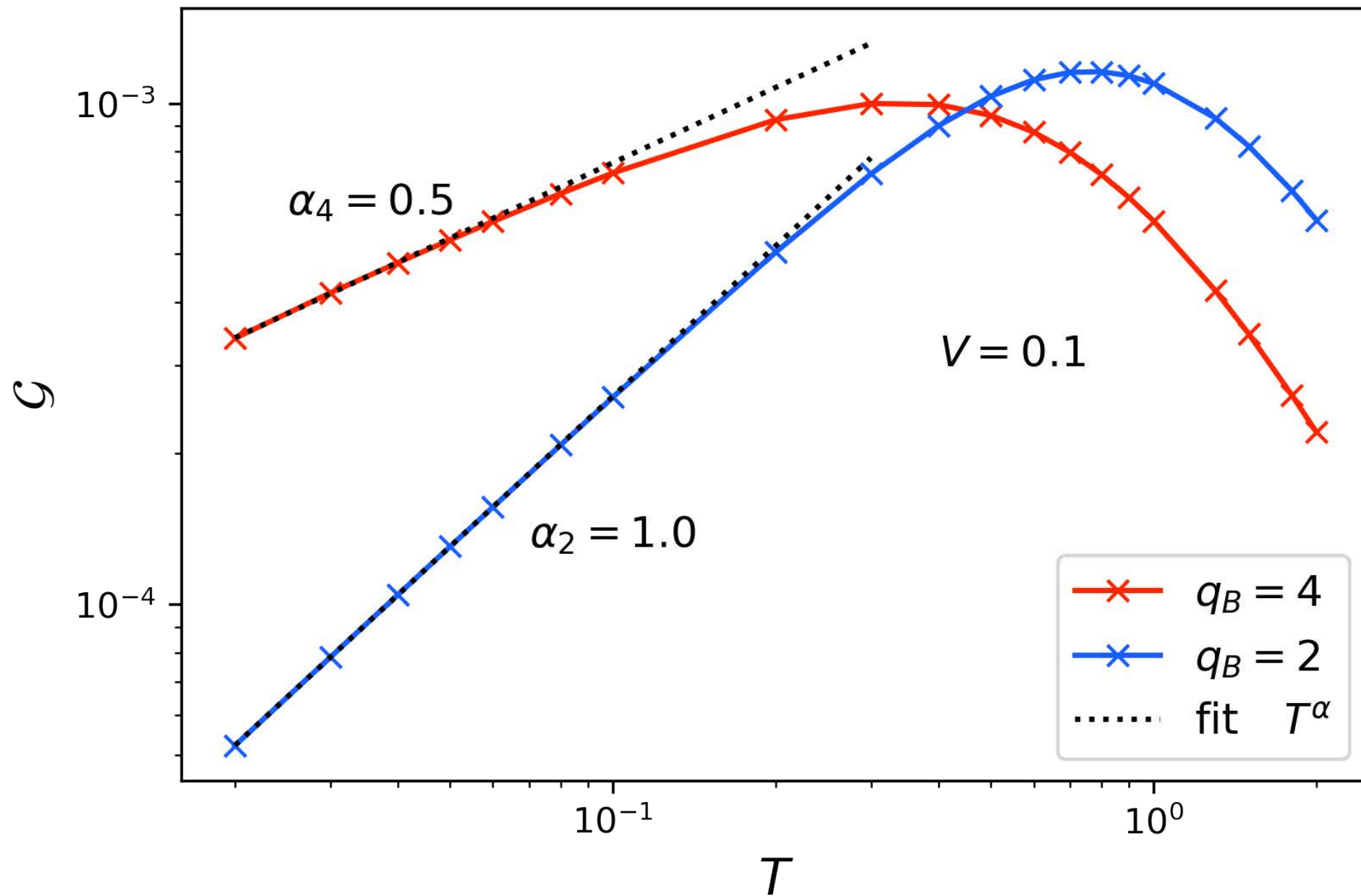
$$\mathcal{G}_2 \sim \mathcal{G}_Q(T) \sim \frac{\pi^2 k_B^2}{3h} T$$

SYK4 Baths: Weak System-Bath Coupling

Thermal Conductance

$$\mathcal{G} \sim \mathcal{J} / \Delta T$$

$$T_{L/R} = T \pm \Delta T$$



📌 Thermal conductance acquires anomalous scaling

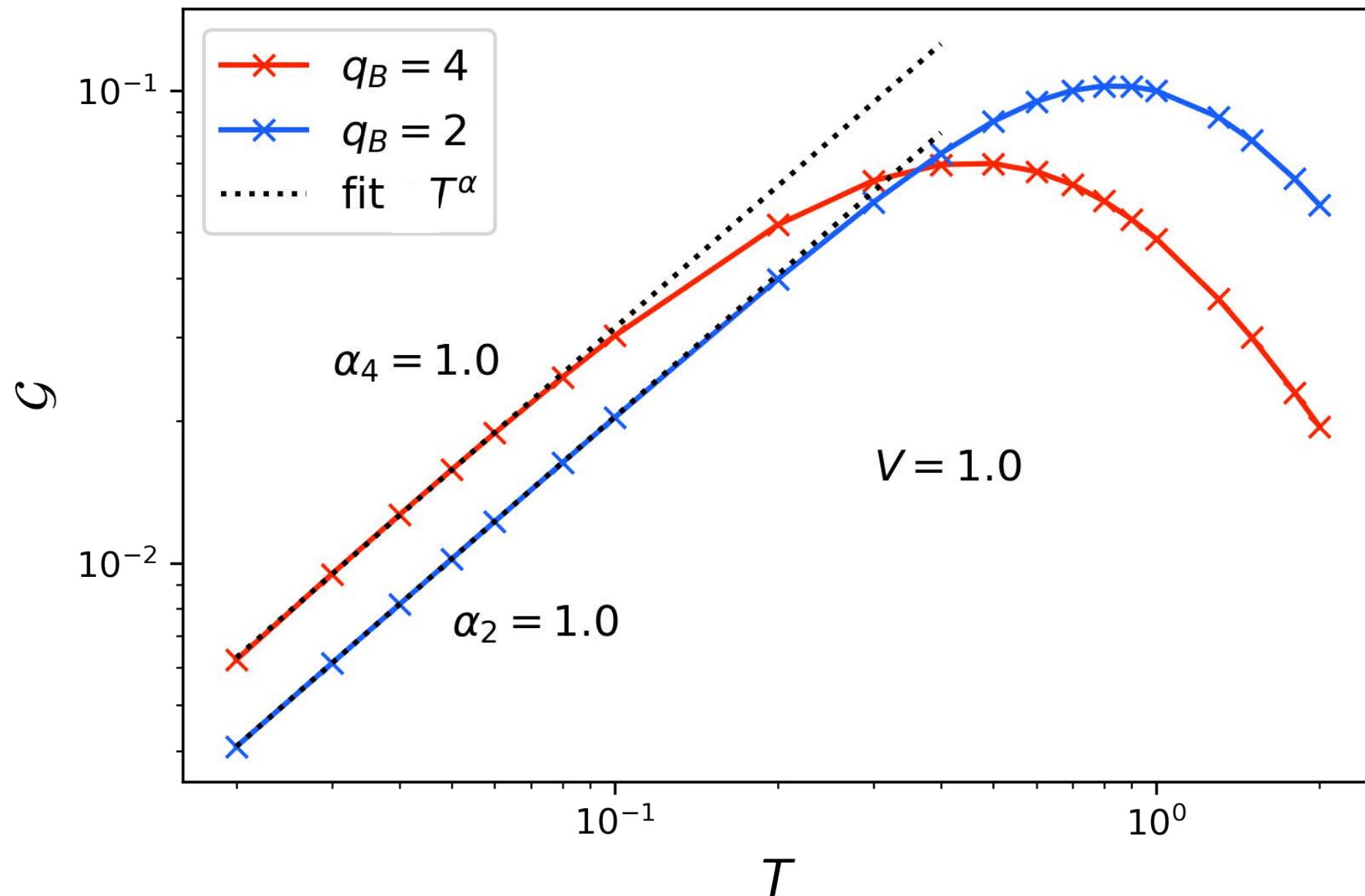
$$\mathcal{G}_4(T) \sim \sqrt{T}$$

SYK+FL Bath: Kruchov, Patel, Kim, Sachdev'20

📌 Enhancement of Thermal Conductance due to the interacting bath

SYK4 Baths: Strong System-Bath Coupling

Thermal Conductance $\mathcal{G} \sim \mathcal{J}/\Delta T$ $T_{L/R} = T \pm \Delta T$



Linear-T behaviour is restored $\mathcal{G}_4(T) \sim T$

FL? No! Rather a different mechanism at play!

Insights from Spectral Functions

📌 Weak-Coupling: system is “decoupled” at low frequency...

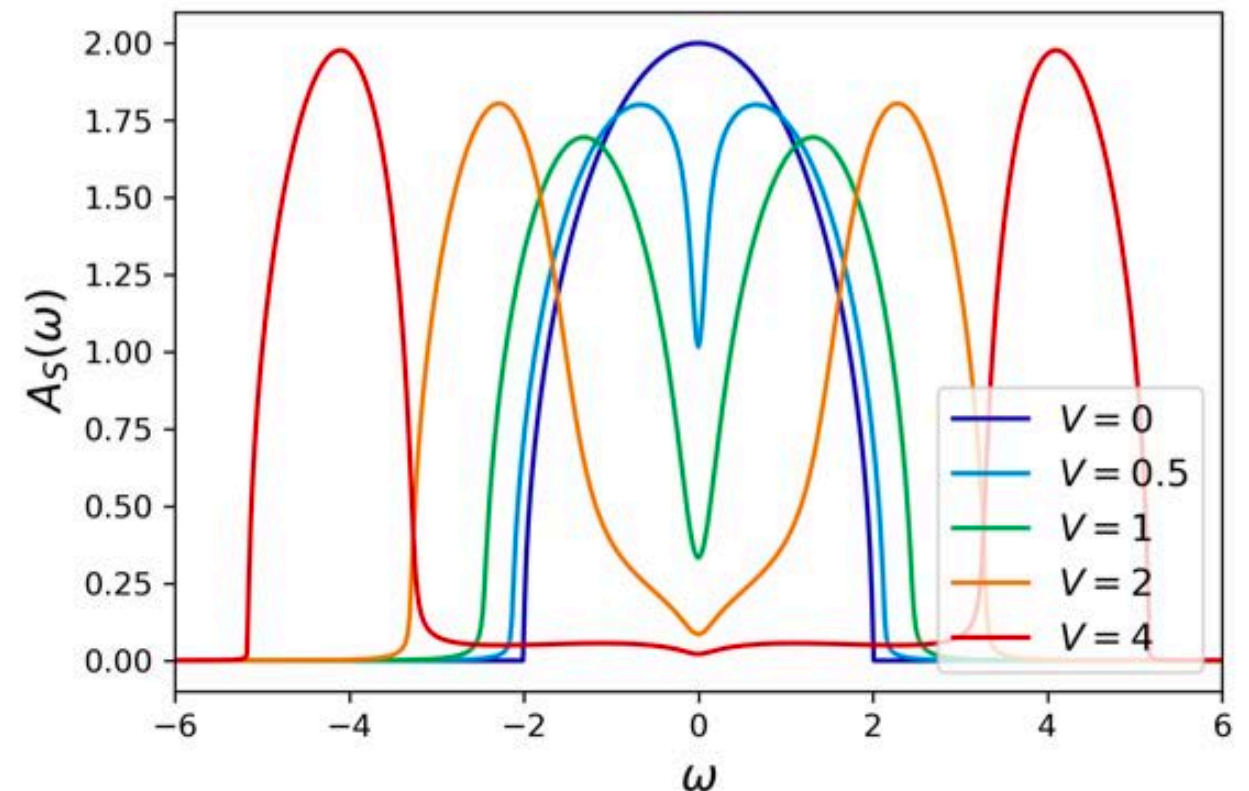
$$A_S^{eq}(\omega \rightarrow 0) \sim 2/\Gamma \quad A_B^{eq}(\omega) = 2 \left(\frac{\pi}{J^2} \right)^{1/4} \frac{1}{\sqrt{2\pi T}} \operatorname{Re} \left(\frac{\Gamma\left(\frac{1}{4} - \frac{i\omega}{2\pi T}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\omega}{2\pi T}\right)} \right)$$

...Enhanced Bath Spectral function leads to enhanced thermal conductance

📌 Strong Coupling: System spectral function is largely renormalised

$$A_S^{eq}(\omega) = \frac{1}{V^2} \left(\frac{J^2}{\pi} \right)^{1/4} \sqrt{2\pi T} \operatorname{Re} \left(\frac{\Gamma\left(\frac{3}{4} - \frac{i\omega}{2\pi T}\right)}{\Gamma\left(\frac{1}{4} - \frac{i\omega}{2\pi T}\right)} \right)$$

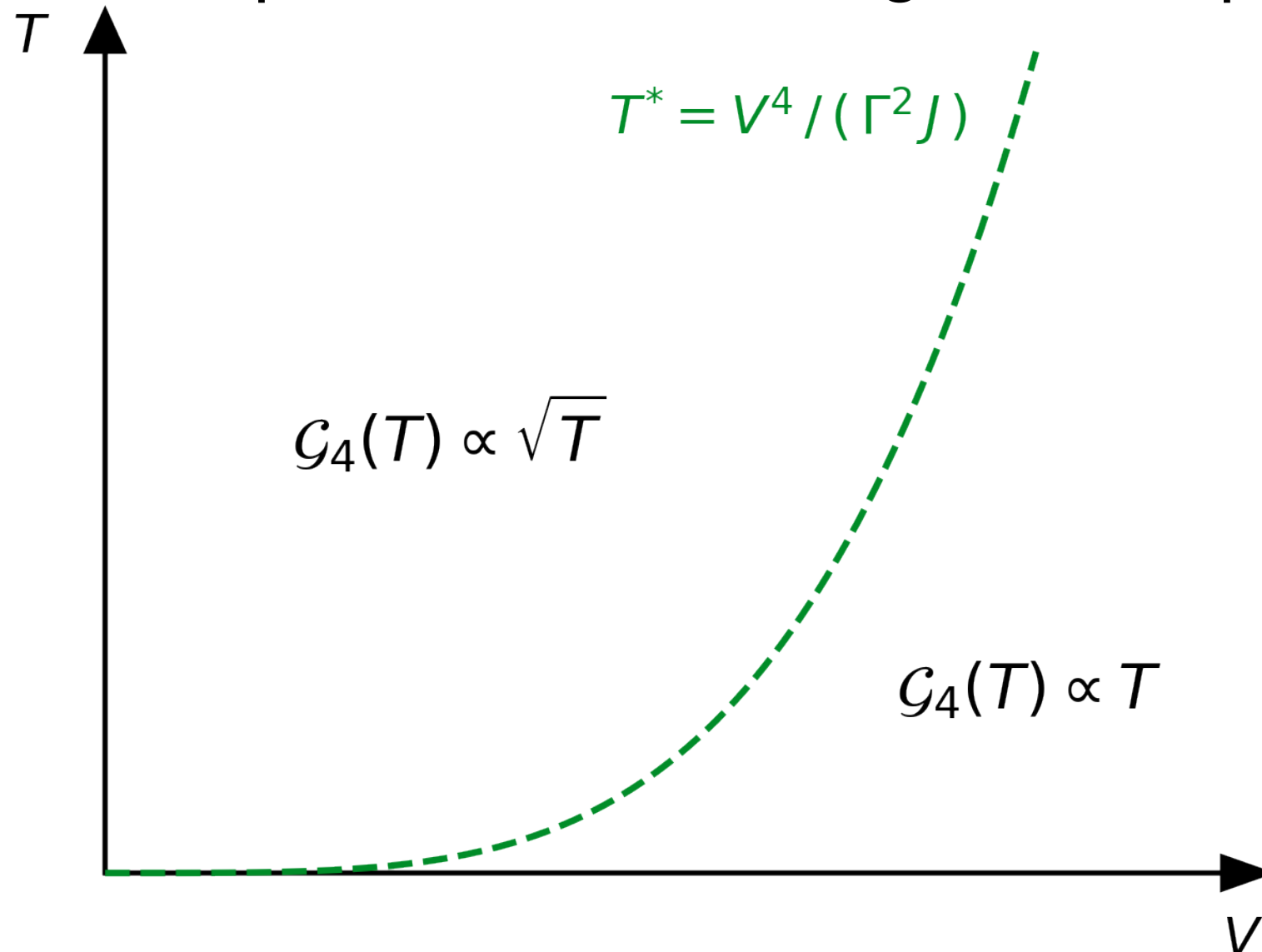
$$A_S^{eq}(\omega \rightarrow 0) \sim \sqrt{\omega}$$



...Suppressed System Spectral function cancels Bath enhancement and gives back to linear-T thermal conductance

Energy Transport Phase-Diagram

Crossover in transport from weak to large bath coupling



Weak-Coupling, $T > T^*$ — broad T range where conductance is enhanced by SYK4 bath

Strong Coupling: conductance is linear in T ...but the system is not a FL!

T^* signals a crossover in the Liapunov exponent

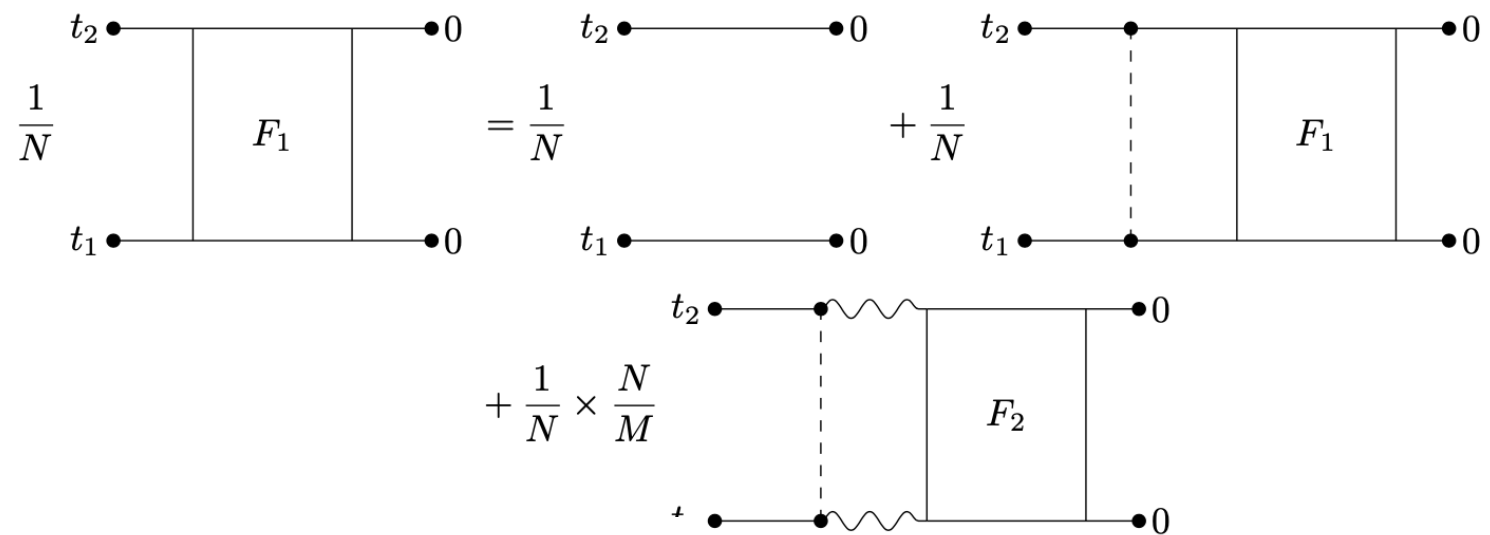
Chaos between System and Strange Bath

$$H = \frac{i}{2} \sum_{i,j=1}^N \Gamma_{ij} \chi_i \chi_j - \frac{1}{4!} \sum_{m,n,p,q=1}^M \psi_m \psi_n \psi_p \psi_q + i \sum_{i=1}^N \sum_{m=1}^M V_{im} \chi_i \psi_m$$

📌 Isolated system SYK2 is non chaotic (Liapunov Exponent=0)

📌 Coupling to the SYK4 bath induce a non trivial correction to OTOC

$$C_1(t_1, t_2) = \frac{1}{N^2} \sum_{i,j=1}^N \left\langle \chi_{2,i}^r(t_1) \chi_{2,j}^a(0) \chi_{1,i}^r(t_2) \chi_{1,j}^a(0) \right\rangle_S = \frac{1}{N} F_1(t_1, t_2) + ..$$



Mixed-Bath OTOC: Chaos between system&bath

$$C_2(t_1, t_2) = \frac{1}{NM} \sum_i^N \sum_{m=1}^M \left\langle \psi_{2,m}^r(t_1) \chi_{2,i}^a(0) \psi_{1,m}^r(t_2) \chi_{1,i}^a(0) \right\rangle_S = \frac{1}{N} F_2(t_1, t_2)$$

📌 Below T^* Liapunov Exponent crosses over to the maximal value

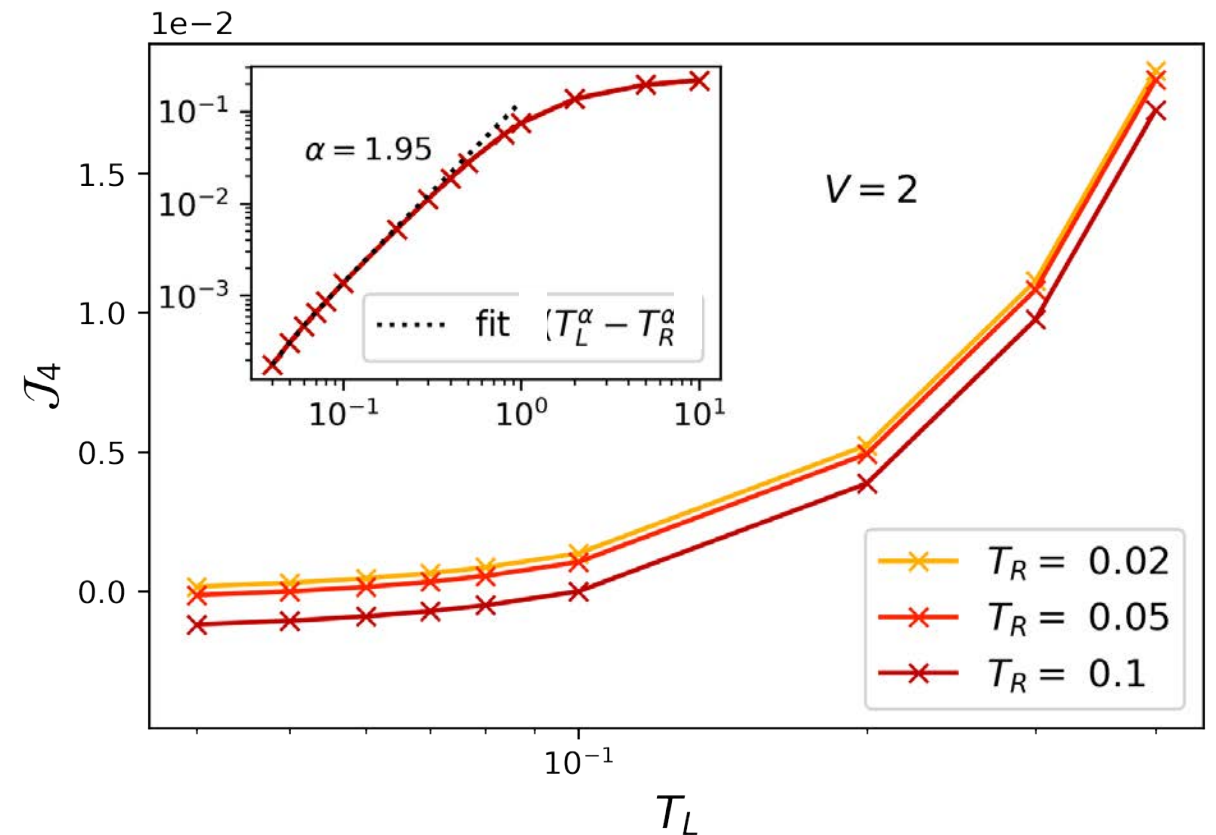
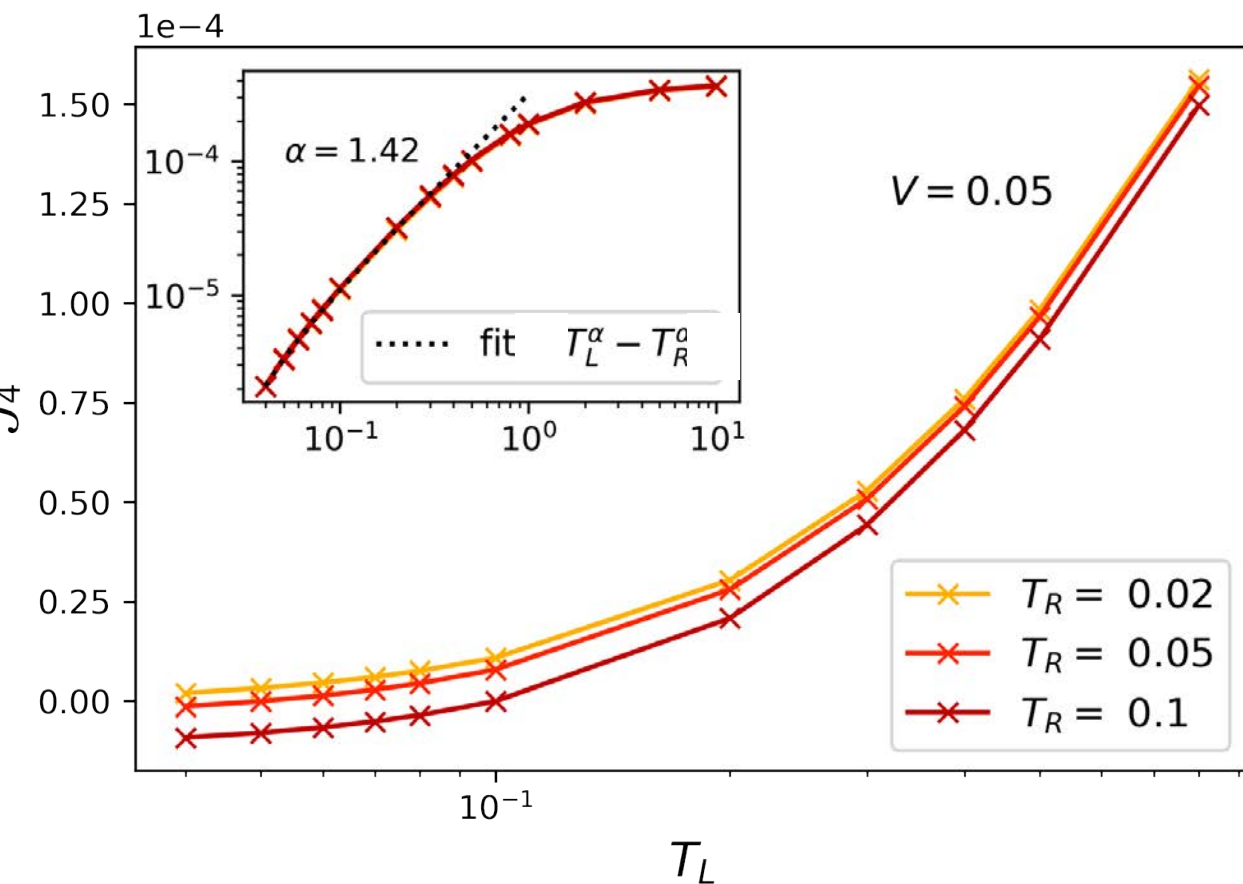
Non-Equilibrium Energy Transport

Energy current beyond linear response: $\Delta T \sim T_L, T_R$

SYK2 Baths $\mathcal{J} = \Phi(T_L) - \Phi(T_R)$ $\Phi(T) \sim T^2$

D. Bernard, B. Doyon (2012)

SYK4 Baths:



Scaling collapse support still the structure $\mathcal{J} = \Phi(T_L) - \Phi(T_R)$

For weak-coupling: $\Phi(T) \sim T^{3/2}$ For large-coupling: $\Phi(T) \sim T^2$

Conclusions&Perspectives

- “Strange” Quantum Matter Far From Equilibrium
- SYK Model as “Strange Quantum Bath”: fast thermalisation dynamics
- Energy Transport: Enhanced Conductance at weak coupling, cross-over to linear T scaling but still no QP (chaos crossover)
- Distribution function in non-equilibrium steady-state: boosting?
- Interplay of energy and particle transport (charged SYK)?
Energy Current Fluctuations?
- Scrambling and Thermalisation of Non-Fermi Liquids?

Thanks!