

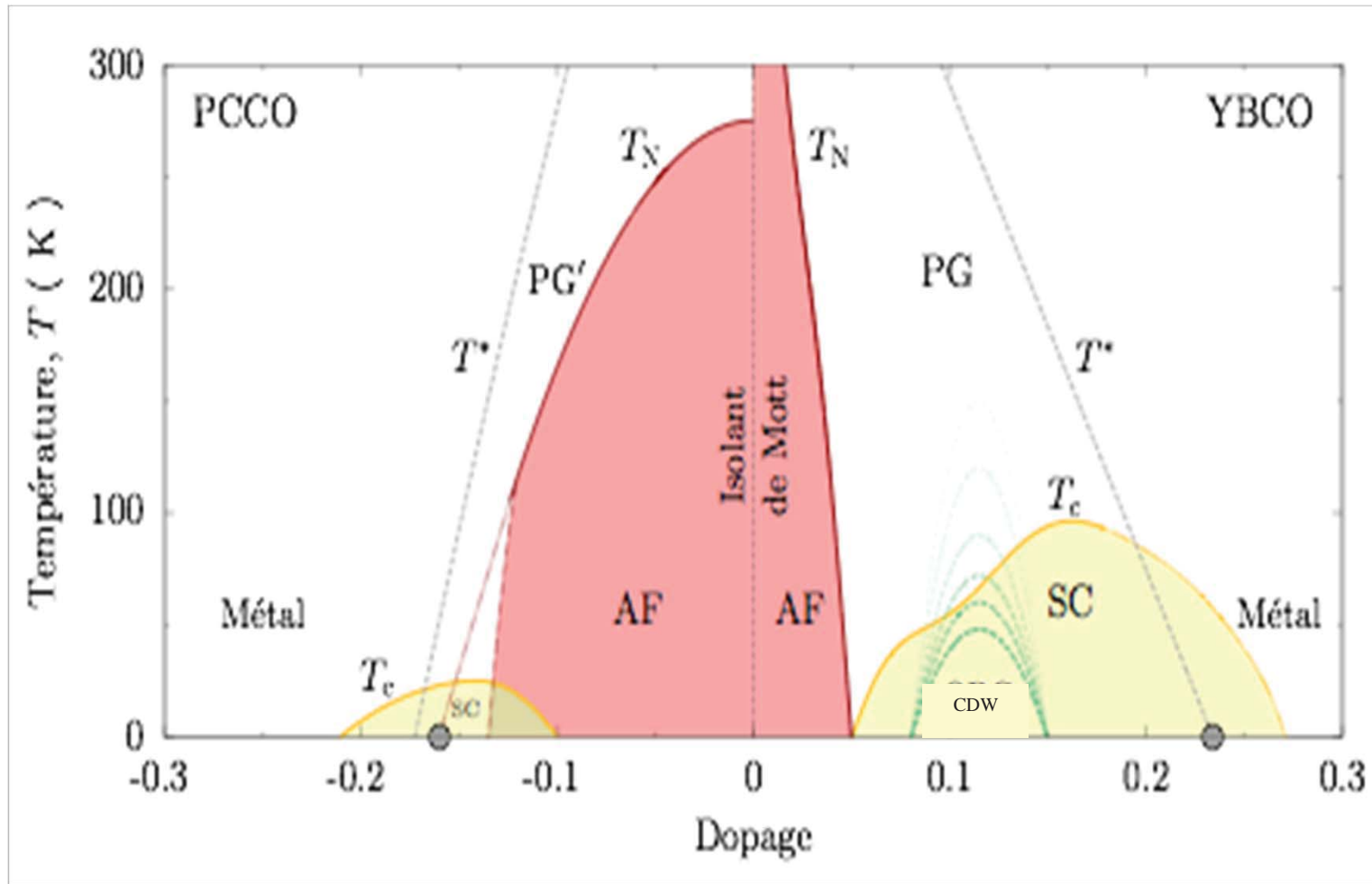
## 6. Charge Density Wave

h-doped

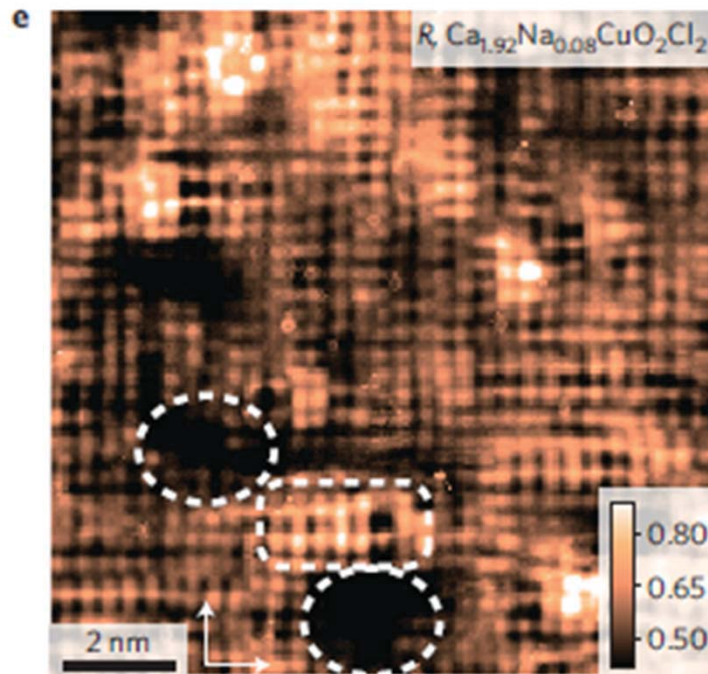


# Our road map

Thèse de Francis Laliberté,  
Université de Sherbrooke



# Intra-unit cell nematic order: STM

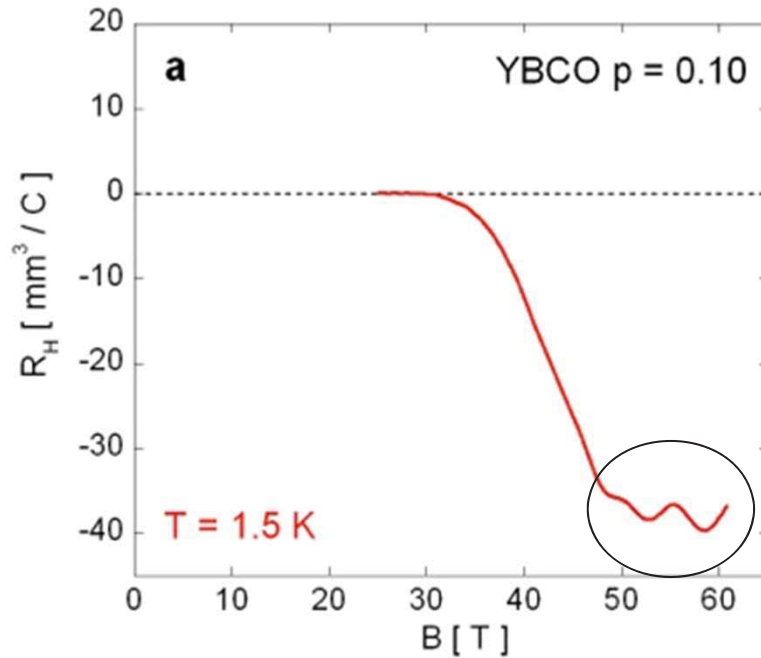


Kohsaka et al. Nature Physics 2012

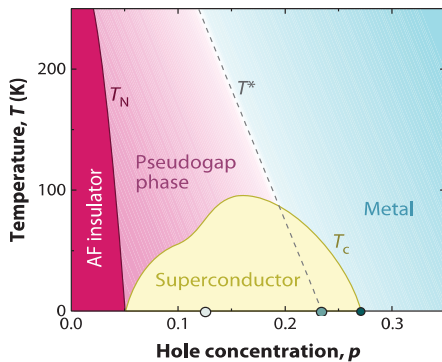
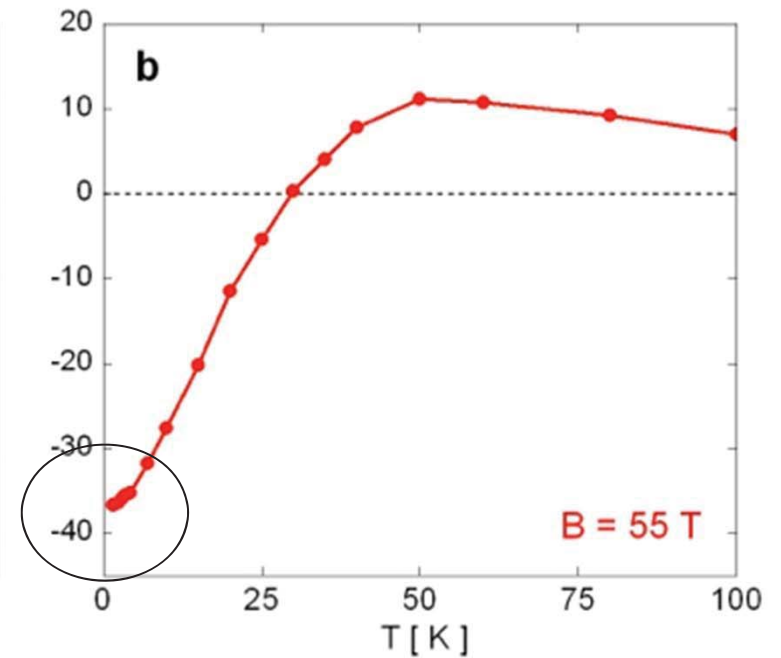


# Quantum oscillations in cuprates: 2007

N. Doiron-Leyraud et al., Nature 2007



D. LeBoeuf et al., Nature 2007



*Quantum oscillations*

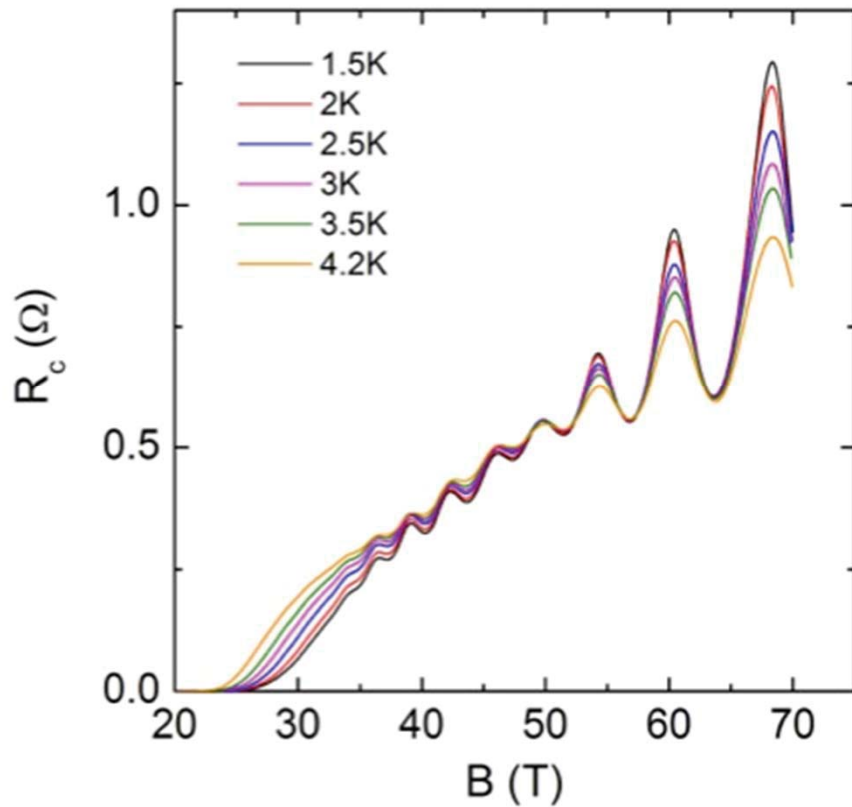
$$R_H < 0$$

**Fermi surface includes a small *electron* pocket !**



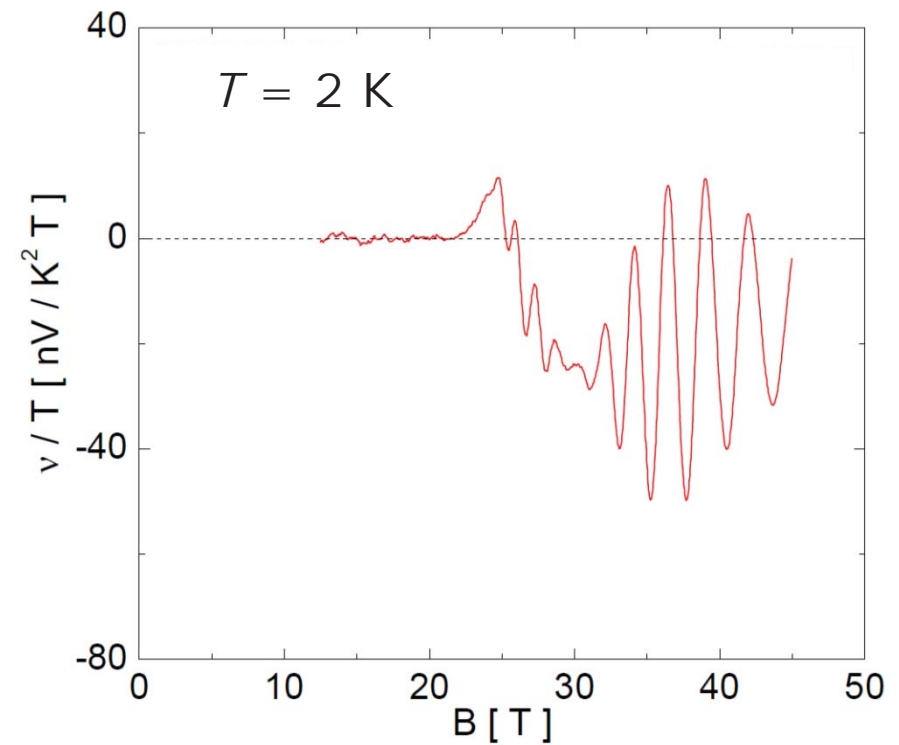
# Quantum oscillations in cuprates: 2013

Resistance



LNCMI, Toulouse

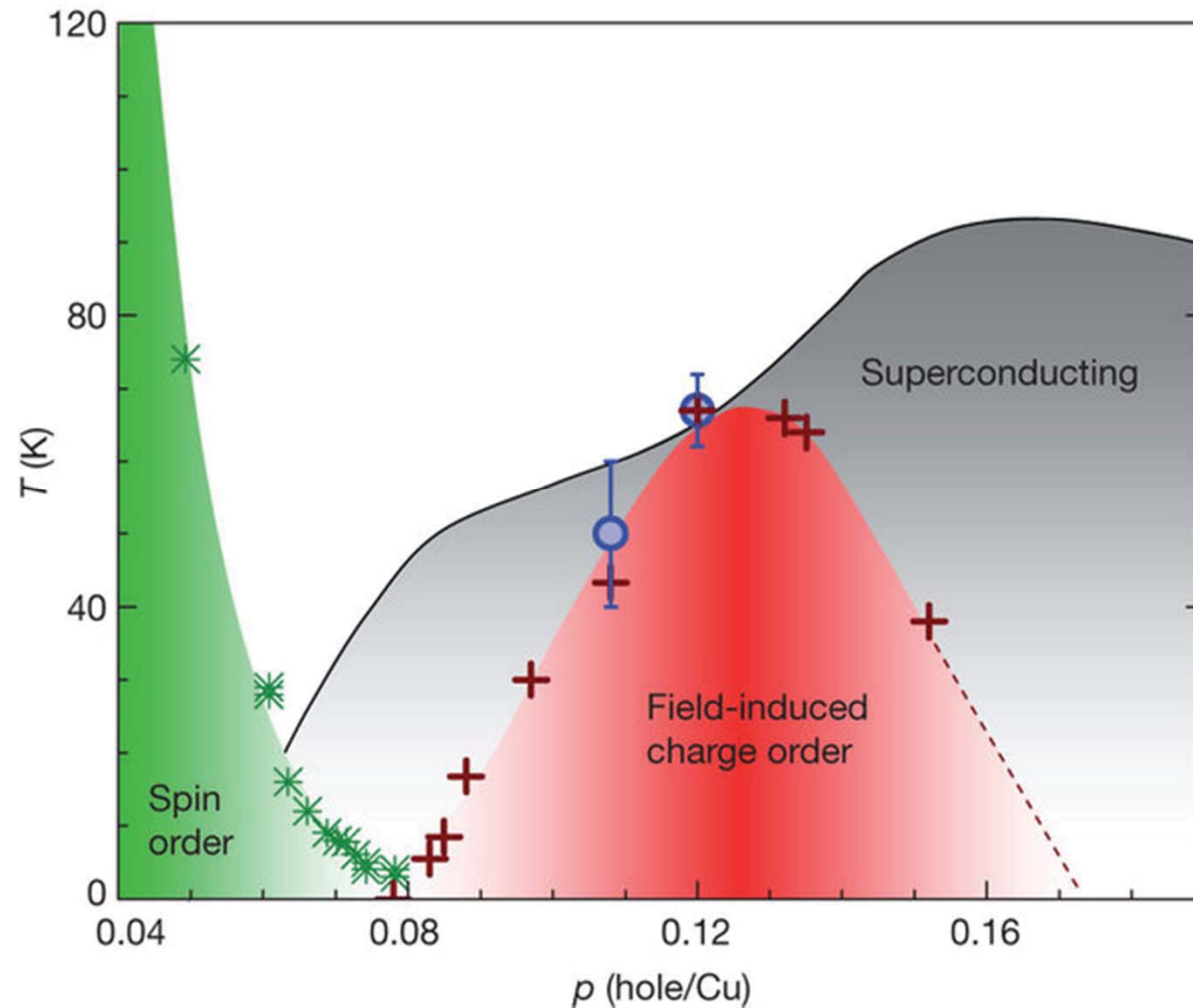
Nernst



NHMFL, Tallahassee



# Stripes and reconstructed Fermi surface



Wu et al. Julien, Nature **477**, 191–194 (2011)

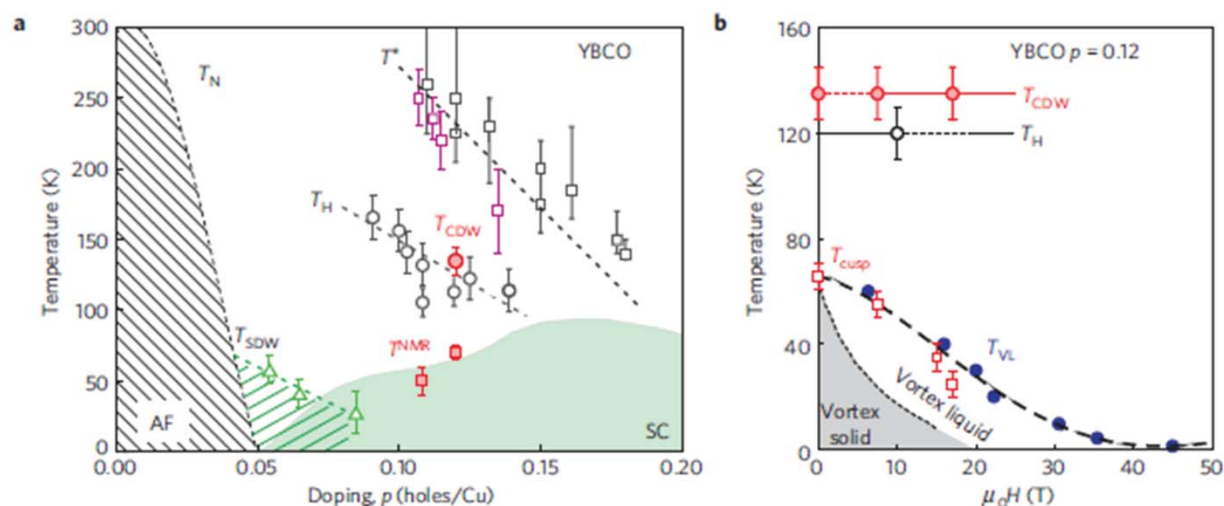
# Competing CDW order

- Wise, W. D. et al. Charge-density-wave origin of cuprate checkerboard visualized by scanning tunnelling microscopy. *Nature Phys.* 4, 696699 (2008).
- Lawler, M. J. et al. Intra-unit-cell electronic nematicity of the high-Tc copper-oxide pseudogap states. *Nature* 466, 347351 (2010).
- Parker, C. V. et al. Fluctuating stripes at the onset of the pseudogap in the high-Tc superconductor  $B2Sr2CaCu2O8Cx$ . *Nature* 468, 677680 (2010).
- Chang, J. et al. Direct observation of competition between superconductivity and charge density wave order in  $YBa2Cu3O6:67$ . *Nature Phys.* 8, 871876 (2012).
- Ghiringhelli, G. et al. Long-range incommensurate charge fluctuations in  $(Y;Nd)Ba2Cu3O6Cx$ . *Science* 337, 821825 (2012).
- Achkar, A. J. et al. Distinct charge orders in the planes and chains of ortho-III-ordered  $YBa2Cu3O6C$  superconductors identified by resonant elastic X-ray scattering. *Phys. Rev. Lett.* 109, 167001 (2012).
- Wu, T. et al. Magnetic-field-induced charge-stripe order in the high-temperature superconductor  $YBa2Cu3Oy$ . *Nature* 477, 192194 (2011).
- LeBoeuf, D. et al. Thermodynamic phase diagram of static charge order in underdoped  $YBa2Cu3Oy$ . *Nature Phys.* 9, 7983 (2013).



# Direct observation of competition between superconductivity and charge density wave order in $\text{YBa}_2\text{Cu}_3\text{O}_{6.67}$

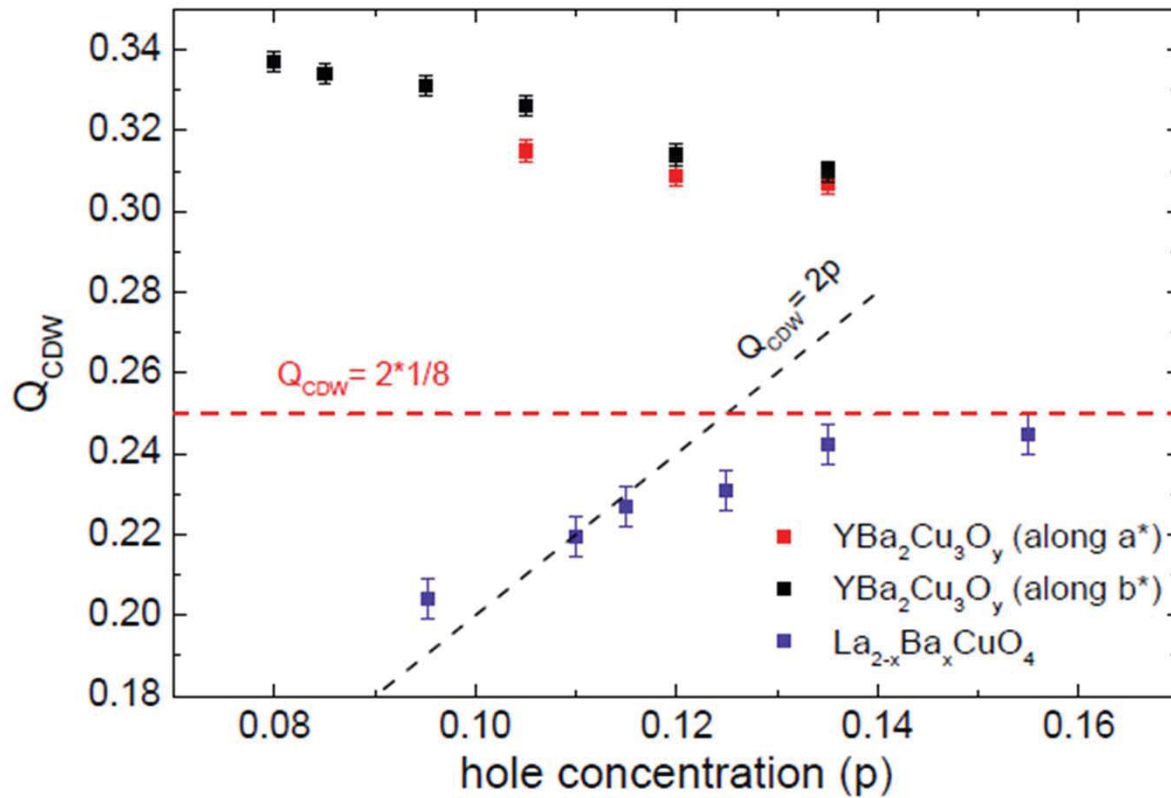
J. Chang<sup>1,2\*</sup>, E. Blackburn<sup>3</sup>, A. T. Holmes<sup>3</sup>, N. B. Christensen<sup>4</sup>, J. Larsen<sup>4,5</sup>, J. Mesot<sup>1,2</sup>, Ruixing Liang<sup>6,7</sup>, D. A. Bonn<sup>6,7</sup>, W. N. Hardy<sup>6,7</sup>, A. Watenphul<sup>8</sup>, M. v. Zimmermann<sup>8</sup>, E. M. Forgan<sup>3</sup> and S. M. Hayden<sup>9</sup>



**Figure 4 | Phase diagram of  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ .** **a**, Doping dependence of the antiferromagnetic ordering temperature  $T_N$ , the incommensurate spin-density wave order  $T_{SDW}$  (green triangles; ref. 21), the superconducting temperature  $T_c$  and the pseudogap temperature  $T^*$  as determined from the Nernst effect<sup>30</sup> (black squares) and neutron diffraction<sup>29</sup> (purple squares). Notice that the Nernst effect<sup>30</sup> indicates a broken rotational symmetry inside the pseudogap region, whereas a translational symmetry preserving magnetic order is found by neutron scattering<sup>29</sup>. Below temperature scale  $T_H$  (black circles), a larger and negative Hall coefficient was observed<sup>26</sup> and interpreted in terms of a Fermi surface reconstruction. Our X-ray diffraction experiments show that in  $\text{YBCO } p = 0.12$  incommensurate CDW order spontaneously breaks the crystal translational symmetry at a temperature  $T_{CDW}$  that is twice as large as  $T_c$ .  $T_{CDW}$  is also much larger than  $T^{NMR}$  (red squares), the temperature scale below which NMR observes field-induced charge order<sup>13</sup>. **b**, Field dependence of  $T_{CDW}$  (filled red circles) and  $T_{cusp}$  (open squares), the temperature below which the CDW is suppressed by superconductivity, compared with  $T_H$  (open black circle) and  $T_{VL}$  (filled blue circles), the temperature where the vortex liquid state forms<sup>26</sup>. Error bars on  $T_{SDW}$ ,  $T_H$ ,  $T^{NMR}$ , and  $T^*$  are explained in refs 21,26,30,33. The error bars on  $T_{CDW}$  and  $T_{cusp}$  reflect the uncertainty in determining the onset and suppression temperature of CDW order from Fig. 2.



# Wave vector



Keimer, Julich summer school 2013



# Theories

S. Sachdev and R. La Placa Phys. Rev. Lett. **111**, 027202 (2013)

D. Chowdhury, S. Sachdev arxiv. 1501.00002

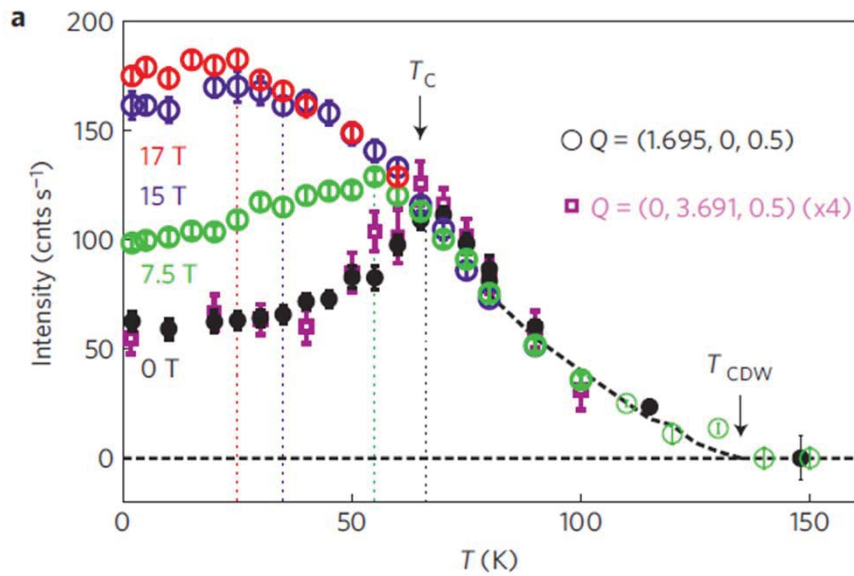
K. B. Efetov, H. Meier, and C. Pepin, Nat Phys **9**, 442 (2013).

Y. Wang and A. Chubukov, Phys. Rev. B **90**, 035149 (2014).

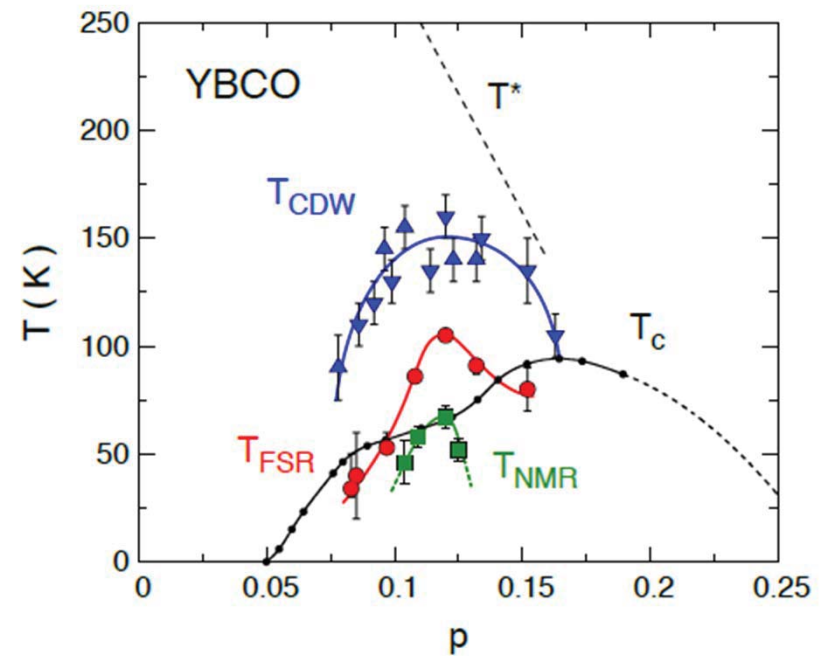
...



# Competition between CDW and SC



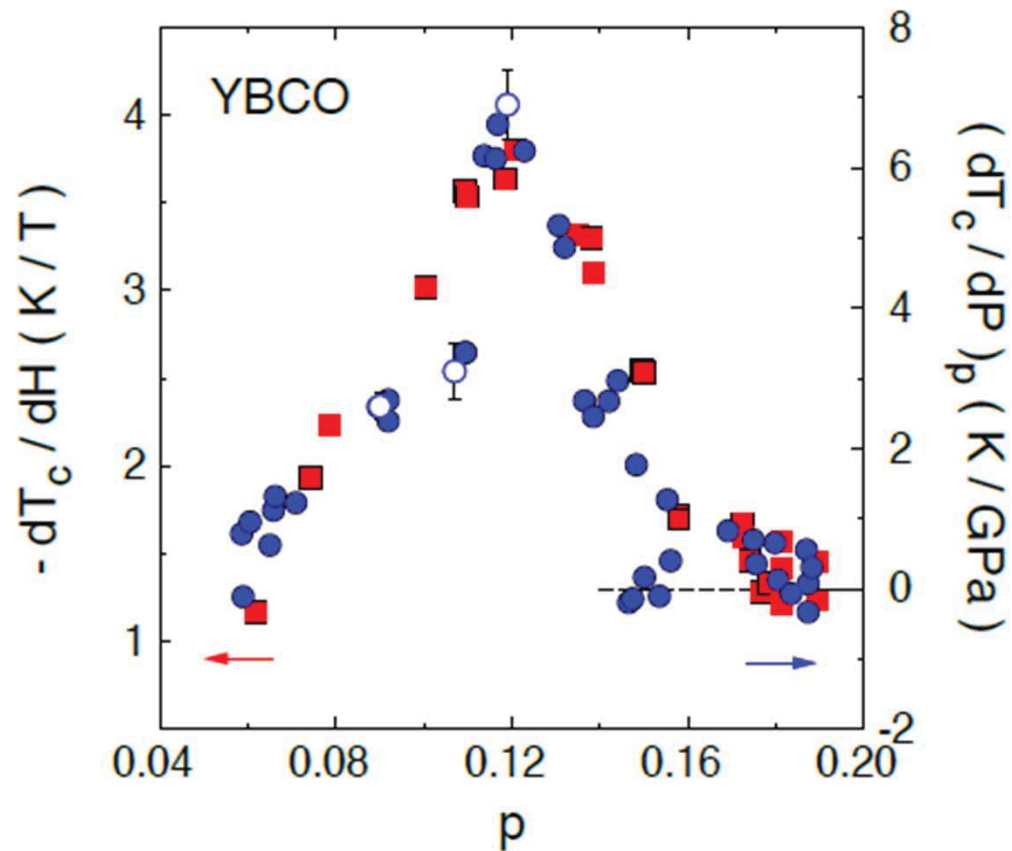
J. Chang *et al.*, *Nat. Phys.* **8**, 871-876 (2012).



Cyr-Choinière et al, arxiv1503.02033



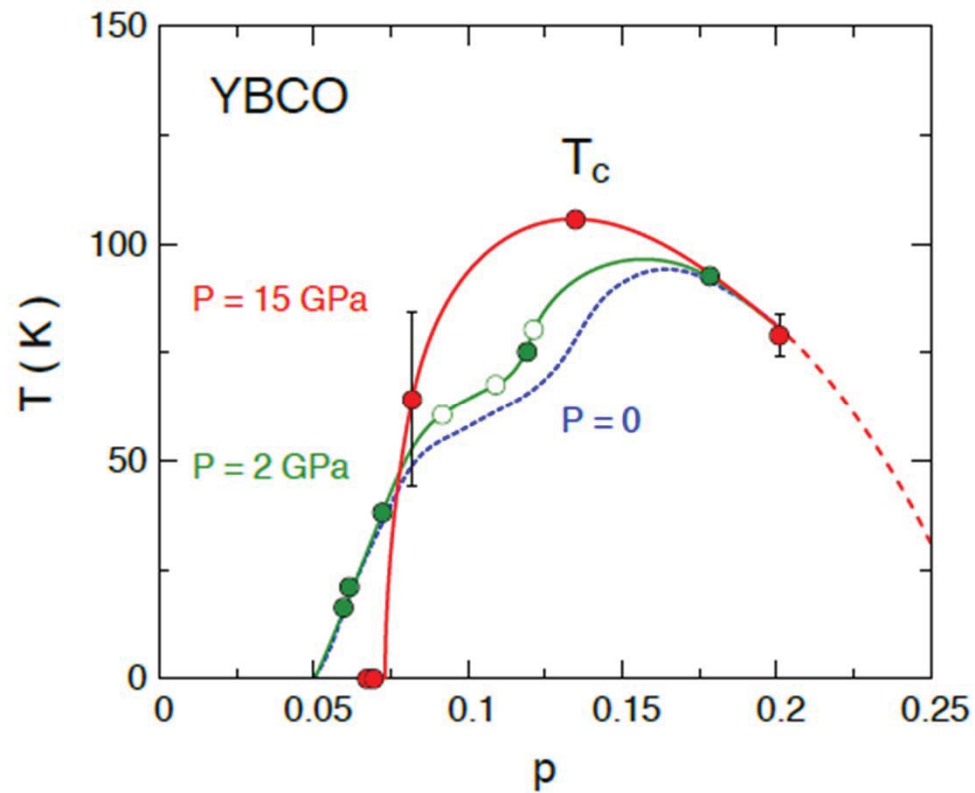
# Tuning SC and CDW



Cyr-Choinière et al, arxiv1503.02033



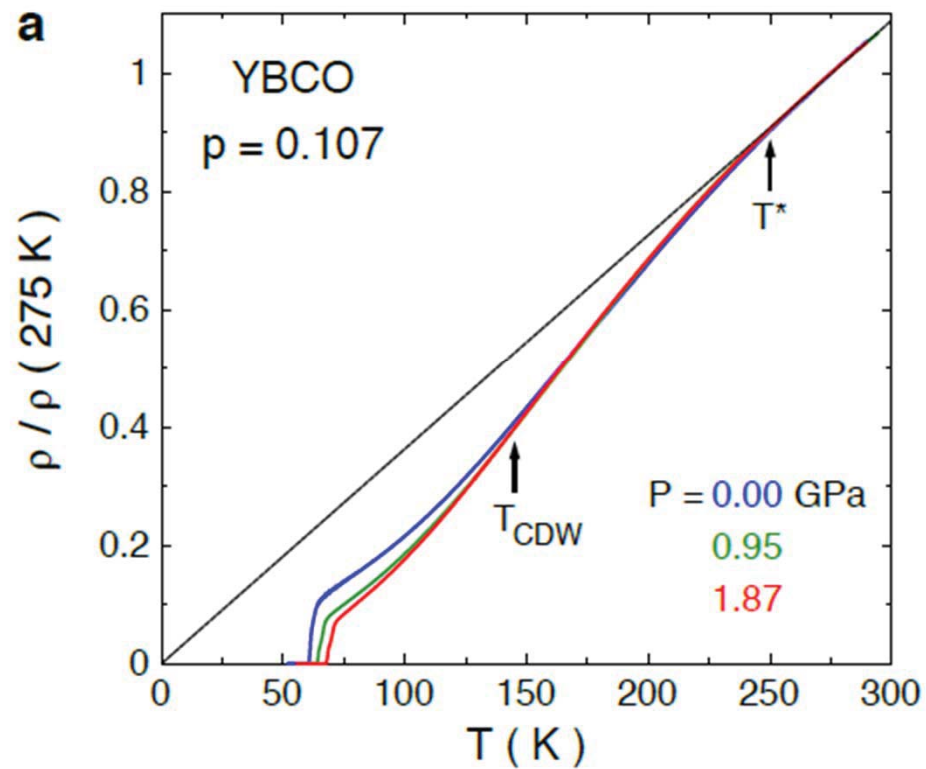
# Getting rid of the CDW



Cyr-Choinière et al, arxiv1503.02033



# $T^*$ not affected



Cyr-Choinière et al, arxiv1503.02033

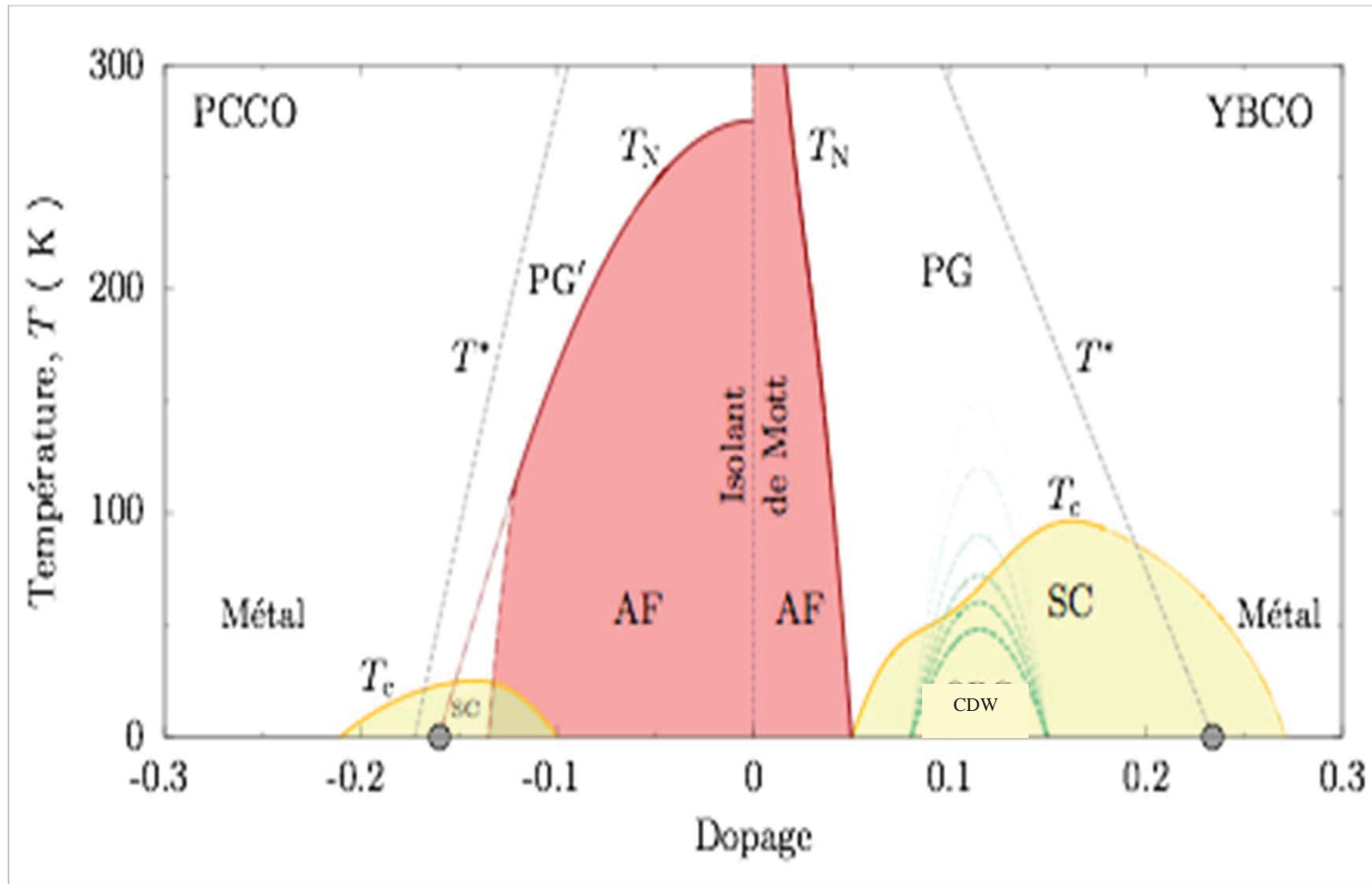


# 7. Pseudogap

h-doped

# Our road map

Thèse de Francis Laliberté,  
Université de Sherbrooke





# Three classes of mechanisms

- Rounded first order transition
- $d = 2$  precursor to a lower temperature broken symmetry phase (e-doped)
- Mott physics (h-doped)

Consider first this Mott hypothesis  
(Cannot be CDW)

- Competing order
  - Current loops: Varma, PRB **81**, 064515 (2010)
  - Stripes or nematic: Kivelson et al. RMP 75 1201(2003); J.C.Davis
  - d-density wave : Chakravarty, Nayak, Phys. Rev. B **63**, 094503 (2001); Affleck et al. flux phase
  - SDW: Sachdev PRB **80**, 155129 (2009) ...
- Or Mott Physics?
  - RVB: P.A. Lee Rep. Prog. Phys. **71**, 012501 (2008)

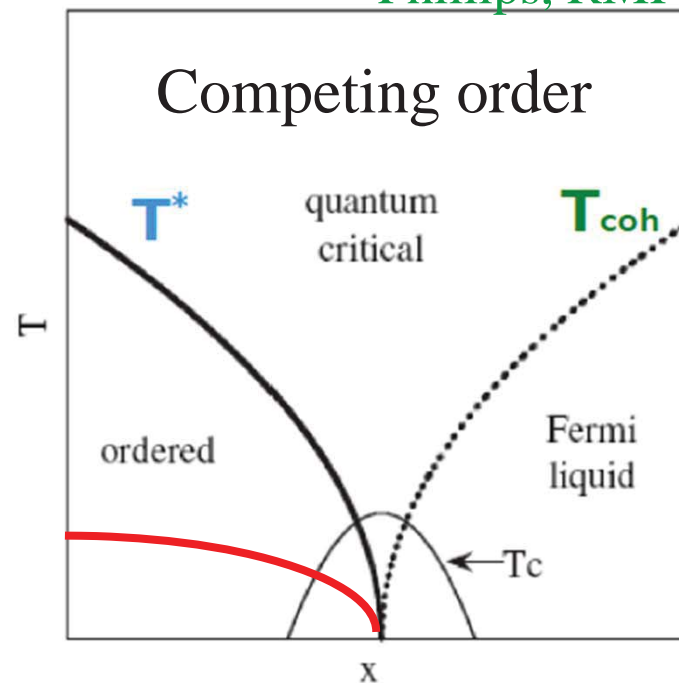
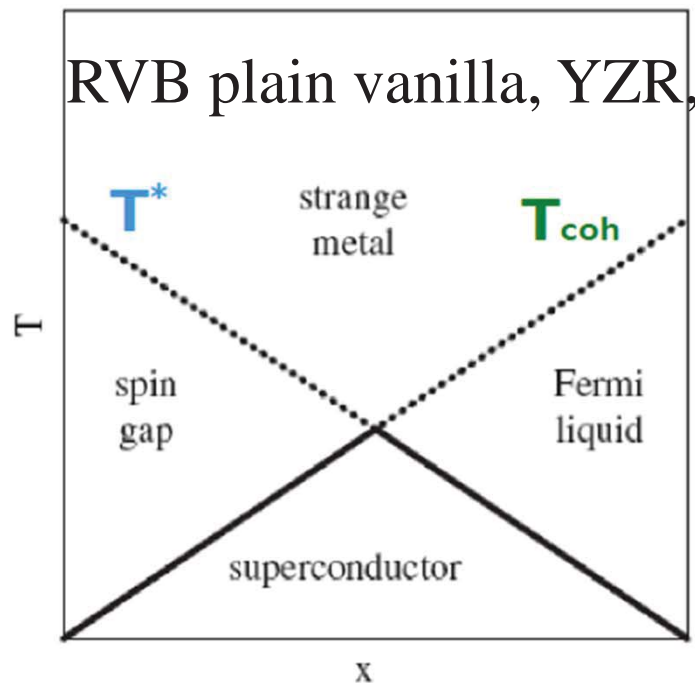


# Three views (caricature)

Norman, Adv. Phys. (2005)

Broun, Nat. Phys. (2006)

Phillips, RMP (2010)



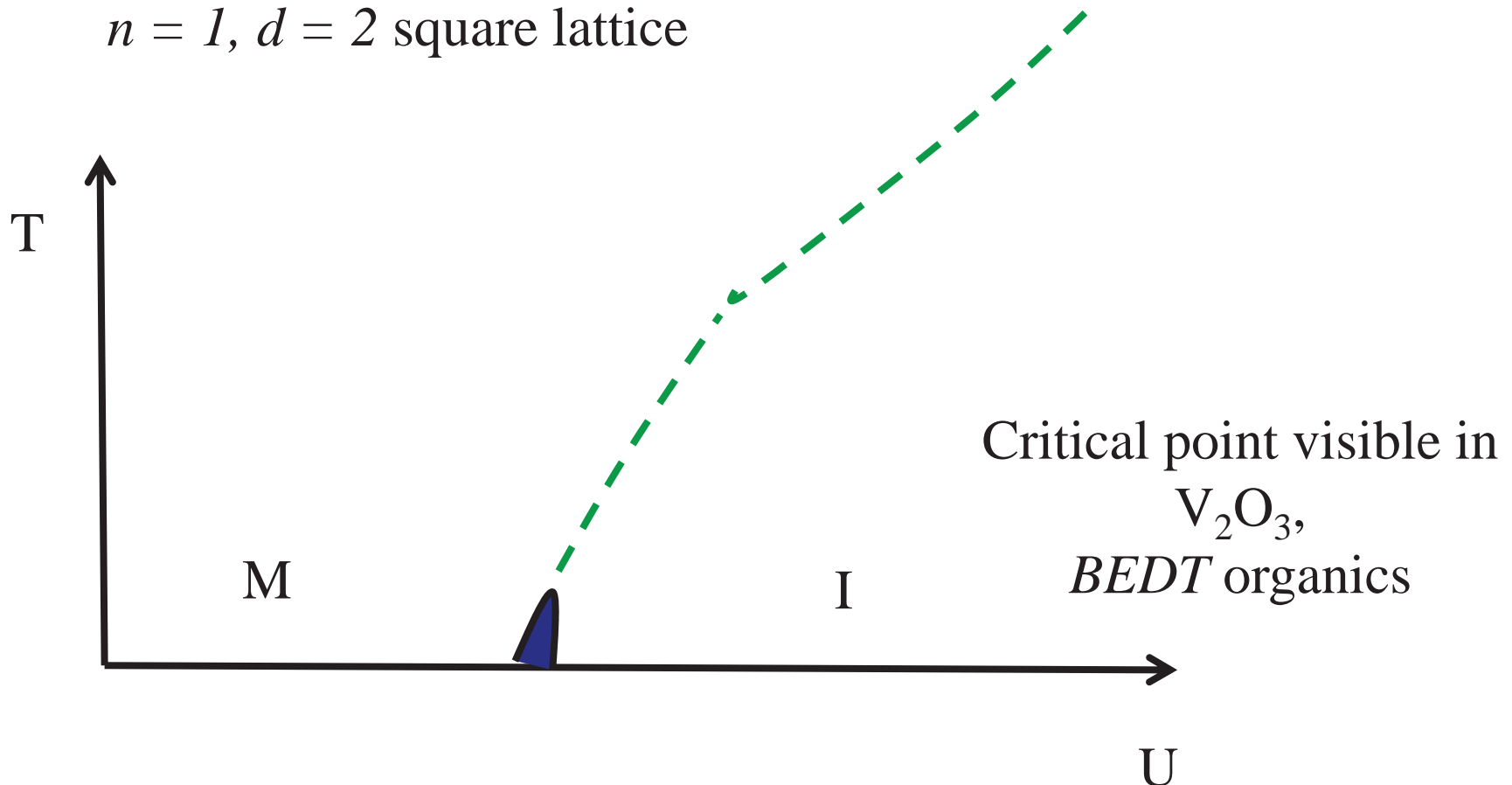
Why  $T_c$  decreases?  
What is the origin of  $T^*$ ?  
What is the strange metal?

Broken symmetry or not.  
What lies beneath the dome.  
Mott Physics away from  $n = 1$



# Local moment and Mott transition

$n = 1, d = 2$  square lattice



Understanding finite temperature phase from a *mean-field theory* down to  $T = 0$



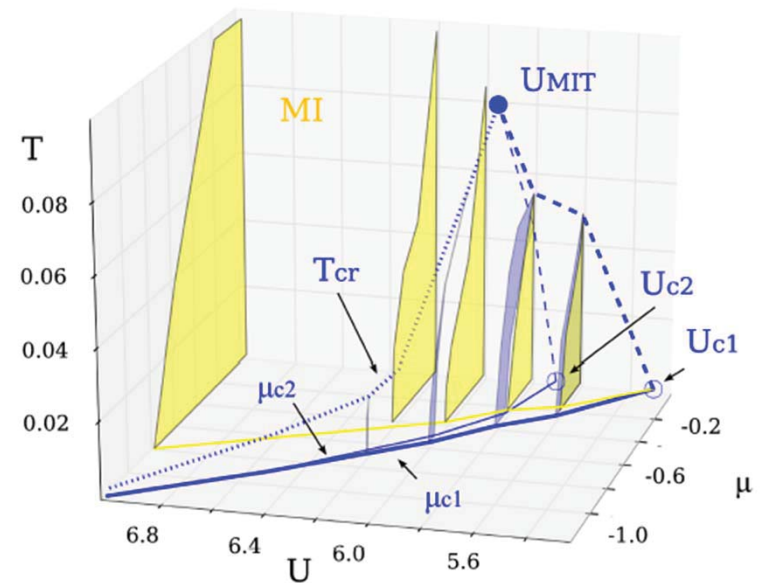
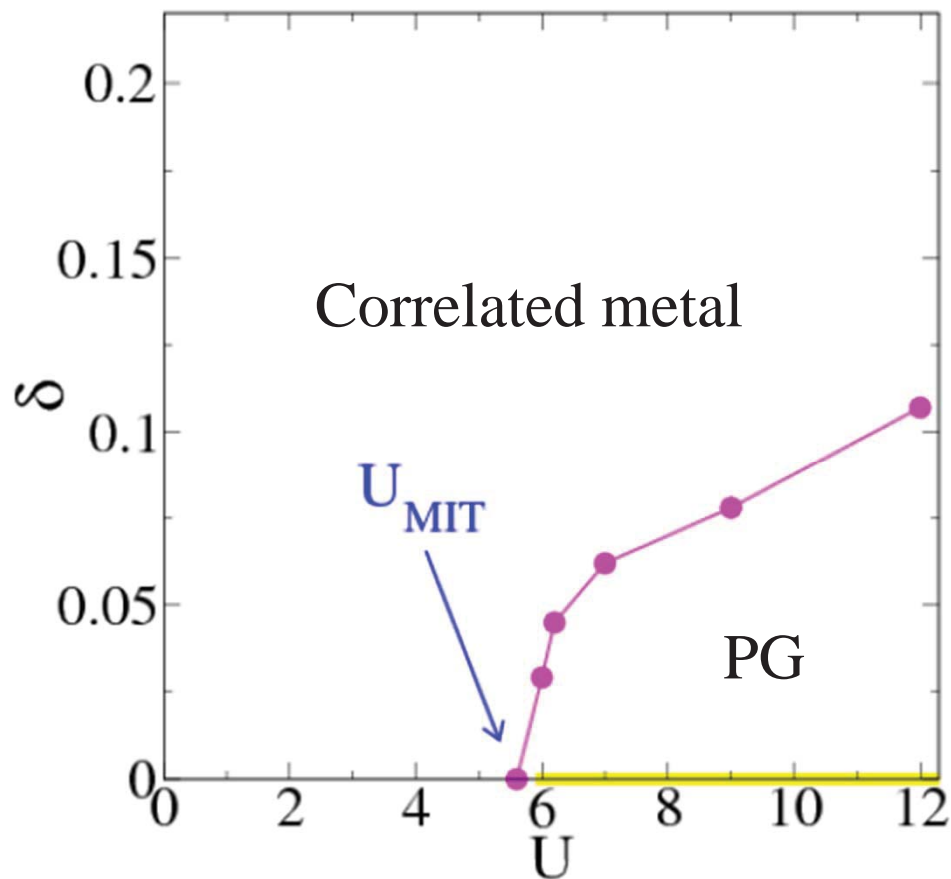
# Interaction-induced Mott transition, $n = 1$

Method	$U_{c1}$	$U_c$	$U_{c2}$	Ref.
VCA+ED 2 x 2 + 8b	5.25	5.5	6.37	Balzer et al. EPL (2009)
CDMFT+CTQMC+H 2 x 2	5.3		5.7	Park et al. PRL (2008)
DCA+CTQMC+H 8	5.7		6.4	Gull et al. cond-mat (2009)
DCA+CTQMC+H 4	!	~4.2	!	Gull et al. EPL (2008)
Dual fermions	!	~6.5	!	Hafermann et al. (2008)
CDMFT+ED 2 x 2 + 8b 15 parameters	?	~5.6	?	Liebsch, Merino... (2008)
CDMFT+ED 2,3,4		~4		Zhang et al. PRB (2007) (3d also)
QMC 6 x 6		6		Vekic et al. (1993)



# Link to Mott transition up to optimal doping

Doping dependence of critical point as a function of  $U$



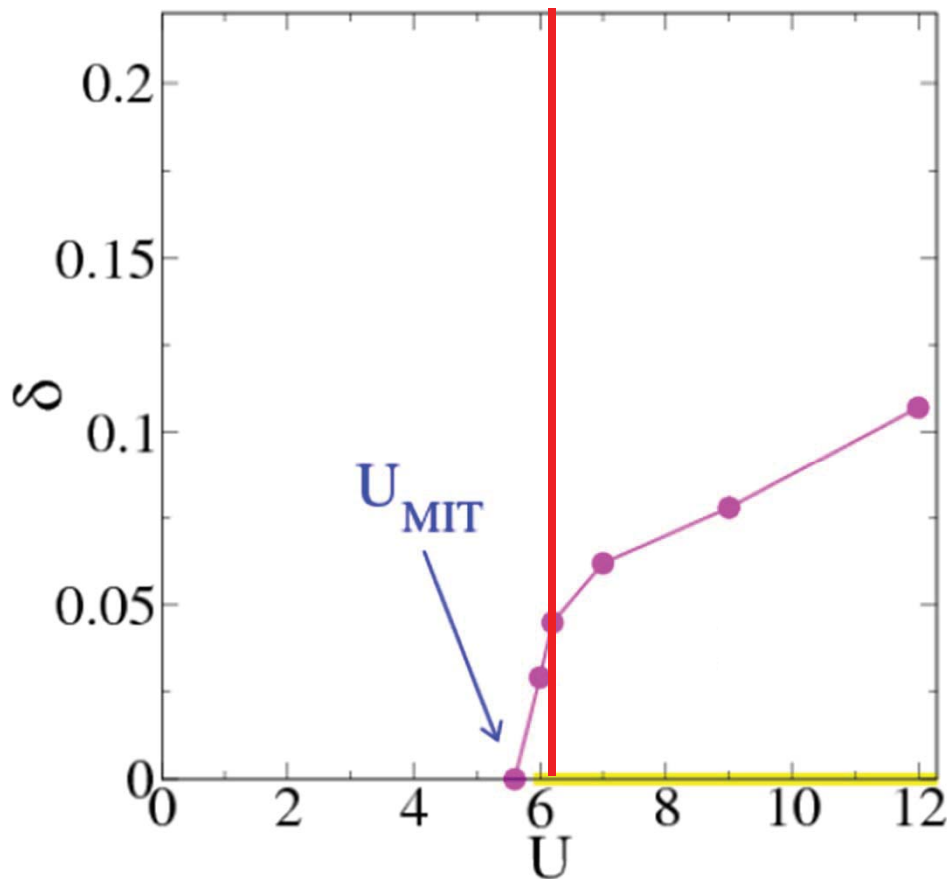
Sordi et al. PRL 2010, PRB 2011



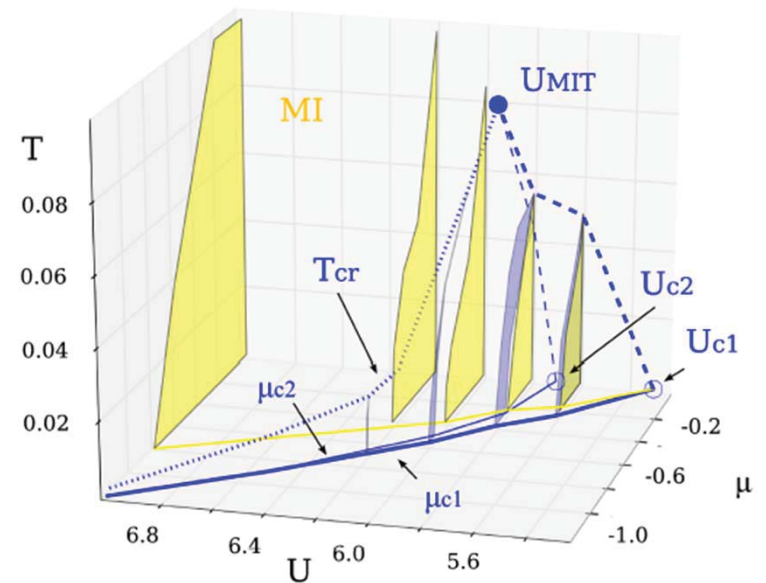
# Link to Mott transition up to optimal doping

## Another emergent transition

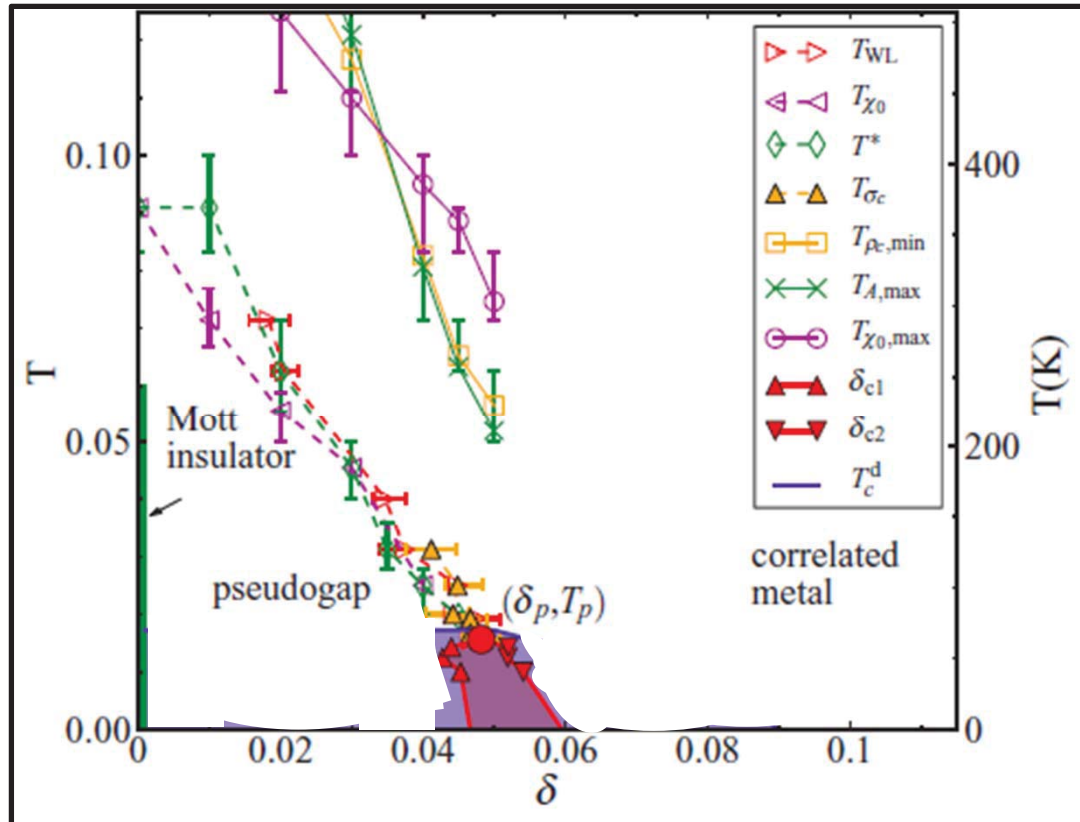
Doping dependence of critical point as a function of  $U$



Smaller  $D$  and  $S$



# Two crossover lines



Sordi et al. PRL 108, 216401 (2012)  
PRB 87, 041101(R) (2013)

# What is the minimal model?

Noninteracting case:

$$\chi_{nn}^{0R}(\mathbf{q}, \omega) = -2 \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{f(\zeta_{\mathbf{k}}) - f(\zeta_{\mathbf{k}+\mathbf{q}})}{\omega + i\eta + \zeta_{\mathbf{k}} - \zeta_{\mathbf{k}+\mathbf{q}}}$$

H. Alloul  
Comptes Rendus Physique,  
15, 519 (2014)

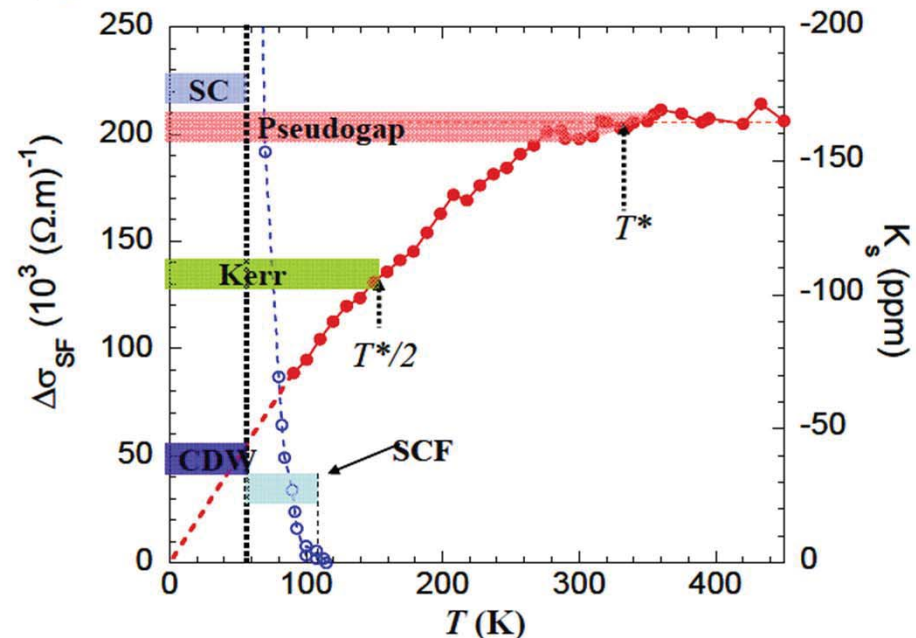
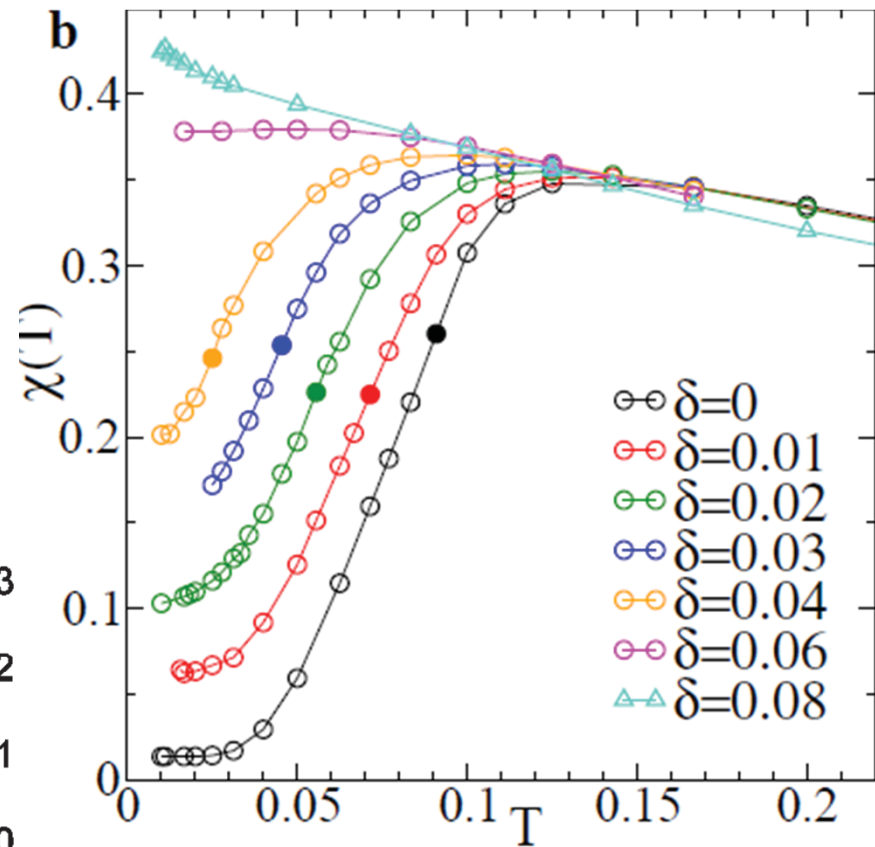
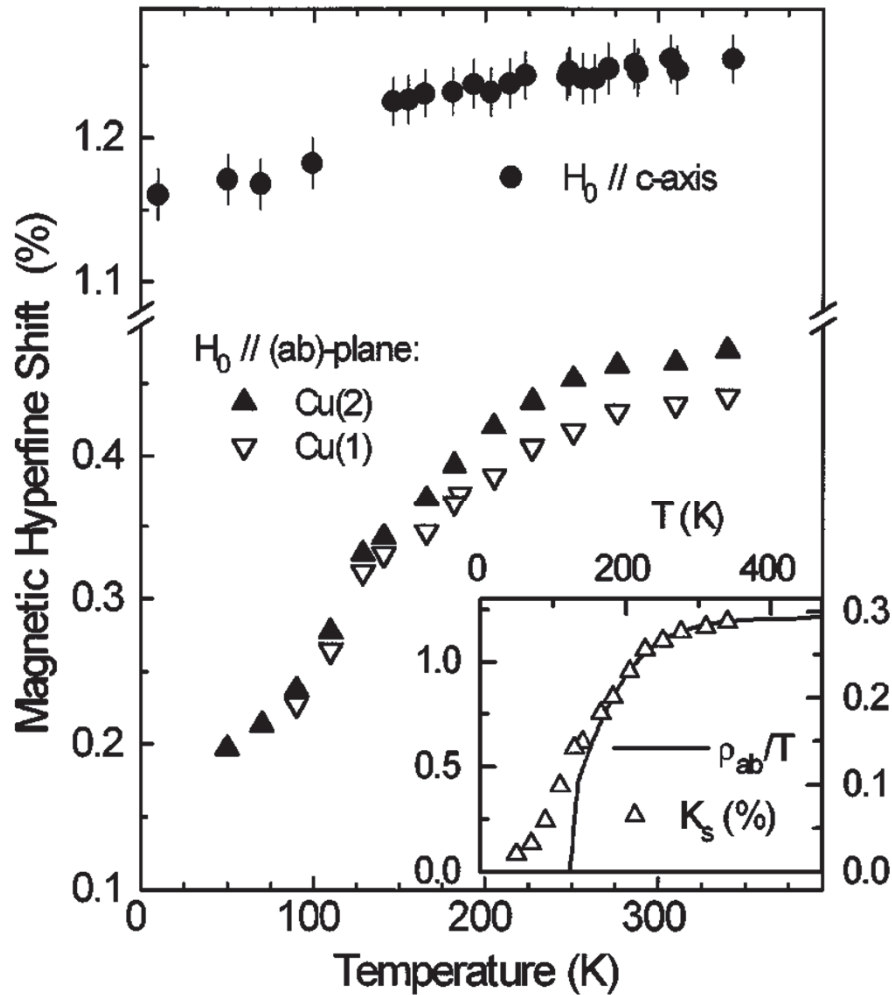


Fig 1 Spin contribution  $K_s$  to the  $^{89}\text{Y}$  NMR Knight shift [11] for  $\text{YBCO}_{6.6}$  permit to define the PG onset  $T^*$ . Here  $K_s$  is reduced by a factor two at  $T \sim T^*/2$ . The sharp drop of the SC fluctuation conductivity (SCF) is illustrated (left scale) [23]. We report as well the range over which a Kerr signal is detected [28], and that for which a CDW is evidenced in high fields from NMR quadrupole effects [33] and ultrasound velocity data [30]. (See text).



# Spin susceptibility



Sordi et al. Scientific Repts. 2012

Underdoped Hg1223

Julien et al. PRL **76**, 4238 (1996)



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Giovanni Sordi



Patrick Sémon



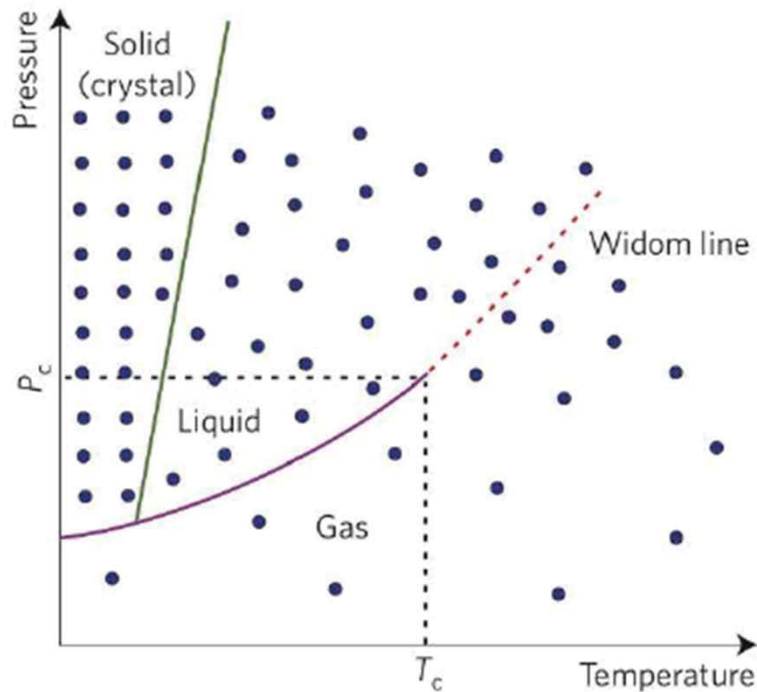
Kristjan Haule

## The Wisdom line

G. Sordi, *et al.* Scientific Reports 2, 547 (2012)



# What is the Widom line?

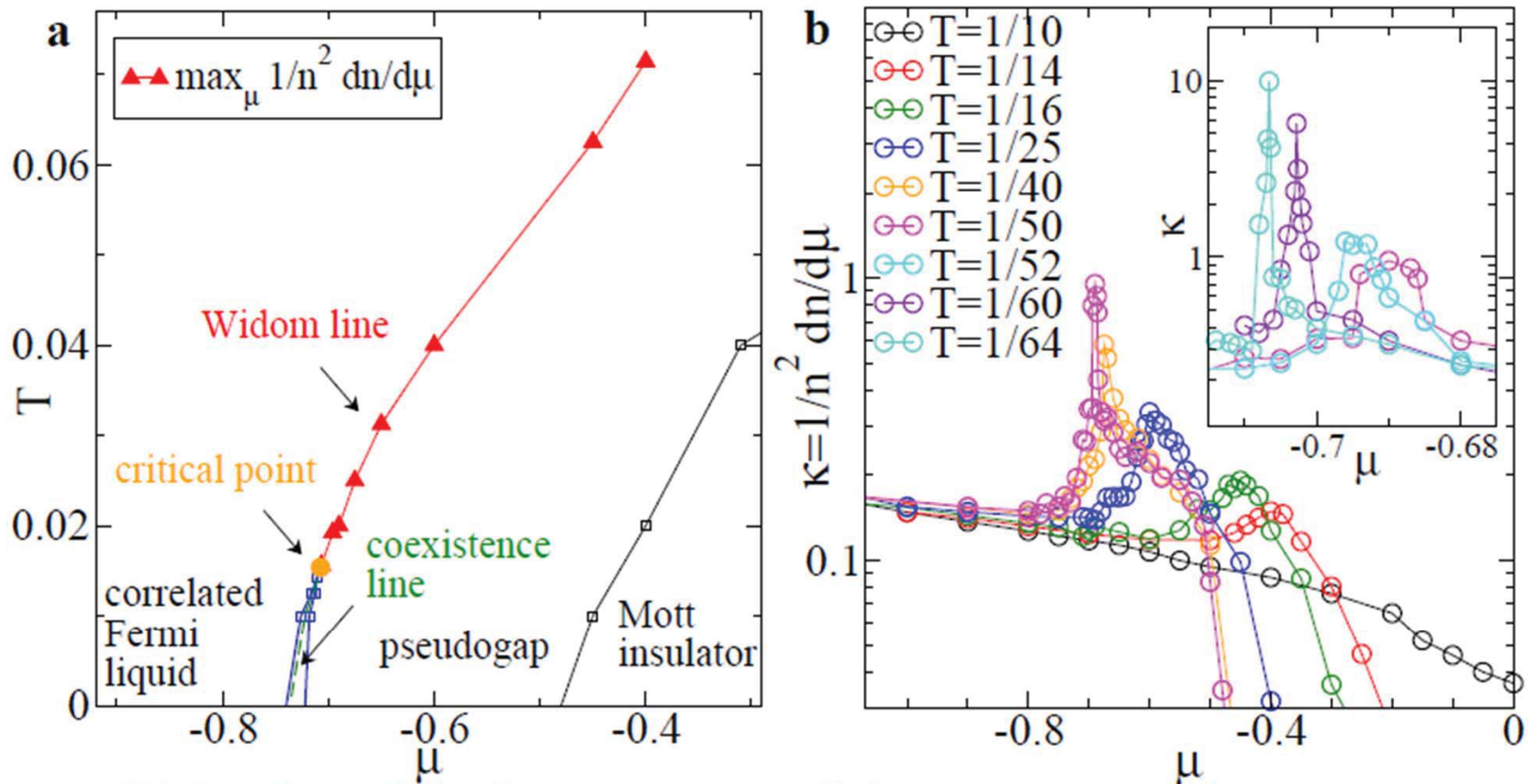


McMillan and Stanley, Nat Phys 2010

- ▶ it is the continuation of the coexistence line in the supercritical region
- ▶ line where the **maxima of different response functions** touch each other asymptotically as  $T \rightarrow T_p$
- ▶ liquid-gas transition in water: max in isobaric heat capacity  $C_p$ , isothermal compressibility, isobaric heat expansion, etc
- ▶ **DYNAMIC crossover arises from crossing the Widom line!**  
water: Xu et al, PNAS 2005, Simeoni et al Nat Phys 2010



# Pseudogap along the Widom line



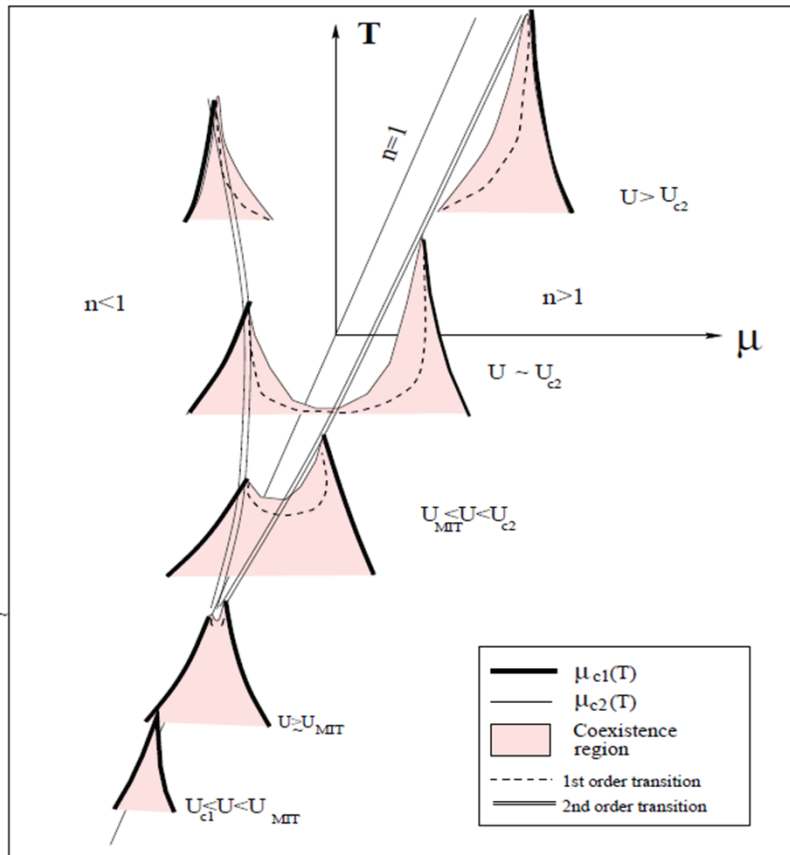
Widom line: defined from **maxima of charge compressibility**

$$\kappa = 1/n^2 (dn/d\mu) T$$

divergence of  $\kappa$  at the (classical) critical point!



# Compressibility divergence at Mott and coexistence



G. Kotliar, S. Murthy, and M. J. Rozenberg, Phys. Rev. Lett. **89**, 046401 (2002).

S. Murthy, Rutgers thesis 2004

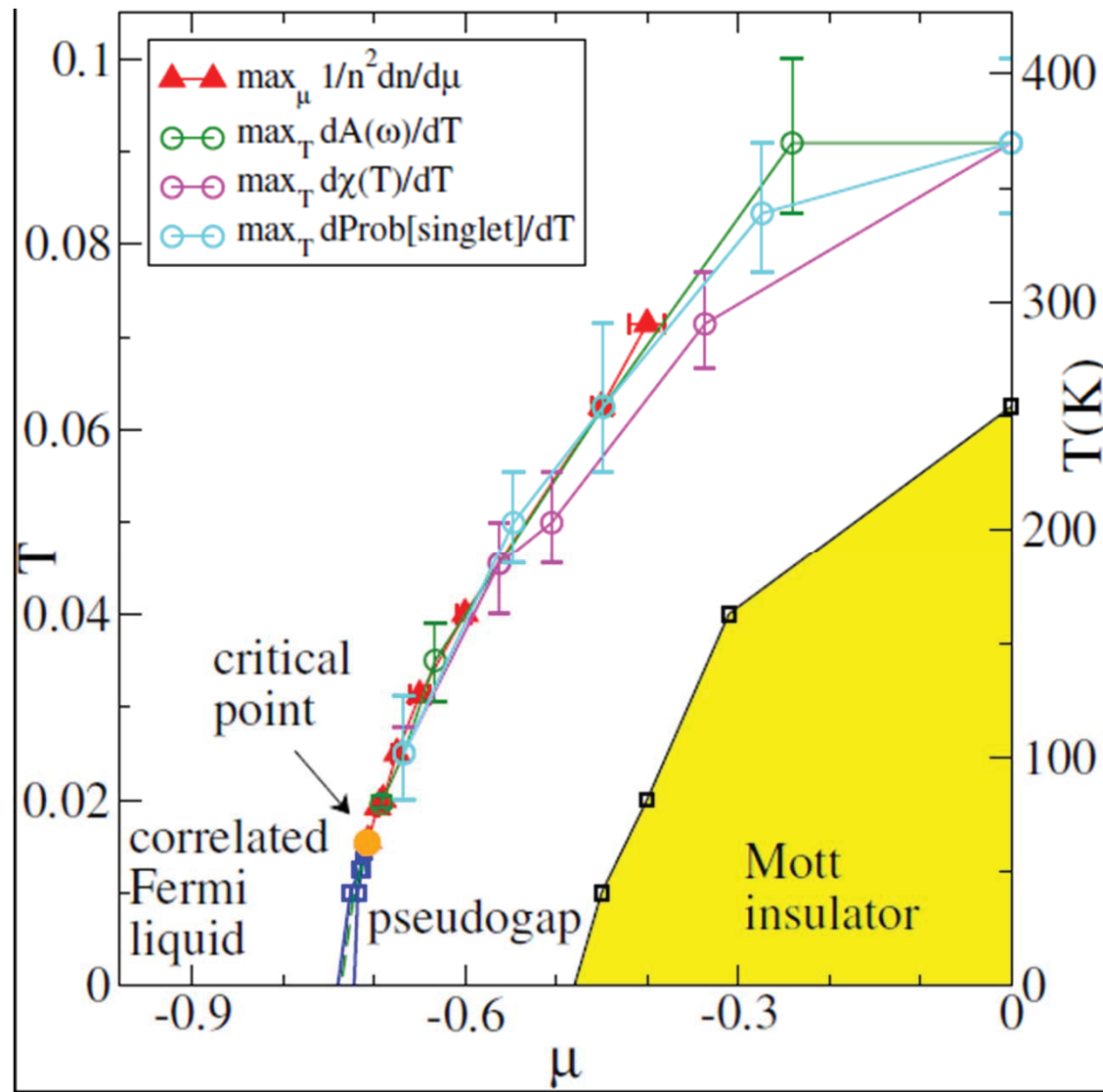
K. Frikach, M. Poirier, et al. PRB **61**, R6491 (2000).

S. R. Hassan, A. Georges, and H. R. Krishnamurthy PRL **94**, 036402 (2005)

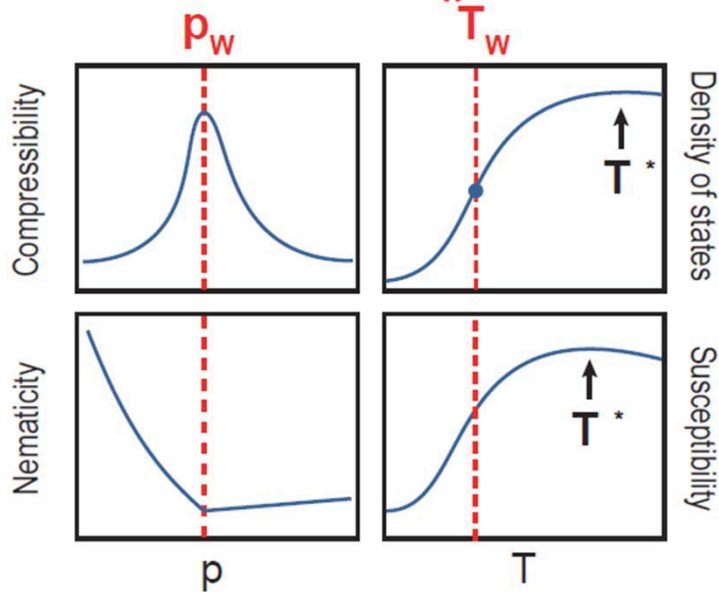
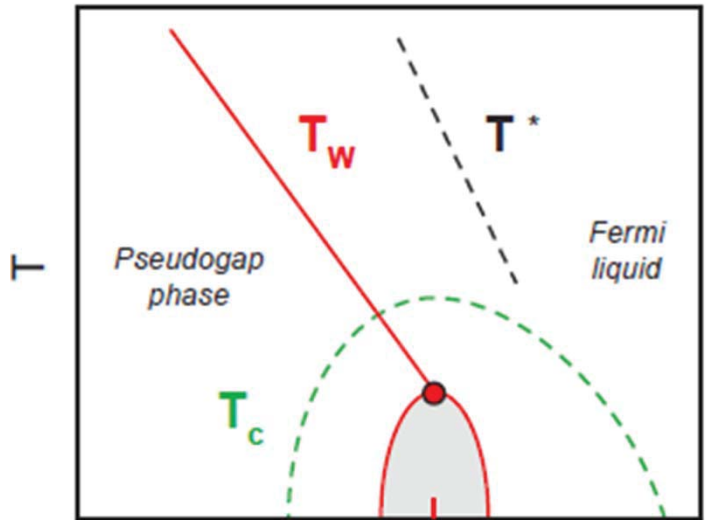
Figure 2.19: Schematic phase diagram for the 2-band case. There is an asymmetry in the triangular peaks as compared to the 1-band case. The cross sections are on the  $T-\mu$  plane for different values of  $U$  as before.  $\mu_{c1}$  and  $U_{c1}$  are the chemical potential and



# Rapid change also in dynamical quantities



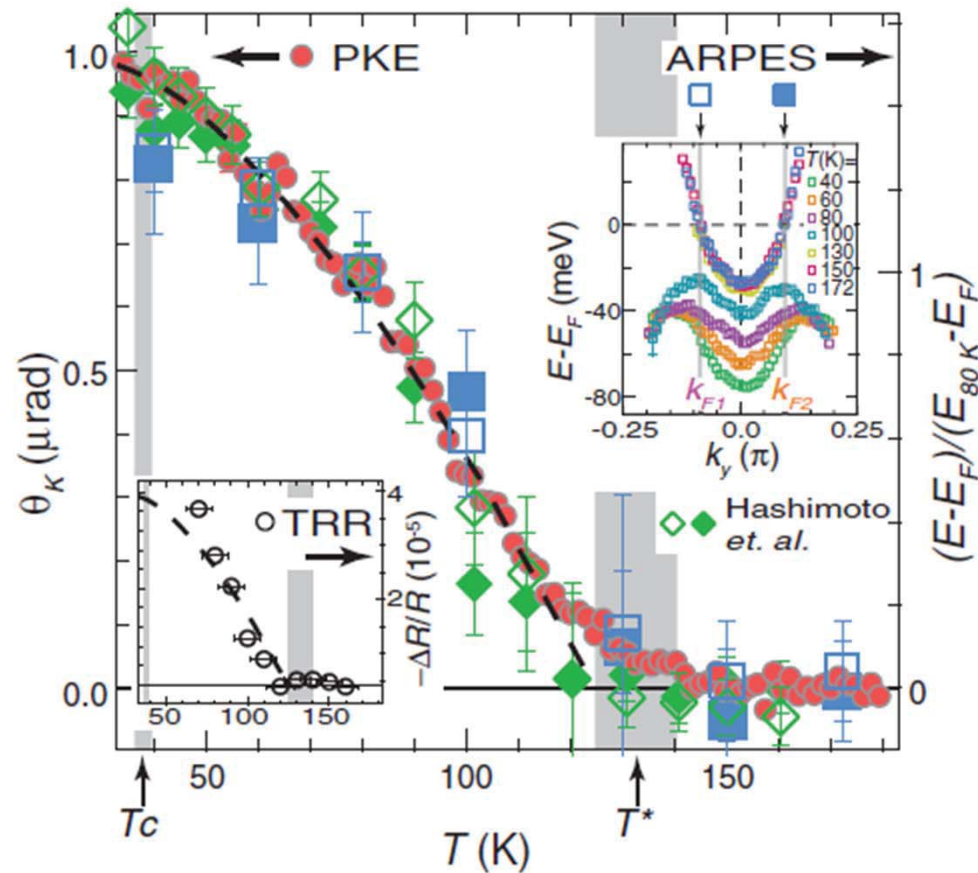
# An alternate point of view : next lecture



- - Is the pseudogap (PG) a crossover or a phase transition ?
- - Relation between CDW and the PG ?
- - Why CDW peaked at 12% doping ?
- - Origin of nematicity ?
- - Why superconducting ?
- - Why a dome of SC ?
- - Does a one-band model capture the key physics ?
- AFM QCP important?
- Lessons from other SC?

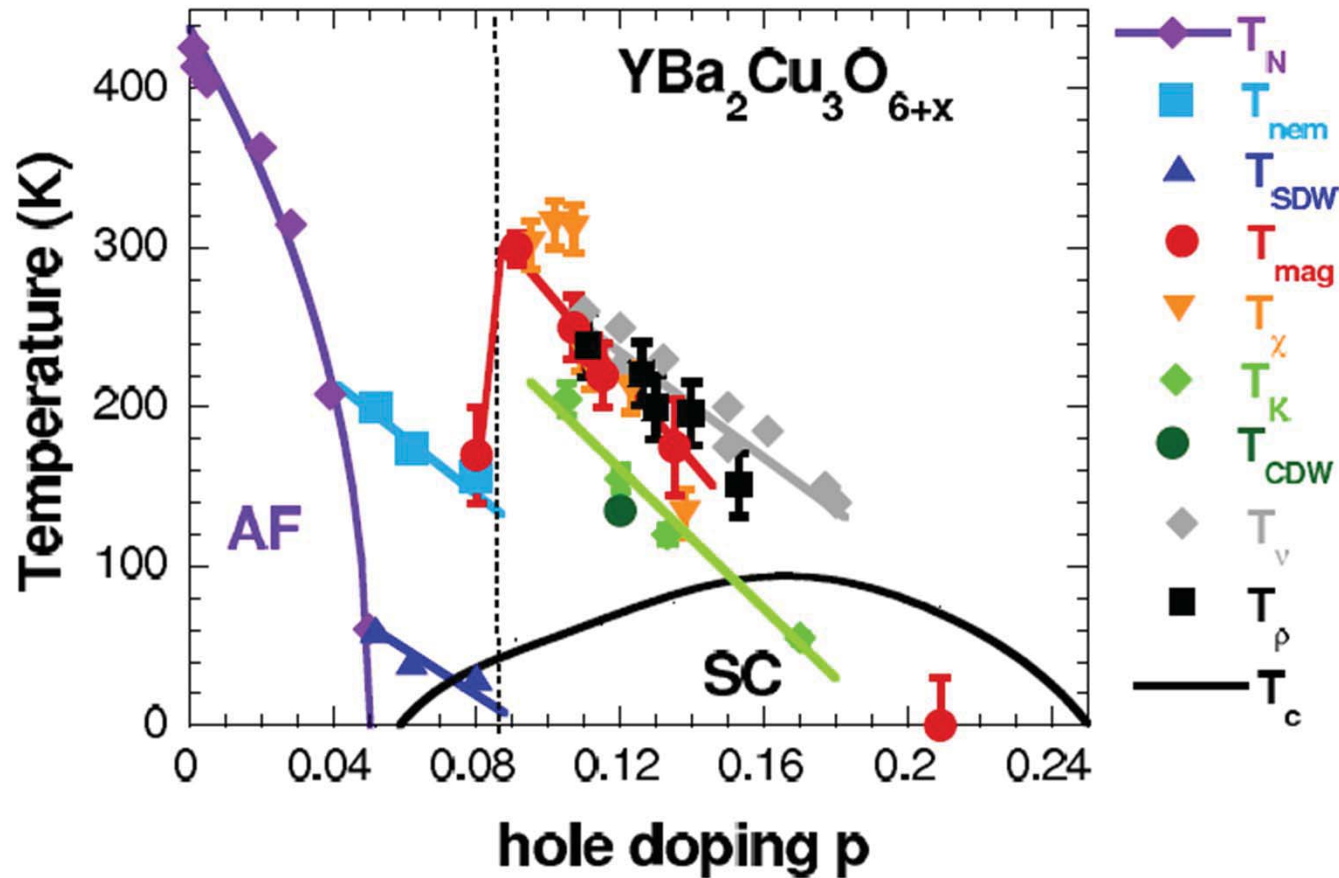
# 3 measurements: Kerr, ARPES, TRR

**Fig. 3.** Temperature dependence of Kerr rotation ( $\theta_K$ ) measured by PKE, in comparison with that of the binding energy position of the EDC maximum at  $k_F$  given by ARPES [reproduced from fig. S1F and (29)]. ARPES results are normalized to the 80 K values (free from the interference of fluctuating superconductivity). The dashed black curve is a guide to the eye for the PKE data, showing a mean-field-like critical behavior close to  $T^*$  [see additional discussion in (27)]. **(Left inset)** Temperature dependence of the transient reflectivity change measured by TRR (right axis). The dashed black curve (left axis) is reproduced from the main panel. Error bars (if not visible) are smaller than the symbol size. **(Right inset)** Dispersion of the EDC maximum at various temperatures above  $T_c$ , summarizing the results of Figs. 2A and 4A and fig. S1, A to E. All data were taken on samples from the same growth and annealing batch, except those reproduced from (29) on differently annealed samples.





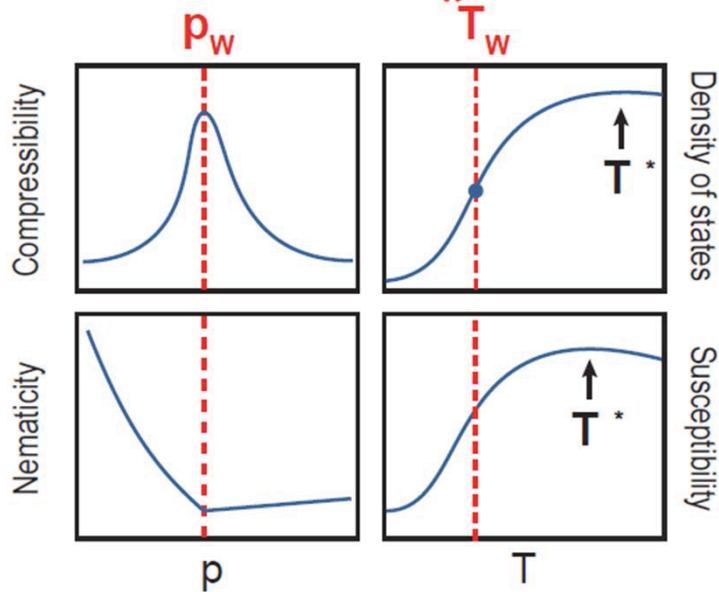
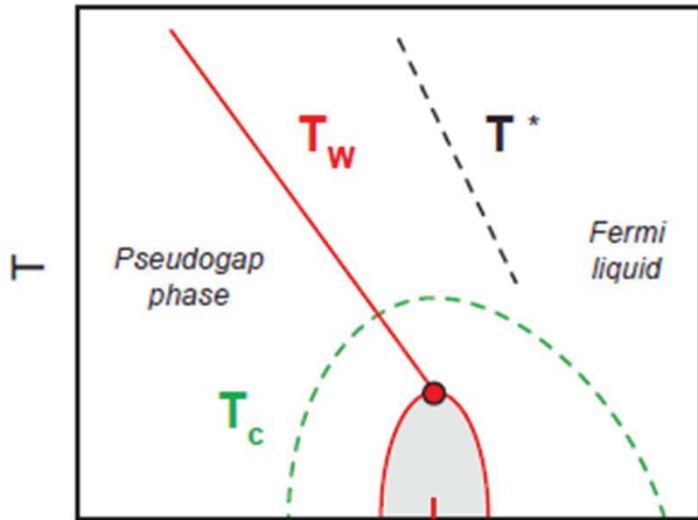
# Intra-Unit-Cell loop order



Y Sidis and P Bourges 2013 *J. Phys.: Conf. Ser.* **449** 012012



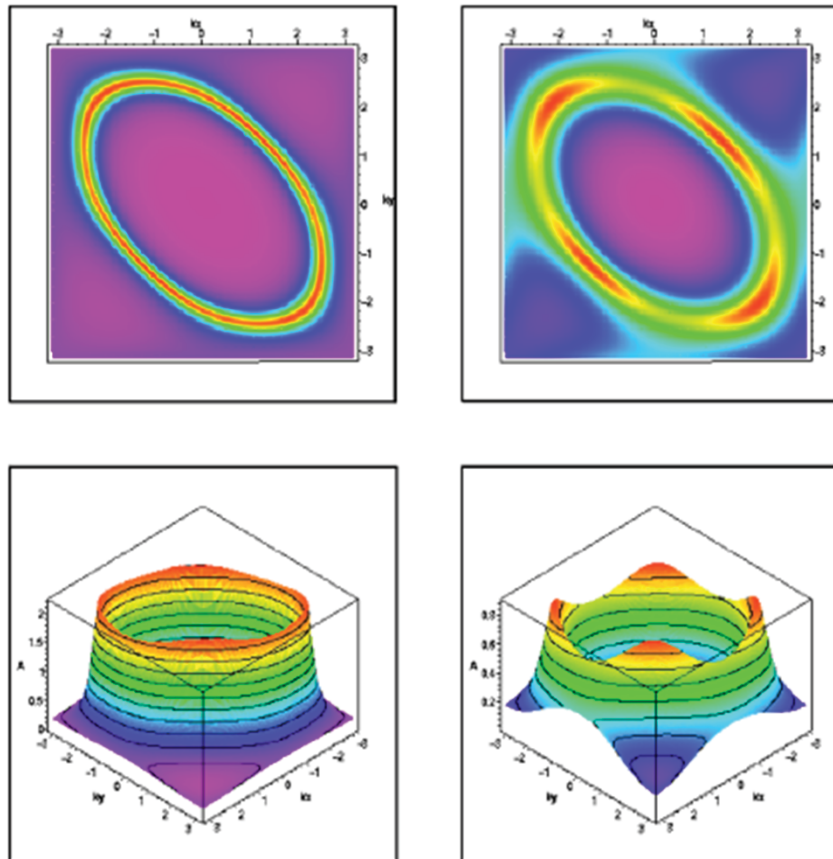
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# Organics

# Pseudogap

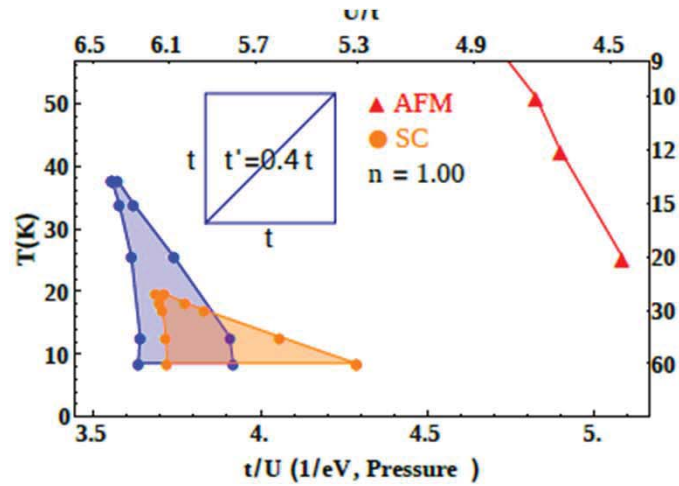
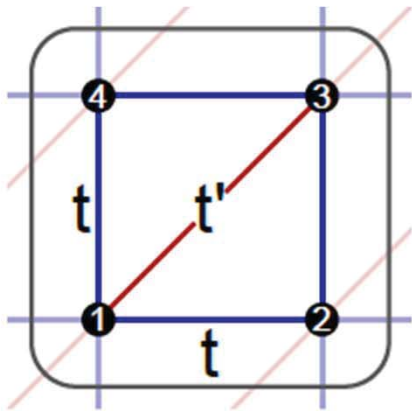


O. Parcollet, G. Biroli  
G. Kotliar  
PRL **92** (2004)

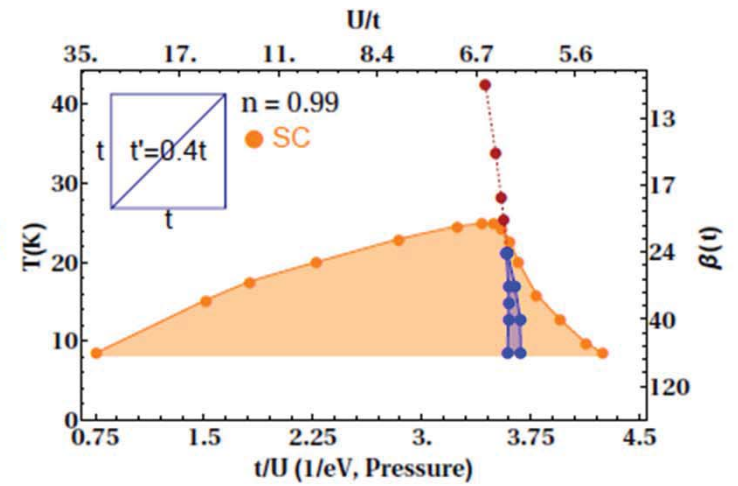
FIG. 4 (color online). Distribution of low energy spectral weight in  $k$  space  $A(k, \omega = 0)$  for  $T/D = 1/44$ ,  $U/D = 2.0$  (left-hand panel) and  $U/D = 2.25$  (right-hand panel). The top panels are color plots to see the Fermi surface and the bottom panels are 3D plots to see the variation of  $A$ . For intermediate  $U$ , cold and hot regions are visible around  $(\frac{\pi}{2}, \frac{\pi}{2})$  and  $(\pi, 0)$ , respectively.



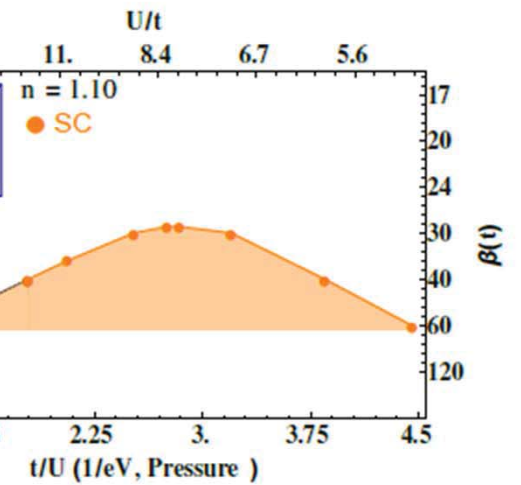
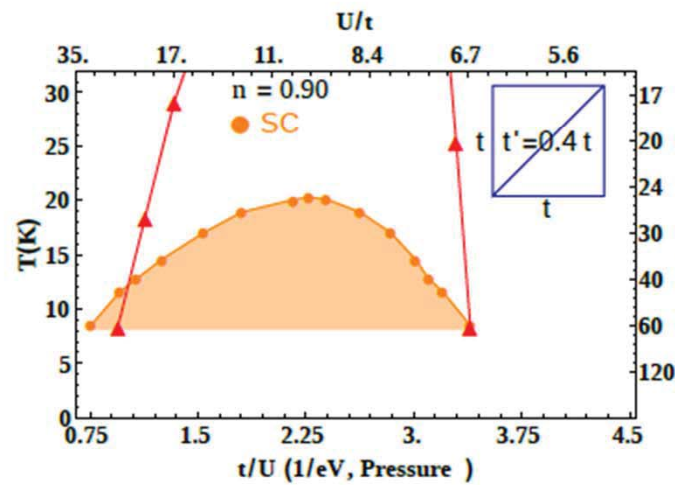
$$t' = 0.4t$$



(a)



(b)

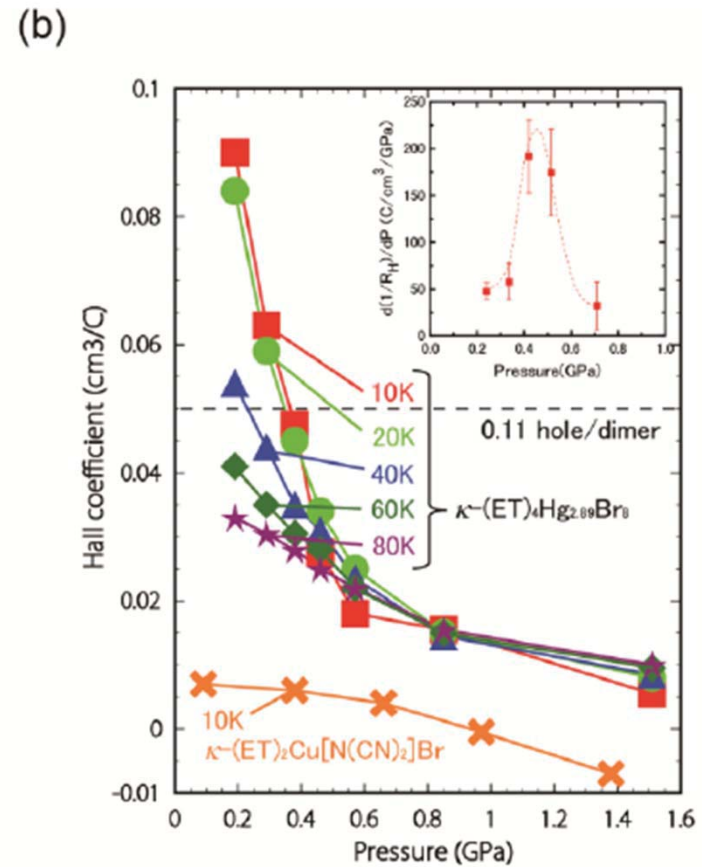
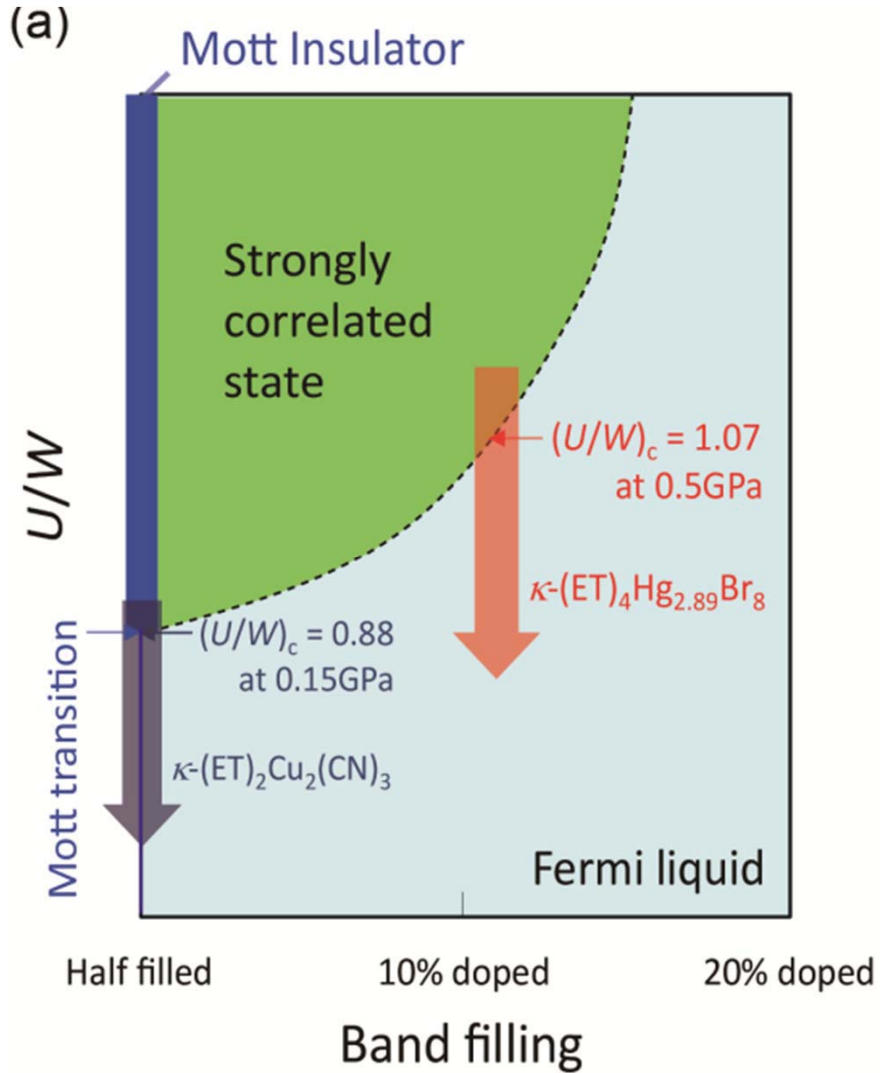


Charles-David Hébert, Patrick Sémon, AMT



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# Doped BEDT

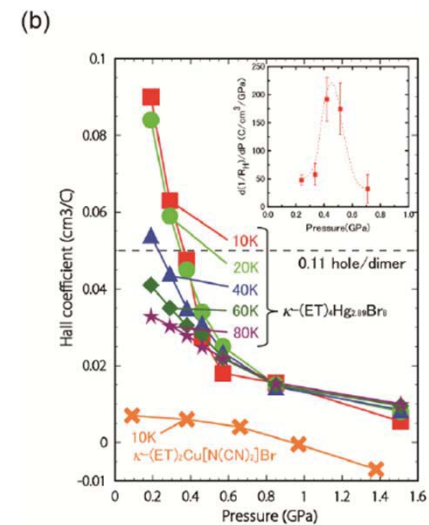
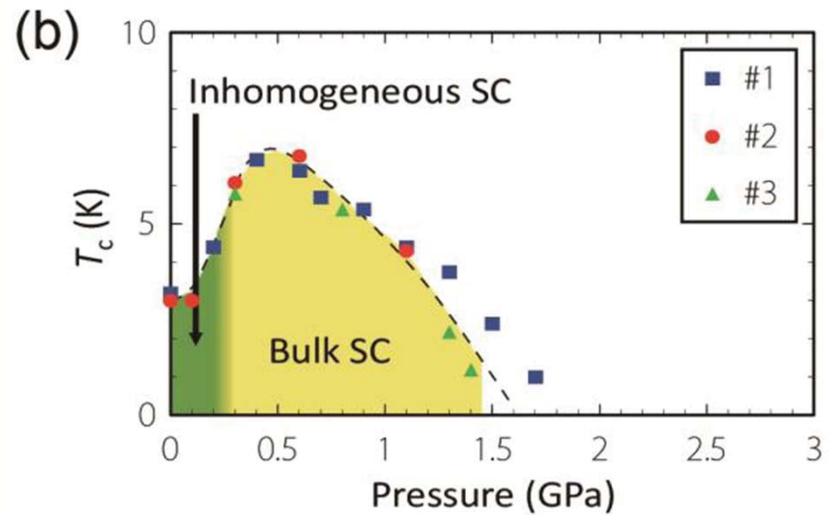
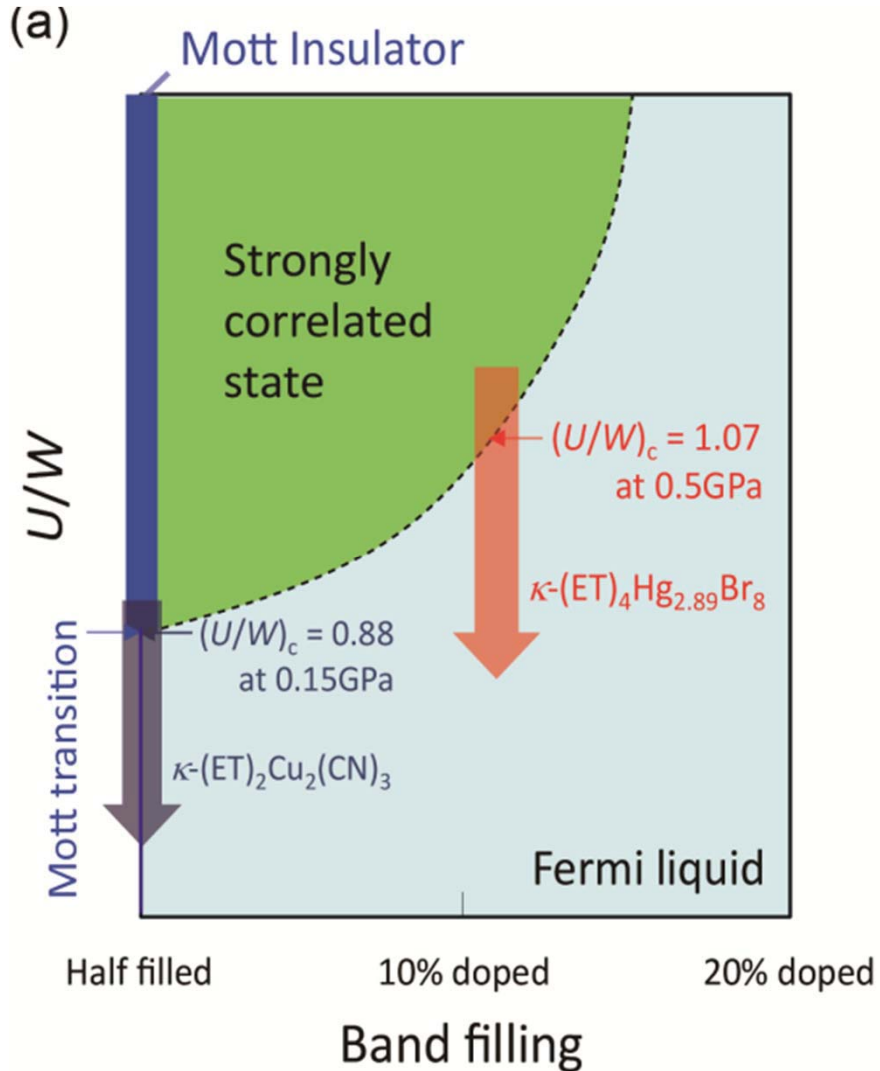


H. Oike, K. Miyagawa, H. Taniguchi, K. Kanoda PRL **114**, 067002 (2015)



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# Doped BEDT

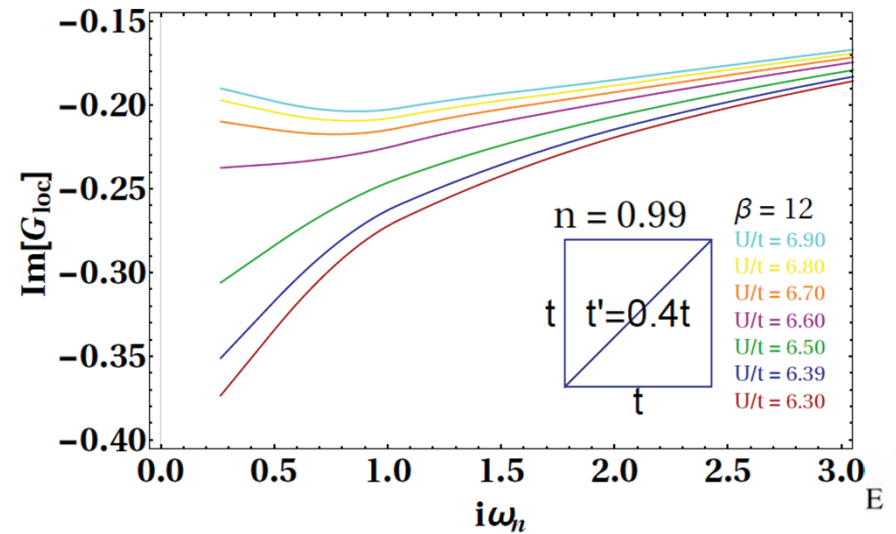
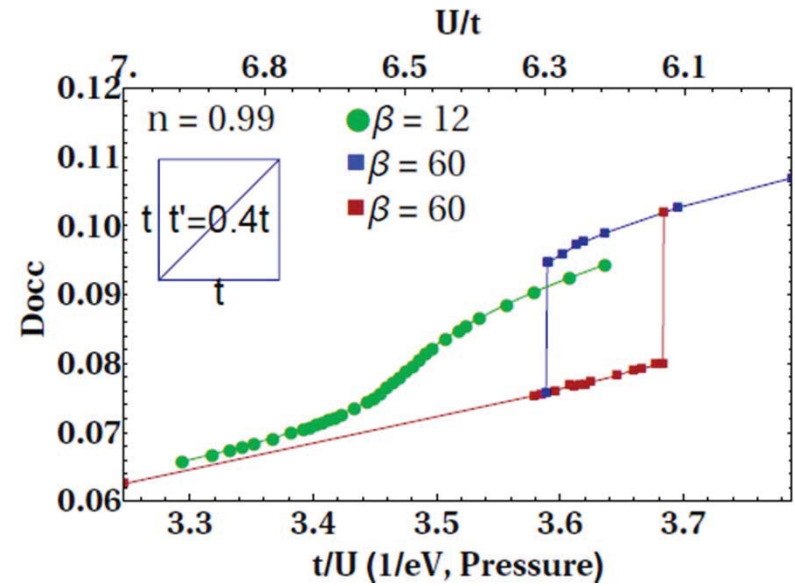
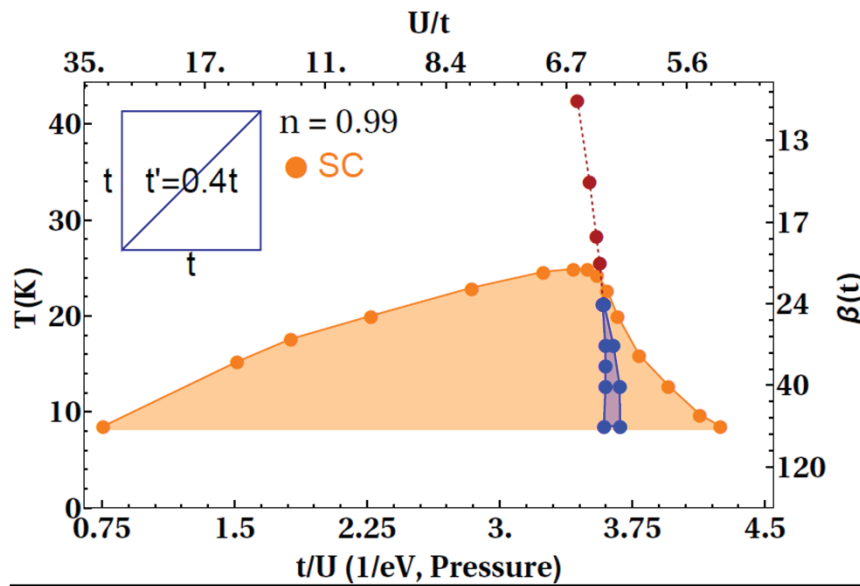


H. Oike, K. Miyagawa, H. Taniguchi, K. Kanoda PRL **114**, 06/002 (2015)



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# Widom line in organics

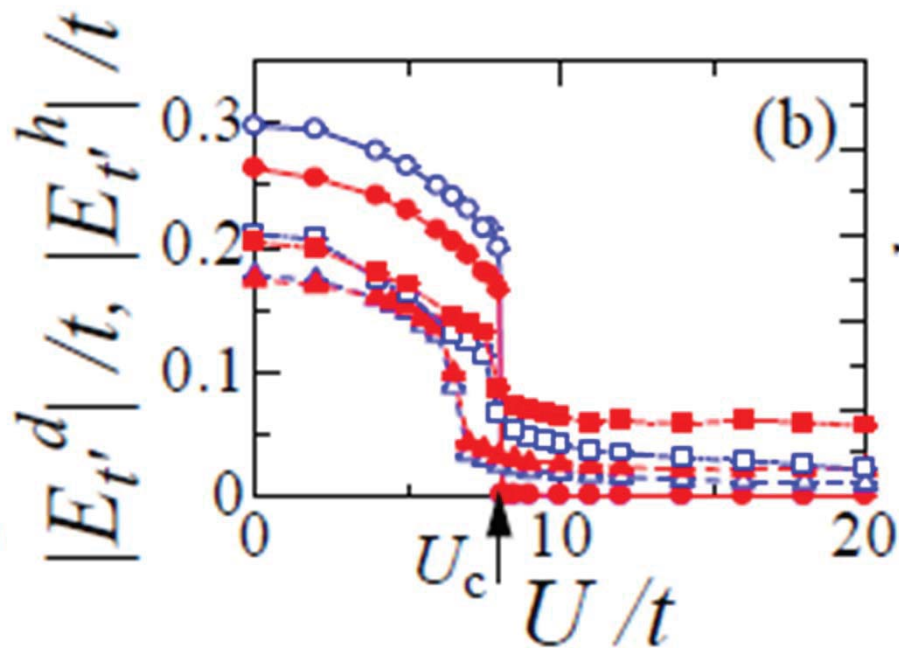


Charles-David Hébert, Patrick Sémon, AMT



# Results from variational MC

$ E_{t(t)}^d $	$ E_{t(t)}^h $	$D$	$\delta$	$L$
○	●	◉	0.0	12
△	▲	◈	0.04	10
□	■	◓	0.083	12
▽	▼	◔	0.12	10



T. Watanabe, H. Yokoyama  
and M. Ogata  
JPS Conf. Proc.  
3, 013004 (2014)



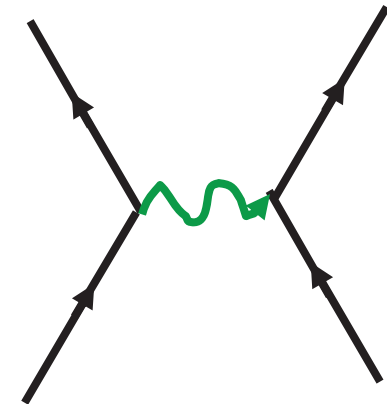
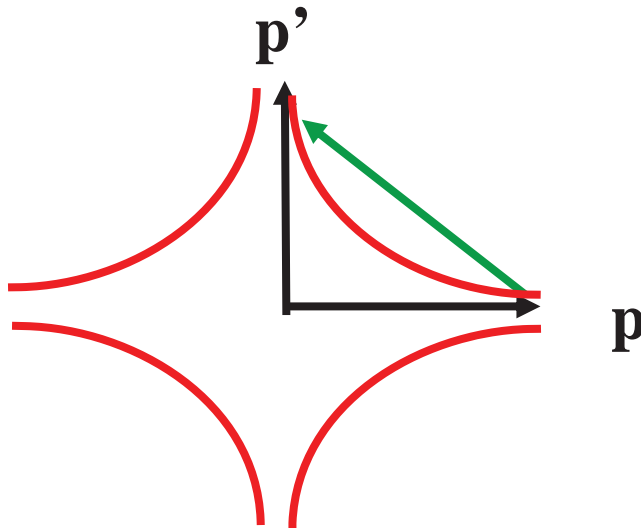
## 8. Superconductivity in general

Analog to weakly and strongly  
correlated antiferromagnets



# Cartoon « BCS » weak-coupling picture

$$\Delta_{\mathbf{p}} = -\frac{1}{2V} \sum_{\mathbf{p}'} U(\mathbf{p} - \mathbf{p}') \frac{\Delta_{\mathbf{p}'}}{E_{\mathbf{p}'}} (1 - 2n(E_{\mathbf{p}'}))$$



Béal–Monod, Bourbonnais, Emery  
P.R. B. **34**, 7716 (1986).

Exchange of spin waves?  
Kohn-Luttinger

D. J. Scalapino, E. Loh, Jr., and J. E. Hirsch  
P.R. B **34**, 8190-8192 (1986).

$T_c$  with pressure

Kohn, Luttinger, P.R.L. **15**, 524 (1965).

P.W. Anderson *Science* **317**, 1705 (2007)



# A cartoon strong coupling picture

$$J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = J \sum_{\langle i,j \rangle} \left( \frac{1}{2} c_i^\dagger \vec{\sigma} c_i \right) \cdot \left( \frac{1}{2} c_j^\dagger \vec{\sigma} c_j \right)$$

$$d = \langle \hat{d} \rangle = 1/N \sum_{\vec{k}} (\cos k_x - \cos k_y) \langle c_{\vec{k},\uparrow} c_{-\vec{k},\downarrow} \rangle$$

$$H_{MF} = \sum_{\vec{k},\sigma} \varepsilon(\vec{k}) c_{\vec{k},\sigma}^\dagger c_{\vec{k},\sigma} - 4Jm\hat{m} - Jd(\hat{d} + \hat{d}^\dagger) + F_0$$

Pitaevskii Brückner:

Pair state orthogonal to repulsive core of Coulomb interaction

P.W. Anderson *Science*  
317, 1705 (2007)

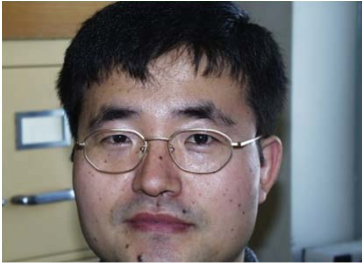
Miyake, Schmitt–Rink, and Varma  
*P.R. B* 34, 6554-6556 (1986)

More sophisticated Slave Boson: Kotliar Liu *PRB* 1988

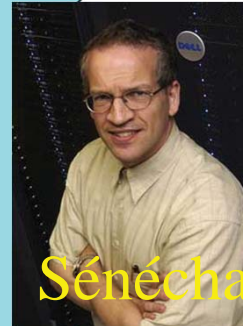
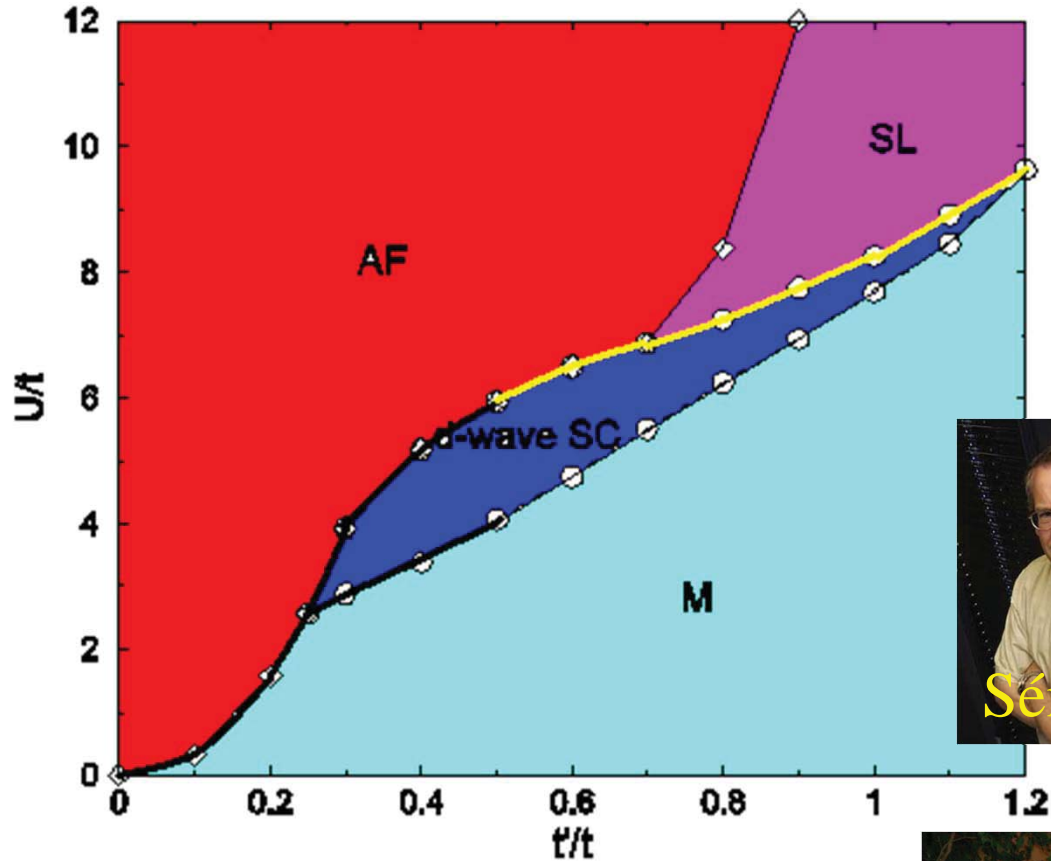
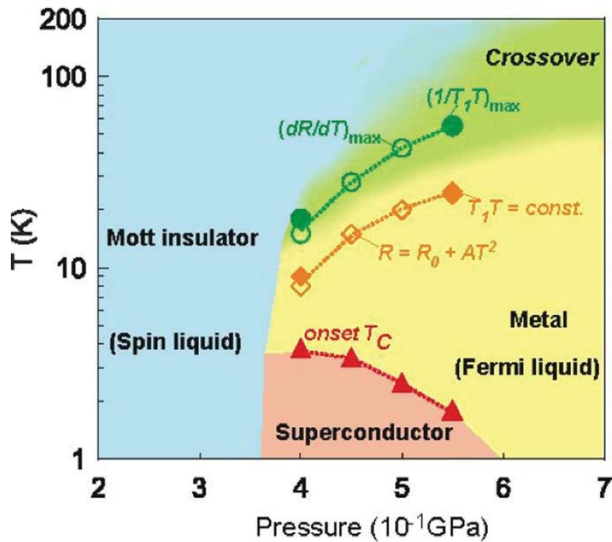


# 8. Superconductivity in the organics





# Theoretical phase diagram BEDT



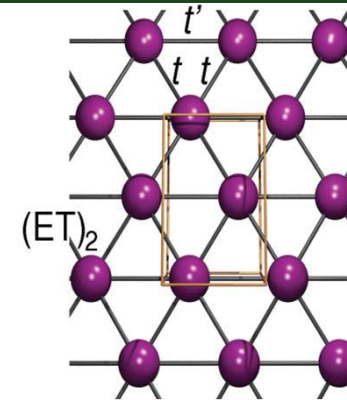
Sénéchal, Sahebsara, Phys. Rev. Lett. **97**, 257004



Y. Shimizu, et al. Phys. Rev. Lett. **91**, (2003)

# Other compounds (R. Valenti et al.)

X	Hueckel		DFT	
	$t'/t$	$U/t$	$t'/t$	$U/t$
CN	1.06	8.2	0.83 (0.85)	7.3 (12)
SCN	0.84	6.8	0.58 (0.83)	6.0
Cl	0.75	7.5	0.44	7.5
Br	0.68	7.2	0.42	5.1



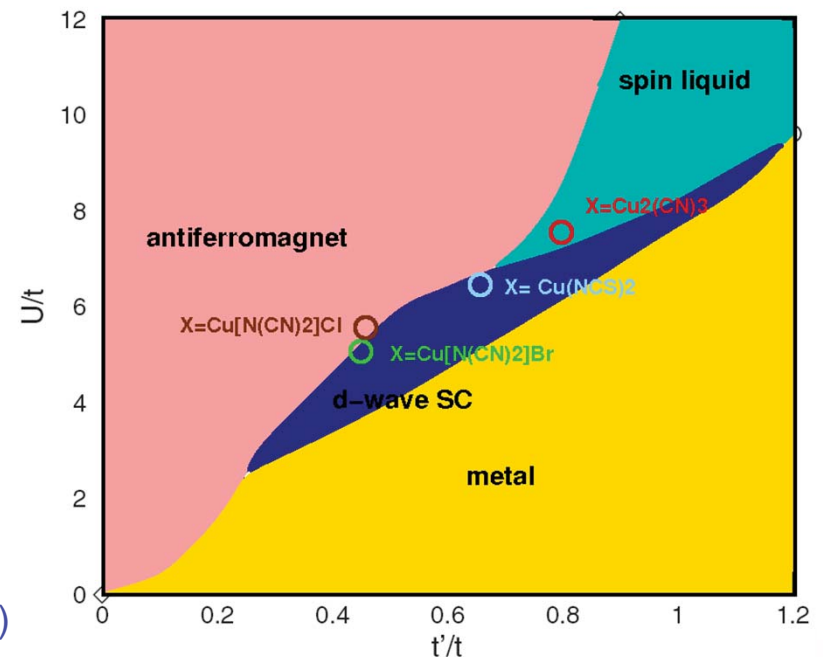
Kandpal et al. PRL (2009)

Nakamura et al. JPSJ (2009)

Komatsu et al. JPSJ (1996)

Kyung, Tremblay PRL (2006)

Tocchio, Parola, Gros, Becca PRB (2009)



# Analogous results with other methods

H. Morita et al., J. Phys. Soc. Jpn. 71, 2109 (2002).

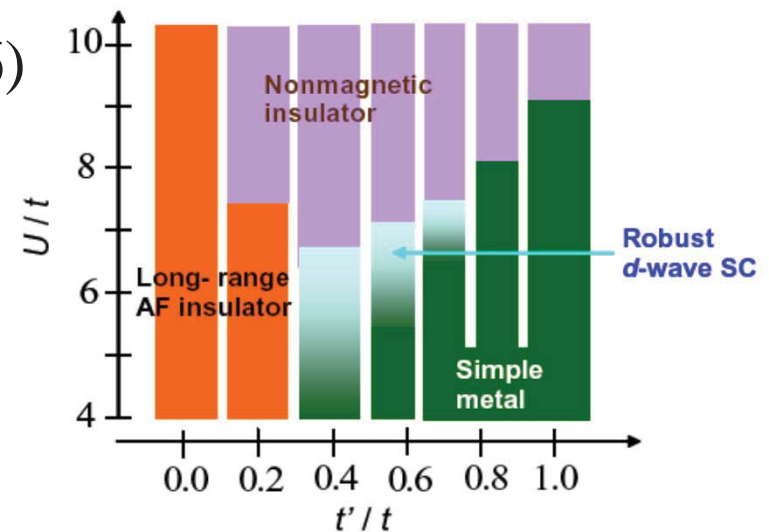
J. Liu et al., Phys. Rev. Lett. 94, 127003 (2005).

S.S. Lee et al., Phys. Rev. Lett. 95, 036403 (2005).

B. Powell et al., Phys. Rev. Lett. 94, 047004 (2005).

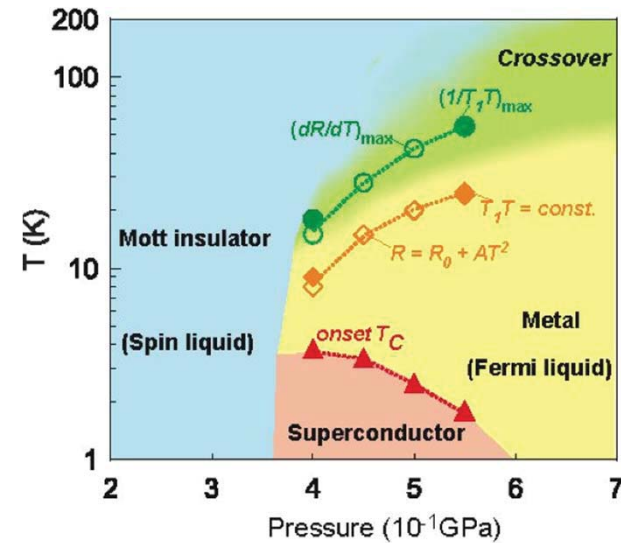
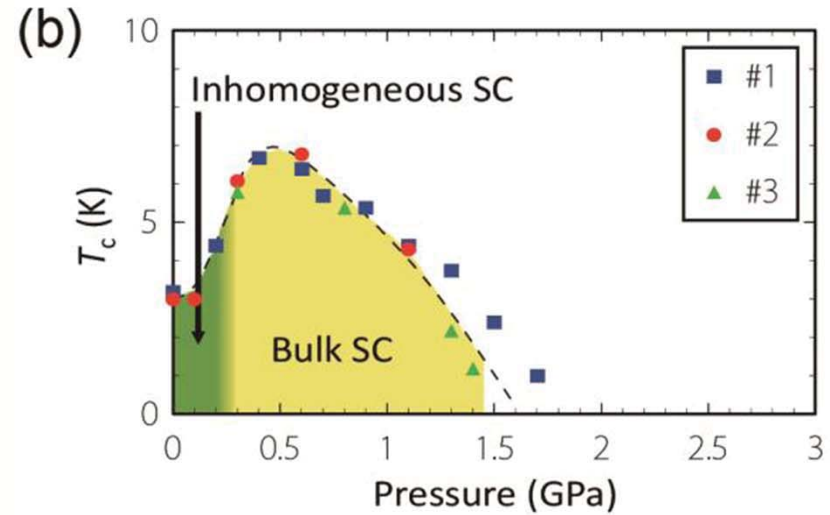
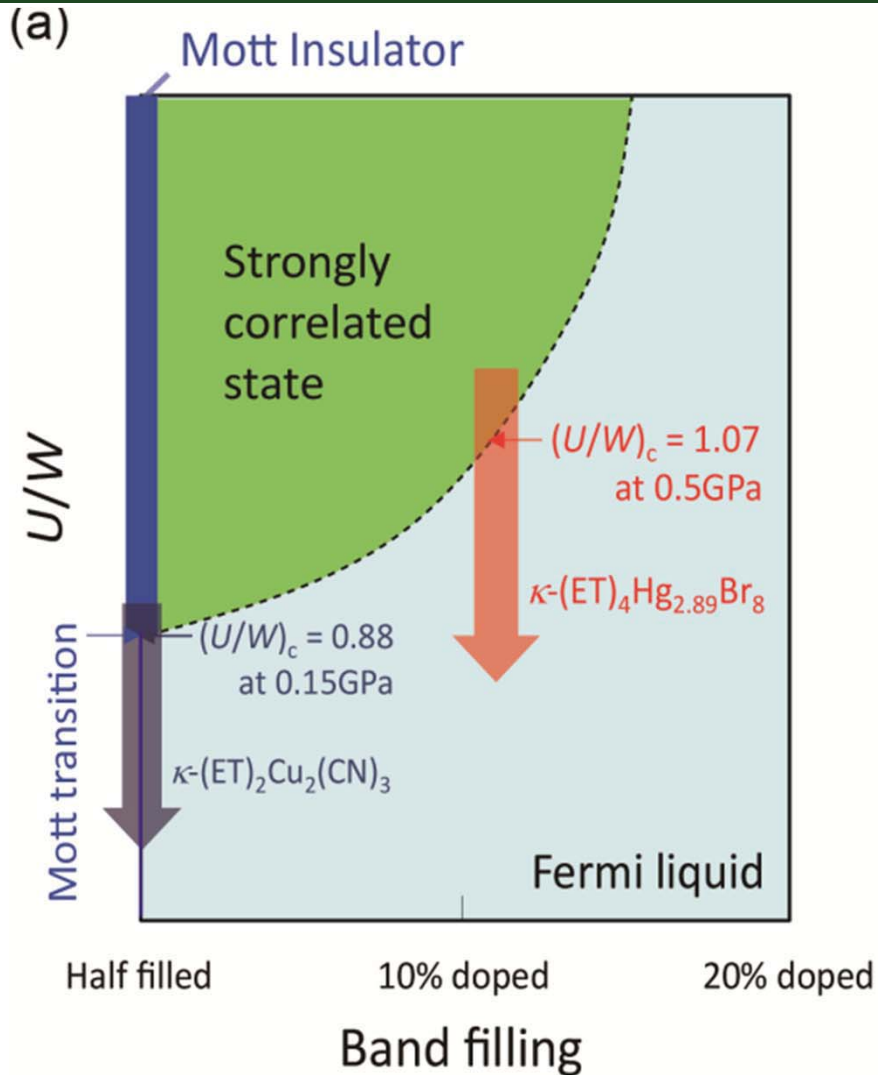
J.Y. Gan et al., Phys. Rev. Lett. 94, 067005 (2005).

T. Watanabe et J. Phys. Soc. Japan (2006)





# Doped BEDT

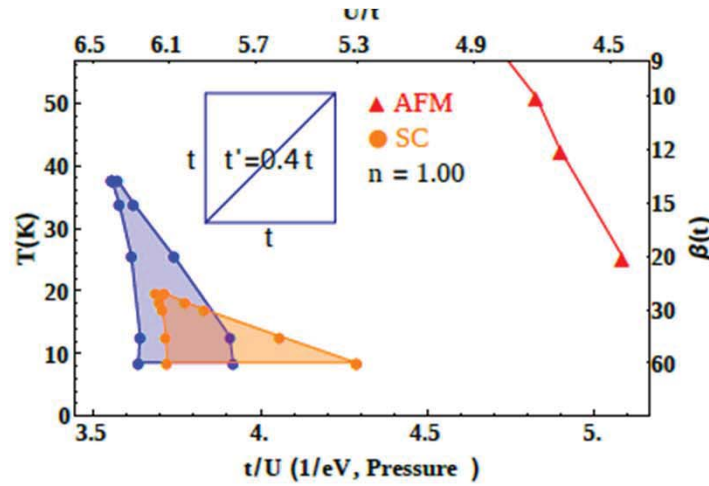
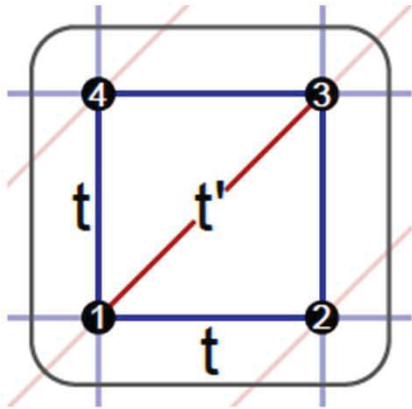


H. Oike, K. Miyagawa, H. Taniguchi, K. Kanoda PRL **114**, 067002 (2015)

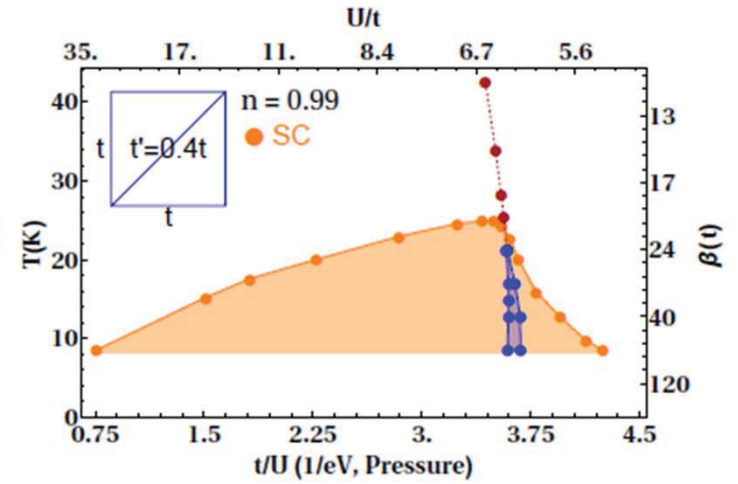


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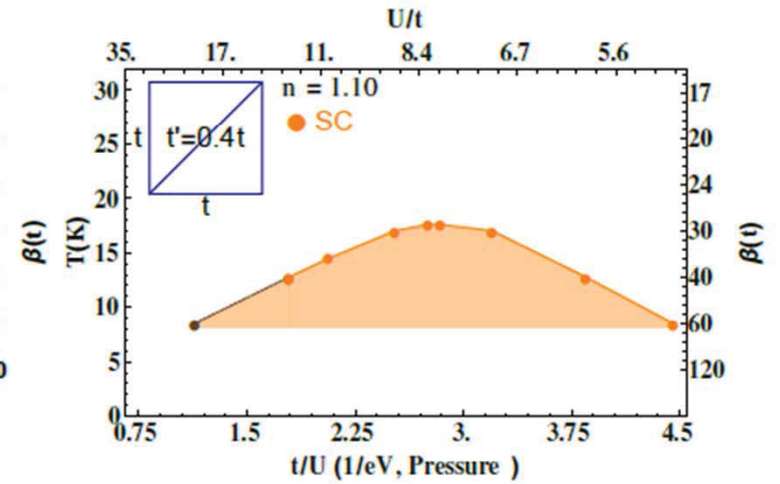
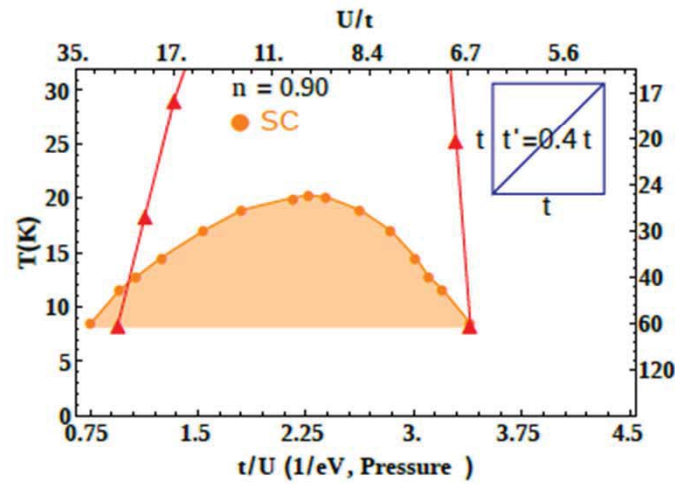
$$t' = 0.4t$$



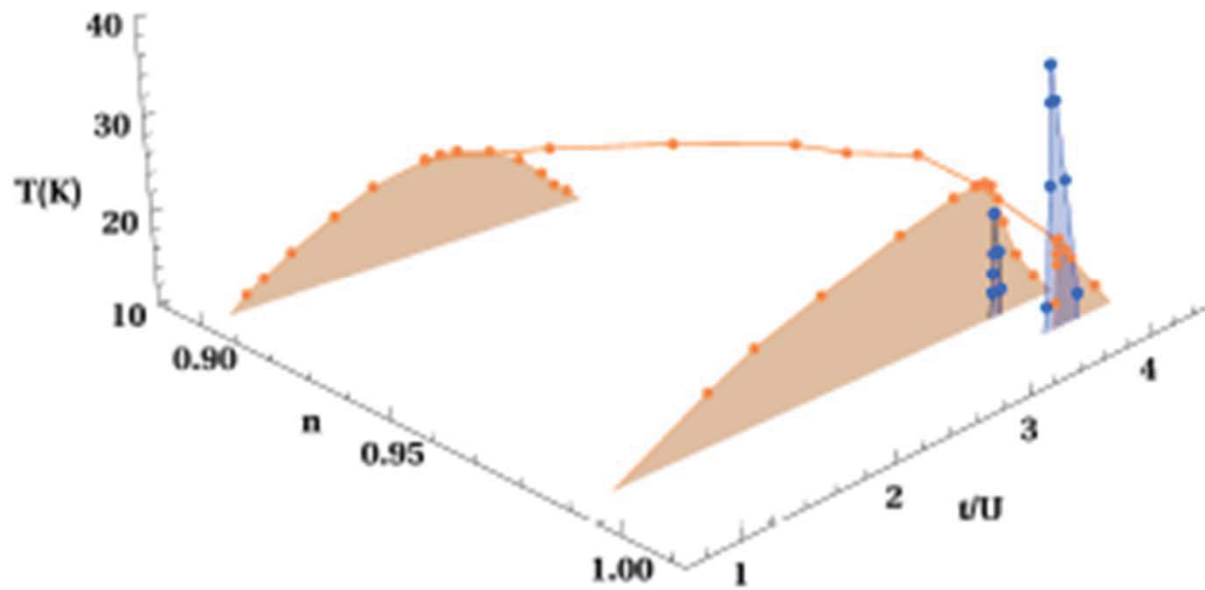
(a)



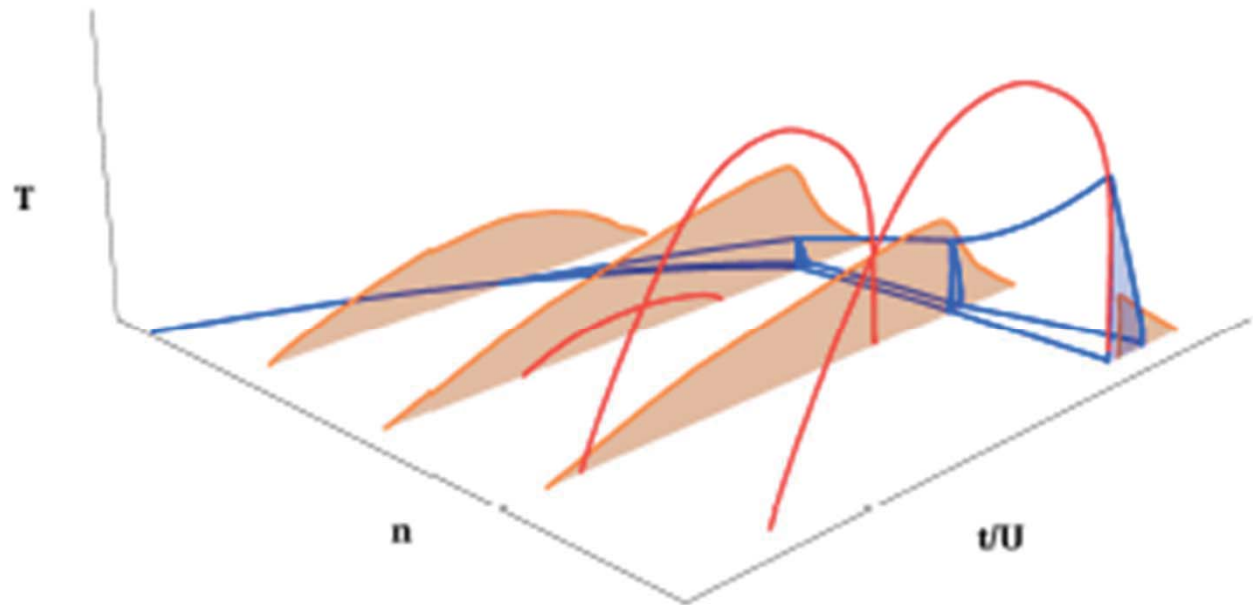
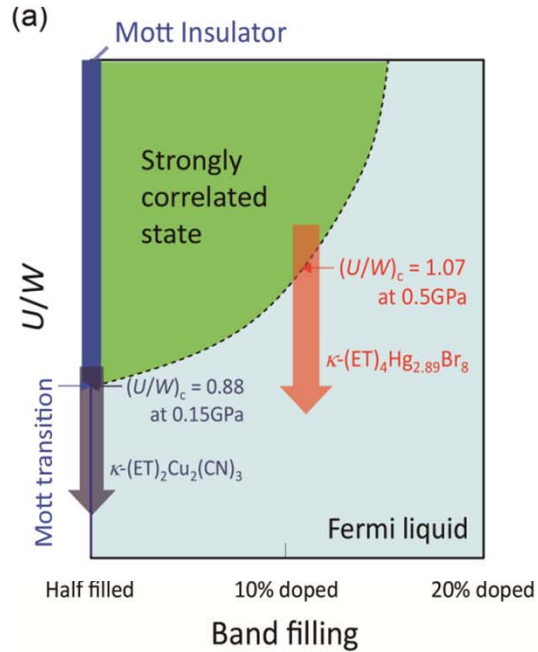
(b)



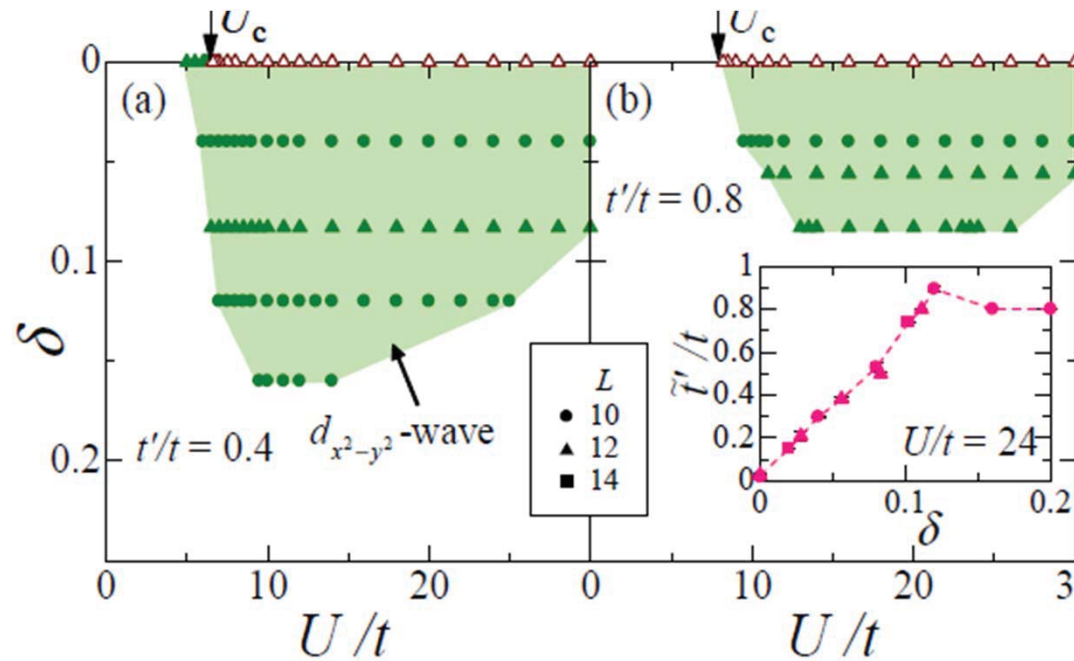
# $t' = 0.4t$ overview



# Generic case highly frustrated case



# Results from variational MC



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JPS Conf. Proc. 3, 013004 (2014)



# Summary : organics

- Agreement with experiment
  - SC: larger  $T_c$  and broader  $P$  range if doped
  - Larger frustration: Decrease  $T_N$  and  $T_c$
  - Normal state metal to pseudogap crossover
- Predictions
  - First order transition at low  $T$  in normal state
    - (or remnants in SC state)
- Physics
  - SC dome without an AFM QCP. Extension of Mott
  - SC from short range  $J$ .
  - $T_c$  decreases at Widom line



# Main collaborators



Giovanni Sordi



Kristjan Haule



David Sénéchal



Bumsoo Kyung



Charles-David Hébert



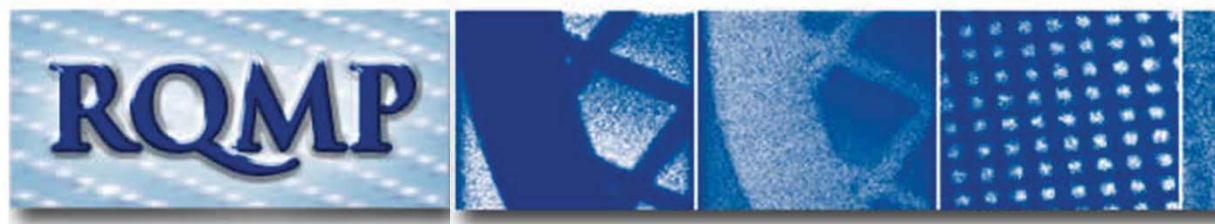
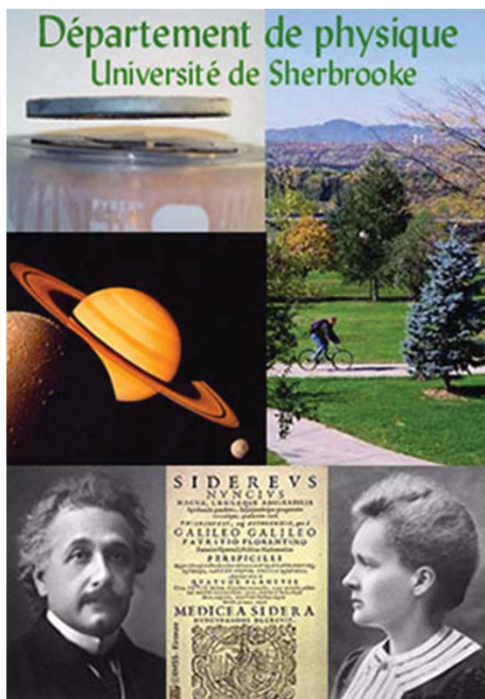
Patrick Sémon



Dominic Bergeron



# André-Marie Tremblay



Le regroupement québécois sur les matériaux de pointe

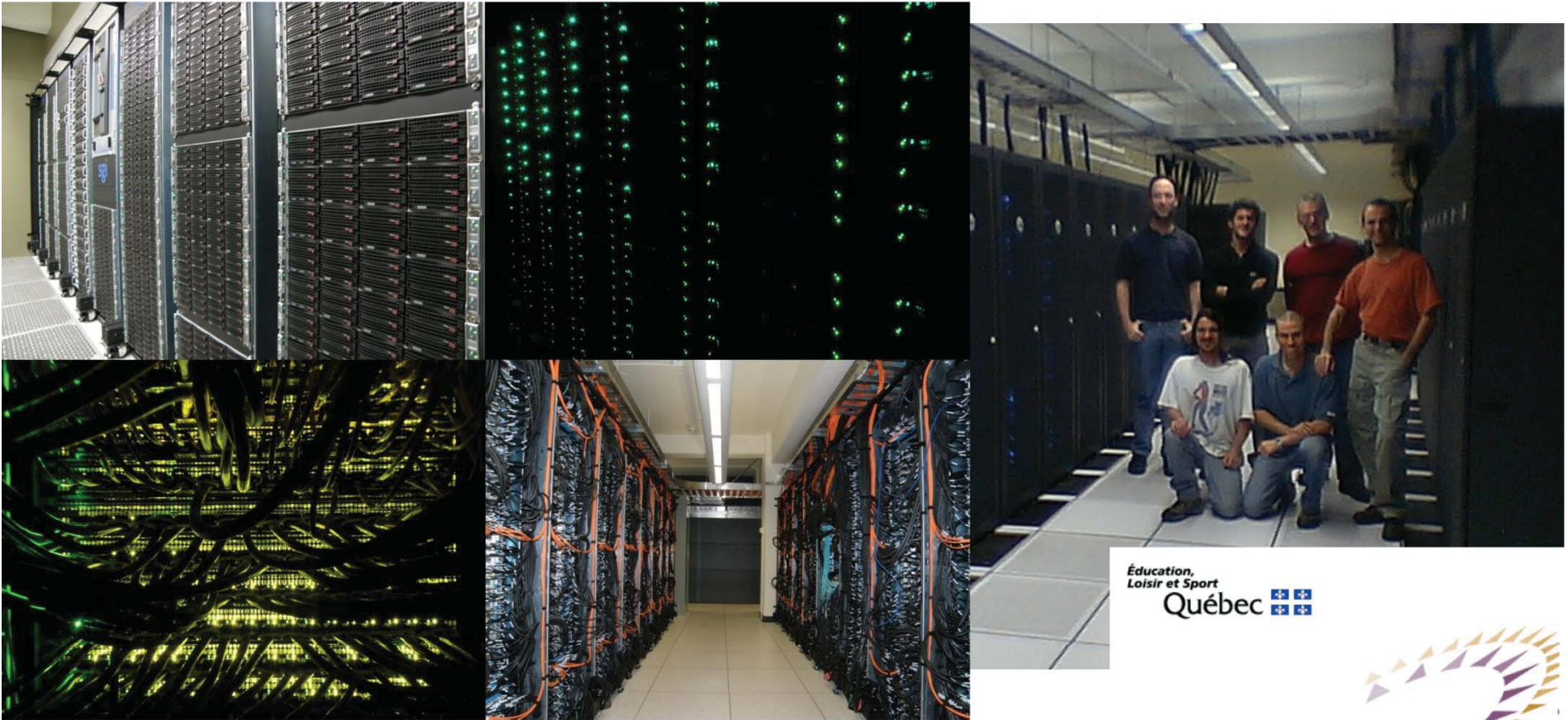


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## Next time

- Emergent finite doping first-order transition (Sordi transition) as an organizing principle
  - Pseudogap
  - Superconductivity
- Strongly correlated superconductivity and retardation.



Merci

Thank you

A.-M.S. Tremblay

*“Strongly correlated superconductivity”*

Chapt. 10 : *Emergent Phenomena in Correlated Matter Modeling and Simulation, Vol. 3*, E. Pavarini, E. Koch, and U. Schollwöck (eds.)

Verlag des Forschungszentrum Jülich, 2013