Quantum critical Planckian metal and SYK physics with spin 1/2 fermions

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Collaborators & references

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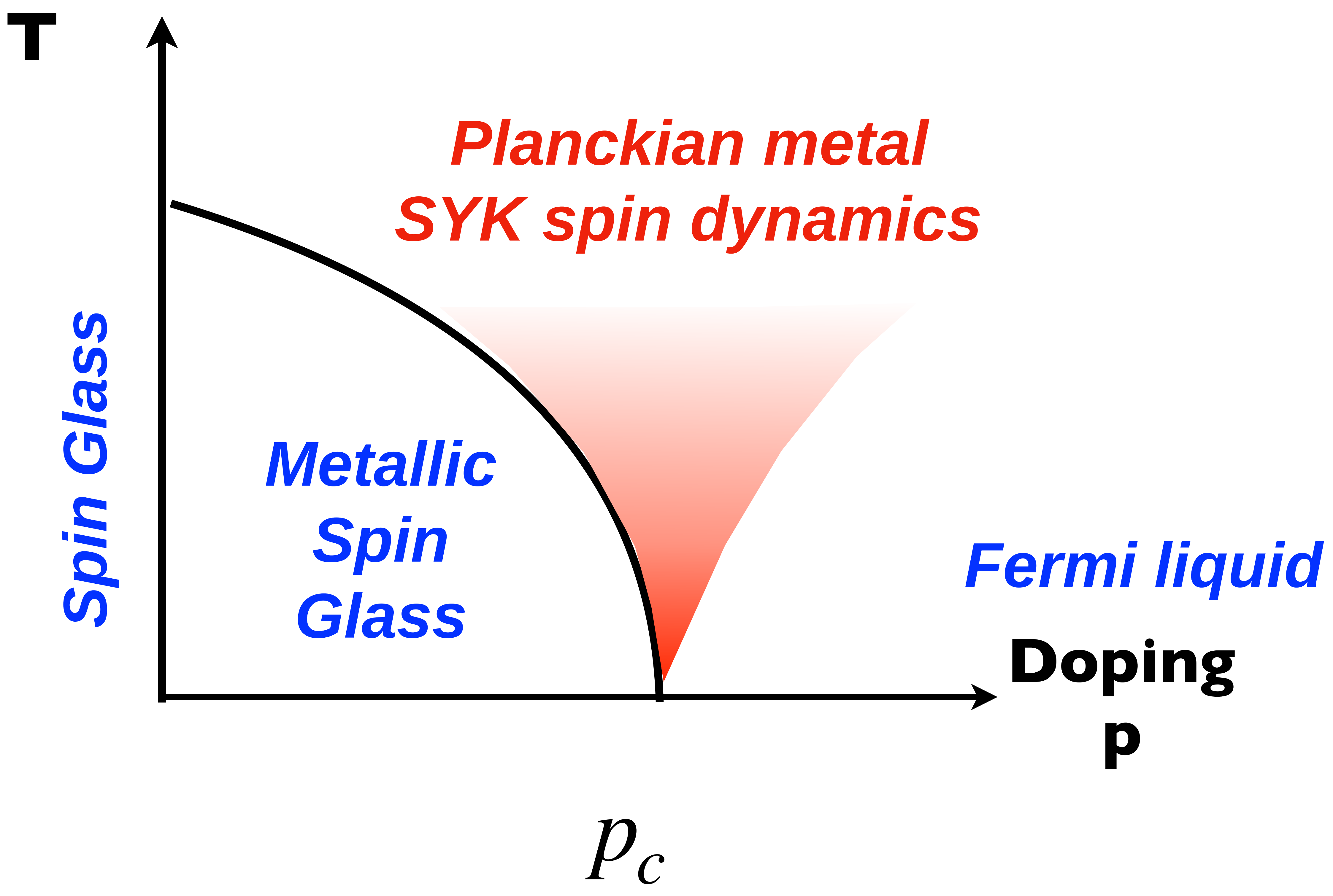
Ph. Dumitrescu, N. Wentzell, A. Georges, ***OP****, Phys. Rev. B 105, L180404 (2022)*

D. Chowdhury, A. Georges, ***OP,*** *S. Sachdev arxiv:2109.05037, To appear in Rev. Mod. Phys.*

Overview

- Incoherent transport in metal close to a quantum spin glass instability
- Disordered SU(2) t-U-J model. No large-M limit.



- SYK spin dynamics
- Linear resistivity at the quantum critical point

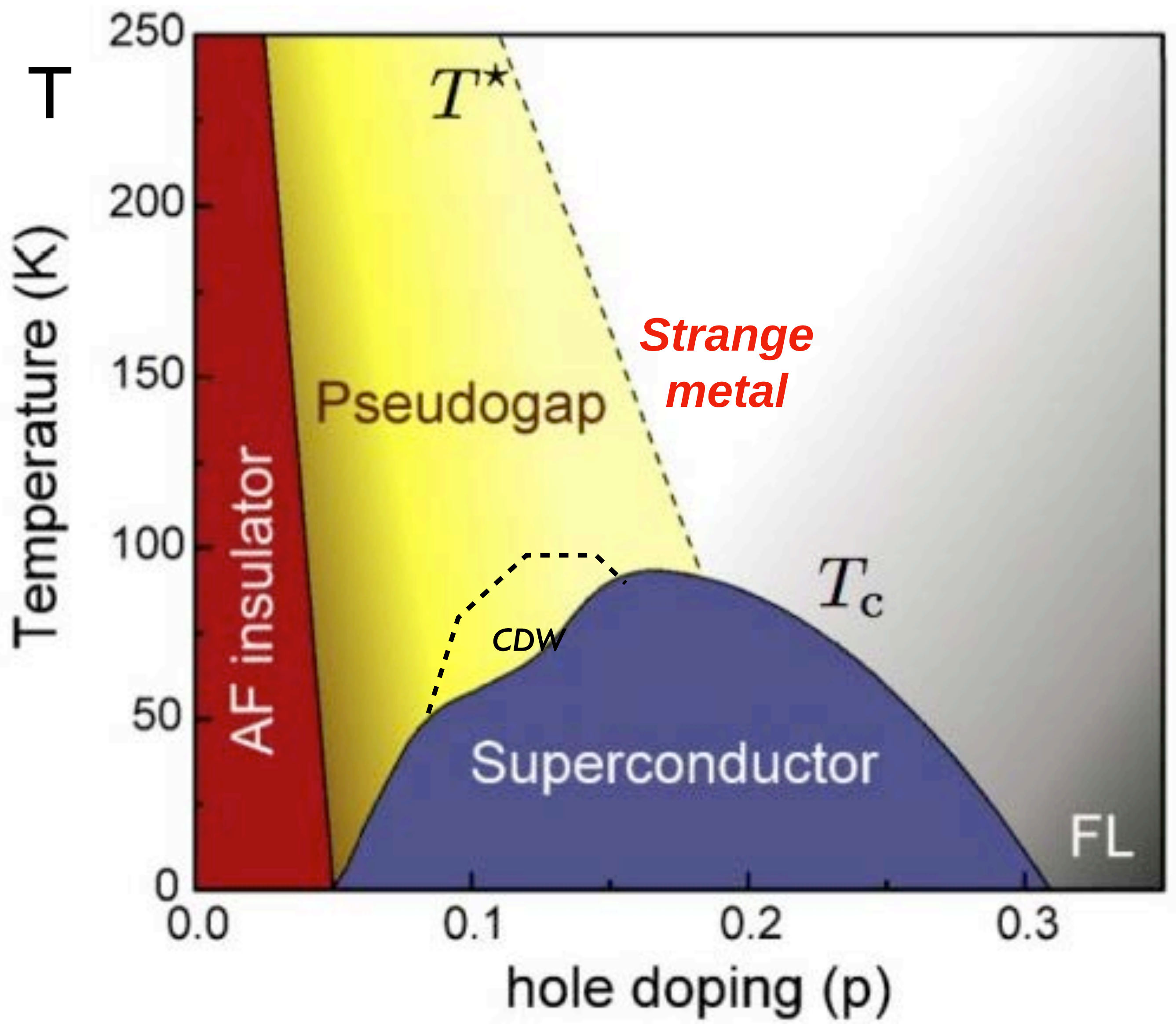
$$\rho \sim T$$

- Lifetime close to the Planckian limit

$$\frac{1}{\tau^*} \simeq c\frac{k_B T}{\hbar}$$

Ph. Dumitrescu, N. Wentzell, A. Georges, ***OP*** *Phys. Rev. B 105, L180404 (2022)*

High temperature superconductors



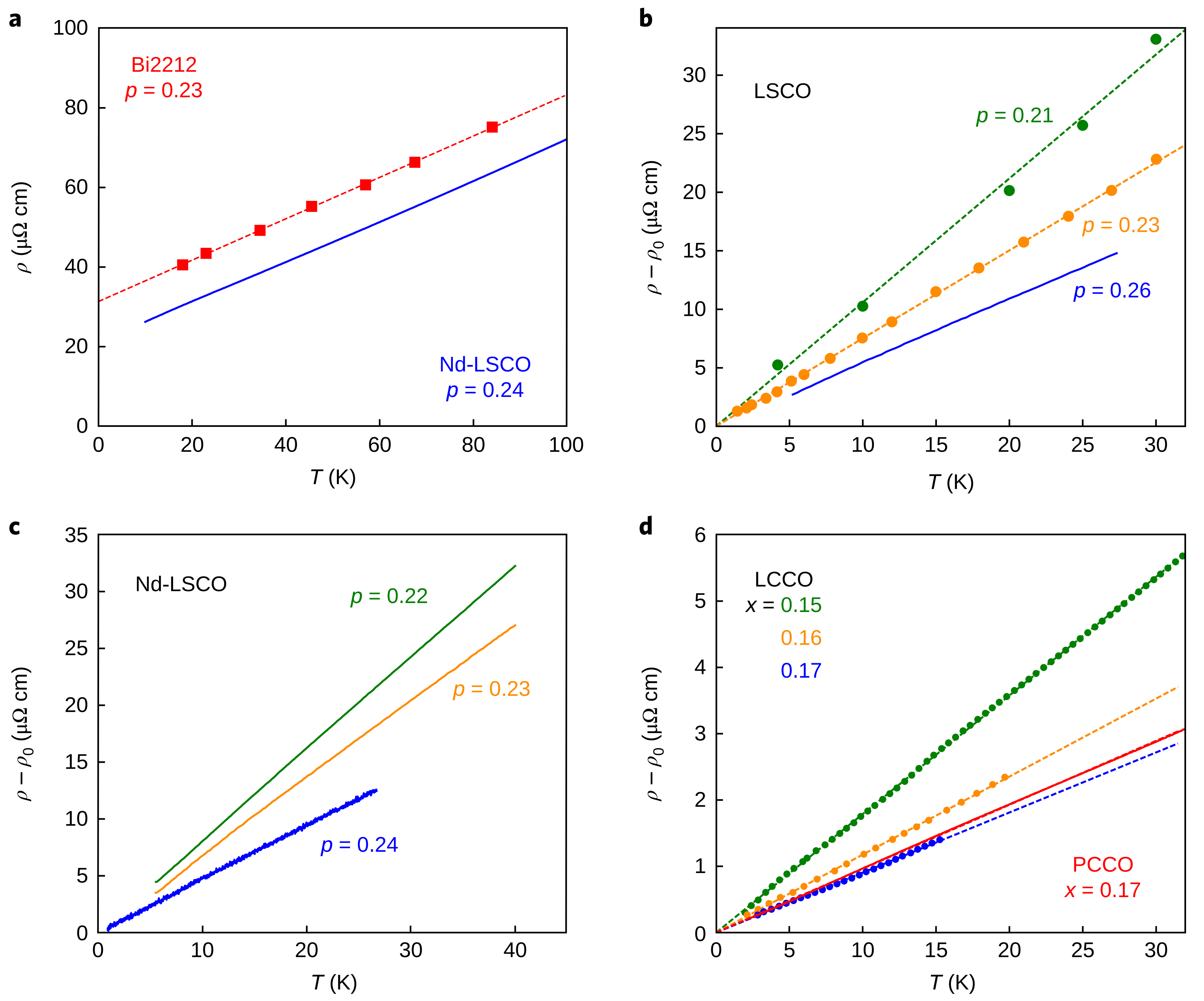
LETTERS
https://doi.org/10.1038/s41567-018-0334-2
nature physics

Universal *T*-linear resistivity and Planckian dissipation in overdoped cuprates

A. Legros[1,2], S. Benhabib[3], W. Tabis[3,4], F. Laliberté[1], M. Dion[1], M. Lizaire[1], B. Vignolle[3], D. Vignolles[3], H. Raffy[5], Z. Z. Li[5], P. Auban-Senzier[5], N. Doiron-Leyraud[1], P. Fournier[1,6], D. Colson[2], L. Taillefer[1,6*] and C. Proust[3,6*]

A. Legros et al. Nature Physics 15, 142 (2019)

$$\rho(T) = \rho_0 + \alpha \frac{h}{2e^2} \frac{T}{T_F}$$

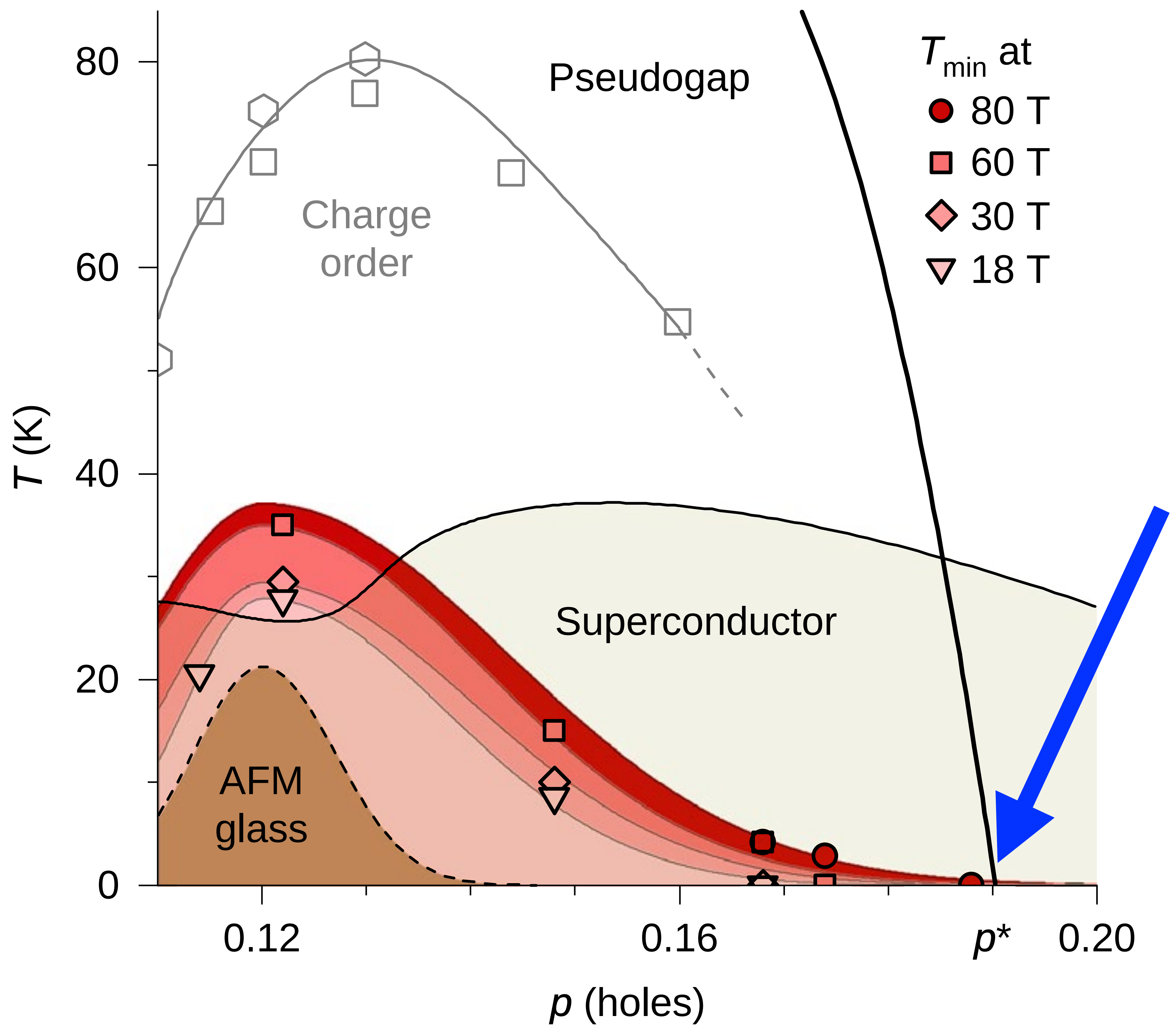


- T- linear resistivity in the strange metal

Table 1 | Slope of *T*-linear resistivity versus Planckian limit in seven materials

Material	Doping[a]	n (10^{27} m^{-3})	m^* (m_0)	A_1/d (Ω K^{-1})	$h/(2e^2T_F)$ (Ω K^{-1})	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

High temperature superconductors



ARTICLES
https://doi.org/10.1038/s41567-020-0950-5
nature physics
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Hidden magnetism at the pseudogap critical point of a cuprate superconductor

Mehdi Frachet[1,9], Igor Vinograd[1,9], Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1] ✉ and Marc-Henri Julien[1] ✉

Frachet et al. Nature Physics 16, 1064 (2020)

LSCO. NMR, ultrasound

- Glassy order up to the boundary of the pseudo gap p*
- Fermi surface reconstruction close to p* (sudden change of number of carriers).
- Relation to strange metal ? T linear resistivity ?

t-U-J model

- Spin 1/2 electrons, on a lattice with local Coulomb repulsion U and disordered J & t

$$H = -\sum_{ij,\sigma}(t_{ij} + \mu\delta_{ij})c^\dagger_{i\sigma}c_{j\sigma} + \sum_{i<j} J_{ij}\boldsymbol{S}_i \cdot \boldsymbol{S}_j + U\sum_i n_{i\uparrow}n_{i\downarrow},$$

- NB : S is electronic spin. Same degrees of freedom

$$\boldsymbol{S}_i = c^\dagger_{i\alpha}\frac{\boldsymbol{\sigma}_{\alpha\beta}}{2}c_{i\beta}$$

- Fully connected model (or hopping on a lattice and use DMFT …)
- J and t with gaussian distribution

$$\overline{t_{ij}} = \overline{J_{ij}} = 0$$

$$\overline{|t_{ij}|^2} = t^2/\mathcal{N} \longleftarrow \textit{Number of sites}$$

$$\overline{|J_{ij}|^2} = J^2/\mathcal{N}$$

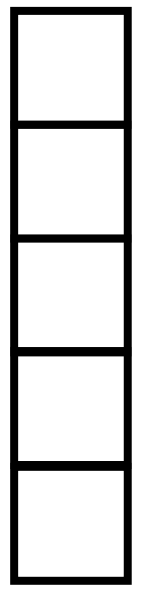
Sachdev-Ye model

Phys. Rev. Lett. 70, 3339 (1993)

$$t = 0, \quad U = \infty$$

$$\sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$S_{\alpha\beta} \sim f^\dagger_\alpha f_\beta$$



$$\sum_\alpha f^\dagger_\alpha f_\alpha = q_0 M$$

- Large M with SU(M) spin instead of SU(2), with fermionic representation

- Gapless spin liquid at T=0. No spin glass ordering at $M = \infty$

- "Marginal Fermi liquid" spin dynamics

$$\chi(t) = \langle \overrightarrow{S}(t) \cdot \overrightarrow{S}(0) \rangle \sim 1/t$$

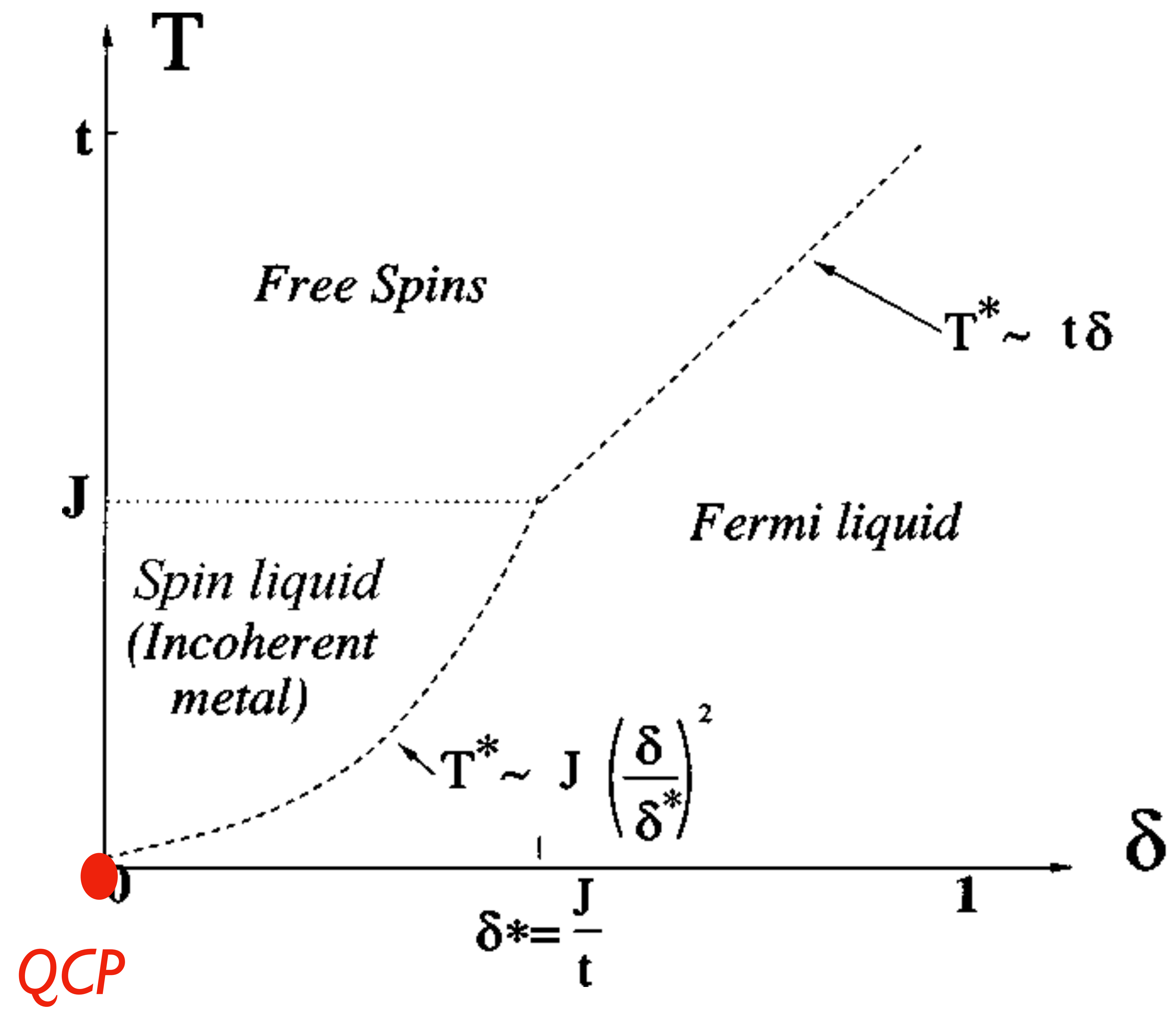
$$J\chi''(\omega, T = 0) \propto \text{sign}(\omega)$$

Doping the SY model : t-J model, large M

O.P. & A.Georges Phys. Rev. B 59, 5341 (1999)

- SU(M) t-J model

$$H = -\sum_{\langle ij\rangle\alpha} t_{ij} P c_{i\alpha}^{\dagger} c_{j\alpha} P + \sum_{\langle ij\rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \,.$$



- SYK/ marginal Fermi liquid spin physics in the quantum critical regime

$$\chi''_{\mathrm{loc}}(\omega, T) = \frac{\sqrt{\pi}}{2J} \tanh \frac{\omega}{2T} \,.$$

- See also, doped SYK version, same large M equations

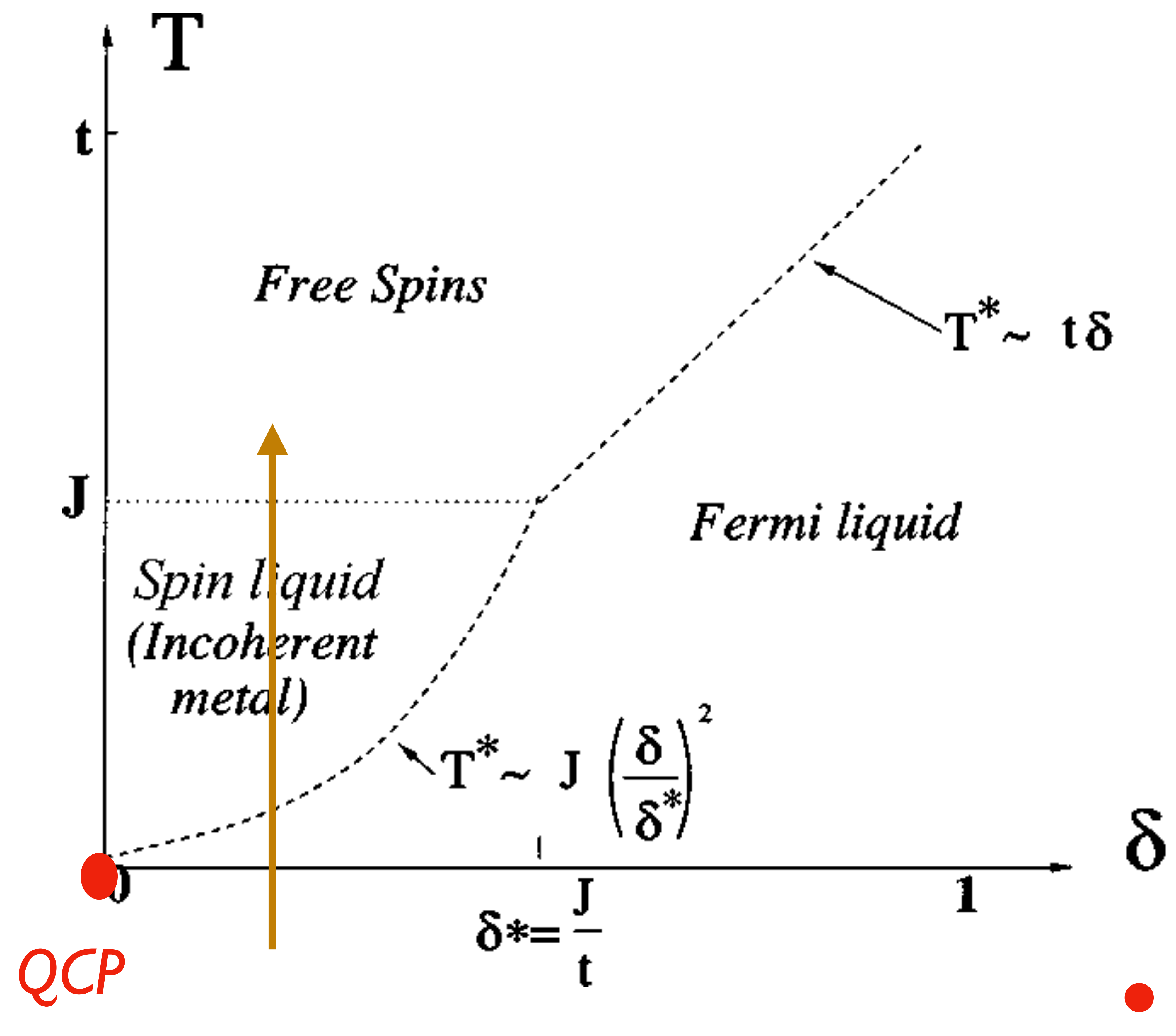
XY Song, CM Jian, and L. Balents
Phys. Rev. Lett. 119, 216601 (2017)

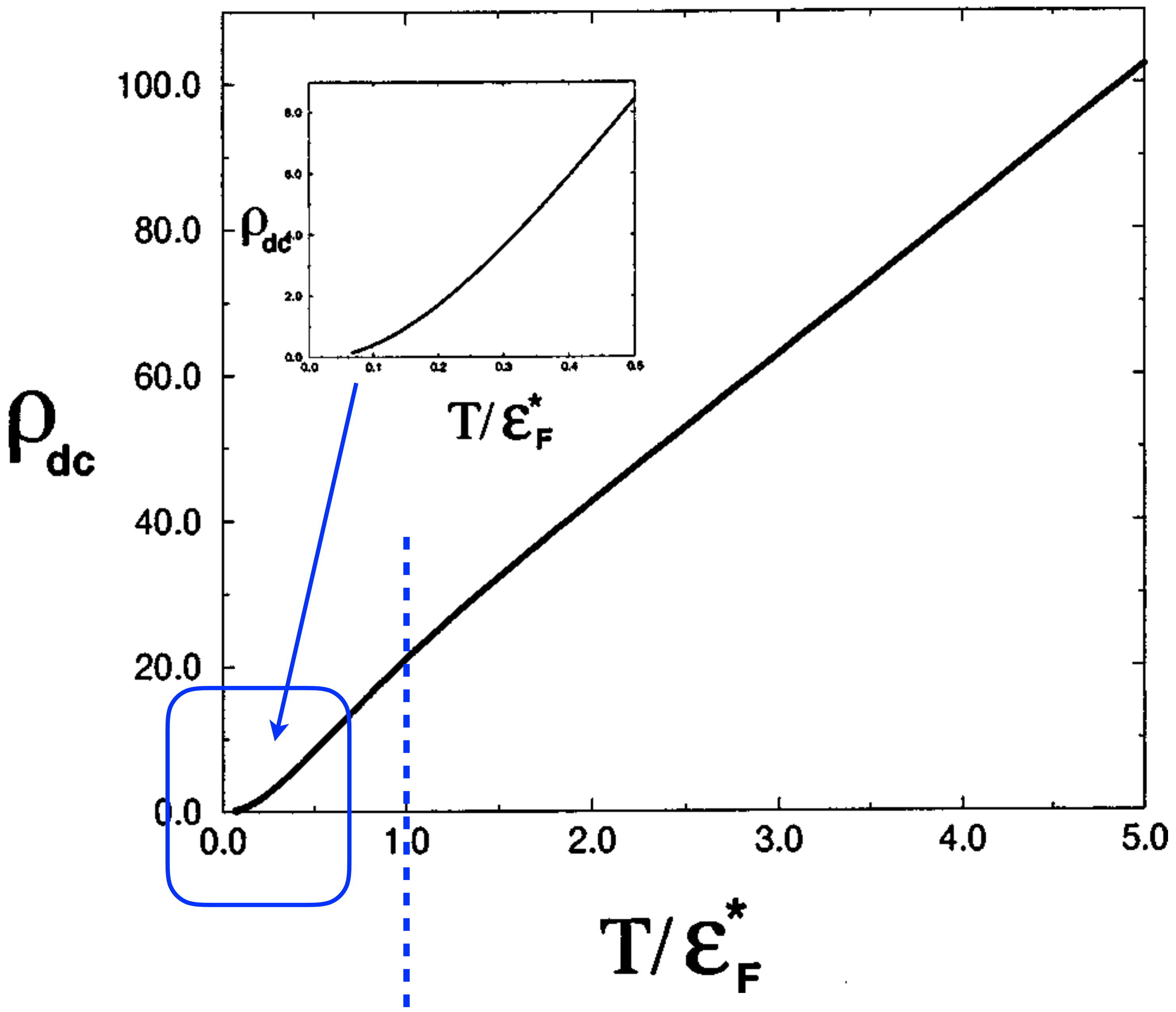
Doping the SY model : t-J model, large M

O.P. & A.Georges Phys. Rev. B 59, 5341 (1999)

- SU(M) t-J model

$$H = -\sum_{\langle ij\rangle \alpha} t_{ij} P c^{\dagger}_{i\alpha} c_{j\alpha} P + \sum_{\langle ij\rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j .$$





Fermi liquid

$$\rho_{dc}(T) \propto \left(\frac{T}{\epsilon_F^*}\right)^2$$

$$T \ll \epsilon_F^*$$

Hot/bad metal

$$\rho_{dc}(T) \propto \frac{T}{\epsilon_F^*}$$

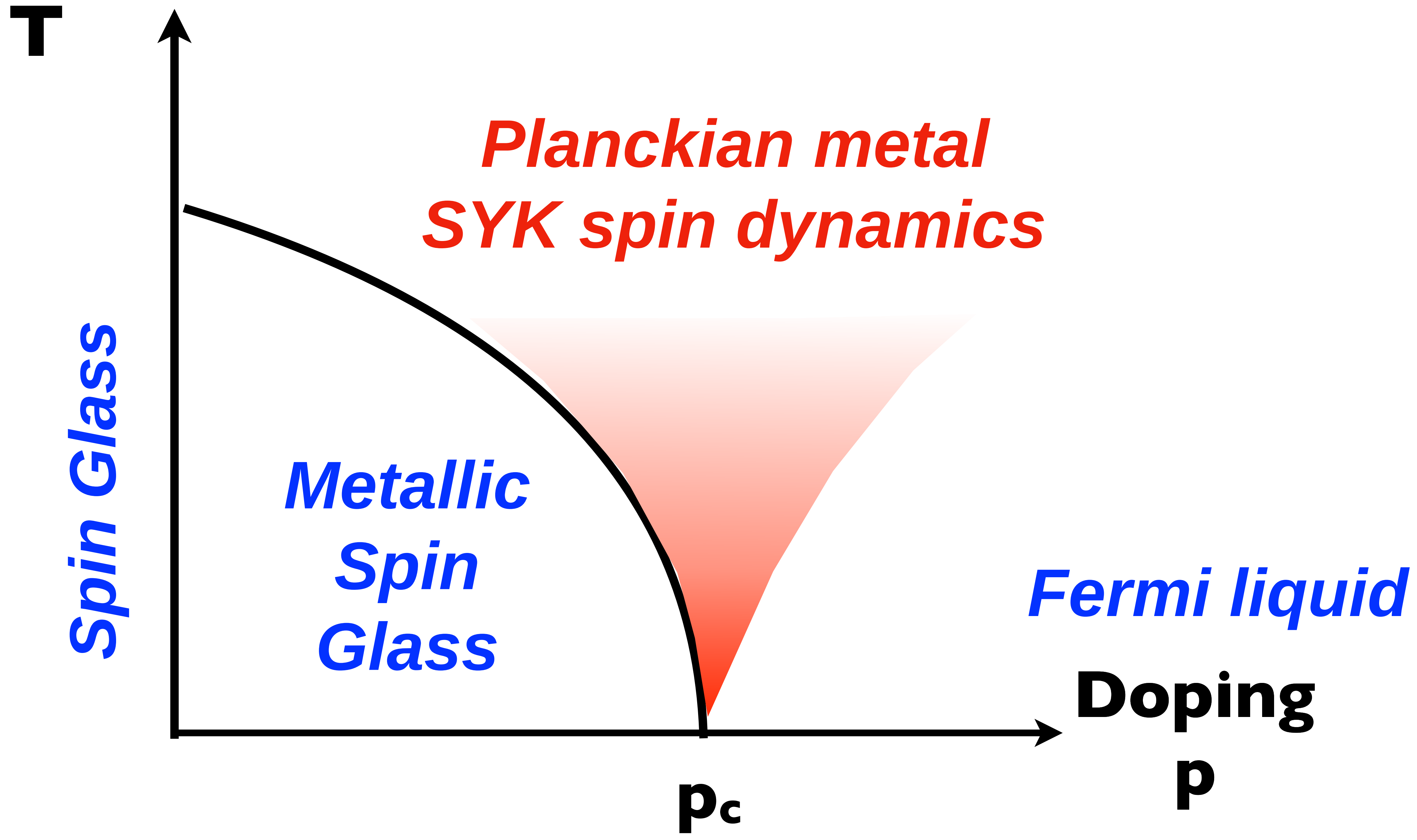
$$\epsilon_F{}^* \ll T \ll J$$

- Not a Planckian metal. QCP at δ=0 is an insulator !

The SU(2) model has richer physics

$$H = -\sum_{ij,\sigma}(t_{ij} + \mu\delta_{ij})c^\dagger_{i\sigma}c_{j\sigma} + \sum_{i<j} J_{ij}\boldsymbol{S}_i \cdot \boldsymbol{S}_j + U\sum_i n_{i\uparrow}n_{i\downarrow},$$

- At low T, zero doping : spin glass, not a spin liquid *Grempel Rozenberg (1998), Arrachea-Rozenberg (2002)*
- QCP is now at finite doping.



How to solve the SU(2) case ?

- Thermodynamic limit, disordered averaged.
  Ph. Dumitrescu, N.Wentzell, A. Georges, OP Phys. Rev. B 105, L180404 (2022). (See also Otzuki,Vollhardt(2013))
- Exact diagonalization on finite systems,
  H. Shackleton,A.Wietek,A. Georges, S. Sachdev Phys. Rev. Lett. 126, 136602 (2021)
- Analytical insights (RG, …)
  A. Sengupta PRB (2000),
  Joshi et al PRX 10, 021033 (2020).
  D. Chowdhury,A. Georges, OP, S. Sachdev arxiv:2109.05037,To appear in Rev. Mod. Phys.

Disordered averaged action

- Thermodynamics limit, with replica trick (replica diagonal solution). Exact action.
  Paramagnetic phase.

$$S_{\text{eff}} = \int \mathrm{d}\tau \left[\sum_{\sigma} c^{\dagger}_{\tau,\sigma} \left[\partial_\tau - \mu\right] c_{\tau,\sigma} + U n_{\uparrow}(\tau) n_{\downarrow}(\tau) \right] + \int \mathrm{d}\tau \mathrm{d}\tau' \left[\Delta(\tau-\tau') c^{\dagger}_{\tau,\sigma} c_{\tau',\sigma} - \frac{J^2}{2} Q(\tau-\tau') \boldsymbol{S}(\tau)\cdot\boldsymbol{S}(\tau') \right]$$

$$G(\tau) = -\langle T_\tau c(\tau) c^{\dagger}(0)\rangle \qquad \Delta(\tau) = t^2 G(\tau) \qquad Q(\tau-\tau') = \tfrac{1}{3}\langle \boldsymbol{S}(\tau)\cdot\boldsymbol{S}(\tau')\rangle$$

Electronic Green function — *Electronic bath* — *Retarded spin spin*

- For SU(M), $M \to \infty$: "slave" boson + saddle point method gives a nonlinear equation for G

$$c^{\dagger} \sim f^{\dagger} b$$

$$G_f(i\omega_n)^{-1} = i\omega_n + \tilde{\mu} - (t\delta)^2 G_f(i\omega_n) - \Sigma(i\omega_n)$$

$$\Sigma_f(\tau) = J^2 G_f(\tau) G_f(\beta-\tau)$$

- In the SU(2) case, we want an exact solution of the action

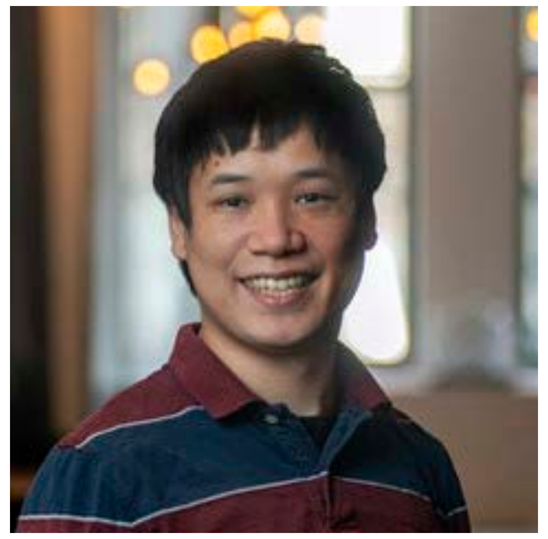
Digression : Parsimonious representation of G(τ)

arXiv:2107.13094 [pdf, other] math.NA cond-mat.str-el

Discrete Lehmann representation of imaginary time Green's functions

Authors: Jason Kaye, Kun Chen, Olivier Parcollet

Jason Kaye



Kun Chen

- G(τ) can be expressed, at precision ε, as a finite sum of N universal exponentials …

$$G(\tau) \approx \sum_{i=1}^{N} g_i e^{-\omega_i \tau} \qquad\qquad N \sim O(\log(\beta\omega_{max})\ \log(1/\epsilon))$$

High energy cutoff

- SYK large-M or similar (e.g. NCA) equations = non-linear equation for g_i
- Similar orthogonal basis (IR) *H. Shinaoka (2016)*
- Advantages over usual (e.g. Matsubara, orthogonal polynomials) representation: maximally compact, adjustable a priori with ε, no truncation needed.

Solve quantum impurity models

$$S_{\text{eff}} = \int \mathrm{d}\tau \left[\sum_\sigma c^\dagger_{\tau,\sigma} \left[\partial_\tau - \mu\right] c_{\tau,\sigma} + U n_\uparrow(\tau) n_\downarrow(\tau) \right] + \int \mathrm{d}\tau \mathrm{d}\tau' \left[\Delta(\tau - \tau') c^\dagger_{\tau,\sigma} c_{\tau',\sigma} - \frac{J^2}{2} Q(\tau - \tau') \boldsymbol{S}(\tau) \cdot \boldsymbol{S}(\tau') \right]$$

- The central building block of **quantum embeddings** methods, e.g.
  - Dynamical Mean Field Theory and extensions
    A.Georges Rev. Mod. Phys. 68, 13 (1996),G. Kotliar, Rev. Mod. Phys. 78, 865 (2006)
  - Vertex based methods (Trilex, Quadrilex, DGA)
  - Quantum chemistry, SEET, …

- A large toolbox of algorithms:
  - Continuous Time QMC, diagrammatic QMC, DMRG, Tensor networks, METTS, NRG ….

One "Continuous Time" Quantum Monte Carlo

$$S_{\mathrm{eff}} = \int \mathrm{d}\tau \left[\sum_\sigma c^\dagger_{\tau,\sigma} \left[\partial_\tau - \mu\right] c_{\tau,\sigma} + U n_\uparrow(\tau) n_\downarrow(\tau) \right] + \int \mathrm{d}\tau \mathrm{d}\tau' \left[\Delta(\tau-\tau') c^\dagger_{\tau,\sigma} c_{\tau',\sigma} - \frac{J^2}{2} Q(\tau-\tau') \boldsymbol{S}(\tau)\cdot\boldsymbol{S}(\tau') \right]$$

- Principle (CT-INT): expand the partition function in interactions U and Q (*Rubtsov 2004*)

$$Z = \sum_{n\geq 0}\sum_{p\geq 0} \frac{(-U)^n J^{2p}}{n!\,p!} \int_0^\beta \prod_{i=1}^n d\tau_i \prod_{j=1}^p d\tau'_j d\tau''_j \sum_{a_j=x,y,z} \left\langle \mathcal{T}_\tau \prod_{i=1}^n n_\uparrow(\tau_i) n_\downarrow(\tau_i) \prod_{j=1}^p S^{a_j}(\tau'_j) S^{a_j}(\tau''_j) \right\rangle_0 .$$

- Imaginary time.
  Samples all integrals with a Monte Carlo. Compute G(τ) and <S(τ)S(0)>
  Typical $n \sim \beta U$

- Q induces a sign problem, but strongly reduced by optimizing the quadratic starting point

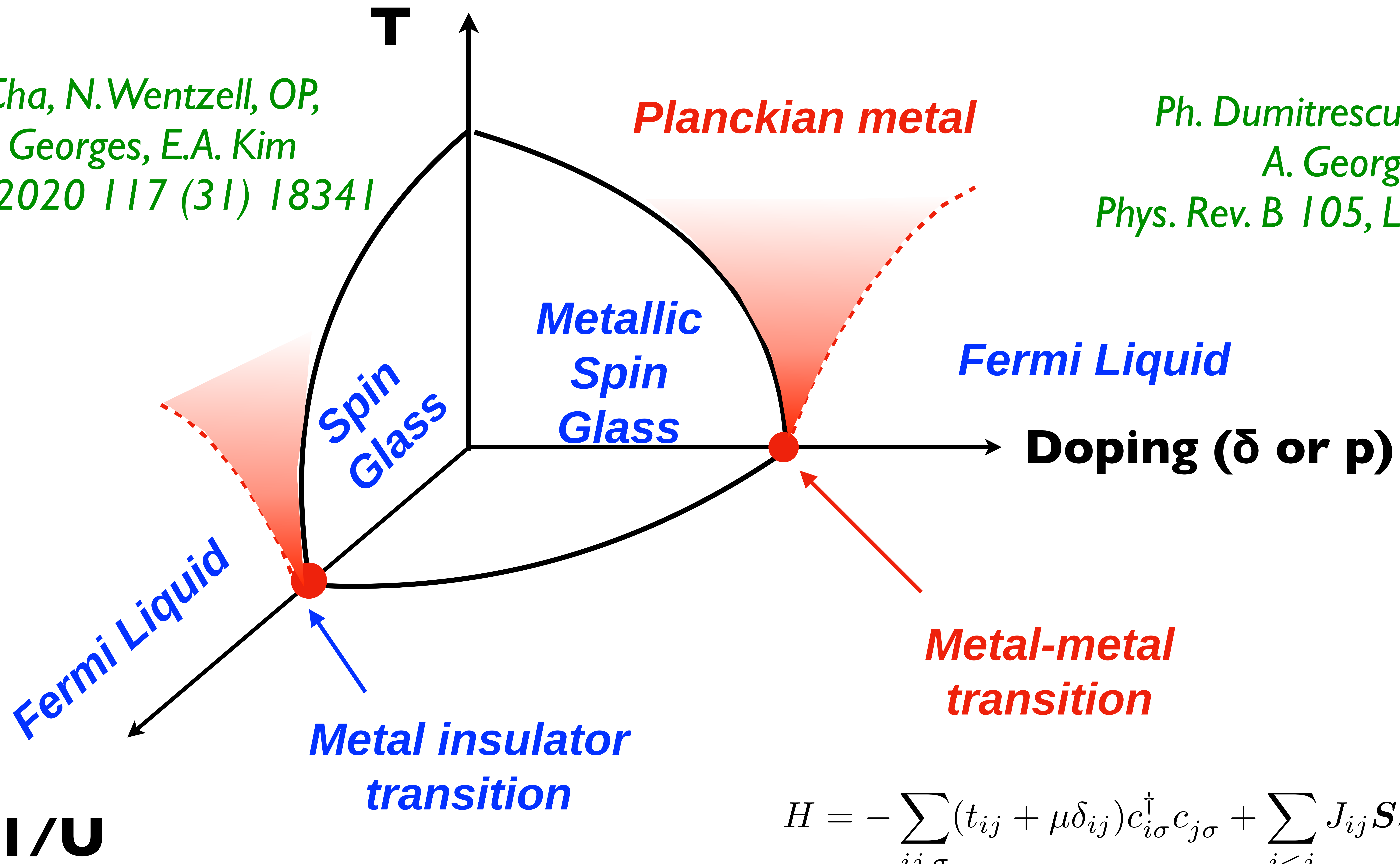
$$n_\uparrow n_\downarrow \to (n_\uparrow - \alpha_\uparrow)(n_\downarrow - \alpha_\downarrow)$$

- Main limitations : very low temperatures/energy scales, close to QCP.

Sketch of the phase diagram

P. Cha, N. Wentzell, OP,
A. Georges, E.A. Kim
PNAS 2020 117 (31) 18341

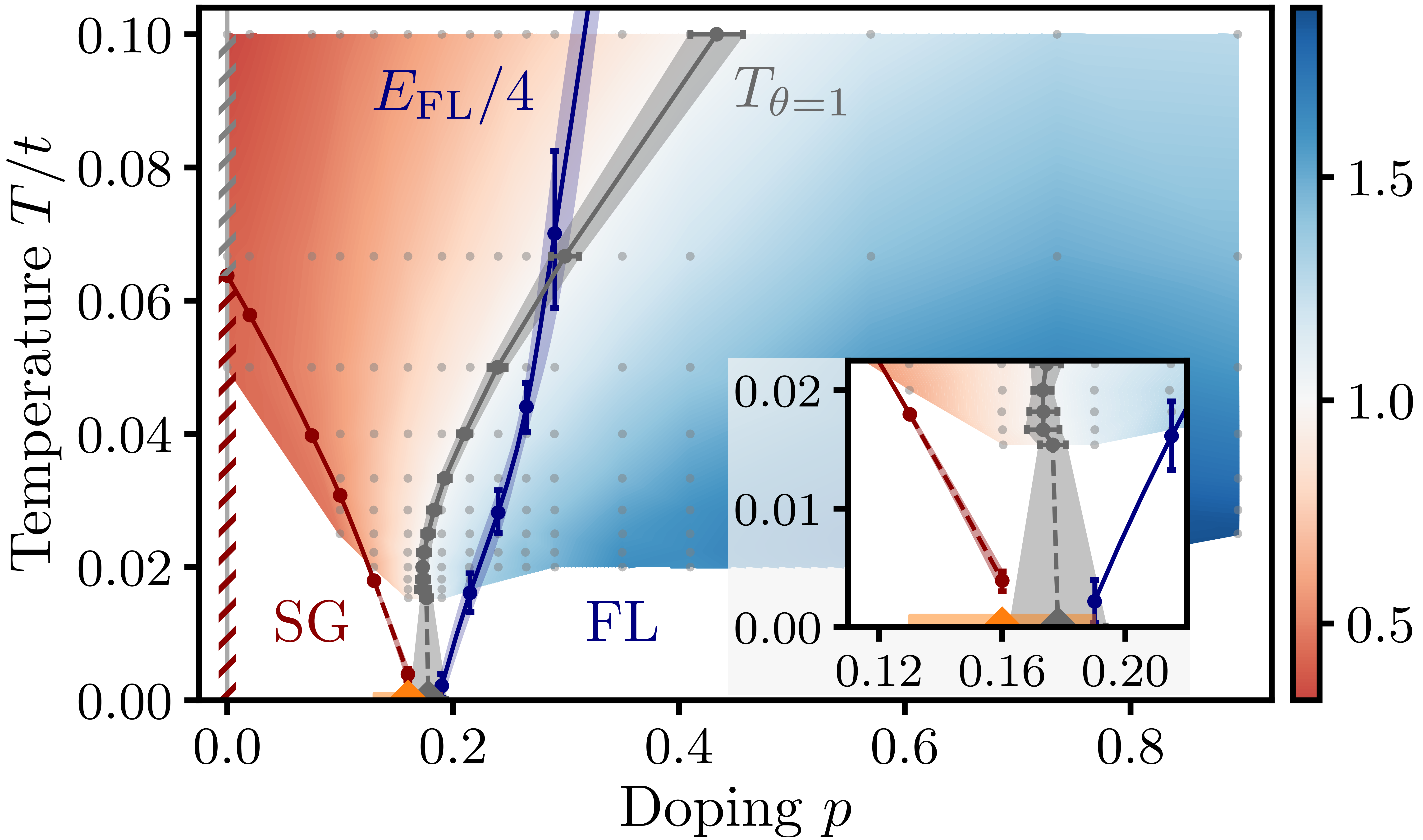
Ph. Dumitrescu, N. Wentzell,
A. Georges, OP
Phys. Rev. B 105, L180404 (2022)



$$H = -\sum_{ij,\sigma}(t_{ij} + \mu\delta_{ij})c^{\dagger}_{i\sigma}c_{j\sigma} + \sum_{i<j} J_{ij}\boldsymbol{S}_i \cdot \boldsymbol{S}_j + U\sum_i n_{i\uparrow}n_{i\downarrow},$$

Phase diagram (doping driven QCP)

- J = 0.5t, U = 4t

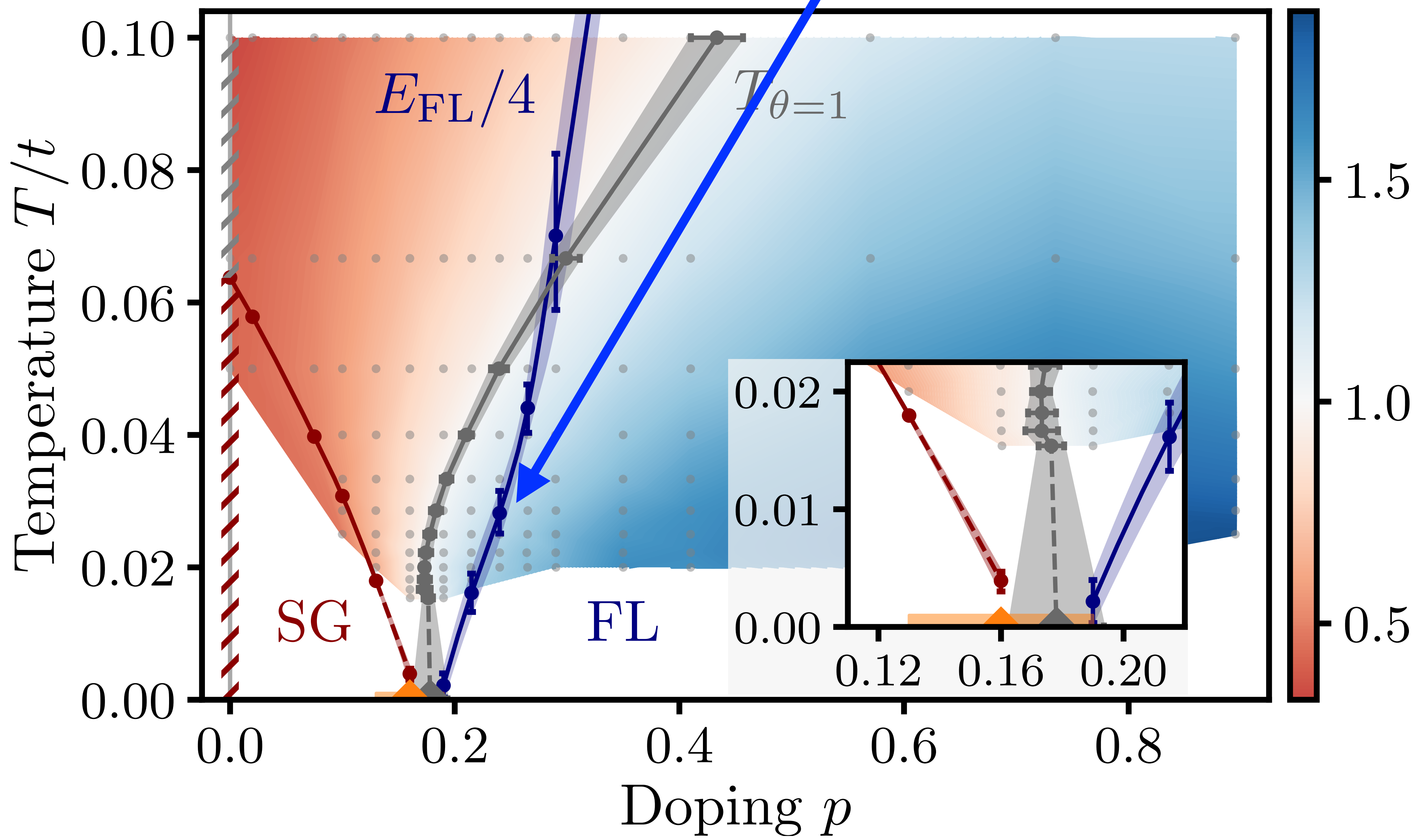


Fermi liquid collapse

- Characteristic energy scale E_{FL} vanishes at the QCP.

- Low T, low frequency Fermi liquid expansion

$$Im\Sigma(i\omega_n) = \left(1 - \frac{1}{Z}\right)\omega_n + \frac{\omega_n^2 - (\pi T)^2}{\boxed{E_{\mathrm{FL}}}} + O(T^3)$$



Metallic spin glass

- Here, we solve only <u>in the paramagnetic phase</u>.
- For p< pc, emerging local moment ***m*** …
  - Characterized by a plateau at large imaginary time *Grempel & Rozenberg (98)*

$$Q(\tau-\tau') = \frac{1}{3}\langle \boldsymbol{S}(\tau)\cdot\boldsymbol{S}(\tau')\rangle$$

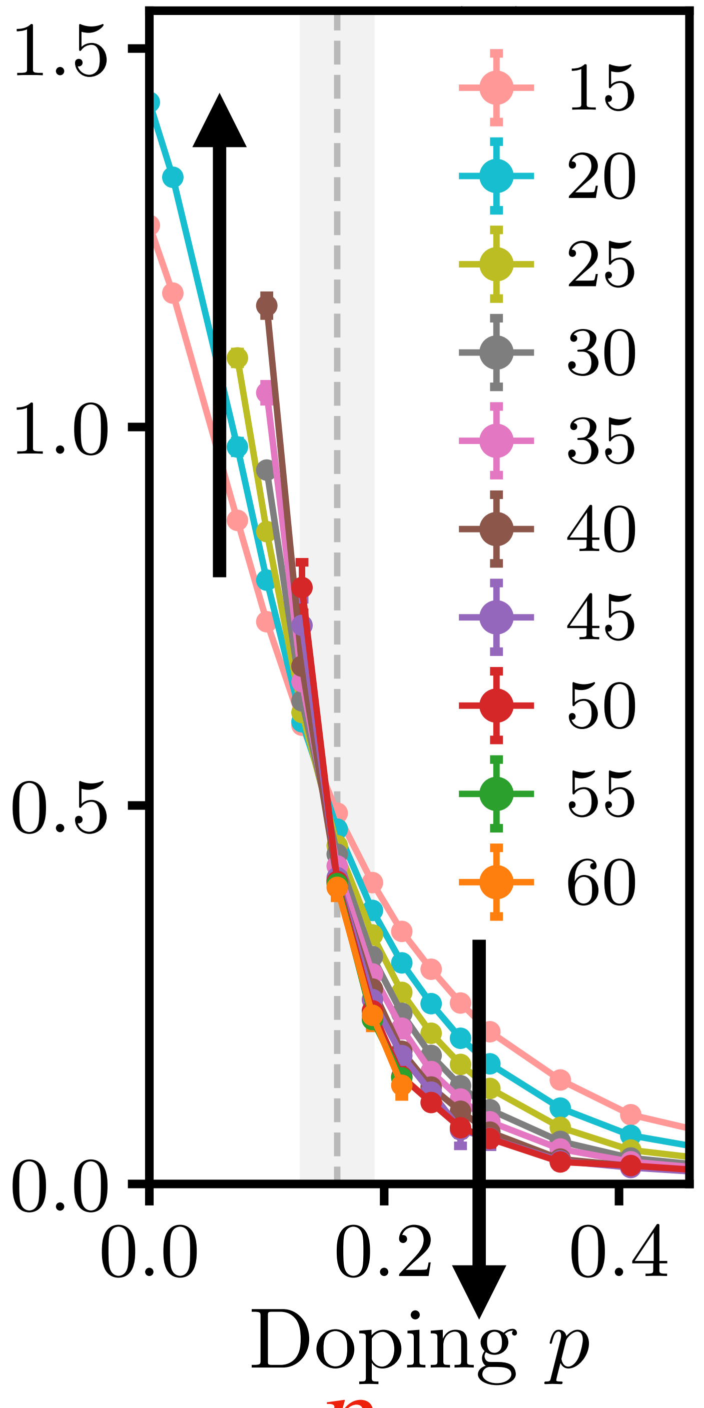
$$Q(\tau) \sim m^2$$

$$Q(i\nu_n) = \beta m^2 \delta_{n,0} + Q_{\text{reg}}(i\nu_n)$$

$$\beta m^2 = Q(i\nu_0) - Q(\nu = 0)$$

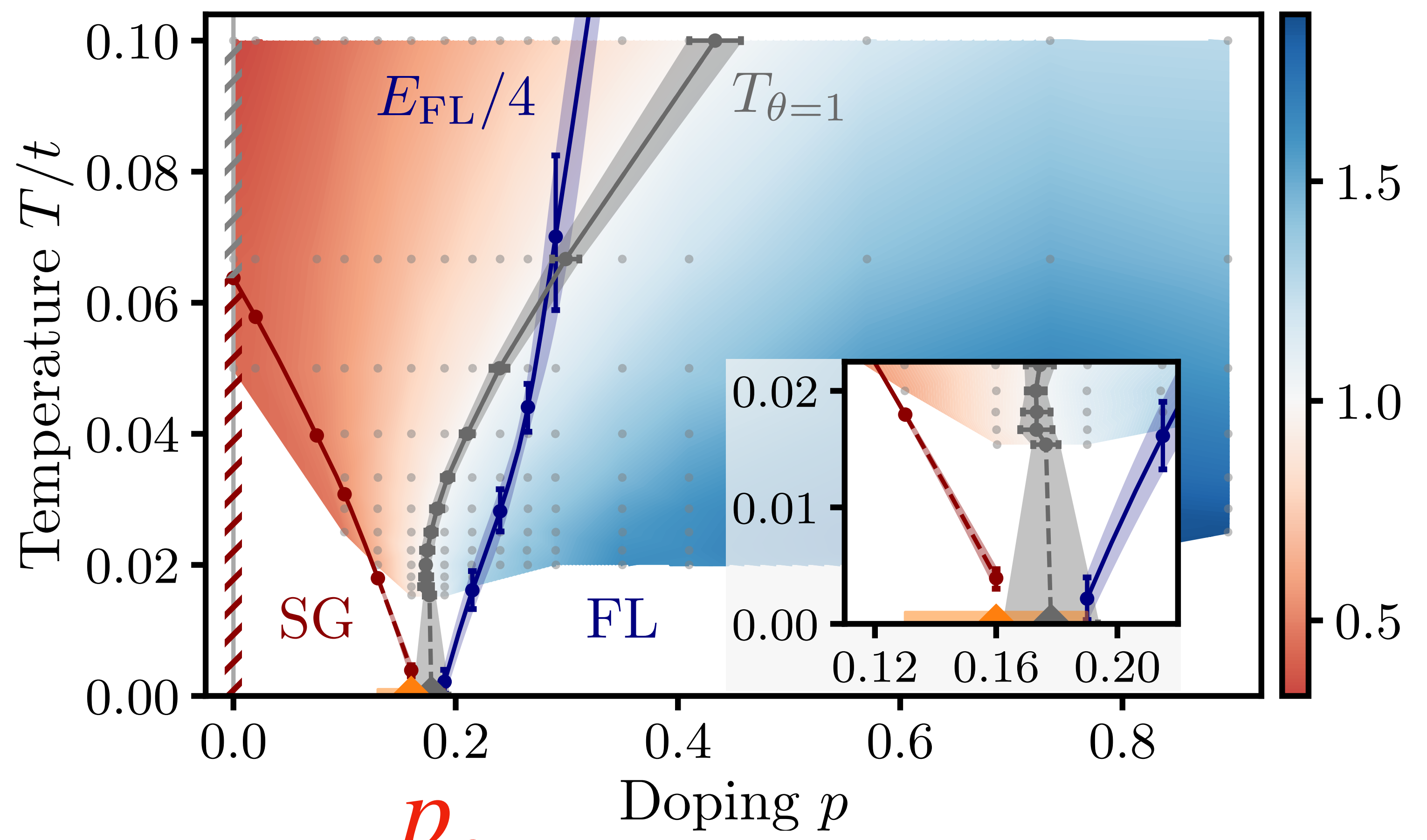
Extrapolated

βm^2



p_c

$T \searrow 0$



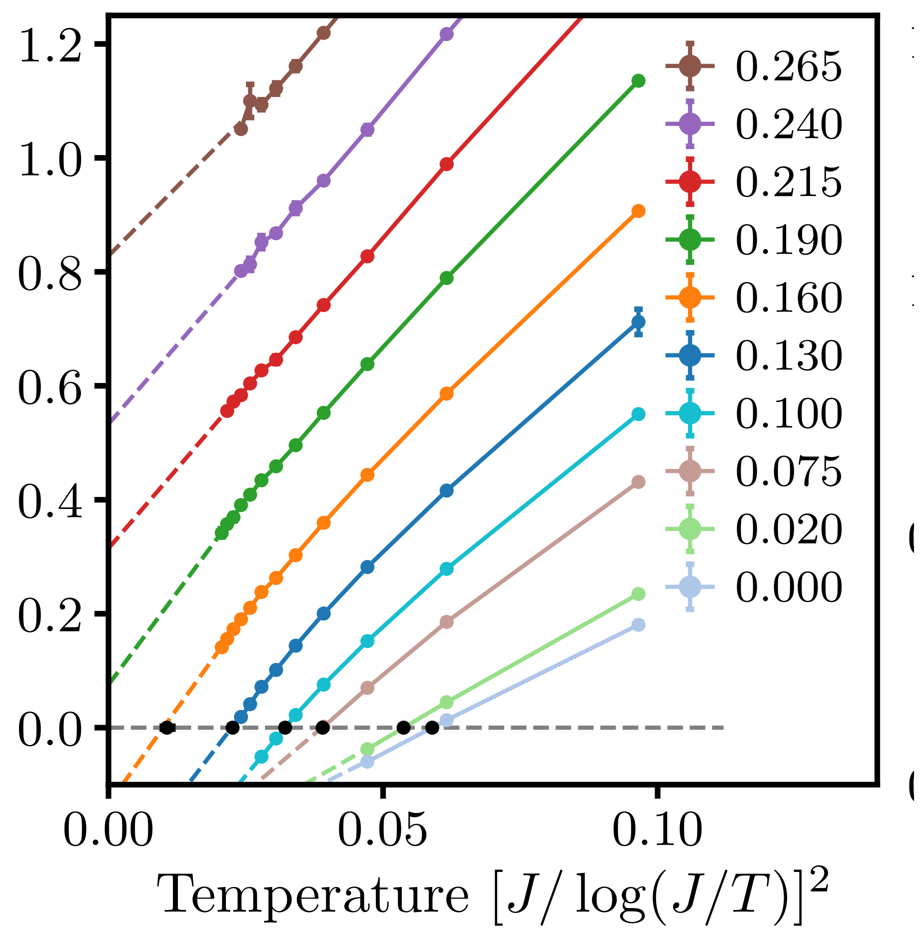
p_c

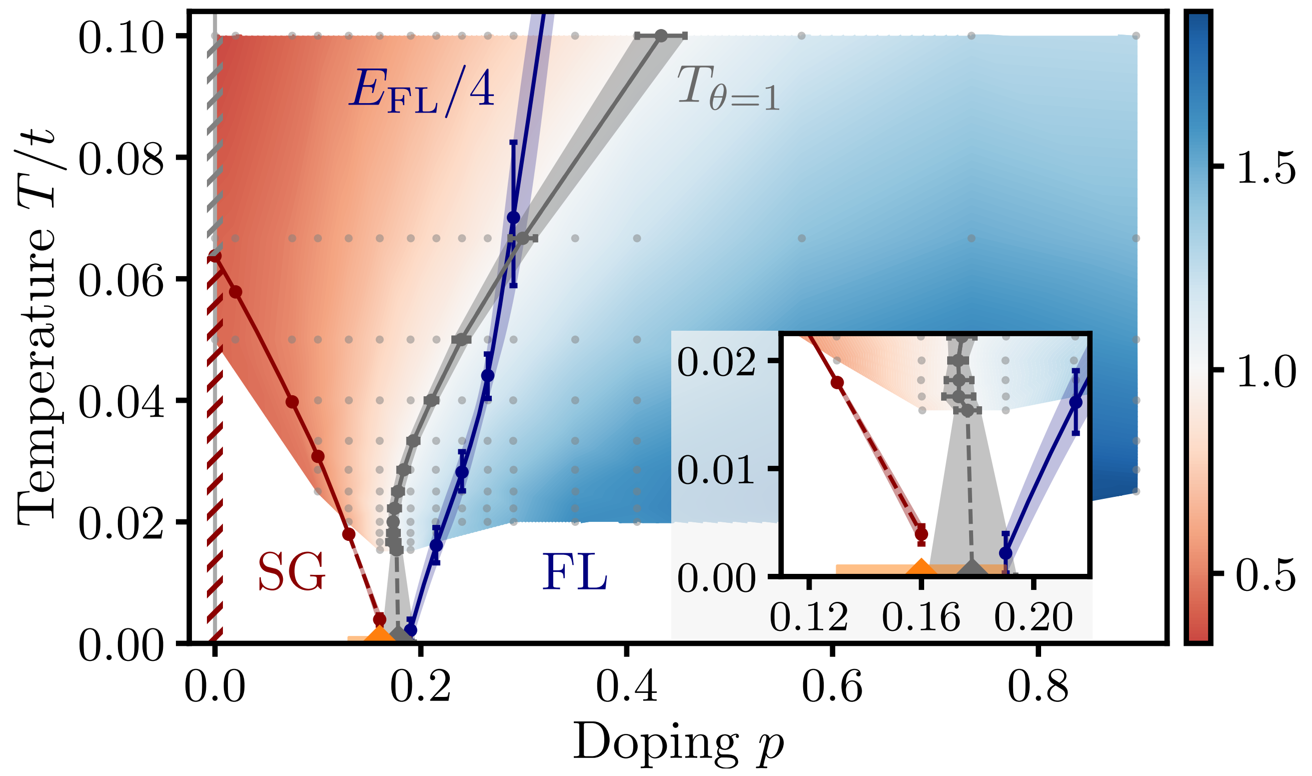
Metallic spin glass

- For p< p$_c$, emerging local moment ***m*** …
- … which orders into a quantum spin glass

Spin glass susceptibility

$$\chi_{SG}^{-1} = \chi_{loc}^{-1} - J^2 \qquad \chi_{loc} = \int_0^\beta d\tau Q(\tau)$$



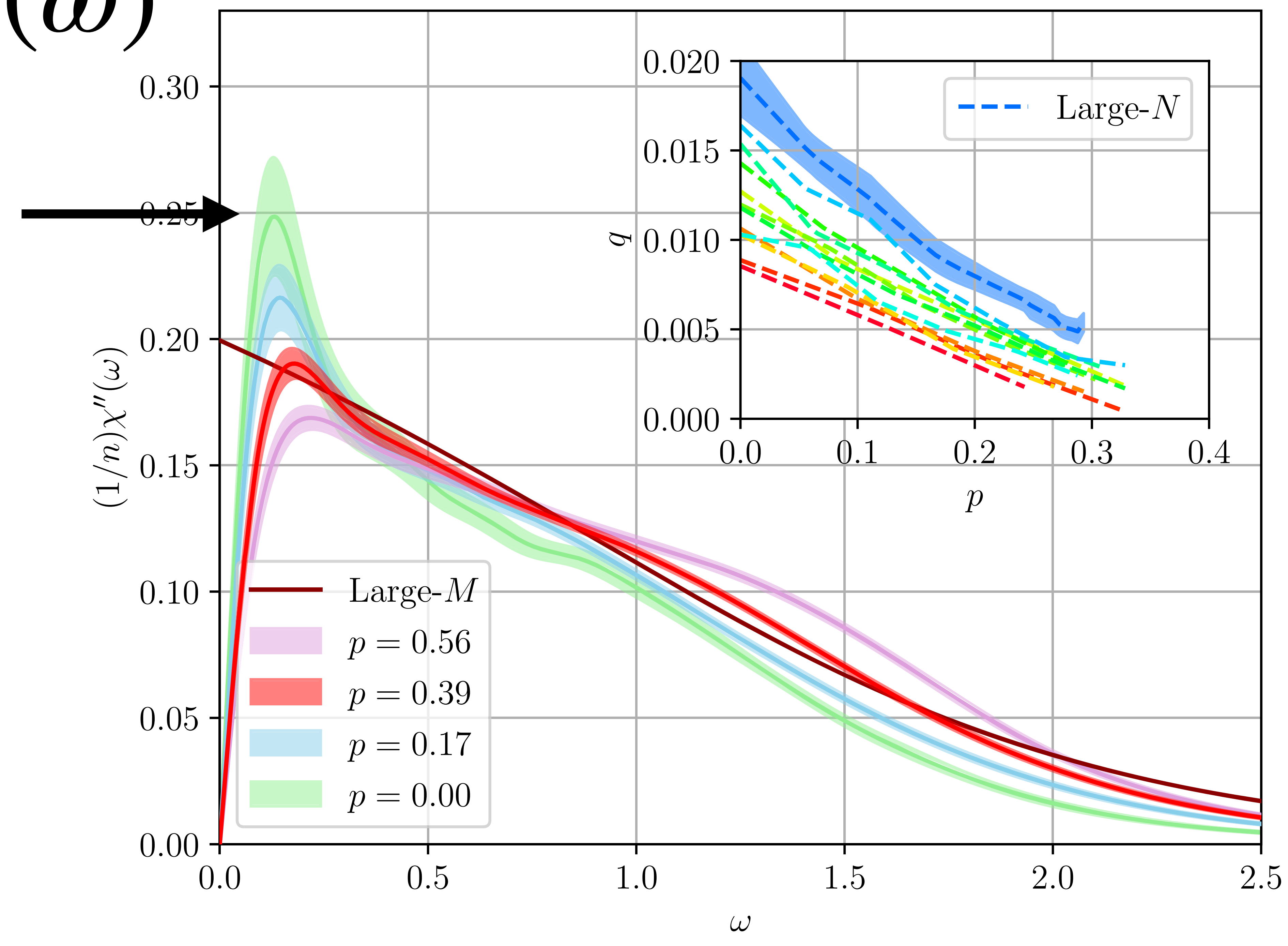


- *Direct solution in the metallic spin glass phase with Parisi replica symmetry breaking ansatz ?*

- Exact Diagonalization of finite system

$$\lim_{t\to\infty}(1/N)\sum_i \langle \mathbf{S}_i(0)\mathbf{S}_i(t)\rangle = q \neq 0,$$

$\chi''(\omega)$



H. Shackleton, et al. PRL 126, 136602 (2021)

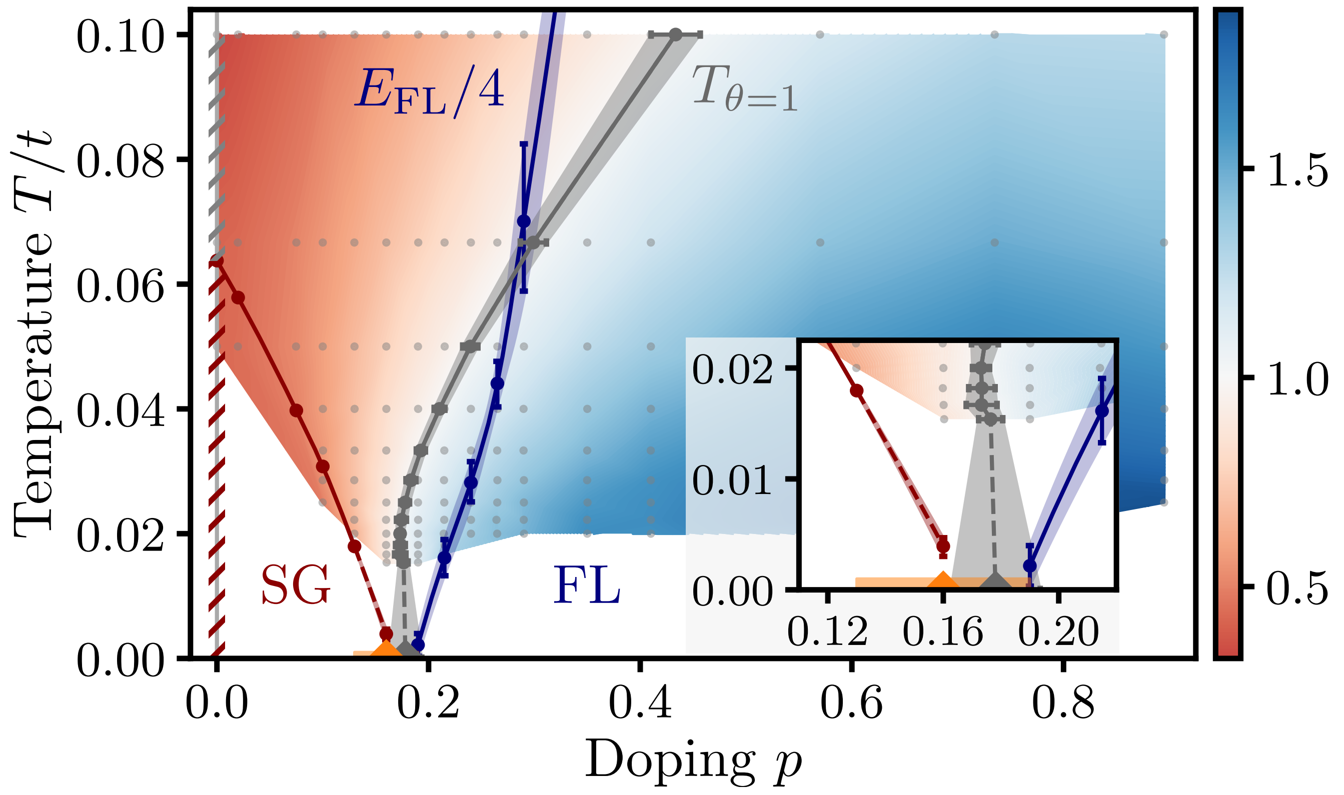
Fermi surface reconstruction at the QCP

See also Otzuki, Vollhardt (2013)

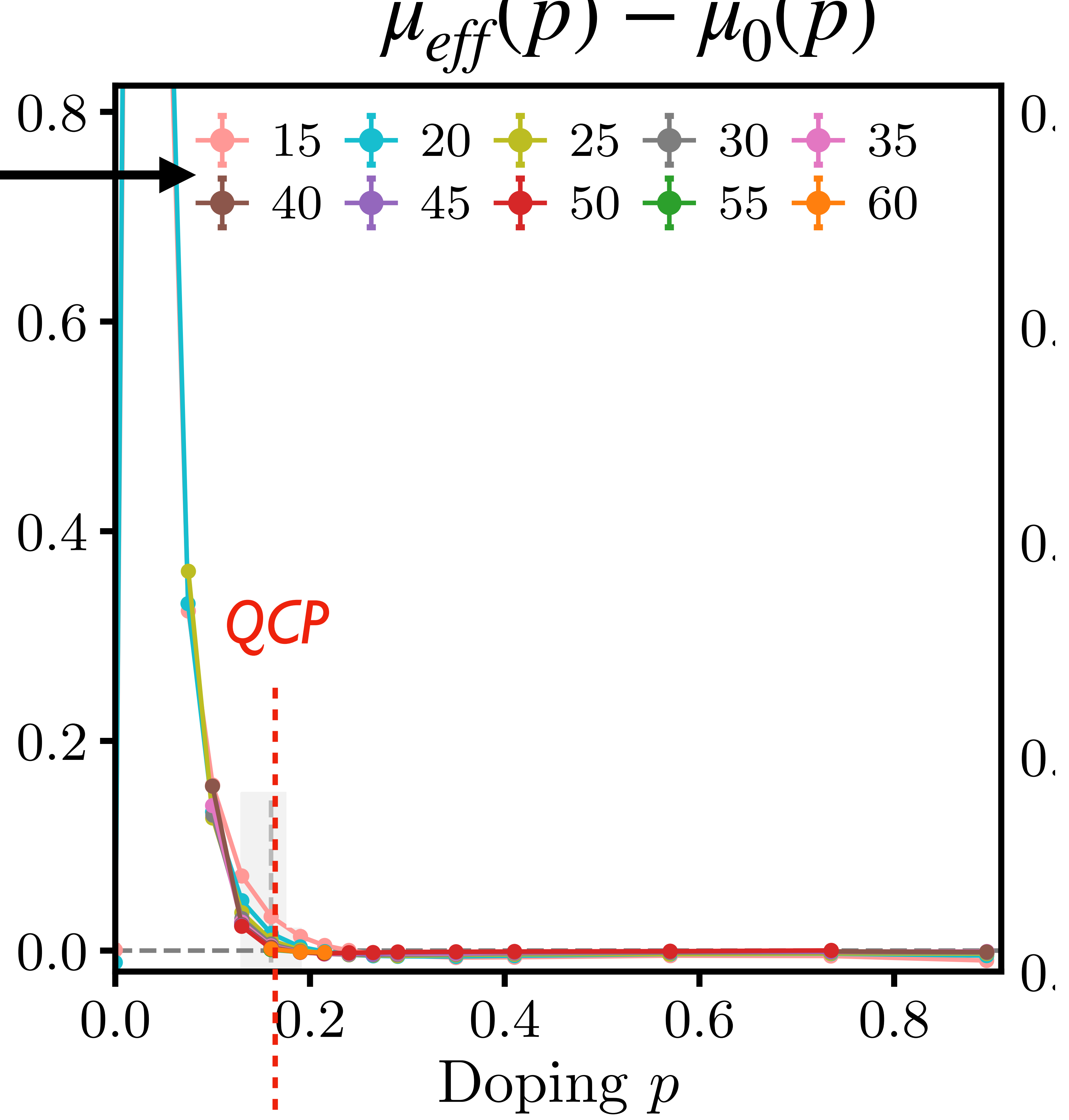
- Luttinger theorem : volume of Fermi surface independent of interaction
- Takes a simple form here, as Σ is local

$$\mu_{\rm eff}(p) \equiv \mu - Re\Sigma(\omega = 0, T = 0)$$
$$= \mu_0(p)$$

Chemical potential of non interacting model



β



Luttinger theorem violated
Reconstruction of the Fermi surface

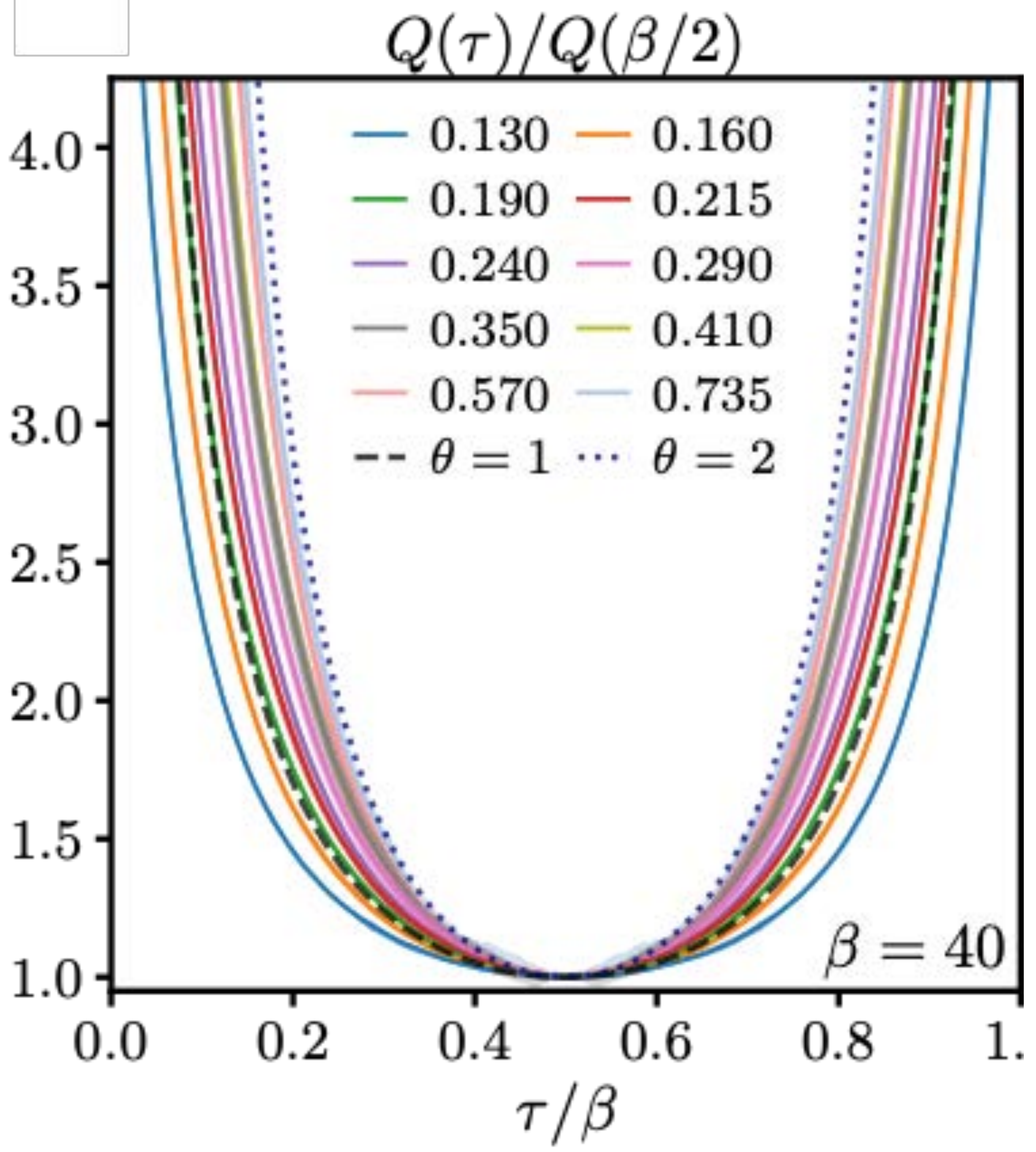
Luttinger theorem ok

Critical scaling : spin dynamics

- Conformal invariant form

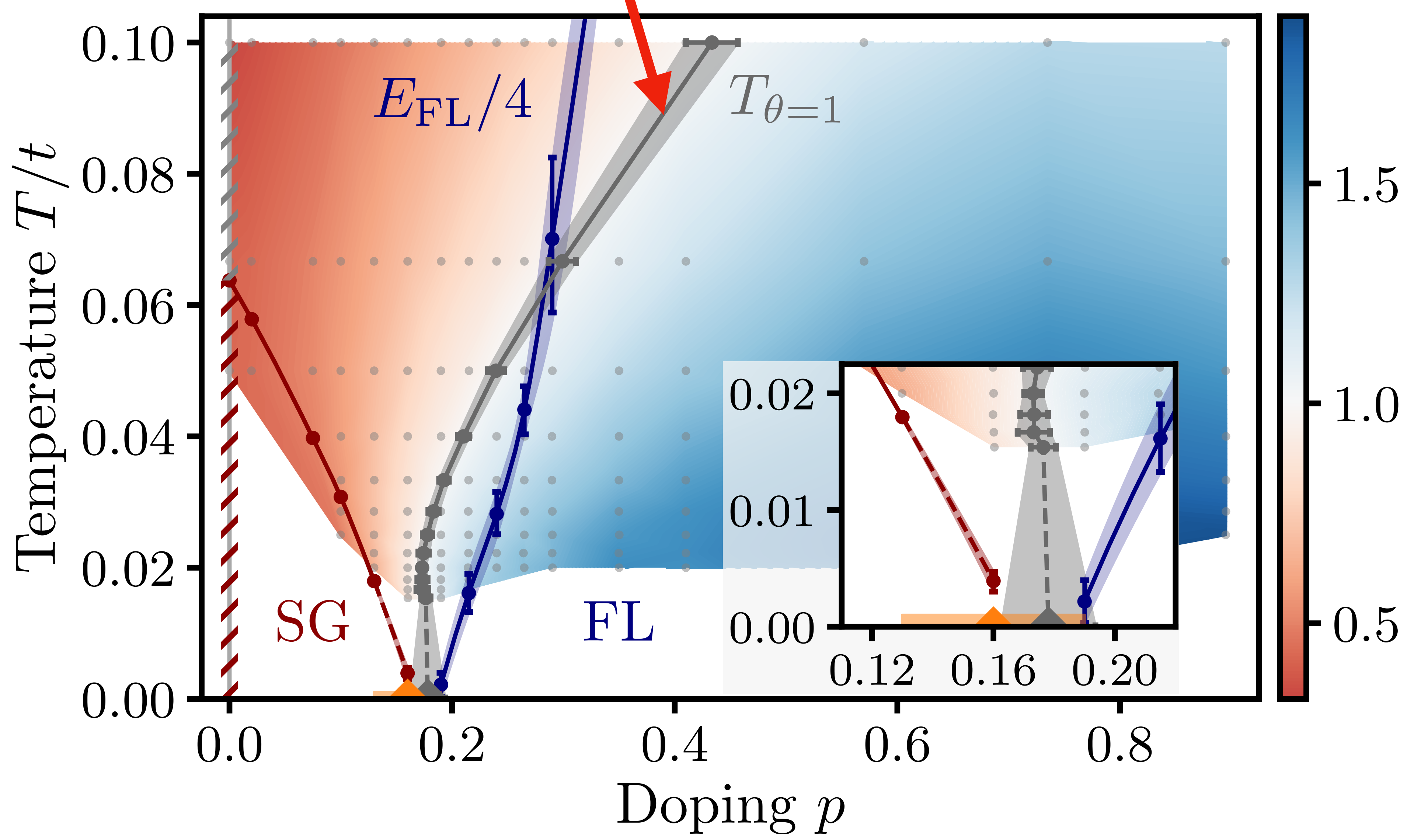
$$Q(\tau-\tau') = \frac{1}{3}\langle \boldsymbol{S}(\tau)\cdot\boldsymbol{S}(\tau')\rangle$$

$$\boxed{Q(\tau) \sim \frac{1}{[\sin(\pi\tau/\beta)]^{\theta}}}$$



- θ=2 (Fermi liquid), θ=1(QCP)
- Phase diagram color map : θ

See also, *Joshi et al PRX 10, 021033 (2020)*



Planckian behavior
Linear resistivity

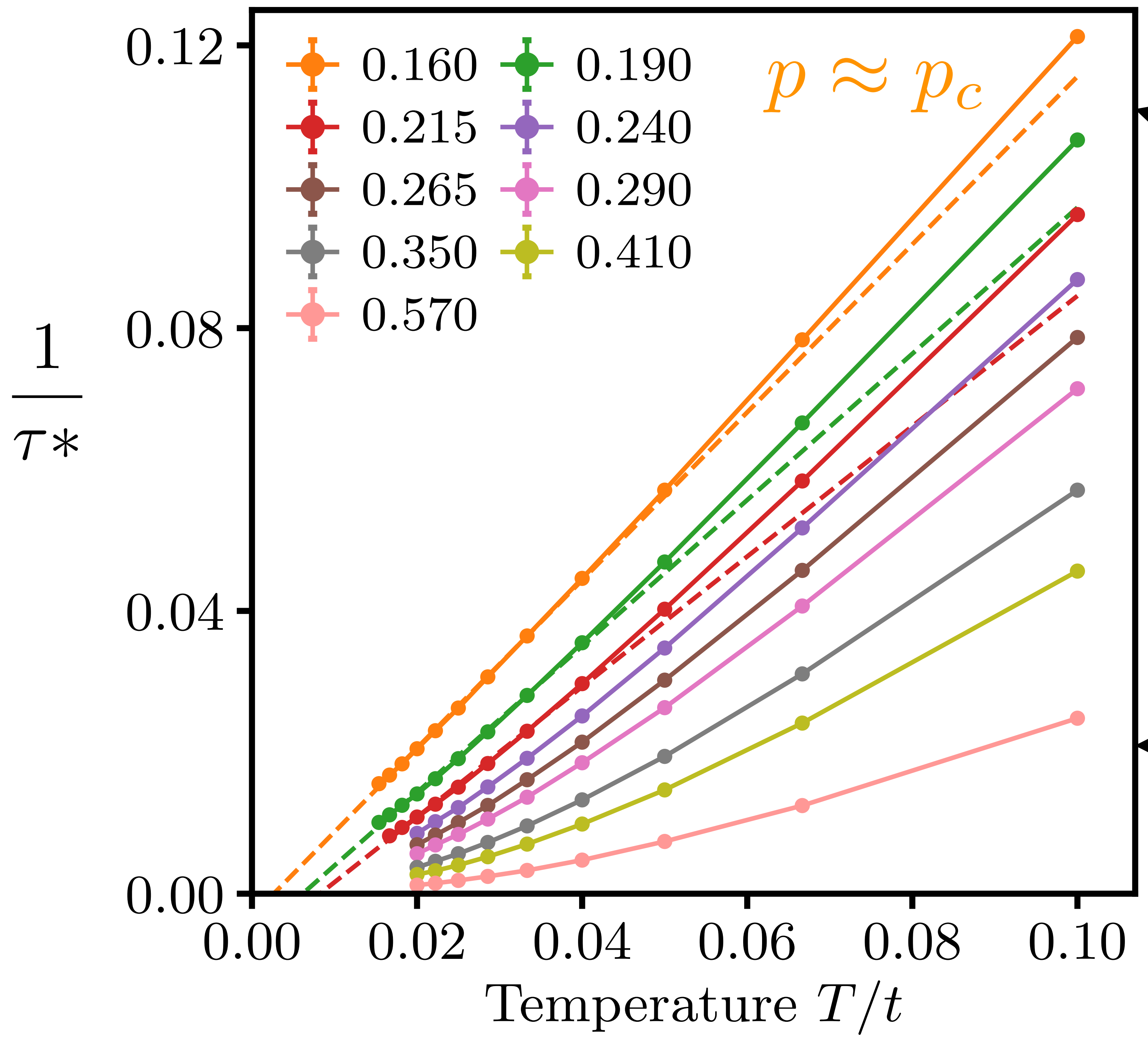
Single particle lifetime

- Quasiparticle lifetime in the Fermi liquid

$$\frac{1}{\tau*} = -ZIm\Sigma(\omega = 0)$$

Extrapolated to ω=0

Doping p



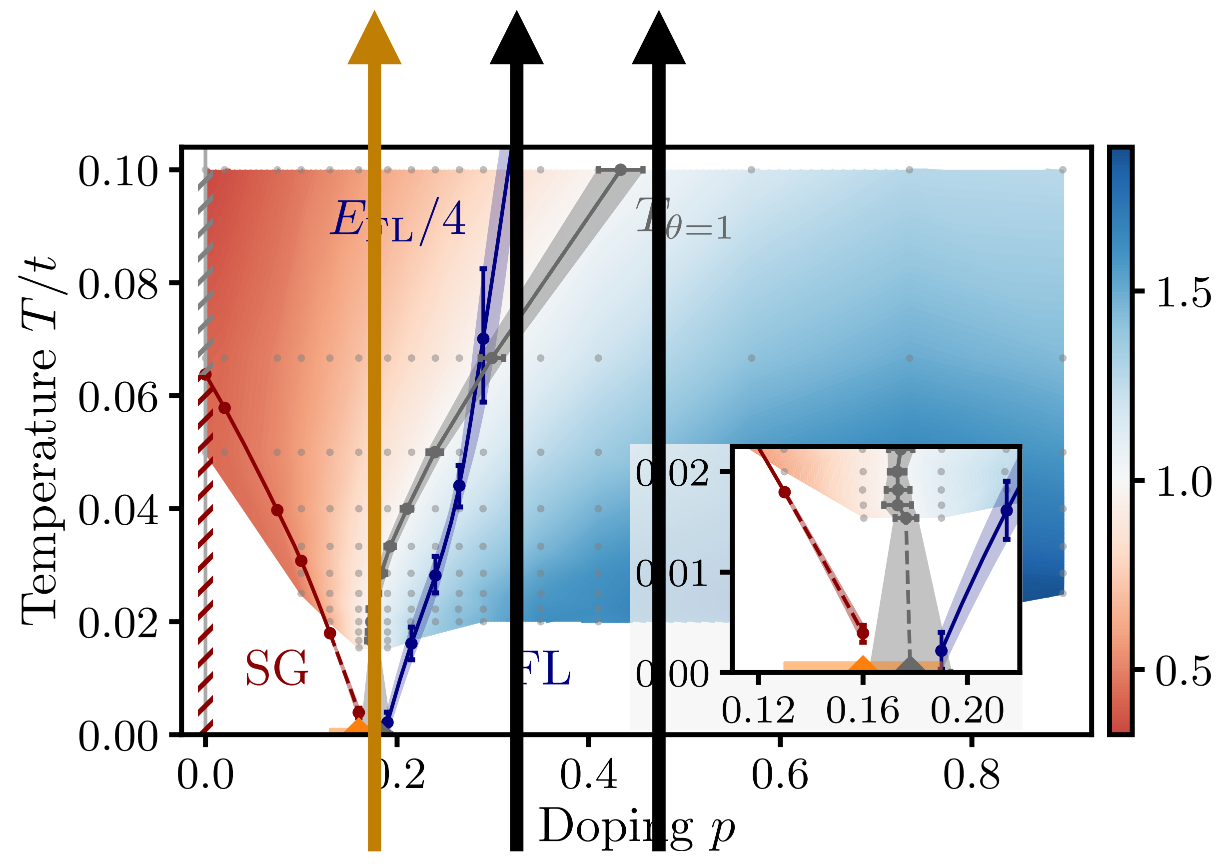
$$\frac{1}{\tau*} \simeq c\,\frac{k_B T}{\hbar}$$

p = p

$p = p_c$
Planckian

$$\frac{1}{\tau*} \propto T^2$$

$p >> p_c$
Fermi Liquid



- NB : Z factor is important to get the constant *c* of order 1

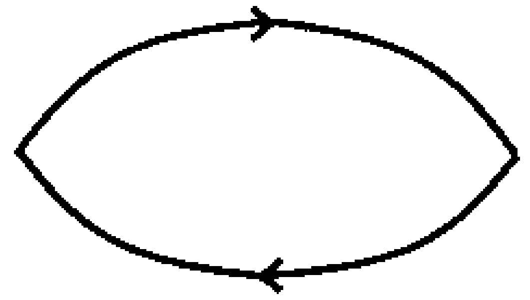
Transport. Resistivity

- Transport time τ_{tr}

$$1/\tau_{transport} = -2Im\Sigma_{extrap}(0)$$

Kubo formula

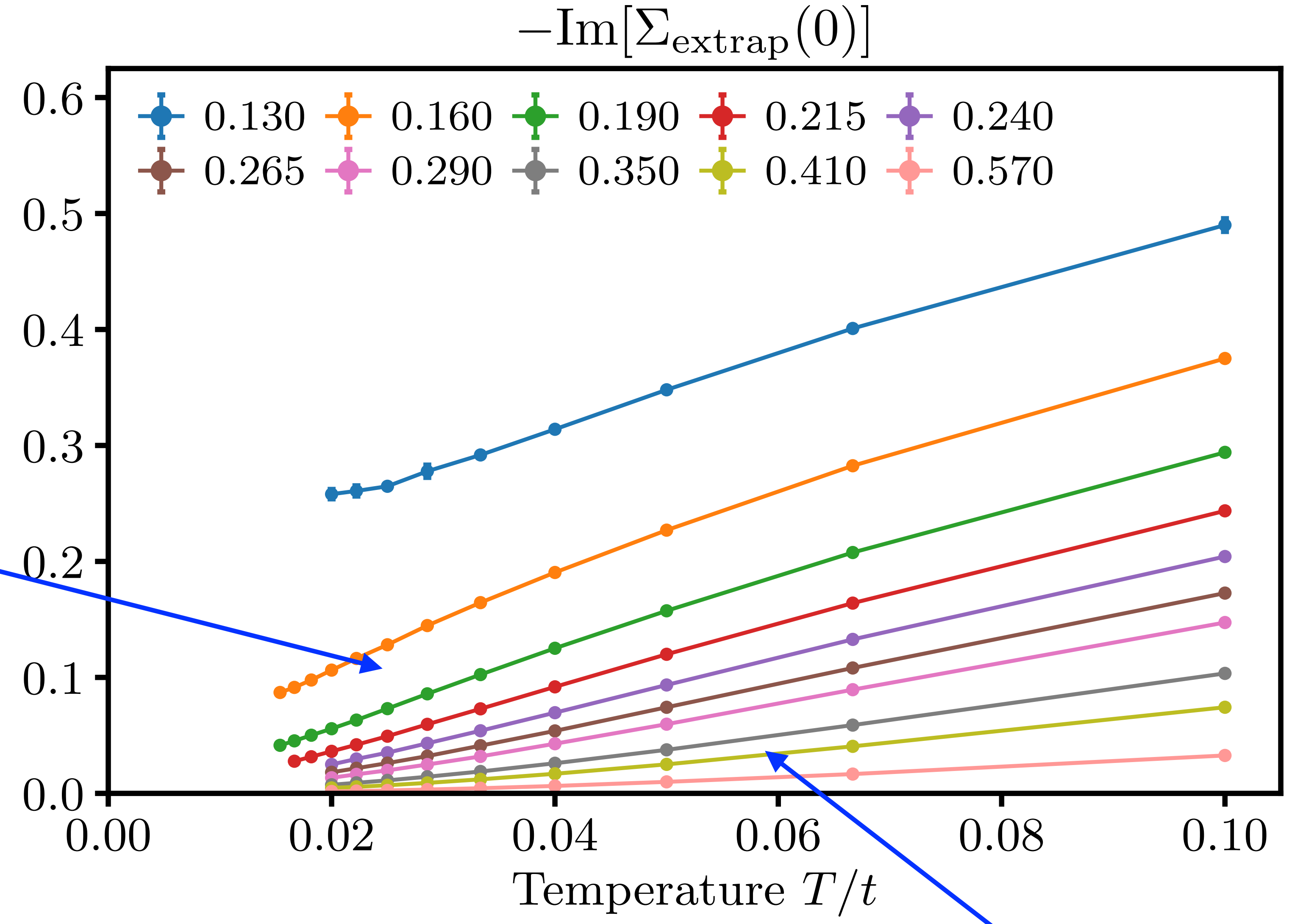
- Kubo formula : no vertex corrections in this model (DMFT)



$$\sigma \sim GG$$

- τ^* and $\tau_{transport}$ are different

$$\sigma_{\rm DC} = \frac{2\pi e^2}{\hbar}\int d\omega \frac{\beta}{4\cosh^2(\beta\omega/2)}\int d\epsilon\, \phi(\epsilon) A(\epsilon,\omega)^2$$



Linear close to p_c

Fermi liquid

$$A(\epsilon,\omega) = \frac{-\Sigma''(\omega)/\pi}{(\omega+\mu-\epsilon-\Sigma'(\omega))^2+\Sigma''(\omega)^2}$$

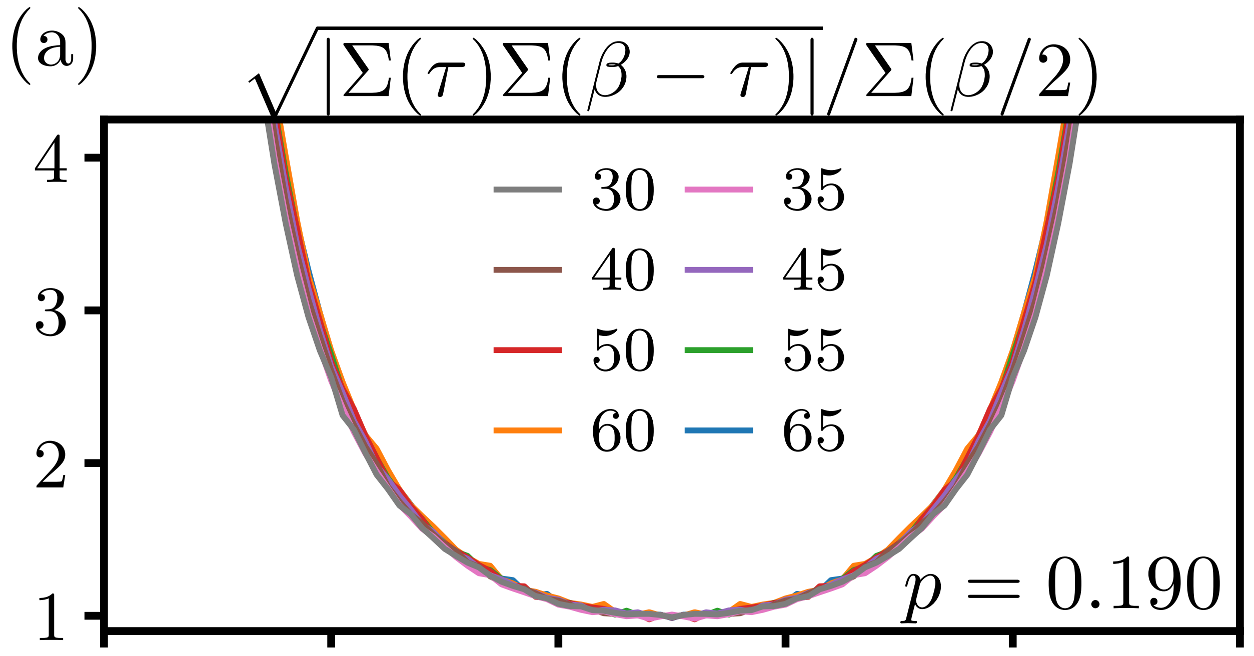
$$\sigma_{DC} \sim C\phi(\epsilon_F)\tau_{transport}$$

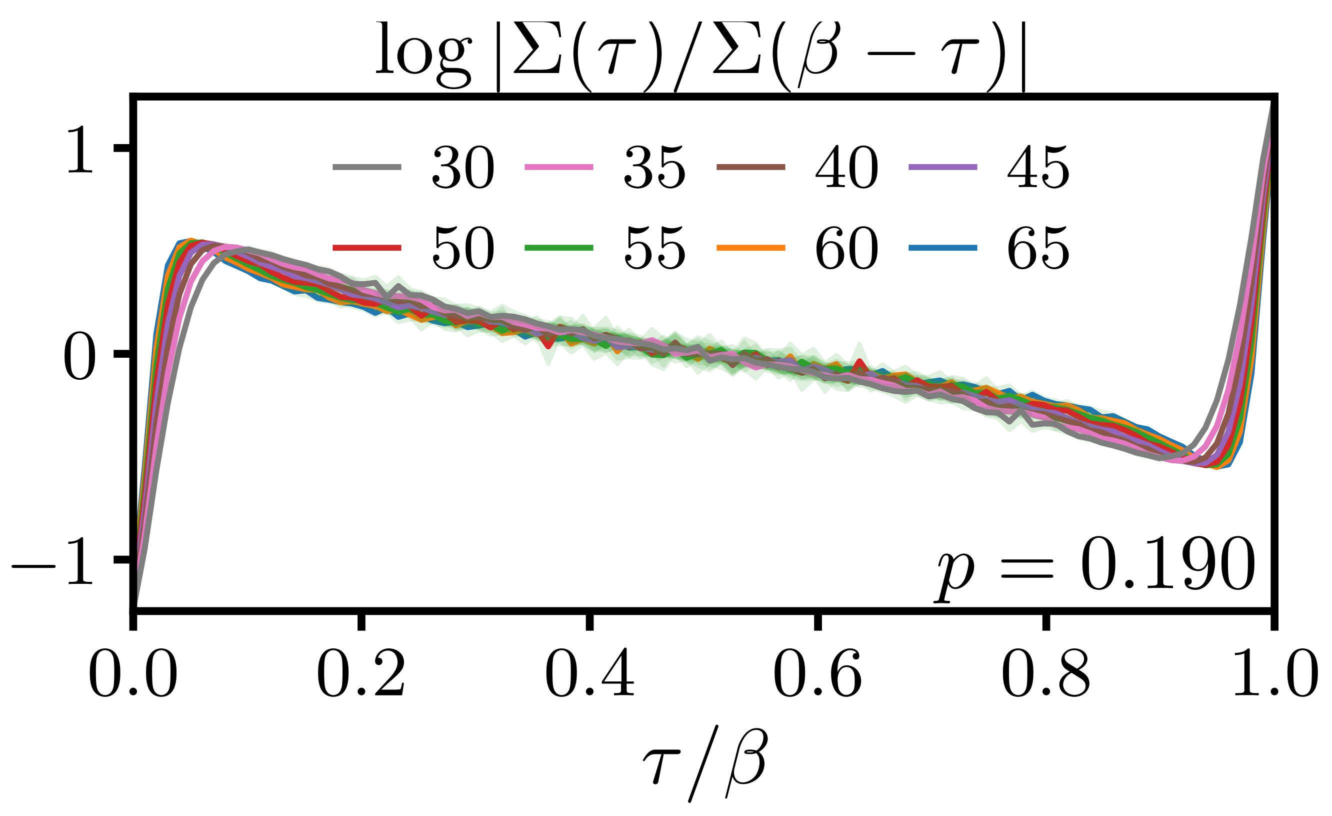
Critical scaling : Fermionic properties

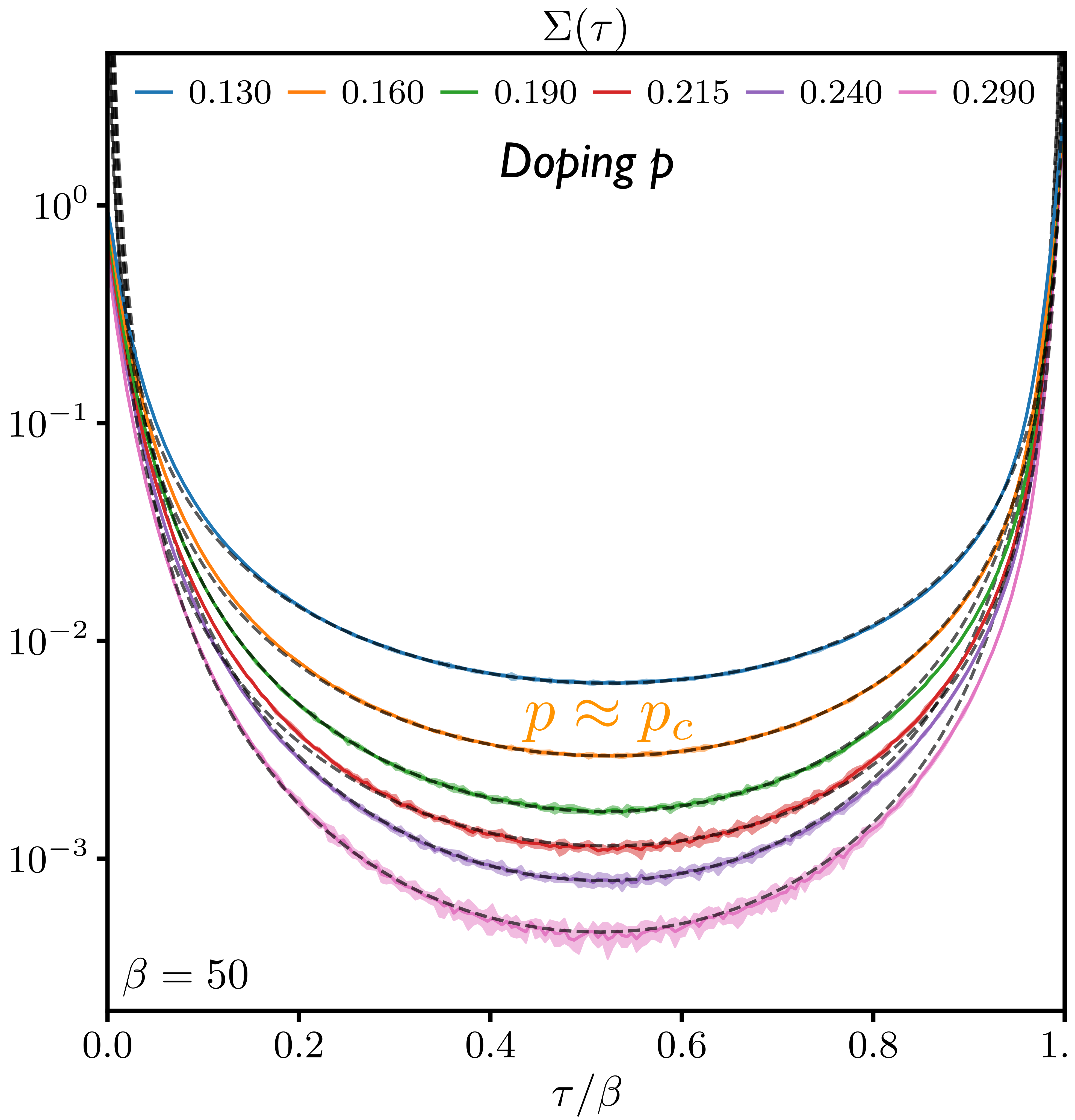
- Conformal form fit for the self-energy

$$\Sigma(\tau) \sim \frac{\exp[\alpha(\tau/\beta - 1/2)]}{[\sin(\pi\tau/\beta)]^{\nu+1}}$$

- Asymmetry α (skewness) in the scaling form



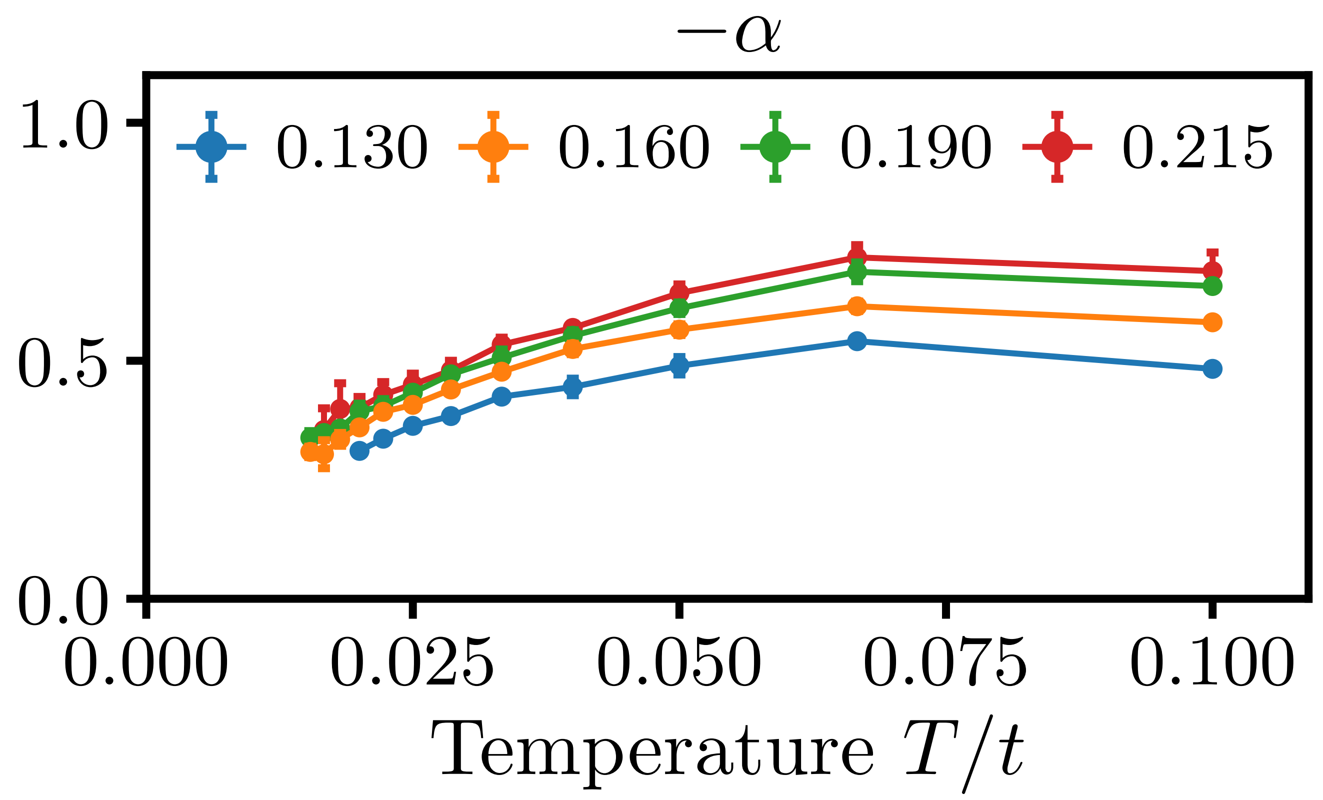


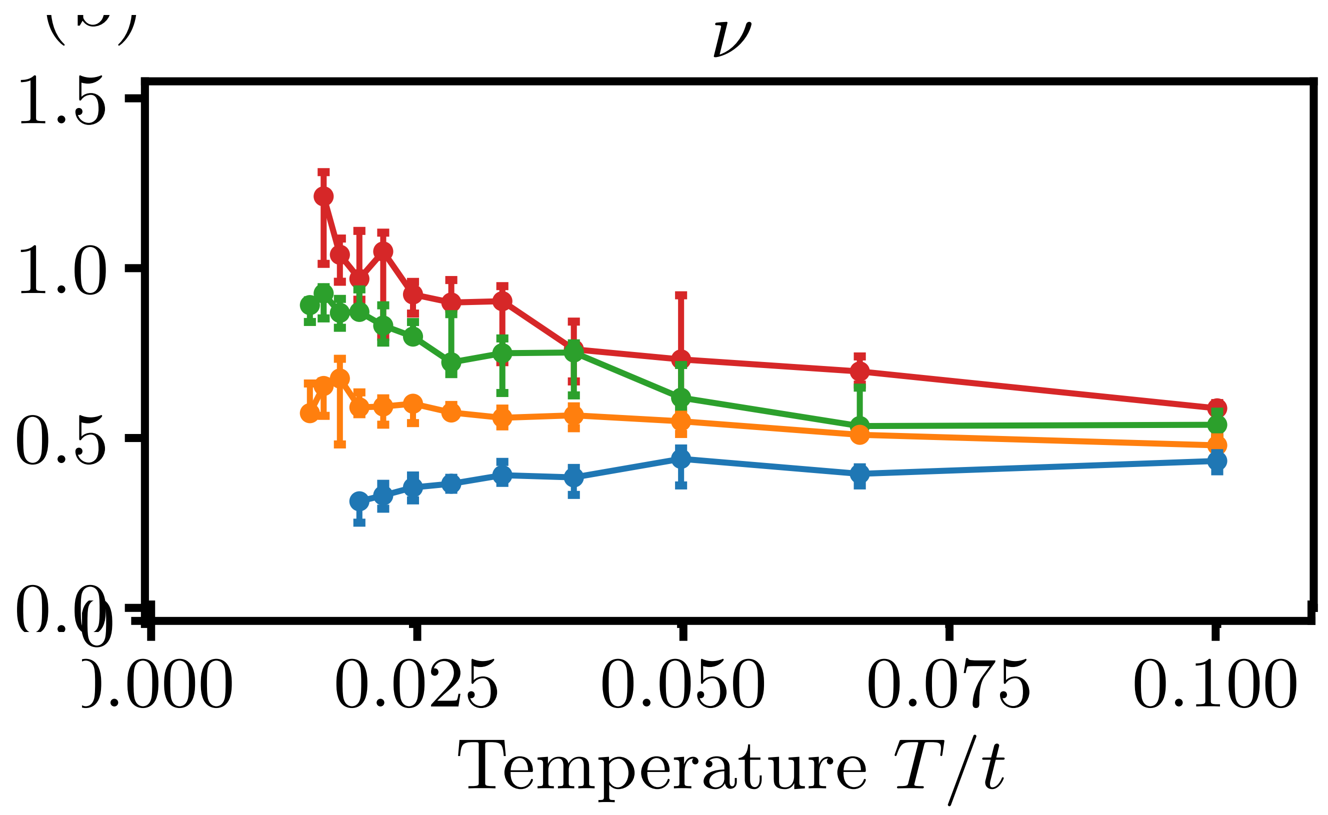


Critical scaling : Fermionic properties

- Fit of asymmetry α, ν

$$\Sigma(\tau) \sim \frac{\exp[\alpha(\tau/\beta - 1/2)]}{[\sin(\pi\tau/\beta)]^{\nu+1}}$$





At QCP

$\nu \approx 0.6 - 0.8$

- Open questions :
  - Does the asymmetry α stay finite at T = 0 at the QCP ?
  - Relationship between α and the entropy at T=0 (like in the large-M models) ?

Single particle lifetime : T-dependency

- ω/T scaling in real frequencies

$$\mathrm{Im}\Sigma(\omega) = -T^{\nu}\sigma(\omega/T)$$

$$Im\Sigma(\omega = 0) \sim T^{\nu}$$

- Compute the real part the self energy, Z, for ν < 1

$$1 - \frac{1}{Z(T,\omega)} \equiv \frac{1}{\omega}\left[\mathrm{Re}\Sigma(\omega,T) - \mathrm{Re}\Sigma(0,T)\right]$$

$$Z(T, \omega = 0) \sim T^{1-\nu}$$

- T linearity of single particle lifetime is independent of ν

$$\frac{1}{\tau^*} = -ZIm\Sigma(\omega = 0) \sim T$$

Compressibility

- The mechanism for T linearity is quite different from a bad/hot metal
- Einstein relation

$$1/\rho_{dc} = \chi_e D$$

Compressibility   Diffusion constant

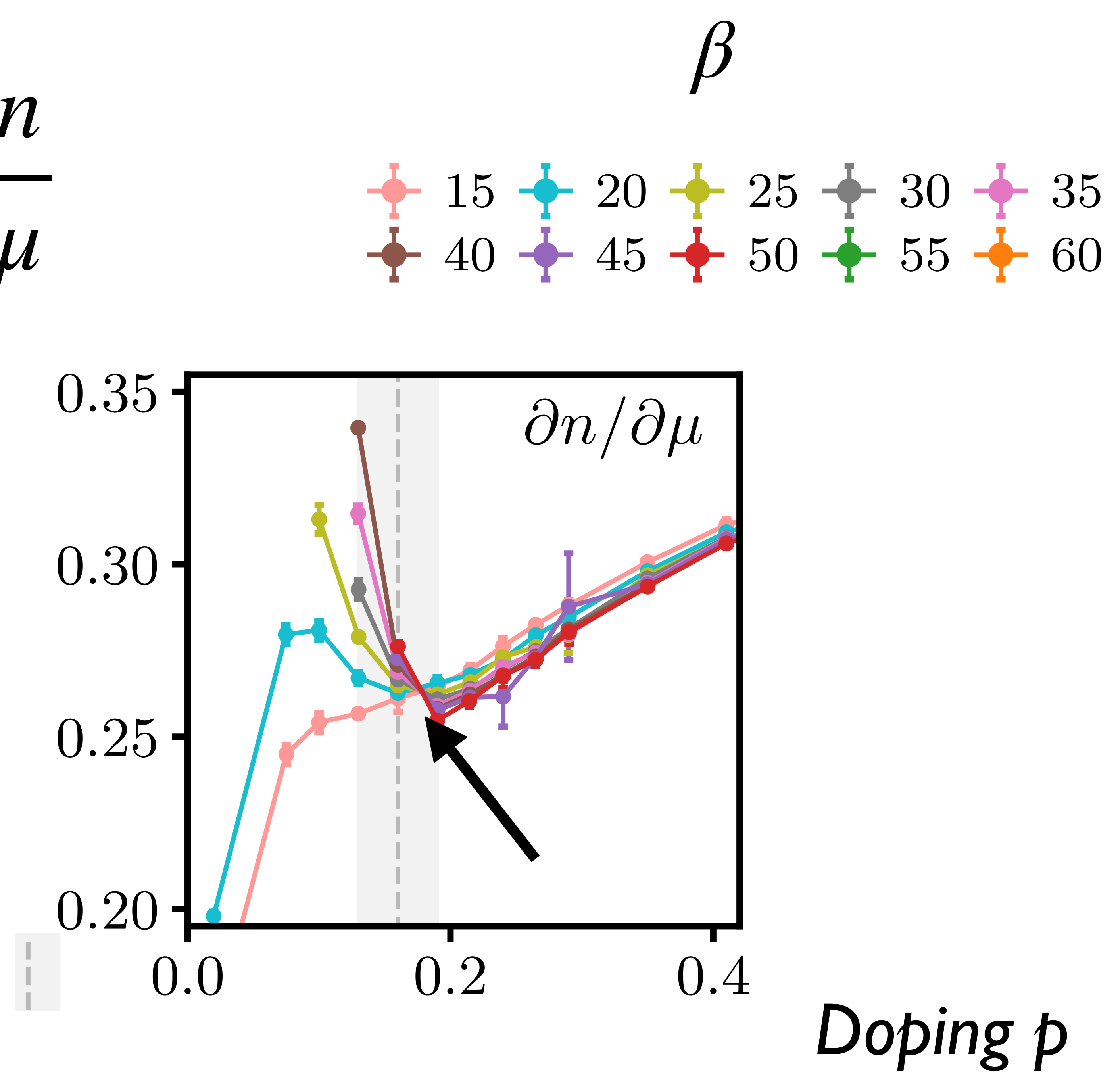
- Generically at very high T (e.g. cold atoms)

$$D \sim \text{const}, \qquad \chi_e(T) \sim 1/T$$

- Here, χ_e has little dependence on T at the QCP

$$\chi_e(T) \sim \text{const}, \qquad D \sim 1/T$$

$$\chi_e = \frac{\partial n}{\partial \mu}$$



Conclusion

- SU(2) model has a richer physics than the simple large M limit.
- Modern algorithms are essential to solve SU(2) models.

- Open questions :
  - Solution in the spin glass phase. Real time, aging, spin glass dynamics ?
  - Residual entropy at the QCP at $T = 0$. Relation with the spectral asymmetry ?
  - Precise scaling $\Sigma(\omega)$ at low ω
    A challenge for a new generation of high precision algorithms,
    very low T, in real time (e.g. real time Quantum Quasi Monte Carlo).
  - SU(2) exact solution for models beyond SYK.

Thank you for your attention!