Scaling properties of the optical conductivity of quantum critical cuprates

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B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, DvdM & A. Georges
arXiv:2205.04030 (2022)

We measure the optical conductivity: σ(ω) = j(ω) / E(ω)

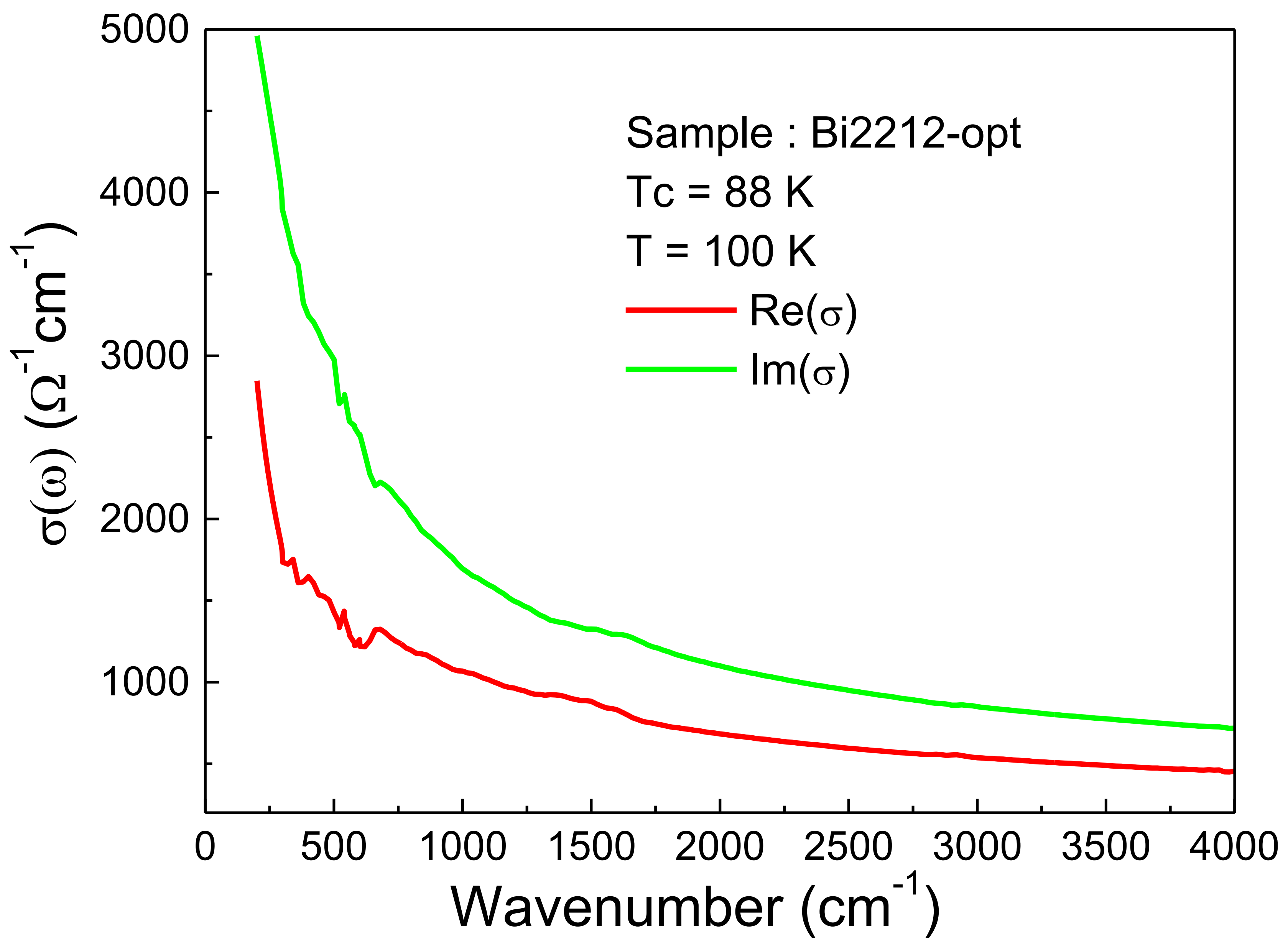
Kubo formula (1957)

$$\sigma(\omega)=\frac{iNe^2}{mV\omega}+\frac{1}{V\omega}\int_0^{+\infty}e^{i\omega t}\left\langle\left[\hat{j}_0(t),\hat{j}_0(0)\right]\right\rangle dt$$

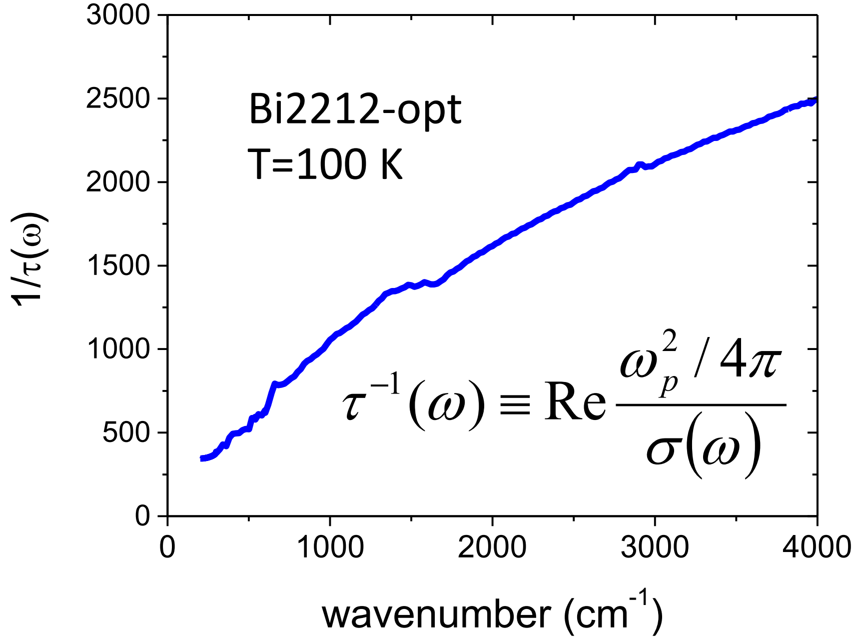
Lehman (spectral) representation:

$$\sigma(\omega)=\frac{2i\omega}{V}\sum_n\left\langle n\middle|\hat{j}_0\,\frac{e^{\beta(\Omega-E_n)}\hbar^3\left(H-E_n\right)^{-1}}{\hbar^2\omega^2-\left(H-E_n\right)^2+i\hbar\omega 0^+}\,\hat{j}_0\middle|n\right\rangle$$

Current operator : $\hat{j}_k = \frac{e}{\hbar}\sum_{q\sigma}\frac{\varepsilon_q-\varepsilon_{q+k}}{k}c^t_{q,\sigma}c_{q+k,\sigma}$

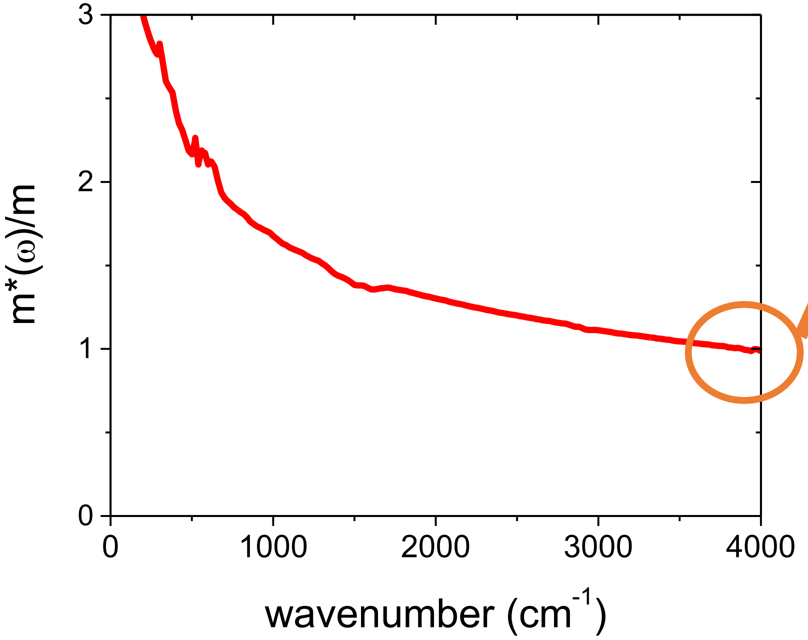


Sample : Bi2212-opt
Tc = 88 K
T = 100 K
Re(σ)
Im(σ)
σ(ω) (Ω-1cm-1)
Wavenumber (cm-1)
0
500
1000
1500
2000
2500
3000
3500
4000
1000
2000
3000
4000
5000

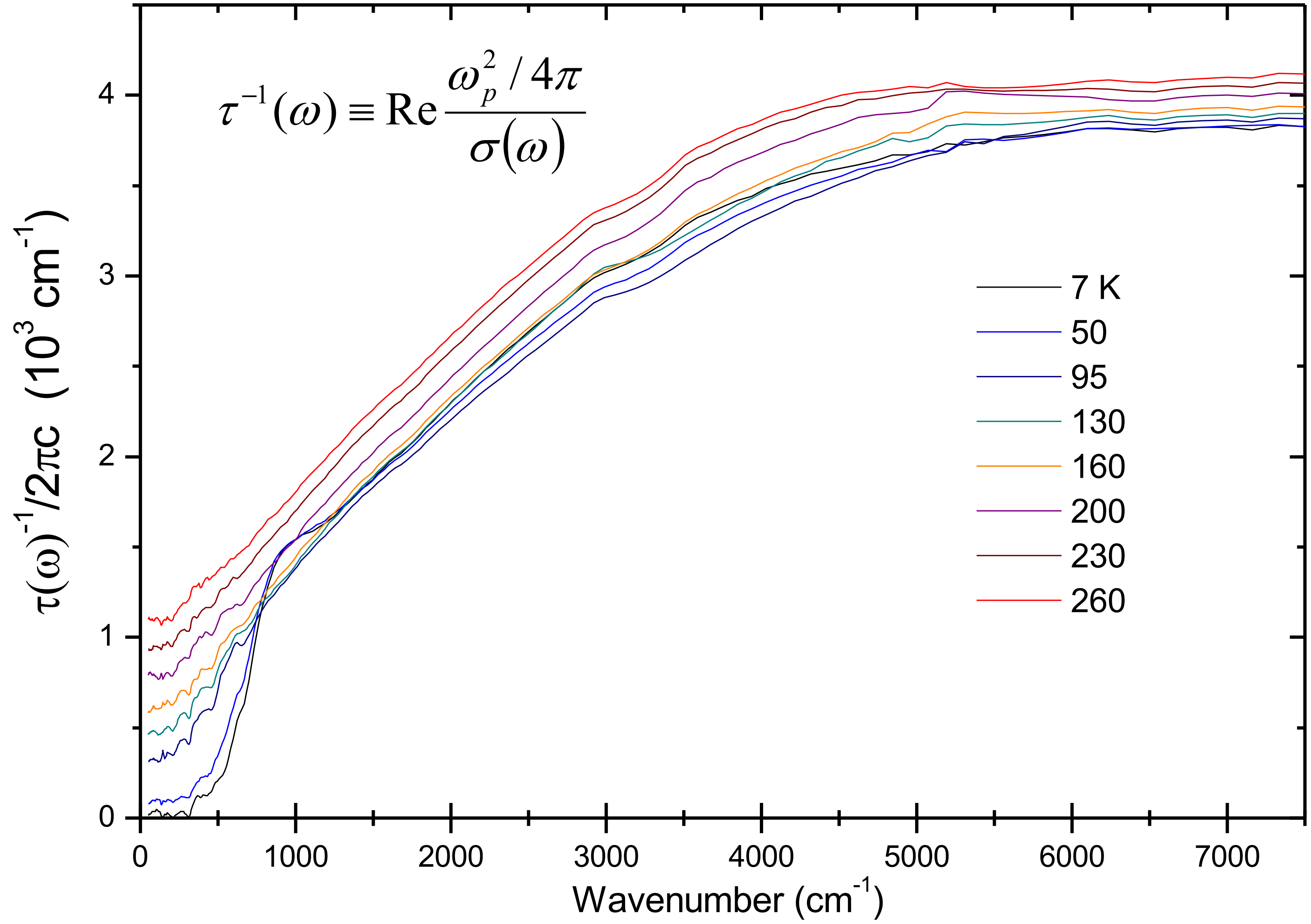


$$\sigma(\omega) = \frac{\omega_p^2/4\pi}{\tau^{-1}(\omega) - i\omega m^*(\omega)/m}$$

$$\omega_p/2\pi c = 19360 cm^{-1}$$



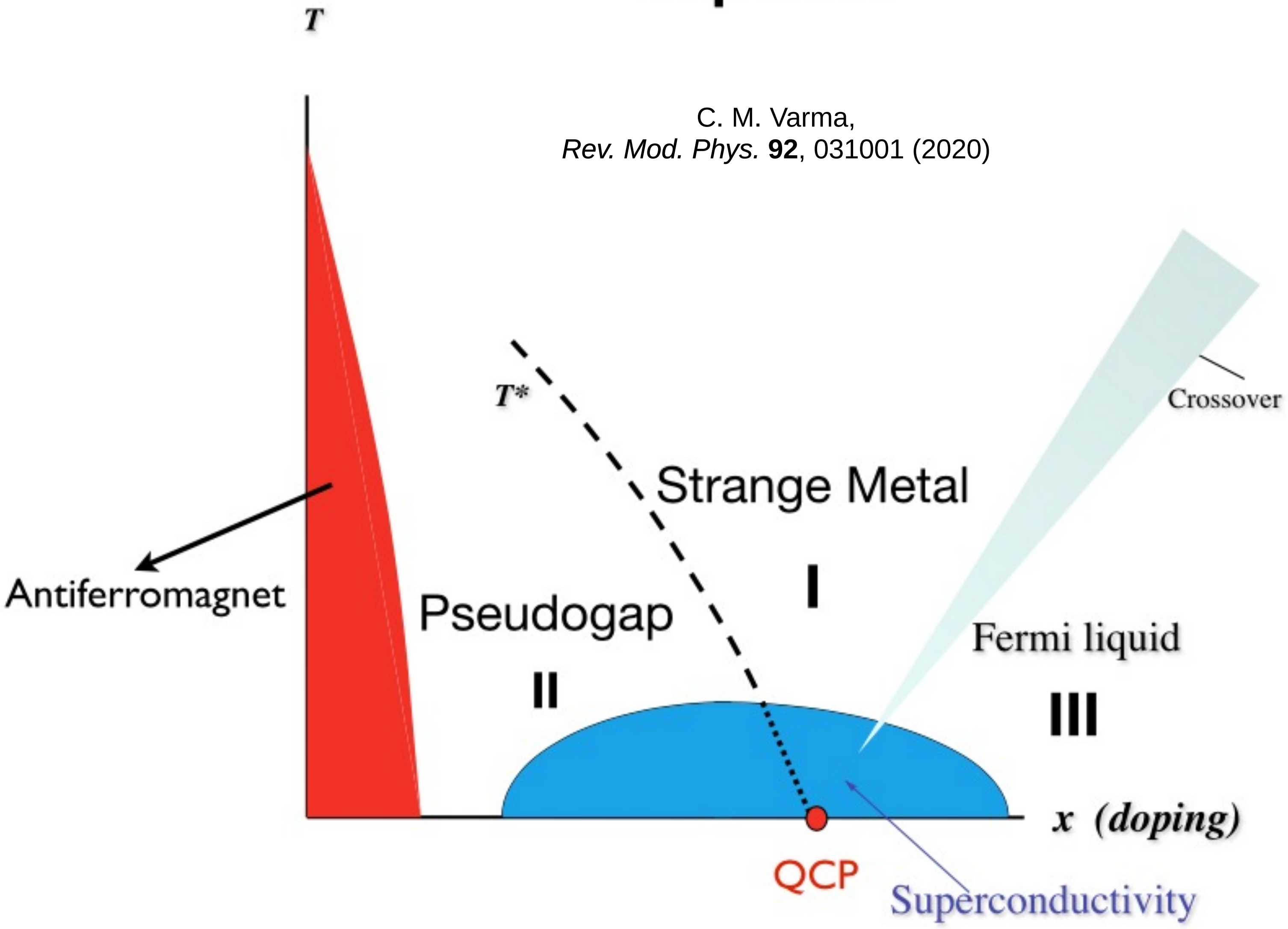
G. Thomas (1988)
Z. Schlesinger (1990)
A. El Azrak et al, PRB 49, 9846 (1994).
C. Baraduc, A. El Azrak, and N. Bontemps, J. Supercond. 9, 3 (1996).
and many other experiments



DvdM, H. J. A. Molegraaf, J. Zaanen, Z. Nussinov, F. Carbone, A. Damascelli, H. Eisaki, M. Greven, P. H. Kes, & M. Li; *Nature* **425**, 271 (2003)

Cuprates

C. M. Varma,
Rev. Mod. Phys. **92**, 031001 (2020)

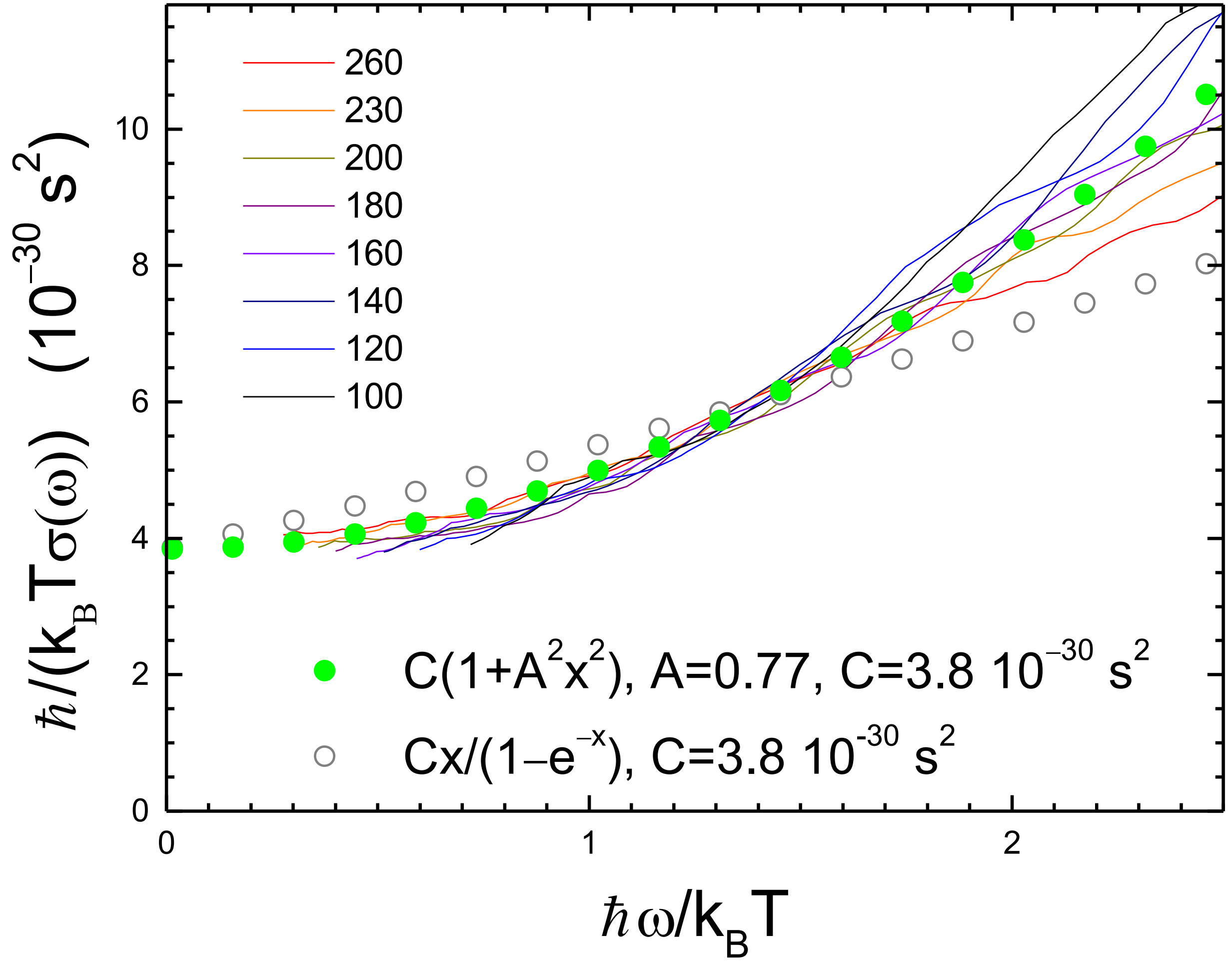


Region 1: ω < T

Expected: $1/\sigma(\omega,T) \propto T^{\nu} F(\omega/T)$, with $\nu \leqslant 1$

This requires that: $1/\tau(\omega,T) = T^{\nu} f_{\tau}(\omega/T)$

$$m^*(\omega,T) - m^*(0,T) = T^{\nu-1} f_m(\omega/T)$$



$$\frac{k_B T}{\hbar}\sigma_1(\omega) = A\frac{\omega_{PR}^2/4\pi}{1 + A^2(\hbar\omega/k_B T)^2}$$

$$A = 0.77$$

$$\omega_{PR}/2\pi c = 9600 cm^{-1}$$

$$\left(\frac{\omega_{PR}}{\omega_P}\right)^2 = 0.25$$

$\nu = 1$

$f_{\tau}(x) = A x$

$f_m(x) = 1$

DvdM, H. J. A. Molegraaf, J. Zaanen, Z. Nussinov, F. Carbone, A. Damascelli, H. Eisaki, M. Greven, P. H. Kes, & M. Li; *Nature* **425**, 271 (2003)

Region 2: $\Omega > \omega > T$ corresponds to quantum critical dynamics
This implies time-scale invariance

$$\sigma(p\omega) = \Lambda\, \sigma(\omega)$$
$$\sigma(\omega) = |C|\, e^{i\phi} (-i\, \omega)^{\eta-2}$$

Time reversal : $\sigma(\omega)= \sigma^*(-\omega)$

Together: $\phi=0$

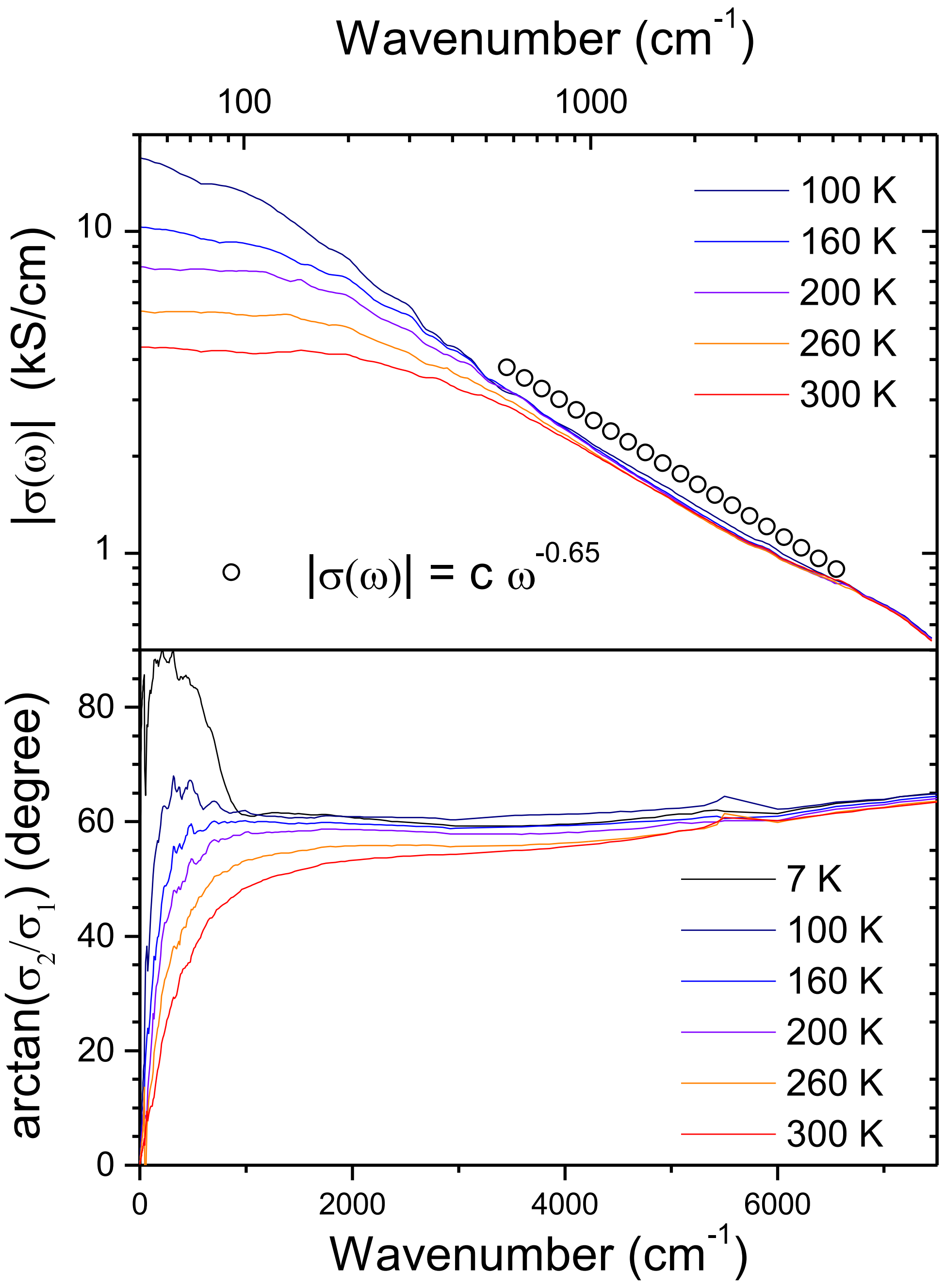
$$\sigma(\omega) = |C|\, (-i\, \omega)^{\eta -2}$$

Originally proposed in the context of spin-charge separation
by P.W. Anderson, PRB55, 11785 (1997)

$$\phi_\sigma = \arctan\,(\sigma_2/ \sigma_1) = \pi - \pi\, \eta\, / \,2$$

$$d \ln|\sigma| \,/\, d \ln\omega = \eta - 2$$

Region 2: $\Omega > \omega > T$



$\Rightarrow$ $\eta-2=-0.65$

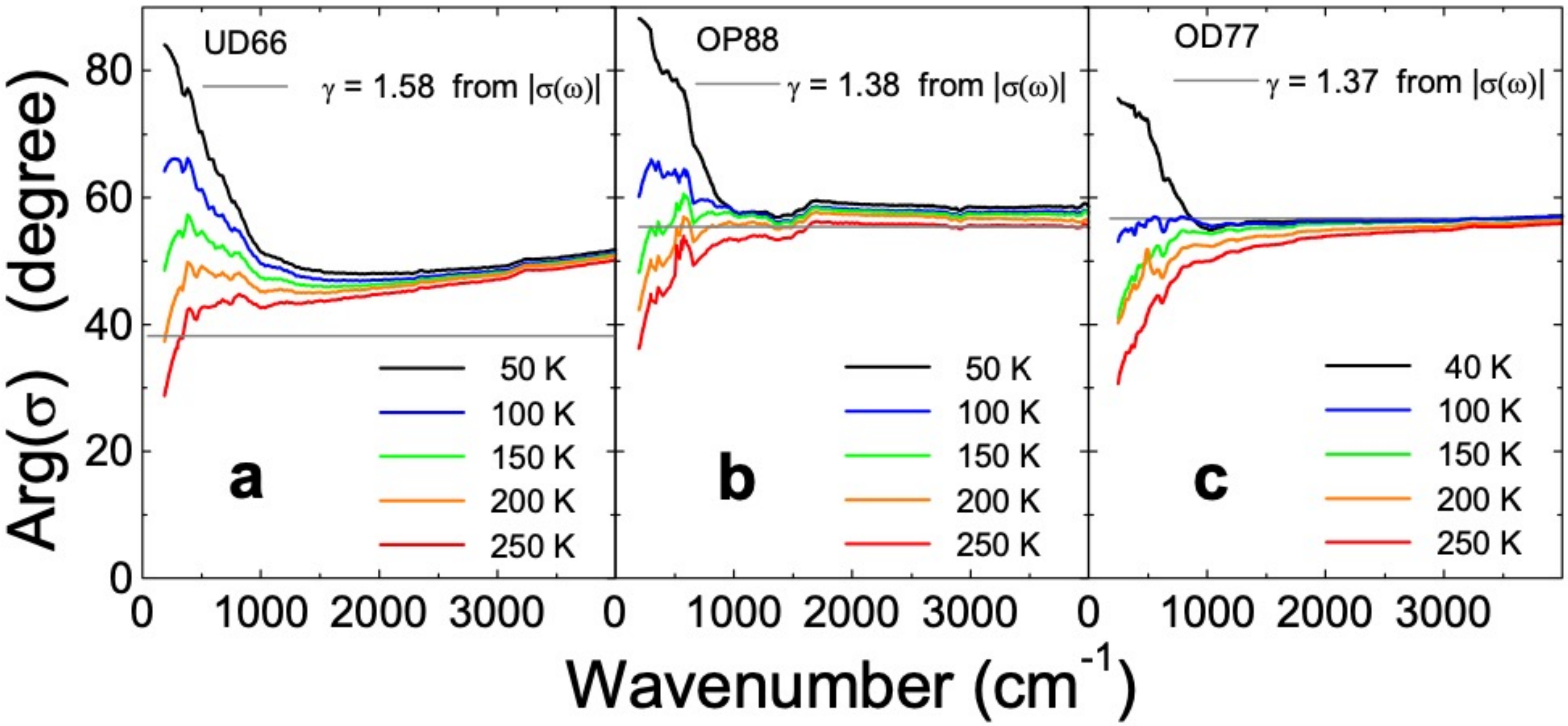
$\eta=1.35$

$\Rightarrow$ $\mathrm{Arg}\sigma = 60^{o} = \pi(1-\eta/2)$

$\eta = 1.33$

DvdM, H. J. A. Molegraaf, J. Zaanen, Z. Nussinov, F. Carbone, A. Damascelli, H. Eisaki, M. Greven, P. H. Kes, & M. Li; *Nature* **425**, 271 (2003)

Region 2: $\Omega > \omega > T$



DvdM, H. J. A. Molegraaf, J. Zaanen, Z. Nussinov, F. Carbone, A. Damascelli, H. Eisaki, M. Greven, P. H. Kes, & M. Li; *Nature* **425**, 271 (2003)

Region 3: ω > Ω: UV-regularization

f-sumrule

$$\lim_{\omega\to\infty}\sigma(\omega)=i\frac{ne^2}{m\omega}$$

Thermodynamics

$$\operatorname{Re}\sigma(\omega)>0$$

Causality

$$\operatorname{Im}\sigma(\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\operatorname{Re}\sigma(x)}{\omega-x}dx$$

Time reversal symmetry

$$\sigma(-\omega)=\sigma^*(\omega)$$

Example:
DvdM, PRB60, R768 (1999)

$$\sigma(\omega,T)=\frac{\omega_p^2/4\pi}{(-i\omega)^{2-\eta}(\Omega-i\omega)^{\eta-1}}$$

Summary

HTSC for optimal doping:

1) **Region 1** $(\hbar\omega < 1.5 k_B T):\qquad \tau_R = A\hbar / k_B T\,,\quad A = 0.77$

2) **Region 2:**

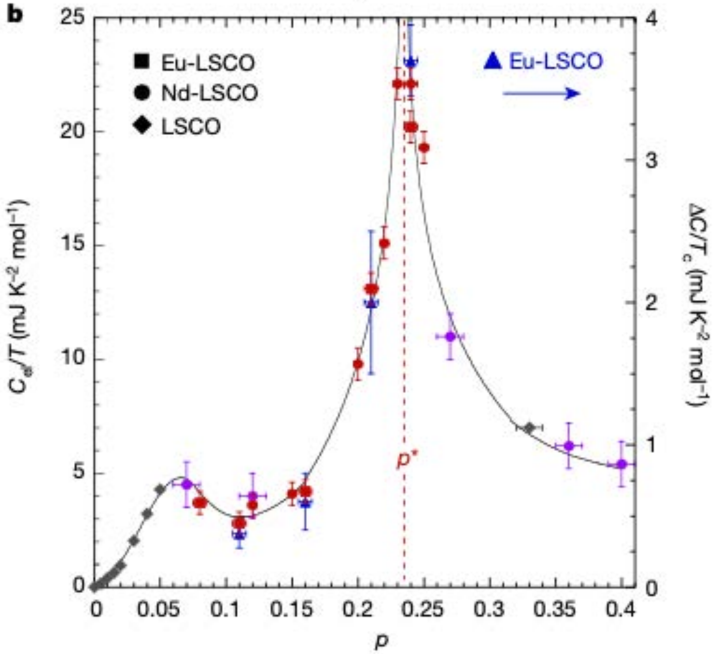
a: $\sigma(\omega)$ is proportional to $(i\omega)^{\eta-2}$

b: Phase of $\sigma(\omega)$ is $\pi(1-\eta/2)$, independent of frequency

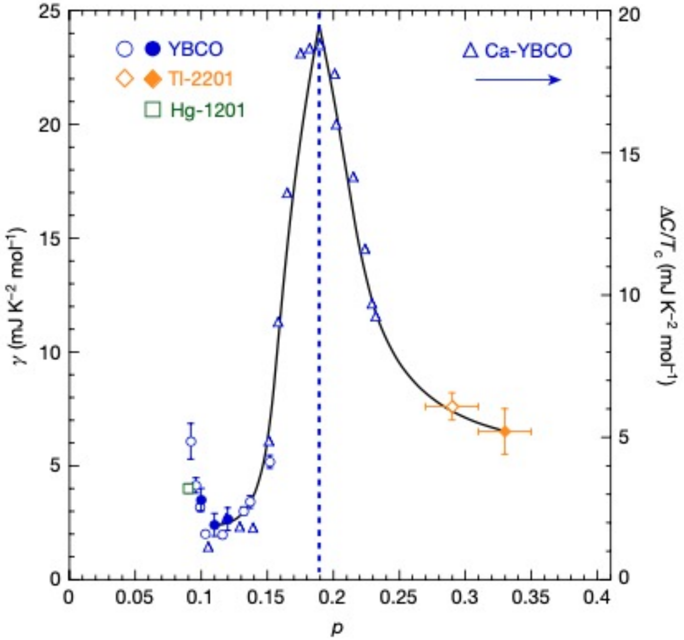
c: $\eta = 4/3 \pm 0.02$

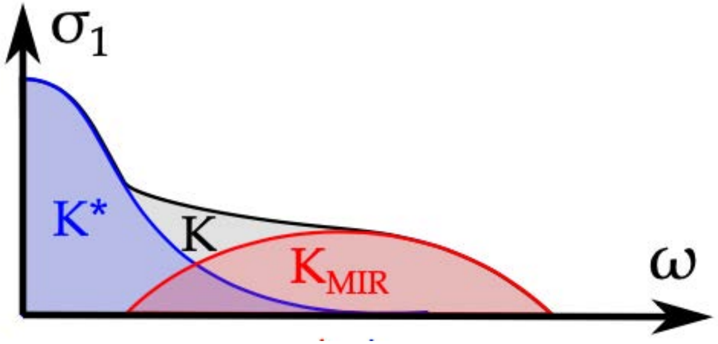
3) **Region 3:**

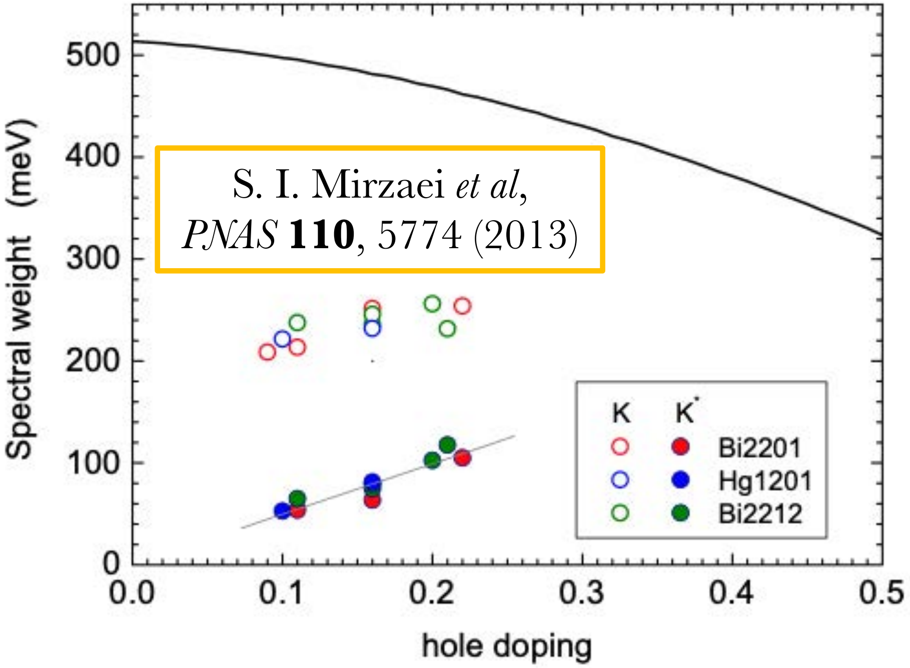
UV regularization becomes noticeable for $\omega > 0.7$ eV



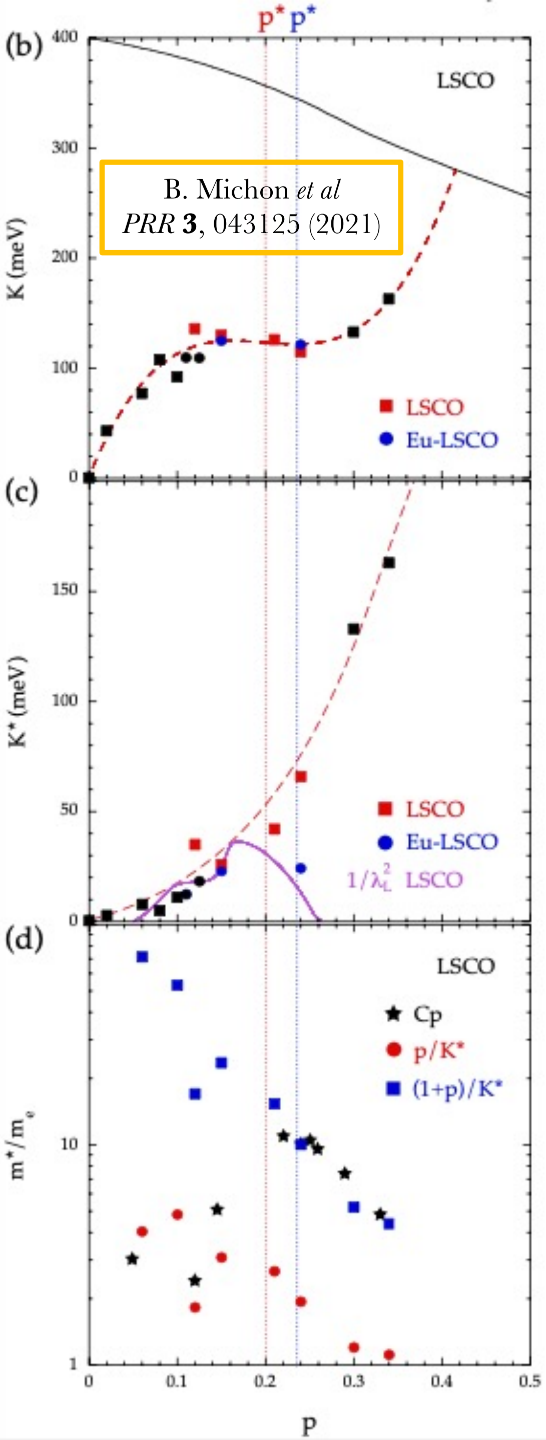
B. Michon *et al*,
Nature **567**, 218 (2019)



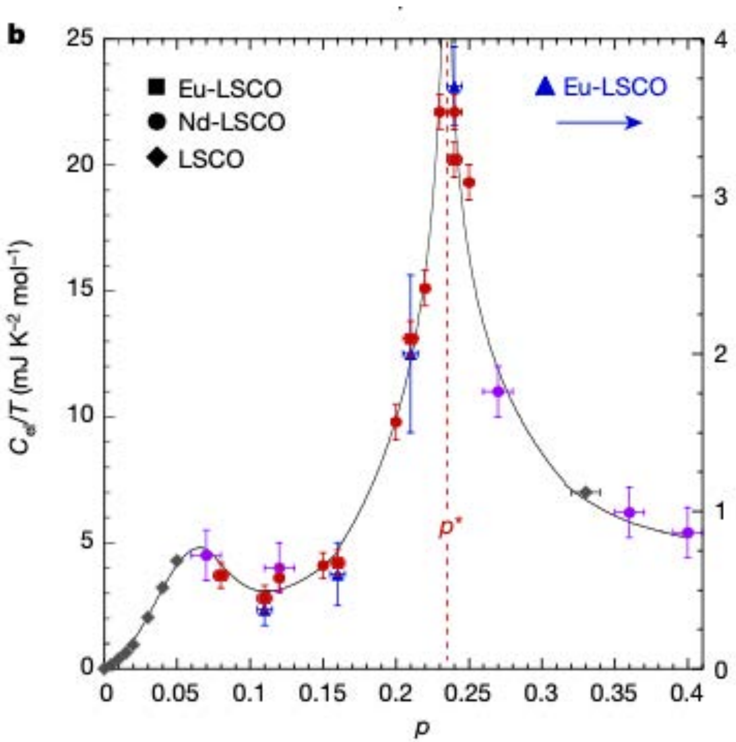




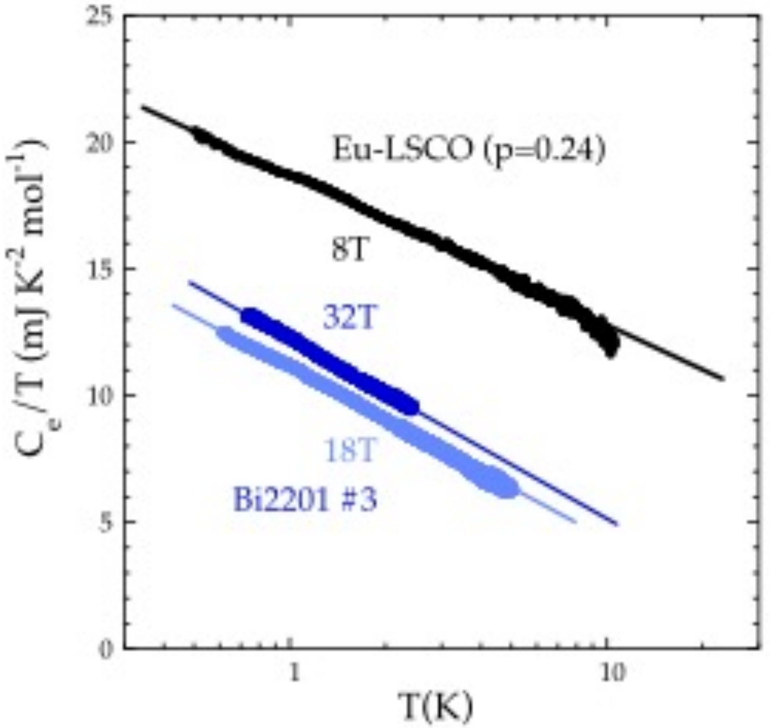
S. I. Mirzaei *et al*,
PNAS **110**, 5774 (2013)



B. Michon *et al*
PRR **3**, 043125 (2021)



B. Michon *et al,*
Nature **567**, 218 (2019)



C. Girod *et al,*
Phys. Rev. B **103**, 214506 (2021)

because in the compounds measured $La_{2-p}A_pCuO_4$, with $A =$ Nd or Eu, T_c is low enough to be completely suppressed with fields of about 15 T.

Close to quantum criticality, the electronic specific heat fits

$$\frac{C_{el}}{k_B T}(p_c) = \gamma\left[1 + \tilde{g}\ln\left(\frac{\tilde{T}_x}{T}\right)\right]. \qquad (1)$$

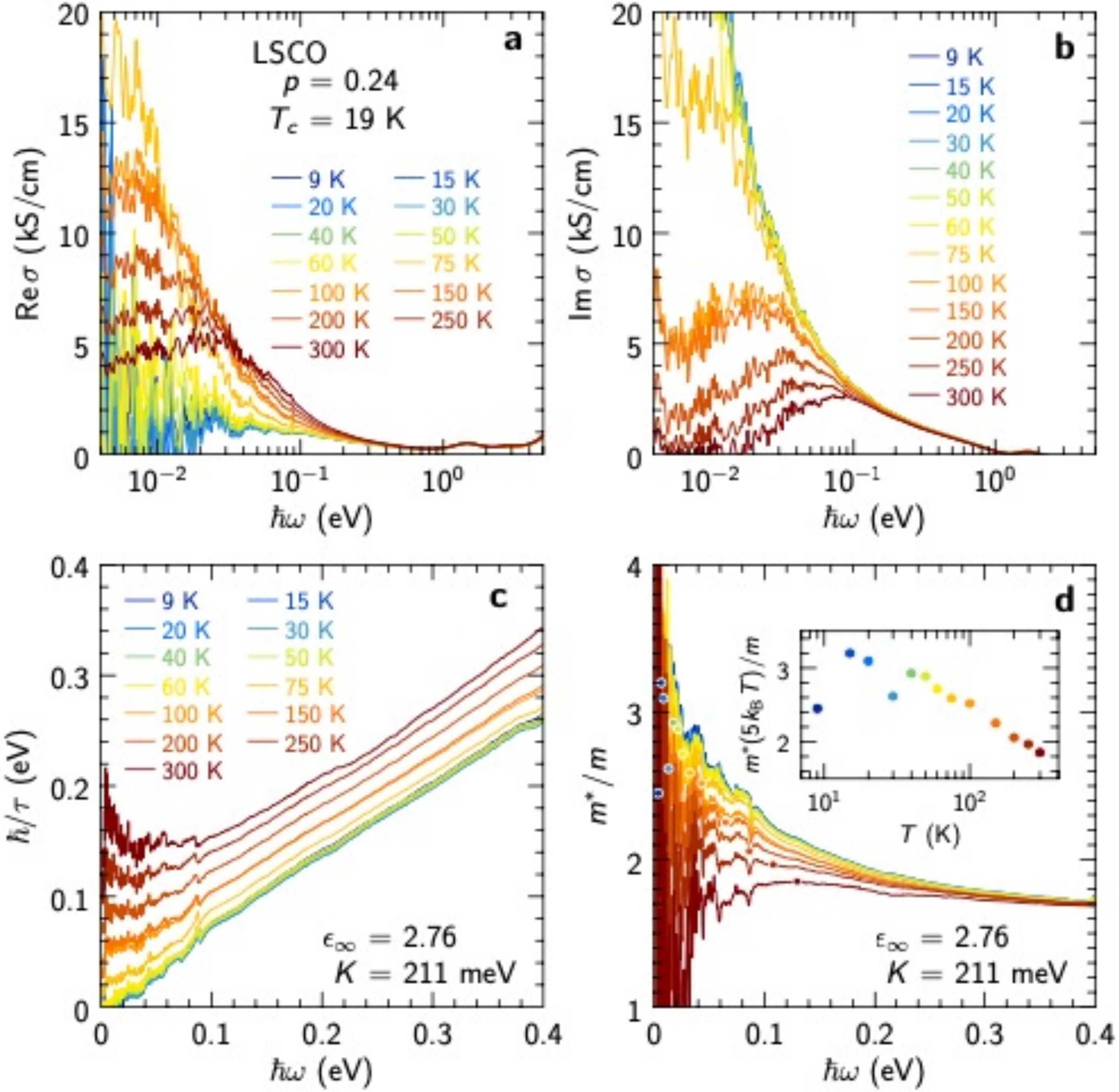
The logarithmic enhancement of the specific heat is equivalent to the basic postulates of a marginal Fermi liquid (Varma *et al.*, 1989), that the quasiparticle residue goes to zero at the critical point as

$$z_{\bar{p}}(\omega, T) = \frac{1}{1 + g_{\bar{p}}\ln(\pi T_{x\bar{p}}/x)}, \qquad x = \max(\pi T, \omega). \qquad (2)$$

Following the summary of the theory in Sec. III, I assume that both the coupling constant g and the cutoff T_x may have weak dependence on the direction of the momentum **p** at the Fermi surface. The experimental $\tilde{g}$ and $\tilde{T}_x$ in the specific heat may be taken as the averages of the parameters in $z_{\bar{p}}$.

What is plotted in Fig. 2 is not the total specific heat divided by T but C_{el}/T obtained by subtracting from the total specific

C. M. Varma,
Rev. Mod. Phys. **92**, 031001 (2020)



a
LSCO
$p = 0.24$
$T_c = 19$ K
9 K
15 K
20 K
30 K
40 K
50 K
60 K
75 K
100 K
150 K
200 K
250 K
300 K
$\mathrm{Re}\,\sigma$ (kS/cm)
$\hbar\omega$ (eV)
b
$\mathrm{Im}\,\sigma$ (kS/cm)
c
$\hbar/\tau$ (eV)
$\epsilon_\infty = 2.76$
$K = 211$ meV
d
m^*/m
$m^*(5k_B T)/m$
T (K)

Ansatz: O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta, *Phys. Rev. B* **58**, 3794 (1998)
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **70**, 3339 (1993)
A. Kitaev (2015)
O. Parcollet and A. Georges, *Phys. Rev. B* **59**, 5341 (1999)
P. T. Dumitrescu, N. Wentzell, A. Georges, and O. Parcollet, *arXiv:***2103**.08607 (2021)
A. A. Patel, H. Guo, I. Esterlis, and S. Sachdev, *arXiv:***2203**.04990 (2022)

$$-\mathrm{Im}\,\Sigma(\varepsilon) = g\pi k_B T S\left(\frac{\varepsilon}{k_B T}\right) \qquad \Sigma(z) = g k_B T \int_\Lambda dx\, \frac{S(x)}{z/k_B T - x}$$

UV cutoff = Λ

$$\mathrm{Re}\left[\Sigma(\varepsilon) - \Sigma(0)\right] = -2g\varepsilon \ln(a\Lambda/k_B T) \qquad \frac{m^*_{\mathrm{qp}}}{m} = \frac{1}{Z} = 1 + 2g\ln\left(a\frac{\Lambda}{k_B T}\right)$$

$$\sigma(\omega) = \frac{i\Phi(0)}{\omega}\int_{-\infty}^{\infty} d\varepsilon\, \frac{f(\varepsilon) - f(\varepsilon + \hbar\omega)}{\hbar\omega + \Sigma^*(\varepsilon) - \Sigma(\varepsilon + \hbar\omega)}$$

dc limit: $\rho = AT, \quad A = \dfrac{4\pi^3 k_B}{7\zeta(3)\hbar}\dfrac{g}{\Phi(0)} = \dfrac{4\pi^3 \hbar k_B d_c}{7\zeta(3)e^2}\dfrac{g}{K}$

(I) $\hbar\omega \lesssim k_B T$ ω/T scaling: $1/\tau \sim T f_\tau(\omega/T)$

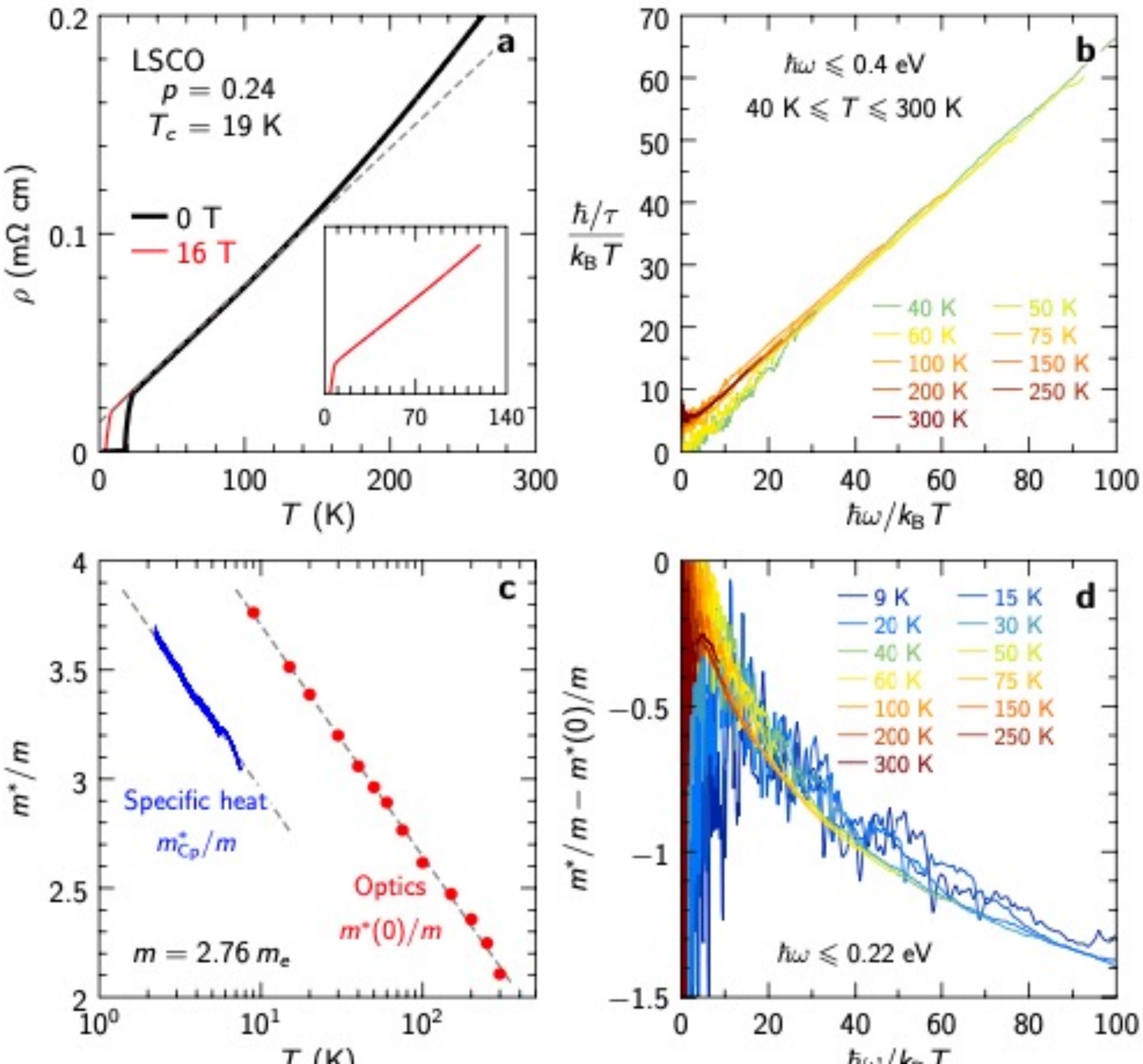
$$m^*(\omega) - m^*(0) \sim f_m(\omega/T)$$

(I) $\hbar\omega \lesssim k_B T$

$$\frac{m^*_{qp}}{m} = \frac{1}{Z} = 1 + 2g\ln\left(a\frac{\Lambda}{k_B T}\right)$$

$$\sigma(\omega) = \frac{\omega_{PR}^2 \tau \ /4\pi}{1 + i\omega\tau} \qquad \hbar/\tau = 4\pi g k_B T$$

fitting => g=0.23 Λ=0.4 eV



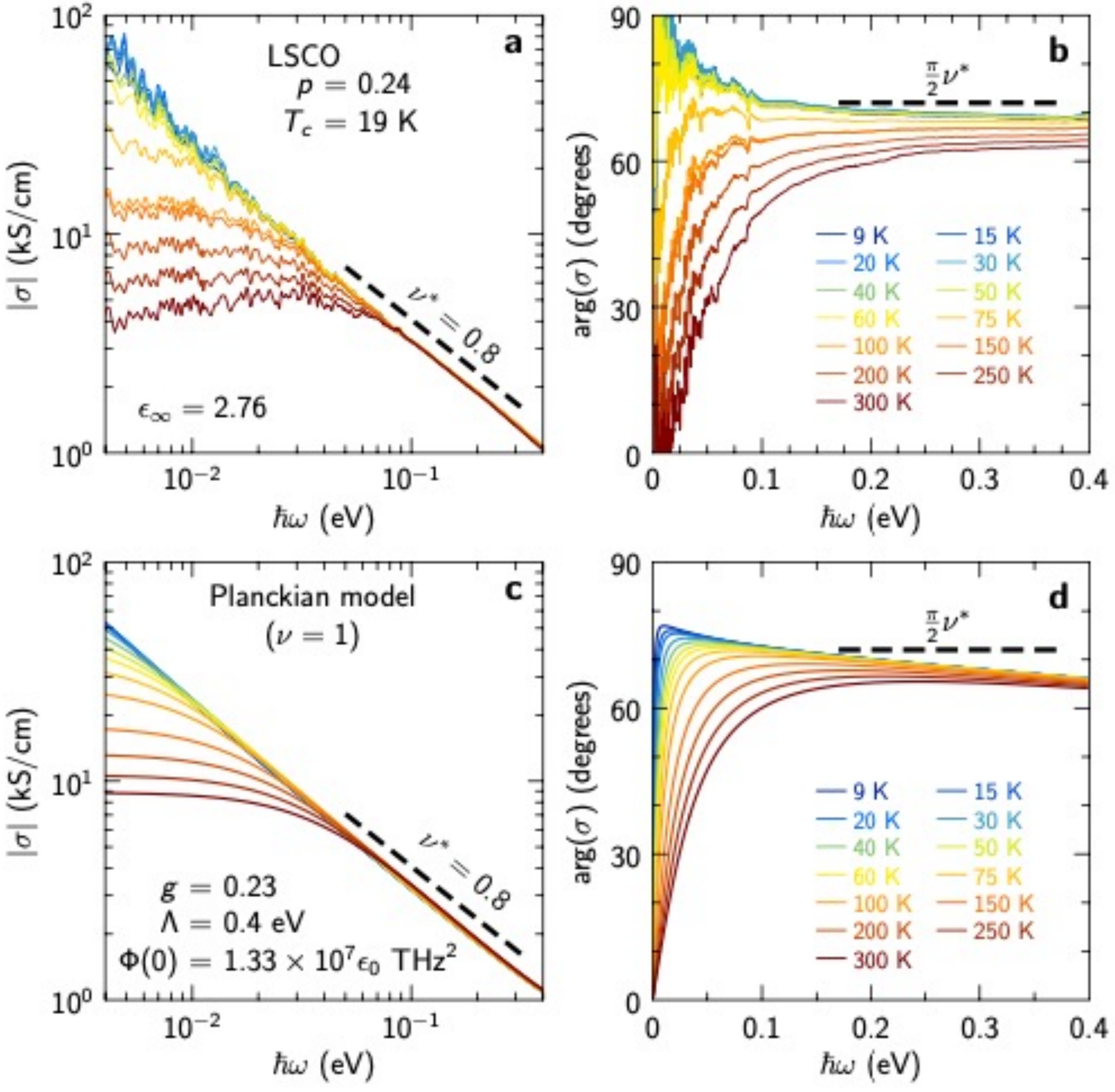
Ansatz: $-\mathrm{Im}\,\Sigma(\varepsilon) = g\pi k_B T S\left(\frac{\varepsilon}{k_B T}\right)$ $\Sigma(z) = g k_B T \int_\Lambda dx \frac{S(x)}{z/k_B T - x}$

(I) $\hbar\omega \lesssim k_B T$ $\sigma(\omega) = \frac{\omega_{PR}^2 \tau /4\pi}{1+i\omega\tau}$ $\hbar/\tau = 4\pi g k_B T$ fitting => g=0.23 Λ=0.4 eV

(II) $k_B T \lesssim \hbar\omega \lesssim \Lambda$. $\sigma(\omega) \approx \frac{\Phi(0)}{-i\omega} \frac{1}{1+2g\left[1-\ln\left(\frac{\hbar\omega}{2\Lambda}\right)\right]+i\pi g}$

$|\sigma| \sim |\omega|^{-\nu^*}$ $\nu^* \equiv -\left.\frac{d\ln|\sigma|}{d\ln\omega}\right|_{\hbar\omega=\Lambda/2} = 1 - \frac{2g[1+2g(1+\ln 4)]}{\pi^2 g^2 + [1+2g(1+\ln 4)]^2}$ =0.8

(III) $\hbar\omega \gtrsim \Lambda$. $|\sigma| \sim 1/\omega, \arg(\sigma) \to \pi/2$



LSCO
$p = 0.24$
$T_c = 19$ K
$\epsilon_\infty = 2.76$
$\nu^* = 0.8$
a
b
$\frac{\pi}{2}\nu^*$
9 K
15 K
20 K
30 K
40 K
50 K
60 K
75 K
100 K
150 K
200 K
250 K
300 K
Planckian model
$(\nu = 1)$
$g = 0.23$
$\Lambda = 0.4$ eV
$\Phi(0) = 1.33 \times 10^7 \epsilon_0$ THz^2
c
d
$|\sigma|$ (kS/cm)
$\arg(\sigma)$ (degrees)
$\hbar\omega$ (eV)

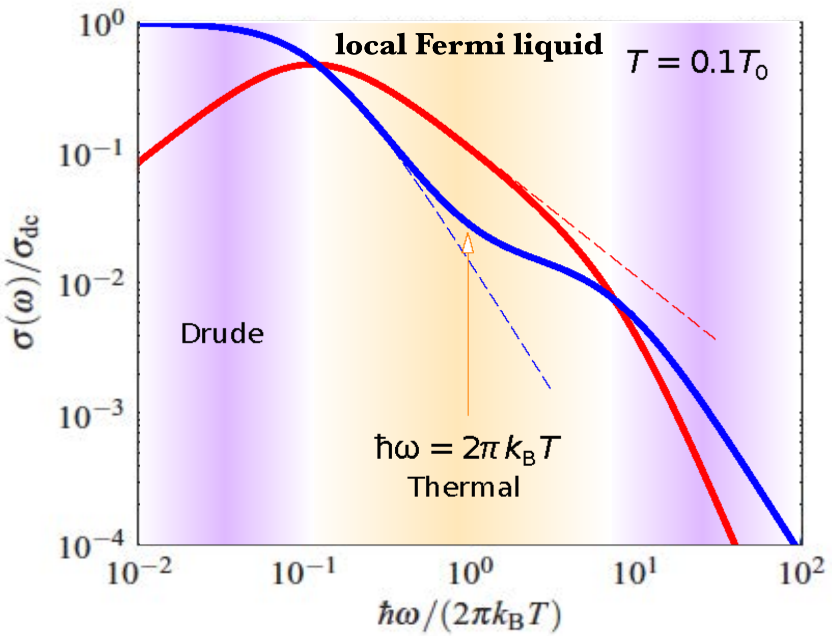
Conclusions

- Compelling evidence for the quantum critical behavior of electrons in cuprate superconductors
- Remarkable consistency between experimental observations based on optical spectroscopy, resistivity and specific heat, all being consistent with $\nu=1$ Planckian behavior and ω/T scaling
- Explanation of the longstanding puzzle of an apparent power law of the optical spectrum over an intermediate frequency range and related the non-universal apparent exponent to the inelastic coupling constant

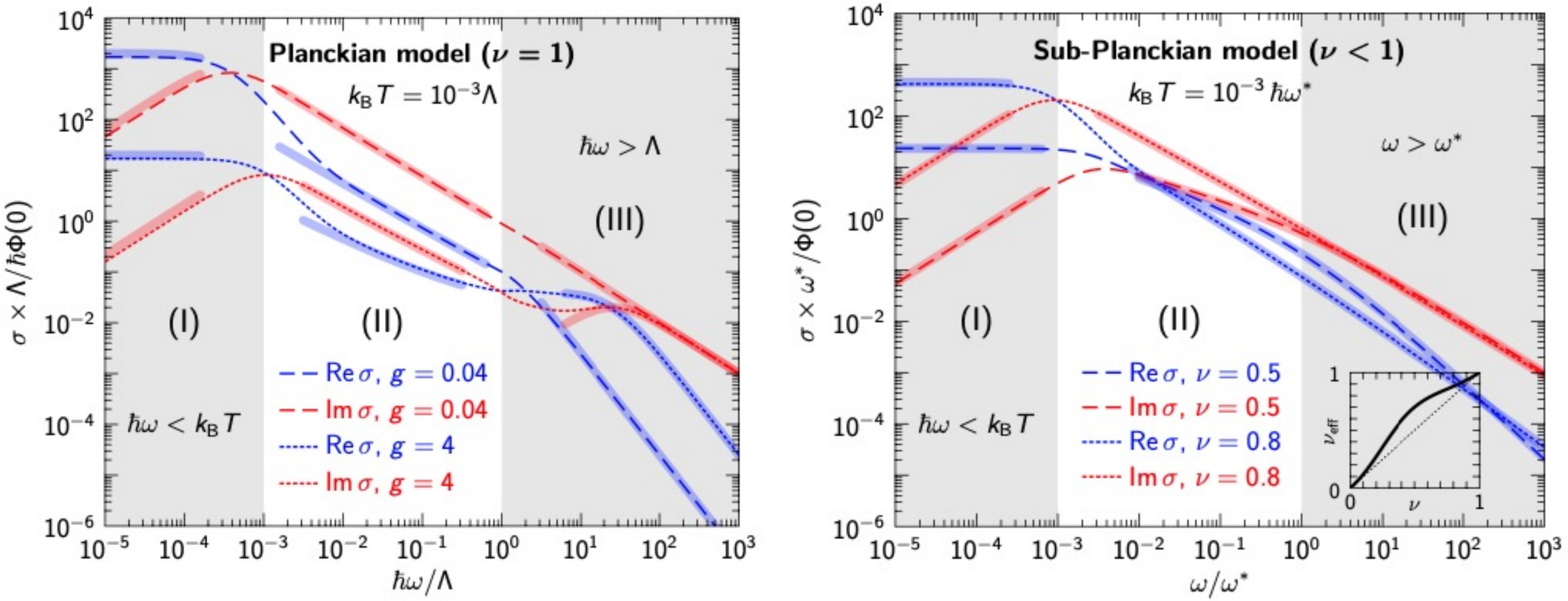
Outlook

- Extend measurements and analysis to other cuprate compounds at doping levels close to the pseudogap quantum critical point
- Explain the nature of the associated quantum critical point, and its relation to the enigmatic pseudogap phase

The slides on the following pages (pp 22-43) are not part of the talk, but turn out handy during the disscussion



Berthod, Mravlje, Deng, Žitko,
vdMarel, Georges,
PRB ***87****, 115109 (2013)*

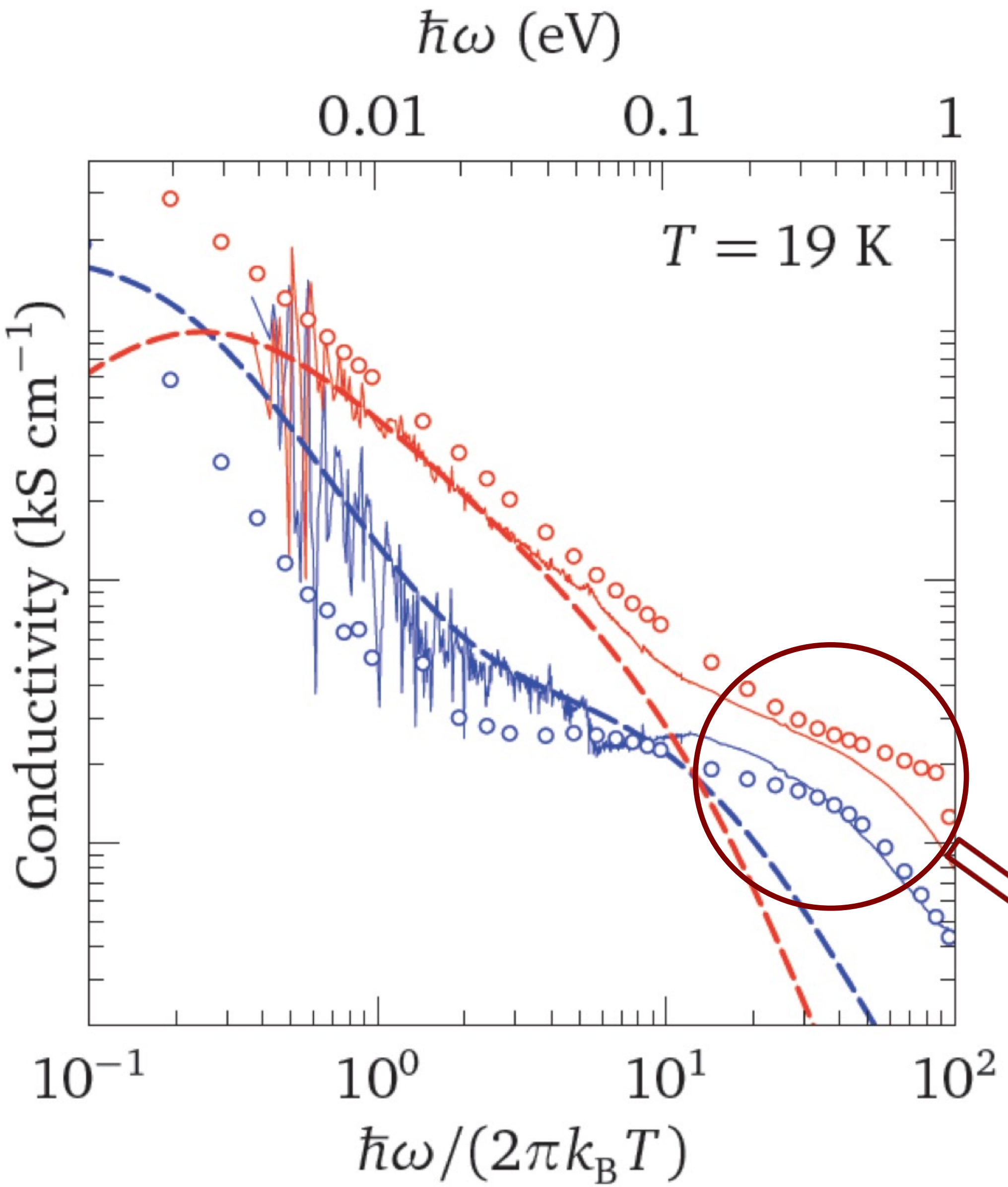


Michon, Berthod, Rischau, Ataei, Chen,
Komiya, Ono, Taillefer, DvdM & Georges
arXiv:2205.04030 (2022)

$$\frac{\sigma(\omega,T)}{\sigma_{DC}} = F\left[\frac{\hbar\omega}{k_B T}, \frac{\hbar\omega T_0}{k_B T^2}\right]$$

$$F[x,y] = \frac{6}{\pi^2 x}\int_{-\infty}^{\infty} du \frac{\left[e^{\pi(u-x)}+1\right]^{-1} - \left[e^{\pi(u+x)}+1\right]^{-1}}{1+x^2-iy+u^2}$$

Re σ(ω) + i Im σ(ω)



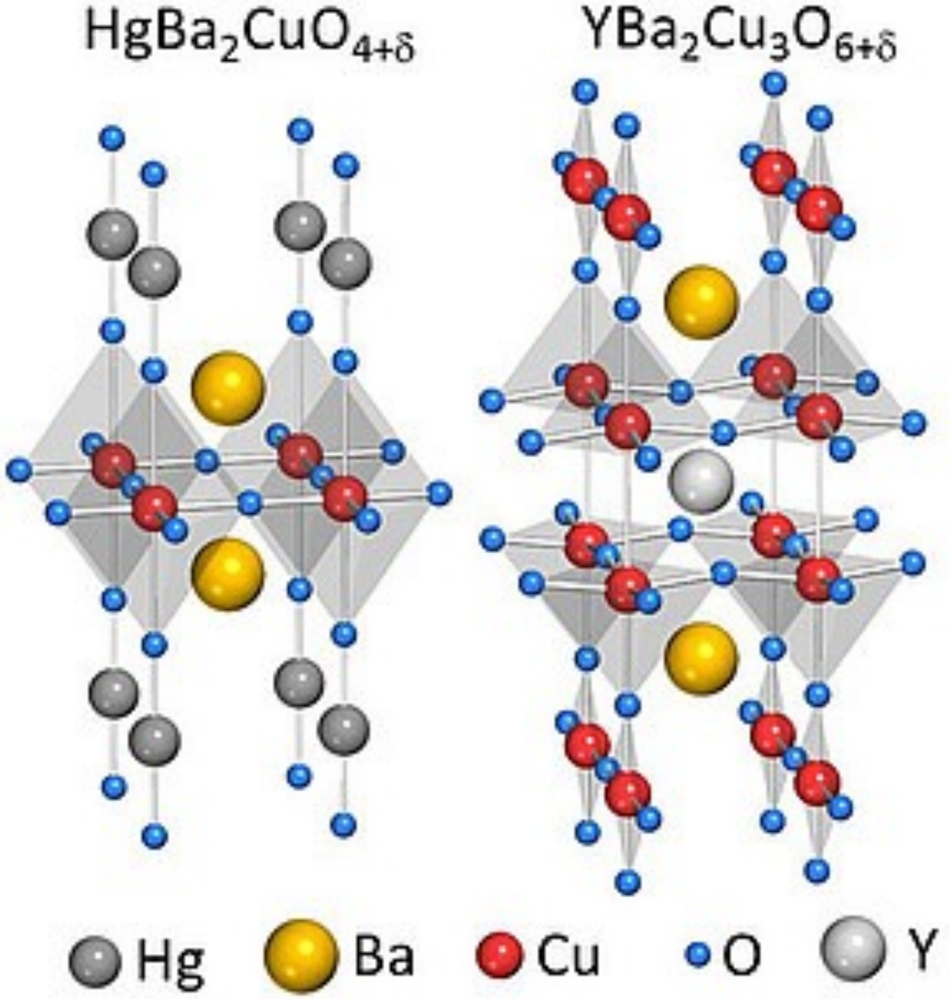
Plain Lines:
Experiments, Sr_2RuO_4

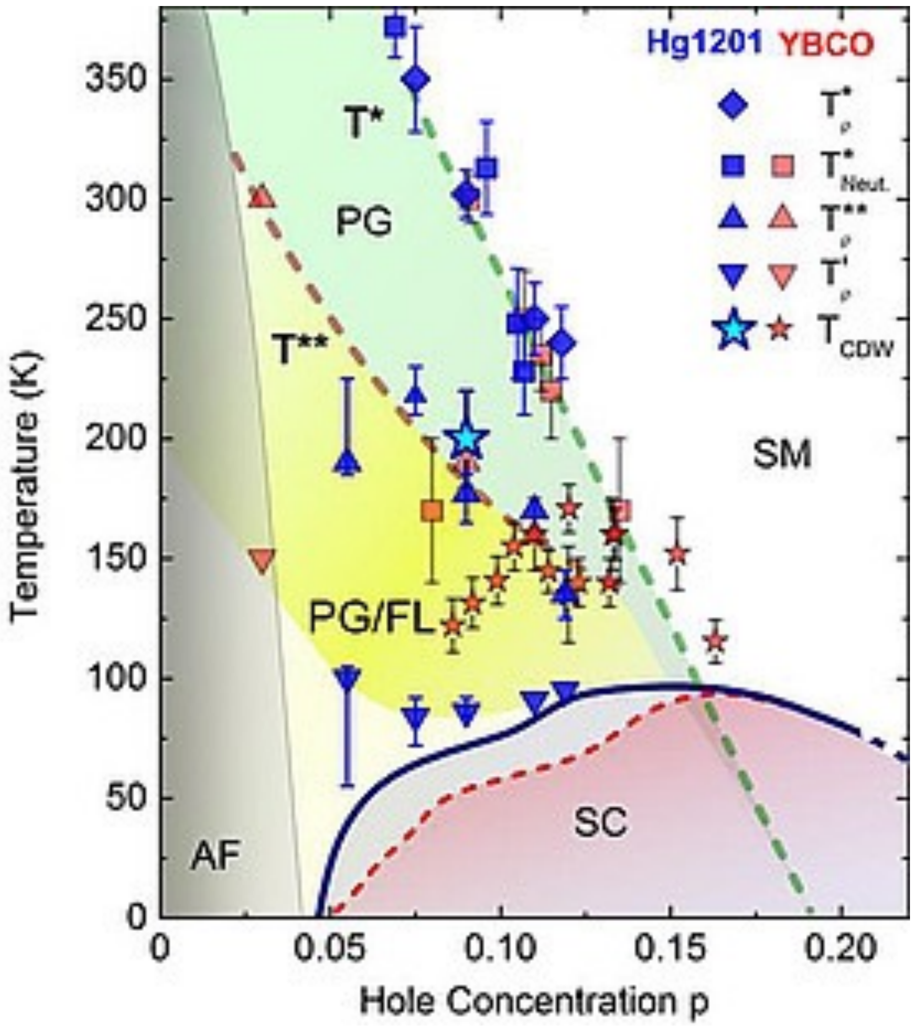
Dashed lines :
universal FL form
→ Beautiful agreement
→ At low T, low ω

Dots:
LDA+DMFT
calculation for this
material

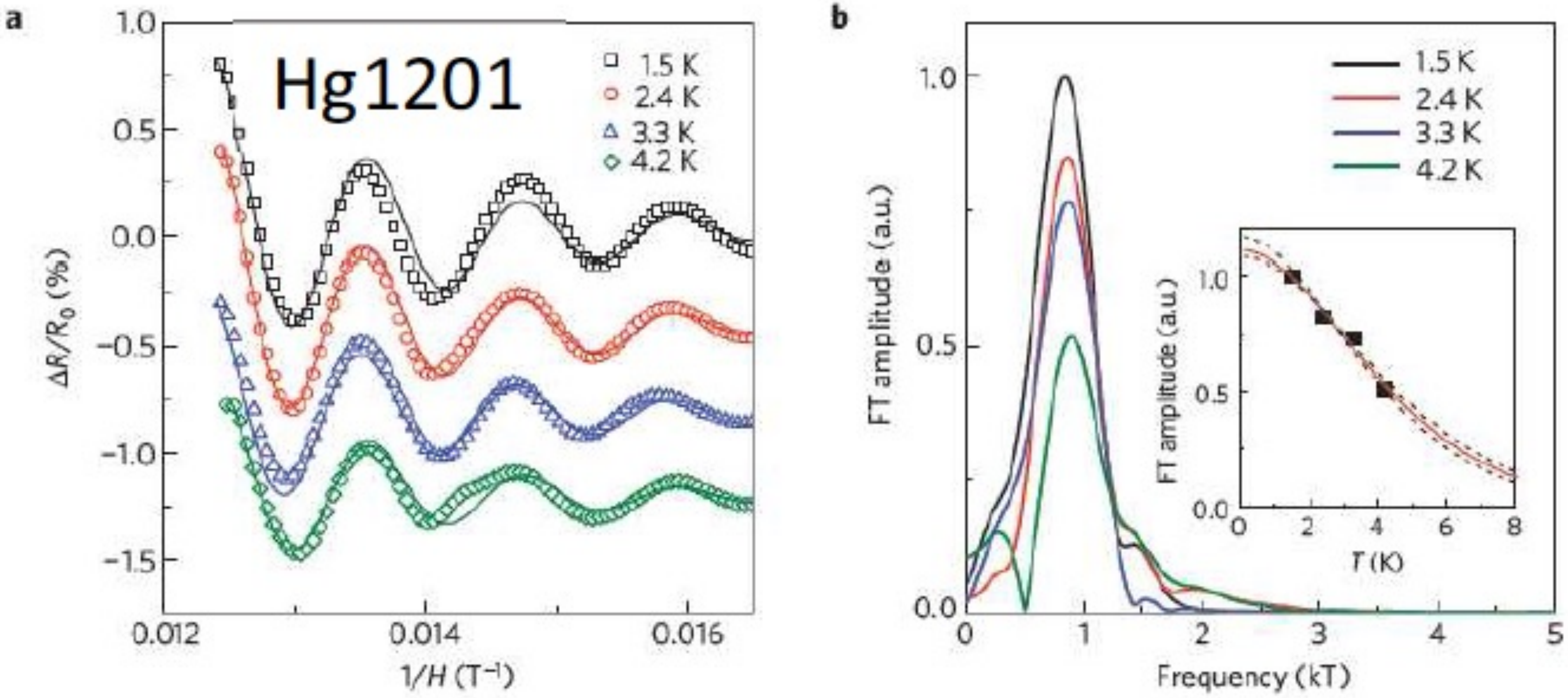
Clear deviations from
FL for ω above ~ 0.1 eV
Very well described
by DMFT !

PRL ***113***, *087404 (2014)*



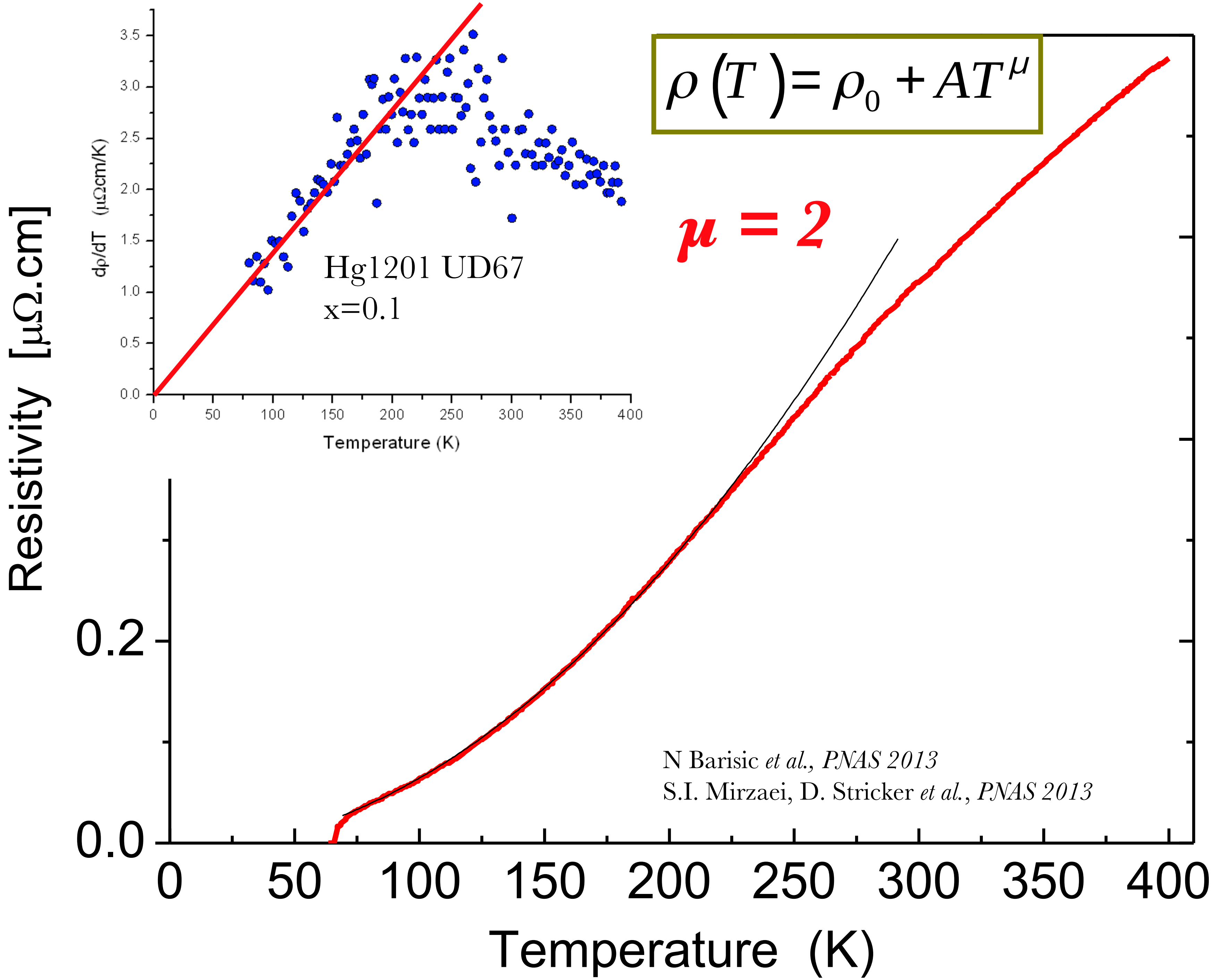


W. Tabis et al., Nature Communications 5, 5875 (2014).



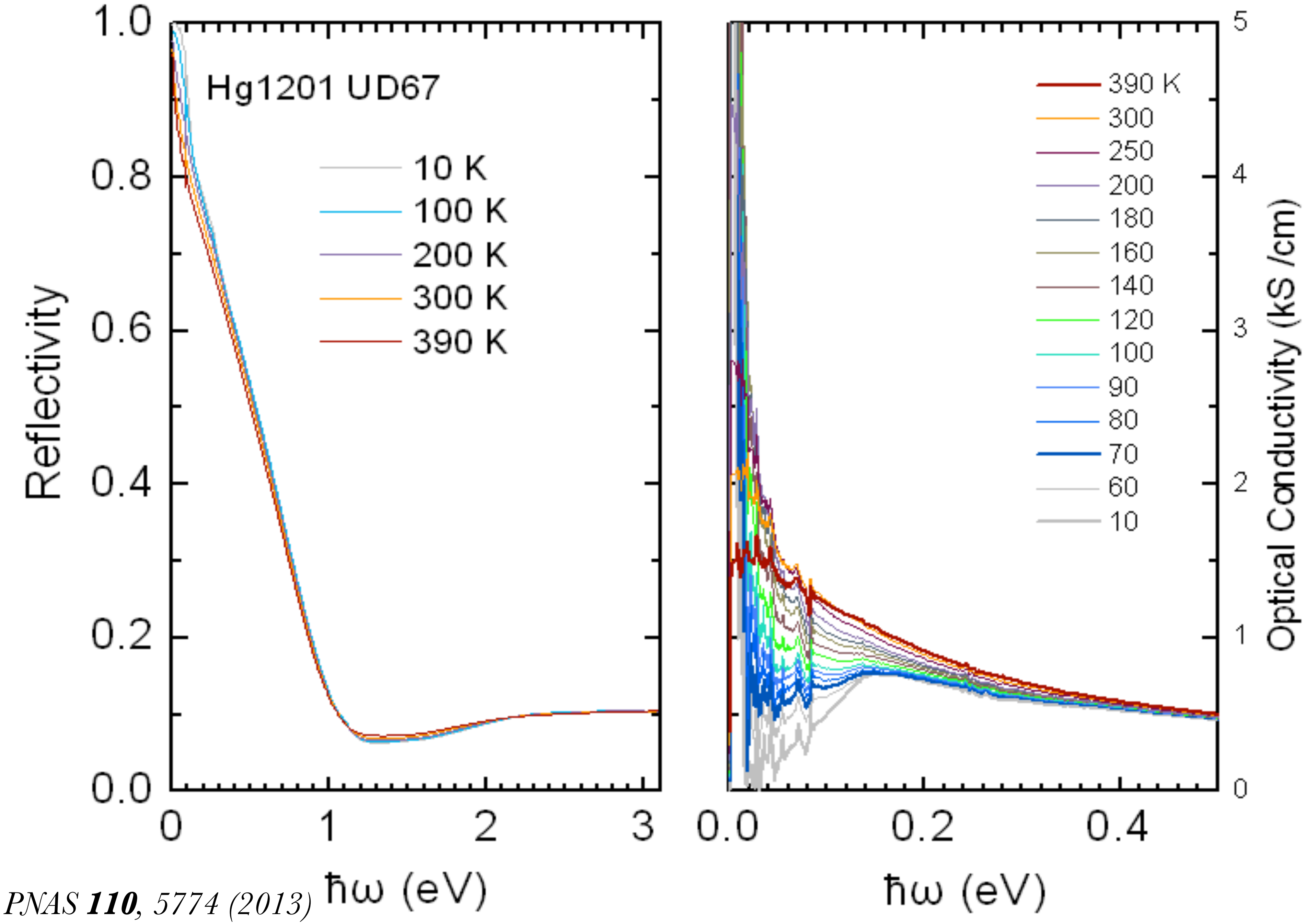
Barisic et al. Nat. Phys. 9 761 (2013)

Zero-field $T_c = 72$ K, $p \approx 0.09$



$\rho(T) = \rho_0 + AT^{\mu}$
μ = 2
Hg1201 UD67
x=0.1
dρ/dT (μΩcm/K)
Temperature (K)
Resistivity [μΩ.cm]
Temperature (K)
N Barisic *et al., PNAS 2013*
S.I. Mirzaei, D. Stricker *et al., PNAS 2013*

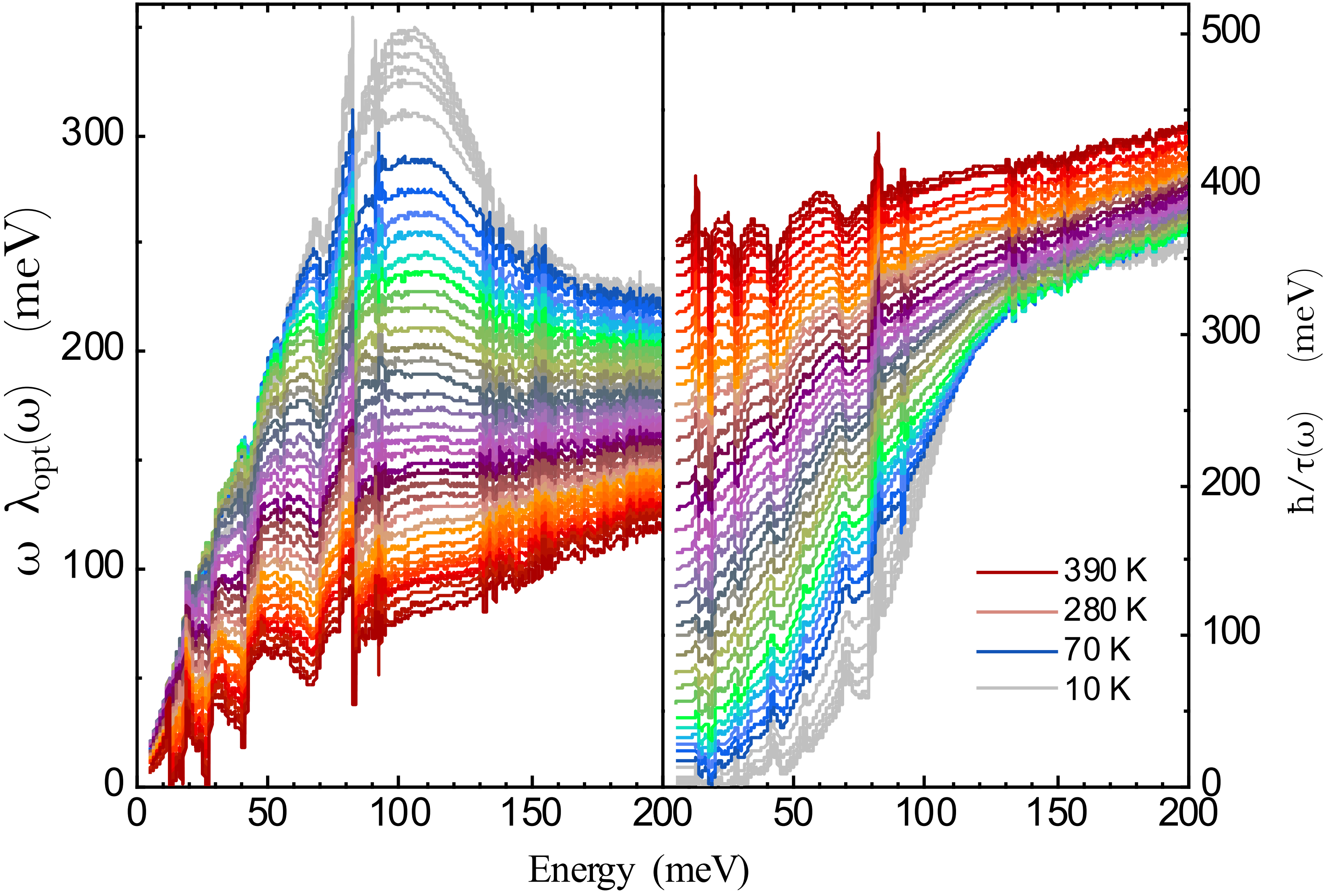
Hg1201 UD67



PNAS ***110***, *5774 (2013)*

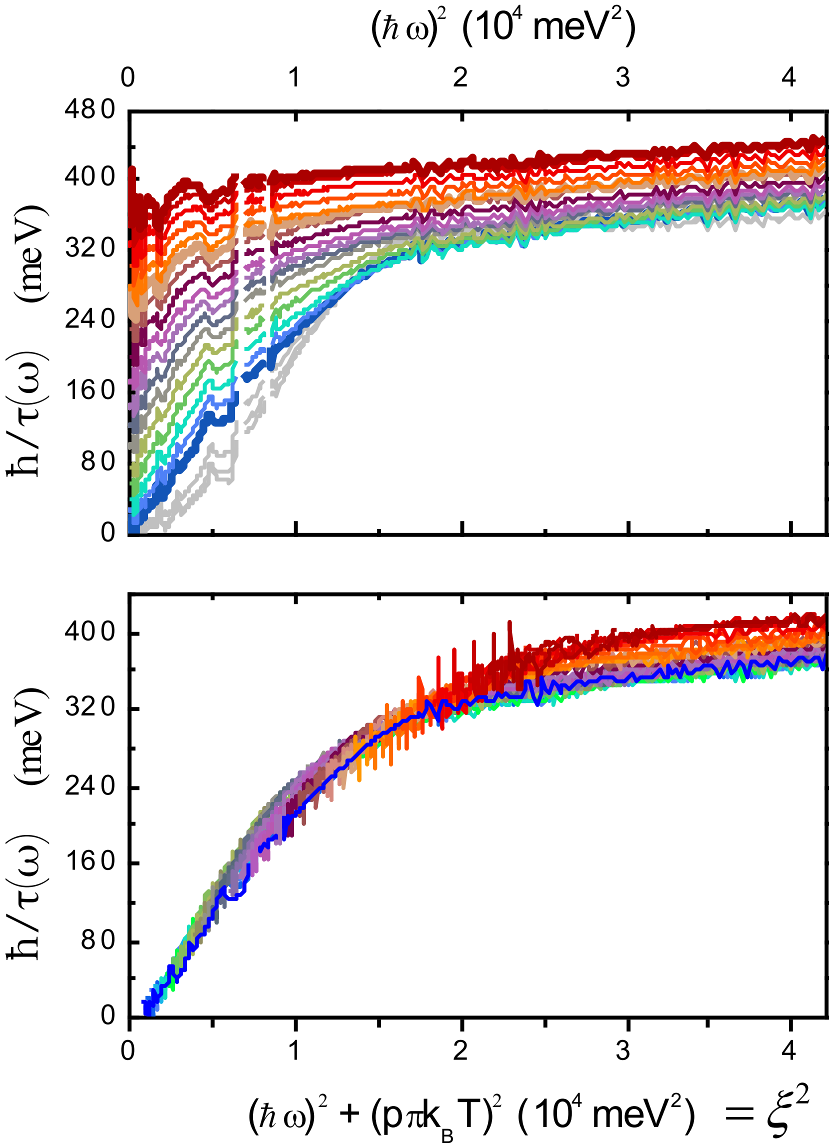
$HgBa_2CuO_4$: Energy dependend Relaxation rate

$$\frac{\omega_p^2}{4\pi i\sigma(\omega,T)} = \omega\left[1+\lambda_{opt}(\omega)\right] + \frac{i}{\tau_{opt}(\omega)}$$



Fermi-liquid

Optical signature: scaling collapse



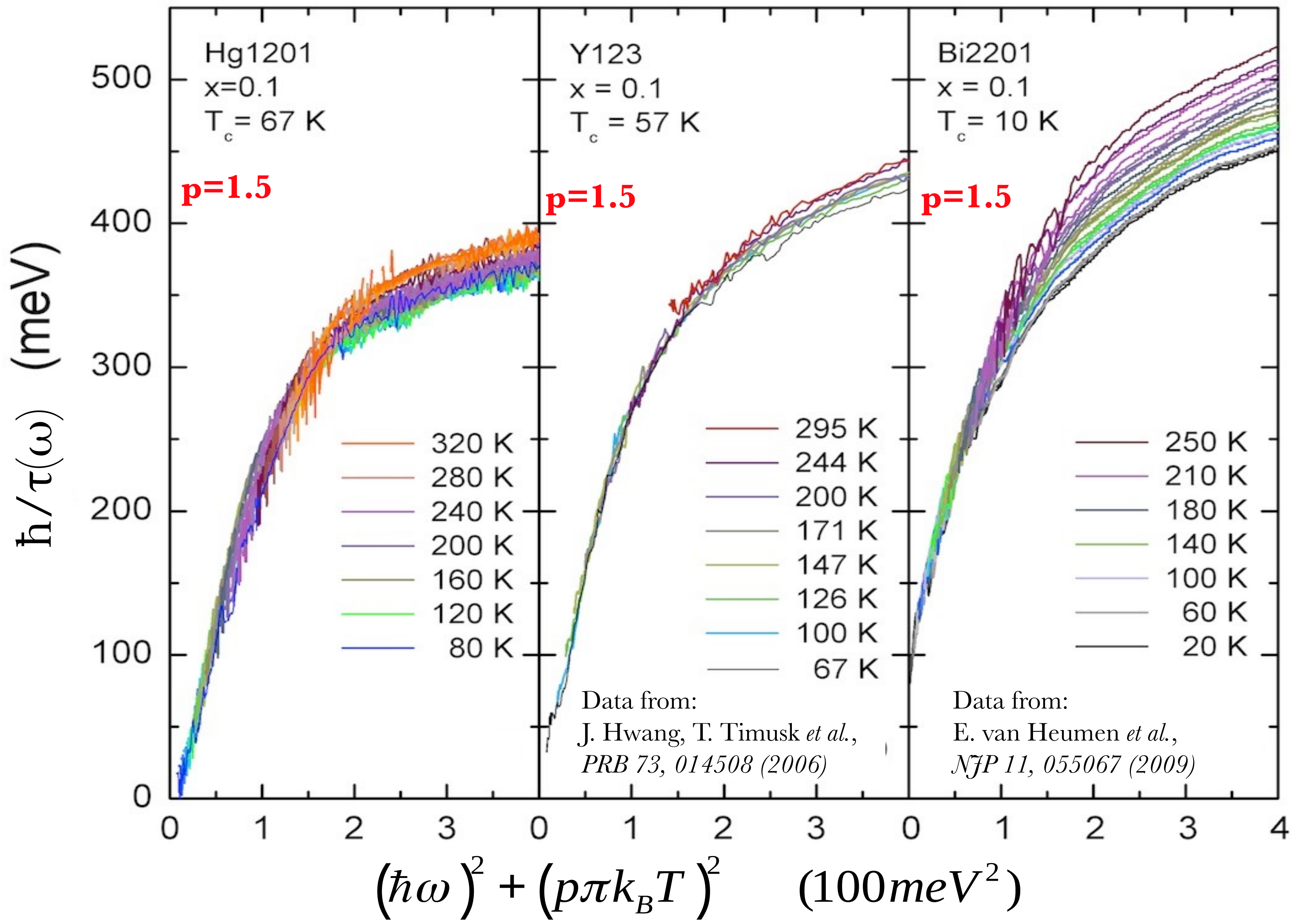
$$\frac{1}{\tau_{opt}} = \frac{1}{\tau\left(T\right)} + A\omega^{\eta}$$

η = 2

$$\frac{1}{\tau_{opt}} \propto \left(\hbar\omega\right)^2 + \left(\mathbf{p}\pi k_B T\right)^2$$

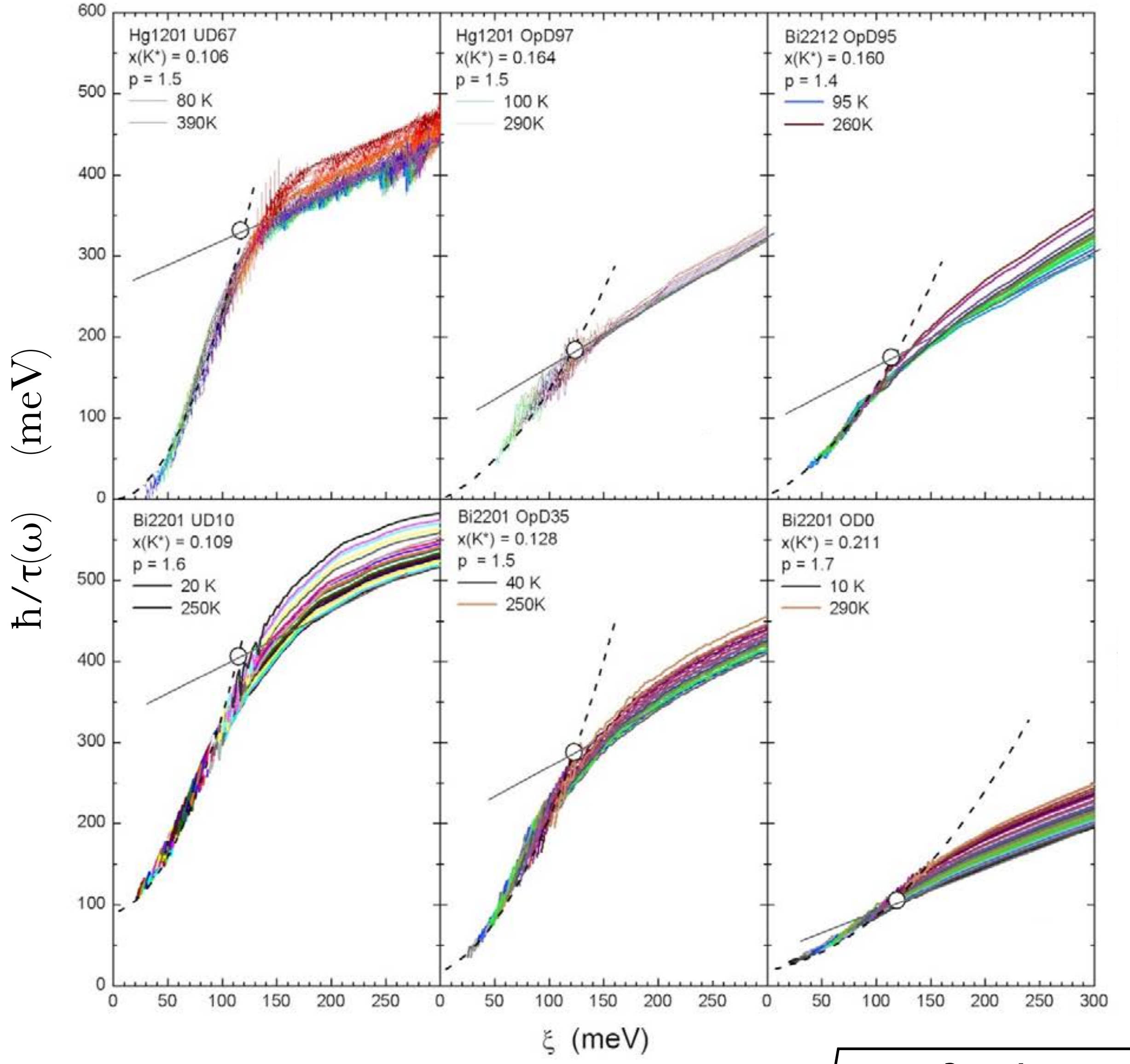
p = 1.5

PNAS ***110***, *5774 (2013)*

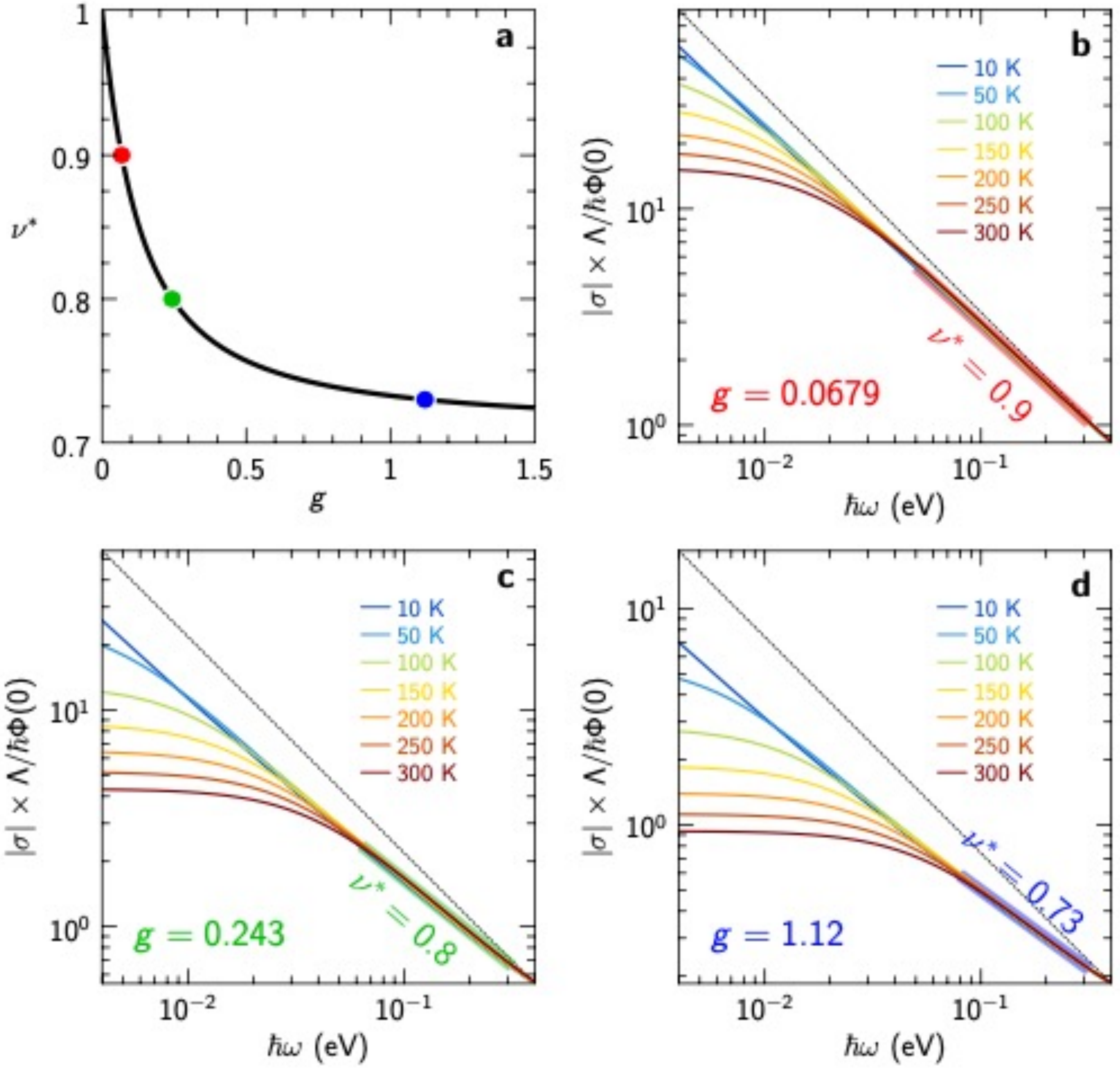


PNAS ***110***, *5774 (2013)*

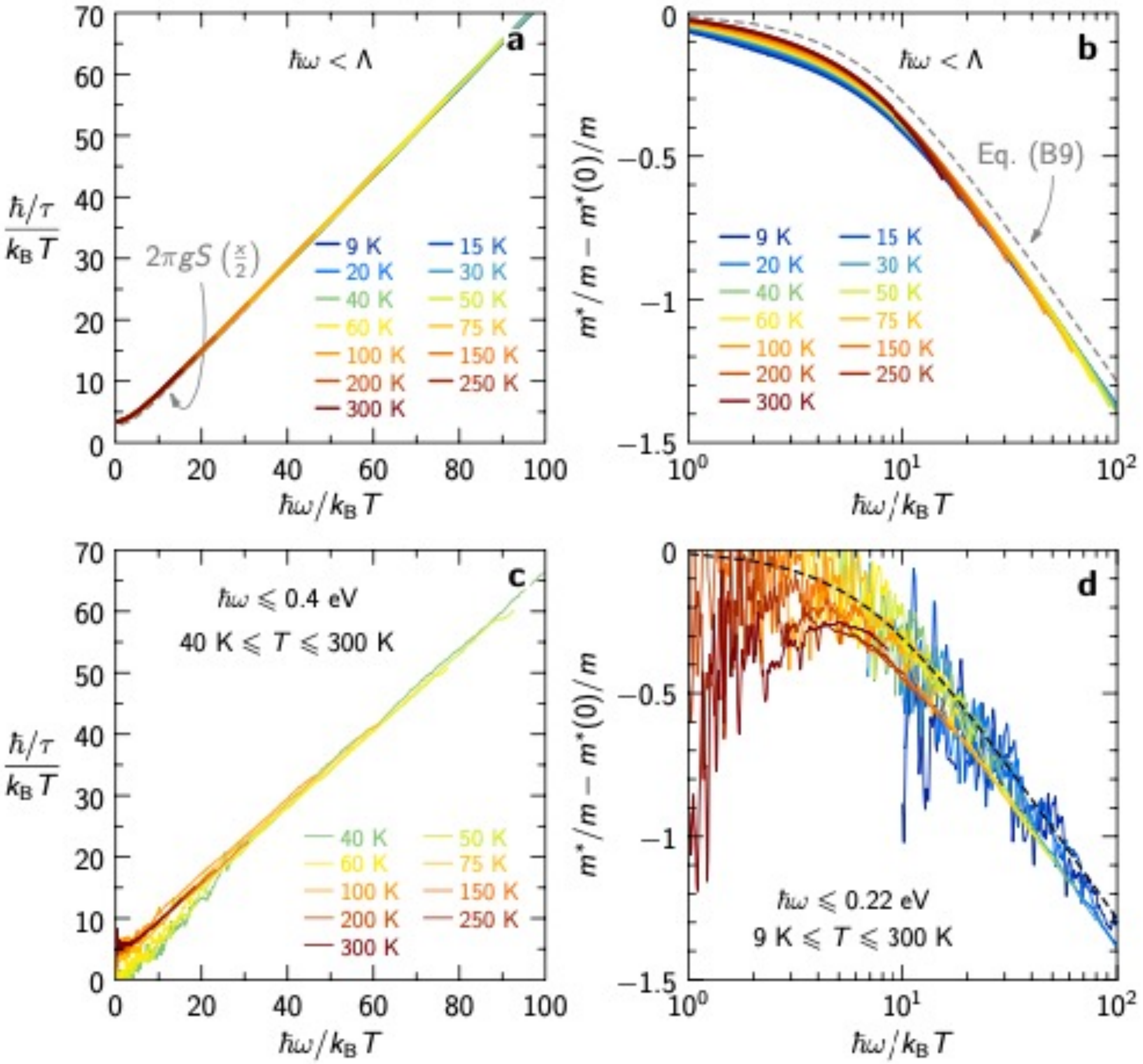
Doping dependence of the scaling collapse



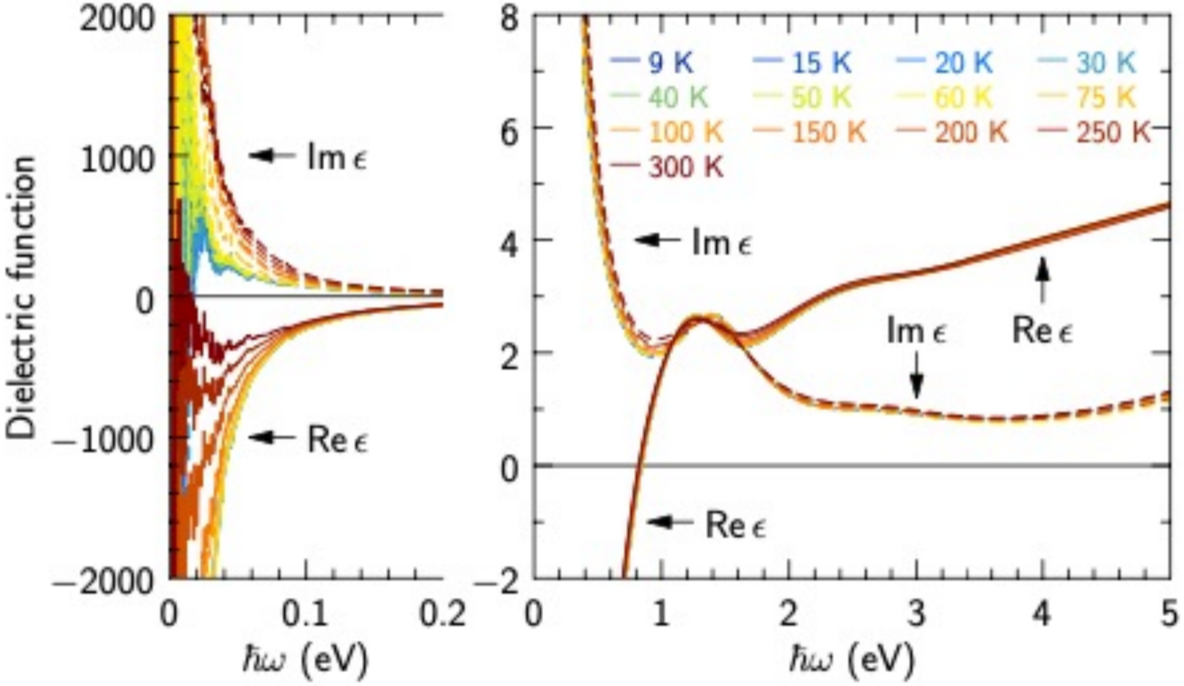
$$\xi \equiv \sqrt{\left(\hbar\omega\right)^2 + \left(p\pi k_B T\right)^2}$$



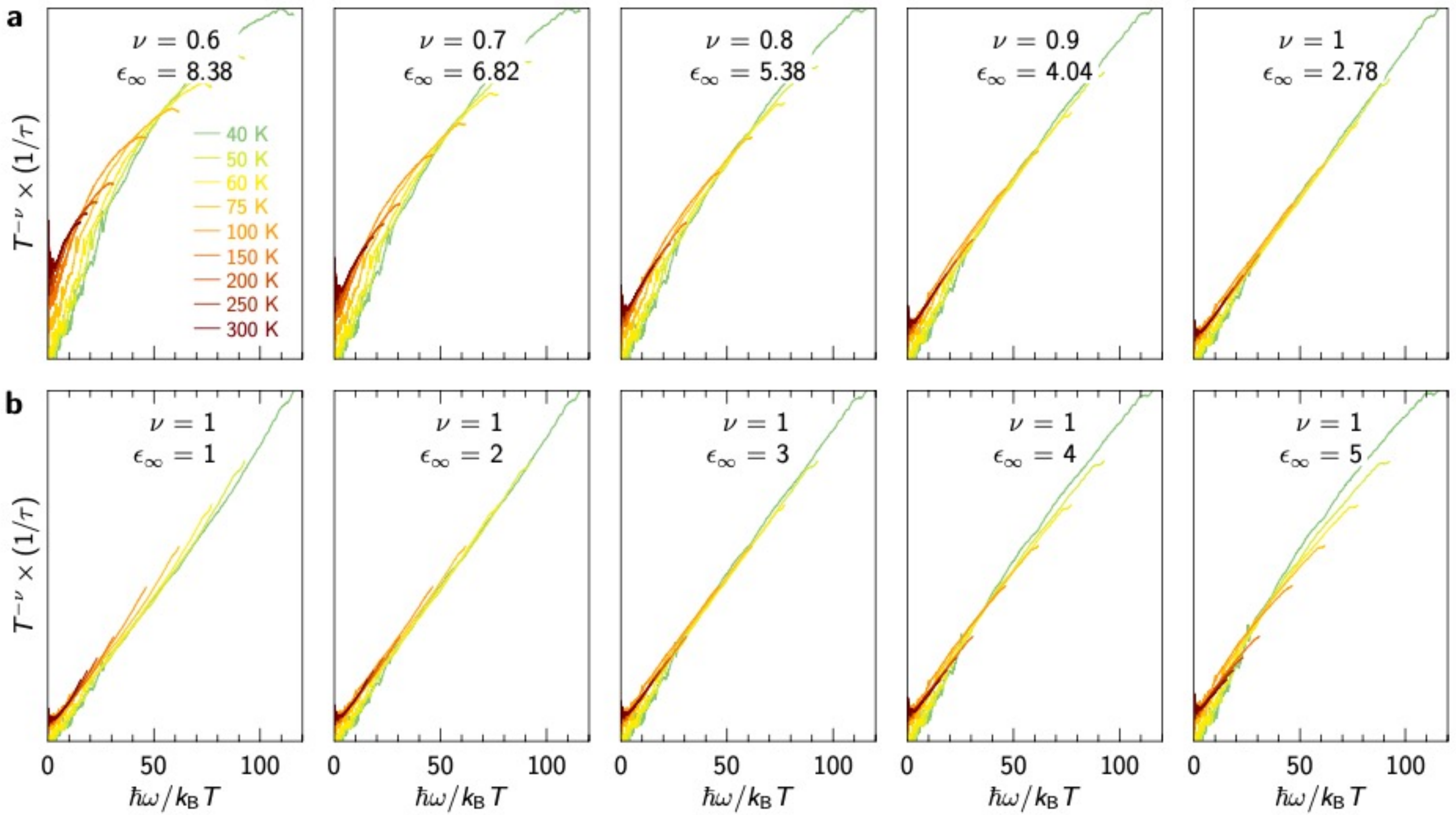
a
b
c
d
ν^*
g
$\hbar\omega$ (eV)
$|\sigma| \times \Lambda/\hbar\Phi(0)$
10 K
50 K
100 K
150 K
200 K
250 K
300 K
$g = 0.0679$
$\nu^* = 0.9$
$g = 0.243$
$\nu^* = 0.8$
$g = 1.12$
$\nu^* = 0.73$

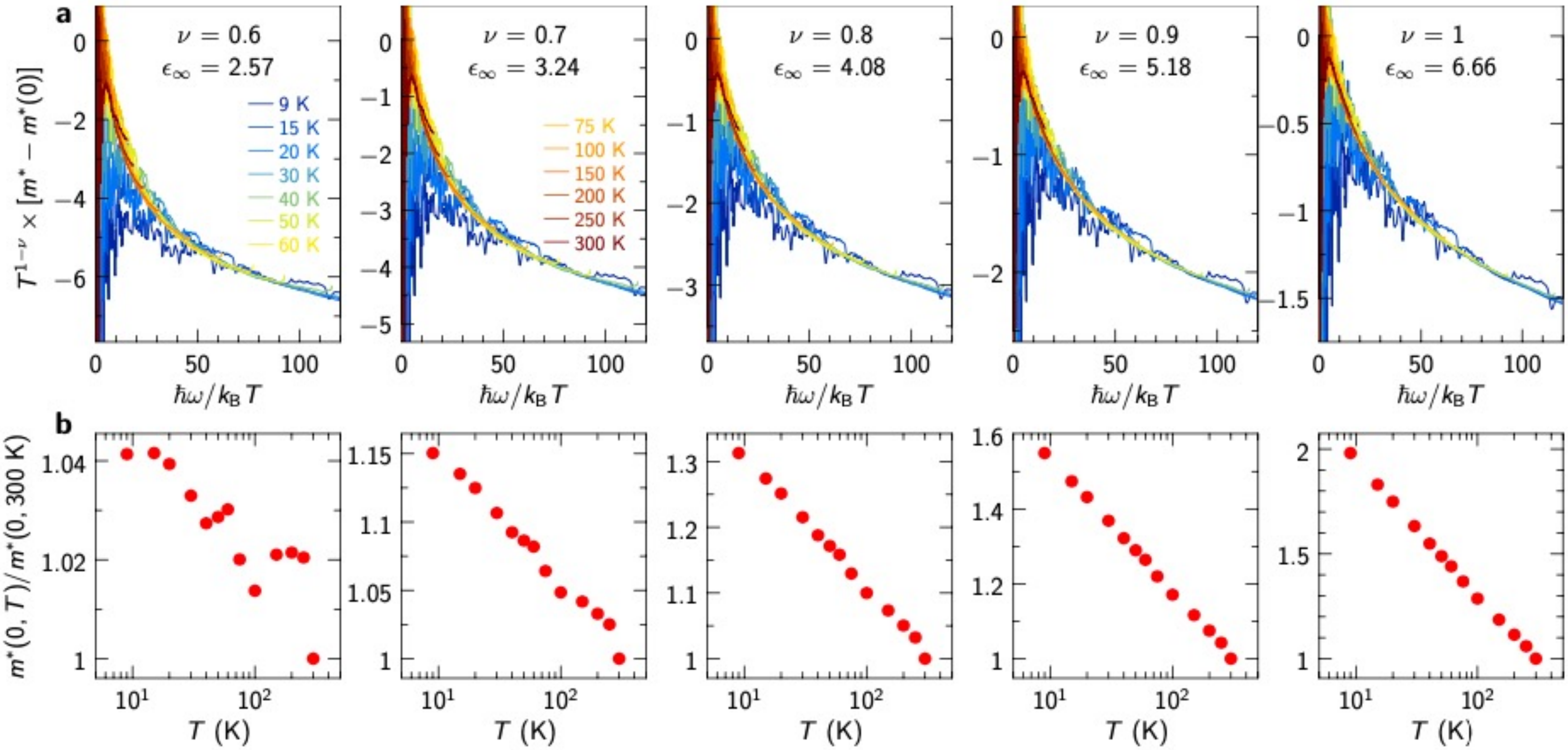


a
$\hbar\omega < \Lambda$
$2\pi g S\left(\frac{x}{2}\right)$
9 K
15 K
20 K
30 K
40 K
50 K
60 K
75 K
100 K
150 K
200 K
250 K
300 K
$\frac{\hbar/\tau}{k_B T}$
$\hbar\omega/k_B T$
b
$\hbar\omega < \Lambda$
Eq. (B9)
$m^*/m - m^*(0)/m$
c
$\hbar\omega \leqslant 0.4$ eV
40 K $\leqslant T \leqslant$ 300 K
d
$\hbar\omega \leqslant 0.22$ eV
9 K $\leqslant T \leqslant$ 300 K

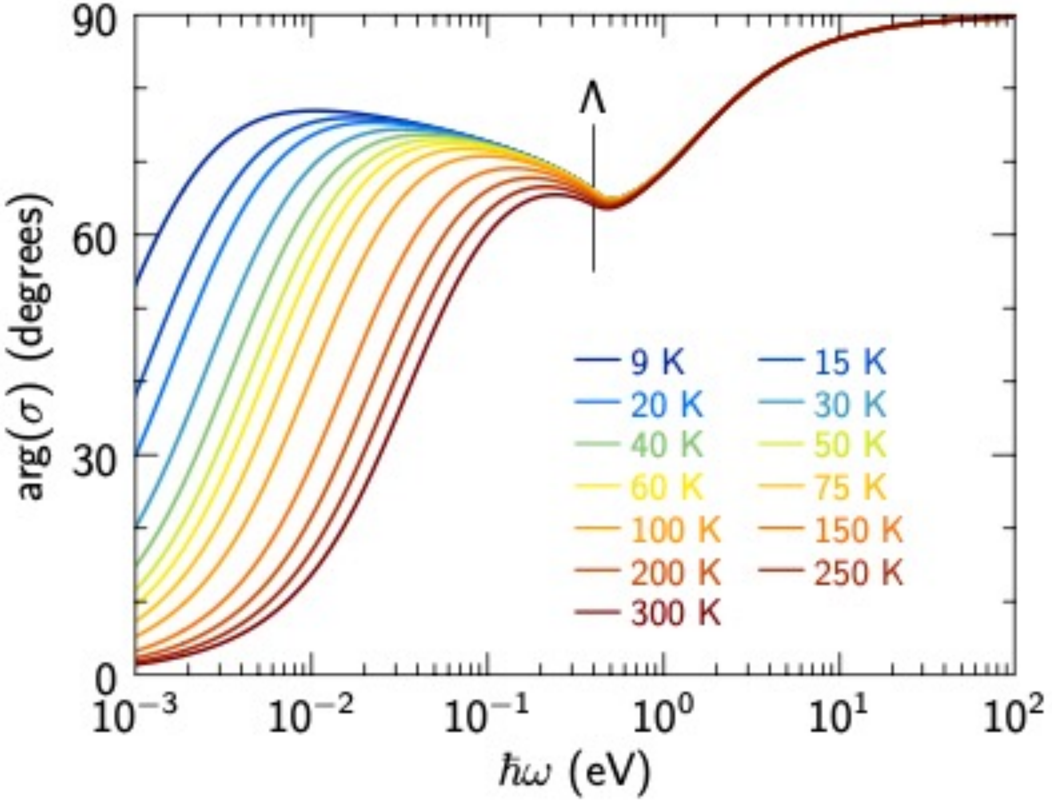


Dielectric function
2000
1000
0
−1000
−2000
Im ϵ
Re ϵ
0
0.1
0.2
ħω (eV)
8
6
4
2
0
−2
9 K
15 K
20 K
30 K
40 K
50 K
60 K
75 K
100 K
150 K
200 K
250 K
300 K
Im ϵ
Im ϵ
Re ϵ
Re ϵ
0
1
2
3
4
5
ħω (eV)

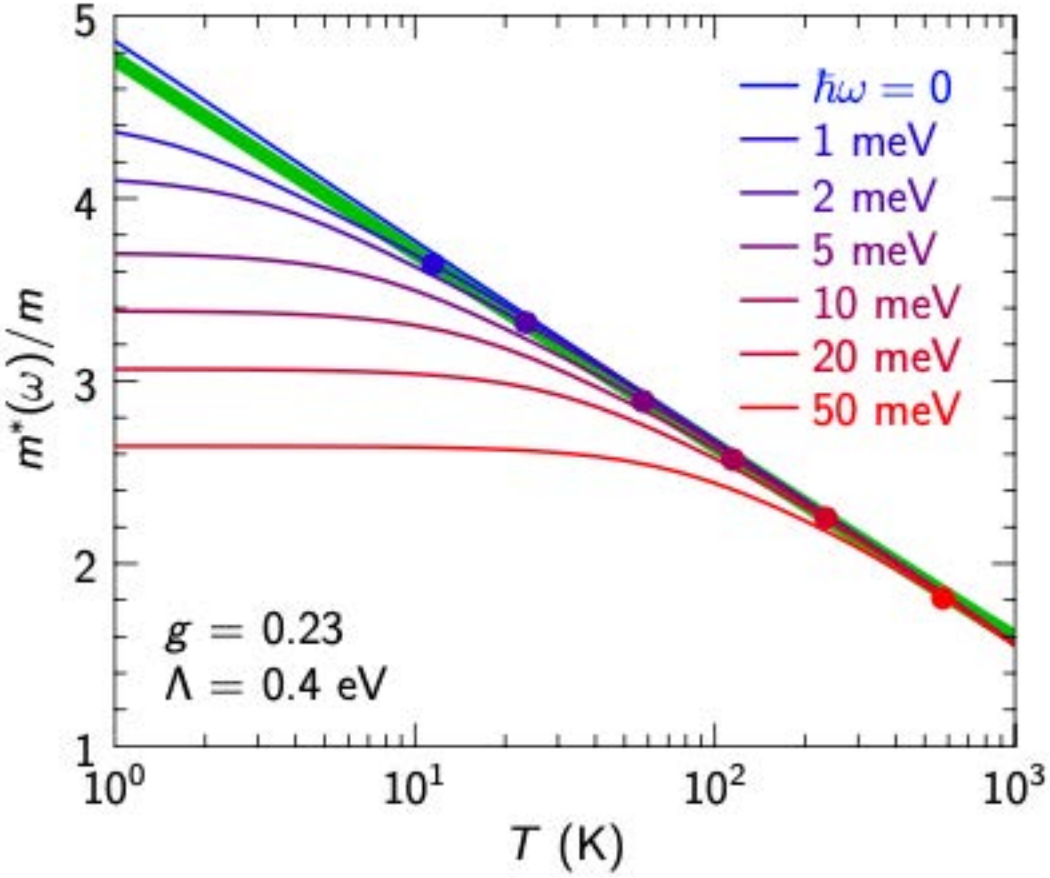

a
$\nu = 0.6$
$\epsilon_\infty = 8.38$
$\nu = 0.7$
$\epsilon_\infty = 6.82$
$\nu = 0.8$
$\epsilon_\infty = 5.38$
$\nu = 0.9$
$\epsilon_\infty = 4.04$
$\nu = 1$
$\epsilon_\infty = 2.78$
40 K
50 K
60 K
75 K
100 K
150 K
200 K
250 K
300 K
$T^{-\nu} \times (1/\tau)$
b
$\nu = 1$
$\epsilon_\infty = 1$
$\nu = 1$
$\epsilon_\infty = 2$
$\nu = 1$
$\epsilon_\infty = 3$
$\nu = 1$
$\epsilon_\infty = 4$
$\nu = 1$
$\epsilon_\infty = 5$
$T^{-\nu} \times (1/\tau)$
0
50
100
$\hbar\omega / k_B T$



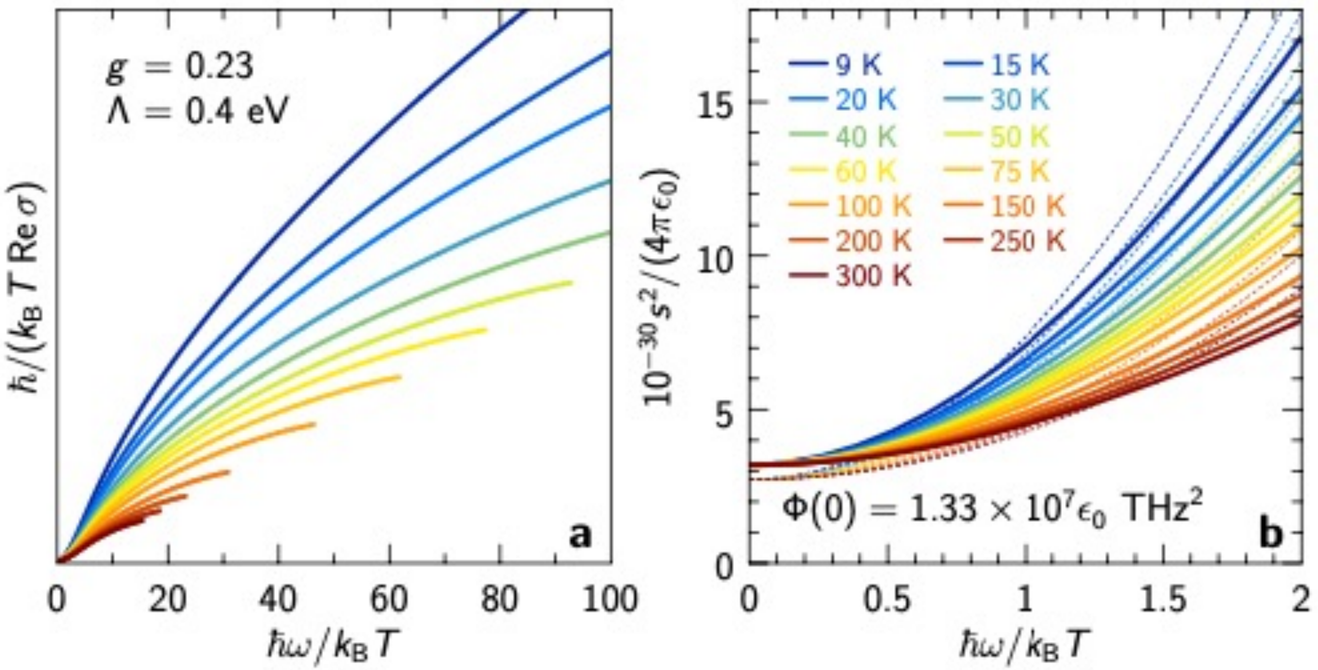
a
b
$\nu = 0.6$
$\epsilon_\infty = 2.57$
$\nu = 0.7$
$\epsilon_\infty = 3.24$
$\nu = 0.8$
$\epsilon_\infty = 4.08$
$\nu = 0.9$
$\epsilon_\infty = 5.18$
$\nu = 1$
$\epsilon_\infty = 6.66$
9 K
15 K
20 K
30 K
40 K
50 K
60 K
75 K
100 K
150 K
200 K
250 K
300 K
$T^{1-\nu} \times [m^* - m^*(0)]$
$\hbar\omega / k_B T$
$m^*(0, T)/m^*(0, 300\ K)$
T (K)



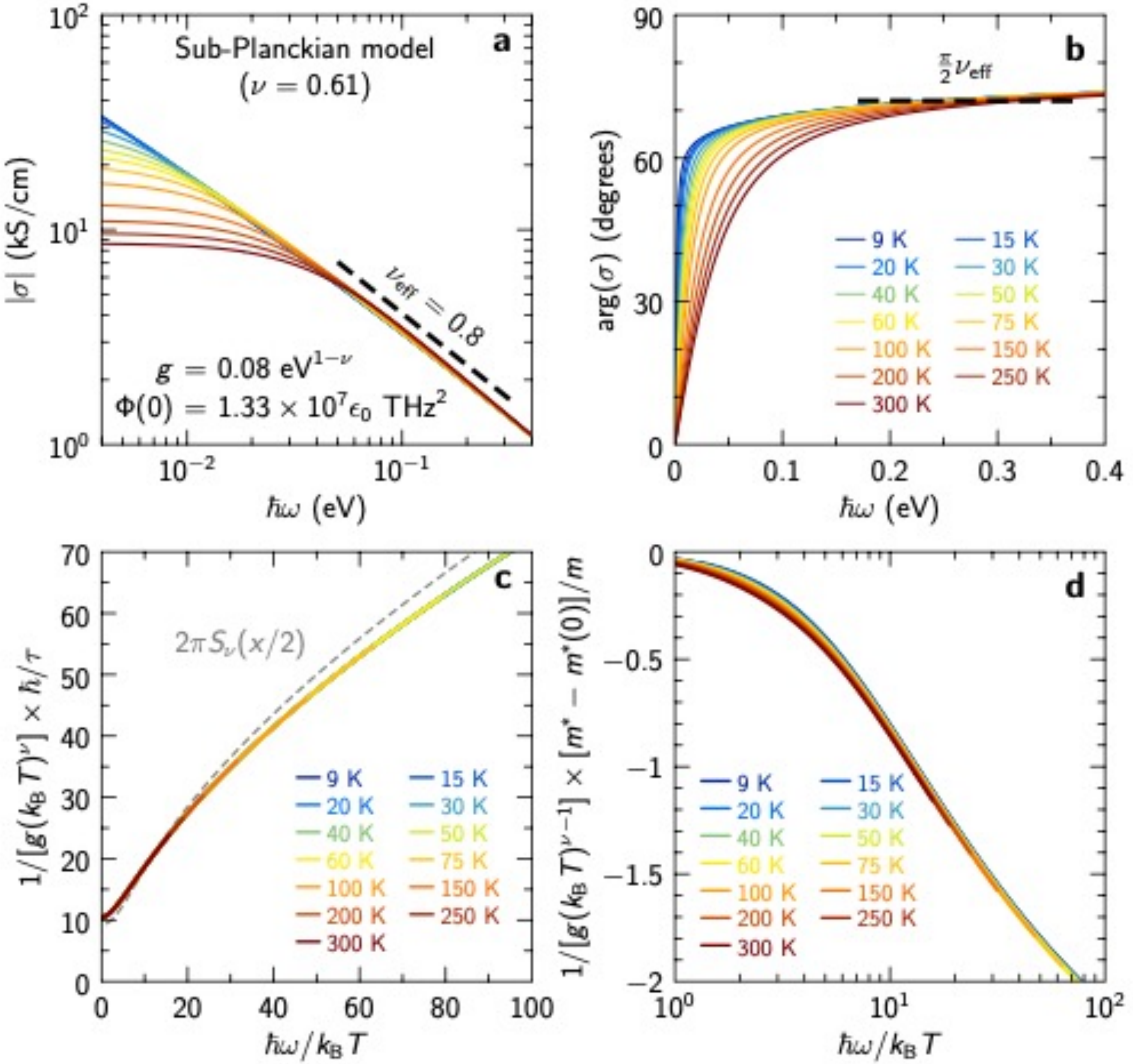
90
60
30
0
arg(σ) (degrees)
Λ
9 K
15 K
20 K
30 K
40 K
50 K
60 K
75 K
100 K
150 K
200 K
250 K
300 K
10−3
10−2
10−1
100
101
102
ℏω (eV)



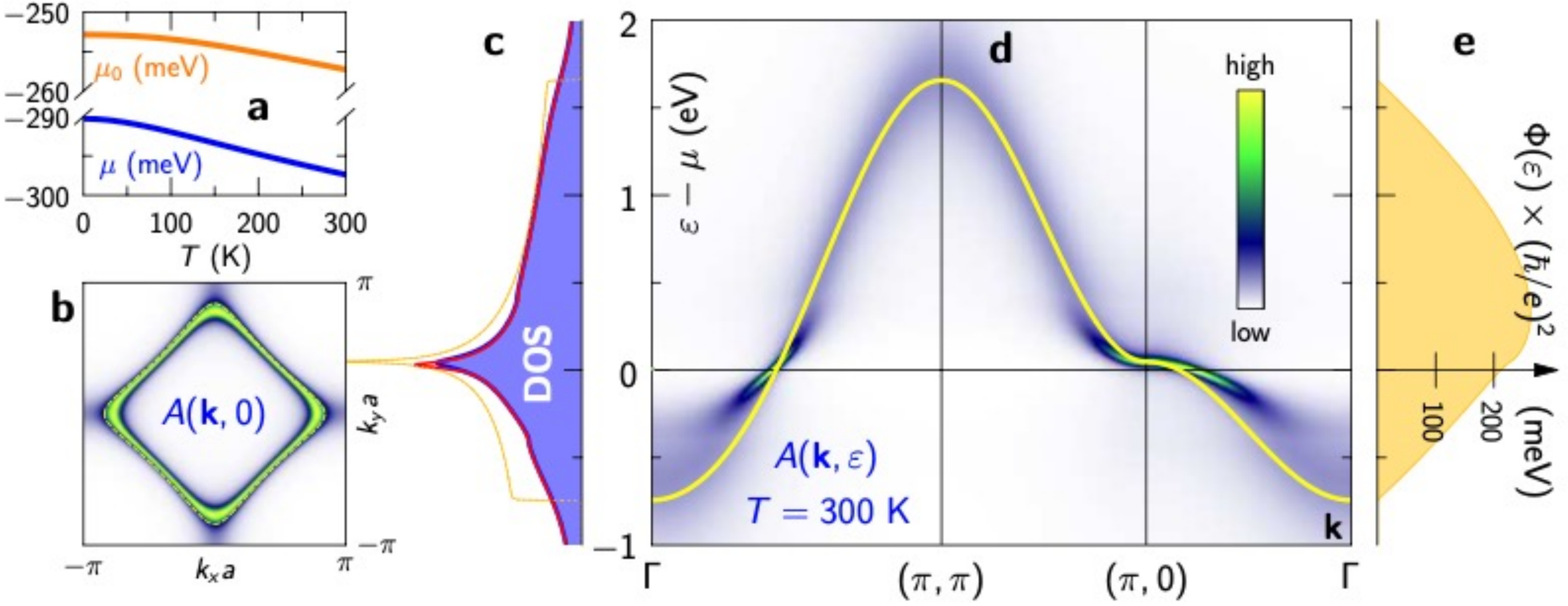
$\hbar\omega = 0$
1 meV
2 meV
5 meV
10 meV
20 meV
50 meV
$g = 0.23$
$\Lambda = 0.4$ eV
$m^*(\omega)/m$
T (K)
1
2
3
4
5
10^0
10^1
10^2
10^3



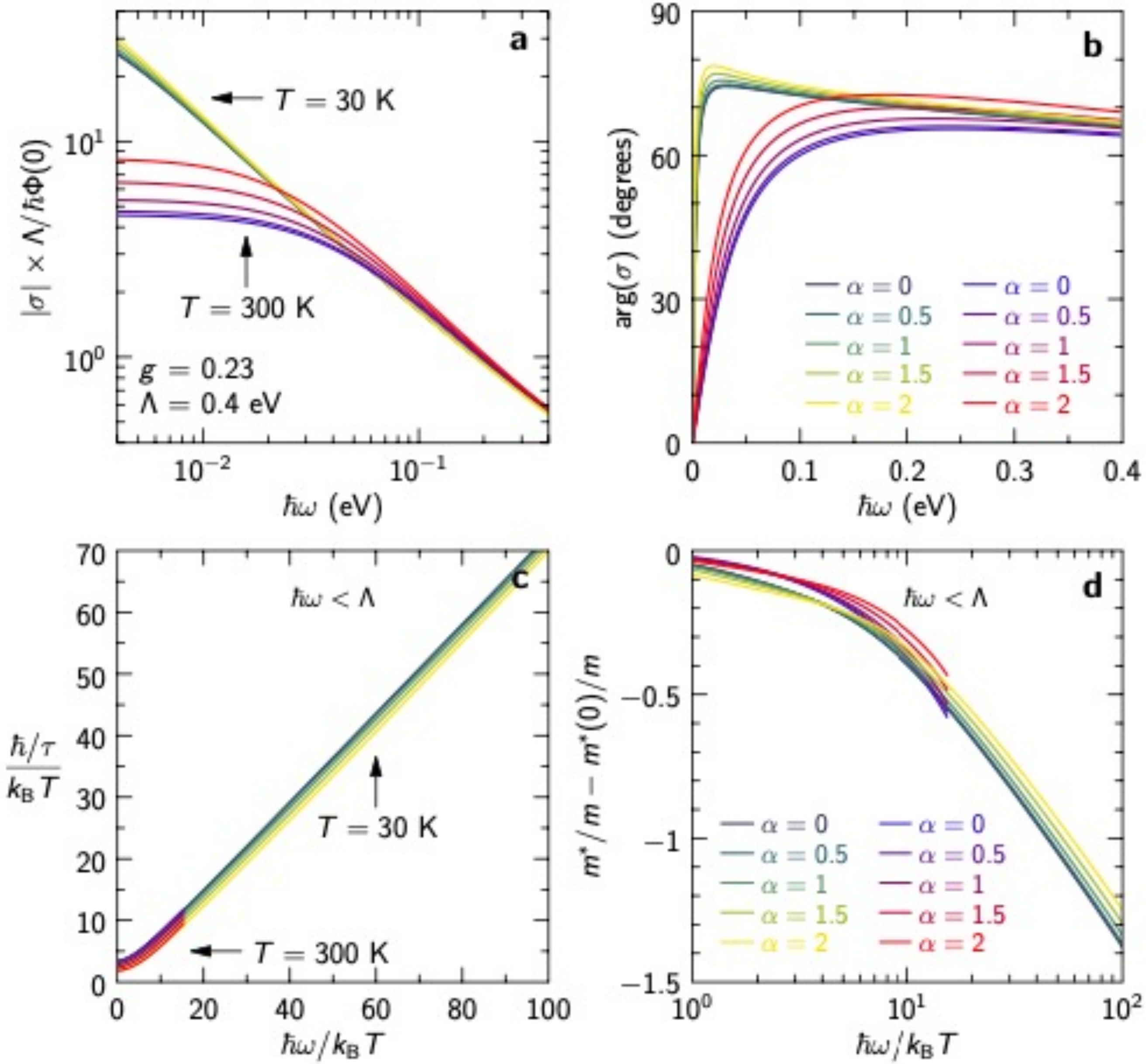
g = 0.23
Λ = 0.4 eV
a
ħ/(k_B T Re σ)
ħω/k_B T
0
20
40
60
80
100
9 K
15 K
20 K
30 K
40 K
50 K
60 K
75 K
100 K
150 K
200 K
250 K
300 K
Φ(0) = 1.33 × 10^7 ε_0 THz^2
b
10^−30 s^2/(4πε_0)
0
5
10
15
0
0.5
1
1.5
2
ħω/k_B T



Sub-Planckian model
($\nu = 0.61$)
a
$\nu_{\text{eff}} = 0.8$
$g = 0.08$ eV$^{1-\nu}$
$\Phi(0) = 1.33 \times 10^7 \epsilon_0$ THz^2
$|\sigma|$ (kS/cm)
$\hbar\omega$ (eV)
b
$\frac{\pi}{2}\nu_{\text{eff}}$
arg(σ) (degrees)
$\hbar\omega$ (eV)
9 K
15 K
20 K
30 K
40 K
50 K
60 K
75 K
100 K
150 K
200 K
250 K
300 K
c
$2\pi S_\nu(x/2)$
$1/[g(k_B T)^\nu] \times \hbar/\tau$
$\hbar\omega/k_B T$
d
$1/[g(k_B T)^{\nu-1}] \times [m^* - m^*(0)]/m$
$\hbar\omega/k_B T$



a
−250
−260
−290
−300
μ_0 (meV)
μ (meV)
0
100
200
300
T (K)
b
$A(\mathbf{k}, 0)$
$-\pi$
π
$k_x a$
$k_y a$
c
DOS
d
2
1
0
−1
$\varepsilon - \mu$ (eV)
high
low
$A(\mathbf{k}, \varepsilon)$
$T = 300$ K
k
Γ
(π, π)
$(\pi, 0)$
Γ
e
$\Phi(\varepsilon) \times (\hbar/e)^2$
100
200
(meV)



a
$T = 30$ K
$T = 300$ K
$g = 0.23$
$\Lambda = 0.4$ eV
$|\sigma| \times \Lambda/\hbar\Phi(0)$
$\hbar\omega$ (eV)
b
$\arg(\sigma)$ (degrees)
$\hbar\omega$ (eV)
$\alpha = 0$
$\alpha = 0.5$
$\alpha = 1$
$\alpha = 1.5$
$\alpha = 2$
$\alpha = 0$
$\alpha = 0.5$
$\alpha = 1$
$\alpha = 1.5$
$\alpha = 2$
c
$\hbar\omega < \Lambda$
$T = 30$ K
$T = 300$ K
$\frac{\hbar/\tau}{k_B T}$
$\hbar\omega/k_B T$
d
$\hbar\omega < \Lambda$
$m^*/m - m^*(0)/m$
$\hbar\omega/k_B T$
$\alpha = 0$
$\alpha = 0.5$
$\alpha = 1$
$\alpha = 1.5$
$\alpha = 2$
$\alpha = 0$
$\alpha = 0.5$
$\alpha = 1$
$\alpha = 1.5$
$\alpha = 2$