

# Microscopic Dynamics of Thermoelectric Materials

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**Devashibhai Adroja; Tomasz Cichorek; Naoya Takeda**

ISIS, RAL, UK; Niigata University, Japan; Polish Academy of Sciences, Wroclaw, Poland

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**Andriy Grytsiv, Peter Rogl**

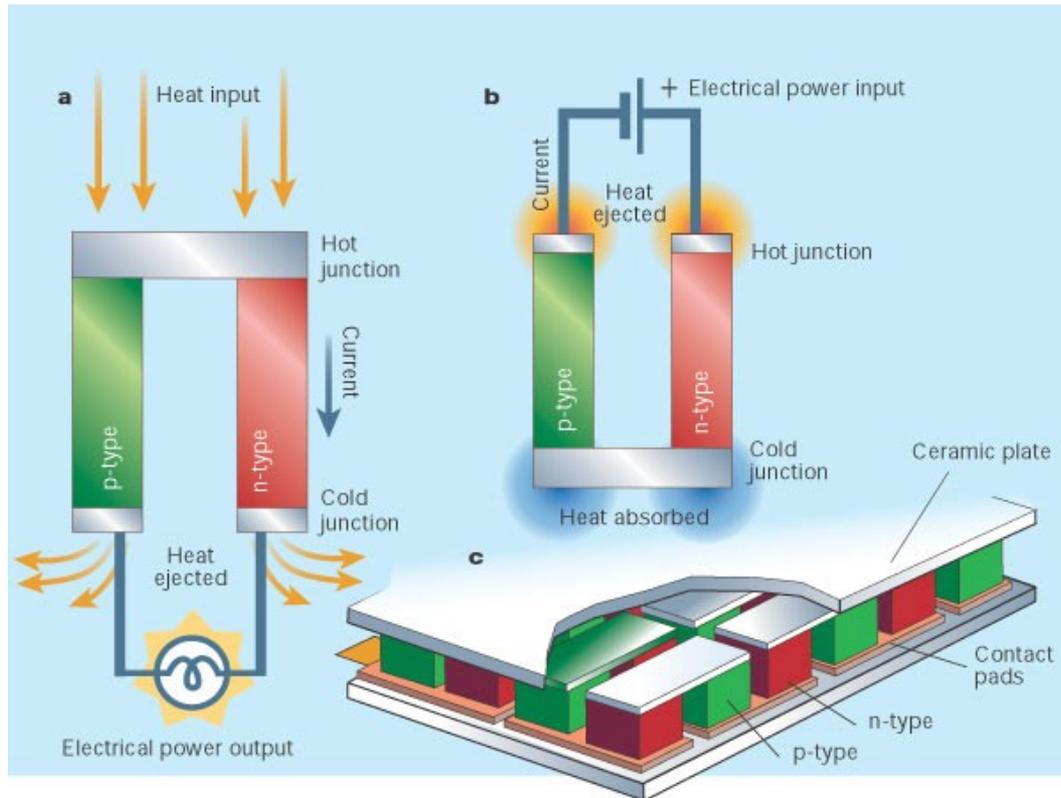
Institut of Physical Chemistry, University of Vienna, Austria

**Zenji Hiroi**

University of Tokyo, Japan

- Introduction into the Phonon Glass Electron Crystal concept and rattling modes idea
- The merit of inelastic and elastic neutron scattering and the approach to the study of polycrystalline matter
- Results for filled skutterudite compounds
- Comparison with other open-framework compounds
- Summary and Outlook

Cronin B. Vining, *Nature* 413, 577, (2001)



- a) Conversion of thermal to electrical energy (Seebeck effect)
- b) Cooling (pumping heat) by application of electrical power (Peltier effect)
- c) Thermoelectric module

Thermoelectric materials are efficient converters of thermal to electrical energy !

Figure of merit :

$$Z = S^2 a / k \quad \text{with} \quad k = k_{\text{electr.}} + k_{\text{lattice}}$$

$S$  = Seebeck coefficient ( $dV/dT$ ),  $a$  = electrical conductivity,  $k$  = thermal conductivity

Cronin B. Vining, *Nature* 413, 577, (2001)

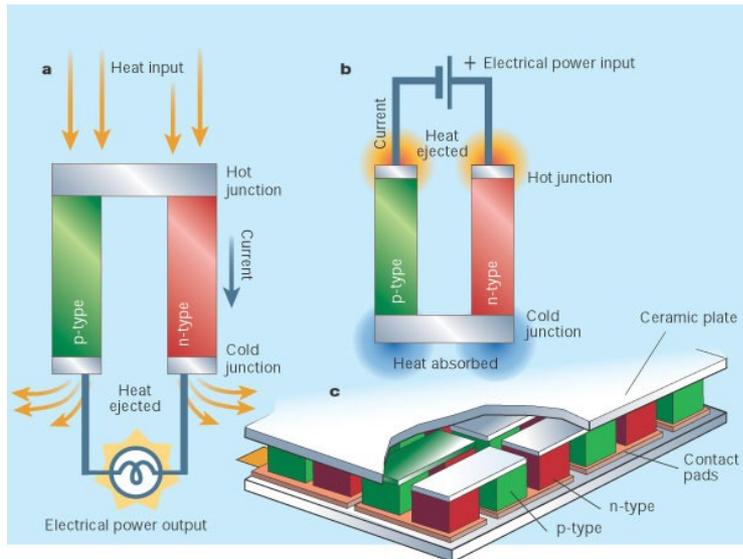
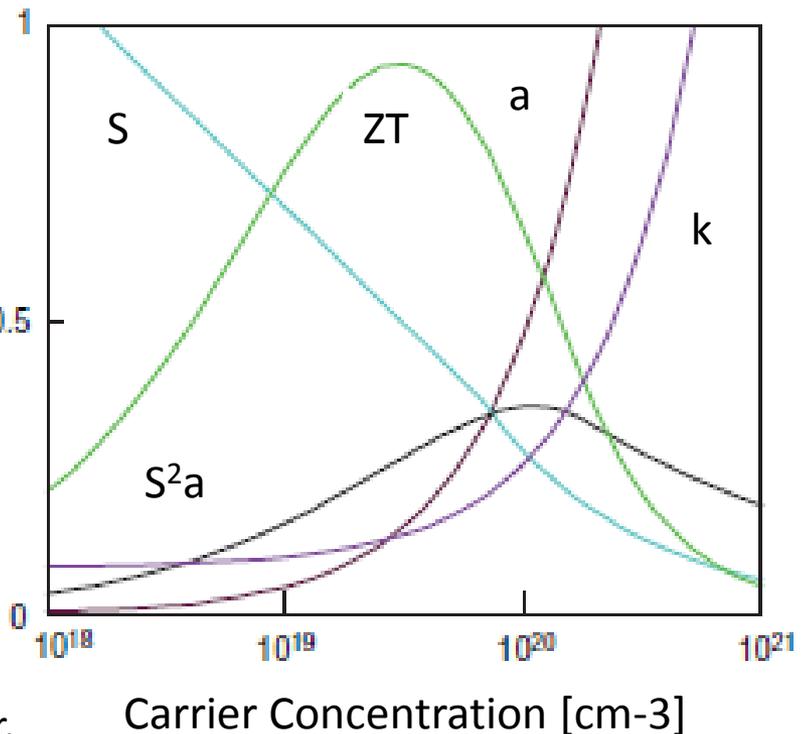


Figure of merit :  $Z = S^2a/k$

Thermal conductivity :

$$k = k_{\text{electr.}} + k_{\text{lattice}}$$

Dimensionless FoM :  $ZT$



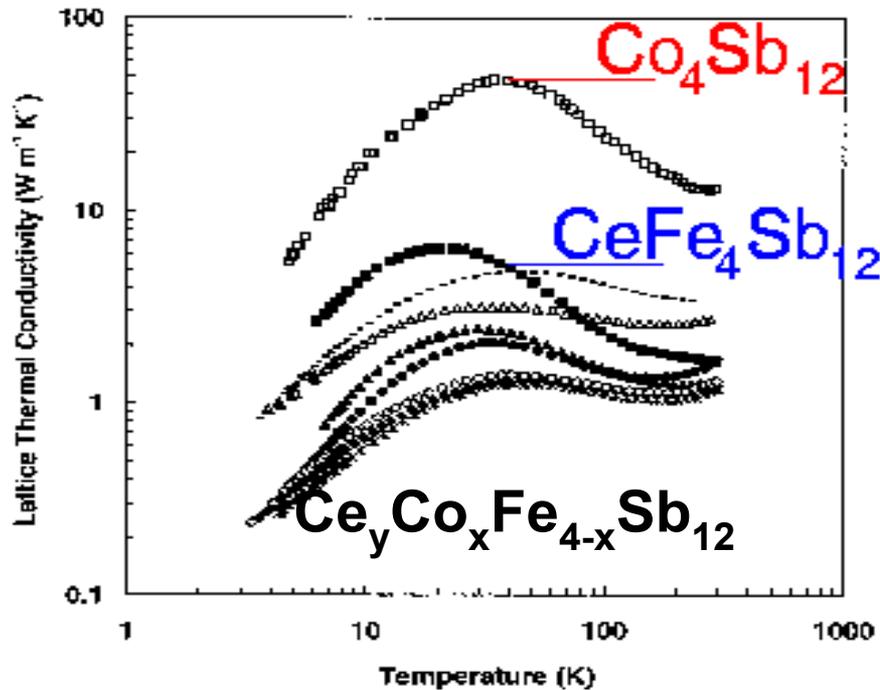
$S^2a$  and  $k_{\text{electr.}}$  limited by  $n \sim 10^{19-20}$

**Working Hypothesis :**

Manipulate  $k_{\text{lattice}}$  without perturbing Bloch symmetry for electronic transport!

G.J. Snyder & E.S. Toberer, *Nature Materials* 7, 105, (2008)

Carrier Concentration  $[cm^{-3}]$



Lattice thermal conductivity of skutterudite compounds :

Ternary compound exhibits a lattice thermal conductivity suppressed by at least one order of magnitude. Effect must be mediated by the vibrational properties (phonons ?) of the crystals.

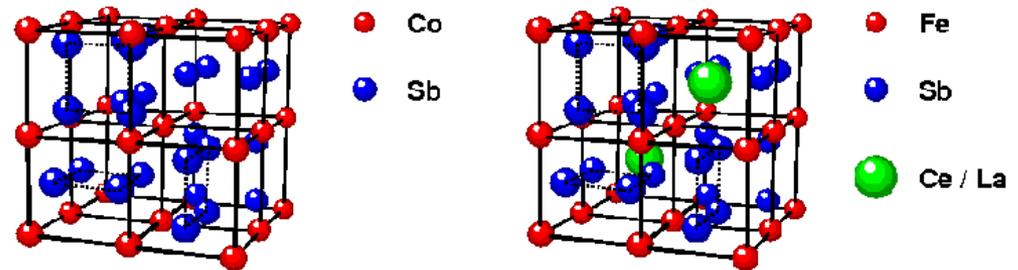
G.P. Meisner et al., *PRL* 80, 3551 (1998)

## MT<sub>4</sub>X<sub>12</sub>- Skutterudite

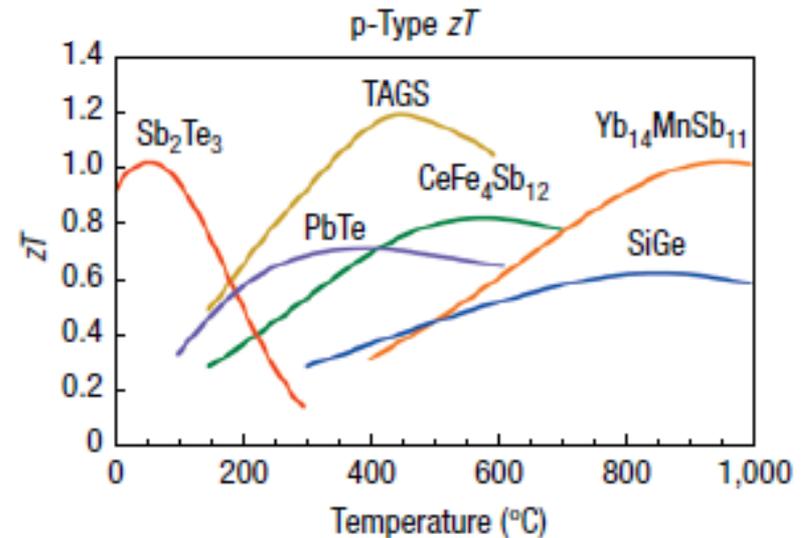
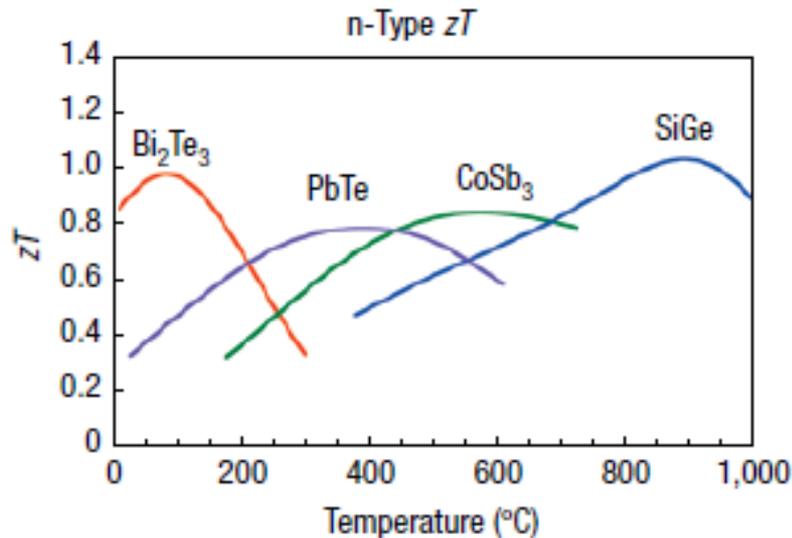
Space group: Im-3

Cubic structure with M on bcc-sites

$d = 9.2 \text{ \AA}$  for LaFe<sub>4</sub>Sb<sub>12</sub>



G.J. Snyder & E.S. Toberer, Nature Materials 7, 105 - 114 (2008)

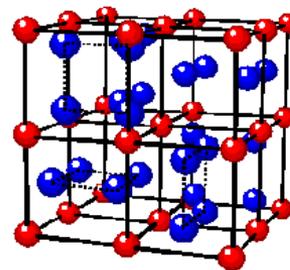


## $\text{MT}_4\text{X}_{12}$ - Skutterudite

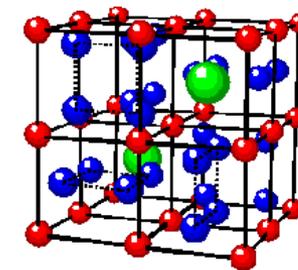
Space group:  $\text{Im}-3$

Cubic structure with M on bcc-sites

$d = 9.2 \text{ \AA}$  for  $\text{LaFe}_4\text{Sb}_{12}$



● Co  
● Sb



● Fe  
● Sb  
● Ce / La

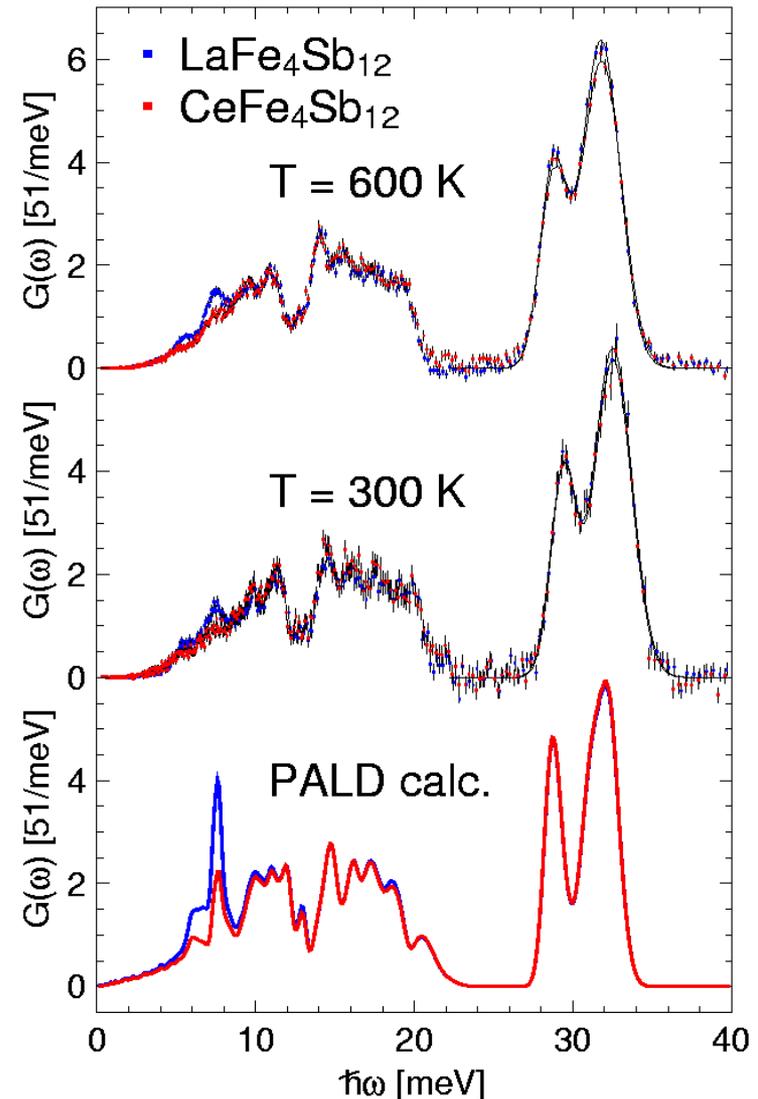
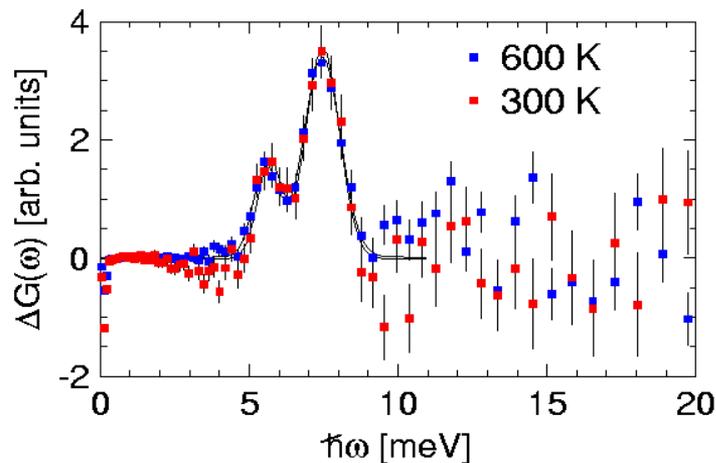
## INS contrast experiment at IN6@ILL

|    | S.C.S. | A.M.U. | S.P. |
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| La | 9.7    | 138.9  | 7.0  |
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S.C.S. = scattering cross section in barns

A.M.U. = atomic mass unit

Scattering Power S.P. = C.S./A.M.U.\*100



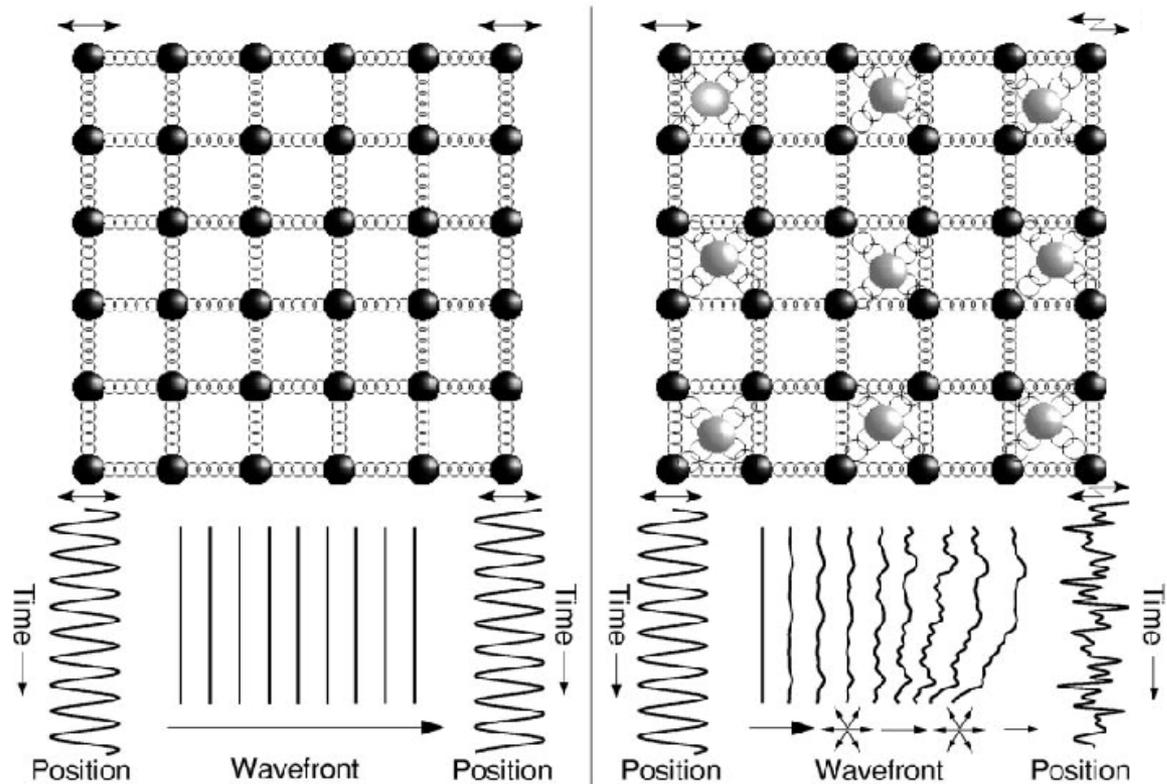
M.M. Koza et al., *Nature Materials* 2008

V. Keppens et al., *Nature* 1998, R.P. Hermann et al., *PRL* 2003, G.J. Long et al., *PRB* 2005

R.P. Hermann et al., *Am. J Phys.* **73**, 110, (2005)

From Literature :

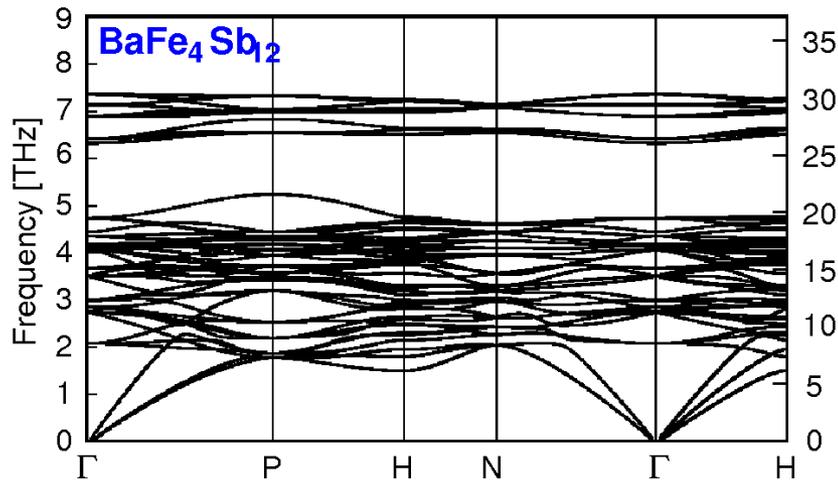
- Einstein mode
- Rattling mode
- Isolated oscillator
- Off-center motion
- Local vibrational mode
- Incoherent phonons
- Independent vibrations
- Decoupled dynamics
- ...



B.C. Sales et al., *PRB* **56**, 15081, (1997)

## Phonon Glass and Electron Crystal - PGEC !

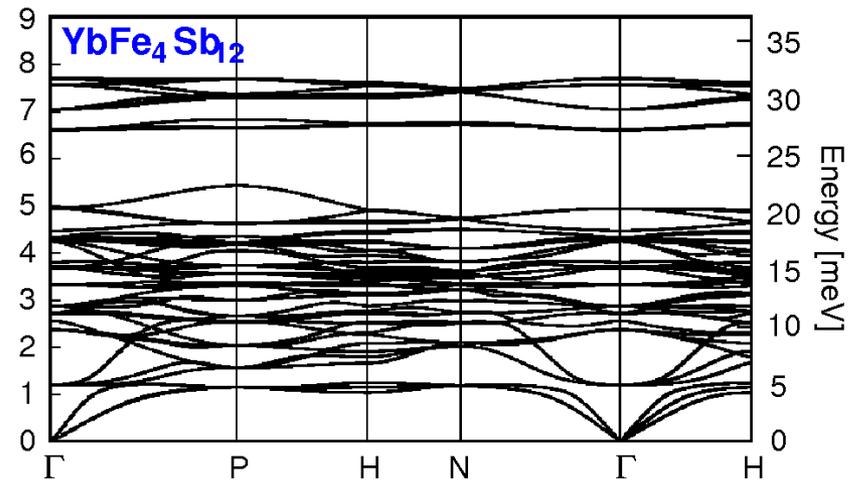
# Ab initio modelling of skutterudite compounds : Phonon dispersion from lattice dynamics calculations



“**Co<sub>4</sub>Sb<sub>12</sub>** and MFe<sub>4</sub>Sb<sub>12</sub>, M = Ca, Sr, Ba

Experimental work :

- “M. Rotter et al. *PRB* 77, 144301 (2009)
- \*R.P. Hermann et al., *PRL* 90, 135505 (2003)
- ^G.J. Long et al., *PRB* 71, 140302(R) (2005)

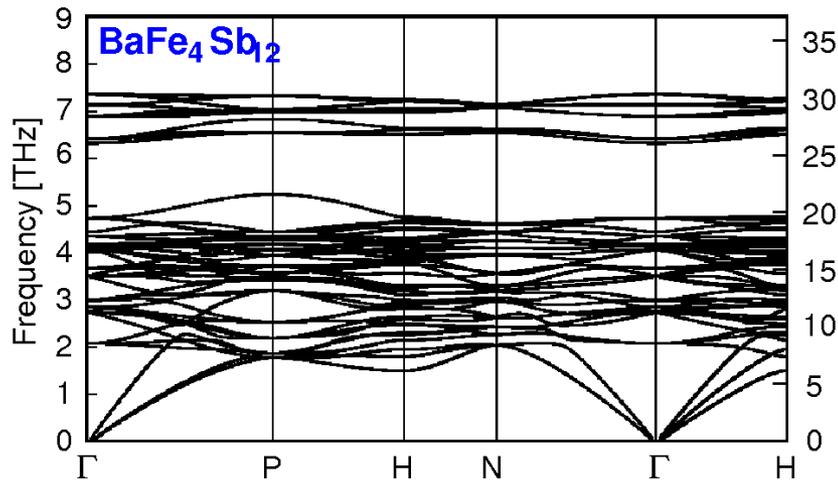


**MFe<sub>4</sub>Sb<sub>12</sub>**, M = **La, Ce**, ^Eu, Nd, Yb, \*Tl

LD calculation work :

- J.L Feldman et al. *PRB* 1996, 2000, 2003, 2006
- P. Ghosez et al., *JPCM* 19, 096002 (2007)
- A.L. Martinotto et al., *PRB* 81, 104112 (2010)

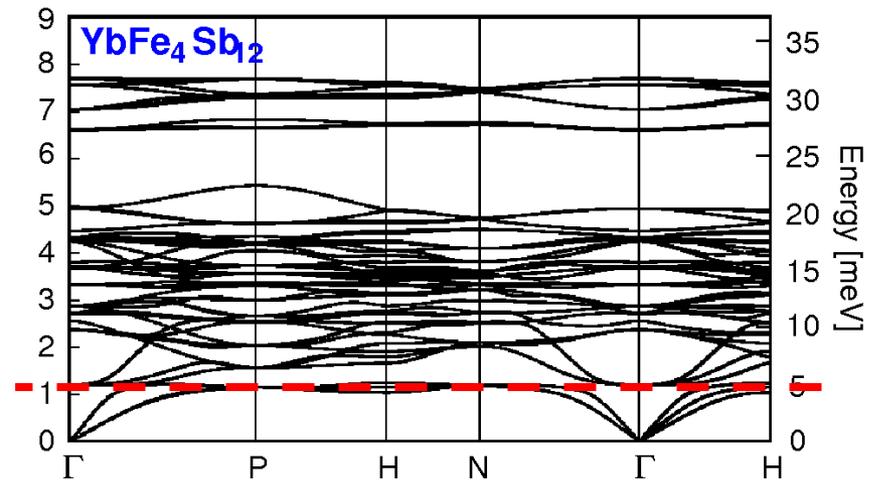
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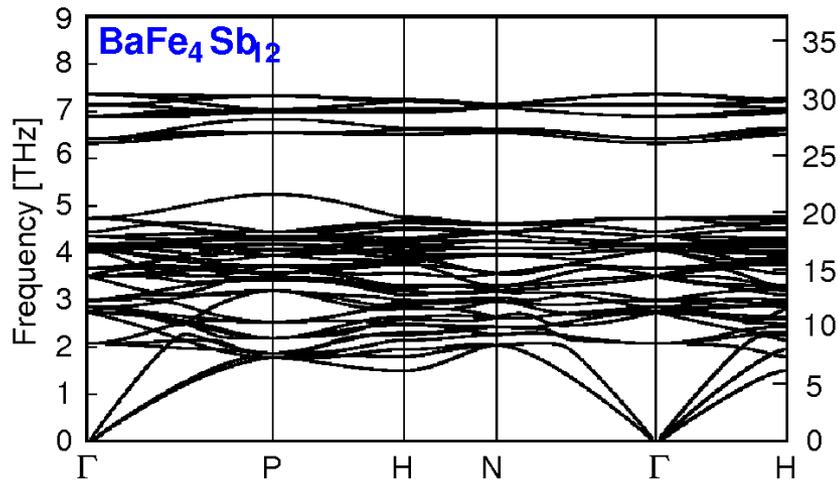
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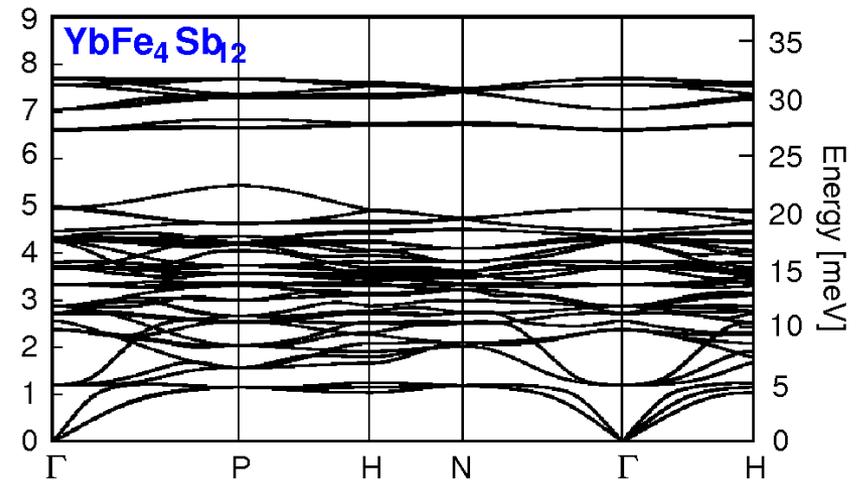
- J.L Feldman et al. *PRB* 1996, 2000, 2003, 2006
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**Rattling mode, Einstein mode, isolated oscillator, local vibrational mode, incoherent phonons, independent vibrations, decoupled dynamics, ...**

# Ab initio modelling of skutterudite compounds : Phonon dispersion from lattice dynamics calculations



“**Co<sub>4</sub>Sb<sub>12</sub>** and MFe<sub>4</sub>Sb<sub>12</sub>, M = Ca, Sr, Ba



**MFe<sub>4</sub>Sb<sub>12</sub>**, M = **La, Ce**, ^Eu, Nd, Yb, \*Tl

The Callaway Model :

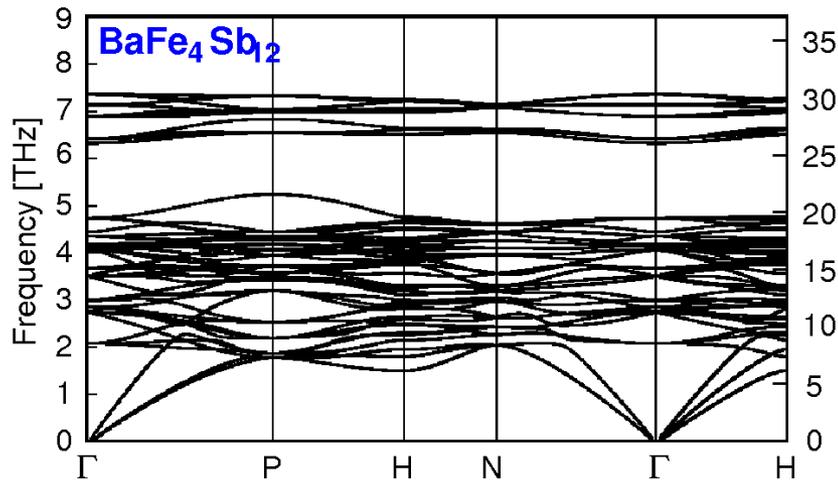
$$\kappa_{\text{lattice}} = 1/3 \cdot C \cdot v \cdot l$$

specific heat  $C$  - via density of states from dispersion relations

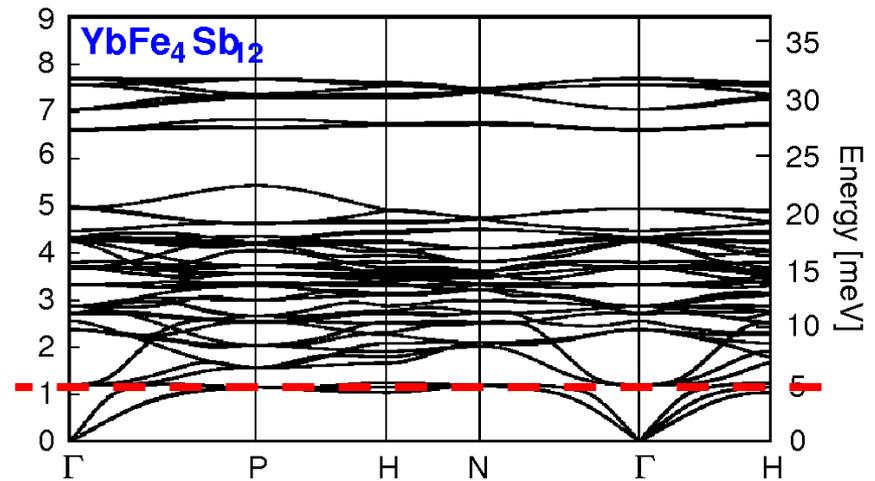
velocity of sound  $v$  - from dispersion relations

mean free path  $l$ , phonon life time  $\tau = l/v$  - **infinite !!!**

# Ab initio modelling of skutterudite compounds : Phonon dispersion from lattice dynamics calculations



“**Co<sub>4</sub>Sb<sub>12</sub>** and MFe<sub>4</sub>Sb<sub>12</sub>, M = Ca, Sr, Ba

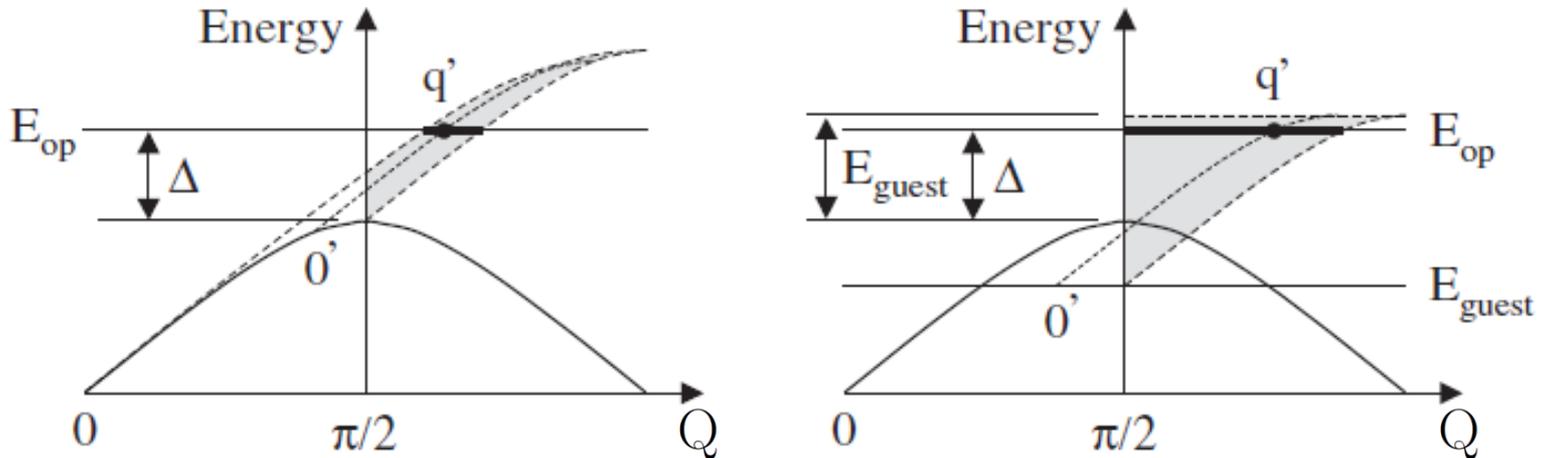


**MFe<sub>4</sub>Sb<sub>12</sub>**, M = **La, Ce**, ^Eu, Nd, Yb, \*Tl

A set of normal modes is sufficient to characterize the vibrational response, the associated thermodynamic and material properties.

Applies as well to the rattling mode, Einstein mode, isolated oscillator, local vibrational mode, incoherent phonons, independent vibrations, decoupled dynamics, ...

‘Conceptual diagram of phonon dispersion relations. ...’

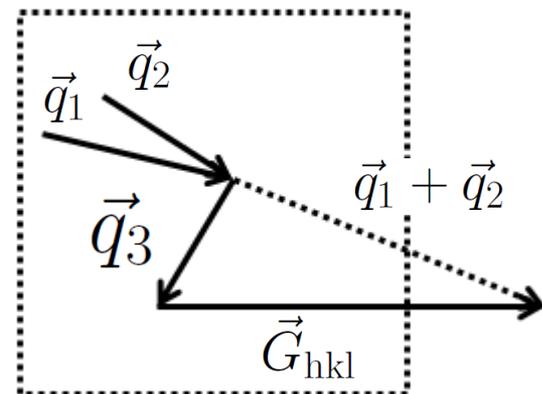


C.H. Lee et al., *J. Phys. Soc. Jpn.* 75, 123602, (2006)

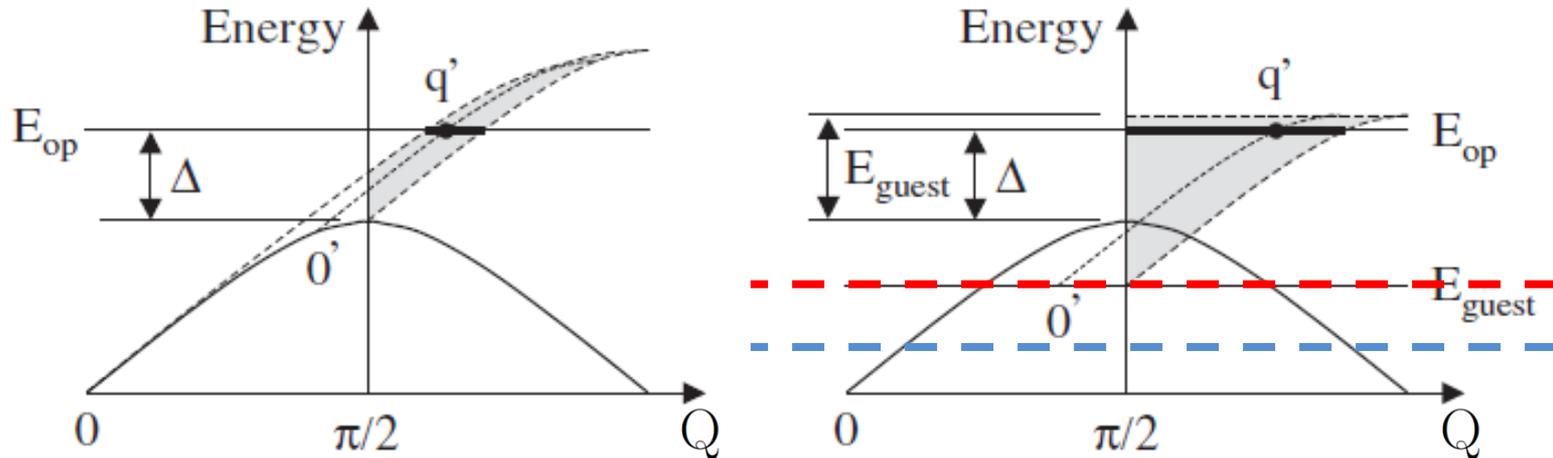
Umklapp scattering :

$$\hbar\omega_1 + \hbar\omega_2 = E_{\text{op}}$$

$$\vec{q}_1 + \vec{q}_2 = \vec{q}_3 + \vec{G}_{\text{hkl}}$$



‘Conceptual diagram of phonon dispersion relations. ...’



C.H. Lee et al., *J. Phys. Soc. Jpn.* 75, 123602, (2006)

Umklapp scattering :

$$\hbar\omega_1 + E_{\text{guest}} = E_{\text{op}}$$

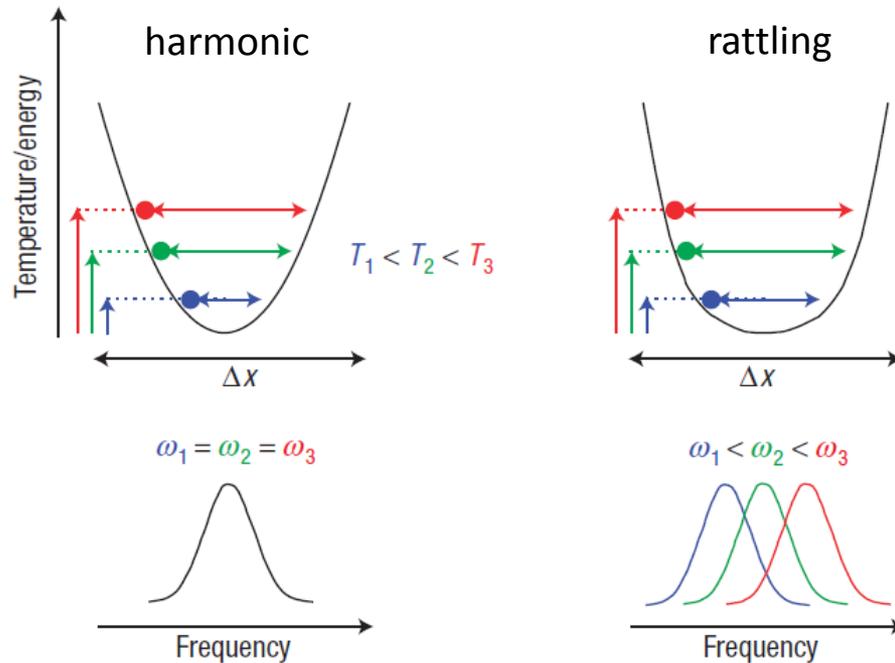
Condition to meet :

$$E_{\text{guest}} \geq \Delta$$

Puzzle to solve :

$$E_{\text{guest}} < \Delta$$

## Particle in a box : isolated oscillator concept



normal modes

$$\frac{\partial^2 U(x)}{\partial x^2} = \text{const.}$$

thermal expansion

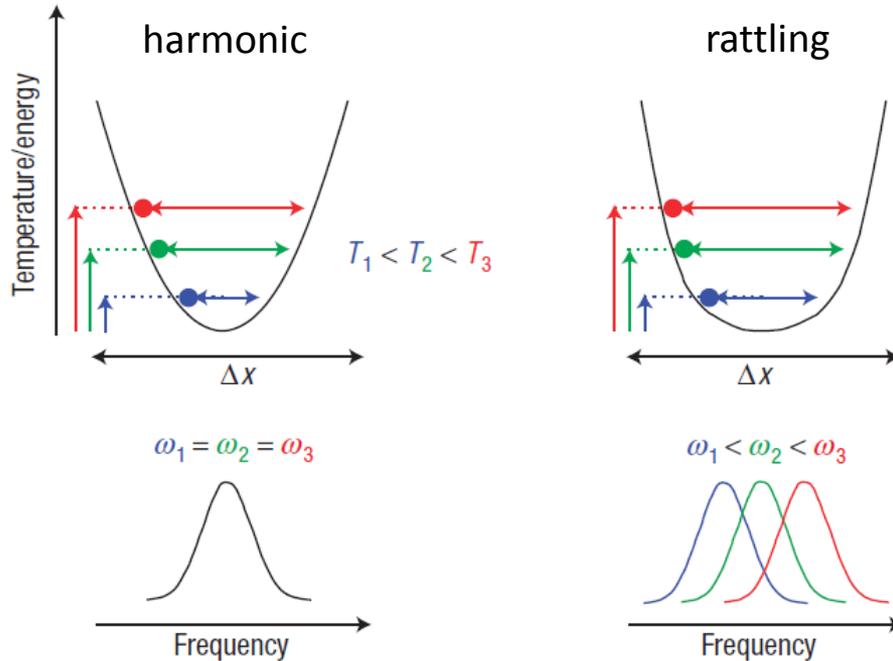
~~$$\frac{\partial^3 U(x)}{\partial x^3} < 0$$~~

rattling term

$$\frac{\partial^4 U(x)}{\partial x^4} > 0$$

$$U(x) = U_0 + \frac{1}{2} \frac{\partial^2 U(x)}{\partial x^2} x^2 + \frac{1}{6} \frac{\partial^3 U(x)}{\partial x^3} x^3 + \frac{1}{24} \frac{\partial^4 U(x)}{\partial x^4} x^4 + \dots$$

## Particle in a box : isolated oscillator concept



normal modes

$$\frac{\partial^2 U(x)}{\partial x^2} = \text{const.}$$

thermal expansion

$$\frac{\partial^3 U(x)}{\partial x^3} < 0$$

rattling term

$$\frac{\partial^4 U(x)}{\partial x^4} > 0$$

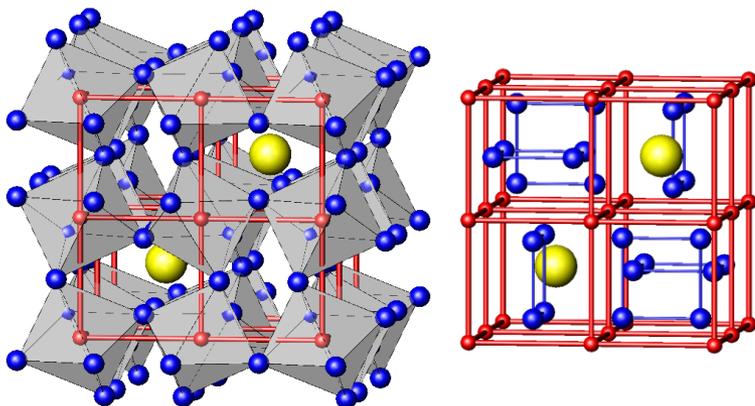
$$U(x) = U_0 + \frac{1}{2} \frac{\partial^2 U(x)}{\partial x^2} x^2 + \frac{1}{6} \frac{\partial^3 U(x)}{\partial x^3} x^3 + \frac{1}{24} \frac{\partial^4 U(x)}{\partial x^4} x^4 + \dots$$

$$\frac{\Delta \omega}{\Delta T} = 0$$

$$\frac{\Delta \omega}{\Delta T} < 0$$

$$\frac{\Delta \omega}{\Delta T} > 0$$

- Do rattling modes (RMs) exist ? What are RMs ?
  - weakly-bonded, isolated, individual oscillators, ...
- Are RMs responsible for PGEC ?
- Are RMs the adequate means to break thermal transport ?
- How weak must a coupling be to generate a RM ?
- How strong must a coupling be to perturb thermal transport ?
- What are the conditions to turn a classical RM into a quantum mechanical oscillator ?
- ...



## MFe<sub>4</sub>Sb<sub>12</sub> - Skutterudite

M = Ca<sup>2+</sup>, Sr<sup>2+</sup>, Ba<sup>2+</sup>, La<sup>3+</sup>, Ce, Eu, Nd, Yb<sup>2+</sup>, Tl, ...

Space group: Im-3

Cubic structure with M on bcc-sites

d = 9.2 Å

## LaT<sub>4</sub>X<sub>12</sub> - Skutterudite

T = Fe, Ru, Os

X = As, Sb

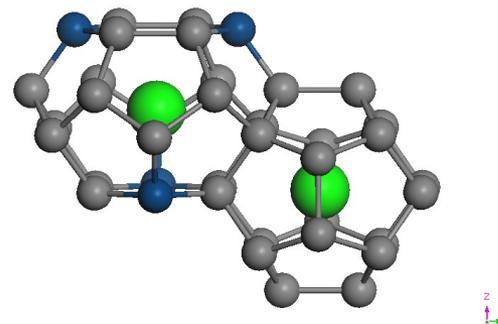
## Clathrates, Clathrate-hydrates

Ba<sub>8</sub>Ge<sub>43</sub>, Ba<sub>8</sub>M<sub>x</sub>Ge<sub>46-x</sub> M = Zn, Pd, ...

Ba<sub>8</sub>Si<sub>46</sub>, Ba<sub>24</sub>Si<sub>100</sub>

M<sub>8</sub>Sn<sub>44</sub> M = K, Rb, Cs

d = 11 – 22 Å

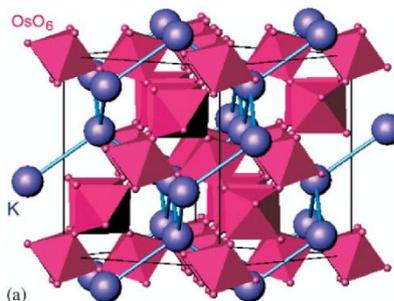


## MV<sub>2</sub>Al<sub>20</sub>

M = Sc, Y, Al, Ga, La, Ce

Space group: Fd-3m

d = 14.5 Å



## Beta - Pyrochlores

MOs<sub>2</sub>O<sub>6</sub>, M = K, Rb, Cs

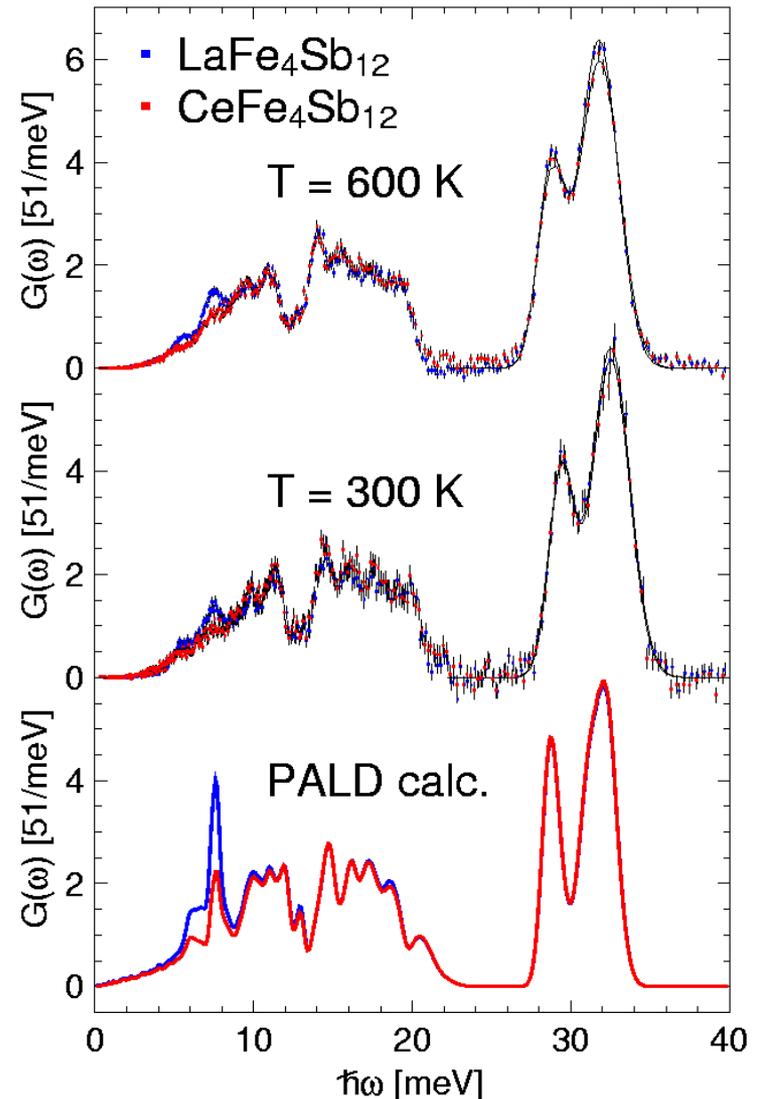
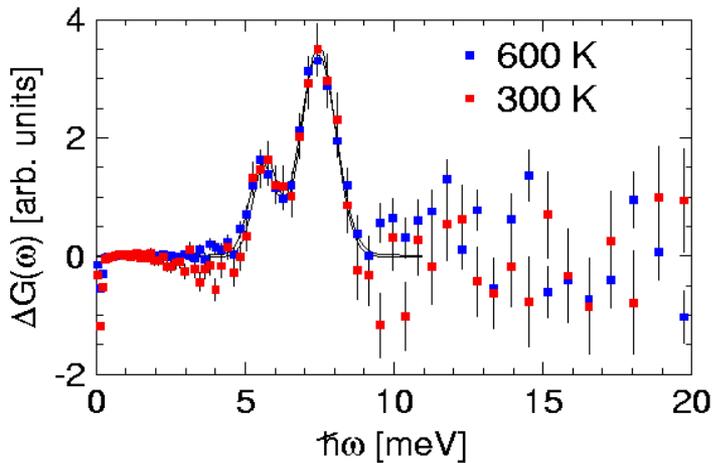
Space group: Fd-3m

d = 10.1 Å

## INS contrast experiment at IN6@ILL

|    | S.C.S. | A.M.U. | S.P. |
|----|--------|--------|------|
| La | 9.7    | 138.9  | 7.0  |
| Ce | 2.9    | 140.1  | 2.1  |
| Fe | 11.6   | 55.9   | 20.8 |
| Sb | 3.9    | 121.8  | 3.2  |

S.C.S. = scattering cross section in barns  
 A.M.U. = atomic mass unit  
 Scattering Power S.P. = C.S./A.M.U. \* 100



M.M. Koza et al., *Nature Materials* 2008

V. Keppens et al., *Nature* 1998, R.P. Hermann et al., *PRL* 2003, G.J. Long et al., *PRB* 2005

## Mode energies in meV

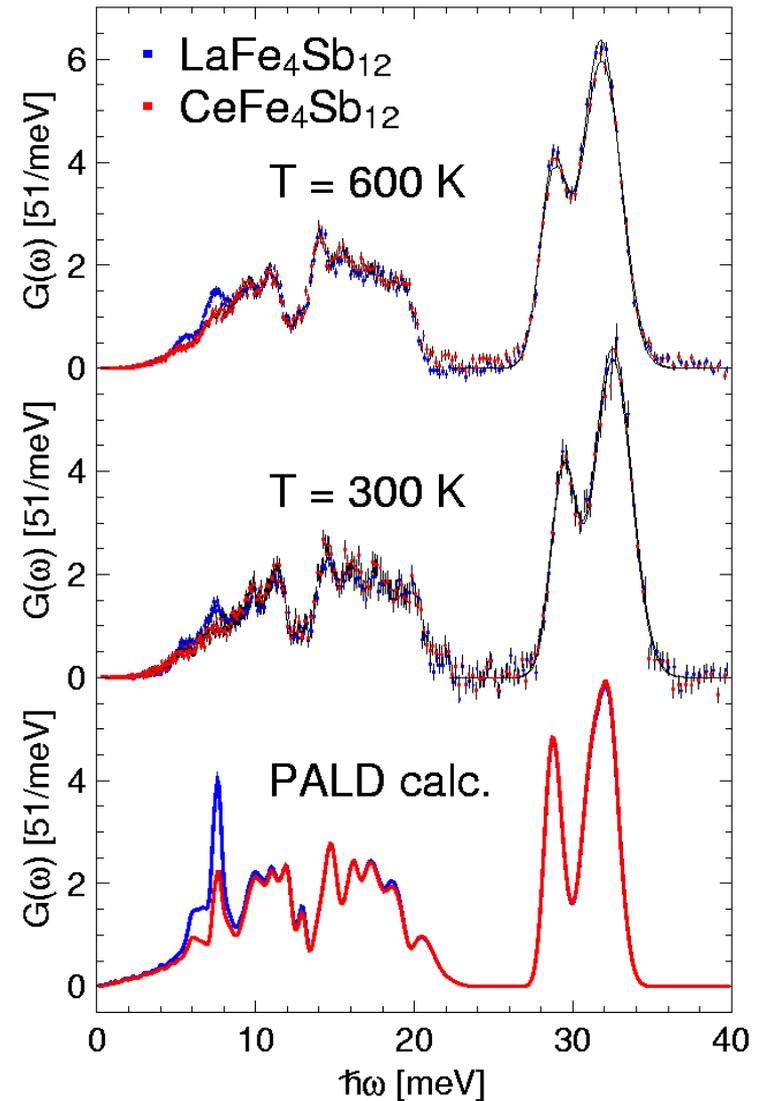
| INS  | Raman* | LD calc.       |
|------|--------|----------------|
| 5.5  | -      | -              |
| 7.5  | -      | 7.2            |
| 9.5  | -      | -              |
| 11.5 | 11.5   | 11.4/11.5      |
| 13.0 | 12.5   | 12.5/12.6      |
| 14.5 | 15.1   | 15.0/15.4      |
| 16.0 | 16.2   | 15.9           |
| 17.5 | 18.1   | 17.3/17.9/18.2 |
| 19.0 | 19.1   | 18.4/18.6/18.9 |
| 20.0 | 20.0   |                |
| 21.5 | 21.3   | 21.1           |

\*J.L. Feldman et al., *PRB* 68, 94301 (2003)

Black - Raman active

Blue - IR active

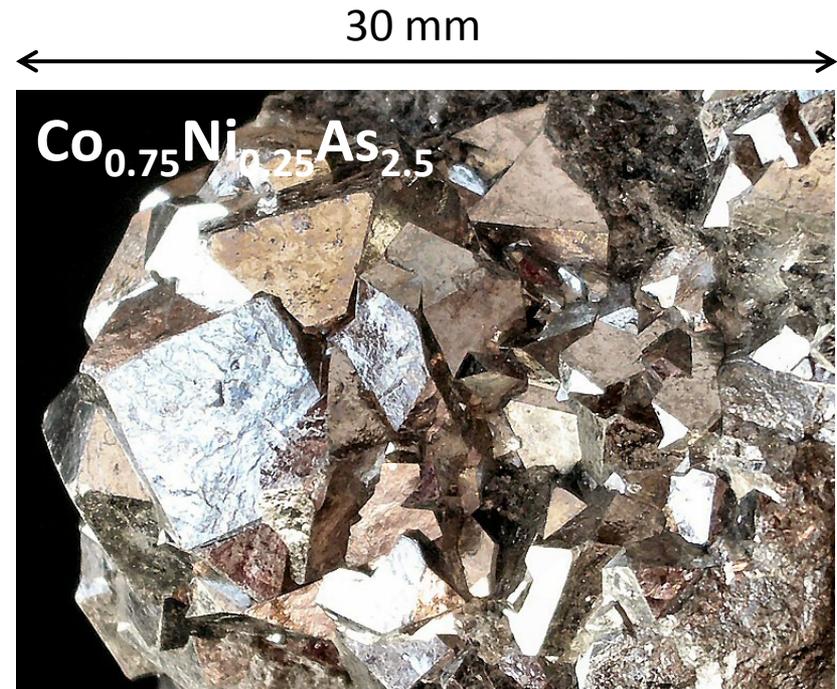
Red - neither ... nor



## Mode energies in meV

| INS  | Raman* | LD calc.       |
|------|--------|----------------|
| 5.5  | -      | -              |
| 7.5  | -      | <del>7.2</del> |
| 9.5  | -      | -              |
| 11.5 | 11.5   | 11.4/11.5      |
| 13.0 | 12.5   | 12.5/12.6      |
| 14.5 | 15.1   | 15.0/15.4      |
| 16.0 | 16.2   | 15.9           |
| 17.5 | 18.1   | 17.3/17.9/18.2 |
| 19.0 | 19.1   | 18.4/18.6/18.9 |
| 20.0 | 20.0   |                |
| 21.5 | 21.3   | 21.1           |

\*J.L. Feldman et al., *PRB* 68, 94301 (2003)

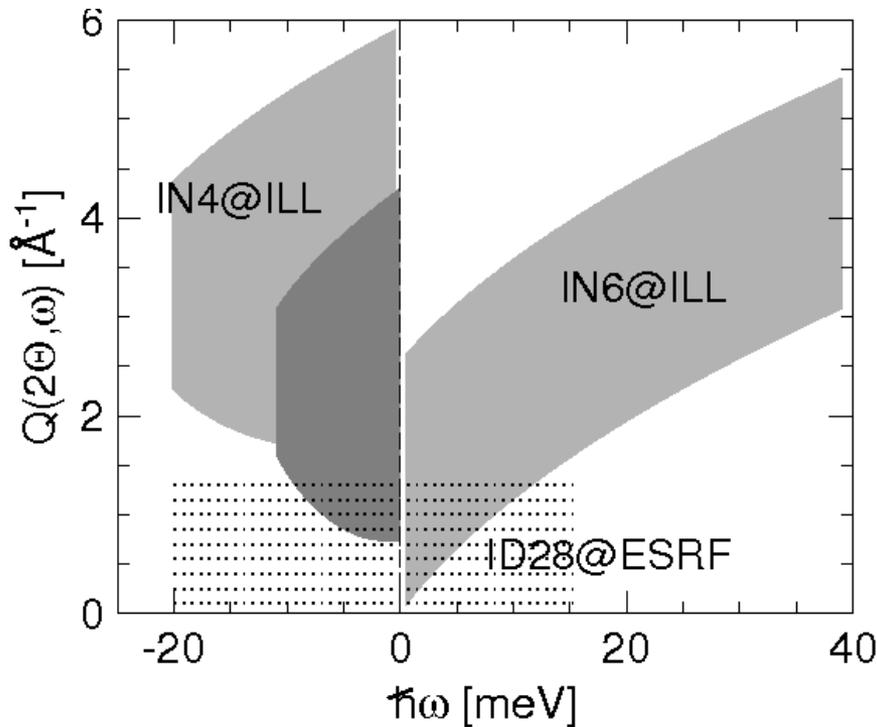


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Compounds form readily as polycrystals and powders.

Their high reflectivity makes optic experiments very difficult.

- High energy resolution of 1-500 microeV.
- Relaxed selection rules governing scattering process and thus capability of monitoring all vibrational eigenstates.
- Moderate and weak absorption effects.
- Capability of sampling large areas of the energy-momentum phase space simultaneously.
- Sensitivity to bulk properties, requiring however samples of sufficient volume  $\sim 1 \text{ cm}^3$ .
- ...



## X-ray spectroscopy

ID28@ESRF

$dE = 1.5 \text{ meV}$

**Plus any other experimental technique that helps to verify/improve calculations !**

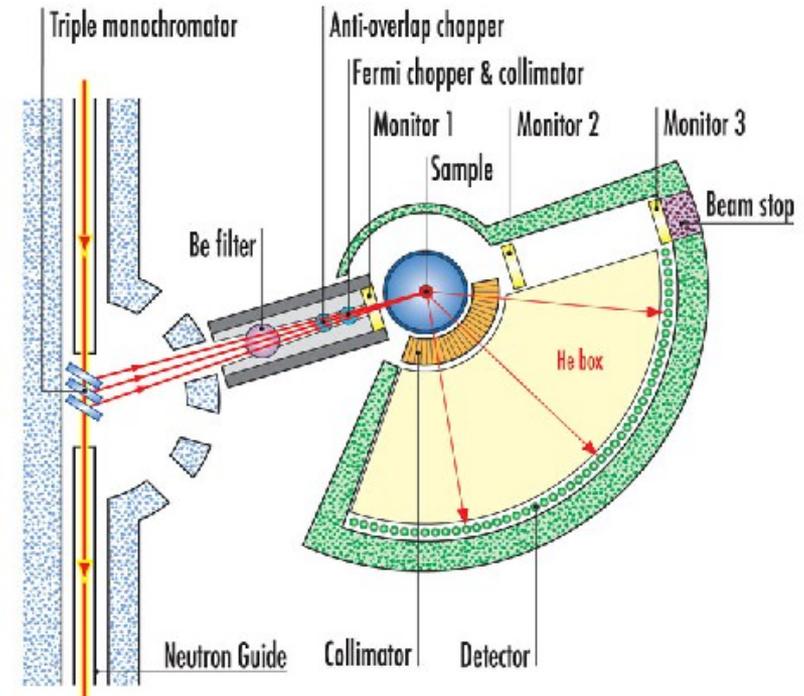
## Time of flight spectroscopy

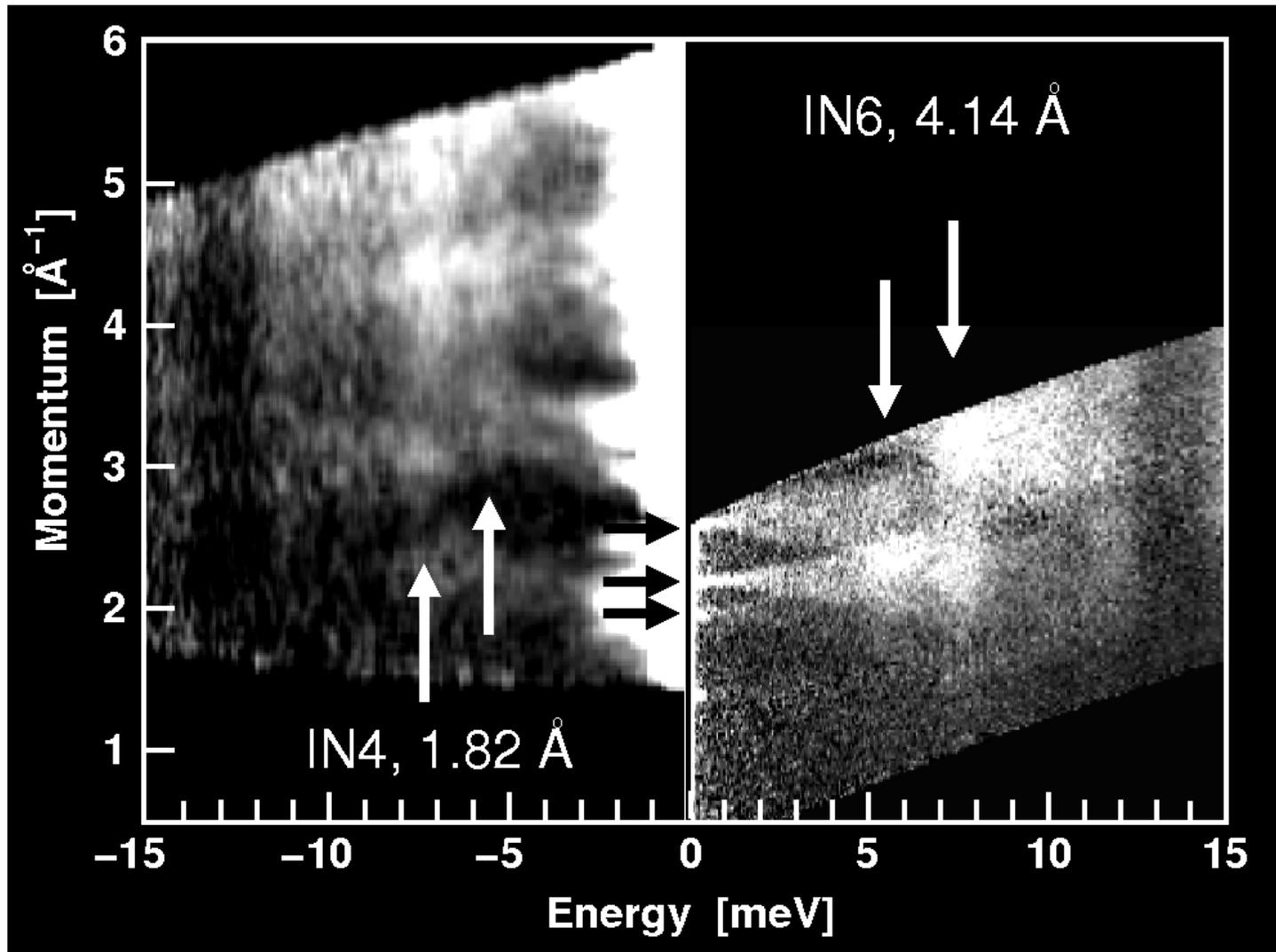
IN6@ILL 'cold' spectrometer :

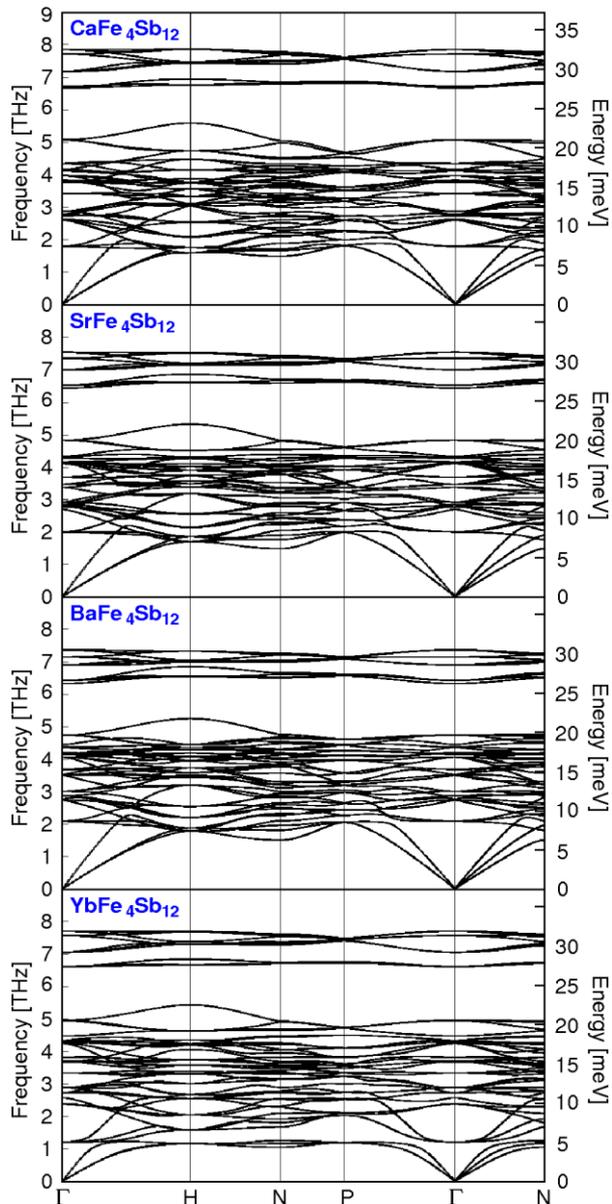
$dE = 0.1 - 0.2 \text{ meV}$

IN4ILL 'thermal' spectrometer :

$dE = 0.5 - 1.0 \text{ meV}$



$S(Q, \omega)$  of  $\text{LaFe}_4\text{Sb}_{12}$ 



All energy and force calculations performed with Vienna Ab-initio Simulation Package (VASP DFT) with PAW-GGA (LDA) potentials.

All lattice dynamics phonon calculations performed with the 'direct method' implemented in the package PHONON.

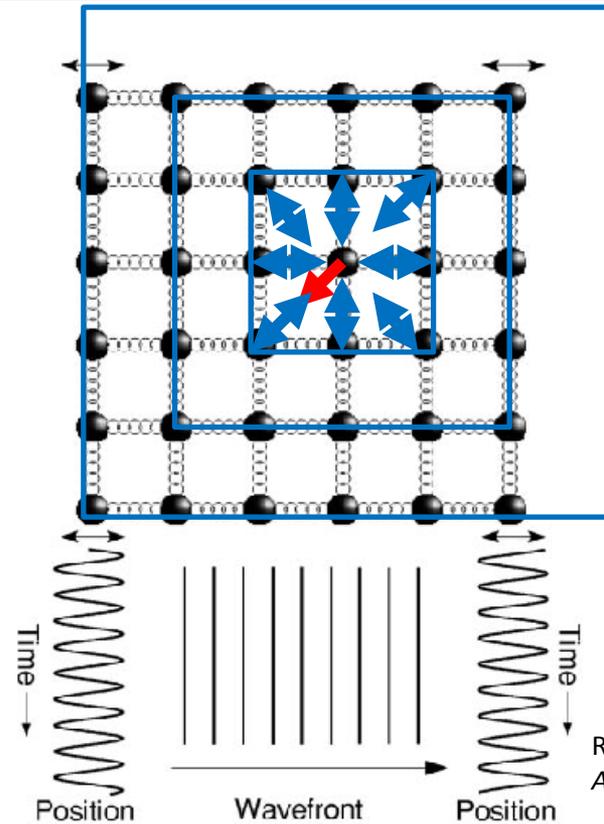
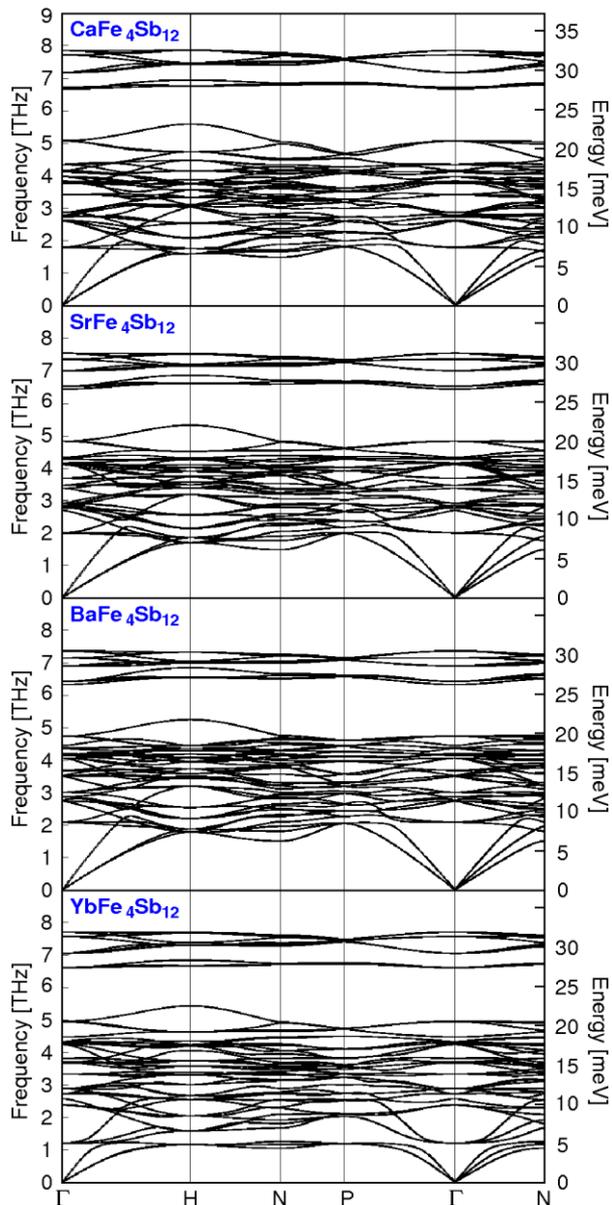
Powder Average Lattice Dynamics (PALD) :

Random generation of Q – points in the spherical coordinate space  $(r, 2\theta, \varphi)$  matching the natural sampling of the phase space by the time of flight neutron scattering technique.

Calculation effort depends on the Q resolution required.

Neutron and x ray specific signals and partial (self- and cross-correlations) contributions can be calculated.

Labor intensive work.

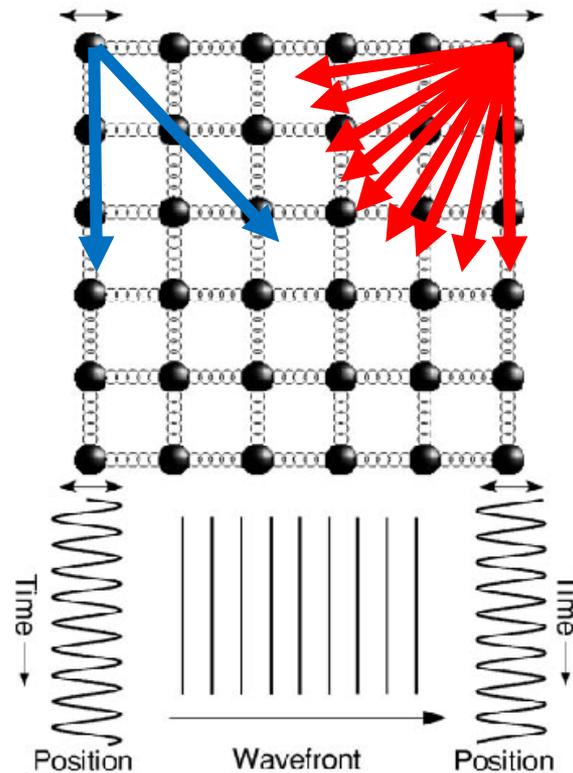
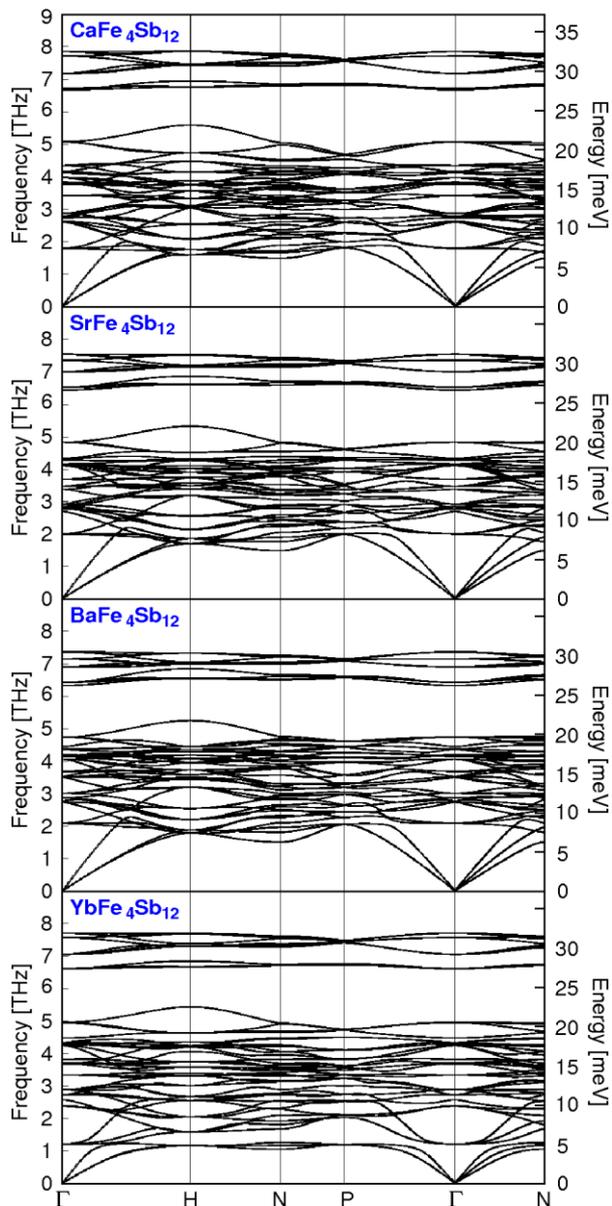


R.P. Hermann et al.,  
*Am. J Phys.* **73**, 110, (2005)

**Displacement of symmetry non-equivalent atoms.**

**Calculation of forces - Hellmann-Feynman theorem.**

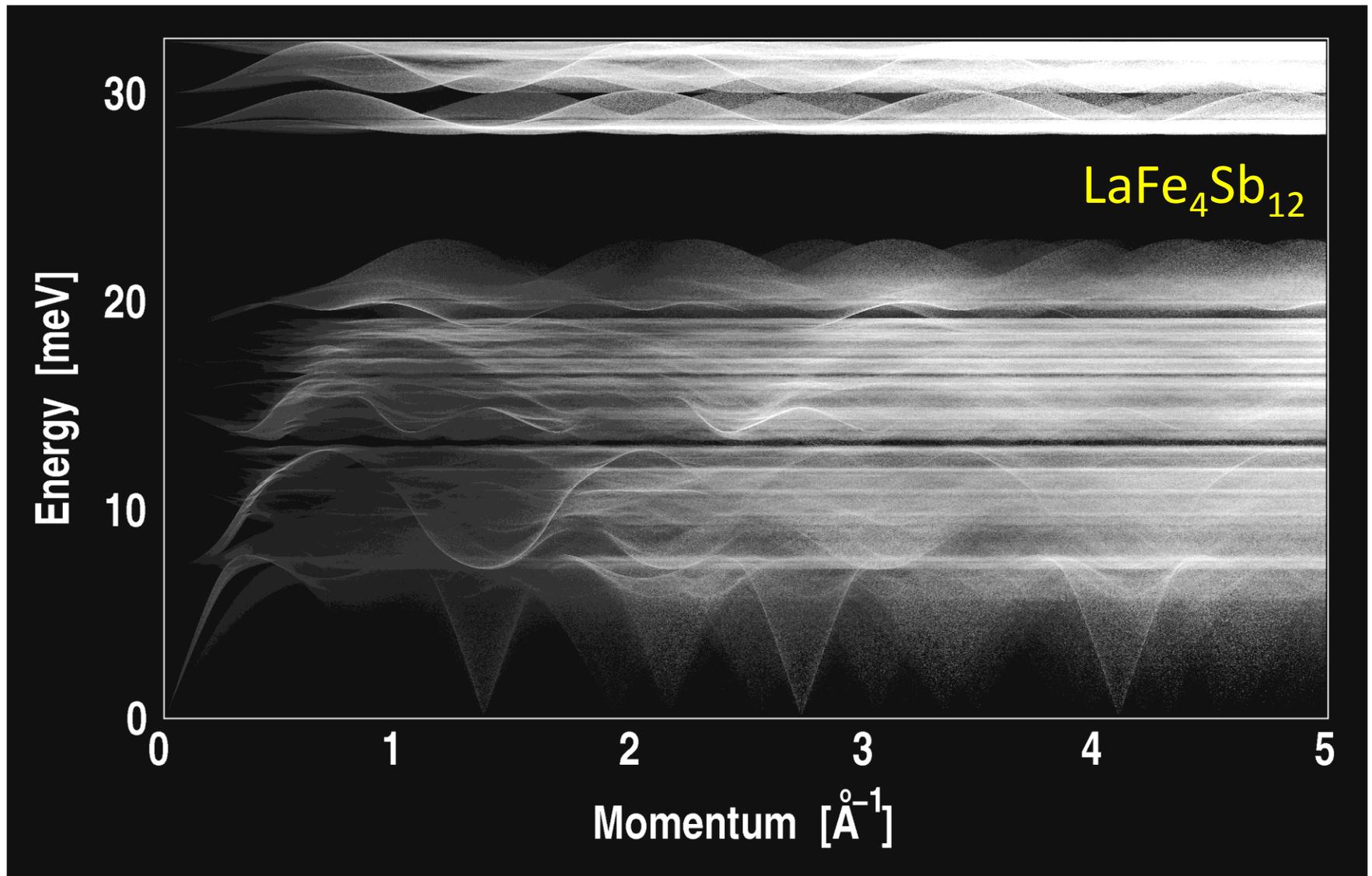
**Precision is dependent on the size of the simulation box size !**



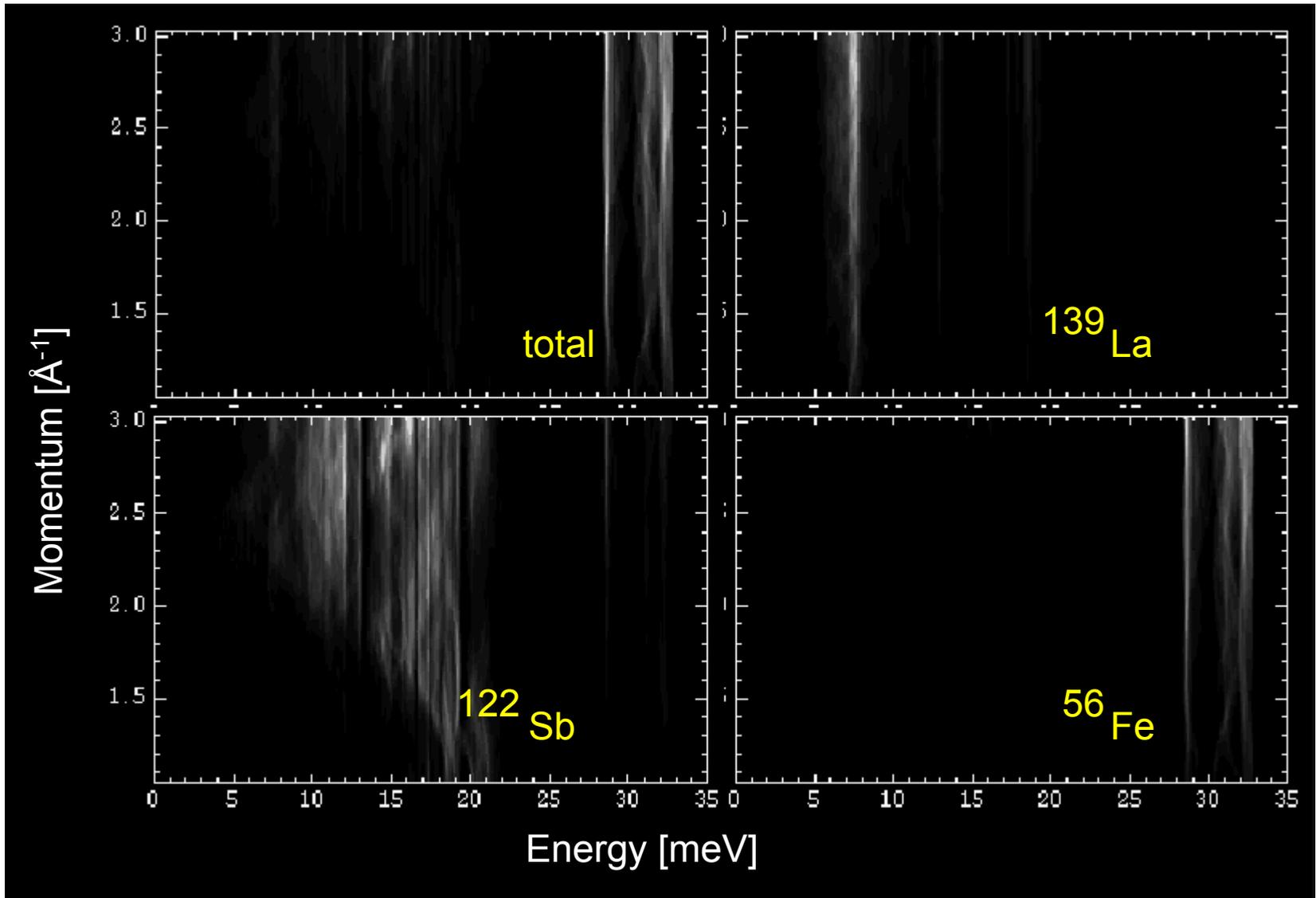
R.P. Hermann et al.,  
*Am. J Phys.* **73**, 110, (2005)

**Standard lattice dynamics calculation is performed along high symmetry directions in reciprocal space.**

**Powder Averaged Lattice Dynamics calculates the dispersion in any possible direction at random.**



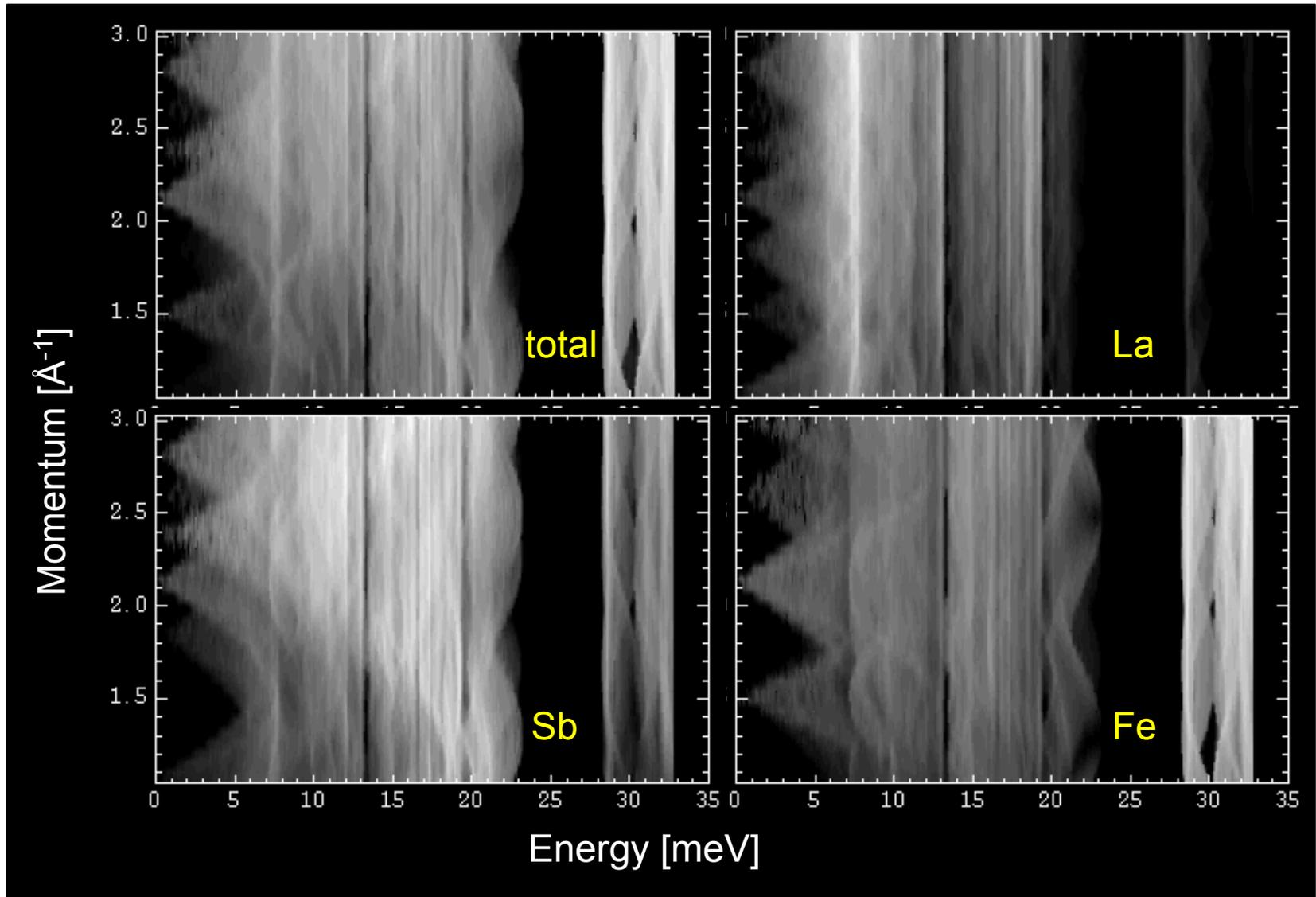
Total  $S(Q, \omega)$  of  $\text{LaFe}_4\text{Sb}_{12}$ , 350 - 400 Gbyte of data plus 50 Gbyte for each partial signal



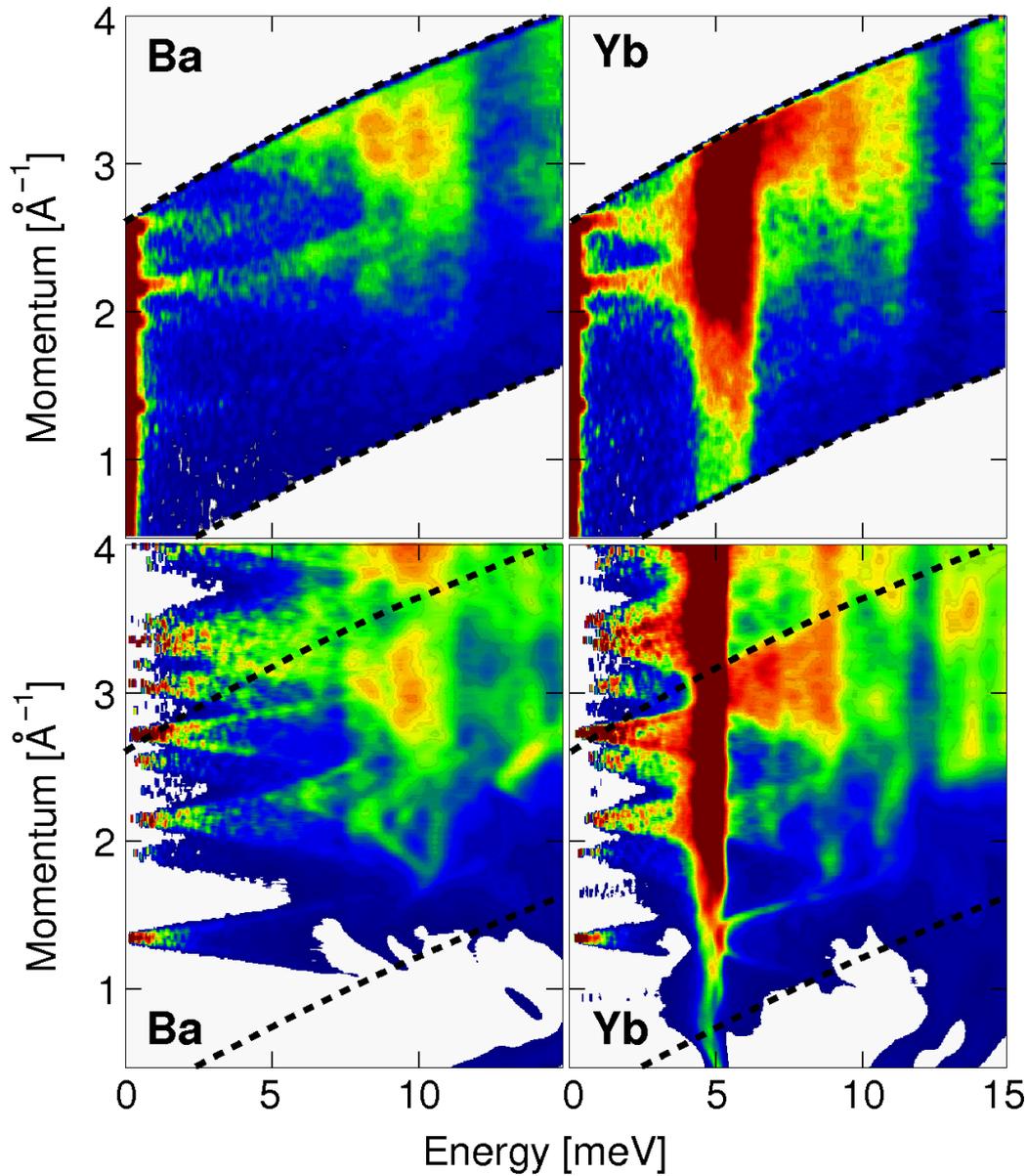
Linear intensity scale !

$$\omega \propto \sqrt{(K/M)}$$

$$S(Q, \omega) \propto \sigma/M$$

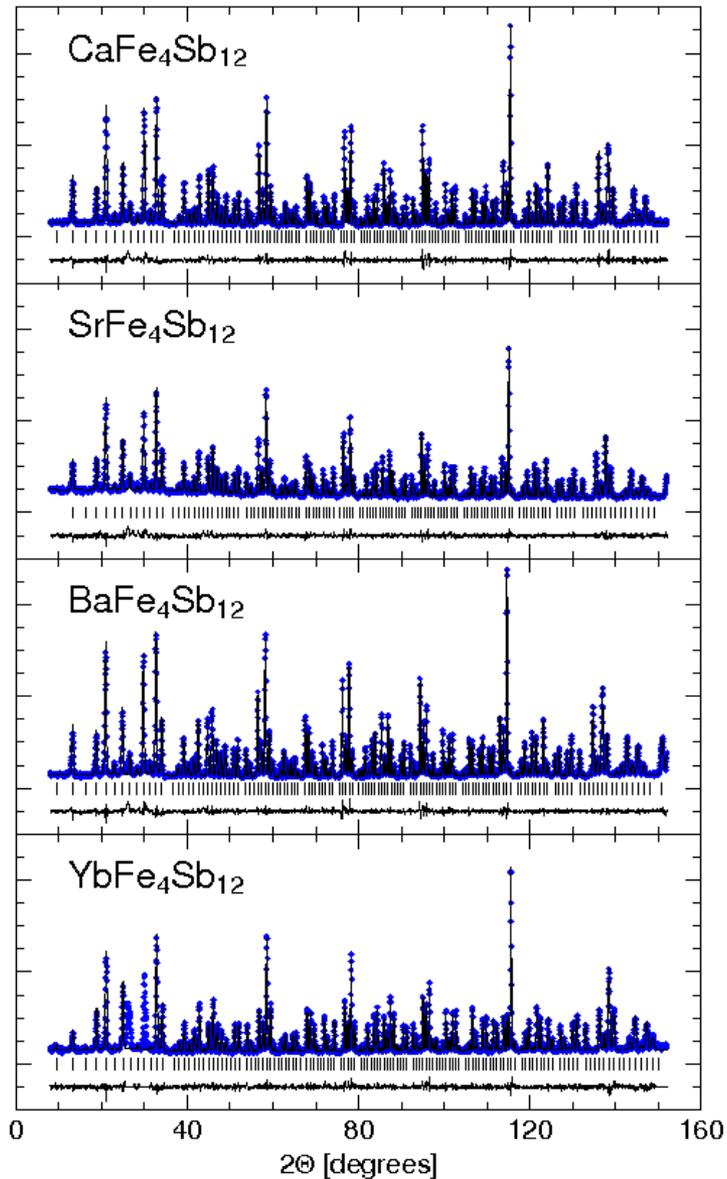


Logarithmic intensity scale !

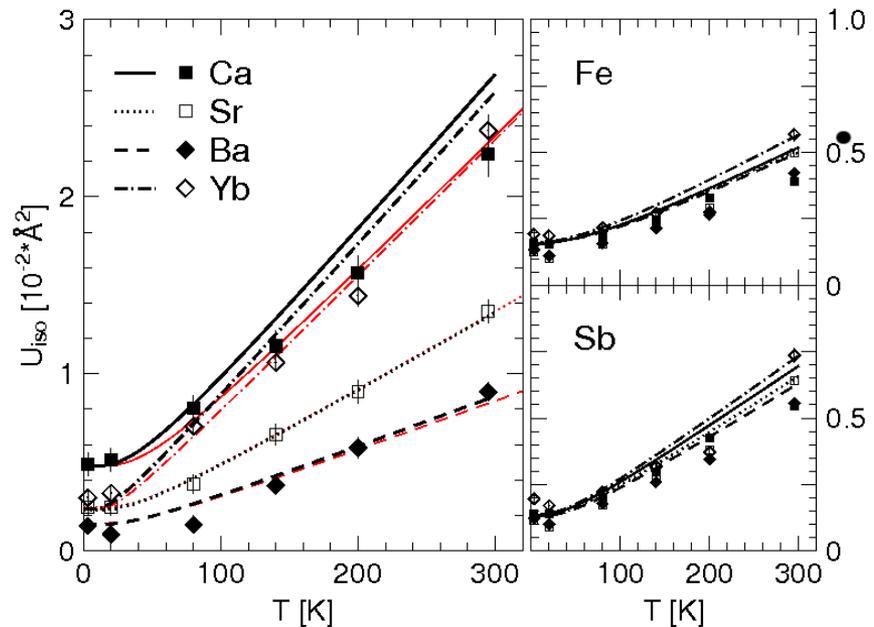

 $S(Q, w)$  of  $MFe_4Sb_{12}$ 

|    | $\sigma_{\text{tot}}$ | A.M.U. | S.P. |
|----|-----------------------|--------|------|
| Ca | 2.8                   | 40.1   | 7.0  |
| Sr | 6.3                   | 87.6   | 7.2  |
| Ba | 3.4                   | 137.3  | 2.5  |
| La | 9.7                   | 138.9  | 7.0  |
| Ce | 2.9                   | 140.1  | 2.1  |
| Yb | 23.4                  | 174.0  | 13.5 |
| Fe | 11.6                  | 55.9   | 20.8 |
| Sb | 3.9                   | 121.8  | 3.2  |

$$\text{S.P.} = \sigma_{\text{tot}} / \text{A.M.U.} * 100$$

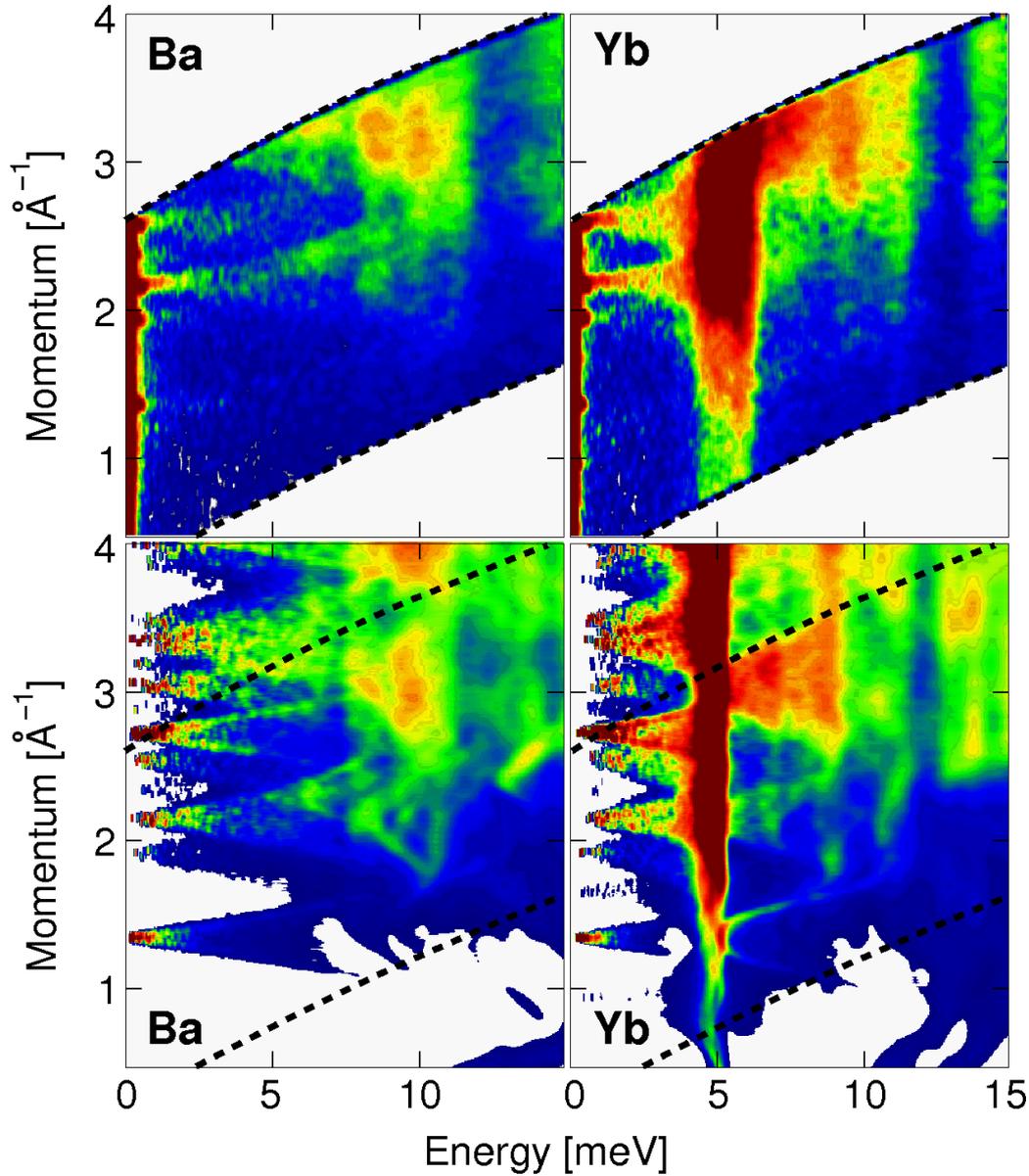


## Mean Square Displacement :



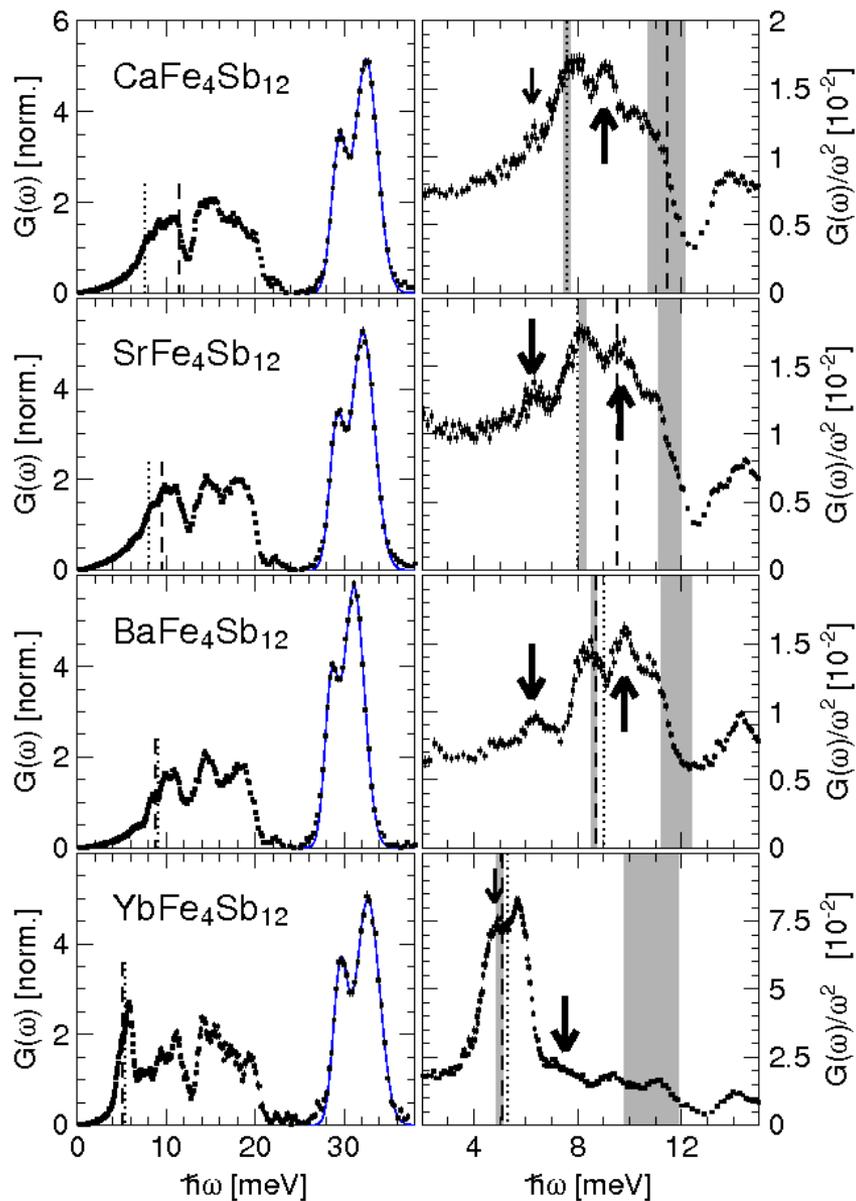
## Lattice Parameters and Ionic Radii in Angstrom :

|            | Ca   | Sr   | Ba   | Yb   |
|------------|------|------|------|------|
| A.I. calc. | 9.19 | 9.21 | 9.24 | 9.18 |
| T = 3 K    | 9.14 | 9.16 | 9.18 | 9.13 |
| Radius     | 1.26 | 1.40 | 1.56 | 1.28 |



Generalized Density of States from momentum integrated signal :  
incoherent approximation

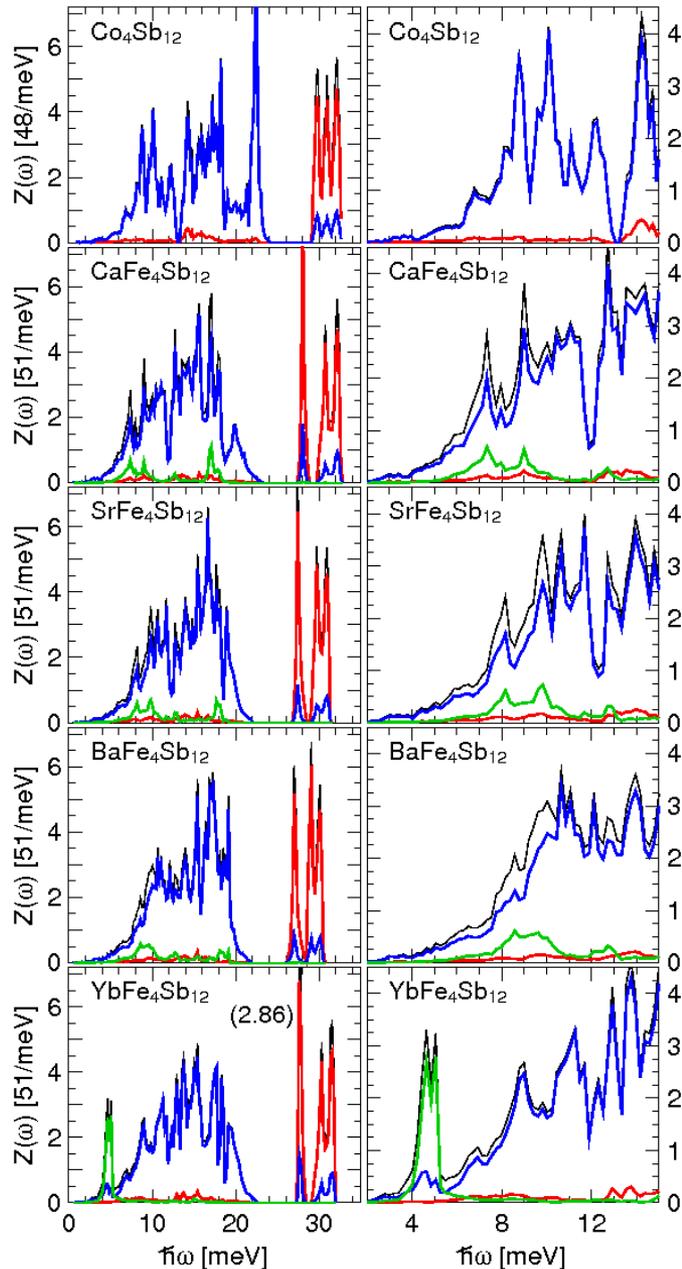




## Characteristic Energies in meV :

|             | Ca   | Sr   | Ba   | Yb   |
|-------------|------|------|------|------|
| $G(\omega)$ | -    | 6.5  | 6.5  | 4.9  |
|             | -    | -    | -    | 5.7  |
|             | 8.0  | 8.2  | 8.4  | 7.5  |
|             | 9.2  | 9.9  | 9.9  | 9.4  |
| $U(r)$      | 11.5 | 11.1 | 10.9 | 11.1 |
|             | 9.6  | 8.8  | 8.8  | 4.6  |
| MSD         | 10.8 | 9.6  | 9.7  | 5.2  |
|             | 7.6  | 8.0  | 9.0  | 5.3  |

\*W. Schnelle et al., *PRB* 77, 094421 (2008)

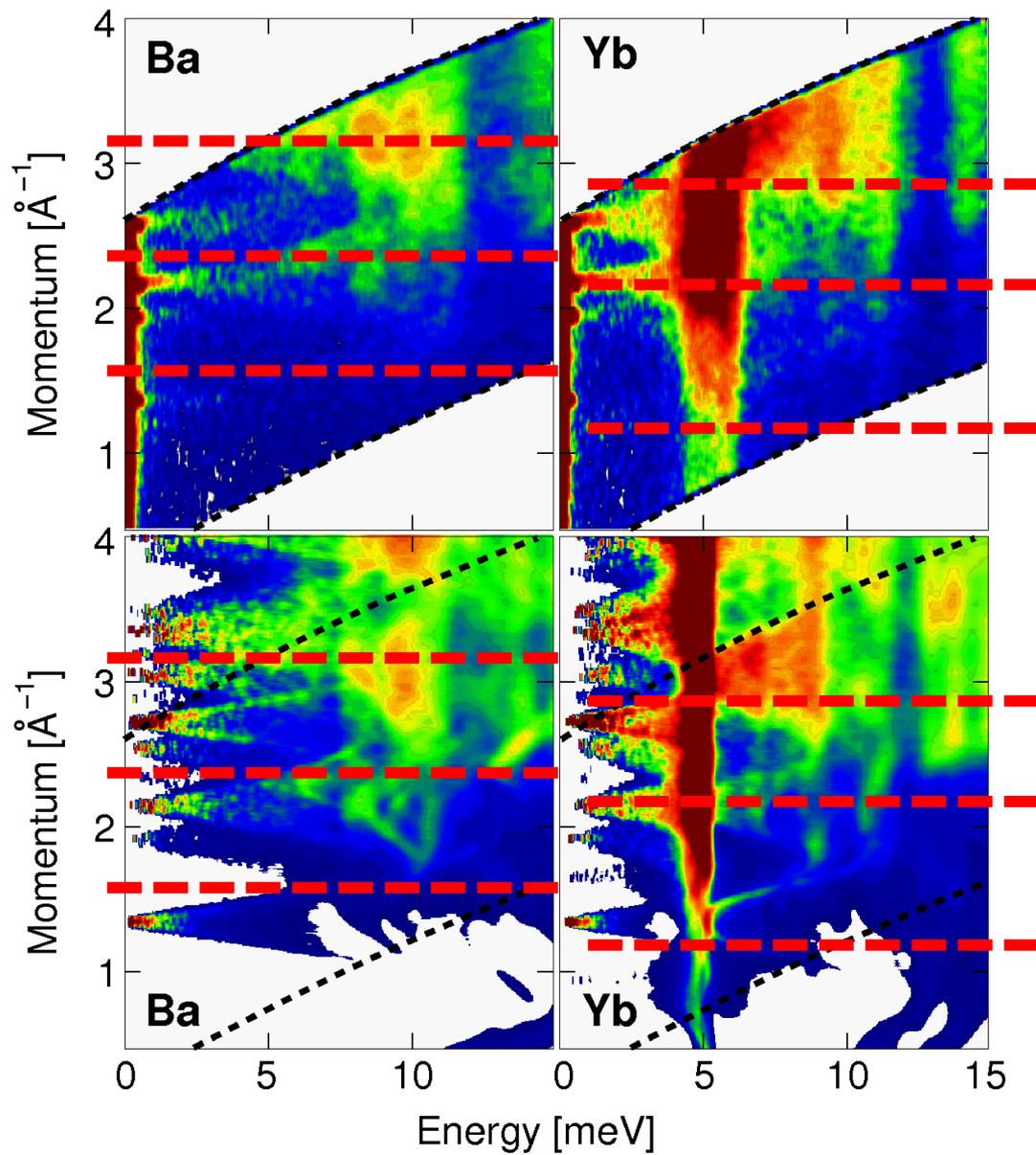


## Characteristic Energies in meV :

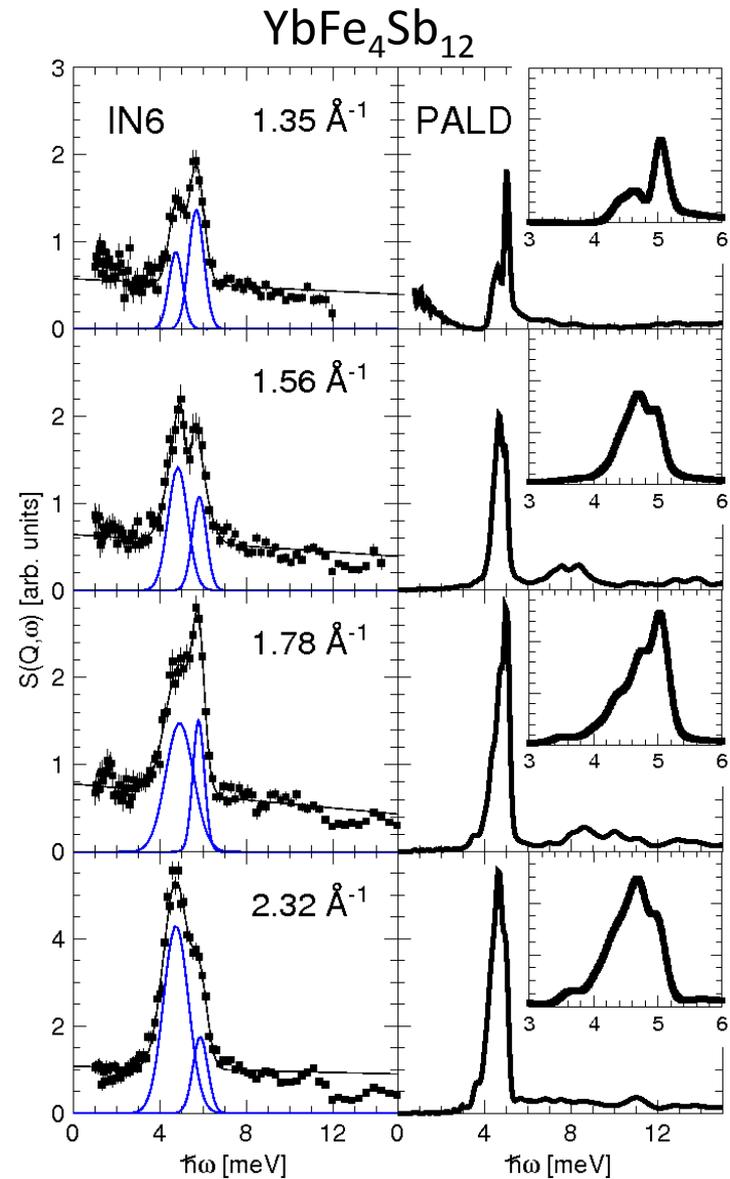
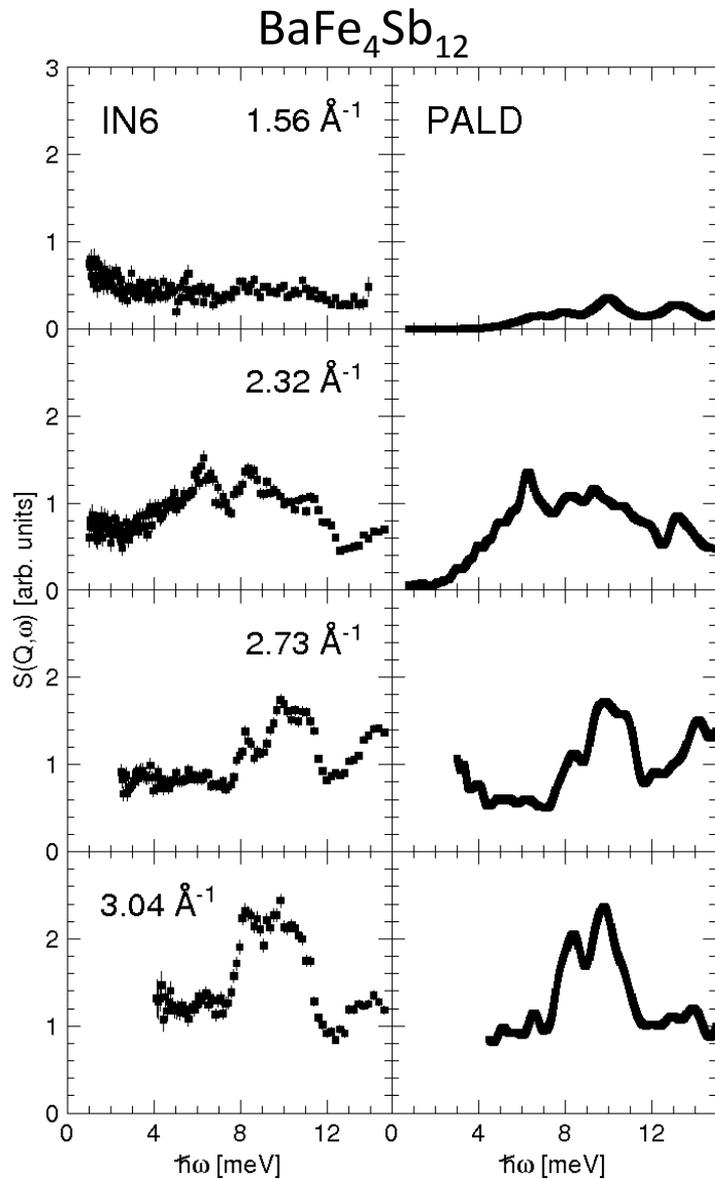
|             | Ca   | Sr   | Ba   | Yb   |
|-------------|------|------|------|------|
| $G(\omega)$ | -    | 6.5  | 6.5  | 4.9  |
|             | -    | -    | -    | 5.7  |
|             | 8.0  | 8.2  | 8.4  | 7.5  |
|             | 9.2  | 9.9  | 9.9  | 9.4  |
|             | 11.5 | 11.1 | 10.9 | 11.1 |
| $U(r)$      | 9.6  | 8.8  | 8.8  | 4.6  |
| MSD         | 10.8 | 9.6  | 9.7  | 5.2  |
| $c(T)^*$    | 7.6  | 8.0  | 9.0  | 5.3  |

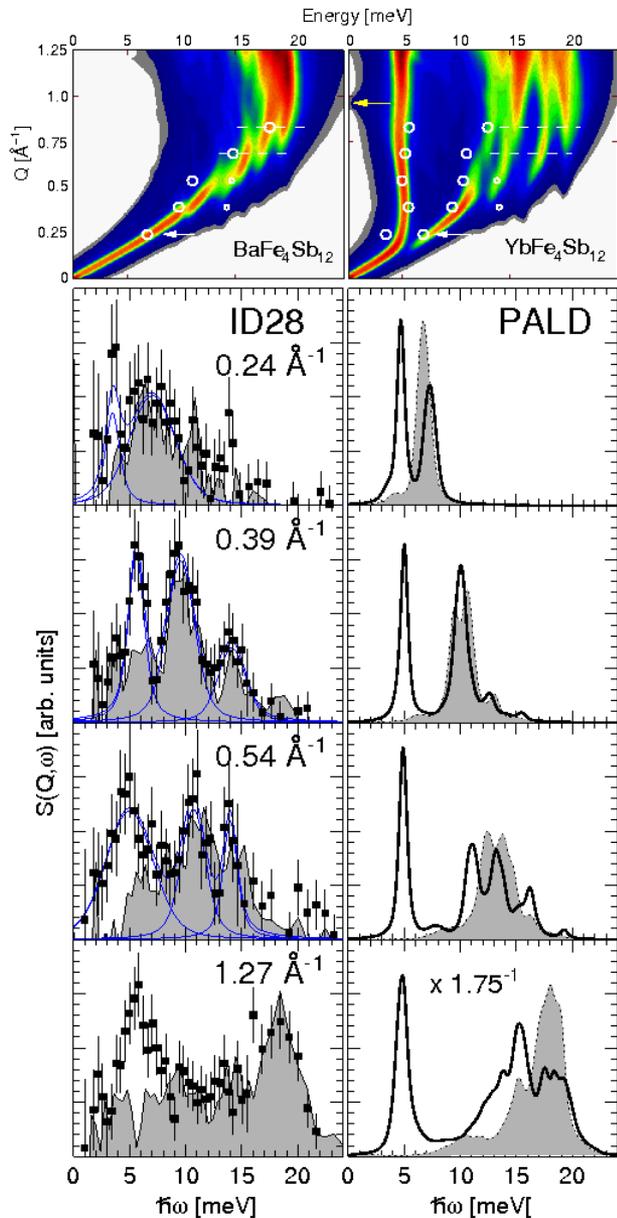
\*W. Schnelle et al., *PRB* 77, 094421 (2008)

Strong localization present in  $MFe_4Sb_{12}$   
with  $M = La, Eu, Nd, Yb, Tl$  .



Spectra at constant momentum  $Q$ .





Mode energies in meV :

| Q    | YbFe <sub>4</sub> Sb <sub>12</sub> |         | BaFe <sub>4</sub> Sb <sub>12</sub> |
|------|------------------------------------|---------|------------------------------------|
|      | E1                                 | E2      | E1                                 |
| 2.42 | 3.5(1)                             | 7.0(4)  | 6.8(2)                             |
| 3.92 | 5.6(1)                             | 9.6(1)  | 9.8(1)                             |
| 5.36 | 5.0(1)                             | 10.7(1) | 11.1(3)                            |
| 6.85 | 5.3(2)                             | 11.0(5) | 14.9(4)                            |
| 8.27 | 5.8(4)                             | 13(2)   | 18.3(2)                            |

Longitudinal velocity of sound :

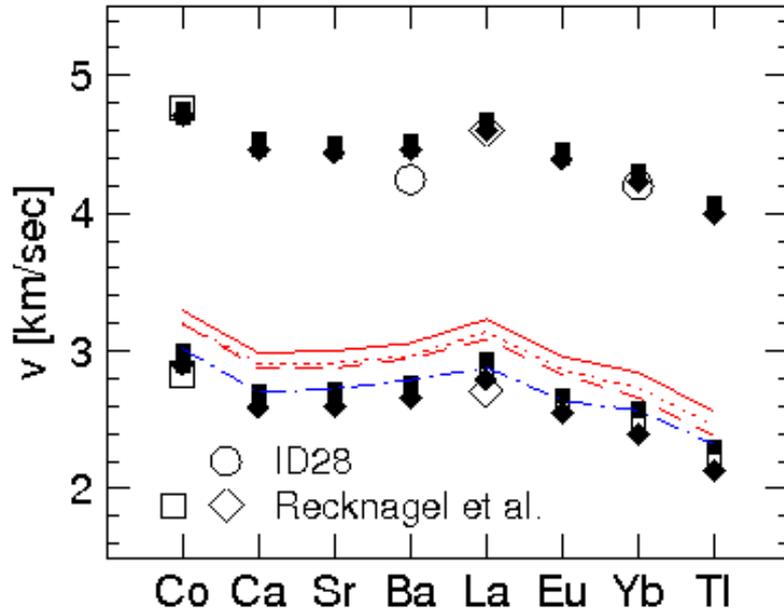
YbFe<sub>4</sub>Sb<sub>12</sub> : 4400(250) m/sec

BaFe<sub>4</sub>Sb<sub>12</sub> : 4270(130) m/sec

Scattering power :  $Z^2$

Absorption :  $Z^4$

## Velocity of sound of $\text{Co}_4\text{Sb}_{12}$ and $\text{MFe}_4\text{Sb}_{12}$



|                                 | $C_{11}/\text{GPa}$ | $C_{44}/\text{GPa}$ | $T_{\text{debye}}/\text{K}$ |
|---------------------------------|---------------------|---------------------|-----------------------------|
| $^+\text{Co}_4\text{Sb}_{12}$   | 164                 | 59                  | 319                         |
| $^*\text{Co}_4\text{Sb}_{12}$   |                     | 61                  | 327                         |
| $^*\text{CaFe}_4\text{Sb}_{12}$ | 132                 | 49                  | 300                         |
| $^*\text{LaFe}_4\text{Sb}_{12}$ |                     | 57                  | 315                         |
| $^+\text{LaFe}_4\text{Sb}_{12}$ | 162                 | 58                  | 310                         |

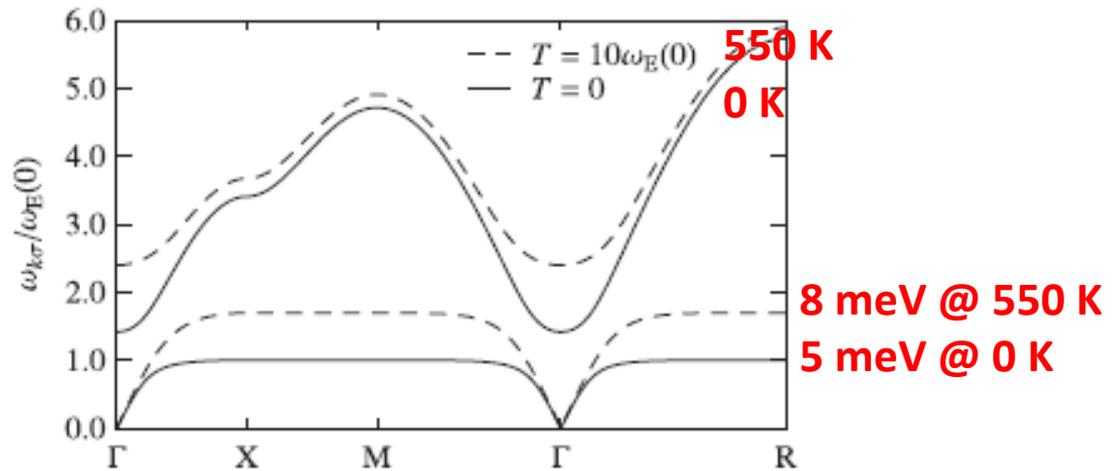
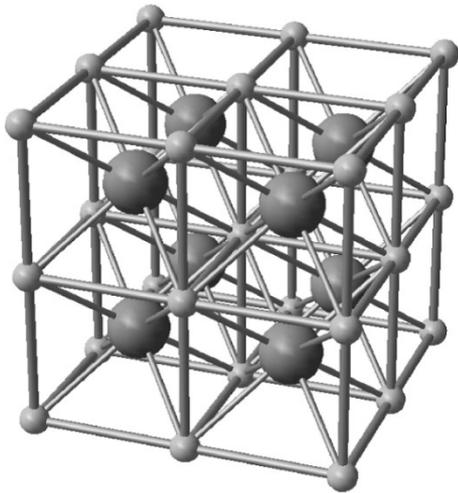
$^+$ V.Keppens et al., Nature, 1998

$^*$ L.Zhang et al., Mat.Sci.&Eng. B, 2010

Material properties : elastic constants, bulk and Young's moduli and Poisson ratio :

|    | $d$<br>[Å] | $\bar{v}_{\text{DL}}$<br>[m/sec.] | $T_{\text{DL}}$<br>[K] | $v_t$<br>[m/sec] | $v_l$<br>[m/sec] | $T_{\text{Debye}}$<br>[K] | $c_{11}$<br>[GPa] | $G$<br>[GPa] | $E$<br>[GPa] | $\nu$   | $B$<br>[GPa] |
|----|------------|-----------------------------------|------------------------|------------------|------------------|---------------------------|-------------------|--------------|--------------|---------|--------------|
| Co | 9.096      | 3094                              | 322                    | 2929(45)         | 4719(20)         | 336(4)                    | 167(1)            | 64(2)        | 152(3)       | 0.19(1) | 81(1)        |
| Ca | 9.180      | 2679                              | 278                    | 2637(44)         | 4487(31)         | 307(4)                    | 149(2)            | 51(2)        | 127(3)       | 0.24(1) | 80(1)        |
| Sr | 9.210      | 2707                              | 281                    | 2646(50)         | 4462(30)         | 307(4)                    | 150(2)            | 53(2)        | 130(4)       | 0.23(1) | 80(1)        |
| Ba | 9.240      | 2769                              | 288                    | 2703(47)         | 4479(28)         | 312(4)                    | 154(2)            | 56(2)        | 136(4)       | 0.21(1) | 80(1)        |
| La | 9.169      | 2857                              | 297                    | 2856(58)         | 4632(33)         | 331(5)                    | 169(2)            | 64(3)        | 153(5)       | 0.19(1) | 83(1)        |
| Eu | 9.172      | 2659                              | 276                    | 2603(53)         | 4418(28)         | 303(4)                    | 154(2)            | 54(2)        | 132(4)       | 0.23(1) | 83(1)        |
| Yb | 9.176      | 2541                              | 264                    | 2475(72)         | 4263(30)         | 289(6)                    | 145(2)            | 49(3)        | 122(5)       | 0.25(2) | 80(2)        |
| Tl | 9.211      | 2300                              | 239                    | 2216(70)         | 4034(29)         | 259(6)                    | 131(2)            | 39(2)        | 101(5)       | 0.28(2) | 78(2)        |

A. Yamakage et al., *J. Phys. Soc. Japan* 78, 064602 (2009)

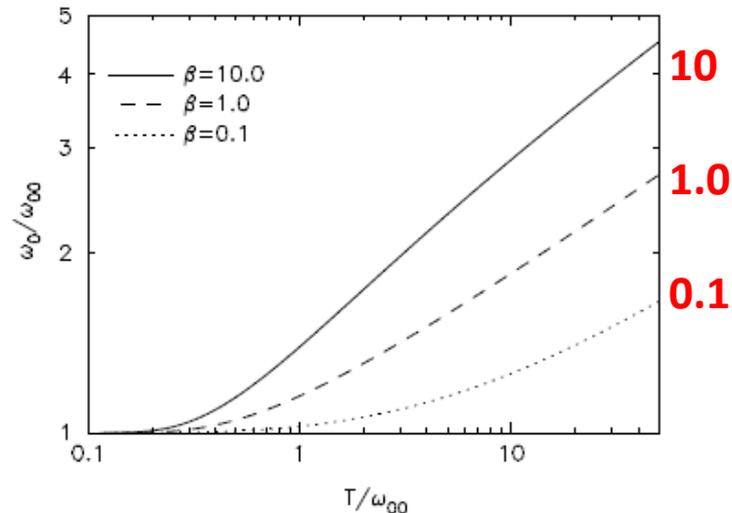


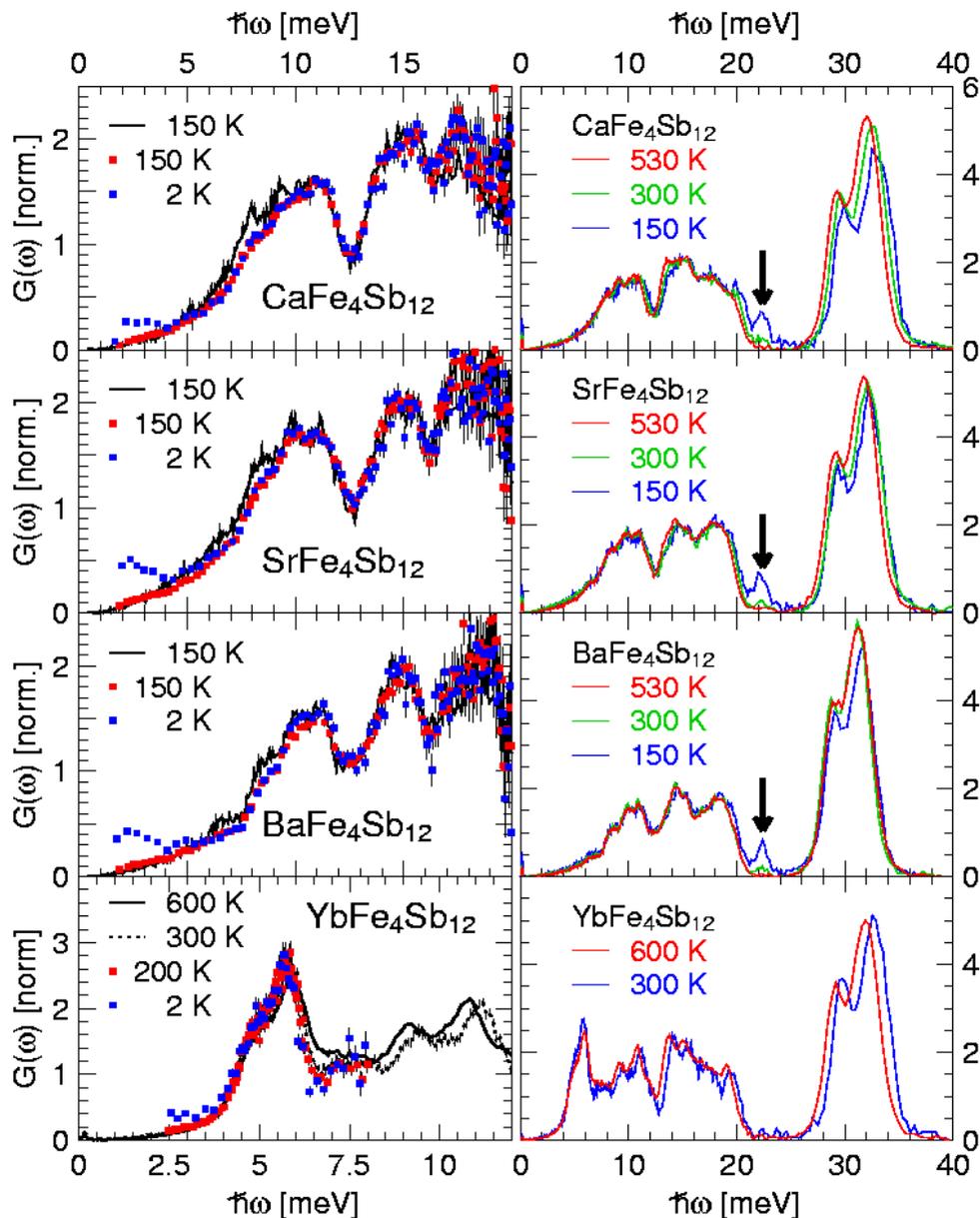
T. Dahm and K. Ueda, *PRL* 99, 187003 (2007)

Temperature dependence of  
guest-dominated modes

$$U(x) \approx A \cdot x^2 + B \cdot x^4$$

$$\beta \propto B/M^2/\omega_{00}^3$$





normal modes

$$\frac{\partial^2 U(x)}{\partial x^2} = \text{const.} \quad \frac{\Delta\omega}{\Delta T} = 0$$

thermal expansion

$$\frac{\partial^3 U(x)}{\partial x^3} < 0 \quad \frac{\Delta\omega}{\Delta T} < 0$$

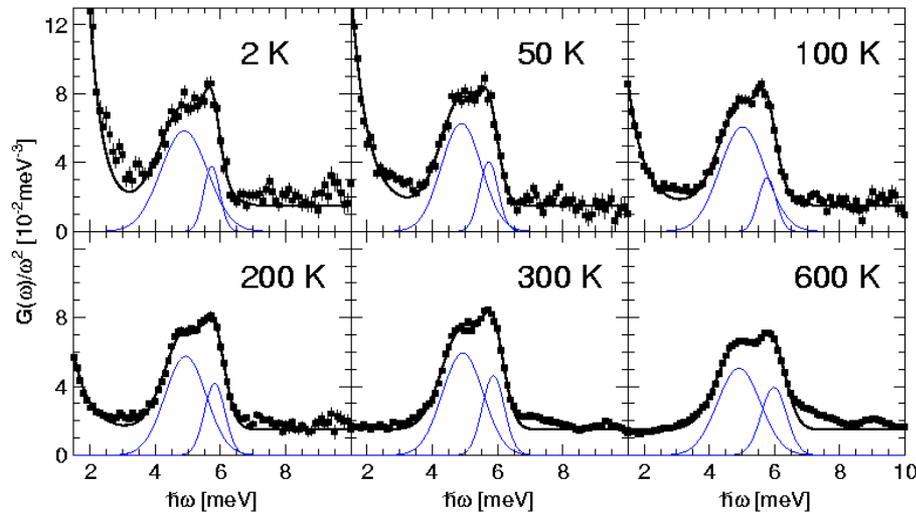
rattling term

$$\frac{\partial^4 U(x)}{\partial x^4} > 0 \quad \frac{\Delta\omega}{\Delta T} > 0$$

Energy shift between 300 and 550/600 K, ionic radii of cations and lattice parameters at 2 K

|                           | Ca   | Sr   | Ba   | Yb   |
|---------------------------|------|------|------|------|
| $\Delta\omega/\text{meV}$ | 0.6  | 0.2  | 0.1  | 0.8  |
| $R/\text{\AA}$            | 1.26 | 1.40 | 1.56 | 1.28 |
| $d/\text{\AA}$            | 9.14 | 9.16 | 9.18 | 9.13 |

Temperature dependence of Yb - dominated modes in  $\text{YbFe}_4\text{Sb}_{12}$  :

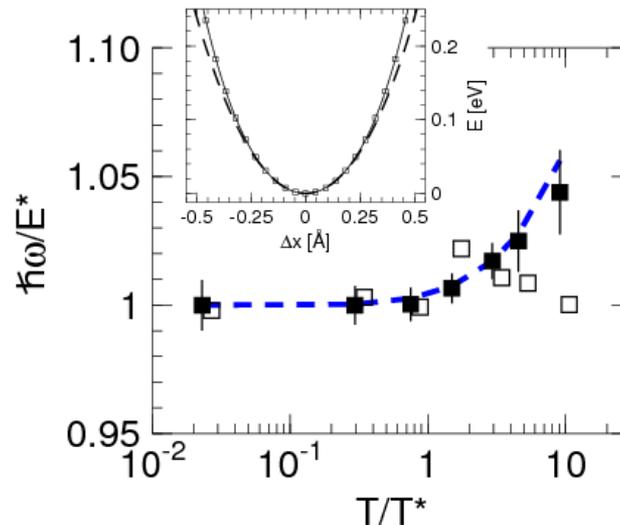


Relative changes of characteristic energies :

$$A = 0.88 \text{ eV/\AA}^2$$

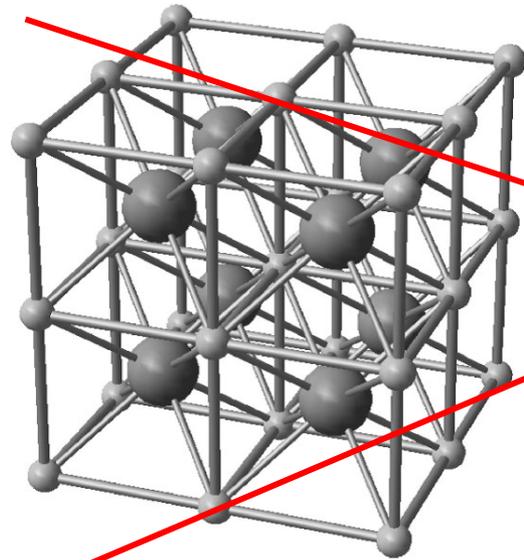
$$B = 1.16 \text{ eV/\AA}^4$$

$$\beta = 0.013(3)$$

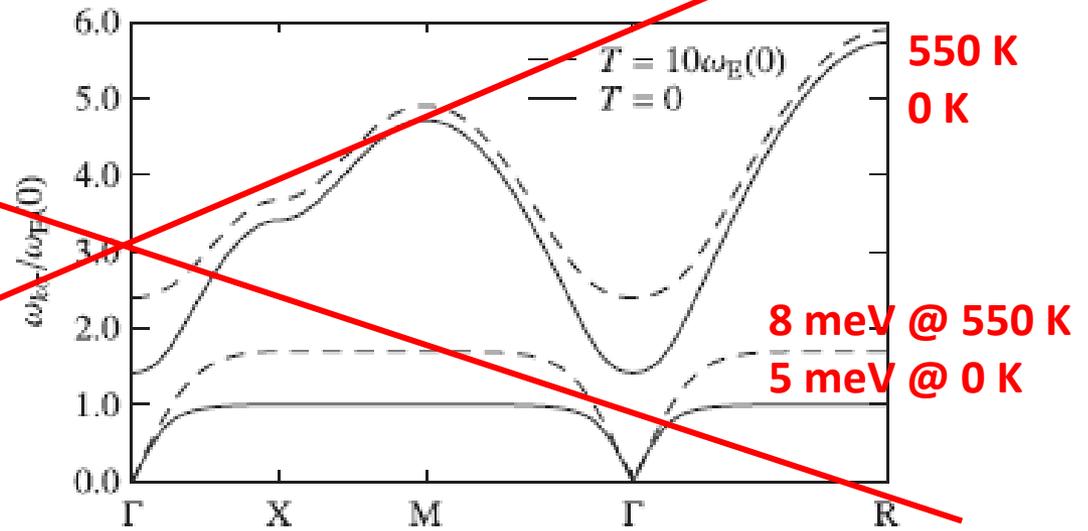


$$\frac{\Delta\omega}{\Delta T} > 0$$

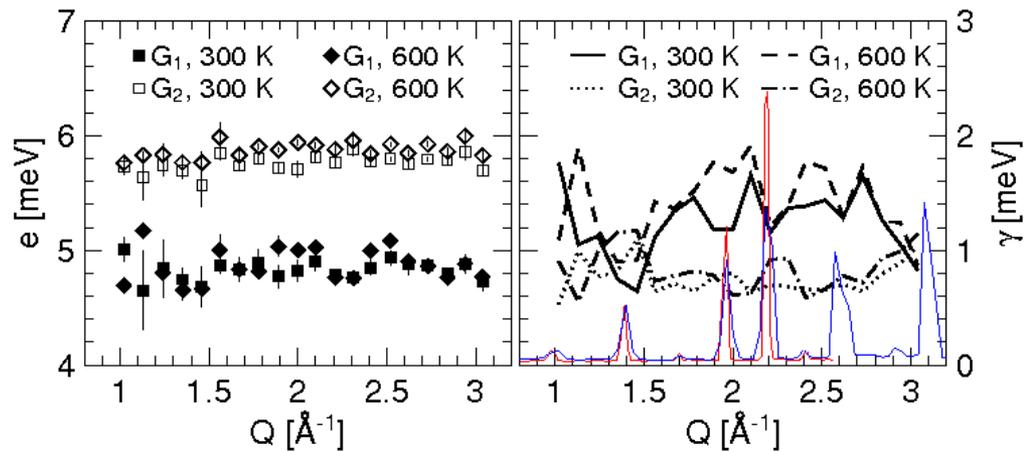
Modelling anharmonicity in filled skutterudites :



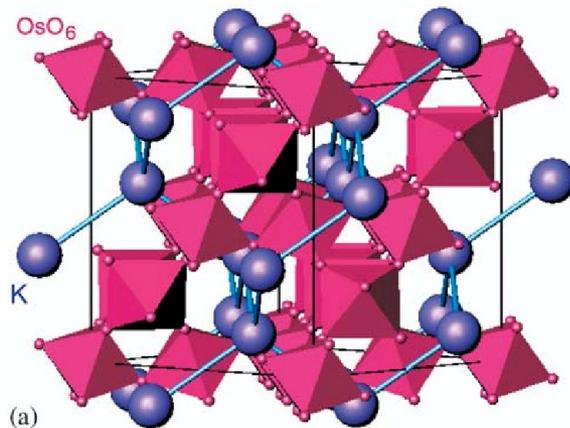
A. Yamakage et al., *J. Phys. Soc. Japan* 78, 064602 (2009).



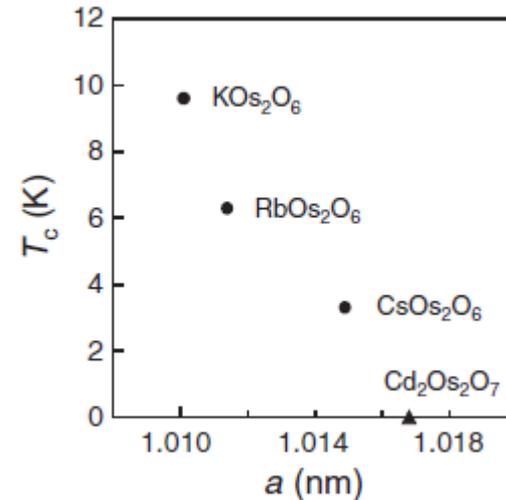
Q-resolved temperature dependence of low-energy modes derived from IN6@ILL data :



Beta – Pyrochlores :  $\text{MOs}_2\text{O}_6$ ,  $M = \text{K, Rb, Cs}$ , space group:  $\text{Fd-3m}$



Shigeki YONEZAWA\*, Yuji MURAOKA and Zenji HIROI  
 Journal of the Physical Society of Japan  
 Vol. 73, No. 7, July, 2004, pp. 1655–1656

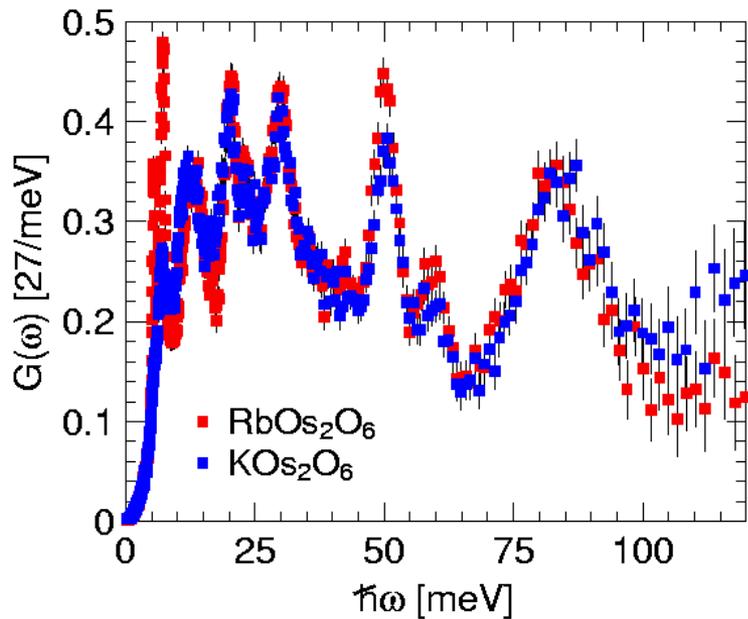


PHYSICAL REVIEW B 78, 104307 (2008)

**Generalized density-of-states and anharmonicity of the low-energy phonon bands from coherent inelastic neutron scattering response in the pyrochlore osmates  $\text{AOs}_2\text{O}_6$  ( $A = \text{K, Rb, Cs}$ )**

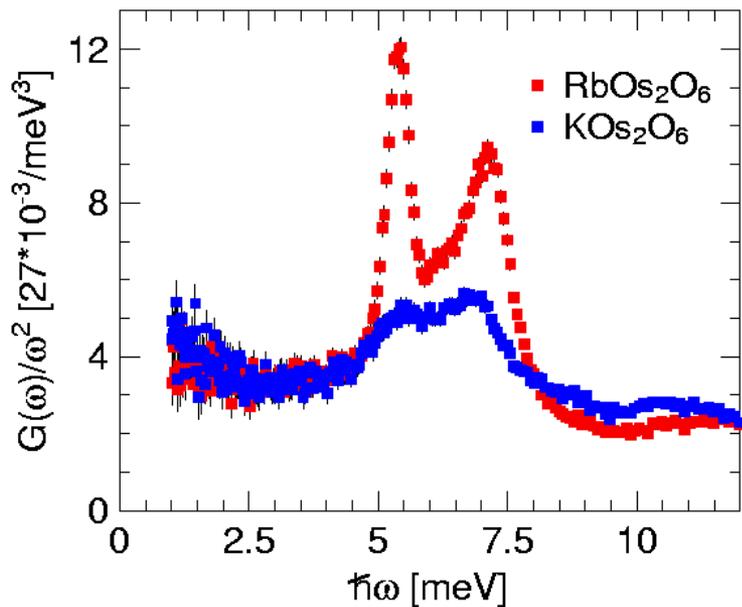
Hannu Mutka,\* Michael Marek Koza, and Mark Robert Johnson  
 Institut Laue Langevin, 6 Rue Jules Horowitz, B.P. 156, 38042 Grenoble Cedex 9, France

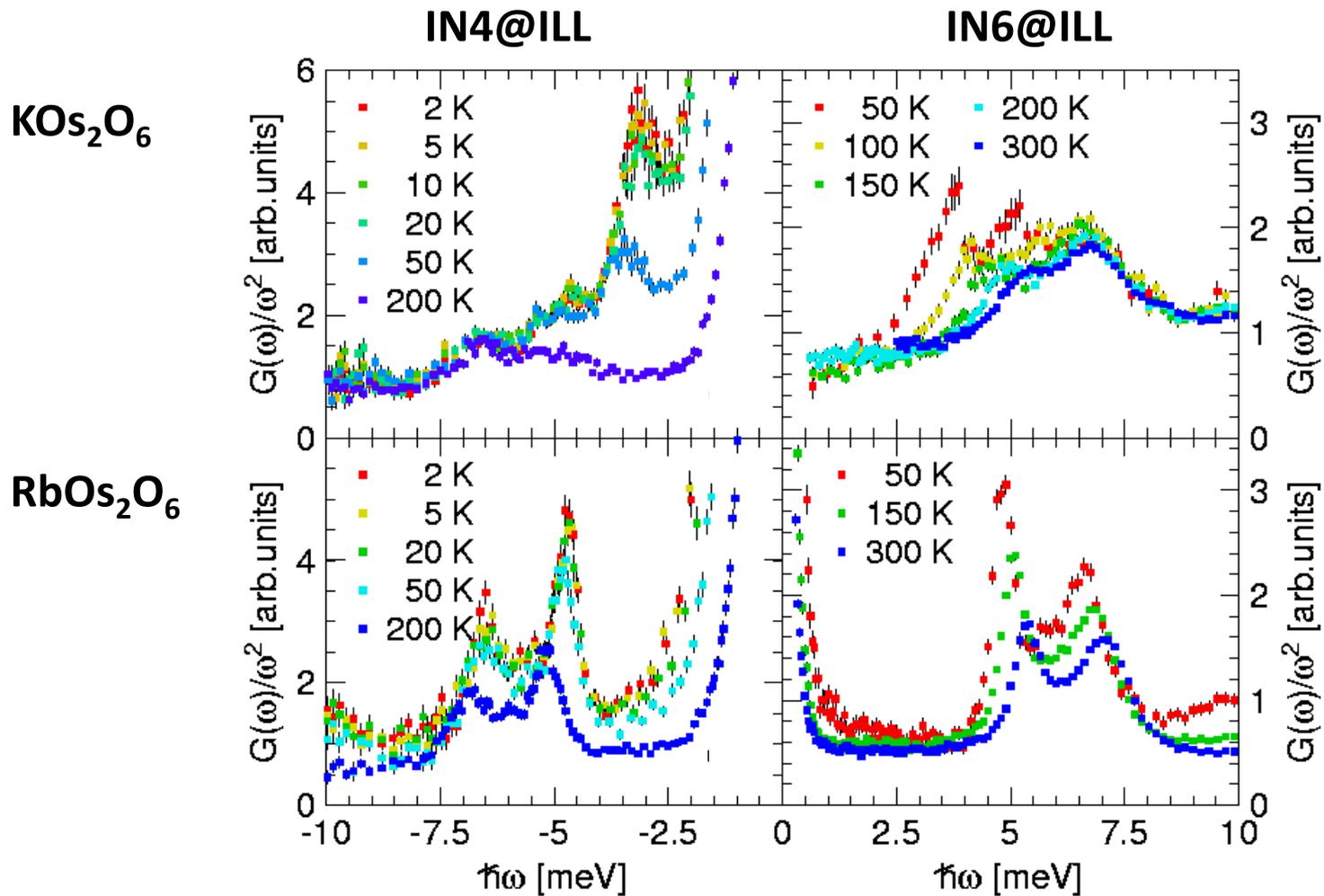
Zenji Hiroi, Jun-Ichi Yamaura, and Yohei Nagao  
 Institute for Solid State Physics, University of Tokyo, Kashiwanoha, Kashiwa, Chiba 277-8581, Japan  
 (Received 22 July 2008; published 30 September 2008)



INS contrast experiment at IN6@ILL

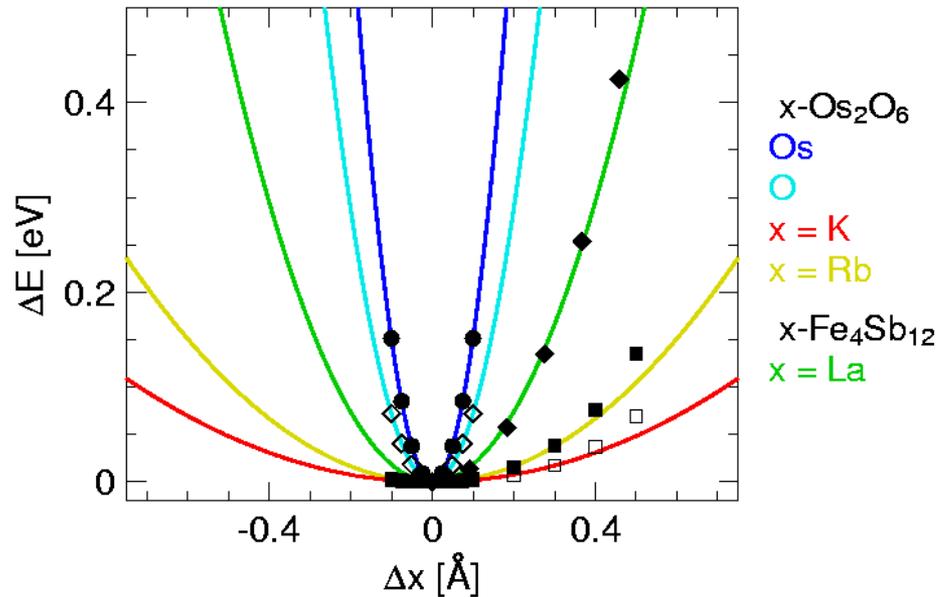
|    | $\sigma_{\text{tot}}$ [barns] | A.M.U. | S.P  |
|----|-------------------------------|--------|------|
| Os | 14.7                          | 190.2  | 7.7  |
| O  | 4.23                          | 16.0   | 26.4 |
| K  | 1.96                          | 39.1   | 5.0  |
| Rb | 6.8                           | 85.5   | 8.0  |





T. Dahm and K. Ueda, *PRL* 99, 187003 (2007)  
matching resistivity data :

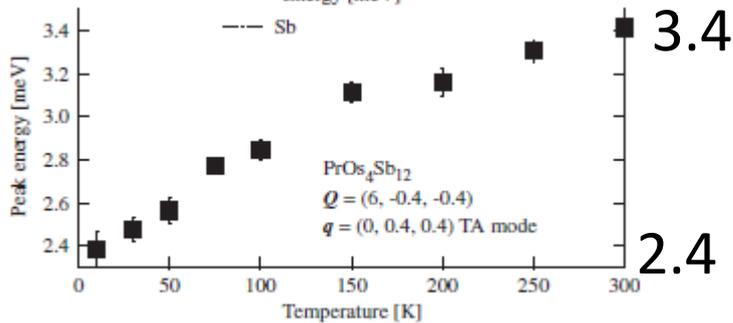
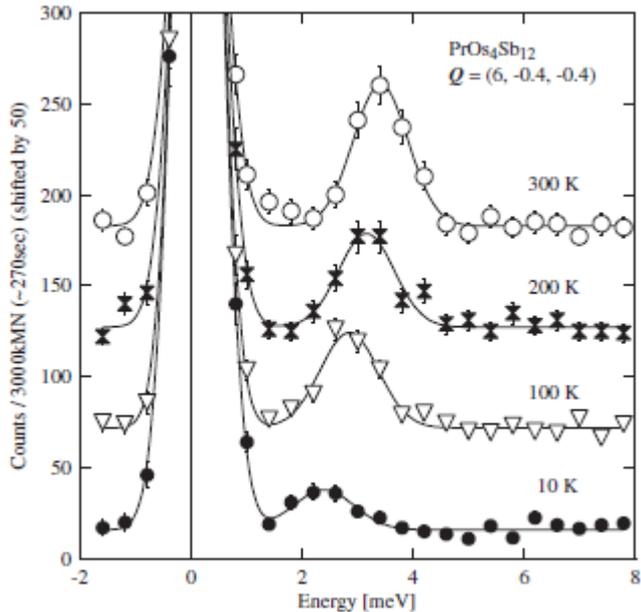
$$\beta = 6.3$$

Potentials calculated *ab initio*

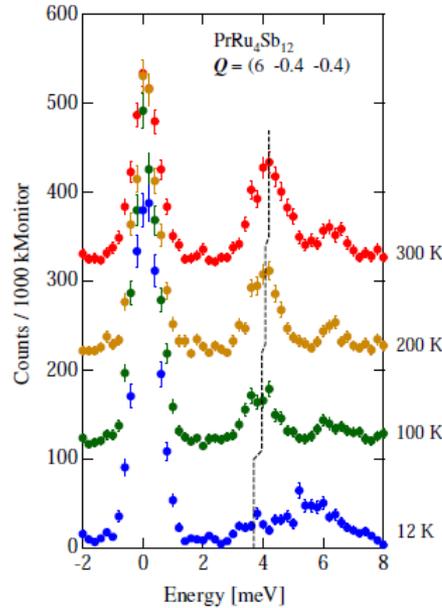
Restoring forces are non-isotropic in particular for Sb in  $M\text{Fe}_4\text{Sb}_{12}$ .

## Restoring Forces

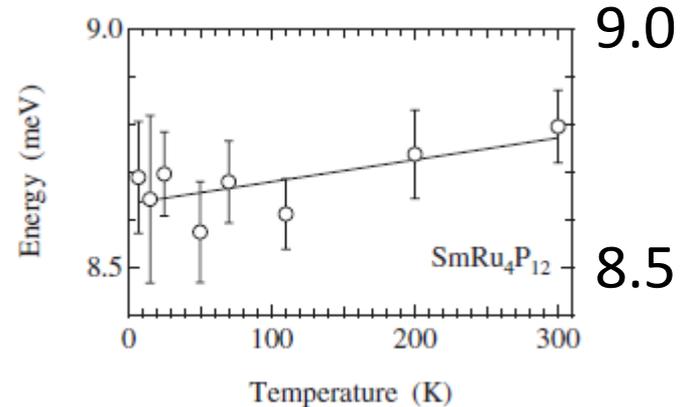
|    | K [eV/Å <sup>2</sup> ] |
|----|------------------------|
| Os | 15.1                   |
| O  | 7.2                    |
| K  | 0.2                    |
| Rb | 0.4                    |
| Ca | 0.9                    |
| Sr | 1.6                    |
| Ba | 2.5                    |
| La | 2.0                    |
| Yb | 0.9                    |
| Fe | 5.1-5.4                |
| Sb | 2.4-5.0                |



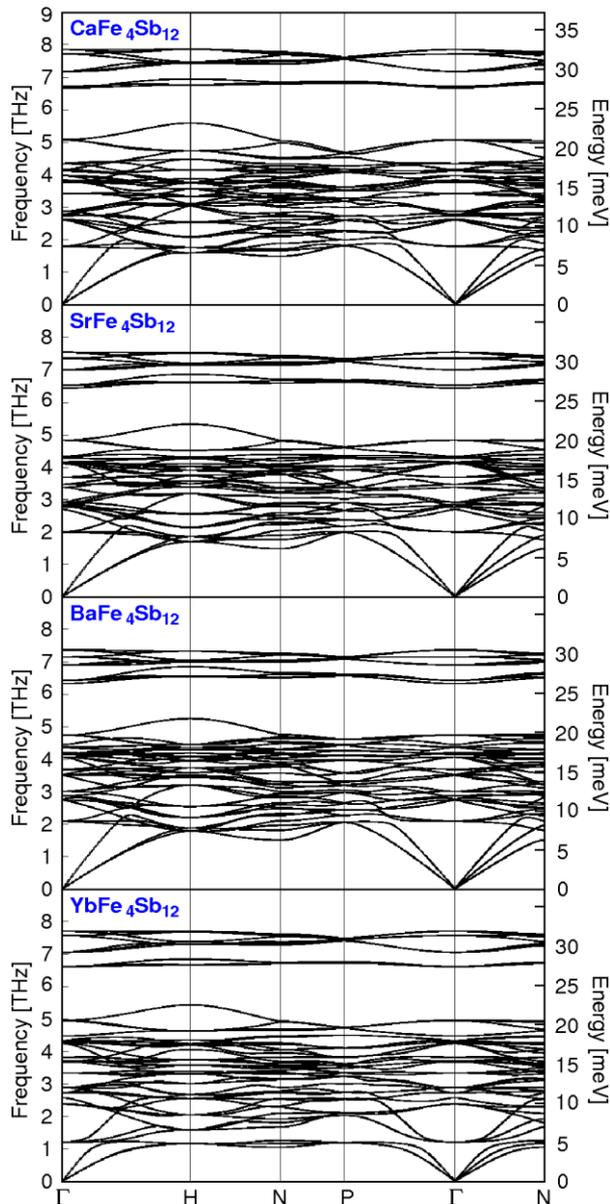
PrOs<sub>4</sub>Sb<sub>12</sub> by K.Iwasa et al.  
 Physica B 378-380, 194, (2006)



PrRu<sub>4</sub>Sb<sub>12</sub>  
 K.Iwasa et al.,  
 Journal of Physics:  
 Conference Series 92,  
 012122, (2007)



SmRu<sub>4</sub>P<sub>12</sub> by S.Tsutsui et al., J.  
 Phys. Soc. Jpn. 77, 33601, (2008)



The dynamics of different  $\text{MFe}_4\text{Sb}_{12}$  compounds ( $\text{M} = \text{Ca}, \text{Sr}, \text{Ba}, \text{La}, \text{Eu}, \text{Nd}, \text{Yb}, \text{Tl}$ ) can be well reproduced by a set of normal modes.

Low energy modes of  $\text{M} = \text{Ca}, \text{Sr}, \text{Ba}, \text{La}, \text{Nd}, \text{Yb}$  containing compounds do not show appreciable anharmonic behaviour.

For  $\text{YbFe}_4\text{Sb}_{12}$  low-energy modes indicate the significance of a quartic anharmonic term.

N. Bernstein et al., *Phys. Rev. B* 81, 134301, (2010)

| Model                      | A-Fe  | A-Sb  | A-La            | B-La           | $\alpha$        | $\alpha^*$ | $\gamma^*$  | $v_s^*$ |  |
|----------------------------|-------|-------|-----------------|----------------|-----------------|------------|-------------|---------|--|
| Unfilled-FSM-cf            | 1.63  | 2.14  |                 |                | 6.71            | 7.76       | 1.48        | 2836    |  |
| Filled-FSM-cf              | 1.80  | 2.06  | $5.03 \pm 0.09$ | $4.1 \pm 1.6$  | 9.31(7.95,7.41) | 10.9(9.73) | 2.00 (1.78) | 2813    | Feldman et al. <i>PRB</i> 2000                 |
| Unfilled-FS-cf             | 1.54  | 2.00  |                 |                | 5.26            | 6.29       | 1.33        | 2932    |  |
| Filled-FS-cf               | 1.67  | 1.92  | $5.10 \pm 0.03$ | $-7.3 \pm 0.4$ | 7.53            | 8.80       | 1.84        | 2908    | Feldman et al. <i>PRB</i> 1996                 |
| Unfilled-LK                | 2.10  | 2.28  |                 |                | 1.74            | 3.46       | 0.68        | 2462    |  |
| Filled-LK                  | 2.32  | 2.13  | $5.14 \pm 0.02$ | $-6.4 \pm 0.4$ | 4.28            | 6.49       | 1.24        | 2383    | Feldman et al. <i>PRB</i> ...                  |
| Filled-LDA                 | 1.55* | 1.94* | 5.12*           |                |                 |            |             | 2912    |  |
| Unfilled Exp. <sup>a</sup> |       |       |                 |                |                 | 6.36       | 0.95,1.5    | 3082    |  |
| Filled Exp. <sup>b</sup>   | 1.14  | 1.79  | 4.69            |                |                 | 9          | 1.5         | 2946    | Chakoumakos et al.<br><i>AC</i> 55, 341 (1995) |

Thermodynamic expansivity  $\alpha$ , Gruneisen parameter  $\gamma$  and velocity of sound  $v$  at  $T = 300$  K from MD and LD calculations.

## Result:

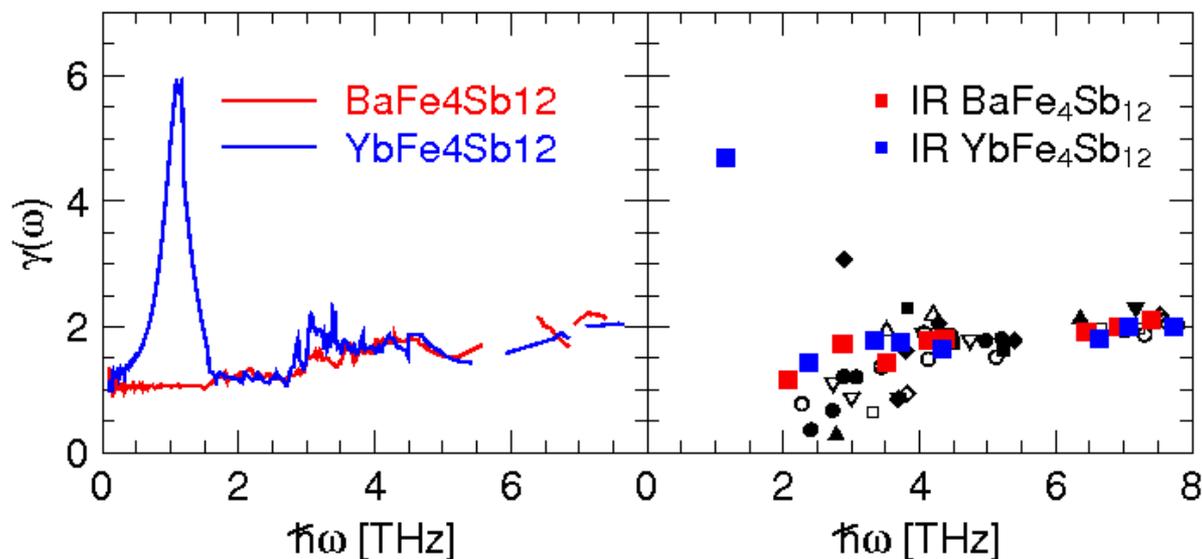
- Simulated  $\text{LaFe}_4\text{Sb}_{12}$  exhibits a thermal conductivity reduced by a factor of 2 in comparison to  $\text{Co}_4\text{Sb}_{12}$ .
- A reduction by 5 is correct.
- However, anharmonicity - cubic as well as quartic - is required to approximate the perturbation of lattice thermal conductivity.

N. Bernstein et al., *Phys. Rev. B* 81, 134301, (2010)

| Model                      | A-Fe  | A-Sb  | A-La            | B-La           | $\alpha$        | $\alpha^*$ | $\gamma^*$  | $v_s^*$ |
|----------------------------|-------|-------|-----------------|----------------|-----------------|------------|-------------|---------|
| Unfilled-FSM-cf            | 1.63  | 2.14  |                 |                | 6.71            | 7.76       | 1.48        | 2836    |
| Filled-FSM-cf              | 1.80  | 2.06  | $5.03 \pm 0.09$ | $4.1 \pm 1.6$  | 9.31(7.95,7.41) | 10.9(9.73) | 2.00 (1.78) | 2813    |
| Unfilled-FS-cf             | 1.54  | 2.00  |                 |                | 5.26            | 6.29       | 1.33        | 2932    |
| Filled-FS-cf               | 1.67  | 1.92  | $5.10 \pm 0.03$ | $-7.3 \pm 0.4$ | 7.53            | 8.80       | 1.84        | 2908    |
| Unfilled-LK                | 2.10  | 2.28  |                 |                | 1.74            | 3.46       | 0.68        | 2462    |
| Filled-LK                  | 2.32  | 2.13  | $5.14 \pm 0.02$ | $-6.4 \pm 0.4$ | 4.28            | 6.49       | 1.24        | 2383    |
| Filled-LDA                 | 1.55* | 1.94* | $5.12^*$        |                |                 |            |             | 2912    |
| Unfilled Exp. <sup>a</sup> |       |       |                 |                |                 | 6.36       | 0.95,1.5    | 3082    |
| Filled Exp. <sup>b</sup>   | 1.14  | 1.79  | 4.69            |                |                 | 9          | 1.5         | 2946    |



LD calculated mode Gruneisen parameter  $[\text{dln}(w)/\text{dln}(V)]$  :



Optimized harmonic dynamics

Optimization of the anharmonic content

- Our approach to the problem of powdered materials : different scattering techniques, *ab initio* + PALD calculations, complementary experimental tools.
- Experimental results on filled skutterudites can be reproduced by *ab initio* lattice dynamics calculations.
- Response to T and p of low-energy modes indicates the relevance of a quartic (rattling) term in bonding potential. However, effect is not strongly pronounced and not globally present!
- Cubic and quartic anharmonic terms can be tuned appreciably in  $\text{MOs}_2\text{O}_6$  and  $\text{MV}_2\text{Al}_{20}$  by cation M variation.
- Computation work requires super-cells of 200-500 atoms to match the inelastic response from experiments limiting other .
- We are more or less prepared to approach the thermal conductivity modelling .

**Thank You for Your Attention !**