# Hund's metals: overview, NRG insights, and the role of spin-orbit coupling

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## Outline

- Hund's metals
  - Ruthenates before 2009
  - J steps in ; Basic picture: Janus
  - Hund's impurity model : RG,NRG results
  - Characterizing the incoherent state
- Role of SOC

#### People

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#### Ruthenates before 2009

### Sr<sub>2</sub>RuO<sub>4</sub>: el. structure



In ionic picture, 4 electrons on Ru; crystal field splitting  $\rightarrow$ t<sub>2g</sub> orbitals: xy and degenerate xz, yz

Wide xy band ( $\Upsilon$  sheet)

Fermi surfaces of DFT, quantum oscillations, ARPES **agree well**  Oguchi, PRB'95 Singh, PRB'95



Damascelli, Shen et al., PRL'00

Mackenzie et al, PRL'96

	α	β	γ
Frequency $F(kT)$	3.05	12.7	18.5
Average $k_F$ (Å <sup>-1</sup> )	0.302	0.621	0.750
$\Delta k_F / k_F (\%)$	0.21	1.3	< 0.9
Cyclotron mass $(m_e)$	3.4	6.6	12.0
Band calc. $F(kT)$	3.4	13.4	17.6
Band calc. $\Delta k_F/k_F$ (%)	1.3	1.1	0.34
Band mass $(m_e)$	1.1	2.0	2.9

#### Low coherence scale in ruthenates

 U<W, yet strong correlations : large mass, coherence-incoherence crossover at low T\* &



## Other ruthenates:also FL at a low T, and bad-metal/NFL behavior above





#### Capogna et al. PRL'02

#### Table 2 Ruthenates in a nutshell<sup>a</sup>

#### Lee et al. PRB'02

Compound	Magnetic order	$\gamma/\gamma_{\rm LDA}$	$\rho \propto T^2$	Remarks
Sr <sub>2</sub> RuO <sub>4</sub>	PM	4	<25 K	Unconventional SC $< 1.5$ K
SrRuO <sub>3</sub>	FM < 160 K	4	<15 K	$\sigma \propto \omega^{-0.5}$
Sr <sub>3</sub> Ru <sub>2</sub> O <sub>7</sub>	PM	10	<10 K	Metamagnetic quantum-critical point and nematicity
CaRuO <sub>3</sub>	РМ	7	$T^{1.5}>2\ K$	$\sigma \propto \omega^{-0.5},  \gamma = \gamma_{\mathrm{FL}} + \log(T)$

Georges • de' Medici • Mravlje

#### Annu. Rev. Condens. Matter Phys. 2013. 4:137-78

3d -> 4d

Orbitals become extended

Ruthenates : U~2eV W~2-3eV

Al Ka

SrRuO.

(a)

Interactions diminish



Evidence against strong correlation in 4d transition-metal oxides CaRuO<sub>3</sub> and SrRuO<sub>3</sub>

Kalobaran Maiti\* and Ravi Shankar Singh

Although  $Sr_2RuO_4$  is a fully confirmed Fermi liquid at low temperatures, its properties at temperatures of approximately 30 K and above are more anomalous, raising the question of what should set such a low "crossover" scale in a material with a relatively high Fermi temperature of greater than 1000 K [12]. The behavior of  $Sr_3Ru_2O_7$  was

in thin films of CaRuO<sub>3</sub> [15]. The situation in SrRuO<sub>3</sub> is more interesting still. High frequency measurements at relatively elevated temperatures suggested an anomalous  $\sqrt{\omega}$  frequency dependence, leading to the proposal of a non-Fermi liquid metallic state [16]. Observation of a  $T^2$ 

L. Capogna,<sup>1</sup> A. P. Mackenzie,<sup>1,2</sup> R. S. Perry,<sup>1</sup> S. A. Grigera,<sup>1,2</sup> L. M. Galvin,<sup>1</sup> P. Raychaudhuri,<sup>1</sup> and A. J. Schofield<sup>1</sup> <sup>1</sup>School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, United Kingdom <sup>2</sup>School of Physics and Astronomy, University of St. Andrews, North Haugh, St. Andrews, Fife KY16 9SS, Scotland

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The three-orbital, projected Hamiltonian together with the *ab initio* Coulomb interaction parameters were used as input for the QMC simulation of the effective quantum impurity problem arising in the DMFT. The simulations were performed for an inverse temperature  $\beta=10 \text{ eV}^{-1}$  using 40 imaginary time slices ( $\Delta \tau=0.25$ ). Although the temperature chosen for the QMC calculations appears to be rather high, it is really sufficiently low, because it is much smaller than (i) the lowest atomic excitations (see the Appendix) and (ii) the characteristic low-energy scale ~0.5 eV obtained from the

<sup>15</sup>Z. V. Pchelkina, I. A. Nekrasov, Th. Pruschke, A. Sekiyama, S. Suga, V. I. Anisimov, and D. Vollhardt, Phys. Rev. B **75**, 035122 (2007).

#### Comment on "Evidence for strong electronic correlations in the spectra of Sr<sub>2</sub>RuO<sub>4</sub>"

D. J. Singh

Materials Science and Technology Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-6114, USA (Received 26 March 2007; published 30 January 2008)

It is pointed out that O 2*p* states dominate the electronic structure of  $Sr_2RuO_4$  in the  $\sim -3$  eV region and can explain the observation of a peak in the density of states in photoemission experiments. This contradicts claims that a lower Hubbard band is needed at 3 eV binding energy.

To summarize, accurate LDA calculations show a prominent DOS peak in the around 3 eV binding energy. This corresponds well with a peak observed in photoemission experiments. Therefore, it may be concluded that the lower Hubbard band arising from Coulomb correlations, which was claimed to be essential to explain the observed spectra, may not be needed after all. While the possibility of important Coulomb correlations is not excluded, it seems that the evidence for strong electronic correlations in the spectra of Sr<sub>2</sub>RuO<sub>4</sub>, due to Hubbard interactions claimed in Ref. 15, is not yet established. However, there are other correlation effects that can lead to mass renormalization, including interaction with itinerant electron spin fluctuations,<sup>23,24</sup> which as mentioned, may be important in  $Sr_2RuO_4$ , and which have been shown to be able to yield renormalizations of the observed magnitude using realistic parameters.<sup>11,12</sup> Besides, the physics of 4*d* ox-

<sup>&</sup>lt;sup>15</sup>Z. V. Pchelkina, I. A. Nekrasov, Th. Pruschke, A. Sekiyama, S. Suga, V. I. Anisimov, and D. Vollhardt, Phys. Rev. B **75**, 035122 (2007).

-Im ∑/t

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#### Spin Freezing Transition and Non-Fermi-Liquid Self-Energy in a Three-Orbital Model

Philipp Werner,<sup>1</sup> Emanuel Gull,<sup>2</sup> Matthias Troyer,<sup>2</sup> and Andrew J. Millis<sup>1</sup>

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In conclusion, we have shown that in a model, relevant to transition metal oxides with partly filled *d*-shells, with several electrons in a threefold degenerate level, an apparent spin-freezing transition occurs. While it is possible that the effects could be due to a rapid decrease of the spin coherence scale to values below the range accessible to us, the square-root self-energy and T-linear spin-spin correlation function are strong evidence for an actual T = 0transition. The frozen-moment phase results from a calcu-

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#### Correlated Electronic Structure of $LaO_{1-x}F_xFeAs$

K. Haule, J. H. Shim, and G. Kotliar

In conclusion, we studied the band structure of the newly discovered superconductor  $LaO_{1-x}F_xFeAs$ , and we predict the orbital and momentum resolved spectral function and optical conductivity of the compound. Density functional theory predicts that a set of Fe 3*d* bands are crossing the Fermi level with no clear splitting into the  $e_g$  and  $t_{2g}$  manifold. The Coulomb correlations among the six electrons in the set of five Fe-3*d* orbitals is strong enough to push the compound close to the metal insulator transition.

#### Different way of looking at ruthenates and pnictides was needed

#### J steps in: Janus

#### Coherence scale drops due to Hund's rule coupling J

- LDA+DMFT applied to Sr<sub>2</sub>RuO<sub>4</sub>
- T<sup>\*</sup> determined from T-dep of Γ=-Z ImΣ(0)
- T\* suppresed by J !



J [eV]	$m^*/m_{\rm LDA} _{xy}$	$m^*/m_{\text{LDA}} _{xz}$	$T_{xy}^*$ [K]	$T_{xz}^*$ [K]	<i>T</i> <sub>&gt;</sub> [K]
0.0, 0.1	1.7	1.7	>1000	>1000	>1000
0.2	2.3	2.0	300	800	>1000
0.3	3.2	2.4	100	300	500
0.4	4.5	3.3	60	150	350

Mravlje et al. PRL'11

Masses in agreement with quantum oscillations & specific heat at physical value of J

Т

### Low T\*/E\* in experimental observables

Temperature dependences of NMR



Excellent agreement. Only if J is properly included.



Mravlje et al. PRL'11



## In a restaurant in Paris, Luca told me, J reduces effective repulsion

• Effective interaction

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U_{eff} = E(N+1) + E(N-1) - 2 E(N)
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- U-3J away from half-filling (eff. U diminished by J)
- U+2J at half filling (eff. U **increased** by J)
- Slater all d-states Hamiltonian

	Effective Coulomb interaction $U_{eff}$ for Hund's rule ground-state						
	Full Hamilt	Simple	Kanamori	Kanamori mean field			
$d^1$	$F^{0} - \frac{8}{49}F^{2} - \frac{9}{441}F^{4}$	U <sub>0</sub> - J <sub>H</sub> -C	$U_0-J_H$	U'- J	U'- J		
$d^2$	$F^0 + \frac{1}{49}F^2 - \frac{54}{441}F^4$	$U_0$ - $J_H$ +C	$U_0-J_H$	U'- J	U'- J		
$d^3$	$F^0 + \frac{1}{49}F^2 - \frac{54}{441}F^4$	$U_0$ - $J_H$ +C	$U_0-J_H$	U'- J	U'- J		
$d^4$	$F^{0} - \frac{8}{49}F^{2} - \frac{9}{441}F^{4}$	U <sub>0</sub> - J <sub><math>H</math></sub> -C	$\mathbb{Q}_0$ - $\mathbb{J}_H$	U'- J	U'- J		
$d^5$	$F^0 + \frac{14}{49}F^2 + \frac{126}{441}F^4$	$U_0 + 4J_H$	$U_0 + 4J_H$	U+4J	U+4J		
$d^6$	$F^{0}-\frac{8}{49}F^{2}-\frac{9}{441}F^{4}$	U <sub>0</sub> - J <sub><math>H</math></sub> -C	$U_0-J_H$	U'- J	U'- J		
$d^7$	$F^0 + \frac{1}{49}F^2 - \frac{54}{441}F^4$	U <sub>0</sub> - J <sub>H</sub> +C	$U_0-J_H$	U'- J	U'- J		
$d^8$	$F^0 + \frac{1}{49}F^2 - \frac{54}{441}F^4$	U <sub>0</sub> - J <sub>H</sub> +C	$U_0$ - $J_H$	U'- J	U'- J		
$d^9$	$F^{0} - \frac{8}{49}F^{2} - \frac{9}{441}F^{4}$	U <sub>0</sub> - J <sub>H</sub> -C	U <sub>0</sub> - J <sub><math>H</math></sub>	U'- J	U'- J		

van der Marel, Sawatzky PRB'88 de'Medici PRB'11

### Two weeks later in Antoine's office at Ecole Polytechnique

- Luca says: Ueff reduced → insulator further away
- Jernej says: coherence scale is reduced, thing becomes more correlated
- Antoine says: you guys might be both right!

#### DMFT : Hund's metals

• Quasiparticle weight Z



- Why such behavior?
- A fruitful line of thinking is to consider it as a doped half filled Mott ins.



de'Medici, Giovannetti, Capone, PRL'14 de'Medici, Hasan, Capone, Dai, PRL'09 Ishida, Liebsch, PRB'10 Misawa,Nakamura, Imada, PRL'12

 Here, I will be discussing insights from impurity models, instead

#### Insights from Hund's impurity model investigation: RG,NRG results

#### Kanamori-Kondo model

$$H_{\rm imp} = \frac{1}{2}(U - 3J)N_d(N_d - 1) - 2J\mathbf{S}^2 - \frac{J}{2}\mathbf{L}^2$$

Schrieffer-Wolff

$$H_{\rm K} = -P_n H_{\rm hyb} \left( \sum_{a} \frac{P_{n+1}^a}{\Delta E_{n+1}^a} + \sum_{b} \frac{P_{n-1}^b}{\Delta E_{n-1}^b} \right) H_{\rm hyb} P_n$$

Kondo model

$$\begin{array}{l} H_K = J_p N_f + J_s \mathbf{S} \cdot \mathbf{s} + J_l \mathbf{L} \cdot \mathbf{l} + J_q \mathbf{Q} \cdot \mathbf{q} + \\ J_{ls}(\mathbf{L} \otimes \mathbf{S}) \cdot (\mathbf{l} \otimes \mathbf{s}) + J_{qs}(\mathbf{Q} \otimes \mathbf{S}) \cdot (\mathbf{q} \otimes \mathbf{s}) \end{array}$$
 For Nd=2  $\rightarrow$  S=1, L=1

$$Q_{i,j}^{bc} = \frac{1}{2} \left( L_{i,m}^b L_{m,j}^c + L_{i,m}^c L_{m,j}^b \right) - \frac{2}{3} \delta_{b,c} \delta_{i,j}$$
$$\operatorname{Tr}(Q^{\alpha} Q^{\beta}) = 2 \delta_{\alpha,\beta}$$

Horvat, Zitko, Mravlje PRB'16

Yin, Haule, Kotliar PRB'12 Aron, Kotliar PRB'15 Stadler et al. PRL'15 Stadler et al.Annals of Phys.'19

## Insights from impurity problem: Js small or even ferromagnetic

• Schrieffer-Wolf| $H_{\rm K} = -P_n H_{\rm hyb} \left( \sum_a \frac{P_{n+1}^a}{\Delta E_{n+1}^a} + \sum_b \frac{P_{n-1}^b}{\Delta E_{n-1}^b} \right) H_{\rm hyb} P_n$ 

SU(2) angular momentum sym.

$$H_{\rm imp} = \frac{1}{2}(U - 3J)N_d(N_d - 1) - 2J\mathbf{S}^2 - \frac{J}{2}\mathbf{L}^2$$

For Nd=2  $\rightarrow$  S=1, L=1  $H_K = J_p N_f + J_s \mathbf{S} \cdot \mathbf{s} + J_l \mathbf{L} \cdot \mathbf{l} + J_q \mathbf{Q} \cdot \mathbf{q} + J_{ls} (\mathbf{L} \otimes \mathbf{S}) \cdot (\mathbf{l} \otimes \mathbf{s}) + J_{qs} (\mathbf{Q} \otimes \mathbf{S}) \cdot (\mathbf{q} \otimes \mathbf{s})$ 

$$Q_{i,j}^{bc} = \frac{1}{2} \left( L_{i,m}^b L_{m,j}^c + L_{i,m}^c L_{m,j}^b \right) - \frac{2}{3} \delta_{b,c} \delta_{i,j}$$
$$\operatorname{Tr}(Q^{\alpha} Q^{\beta}) = 2\delta_{\alpha,\beta}$$

Horvat, Žitko, Mravlje PRB'16

SU(3) angular momentum sym.

$$H_{\rm imp} = \frac{1}{2} \left( U - 3J \right) N_d (N_d - 1) - 2J \mathbf{S}^2$$

For Nd=2 
$$\rightarrow$$
 S=1, T fund.rep  
of SU(3)  
$$H_K^{DN} = J_p N_f + J_s \mathbf{S} \cdot \mathbf{s} + J_t \mathbf{T} \cdot \mathbf{t} + J_{ts} (\mathbf{T} \otimes \mathbf{S}) \cdot (\mathbf{t} \otimes \mathbf{s})$$

Yin, Haule, Kotliar PRB'12 Aron, Kotliar PRB'15 Stadler et al. PRL'15

Js small or even ferromagnetic! (ferromagnetic Kondo leads to unscreened moments)  Why ferromagnetic? Fluctuations to N=3 (halffilled) states prefer ferromagnetic arrangement [in contrast to single-orbital!]



#### Kondo coupling constants



- Small splittings between quadrupole and orbital terms.
- Mixed terms are strongest.

Yin, Haule, Kotliar PRB'12 Aron, Kotliar PRB'15 Stadler et al. PRL'15 Horvat, Mravlje, Zitko PRB'16

### RG



$$\beta_i = \mathrm{d}J_i/\mathrm{d}\ln(D)$$

$$H_K = J_p N_f + J_s \mathbf{S} \cdot \mathbf{s} + J_l \mathbf{L} \cdot \mathbf{l} + J_q \mathbf{Q} \cdot \mathbf{q} + J_{ls}(\mathbf{L} \otimes \mathbf{S}) \cdot (\mathbf{l} \otimes \mathbf{s}) + J_{qs}(\mathbf{Q} \otimes \mathbf{S}) \cdot (\mathbf{q} \otimes \mathbf{s})$$

$$\begin{split} \beta_{p} &= 0, \\ \beta_{s} &= -\frac{1}{9} \left( 3J_{ls}^{2} + 5J_{qs}^{2} + 9J_{s}^{2} \right), \\ \beta_{l} &= -\frac{1}{16} \left( 4J_{l}^{2} + 3J_{ls}^{2} + 5 \left( 4J_{q}^{2} + 3J_{qs}^{2} \right) \right), \\ \beta_{q} &= -\frac{3}{8} (4J_{l}J_{q} + 3J_{ls}J_{qs}), \\ \beta_{ls} &= -\frac{1}{6} (3J_{l}J_{ls} + 5J_{ls}J_{qs} + 12J_{ls}J_{s} + 15J_{q}J_{qs}), \\ \beta_{qs} &= -\frac{1}{12} (J_{qs}(18J_{l} + 7J_{qs} + 24J_{s}) + 3J_{ls}^{2} + 18J_{ls}) \\ \end{split}$$

- J<sub>s</sub>,J<sub>l</sub> influenced by mixed terms
- If  $J_{ls}=J_{qs}=0$ , faster running of  $J_l$  in SU(3) limit  $(J_l=J_q)$
- Dynamic restoration of symmetry  $J_{I} \rightarrow J_{q}$  when they differ initially.



#### NRG results

• Kanamori impurity with NRG [S and L SU(2) symmetries]

$$H_{\rm imp} = \frac{1}{2}(U - 3J)N_d(N_d - 1) - 2J\mathbf{S}^2 - \frac{J}{2}\mathbf{L}^2.$$

• Distinct scales for screening of S and L





- Suppression of (both)  $T_{\kappa} with \ J$
- Similar results for Kanamori, Dworin-Narath,Kondo-Kanamori,

Horvat, Žitko, Mravlje PRB'16 Okada, Yosida, PTP'73 Yin, Haule, Kotliar PRB'12



Horvat, Zitko, Mravlje PRB'16

NRG for 3orbital Kanamori Hamiltonian

## LDA+DMFT on Sr<sub>2</sub>RuO<sub>4</sub>

 This behavior found also in DMFT: Hund's metal = quenched orbitals / slowly fluctuating spins



#### **Consequence for photoemission**

• Quasiparticle peak has structure:





## Quasiparticle part of the spectra;DMFT semicirc. J/U=1/6



Observed in optical study of Sr<sub>2</sub>RuO<sub>4</sub> Stricker,JM et al. PRL'14

#### Two-stage decoherence

Entropy in Sr<sub>2</sub>RuO<sub>4</sub> from LDA+DMFT



**PRL'16** 

#### But is this correct?

- Should one associate log 3 entropy with fluctuating spin state?That is, is the intermediate state a fluctuating spin and fully screened orbital?
- How to characterize this properly?

$$H_K = J_p N_f + J_s \mathbf{S} \cdot \mathbf{s} + J_l \mathbf{L} \cdot \mathbf{l} + J_q \mathbf{Q} \cdot \mathbf{q} + J_{ls} (\mathbf{L} \otimes \mathbf{S}) \cdot (\mathbf{l} \otimes \mathbf{s}) + J_{qs} (\mathbf{Q} \otimes \mathbf{S}) \cdot (\mathbf{q} \otimes \mathbf{s})$$





• 
$$\mathbf{J}_{s}, \mathbf{J}_{ts}, \mathbf{J}_{p} \rightarrow \mathbf{0}$$
;  $\mathbf{H} = \mathbf{J}_{t} \mathbf{T} \mathbf{*} \mathbf{t}$   
 $H_{K}^{DN} = J_{p}N_{f} + J_{s}\mathbf{S} \cdot \mathbf{s} + J_{t}\mathbf{T} \cdot \mathbf{t} + J_{ts}(\mathbf{T} \otimes \mathbf{S}) \cdot (\mathbf{t} \otimes \mathbf{s})$ 

- SU(3) T (in fund. rep.) coupled to two (spin up,down) channels of cond els with SU(3)
- Intermediate NFL fixed point is two-channel overscreened SU(3): N=3,K=2  $S_{imp}=\ln \prod_{n=1}^{Q} \frac{\sin[\pi(N+1-n)/(N+K)]}{\sin[\pi n/(N+K)]}$ .

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Overscreened multichannel SU(N) Kondo model: Large-N solution and conformal field theory

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> Gabriel Kotliar and Anirvan Sengupta Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08854 (Received 20 November 1997)



## Is this seen for parameters for Anderson Hamiltonian, too?

 Marginaly – plateau cannot clearly be seen as the two scales are close.



## Summary this part

- Hund's metals have a low coherence scale due to J.
- There is intermediate T state with fluctuating spins and "quenched" orbitals
- This state is influenced by NFL fixed point.
- Incoherent state of ruthenate in DMFT: twochannel overscreened SU(3)

#### SOC

• SOC in ruthenates nonnegligible 0.1eV, larger than  $T_{FL}$ . Its effects?

Incidentally, the quenching of orbital moments at high T may explain why calculations (such as the present one) neglecting the spin-orbit coupling [12,19] may still be accurate for ruthenates down to quite low T, even though the bare value of this coupling is ~0.1 eV [43].

JM & Georges, PRL'16

### RG picture on relevance of $\lambda$



 $(T_{\kappa} \rightarrow T_{\kappa}^{orb})$ , for Hund's metals, but more generally, first scale at which either spin/orbit moment is screened)



#### Returning to $Sr_2RuO_4$ within DMFT-self energies with SOC very similar to the ones without





• But there is also off-diagonal term that leads to enh of SOC

Kim, JM,Ferrero,Parcollet,Georges, PRL'18, Zhang et al. PRL'16



#### Semicircular calculation study



Triebl, Kraberger,JM,Aichhorn, PRB'18



## Summary SOC

- For weak enough SOC its effects on correlations (but not on spectral functions) can be ignored (perhaps up to Tκ<sub>orb</sub>)
- When it becomes large it destroys Hund's metal
- Further work needed to characterize it better

Thank you!