

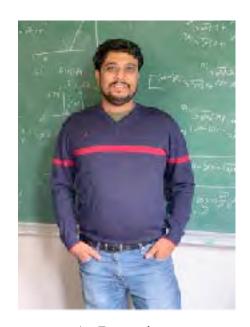






# Charge orders and Strange metals in Cuprates

### Catherine Pépin, (IPhT, CEA-Saclay)



A. Banerjee

- L. Haurie
- E. Pangburn





S. Sarkar & M. Grandadam

College de France, June 2nd, 2022

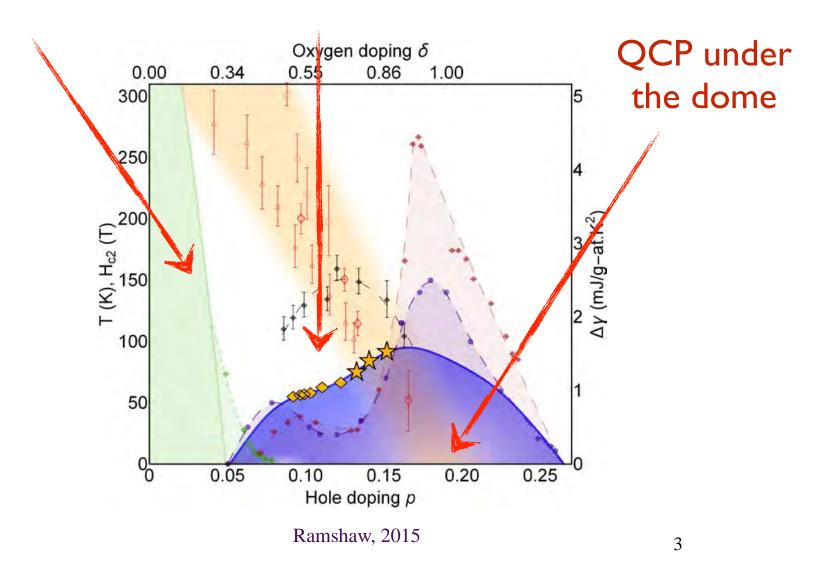
Yvan Sidis, J. C. Séamus Davis, Mohammad Hamidian,
Alain Sacuto, Henri Alloul, Nigel Hussey, Dorothée Colson
Philippe Bourges, Victor Balédent, Dalila Bounoua, Brigitte Leridon,
Cyril Proust, M-H Julien...



Konstantin Borisovich Efetov (April 29, 1950 – August 11, 2021) was a Russian/German theoretical physicist, recognized leader in the theory of condensed matter, and a teacher of a number of actively working theorists.

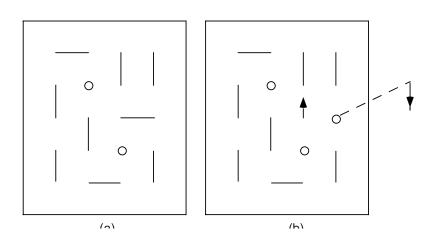
### Mott transition

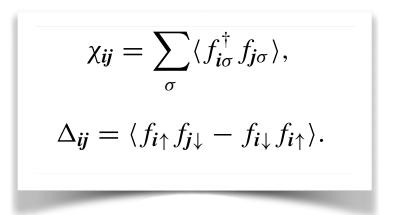
### **Fluctuations**

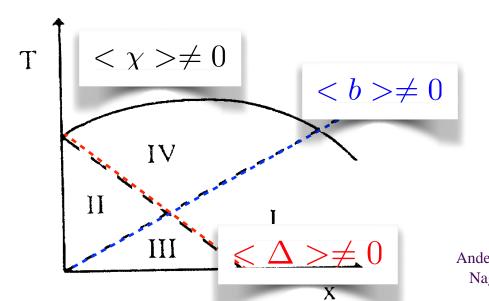


# 1. The context of strong coupling : doping a Mott insulator

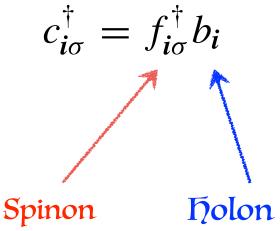
Resonating Valence Bond (RVB) : pairs form and fluctuate







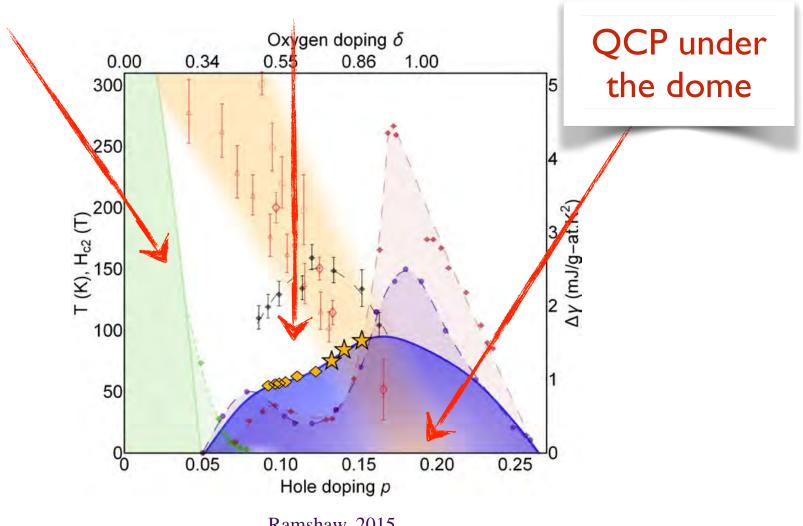
Anderson, Lee, Wen, Nagaosa, Kotliar  $f_{m{i}\uparrow}^{\dagger}f_{m{i}\uparrow}+$ 



$$f_{i\uparrow}^{\dagger} f_{i\uparrow} + f_{i\downarrow}^{\dagger} f_{i\downarrow} + b_i^{\dagger} b_i = 1.$$

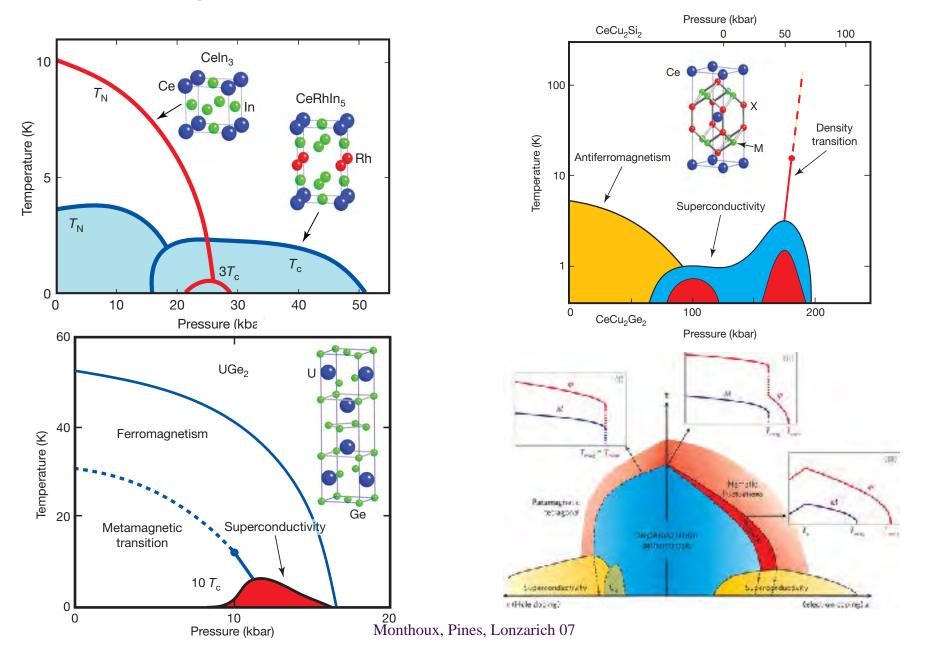
### Mott transition

### **Fluctuations**

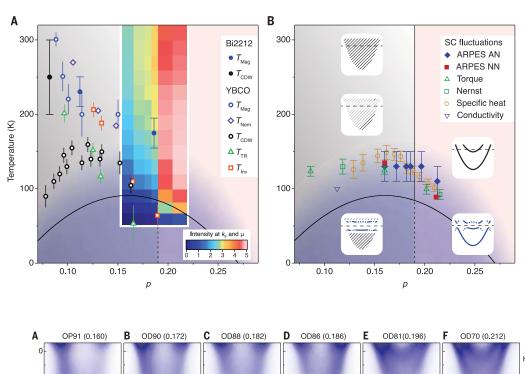


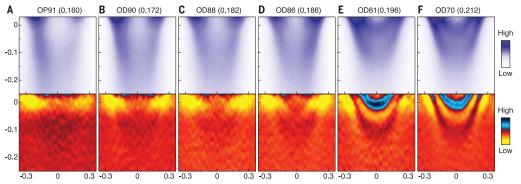
Ramshaw, 2015

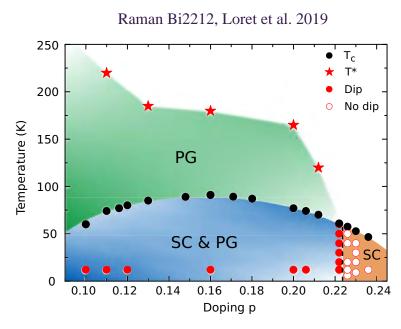
## 2. QCP under the SC dome



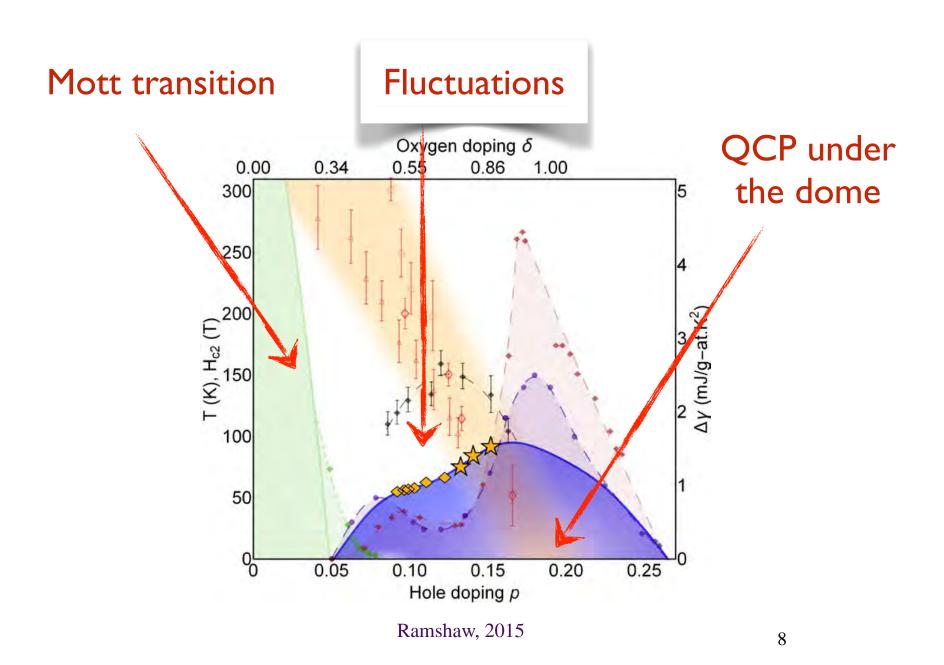
# QCP questionned: an abrupt change at $p^*$ ?

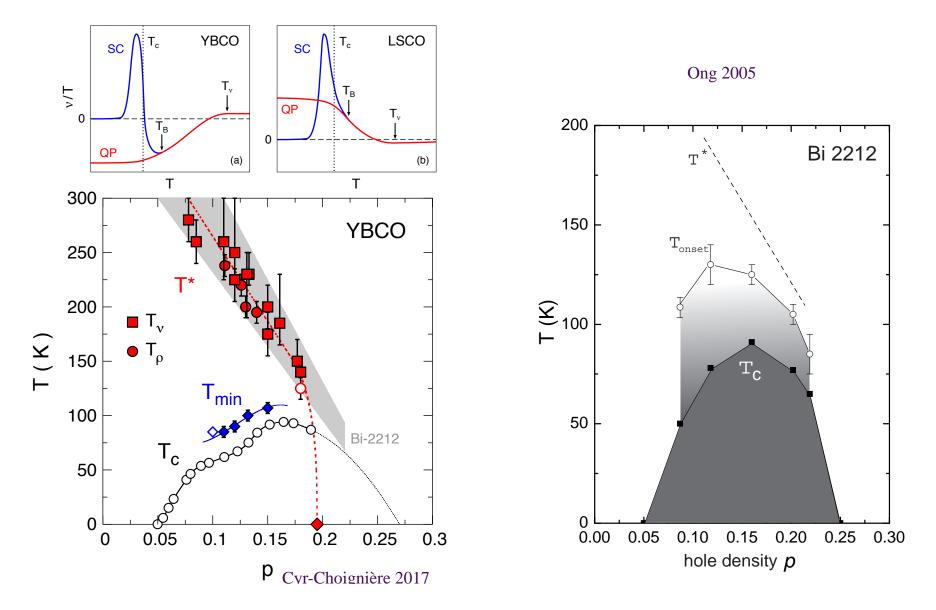






ARPES Bi2212, Chen et al. 2019



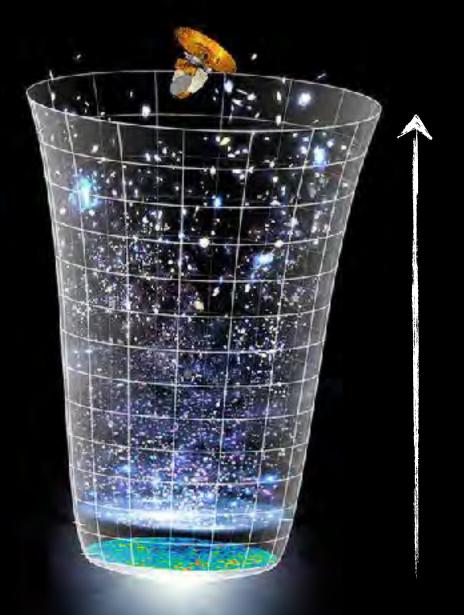


**Nernst effect : resolved controversy** 

Amplitude
Fluctuations ...>

Phase fluctuations ...>

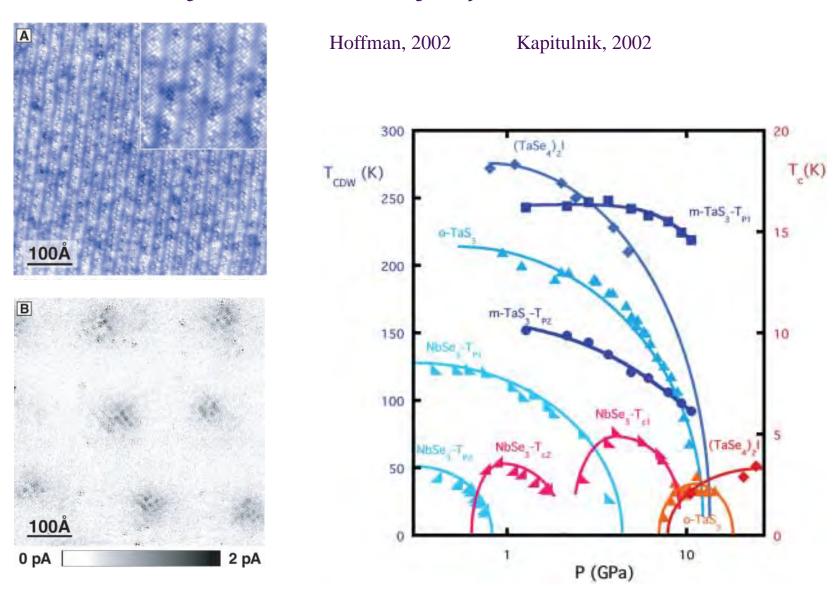
Condensate ....



# Recent Exp. developments Charge Order

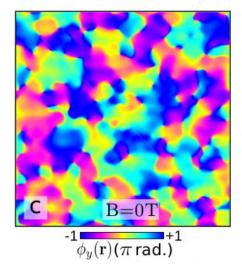
# Presence of competing orders

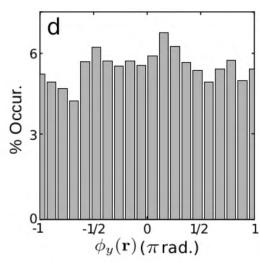
Charge modulations in strong competition with SC state



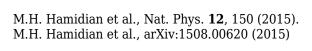
STM measurement of charge density modulation :  $Re\left(\chi_{ij}\right) = \hat{d}|\chi_{ij}|cos\left(\mathbf{Q}\cdot\mathbf{r} + \phi\left(\mathbf{r}\right)\right)$ 

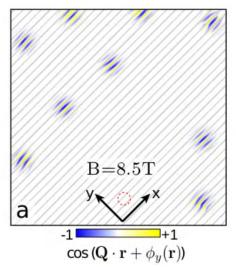
 $B=0\ T$  random phase distribution :

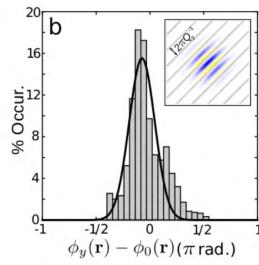




 $B \neq 0 \ T \, \mathrm{centered} \, \, \mathrm{distribution}$  :

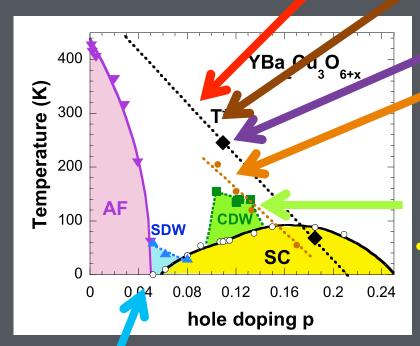






### Charge order Landscape

YBa<sub>2</sub>Cu<sub>3</sub>O<sub>6+x</sub>



#### **Nematicity**

**Inversion symmetry** 

loop currents

anomalous Kerr effect  $T_k < T^*$ 

Xia, PRL 2008

Incipient CDW  $-T_m < T^*$ 

 $Q^* = (\delta, 0)$  and  $(0, \delta)$  with  $\delta \sim 0.3$ 

Chang , Nature Phys. 2012 Ghiringhelli, Science 2012

glassy SDW: T<sub>SDW</sub> << T\* (neutron, µSR, RMN)

Haug, New J. Phys. 2010 T. Wu et al., PRB 2013

Stable CDW under magnetic field & Fermi surface reconstruction (NMR, quantum oscillation, ultrasound)

D. LeBoeuf, *Nature* 2007.

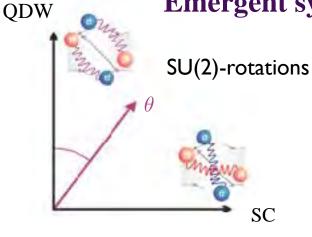
T. Wu et al., *Nature* 2011.

D. LeBoeuf et al., Nature Physics 2013.

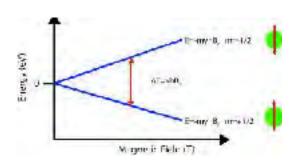
Courtesy Y. Sidis

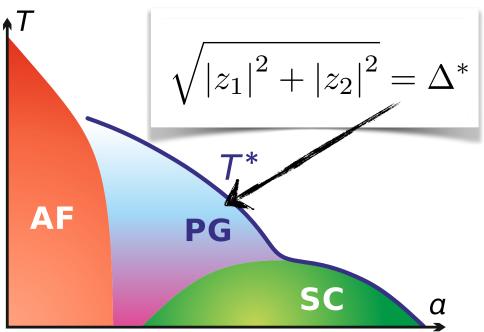
# Emergent symmetry

### **Emergent symmetries in the under-doped regime**



Degenerescence of levels: accidental? symmetry related?





At some energy scale in the phase diagrams SC and Charge sectors are related by and SU(2) symmetry

Sachdev et al (2013) Efetov, Meier, CP (2013)

### Pseudo-gap from quantum criticality

### AFM QCP in d=2

K.B.Efetov, H.Meier, C.P. Nat. Phys. 9, (2013)

### Dispersion linearized around 8 hot spots

$$\mathcal{L} = \chi^{\dagger} \left( \partial_{\tau} + \varepsilon (-i\hbar \nabla) + \lambda \vec{\phi} \vec{\sigma} \right) \chi$$

$$\mathcal{L} = \chi^{\dagger} \left( \partial_{\tau} + \varepsilon (-i\hbar \nabla) + \lambda \vec{\phi} \vec{\sigma} \right) \chi \qquad \langle \phi_{\omega, \mathbf{k}}^{i} \phi_{-\omega, -\mathbf{k}}^{j} \rangle \propto \frac{\delta_{ij}}{(\omega/v_{s})^{2} + (\mathbf{k} - \mathbf{Q})^{2} + a}$$

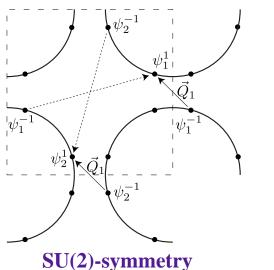
Composite order parameter

$$c_{\mathbf{p}}^{\mathrm{pp}} \left\langle (i\sigma_2)_{\alpha\beta} \psi_{\alpha,\mathbf{p}} \psi_{\beta,-\mathbf{p}} \right\rangle + c_{\mathbf{p}}^{\mathrm{ph}} \left\langle \delta_{\alpha\beta} \psi_{\alpha,\mathbf{p}} \psi_{\beta,-\mathbf{p}}^* \right\rangle,$$

SU(2) symmetry and fluctuations

$$u = \begin{pmatrix} \Delta_{-} & \Delta_{+} \\ -\Delta_{+}^{*} & \Delta_{-}^{*} \end{pmatrix} \quad \text{with} \quad |\Delta_{+}|^{2} + |\Delta_{-}|^{2} = 1$$

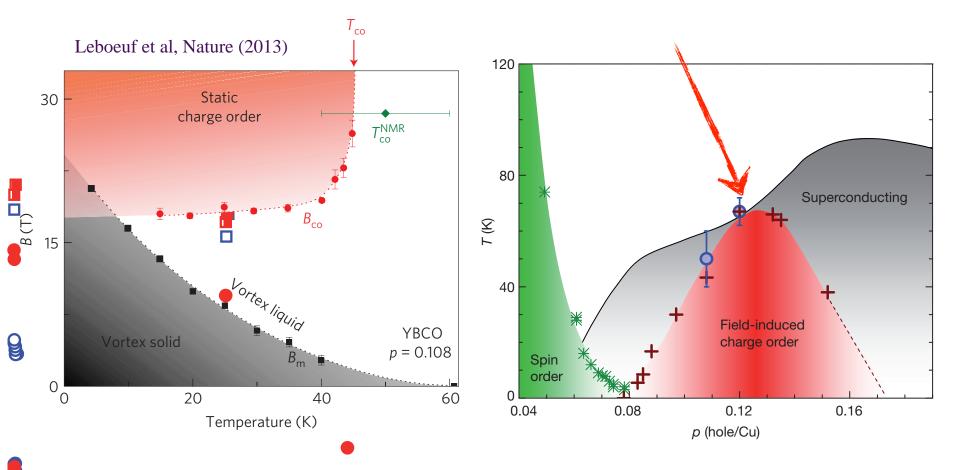
M. Metlitsky and S. Sachdev (2010)

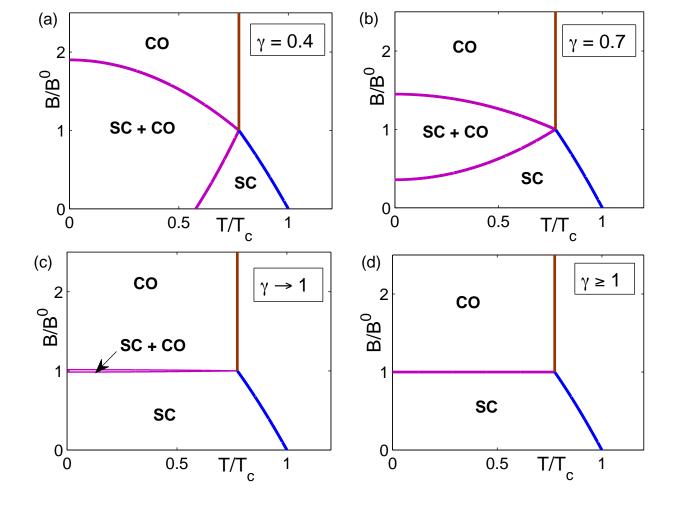




# Magnetic-field-induced charge-stripe order in the high-temperature superconductor YBa<sub>2</sub>Cu<sub>3</sub>O<sub>y</sub>

Tao Wu<sup>1</sup>, Hadrien Mayaffre<sup>1</sup>, Steffen Krämer<sup>1</sup>, Mladen Horvatić<sup>1</sup>, Claude Berthier<sup>1</sup>, W. N. Hardy<sup>2,3</sup>, Ruixing Liang<sup>2,3</sup>, D. A. Bonn<sup>2,3</sup> & Marc-Henri Julien<sup>1</sup>

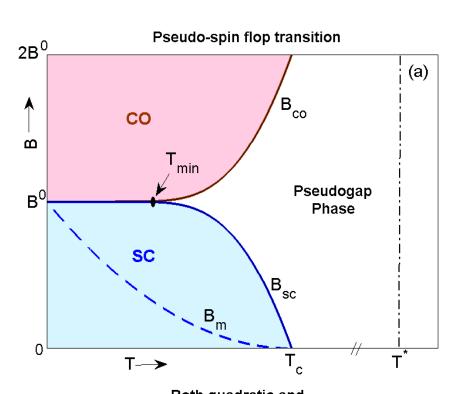


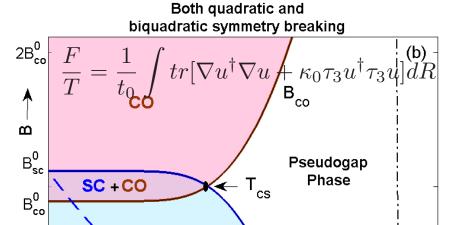


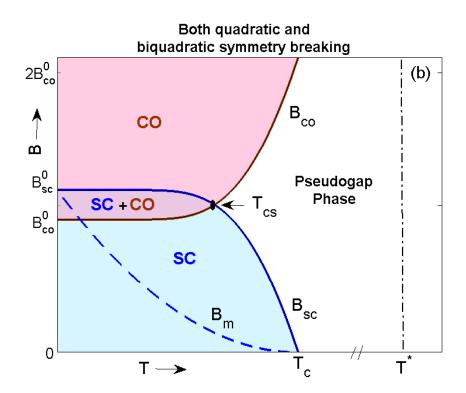
$$f[\psi,\phi] = \alpha_{\psi}|\psi|^2 + \frac{\beta_{\psi}}{2}|\psi|^4 + \alpha_{\phi}|\phi|^2 + \frac{\beta_{\phi}}{2}|\phi|^4 + \gamma|\psi|^2|\phi|^2,$$

$$\alpha_{\phi} = \alpha_{\phi}' + a_{co}T^2$$

## Non Linear Sigma Model





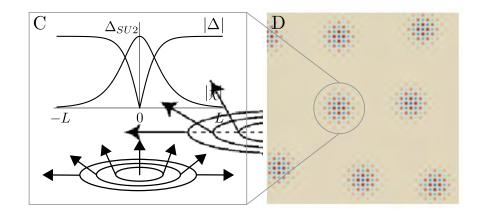


$$\frac{F_{bq}}{T} = \frac{1}{t_0} \int z_0 \left\{ \left( tr[\tau_3 u^{\dagger} \tau_3 u] \right)^2 - 1 \right\} dR$$

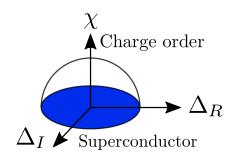
# Topology and local structures

### **Homotopy classes**

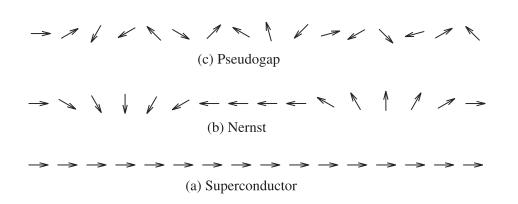
$$\Delta_{-,R}^2 + \Delta_{-,I}^2 + \Delta_{+,R}^2 + \Delta_{+,I}^2 = 1$$
$$\pi_2(S_3) = 0$$

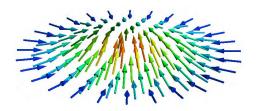


### 0(3) non linear $\sigma$ -model



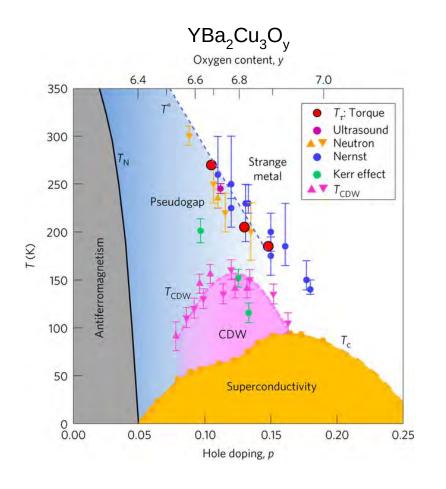
### Vortex structure Phase diagram





# Fractionalization of a PDW

### The phase diagram



#### Slave-boson method:

$$c_{i,\sigma}^{\dagger} = b_i f_{i,\sigma}^{\dagger}$$
 Charge (holon) Spin (spinon)

Constraint : 
$$b_i b_i^\dagger + \sum_{\sigma} f_{i,\sigma} f_{i,\sigma}^\dagger = 1$$

Fictitious gauge transformation : 
$$\begin{cases} f_{i\sigma} \to e^{i\theta} f_{i\sigma} \\ b_i \to e^{i\theta} b_i \end{cases}$$

## Fractionalization of a Pair Density Wave

Modulated particle-particle pair :  $\Delta_{ij}^{PDW} = \left\langle c_{i,\sigma} c_{j,\bar{\sigma}} e^{i {m Q} \cdot {m r}_{ij}} \right\rangle$ 

PDW fractionalization : 
$$\Delta_{ij}^{PDW} = \left[\Delta_{ij}, \chi_{ij}^* \right]$$

$$\Delta_{ij}^* \Delta_{ij} + \chi_{ij}^* \chi_{ij} = 1$$

Uniform particle-particle pair :  $\Delta_{ij}=\langle c_{i,\sigma}c_{j,ar{\sigma}}\rangle$  — Charge (2)

Modulated particle-hole pair :  $\chi_{ij} = \left\langle c_{i,\sigma}^{\dagger} c_{j,\sigma} e^{i {m Q} \cdot {m r}_{ij}} \right\rangle$  Translation symmetry

Phase transformation :  $\left\{ \begin{array}{l} \Delta_{ij} \to e^{i\theta} \Delta_{ij} \\ \chi_{ij} \to e^{i\theta} \chi_{ij} \end{array} \right.$ 

Ansatz :  $|PG\rangle = \left(\hat{\chi}_{ij} + \hat{\Delta}_{ij}\right)|0\rangle$  + constraint

## The phase diagram

### Emerging gauge field: confining transition

$$S = \frac{1}{2} \int d^2x \sum_{a,b=1}^{2} |\omega_{ab}|^2, \text{ with } \omega_{ab} = z_a \partial_{\mu} z_b - z_b \partial_{\mu} z_a,$$

$$z_1 = \Delta, z_2 = \chi, z_1^* = \Delta^*, z_2^* = \chi^*.$$

$$T = T^*$$

 $\theta$  gets to fluctuate

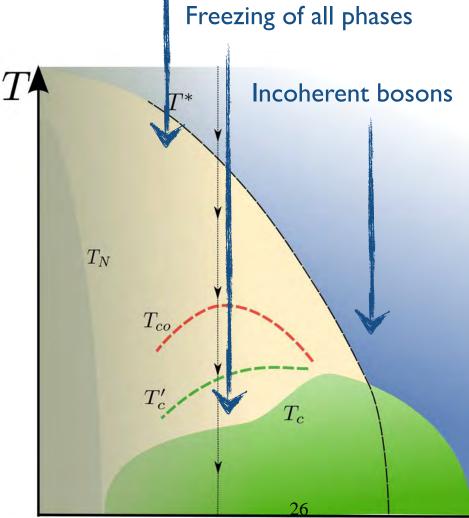
We obtain the constraint

$$|\Delta_{ij}|^2 + |\chi_{ij}|^2 = (E^*)^2$$

$$T = T_c$$

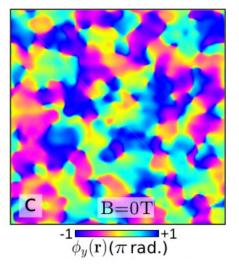
 $\phi$  gets frozen and we have global phase coherence.

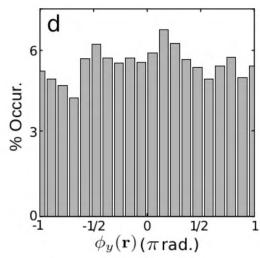
Meissner effect.

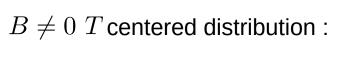


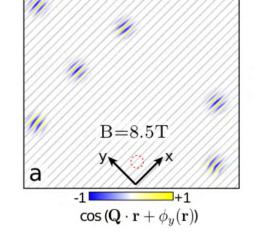
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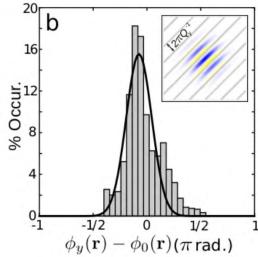
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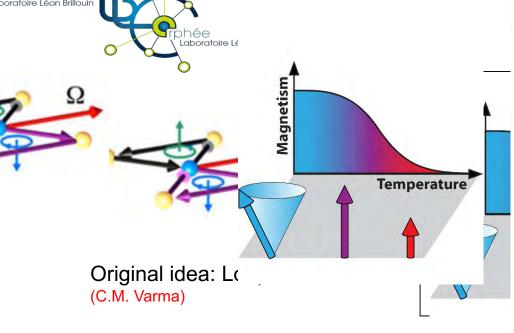






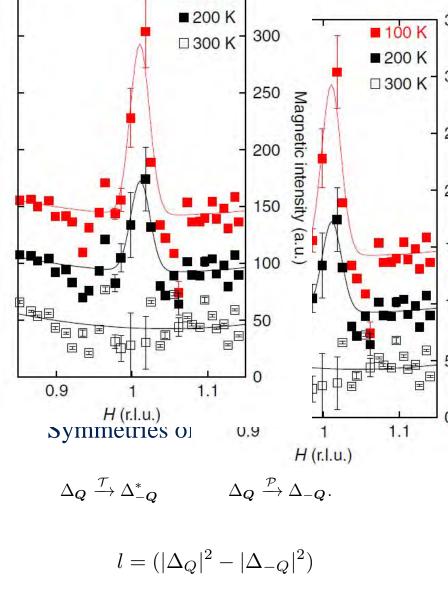


M.H. Hamidian et al., Nat. Phys. **12**, 150 (2015). M.H. Hamidian et al., arXiv:1508.00620 (2015)



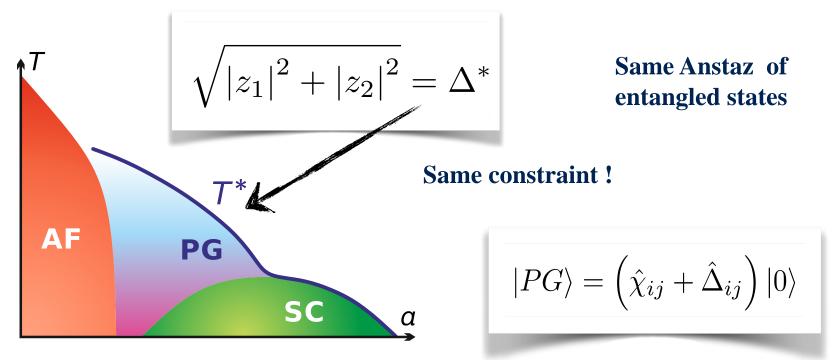
L. Mangin-Thro et al, Nat. Comm 6, 7705 (2015)

The fractionalized PDW supports the symmetry of the Q=0 loop currents as a precursor order parameter



# Analogy with SU(2) emergent syl $\Delta_{ij}^{PDW} = \left[\Delta_{ij}, \chi_i^* ight]$

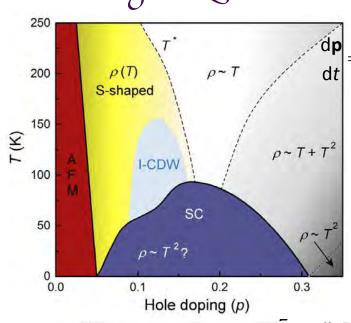
$$\psi = (z_1, z_2)$$
  $\mathcal{L}_{CP^1} = \frac{1}{2g} |D_{\mu}\psi|^2 + V(\psi)$ 



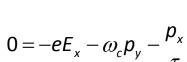
Sachdev et al (2013) Efetov, Meier, CP (2013)

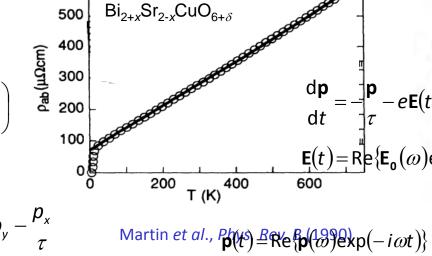
# Strange metals

# Most strongly Correlated/ Entangled QCP?

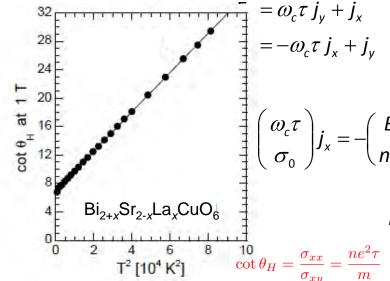


$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -\frac{\mathbf{p}}{\tau} - e\left(\mathbf{E} + \begin{bmatrix} \mathbf{p} \\ m \end{bmatrix}\right)$$





$$0 = -eE_y + \operatorname{pranckian regime for the resistip(ity)}_{\tau}, -eE_0(\omega)$$
minimal viscosity



Black hole models, SYK etc... 
$$(ne^2 m)E_0(\omega) = m$$

$$j(\omega) = m = (1 \tau) - i\omega$$

 $\begin{pmatrix} \omega_c \tau \\ \sigma_0 \end{pmatrix} j_x = -\begin{pmatrix} B \\ ne \end{pmatrix} j_x \text{ At the same time Drude like optical conductivity driven by T}$ 

$$R_H = \frac{E_y}{j_x B} = -\frac{1}{ne}$$
 Van der Marel 90

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}$$

$$\tau \propto 1/T$$

$$\sigma_{xx} = \frac{ne^2\tau}{m}$$

$$\sigma_{xy} = \frac{ne^3B\tau^2}{cm}$$

$$\cot \theta_H = \frac{\sigma_{xx}}{\sigma_{xy}} = \frac{ne^2\tau}{m}$$

$$R_H = \frac{\sigma_{xy}}{\sigma_{xx}^2} = \frac{Bm}{ne}$$

$$\tau^{-1} \sim T^2$$

$$n \sim T ?$$

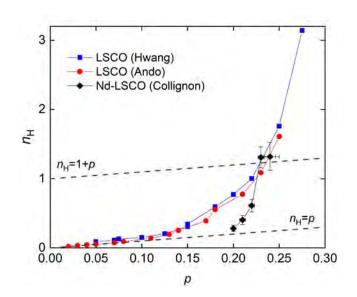
$$n \sim T$$
 ?

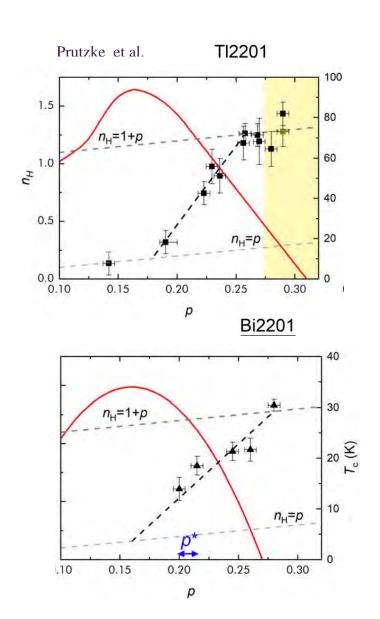
## Transport in the Strange Metal

### Recent controversy

Spectral weight missing in the Strange Metal regime?

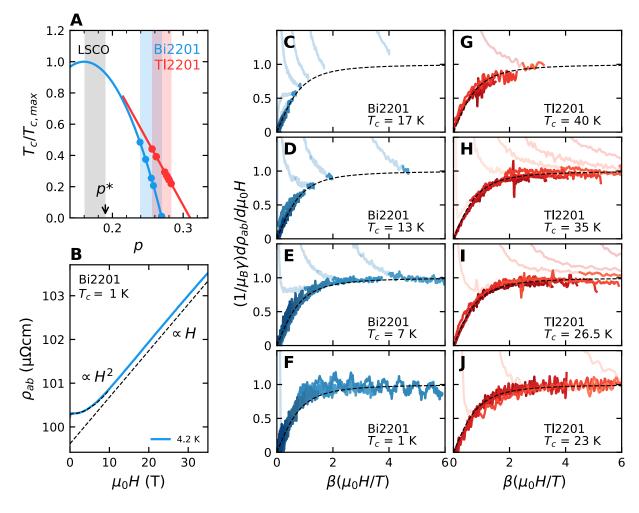
Our answer: two types of carriers, fermions and charge-2 bosons with finite momentum





## H/T scaling in the SM phase

$$\rho(H,T) - \rho(0,0) = \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2}$$



• Isotropic

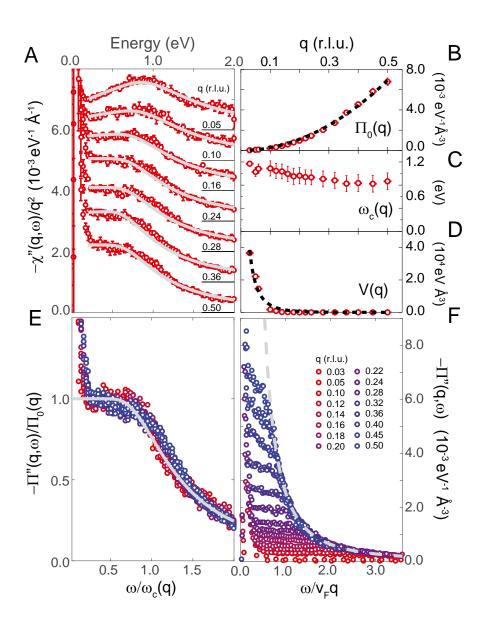
• Incoherent

$$\sigma_{xy} = 0$$

• Planckian limit

Ayres et al., preprint 2020, courtesy N.Hussey

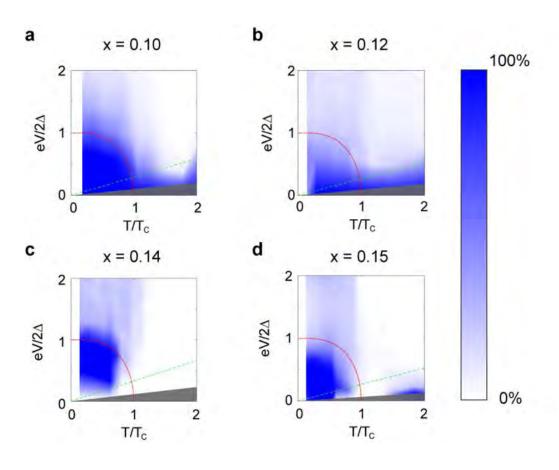
### Presence of « another species » in this regime : MEELS experiment



Mitrano et al., 2018

**Jamming transition** 

# Pair tuneling in LSCO: noise measurement

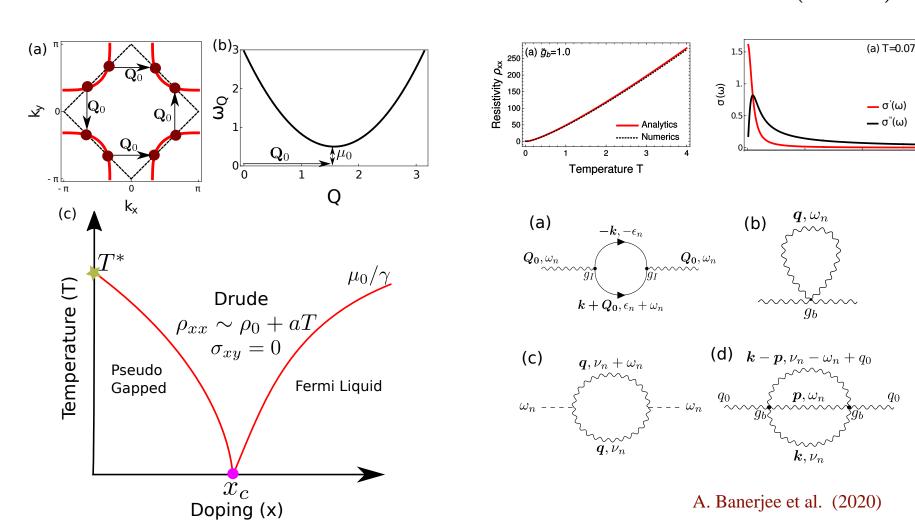


Zhou et al., 2019

# Our proposal : Charged bosons in the Strange Metal phase

$$\mathcal{D}^{-1}(\mathbf{q}, i\omega_n) = \gamma |\omega_n| + \mathbf{q}^2 + \mu(T)$$

$$\sigma_{xx} \left( i\omega \to \omega + i\delta \right) = \frac{\sigma_0^b \tau}{\left( 1 - i \frac{\gamma \omega}{2\mu} \right)},$$



$$\sigma_{xx} = \frac{ne^2}{m} \tau_{xx}$$

$$\cot \theta_H = \frac{\sigma_{xx}}{\sigma_{xy}} = \frac{c}{eB} \frac{\tau_x}{\tau_{xy}^2}$$

$$\sigma_{xy} = \frac{ne^3B}{cm}\tau_{xy}^2$$

(a) 
$$(\pi,\pi)$$
  $(0,0)$   $(0,\pi)$ 



$$\tau_{xx}^{-1} \sim T$$

$$\tau_{xy}^{-1} \sim T^{1.5}$$

Averaging the Hall conductivity around the Fermi surface with hot and cold spots

> Kokalj, McKenzie and Hussey, 2012

# Predictions ARPES

## Why the system would want to do this?

$$H = \sum_{i,j,\sigma} c_{i,\sigma}^{\dagger} t_{ij} c_{j,\sigma} + J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$
$$+ V \sum_{\langle i,j \rangle} n_i n_j$$

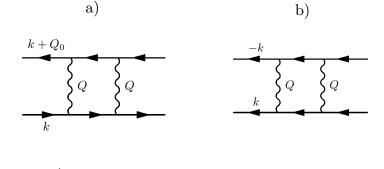
#### Mean-field decoupling

$$\Delta_k = \sum_{\sigma} \sigma c_{k,\sigma} c_{-k,\bar{\sigma}}$$

$$\chi_k^Q = \sum_{\sigma} c_{k,\sigma}^{\dagger} c_{k+Q,\sigma}$$

#### **Energy scales:**

$$\Delta_k \sim 3J - V$$
$$\chi_k^Q \sim 3J + V$$



$$\chi_{ij} = \frac{1}{2} \langle c_{i\sigma}^{\dagger} c_{j\sigma} \rangle \qquad \Delta_{ij}^* = \langle c_i^{\dagger} \uparrow c_j^{\dagger} \downarrow \rangle$$

$$\Psi_{ij} = (\hat{\Delta}_{ij}, \hat{\chi}_{ij})^t \qquad |\Psi_{ij}| = E^*$$

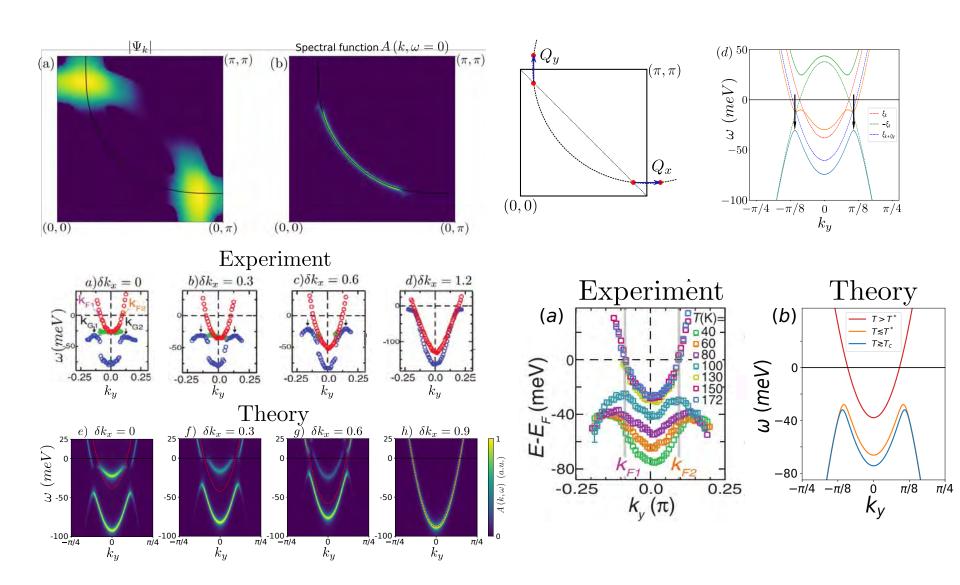
#### Condensation energy

$$E_{PG} = \frac{1}{2\tilde{J}} |\Psi_{k=k_F}|^2 = 0.017 \, eV,$$

$$E_{SC} = \frac{1}{2J_-} |\Delta_{k=k_F}|^2 = 0.014 \, eV,$$

$$E_{CDW} = \frac{1}{2J_+} |\chi_{k=k_F}|^2 = 0.011 \, eV.$$

## Opening a gap in the Fermi surface



M. Grandadam et al. PRB (2020)

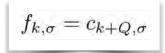
ARPES, Bi2201

## Comparison with CDMFT: « hidden fermion»

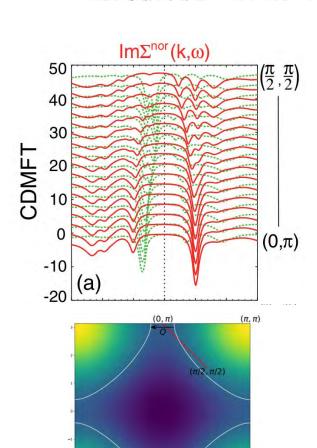
$$H = \sum_{k,\sigma} \epsilon_k c_{k,\sigma}^{\dagger} c_{k,\sigma} + \sum_{k,\sigma} \left( \sigma \Delta_k^* c_{k,\sigma}^{\dagger} c_{-k,-\sigma}^{\dagger} + h.c. \right)$$

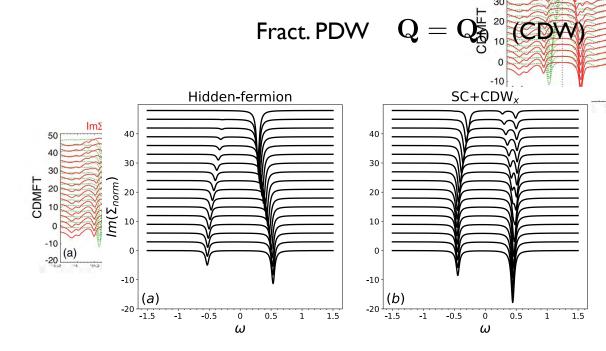
$$+ \sum_{\alpha,k,\sigma} \epsilon_k^{f,\alpha} f_{k,\sigma}^{\alpha\dagger} f_{k,\sigma}^{\alpha} + \sum_{\alpha,k,\sigma} \left( \sigma \Delta_k^{f,\alpha^*} f_{k,\sigma}^{\alpha\dagger} f_{-k,-\sigma}^{\alpha\dagger} + h.c \right)$$

$$+ \sum_{\alpha,k,\sigma} \left( V_k^{\alpha} c^{\dagger} k, \sigma f^{\alpha} k, \sigma + h.c \right)$$



 $\mathbf{Q}=(\pi,\pi_{}^{\scriptscriptstyle 4})$  (AF)





S. Sakai et al. 2021

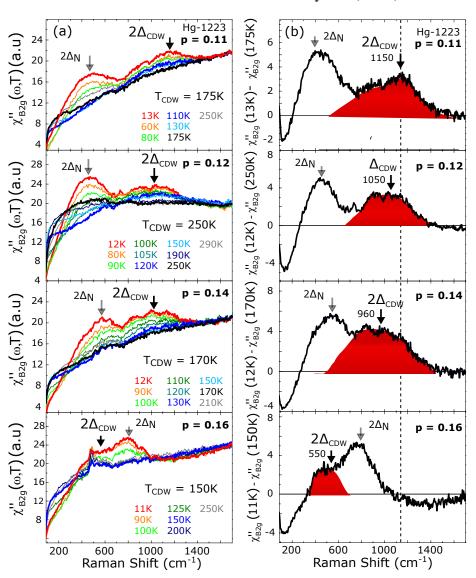
42 M. Grandadam et al. 2021

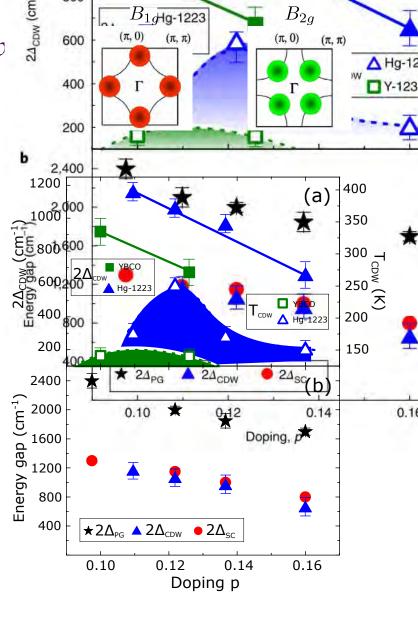
# Raman scattering

### Raman Scattering

B2g, T < Tco

Loret et al. Nat. Physics (2019)

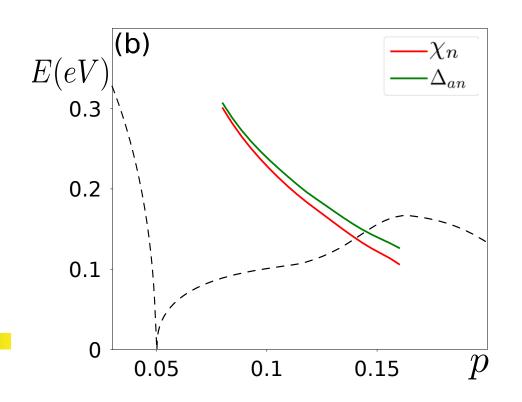


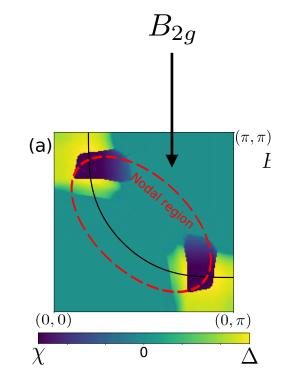


# Solving gap equations

$$\Delta_{k,\omega} = -\frac{1}{\beta} \sum_{q,\Omega} \frac{J_{-}(q,\Omega) \Delta_{k+q}}{(\omega + \Omega)^2 - \xi_{k+q}^2 - \Delta_{k+q}^2},$$

$$\chi_{k,\omega} = -\frac{1}{\beta} \sum_{q,\Omega} \frac{J_{+}(q,\Omega) \chi_{k+q}}{(\omega + \Omega - \xi_{k+q}) (\omega + \Omega - \xi_{k+Q+q}) - \chi_{k+q}^{2}},$$

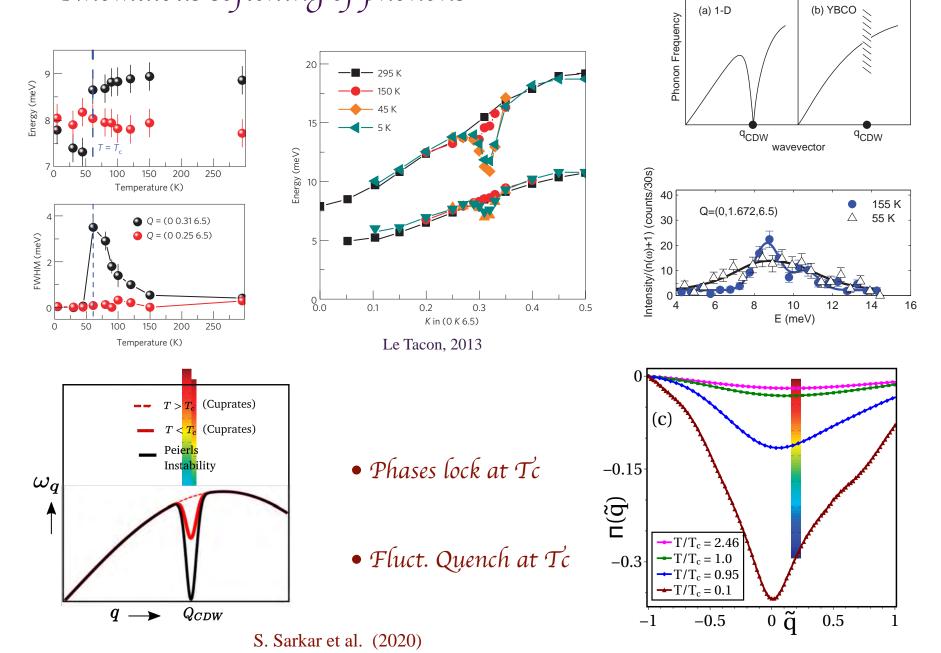


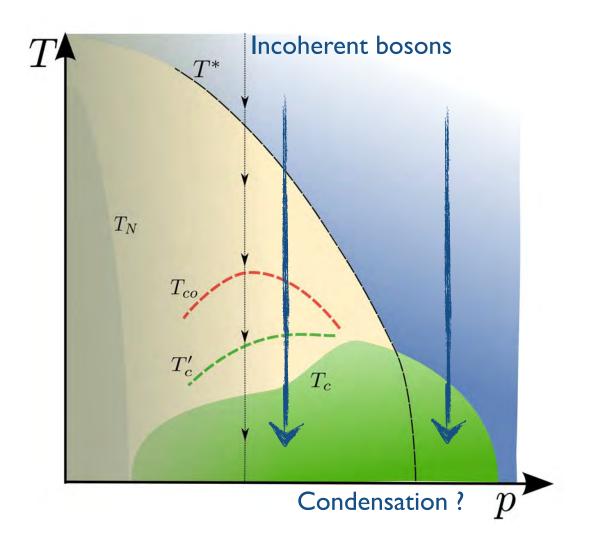


# Same order of magnitude for $\Delta$ and $\chi$

M. Grandadam et al. PRB (2019)

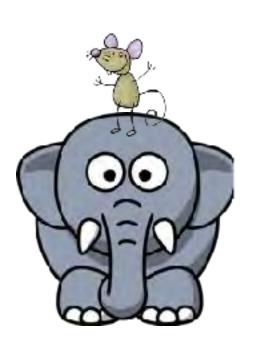
# Phonon Softening





#### **Conclusions**

- Charge orders are a key players in cuprate physics: natural competitor of superconductivity
- Fractionalizing a PDW or a more complex boson
- Entangling particle-hole and particle-particle pairs at T\*
- Explains recent Raman, phonon softening
- ARPES : back-bending, poles in self-energy (cf. DMFT studies)
- Can a charge-2 boson explain the mystery of strange metal and Hall resistivity?
- Exp. predictions with mesoscopic noise, Josephson effects
- Numerical check in strong coupling approaches



## Discussions of the data and a few Refs

Maxence Grandadam, Catherine Pépin

arXiv:2012.11226

Anurag Banerjee, Maxence Grandadam, Hermann Freire, Catherine Pépin arXiv:2009.09877

Saheli Sarkar, Maxence Grandadam, Catherine Pépin arXiv:2009.02975

Maxence Grandadam, Debmalya Chakraborty, Xavier Montiel, Catherine Pépin arXiv:2002.12622

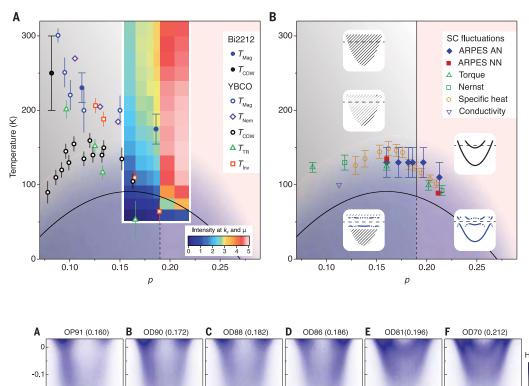
D. Chakraborty, M. Grandadam, M. H. Hamidian, J. C. S. Davis, Y. Sidis, C. Pépin arXiv:1906.01633

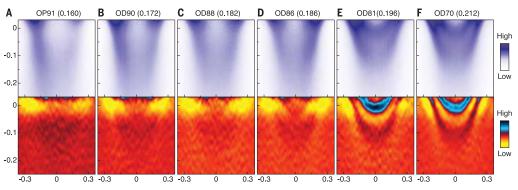
Saheli Sarkar, Debmalya Chakraborty, Catherine Pépin arXiv:1906.08280

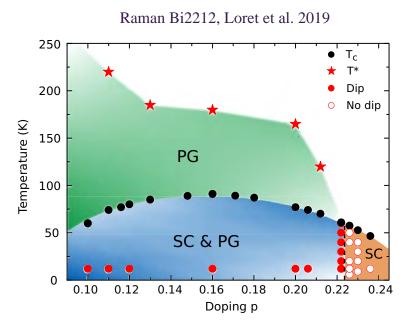
C. Pépin, D. Chakraborty, M. Grandadam, S. Sarkar arXiv:1906.10146

# Quantum Criticality Or Cross-Over?

# QCP questioned : an abrupt change at $p^*$ ?

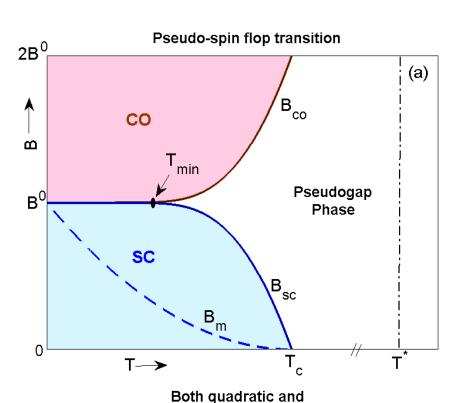


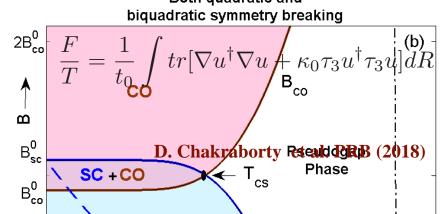


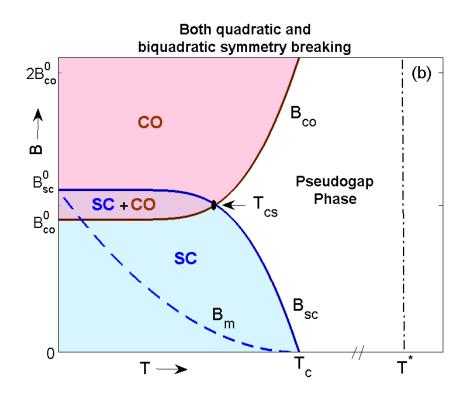


ARPES Bi2212, Chen et al. 2019

# 0(3) Non Linear Sigma Model



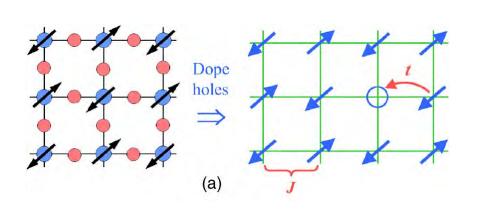


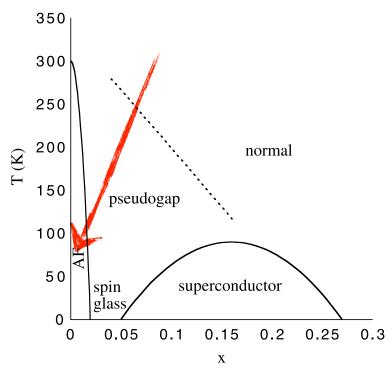


$$\frac{F_{bq}}{T} = \frac{1}{t_0} \int z_0 \left\{ \left( tr[\tau_3 u^{\dagger} \tau_3 u] \right)^2 - 1 \right\} dR$$

# The context of strong coupling : doping a Mott insulator

#### **Resonating Valence Bond (RVB)**





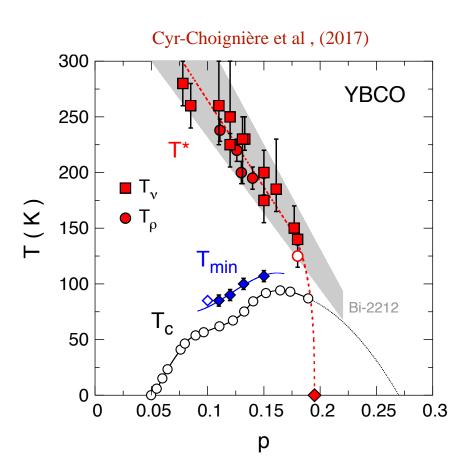
$$H = P \left[ -\sum_{\langle ij \rangle, \sigma} t_{ij} c_{i\sigma}^{\dagger} c_{i\sigma} + J \sum_{\langle ij \rangle} \left( S_i \cdot S_j - \frac{1}{4} n_i n_j \right) \right] P$$

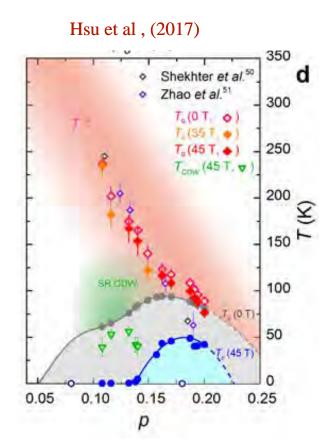
Anderson, Lee, Nagaosa, Rice etc...

P: projection on no double occupancy

The extend of the Cooper pairs phase fluctuations regime Nernst effect (Ong, Behnia), transport (Rullier-Albenque, Sebastian), Squid spectroscopy (Lesueur)...

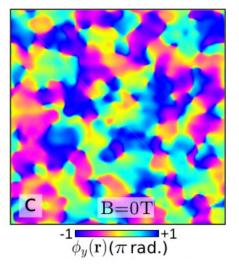
The presence of a partner to SC pairing inhibits the visibility of phase fluctuations in transport and Nernst effect (Orgard, 2017)

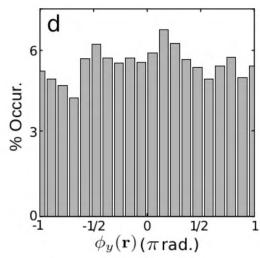


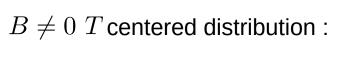


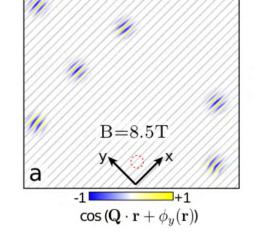
STM measurement of charge density modulation :  $Re\left(\chi_{ij}\right) = \hat{d}|\chi_{ij}|cos\left(\mathbf{Q}\cdot\mathbf{r} + \phi\left(\mathbf{r}\right)\right)$ 

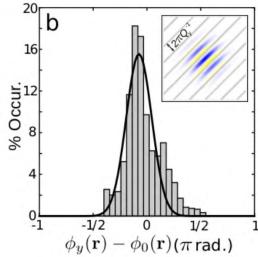
 $B=0\ T$  random phase distribution :



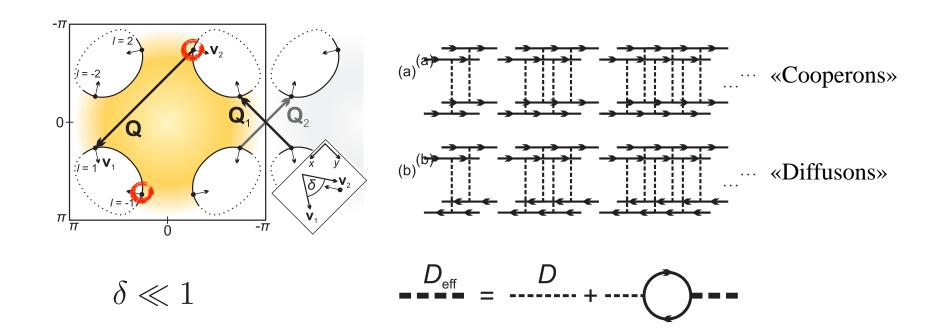








M.H. Hamidian et al., Nat. Phys. **12**, 150 (2015). M.H. Hamidian et al., arXiv:1508.00620 (2015)



Composite order parameter

$$c_{\mathbf{p}}^{\mathrm{pp}} \left\langle (i\sigma_2)_{\alpha\beta} \psi_{\alpha,\mathbf{p}} \psi_{\beta,-\mathbf{p}} \right\rangle + c_{\mathbf{p}}^{\mathrm{ph}} \left\langle \delta_{\alpha\beta} \psi_{\alpha,\mathbf{p}} \psi_{\beta,-\mathbf{p}}^* \right\rangle,$$

SU(2) symmetry and fluctuations

$$u = \begin{pmatrix} \Delta_{-} & \Delta_{+} \\ -\Delta_{+}^{*} & \Delta_{-}^{*} \end{pmatrix}$$
 with  $|\Delta_{+}|^{2} + |\Delta_{-}|^{2} = 1$ 

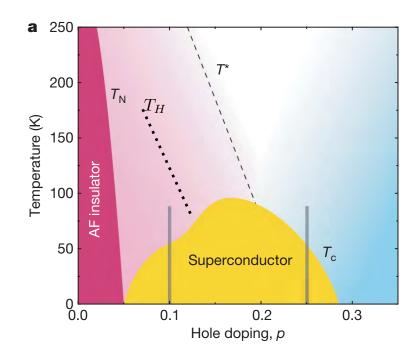
#### <sup>89</sup>Y NMR Evidence for a Fermi-Liquid Behavior in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>6+x</sub>

H. Alloul, T. Ohno, (a) and P. Mendels

Physique des Solides, Université de Paris-Sud, 91405 Orsay, France
(Received 15 May 1989)

We report NMR shift  $\Delta K$  and  $T_1$  data of <sup>89</sup>Y taken from 77 to 300 K in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>6+x</sub> for 0.35 < x < 1, from the insulating to the metallic state. A Korringa law and therefore a Fermi-liquid picture is found to apply for the spin part  $K_s$  of  $\Delta K$ . The spin contribution  $\chi_s(x,T)$  to  $\chi_m$  is singled out, as the T variation of  $\Delta K$  scales linearly with the macroscopic susceptibility  $\chi_m$ . This implies that Cu(3d) and O(2p) holes do not have independent degrees of freedom. Their hybridization, which has a  $\sigma$  character, hardly varies with doping. These results put severe constraints on theoretical models of high- $T_c$  cuprates.

PACS numbers: 74.70.Vy, 75.20.En, 76.60.Cq, 76.60.Es



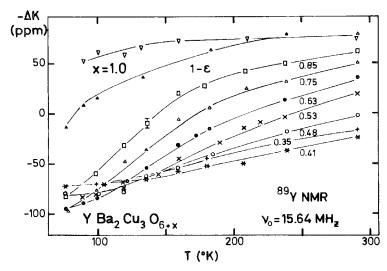
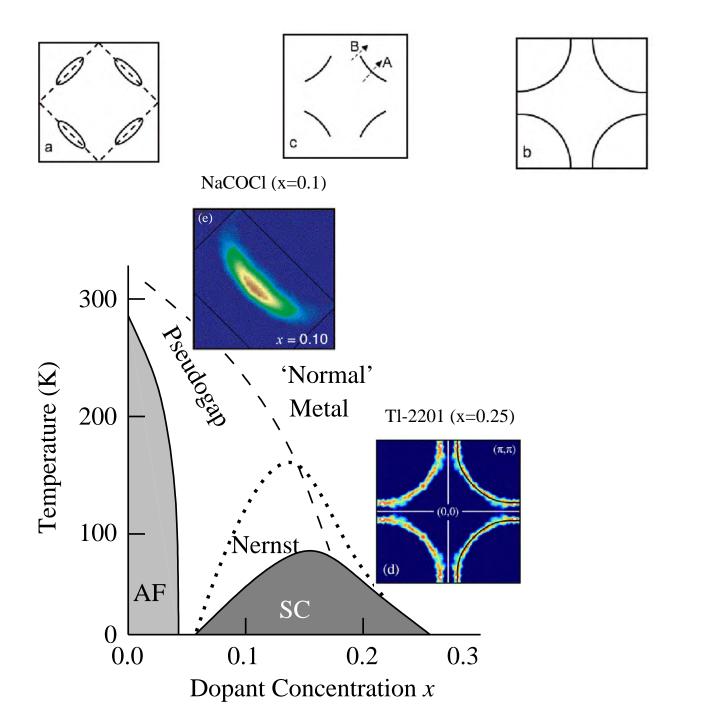
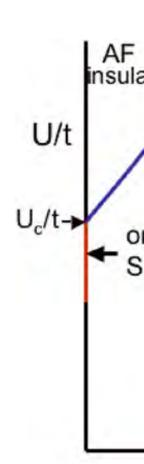
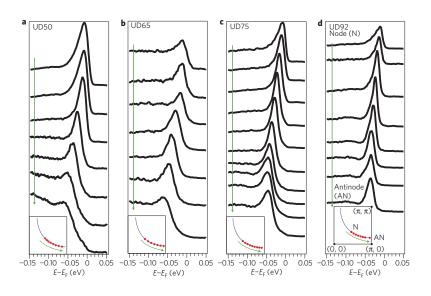


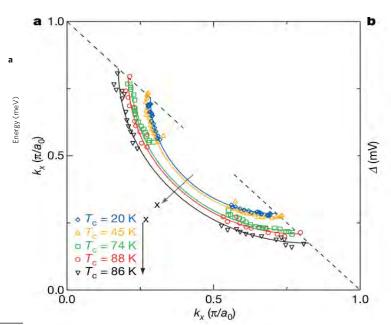
FIG. 1. The shift  $\Delta K$  of the <sup>89</sup>Y line, referenced to YCl<sub>3</sub> plotted vs T, from 77 to 300 K. The lines are guides to the eye.

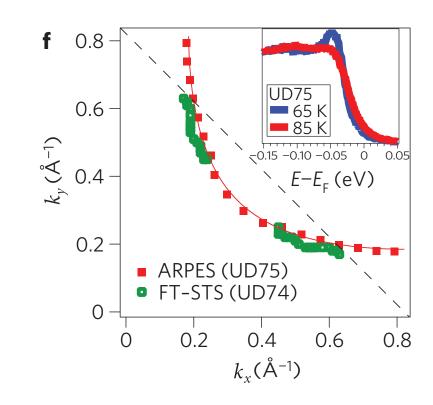




### Fractionalization in the PG phase?







Is fractionalization compatible with the observation of Bogoliubov QP in the anti-nodal region ?

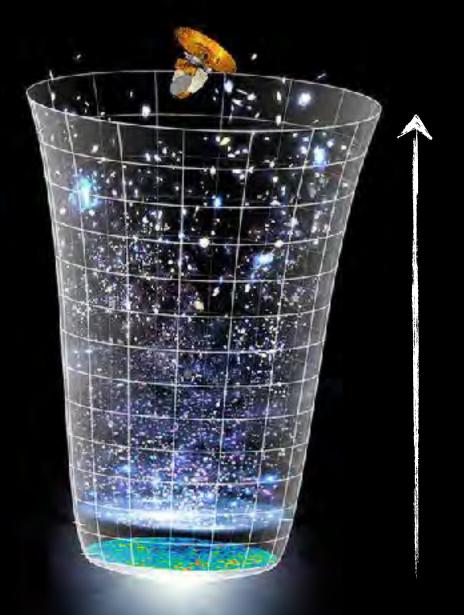
Coherence of the electrons?

D00 T 10 K

Amplitude
Fluctuations ...>

Phase fluctuations ...>

Condensate ....



#### The concept of SU(2) symmetry

C.N. Yang & S-C. Zhang (1989)

#### Pseudo-Spins

$$\eta^{+} = \sum_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}+\mathbf{Q}\downarrow}^{\dagger}$$

$$\eta_{z} = \sum_{\mathbf{k}} \left( c_{\mathbf{k}\uparrow}^{\dagger} c_{\mathbf{k}\uparrow} + c_{\mathbf{k}+\mathbf{Q}\downarrow}^{\dagger} c_{\mathbf{k}+\mathbf{Q}\downarrow} - 1 \right)$$

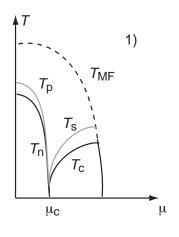
#### l=1 representation

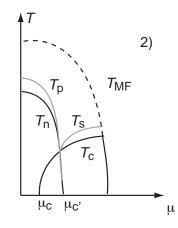
$$\Delta_{1} = -\frac{1}{\sqrt{2}} \sum_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger},$$

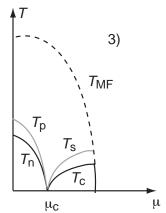
$$\Delta_{0} = \frac{1}{2} \sum_{\mathbf{k},\sigma} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}+\mathbf{Q}\sigma},$$

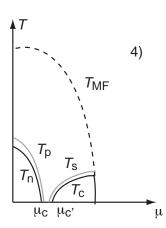
$$\Delta_{-1} = -\Delta_{1}^{\dagger},$$

$$\left[\eta^{\pm}, \Delta_{m}\right] = \sqrt{l\left(l+1\right) - m\left(m\pm1\right)} \Delta_{m\pm1},$$
$$\left[\eta_{z}, \Delta_{m}\right] = m\Delta_{m}.$$

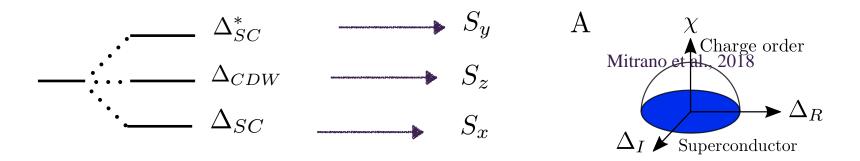








#### 0(3) non linear $\sigma$ -model



Topological structure: Skyrmions in the pseudo spin space

