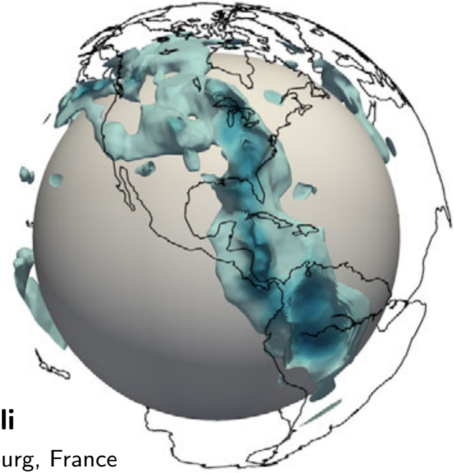
Global seismic tomography
using SOLA–Backus–Gilbert inversion



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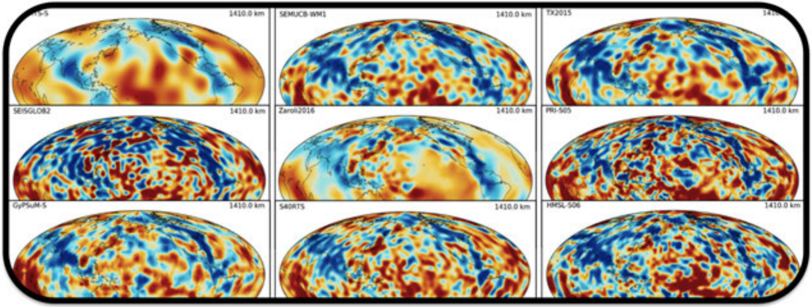
Symposium – Collège de France (2021)
Global scale seismic imaging and dynamics of the Earth's mantle

▶▶ Introduction

- Global tomographic models differ (at least on short wavelengths)
- Interpreting physical processes based on models without accurate **resolution** and **uncertainty** analyses is a hazardous endeavor
- SOLA–Backus–Gilbert inversion can help building a new generation of models accompanied with resolution/uncertainty informations

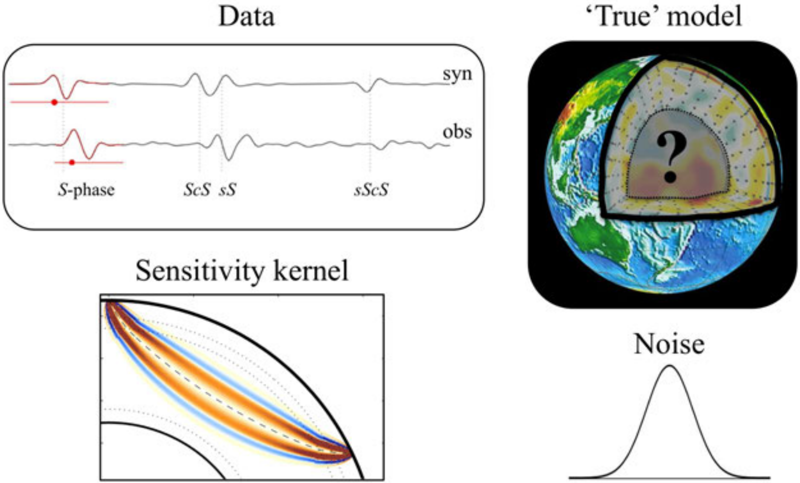


SubMachine

Hosseini et al. (2018)

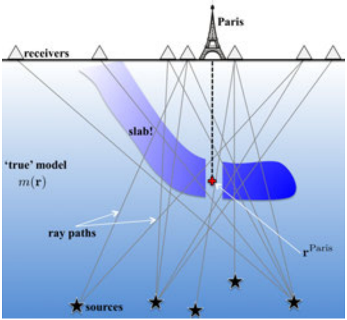
▶▶ Large scale, linear(ized) tomographic problems

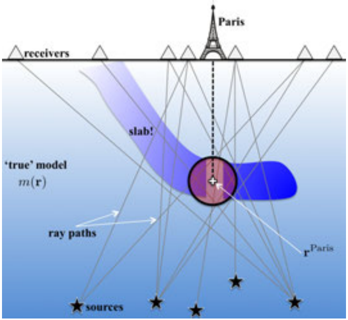
- **General form:** $\boxed{d_i = \int K_i(\mathbf{r})\, m(\mathbf{r})\, d^3\mathbf{r} + n_i}$
- Global tomography: 10^5–10^6 data



►► Reviving Backus–Gilbert theory for seismic tomography

- Backus–Gilbert theory (1967, 68, 70) seeks to determine **Optimally Localized Averages** (OLA) over the continuous 'true' Earth model.
- What is the average value, and the attached uncertainty, of velocity anomalies within some localized volume in the Earth's interior?
- *"[B–G] carefully avoids using any a priori information on the model parameters that could 'bias' the inferences to be drawn from the data."* (Tarantola, 2006)



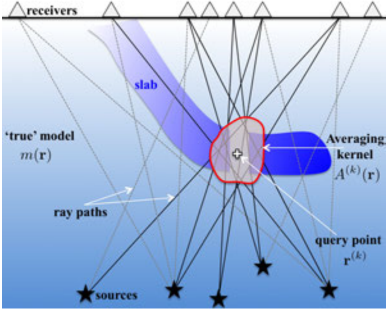


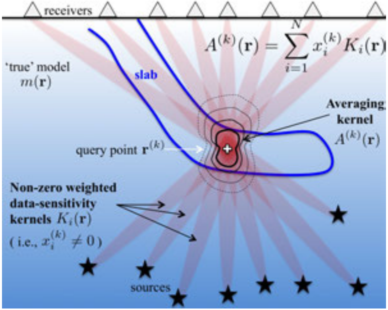
▶▶ Backus–Gilbert tomography in a nutshell (1/3)

- **Linear tomographic problem:** $d_i = \int K_i(\mathbf{r})\, m(\mathbf{r})\, d^3\mathbf{r} + n_i$ (N data)

- $\boxed{\hat{m}^{(k)} = \sum_{i=1}^{N} x_i^{(k)} d_i}$ $= \int \underbrace{\left(\sum_i x_i^{(k)} K_i(\mathbf{r}) \right)}_{\text{averaging kernel } A^{(k)}(\mathbf{r})} m(\mathbf{r})\, d^3\mathbf{r} + \underbrace{\sum_i x_i^{(k)} n_i}_{\text{noise effect}}$

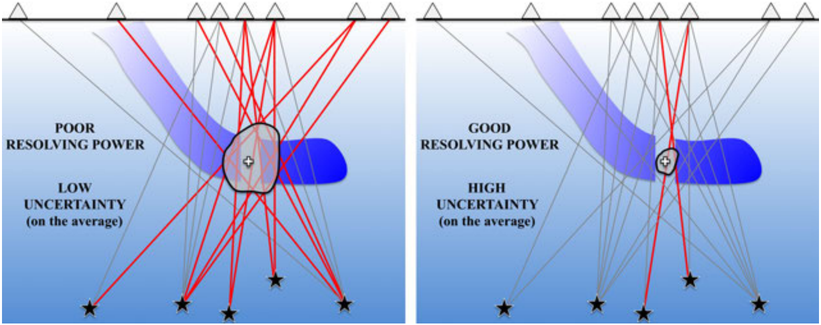
- $\boxed{\hat{m}^{(k)} \approx \underbrace{\int A^{(k)}(\mathbf{r})\, m(\mathbf{r})\, d^3\mathbf{r}}_{\text{local averaging}}}$ $\boxed{\underbrace{\sigma_{\hat{m}^{(k)}}}_{\text{uncertainty}} = \sqrt{\sum_i \left(x_i^{(k)} \sigma_{d_i} \right)^2}}$



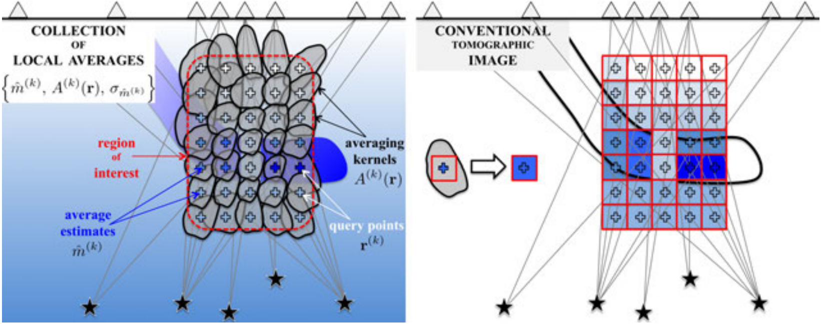


▶▶ Backus–Gilbert tomography in a nutshell (2/3)

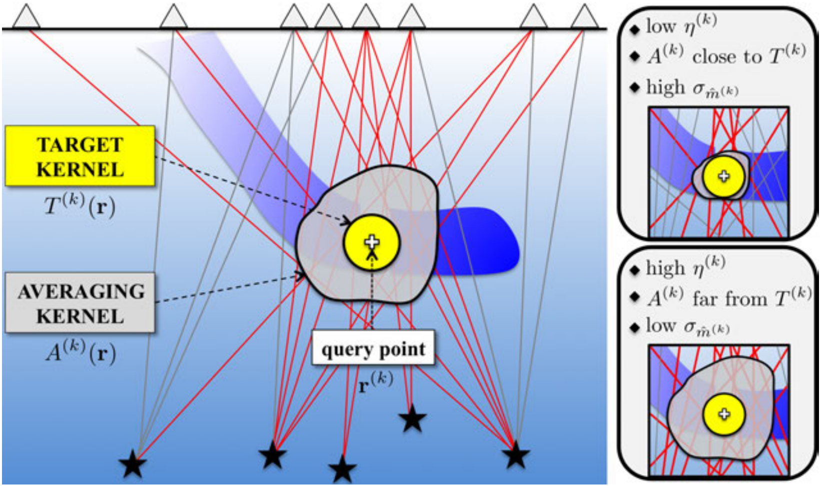
- **Trade-off:** resolution *vs* uncertainty
- Resolving length relates to the size of averaging kernel $A^{(k)}$
- Uncertainty $\sigma_{\hat{m}^{(k)}}$ describes noise effect on local average $\hat{m}^{(k)}$



- **From a collection of local averages to a tomographic 'image'**
- Model appraisal is not an easy task — in particular when local resolution (and uncertainty) differs from one point to another

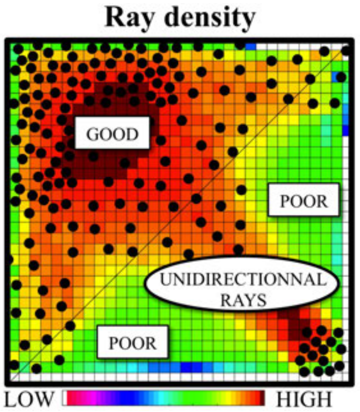


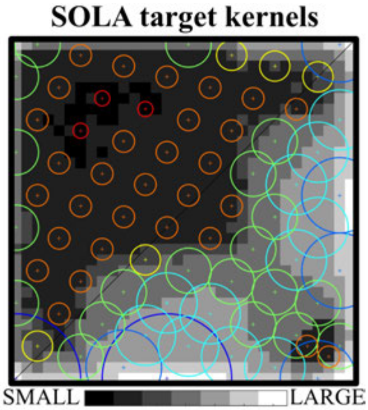
- **Specify *a priori* information on local model resolution**
- Target form $T^{(k)}$ for averaging kernel $A^{(k)}$ (Pijpers & Thompson, 92)
- Different from specifying *a priori* information on the model itself !



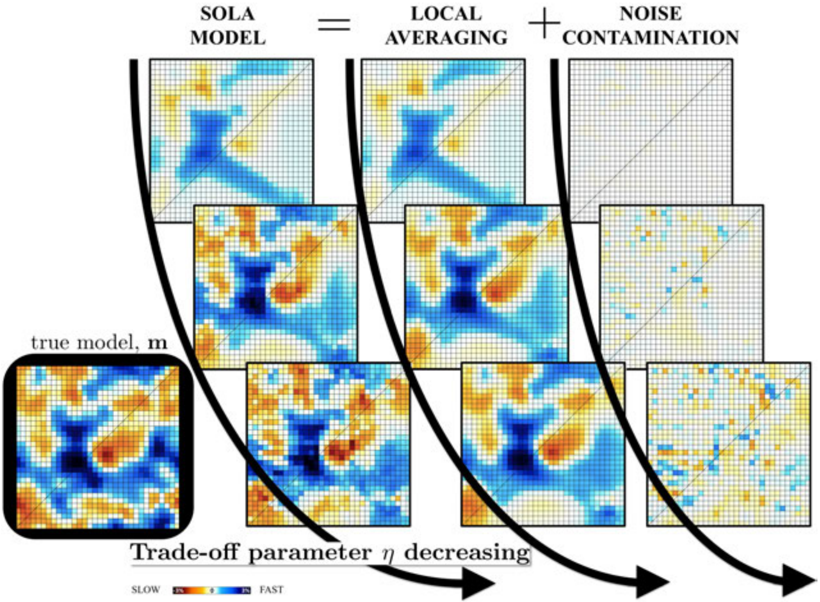
▶▶ SOLA–Backus–Gilbert tomography in a nutshell (2/4)

- **References**:
  - Zaroli (2019, *GJI*) — Continuous SOLA tomography
  - Zaroli (2016, *GJI*) — Discrete SOLA tomography
  - Zaroli *et al.* (2017, *GRL*) — Discrete SOLA *vs* DLS tomography
- **Example:**

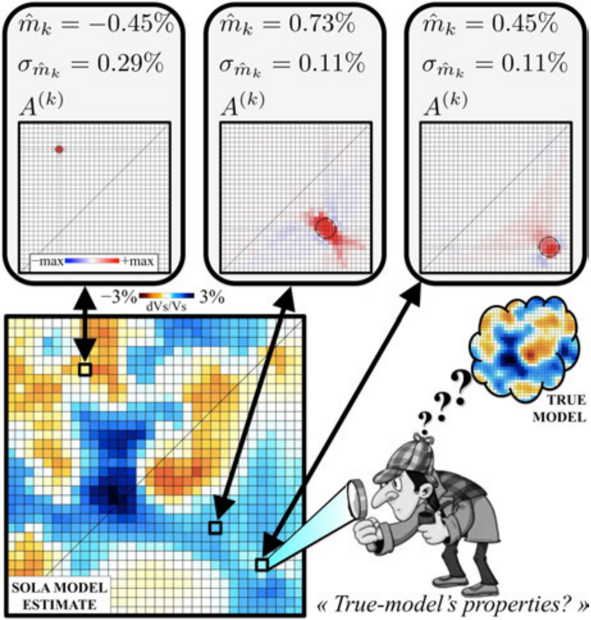




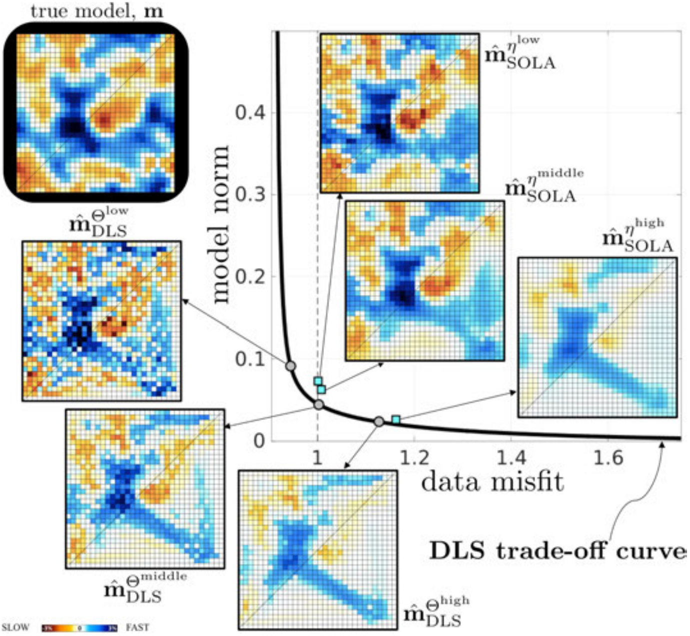
▶▶ SOLA–Backus–Gilbert tomography in a nutshell (3/4)



▶▶ SOLA–Backus–Gilbert tomography in a nutshell (4/4)

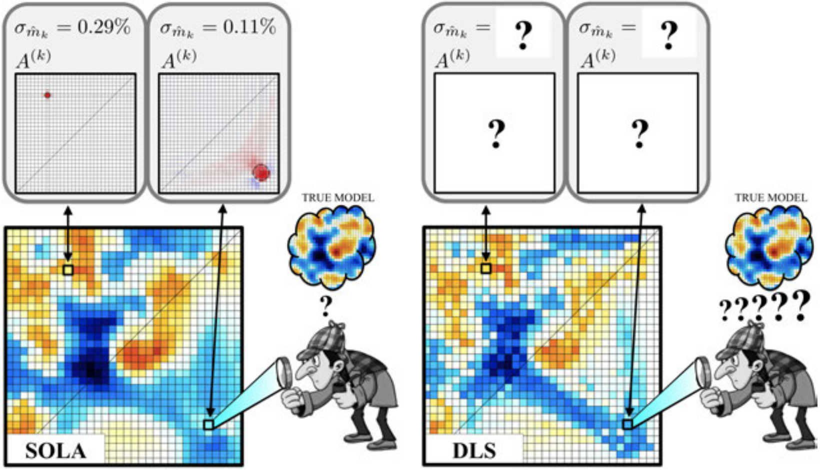


▶▶ SOLA *vs* traditional DLS tomography (1/4)

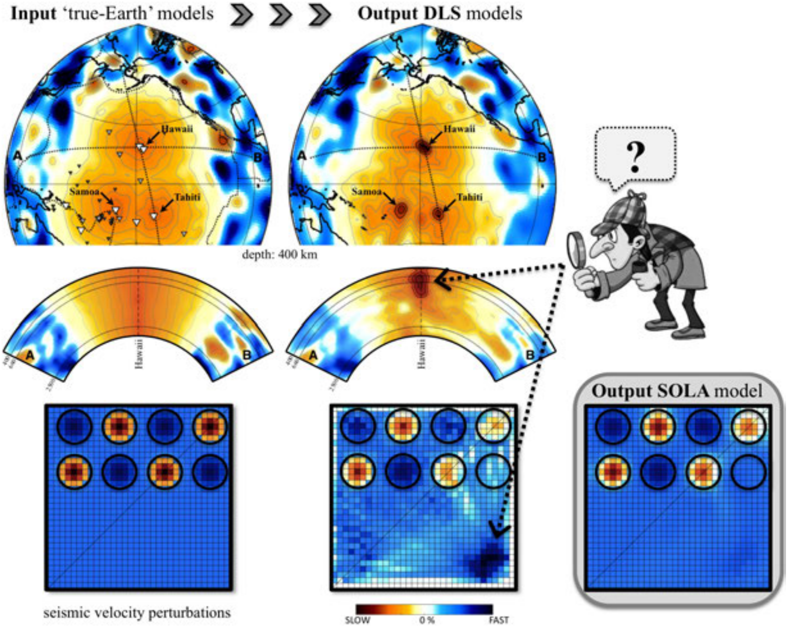


▶▶ SOLA *vs* traditional DLS tomography (2/4)

- Generalized inverse $\Longrightarrow$ Resolution *and* Uncertainty
- SOLA is more efficient than DLS for computing generalized inverses

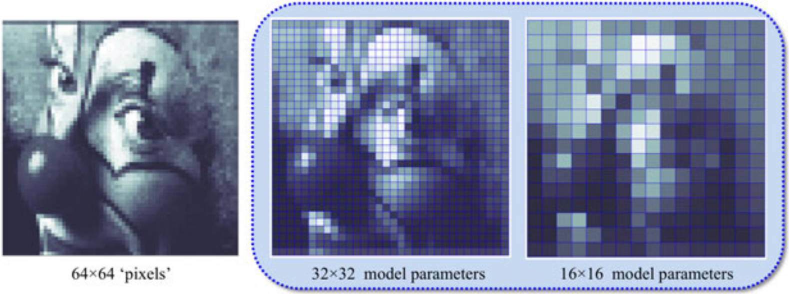


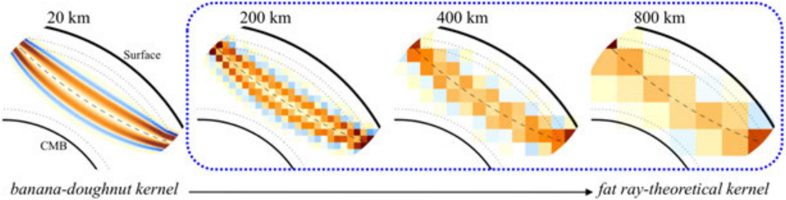
▶▶ SOLA *vs* traditional DLS tomography (3/4)



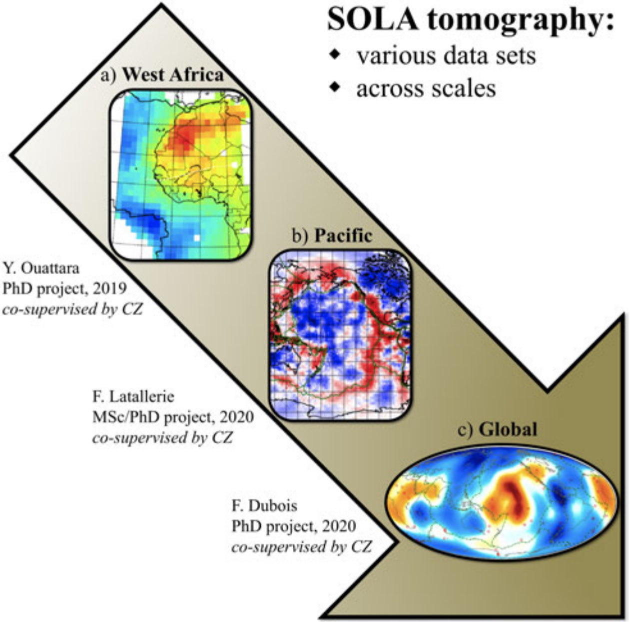
▶▶ SOLA *vs* traditional DLS tomography (4/4)

$$d_i = \int K_i(\mathbf{r}) m(\mathbf{r}) d^3\mathbf{r} + n_i \longrightarrow d_i = \sum_{j=1}^{M} G_{ij} m_j + \boxed{\underline{n}_i}$$



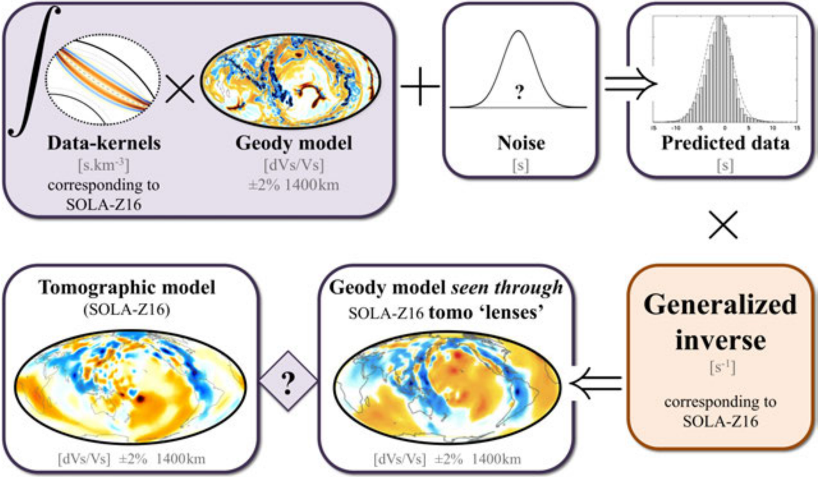


▶▶ Tomographic applications



▶▶ Tomographic–geodynamic comparisons

- SOLA tomography provides a natural framework for 'quantitatively' testing **geodynamic scenarios against tomographic models**

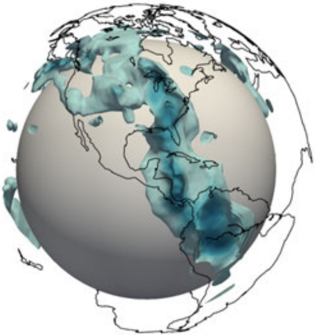


Freissler*, Zaroli, Lambotte, Schuberth (2020)

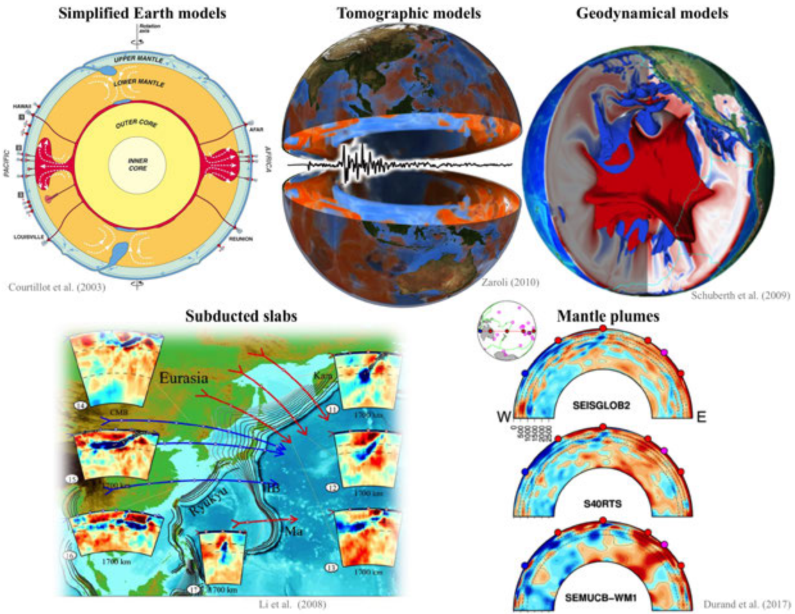
▶▶ Conclusion

SOLA–Backus–Gilbert tomography:

- Models with resolution *and* uncertainty informations
- Direct control on resolution/uncertainty
- Models can be free of averaging-bias effects, and fit the data
- Data-kernels can be fully exploited (if no model space discretization)
- Natural framework for tomographic–geodynamic comparisons

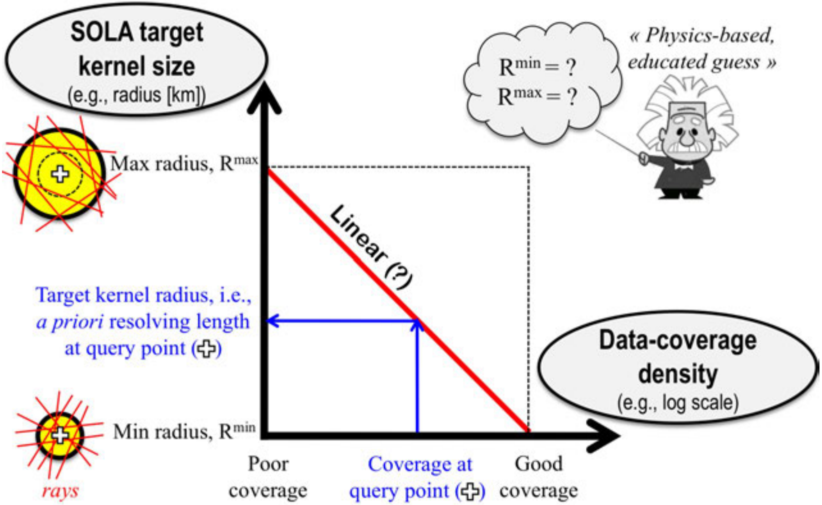


▶▶ Need for ‘quantitative’ comparisons of models...

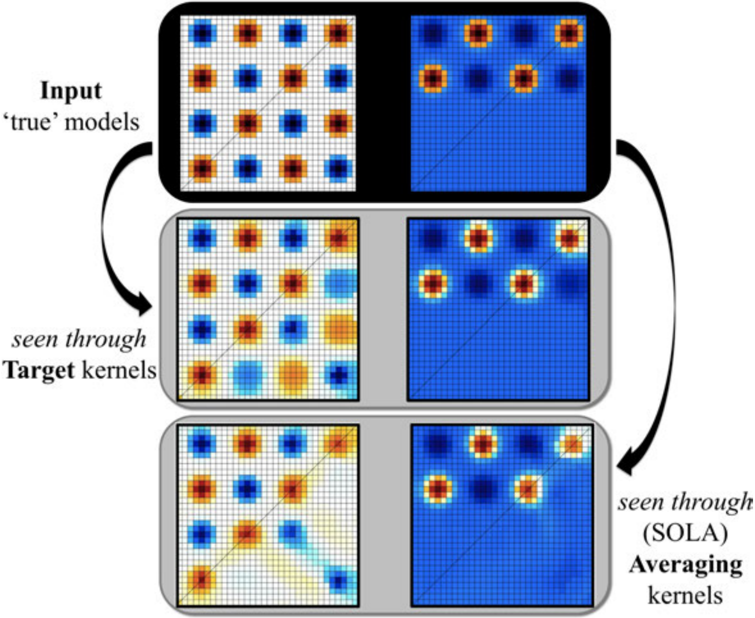


▶▶ Tuning SOLA tomographic inversions

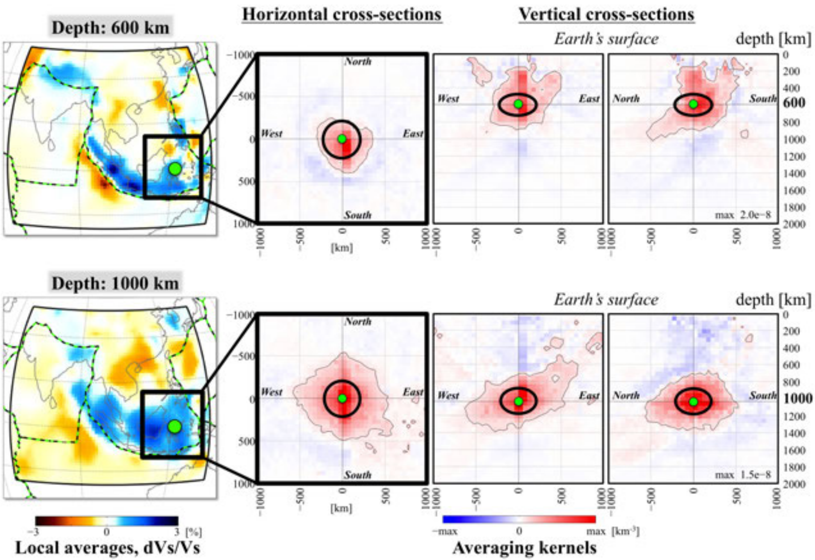
- **Recipe** for target kernels, $T^{(k)}$, and trade-off parameters, $\eta^{(k)}$



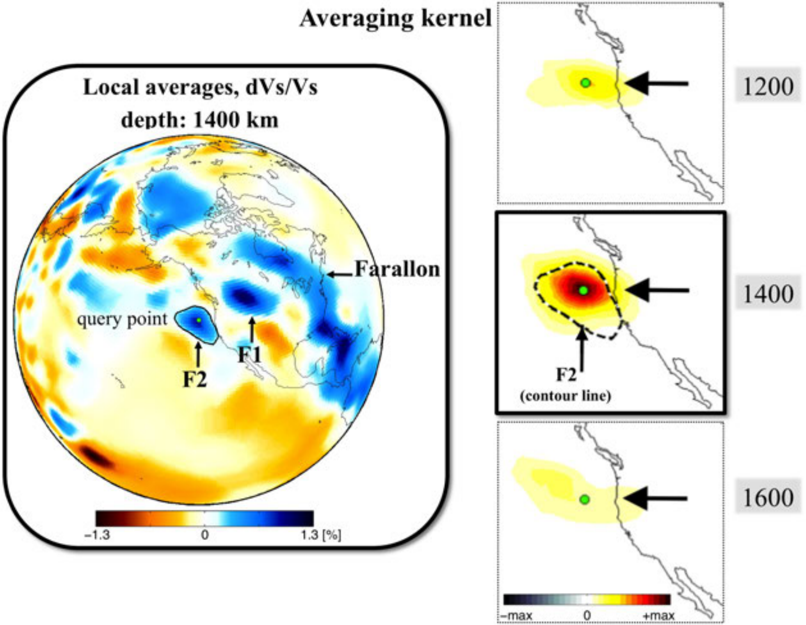
►► Visualizing 'at a glance' local resolution



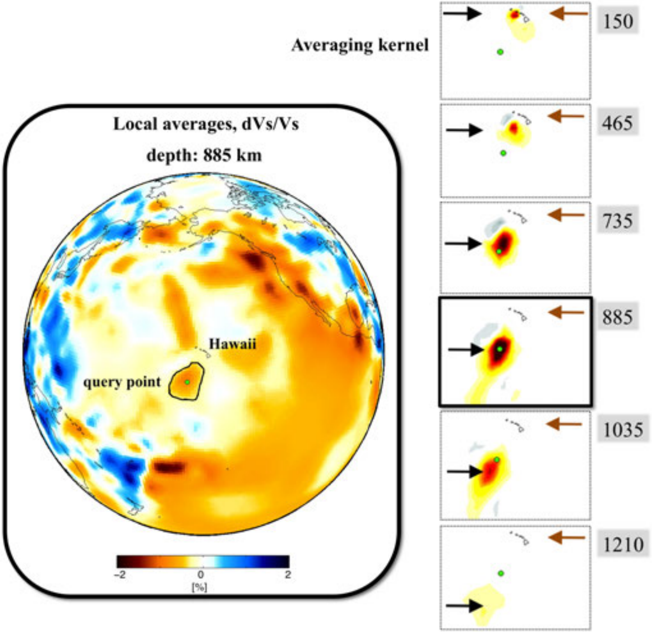
▶▶ Interrogating SOLA models (first example)

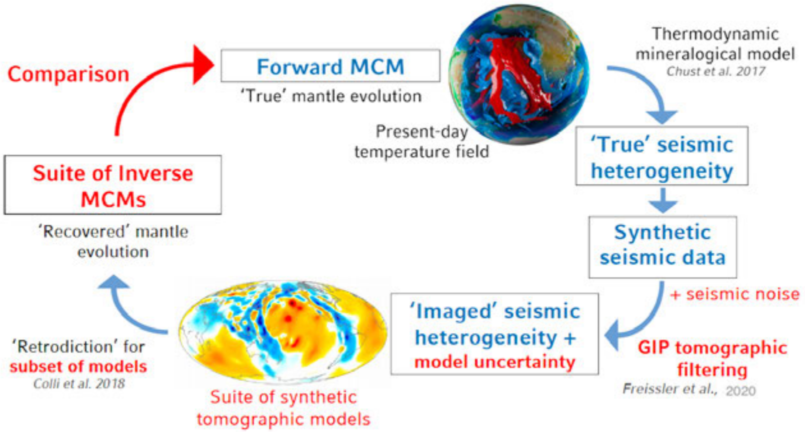


▶▶ **Interrogating SOLA models** (second example)



▶▶ **Interrogating SOLA models** (third example)




Comparison
Forward MCM
'True' mantle evolution
Thermodynamic mineralogical model
Chust et al. 2017
Present-day temperature field
'True' seismic heterogeneity
Suite of Inverse MCMs
'Recovered' mantle evolution
Synthetic seismic data
+ seismic noise
'Imaged' seismic heterogeneity + model uncertainty
GIP tomographic filtering
Freissler et al., 2020
'Retrodiction' for subset of models
Colli et al. 2018
Suite of synthetic tomographic models