Chaire européenne

M. Sandro STRINGARI, professeur

Condensation de Bose-Einstein et superfluidité

There were 9 lectures given, one per week from February 14, 2005 to April 18, 2005 (there was no lecture on March 28). These lectures, whose main contents are listed below, presented a review of the physics of Bose-Einstein condensation in ultracold atomic gases, a field of research that has grown in an impressive way during the last ten years after the first experimental realization of a condensate achieved in 1995. The lectures received a good response with an average attendance of around 80-100 participants. Each lecture was followed by a seminar whose list is given separately. Both the lectures and the seminars are visible at the web site http://www.phys.ens.fr/cours/Sandro/index.html

1. **Bose-Einstein condensation and long range order.** In this lecture some basic concepts of Bose-Einstein condensation (BEC) were discussed. These include the concept of long range order, the behaviour of the eigenvalues of the 1-body density matrix and the concept of order parameter as a classical field. The key results for the critical temperature and for the temperature dependence of the condensate in the presence of harmonic trapping were presented. A series of questions concerning the role of interactions were addressed. For example : do interactions modify the shape of the order parameter ? Do interactions reinforce or weaken BEC ? Do they enhance or decrease the critical temperature ? Can BEC be fragmented (more than one single particle state with macroscopic occupancy) ? Some examples illustrating the relevance of the above questions and some partial answers were presented.

2. **Superfluidity and hydrodynamics.** This lecture addressed the important question of superfluidity and of its manifestations at a macroscopic scale. This lecture first discussed the most famous Landau's criterium for superfluidity (for both Galilean and rotational transformations). A crucial implication of superfluidity, of particular relevance for the physics of trapped Bose-Einstein

condensates, is the irrotational hydrodynamic behaviour whose consequences can be probed experimentally through the measurement of the sound velocity, the frequencies of the collective oscillations, the shape of the atomic cloud after expansion. The lecture finally pointed out the limits of applicability of hydrodynamic theory in the description of phenomena which require a more microscopic approach to the many-body problem. Two examples were addressed : the determination of the critical angular velocity satisfying Landau's criterium and the prediction for the interference fringes prodcuced by two expanding and overlapping condensates.

3. **Equation for the order parameter**. This lecture discussed some general features of Gross-Pitaevskii theory which provides an explicit equation for the space and time dependence of the order parameter. Special emphasis was given to the analogies between the Gross-Pitaevskii equation for the classical field characterizing the order parameter and the Maxwell equations for the classical electromagnetic field. The equation provides systematic results both for the equilibrium properties of the system (equilibrium profile, energy systematics, etc.) as well as for the dynamic properties where, in the linear limit of small oscillations, the theory reproduces the Bogoliubov equations. The key role of the healing length was extensively discussed. This quantity provides a crucial length describing the transition from the macroscopic phenomena, governed by the hydrodynamic theory, to more microscopic features of the system. The lecture also pointed out the success of the theory in providing an accurate description of interference phenomena and Josephson like effects.

4. **Fluctuations of the order parameter**. This lecture addressed the question of the quantum and thermal fluctuations of the order parameter. The quantum fluctuations are calculated using Bogoliubov theory, based on the most famous transformation between particle and quasi-particle operators. Special emphasis was given to the behaviour of the momentum distribution in uniform media, to the quantum depletion of the condensate and to the divergent behaviour of the kinetic energy caused by the failure of the theory in providing the correct behaviour of the momentum distribution at high momenta. Also the behaviour of the density fluctuations and of the static structure factor was discussed. The lecture further addressed the challenging question of the so called beyond mean field effects, not accounted for by Bogoliubov theory. These effects show up in small, but in principle measurable, corrections in the collective frequencies which are sensitive to the changes in the equation of state. The lecture finally addressed the role of the thermal fluctuations which result in a thermal depletion of the condensate. The role of interactions on the shift of the critical temperature for Bose-Einstein condensation was also discussed.

5. **Bose-Einstein condensation in low dimensions**. This lecture first summarized well known theorems of quantum statistical physics concerning the absence of long range order in 1 and 2 dimensions (the so called Mermin-

Wagner-Hohenberg theorem) at finite temperature where the thermal fluctuations of the phase destroy long range order. The extension of such a theorem to zero temperature, where the quantum fluctuations of the phase play a crucial role, was also presented. The behaviour of the one-body density matrix in configurations characterized by the absence of long range order was explicitly discussed with the help of the predictions of quantum hydrodynamics. In the second part of the lecture some key features exhibited by trapped Bose-Einstein condensates in low dimensions were presented, with special attention to the case of one dimensional systems. These include the propagation of solitons and new dispersion laws exhibited by the collective oscillations. Some features beyond the mean field regime, including the most challenging case of the Tonks gas, were also discussed.

6. **Moment of inertia and superfluidity**. This and the following lecture addressed one of the most spectacular features exhibited by superfluids, associated with their rotational properties. Superfluids cannot in fact rotate like a classical body, being subject to the constraint of irrotationality for the velocity flow. In this lecture the consequences of irrotationality on the rotation of a Bose-Einstein condensate were discussed at low angular velocity. The quenching of the moment of inertia and the consequences on the so-called scissors mode and on the expansion of a gas initially trapped by a rotating trap were discussed, with explict comparisons with available experimental results. The stationary solutions of the hydrodynamic equations of motion in the rotating frame were also discussed. In particular the spontaneous breaking of rotational symmetry induced at relatively high angular velocities, when the quadrupole oscillation of the condensate becomes energetically unstable, was discussed in a systematic way.

7. **Quantized vortices**. An important consequence of the irrotationality constraint imposed by superfluidity is the occurrence of quantized vortices where the velocity field obeys the irrotationality criterium every where except on the singular lines of vorticity where the density exactly vanishes. The quantum nature of the problem then results in the quantization of circulation and of angular momentum. Gross-Pitaevskii theory provides the proper description of vortex lines in a dilute Bose-Einstein condensed gas. In particular the size of the vortex core is fixed by the healing length and hence by the interatomic force. Due to the smallness of the healing length the vortex lines become clearly visible only after expansion. Nevertheless the quantization of angular momentum can be directly probed by studying the precession phenomena in the shape oscillations induced by the angular momentum carried by the vortex. The lecture presented also some key features exhibited by arrays of vortex lines giving rise to regular structures, also called Abrikosov lattices. The low energy ecitations of such systems are the so called Tkachenko oscillations of the vortex lattice. The higher frequencies modes can be described by introducing the concept of diffused vorticity and solving the equations of rotational

hydrodynamics. The lecture finally discussed the new regimes available when the system rotate close to the centrifugal limit and occupies the so called lowest Landau level.

8. **Ultracold Fermi gases**. This lecture addressed the problem of quantum degenerate Fermi gases confined in harmonic traps. These systems have been the object of extensive experimental and theoretical research in the recent years and reveal quite different features with respect to trapped bosons. The key concepts of the non interacting Fermi gas were first introduced, with special emphasis to the relevant energy scale fixed, at low temperature, by the Fermi energy and to the isotropy of the momentum distribution which introduces a crucial difference with respect to a Bose-Einstein condensed gas. The role of interactions was hence discussed, both in the case of negative and positive values of the scattering length. In the first case theory predicts the formation of a superfluid phase similar to the one (BCS) occurring in certain superconductors. In the latter case weakly bound molecules can be formed which, at low temperature, give rise to the phenomenon of Bose-Einstein condensation. Due to the presence of a Feshbach resonance it is possible to tune the value of the scattering length by just changing the value of the magnetic field and to explore the crossover between the two regimes. Some important features of the crossover were discussed, with special focus on the the region close to the resonance where the scattering length becomes infinite and the system becomes highly correlated, exhibiting non trivial universal properties. The quest for superfluidity in such systems has been directly addressed and some results concerning the anisotropy of the expansion and the shift of the collective frequencies were explicitly discussed.

9. **Bose-Einstein condensation in periodic potentials**. Periodic potentials can be produced employing counter propagating laser beams which give rise to an optical lattice which favours the localization of atoms near the bottom of the wells produced by the laser. If the depth of the wells is not too large the system still preserves long range order and Bose-Einstein condensation. In these conditions the atomic cloud exhibits very spectacular interference fringes after expansion, which are the analogue of Fraunhofer intereference in ordinary crystals. Another peculiar property of such configuration is given by their band structure and the occurrence of Bloch oscillations. From the point of view of superfluidity the coherence among atoms occupying different wells is at the origin of Josephson like effects and shows up in the occurrence of collective oscillations during which the atoms tunnel coherently through the barriers produced by the optical lattice. These configurations are also well suited to study the phenomena of energetic and dynamic instability, taking place when the system is non longer in its equilibrium configuration. When the height of the barrier becomes too large the coherence through the barriers is lost and the system undergoes a transition to a Mott insulating phase where

atoms are localized in each well and the phase characterizing the Bose-Einstein condensed phase exhibits large fluctuations.

General bibliography related to Bose-Einstein condensation

F. Dalfovo, S. Giorgini and S. Stringari, *Rev. Mod Phys.*, **71** (1999) 463.

M. Inguscio, S. Stringari and C. Wieman, Bose-Einstein Condensation in Atomic Gases, Enrico Fermi Summer School, Course CXL (IOS Press Amsterdam, 1999).

A. Leggett, *Rev. Mod. Phys.*, **73** (2001) 307.

C. Pethick and H. Smith, Bose-Einstein Condensation in Dilute Bose Gases, Cambridge University Press (2002).

E.A. Cornell and C.E. Wieman, *Rev. Mod. Phys.*, **74** (2002) 875 ; W. Ketterle, *Rev. Mod. Phys.*, **74** (2002) 1131.

L. Pitaevskii and S. Stringari, Bose-Einstein Condensation, Oxford University Press (2003).

Seminars organized during the course

14 February	Antony Leggett (Urbana, IL USA) "What is superfluidity ?"
21 February	William Phillips (Nist, Gaithersburg USA) "Experiments with coherent atoms in one dimension"
28 February	Wolfgang Ketterle (MIT, Cambridge, USA) "Bose-Einstein condensation of atoms, molecules and fermion pairs"
7 March	Lev Pitaevskii (Trento, Italy) "Coherence and superfluidity in Bose-Einstein condensates"
14 March	Gora Shlyapnikov (LPTMS Orsay, France) "Low-dimensional trapped gases"
21 March	Sebastien Balibar (ENS Paris, France) "BEC in strongly interacting systems : liquid and solid helium"
4 April	Achim Richter (Darmstaadt, Germany) "Some aspects of collective oscillations and superfluidity in atomic nuclei"
11 April	Rudy Grimm (Innsbruck, Austria) "Bose-Einstein condensation and superfluidity in ultracold Fermi gases"
18 April	Antoine Georges (Ecole Polytechnique, France) "Probing the physics of strong correlations with fermions in optical traps"

Research activity carried out during the year 2004-2005

The research activity has mainly concerned the study of quantum phenomena in ultracold atomic gases.

Main topics investigated during the year 2004-2005 :

— **Collective oscillations, equation of state and structure factor of a superfluid Fermi gas near a Feshbach resonance**. In a collaboration with G. Astrakharchik (BEC Center, Trento), Roland Combescot and Xavier Leyronas (École Normale Supérieure, Paris) we have investigated the behaviour of the collective frequencies of an interacting trapped Fermi gas close to a Feshbach resonance and in particular the crossover from the regime of Bose-Einstein condensed molecules to the unitary regime at resonance where the scattering length becomes infinite. In a further collaboration, involving Roland Combescot (École Normale Supérieure, Paris) and Stefano Giorgini (BEC Center, Trento) we have investigated the behaviour of the static and dynamic structure factor of these interacting Fermi gases.

— **Vortical configurations in ultracold atomic gases**. In a collaboration with the teams of Yvan Castin and Jean Dalibard (École Normale Supérieure, Paris) we have investigated the new features exhibited by the vortex lines of a rapidly rotating Bose-Einstein condensate when the interactions are switched off and the statistical distribution of vortices follows the zeros of an important class of random polynomials. In another collaboration with Alexander Fetter (Stanford, USA), Brian Jackson and Marco Cozzini (BEC Center, Trento) we have investigated the dynamic features of a rotating Bose-Einstein condensate trapped in annular configurations. With Marco Cozzini and Cesare Tozzo (BEC Center, Trento) the Tkachenko oscillations of a vortex lattice have been investigated with special focus on the crossover between the Thomas-Fermi and the lowest Landau level regime.

— **Casimir-Polder and thermal effects on the surface-atom force**. In a collaboration with Mauro Antezza and Lev Pitaevskii (BEC Center, Trento) we have studied the behaviour of the force between a dielectric surface and an atom. New results have been obtained for the asymptotic behaviour at large distances when the system is out of thermal equilibrium. This research has profited of an important collaboration with the team of Eric Cornell at JILA where experiments on the Casimir-Polder force have been recently carried out. In another collaboration with Iacopo Carusotto and Lev Pitaevskii (BEC Center, Trento) and Massimo Inguscio and Giovanni Modugno (LENS, Florence) we have proposed a new interferometric method to measure the Casimir-Polder force using the Bloch oscillations of an ultracold quantum gas.

— **Molecular configurations in the presence of periodic potentials**. In a collaboration with Lev Pitaevskii, Giuliano Orso and Michiel Wouters (BEC Center, Trento) we have investigated the consequences of a periodic potential (optical lattice) on the formation of molecules near a Feshbach resonance, as well as the effects on the tunnelling properties through the barrier.

Publications

M. Cozzini, A.L. Fetter, B. Jackson and S. Stringari (2005), **Oscillations of a Bose-Einstein condensate rotating in a harmonic plus quartic trap.** *Phys. Rev. Lett.*, **94**, 100402.

Abstract : We study the normal modes of a two-dimensional rotating Bose-Einstein condensate confined in a quadratic plus quartic trap. Hydrodynamic theory and sum rules are used to derive analytical predictions for the collective frequencies in the limit of high angular velocities, Ω, where the vortex lattice produced by the rotation exhibits an annular structure. We predict a class of excitations with frequency $\sqrt{6}\,\Omega$ in the rotating frame, irrespective of the mode multipolarity m, as well as a class of low energy modes with frequency proportional to $|m|/\Omega$. The predictions are in good agreement with results of numerical simulations based on the 2D Gross-Pitaevskii equation. The same analysis is also carried out at even higher angular velocities, where the system enters the giant vortex regime.

G. Orso, L.P. Pitaevskii, S. Stringari and M. Wouters (2005), **Formation of molecules near a Feshbach resonance in a 1D optical lattice.** *Phys. Rev. Lett.*, **95**, 060402.

Abstract : We calculate the binding energy of two atoms interacting near a Feshbach resonance in the presence of a 1D periodic potential. The critical value of the scattering length needed to produce a molecule as well as the value of the molecular binding energy in the unitarity limit of infinite scattering length are calculated as a function of the intensity of the laser field generating the periodic potential. The Bloch bandwidth and the effective mass of molecules are shown to depend strongly on the value of the scattering length due to the correlated motion of the two atoms.

I. Carusotto, L. Pitaevskii, S. Stringari, G. Modugno and M. Inguscio (2005), **Sensitive measurement of forces at micron scale using Bloch oscillations.** *Phys. Rev. Lett.*, **95**, 093202.

Abstract : We show that Bloch oscillations of ultracold fermionic atoms in the periodic potential of an optical lattice can be used for a sensitive measurement of forces at the micrometer length scale, e.g. in the vicinity of dielectric surface. In particular, the proposed approach allows to perform a local and direct measurement of the Casimir-Polder force which is, for realistic experimental parameters, as large as 10^{-4} gravity.

G.E. Astrakharchik, R. Combescot, X. Leyronas and S. Stringari (2005), **Equation of state and collective frequencies of a trapped Fermi gas along the BEC-unitarity crossover.** *Phys. Rev. Lett.*, **95**, 030405.

Abstract : We show that the study of the collective oscillations in a harmonic trap provides a very sensitive test of the equation of state of a Fermi gas near a Feshbach resonance. Using a scaling approach, whose high accuracy is proven by comparison with exact hydrodynamic solutions, the frequencies of the lowest compressional modes are calculated at $T = 0$ in terms of a dimensionless parameter characterizing the equation of state. The predictions for the collective frequencies, obtained from the equations of state of mean field BCS theory and of recent Monte-Carlo calculations, are discussed in detail.

Mauro Antezza, Lev P. Pitaevskii, Sandro Stringari (2005), **New asymptotic behaviour of the surface-atom force out of thermal equilibrium.** *Phys. Rev. Lett.*, **95**, 113202.

Abstract : The Casimir-Polder-Lifshitz force felt by an atom near the surface of a substrate is calculated out of thermal equilibrium in terms of the dielectric function of the material and of the atomic polarizability. The new force decays like $1/z^3$ at large distances (i.e. slower than at equilibrium), exhibits a sizable temperature dependence and is attractive or repulsive depending on whether the temperature of the substrate is higher or smaller than the one of the environment. Our predictions can be relevant for experiments with ultracold atomic gases. Both dielectric and metal substrates are considered.

M. Cozzini, S. Stringari, C. Tozzo. **Vortex lattices in Bose-Einstein condensates : from the Thomas-Fermi to the lowest Landau level regime.** *cond-mat/ 0509559.*

Abstract : We consider a periodic vortex lattice in a rotating Bose-Einstein condensed gas, where the centrifugal potential is exactly compensated by the external harmonic trap. By introducing a gauge transformation which makes the Hamiltonian periodic, we solve numerically the 2D Gross-Pitaevskii equation finding the exact mean field ground state. In particular, we explore the crossover between the Thomas-Fermi regime, holding for large values of the coupling constant, and the lowest Landau level limit, corresponding to the weakly interacting case. Explicit results are given for the equation of state, the vortex core size, as well as the elastic shear modulus, which is crucial for the calculation of the Tkachenko frequencies.

Y. Castin, Z. Hadzibabic, S. Stock, J. Dalibard and S. Stringari. **Seeing zeros of random polynomials : quantized vortices in the ideal Bose gas. cond-mat/ 0511330**.

We propose a physical system allowing one to experimentally observe the distribution of the complex zeros of a random polynomial. We consider a degenerate, rotating, quasi-ideal atomic Bose gas prepared in the lowest Landau level. Thermal fluctuations provide the randomness of the bosonic field and of the locations of the vortex cores. These vortices can be mapped to zeros of random polynomials, and observed in the density profile of the gas.

Seminars given in other institutions

— Modena (Palazzo dei Musei, 5 May)
L'avventura dei gas ultrafreddi: condensazione di Bose-Einstein e superfluidita' (lecture).

— Orsay (LPTMS 10 May)
Bloch oscillations with ultra-cold atoms and test of the Casimir-Polder force (seminar).

— Tokyo (Tokyo Institute of Technology, 1 July)
Bloch oscillations with ultra-cold atoms and test of the Casimir-Polder force (seminar).

— Kyoto (14th International Laser Physics Conference, 4-8 July)
Superfluidity of ultracold atomic gases (invited talk).

— Bern (EPS 13 Conference: Beyond Einstein: Physics of the 21st Century, 11-15 July)
Superfluidity of ultracold atomic gases (invited talk).

— Trieste (School on Quantum Phase Transitions and Non-Equilibrium phenomena in Cold Atomic Gases, 11-22 July)
Rotating cold atomic gases: basic theory and experiments (lecture).