Streaming Algorithms for the Set Cover Problem

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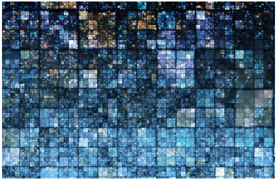
Collège de France, June 2018

Motivation - Big Data



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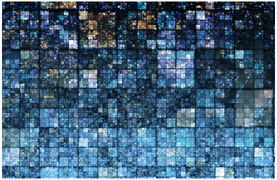

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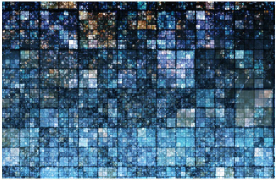

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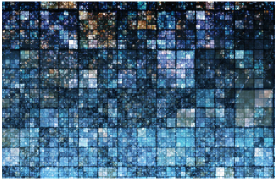

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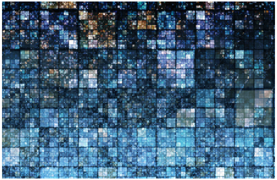

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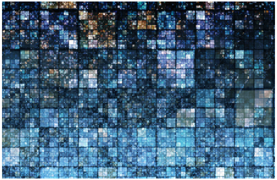

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- Today: algorithms that store a small fraction of the available data

1. The streaming model
2. The set-cover problem
3. (some of the) Results
4. (some of the) Techniques
   - Single pass, semi-streaming algorithm (unweighted case)
   - Matching lower bound(s)
5. Conclusions and open problems

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- Problems:
  - Selection of k^{th} largest element [Munro, Paterson 1980]
  - Estimating frequency moments [Alon, Matias, Szegedy 1996]
  - Finding heavy hitters [Karp, Papadimitriou, Shenker 2003]
  - Counting distinct elements [Kane, Nelson, Woodruff 2010]
  - Checking balanced parentheses [Magniez, Mathieu, Nayak 2010]

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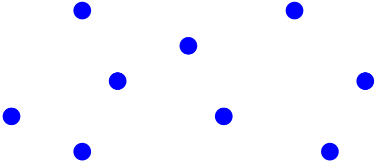

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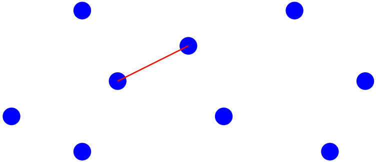

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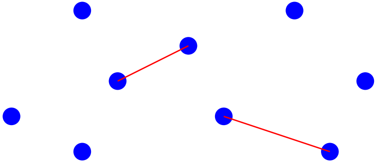

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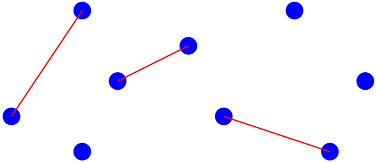

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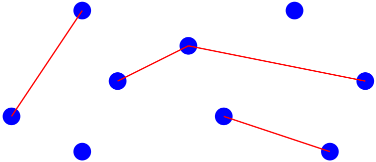

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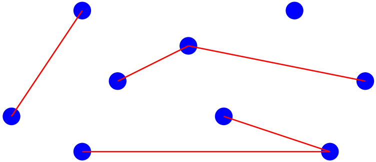

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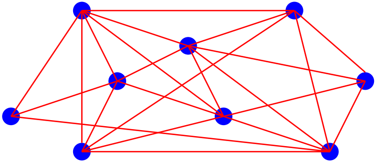

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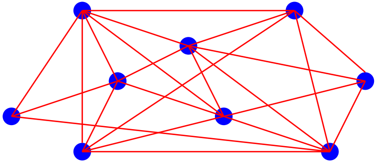

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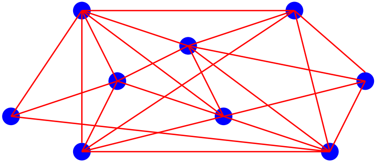

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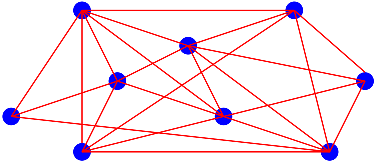

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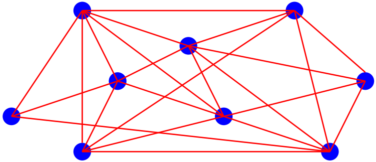

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  - Distances and diameter
    [Feigenbaum, Kannan, McGregor, Suri, Zhang 2005]
  - Constructing spanners and shortest path trees
    [Feigenbaum, Kannan, McGregor, Suri, Zhang 2008]
  - Maximum matching
    [McGregor 2005; Epstein,Levin,Mestre,Segev 2010; Crouch,Stubbs 2014]
  - Constructing spectral sparsifiers [Ahn, Guha 2009; Kelner, Levin 2013]
  - Maximum Independent Set
    [Halldórsson, Halldórsson, Losievskaja, Szegedy 2010]

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- *"... a problem whose study has led to the development of fundamental techniques for the entire field of approximation algorithms"* [Vazirani 2001]

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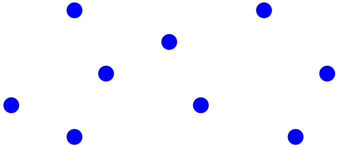

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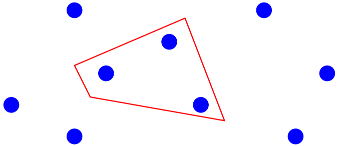

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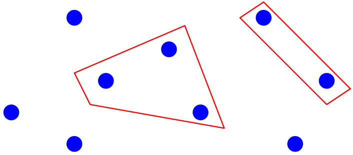

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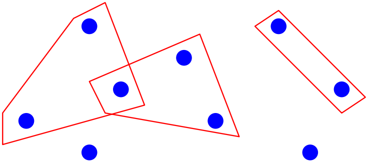

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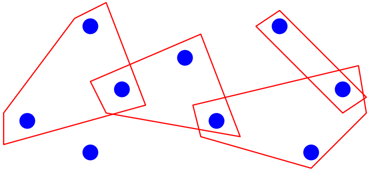

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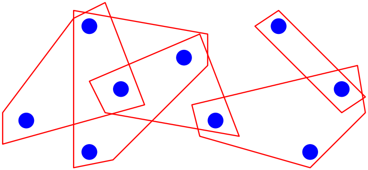

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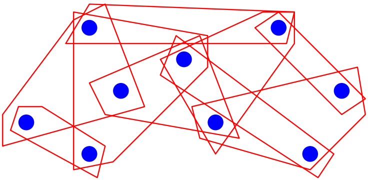

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- Generalization of the set cover problem
  - Given G and δ
  - Find an $F \subseteq E$ that is an edge δ-cover for V, and minimizes $|F|$.

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$\chi(v)$	7	3	$\perp$	3	4	$\perp$	3	3	7	4
v	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}

Cover certificates

- Recall: Each edge (set) e is associated with ID(e).
- "regular" setting:
  - Given $\{\text{ID}(e) : e \in F\}$ easy to check for given $v \in V$ if v covered by F.
- Streaming setting:
  - This cannot be checked given only $\{\text{ID}(e) : e \in F\}$.
- (some) streaming algorithm output a δ-cover certificate χ: partial function from V to ID(E) that satisfies
  - if $\chi(v) = \text{ID}(e)$, then $v \in e$ (soundness)
  - $|\mathrm{Dom}(\chi)| \geq \delta n$ (δ-coverage)
  - objective: minimize $|\mathrm{Im}(\chi)|$

$\chi(v)$	7	3	$\perp$	3	4	$\perp$	3	3	7	4
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Single pass [Emek, Rosén 2014]

Theorem

There is a semi-streaming algorithm that on an input hypergraph $G = (V, E)$ *uses* $O(n \log n)$ *space, and for every* $0 \leq \epsilon < 1$ *produces a* $(1 - \epsilon)$*-cover certificate* χ_ϵ *for* G *such that*

$$|\mathrm{Im}(\chi_\epsilon)| = O\left(\min\left\{1/\epsilon, \sqrt{n}\right\}\right) \cdot |OPT| \ ,$$

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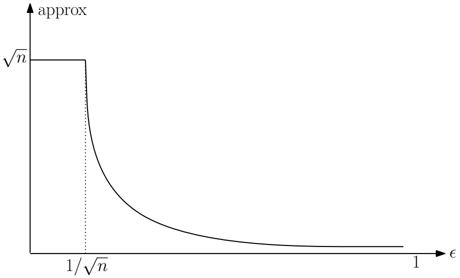

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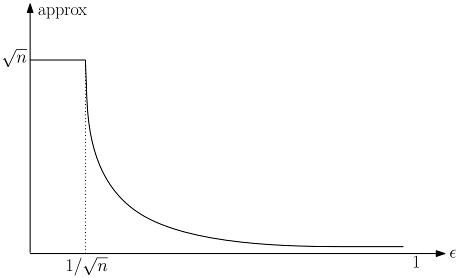

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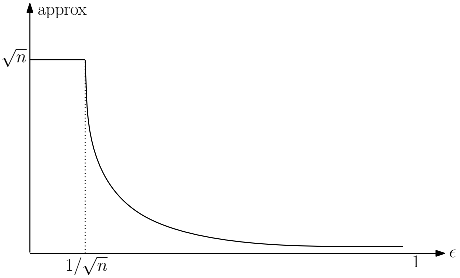

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Theorem

Fix some constant real $\alpha > 0$.
If a randomized streaming algorithm uses memory of size $o(n^{1+\alpha})$, and for an $\epsilon \geq n^{-1/2+\alpha}$ guarantees to output a $(1-\epsilon)$-cover F with $\mathbb{E}[|F|] = \rho_\epsilon \cdot |\mathtt{Opt}|$, then $\rho_\epsilon = \Omega(\frac{\log\log n}{\log n} \cdot \frac{1}{\epsilon})$.

Single Pass, sublinear space [Assadi, Khanna, Li 2016]

Theorem

For any $\alpha = o(\sqrt{n}/\log n)$, and $m = poly(n)$, any randomized single-pass streaming algorithm that α-approximates the set cover problem with probability at least $2/3$ requires $\Omega(mn/\alpha)$ bits of space.

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- Matching deterministic upper bound of set cover
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- These results only for 1-covers

Multiple Passes [Chakrabarti, Wirth 2016]

Theorem

For every $p \geq 1$, there is a p-pass semi-streaming deterministic algorithm for weighted $(1-\epsilon)$ set-cover that returns a cover certificate that approximates the 1-cover up to $O(p \cdot \min\{n^{1/(p+1)}, \epsilon^{-1/p}\})$.

Theorem

Let $c > 0$ be a constant. If A is a randomized p-pass streaming algorithm for $(1-\epsilon)$ set cover, $0 <$ epsilon $< 1/2$, that for all large enough n and m, returns an α-approximation, $\alpha < \frac{1}{8c(p+1)^2} \cdot \min\{n^{1/(p+1)}, \epsilon^{-1/p}\}$, then A uses $\Omega(n^c/p^3)$ space.

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- lower bound on decision problem

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Theorem (HIMV)

For every $p \geq 1$, there is a p-pass randomized algorithm for the set-cover problem that uses $\tilde{O}(mn^{1/p})$ space, and with high probability returns an $O(p)$ approximation.

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For every $p \geq 1$, $\alpha = o(\log n / \log\log n)$, any algorithm that makes p passes, and returns with constant probability an α approximation, uses $\tilde{\Omega}(mn^{1/\alpha}/p)$ space.

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- Lower bound applies to estimating the size.

1. The streaming model
2. The set-cover problem
3. (some of the) Results
4. (some of the) Techniques
   - Single pass, semi-streaming algorithm (unweighted case)
   - Matching lower bound(s)
5. Conclusions and open problems

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  - no additional memory
  - running time $O(n \log n)$.

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$\mathrm{qlt}_{t+1}(v)$	3	3	3	3	3	3	5	5	5	7	8	8
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    i.e., all edges that give high-quality $(> r)$ coverage to at least one node

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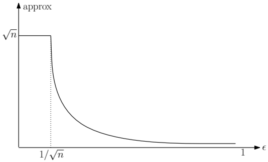
Idea of proof of approximation factor:

- $S(r^*) < n/2^{r^*-1} \le \frac{1}{\epsilon} \cdot |I(r^*)| \cdot \frac{1}{2^{r^*-1}} < \frac{4}{\epsilon} \cdot |\texttt{Opt}|$.

Lower Bound - single pass, semi-streaming

Theorem (1-coverage)

If a randomized streaming algorithm uses memory of size $o(n^{3/2})$, and guarantees to output a (1)-cover certificate χ with $\mathbb{E}[|\mathrm{Im}(\chi)|] = \rho \cdot |\mathtt{Opt}|$, then $\rho = \Omega(\sqrt{n})$.



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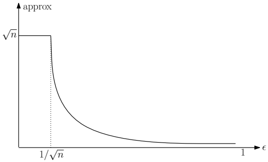

Distribution $\mathcal{G}$ over n-node hypergraphs (based on affine planes)

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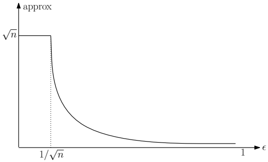

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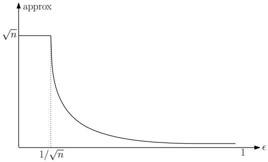

Distribution $\mathcal{G}$ over n-node hypergraphs (based on affine planes)

- $\mathtt{Opt}(G) = O(1)$ for every $G \in \mathcal{G}$
- Every deterministic streaming algorithm with memory $o(n^{3/2})$ that outputs 1-cover certificate χ has $\mathbb{E}_{\mathcal{G}}[|\mathrm{Im}(\chi)|] = \Omega(\sqrt{n})$.

1. The streaming model
2. The set-cover problem
3. (some of the) Results
4. (some of the) Techniques
   - Single pass, semi-streaming algorithm (unweighted case)
   - Matching lower bound(s)
5. Conclusions and open problems

Conclusions

Our results [Emek, Rosén 2014]:

Subsequent work:

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- Tight results on approximation factor in 1-pass semi-streaming ($\tilde{O}(n)$ space) of $(1-\epsilon)$ cover certificates.

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Open Problems

- Extend results for sublinear space to $(1-\epsilon)$ covers.

Open Problems

- Can we approximate the optimal $(1 - \epsilon)$ cover ?

Thank you

Thank You