

Superconductivity and the Pseudogap in the 2D Hubbard Model: Results and implications for cuprates

**Coda: slowly fluctuating density wave order
in pnictides; ?implications? for cuprates**

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Support: NSF-DMR-1308236

Collaborators

Emanuel Gull
U. Michigan

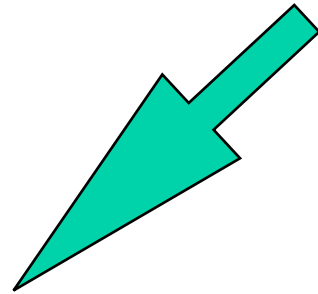


Olivier Parcollet
Saclay

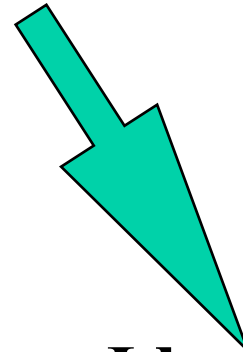
Antoine Georges
Ecole Polytechnique

Nan Lin
Columbia->Finance

Two approaches to cuprate physics



**Revel in the
specifics**

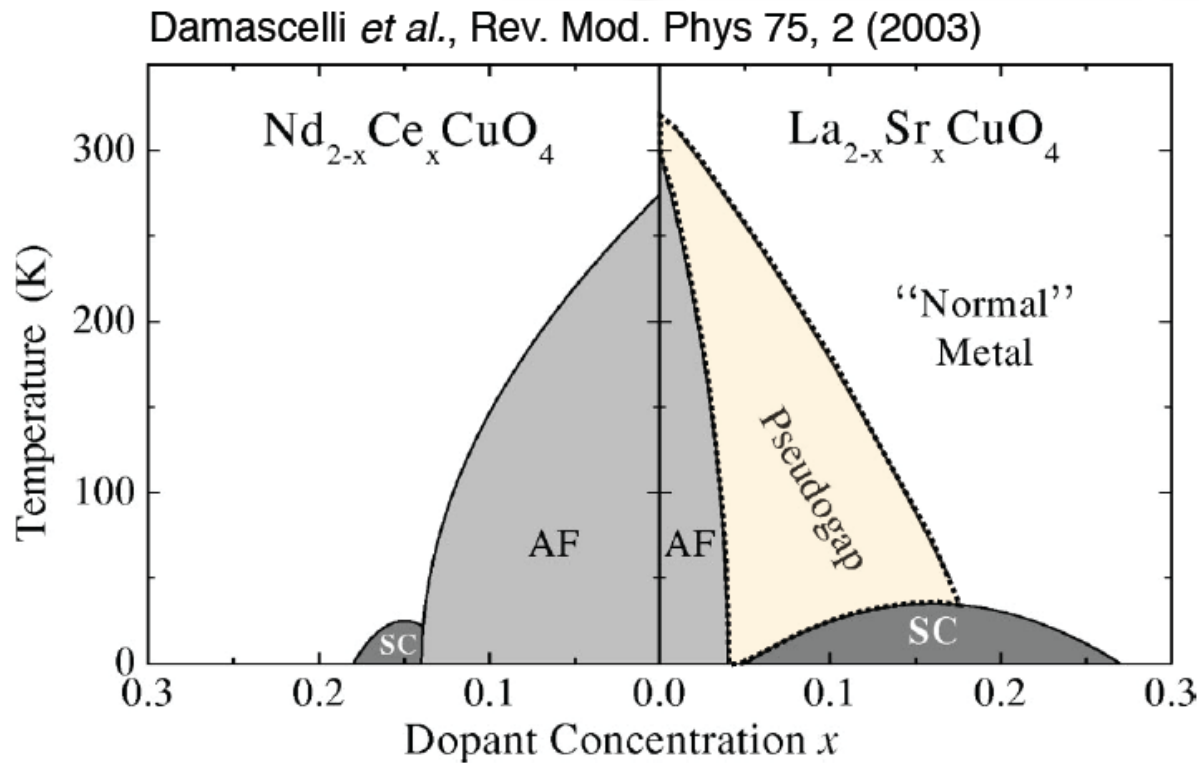


**Identify the
essentials**

**This talk: focus on (what I conceive
of as) the essential behavior—**

superconductivity and pseudogap

Key features of high-T_c cuprates



Mott insulating phase, giving rise (on hole but not electron doping) to an anomalous normal state characterized by important differences between zone-diagonal and zone face states, leading to a pseudogap unstable at lower T to a dx^2-y^2 symmetry superconducting state

The Hubbard Model

$$\mathbf{H} = - \sum_{ij} t_{i-j} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



1988: P.W. Anderson said

The 2d square-lattice Hubbard model captures the essential features of the physics of the cuprates

**2015: We have good
reason to believe
Anderson was correct.**



More specifically



We have reasonable confidence that

The 2d square-lattice Hubbard model has

- momentum space differentiation (anisotropic scattering)
- dx^2-y^2 superconductivity in a superconducting dome with T_c of the correct order of magnitude
- a pseudogap producing many of the features observed in the cuprates

In the Hubbard model the pseudogap and superconductivity are competing phenomena

We believe, but don't have complete and convincing evidence, that the 2d Hubbard model lacks, at least in any strong form

- **CDW order (independent of spin stripes)**
- **nematicity**

IMPLICATION: CDW order and nematicity are 'epiphenomena': things that occur, and are interesting, but are not fundamental to the physics of the cuprates

?Why do we believe this?

References

- **Emanuel Gull, Michel Ferrero, Olivier Parcollet, Antoine Georges, Andrew J. Millis, Phys. Rev. B82 155101 (2010)**
- **E. Gull, O. Parcollet and A. J. Millis, Phys. Rev. Lett. 110 216406 (2013)**
- **E. Gull and A. J. Millis, Physical Review B86 241106 (2012).**
- **Emanuel Gull, Andrew J. Millis. Phys. Rev. B88, 075127 (2013).**
- **E. Gull and A. J. Millis, Phys. Rev. B90, 041110 (2014).**
- **E. Gull and A. J. Millis, Physical Review B91, 085116 (2015).**

See also

Talks and papers by

Tremblay

Kotliar

Civelli

Parcollet





A. Georges

Dynamical Mean Field Theory (DMFT)



G. Kotliar

DMFT: approximation to electron self energy

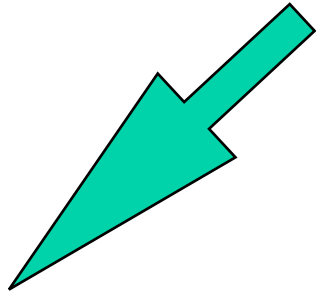
$$\Sigma(\mathbf{k}, \omega) = \sum_{\mathbf{a}=1\dots N} \mathbf{f}_{\mathbf{a}}(\mathbf{k}) \Sigma^{\mathbf{a}}(\omega)$$

The $\mathbf{f}^{\mathbf{a}}(\mathbf{k})$ determine the ‘flavor’ of DMFT

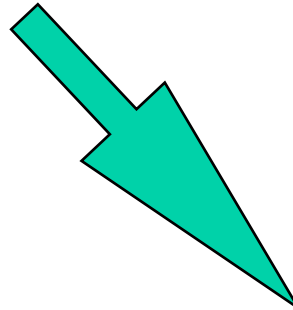
The $\Sigma^{\mathbf{a}}(\omega)$ come from solution of auxiliary problem plus self-consistency condition

$N \rightarrow \infty$ recovers exact solution

Two approaches (related; both important)



**Identify interesting
physics and
approximations that
express it**

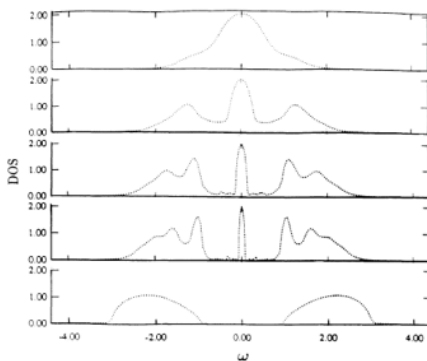


**Attempt to determine
properties of $N \rightarrow \infty$
solution**

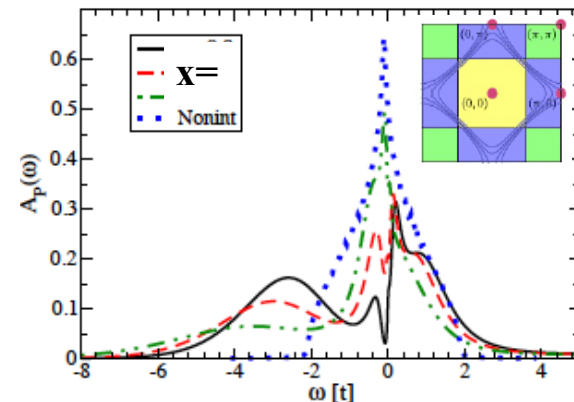
**Approach taken
in this talk**

There is a long story about ‘flavors’ of DMFT and number of approximants (N) needed

**N=1: extensive entropy
midgap states**



**N=2,4: (over?)emphasis
on singlet physics; PG
just a DOS suppression**

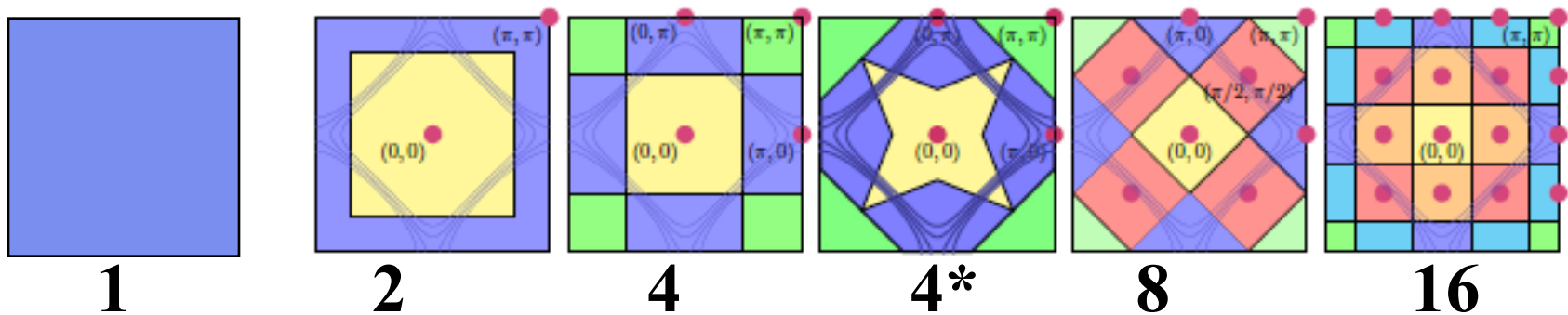


**Here: present results which (we believe) are
generic, representative of N- \rightarrow infinity limit**

Momentum space version of DMFT

M. H. Hettler, M. Mukherjee, M. Jarrell, and H. R. Krishnamurthy
 Phys. Rev. B **61**, 12739 (2000)

tile Brillouin zone: choose N momenta \mathbf{K}_a , draw an equal area patch around each one



$$\Sigma_{\mathbf{p}}(\omega) \rightarrow \Sigma_{\mathbf{p}}^{\text{approx}}(\omega) = \sum_{\mathbf{a}} \phi_{\mathbf{a}}(\mathbf{p}) \Sigma_{\mathbf{a}}(\omega)$$

$\phi_{\mathbf{a}}(\mathbf{p}) = 1$ if \mathbf{p} is in the patch containing \mathbf{K}_a and is 0 otherwise

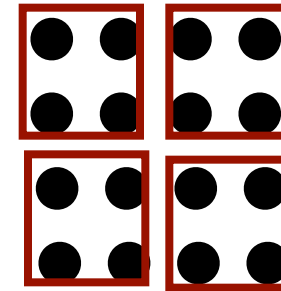
Find Σ_a from N -site quantum impurity model + self consistency condition

Obvious undesirable feature: Momentum-space discontinuities

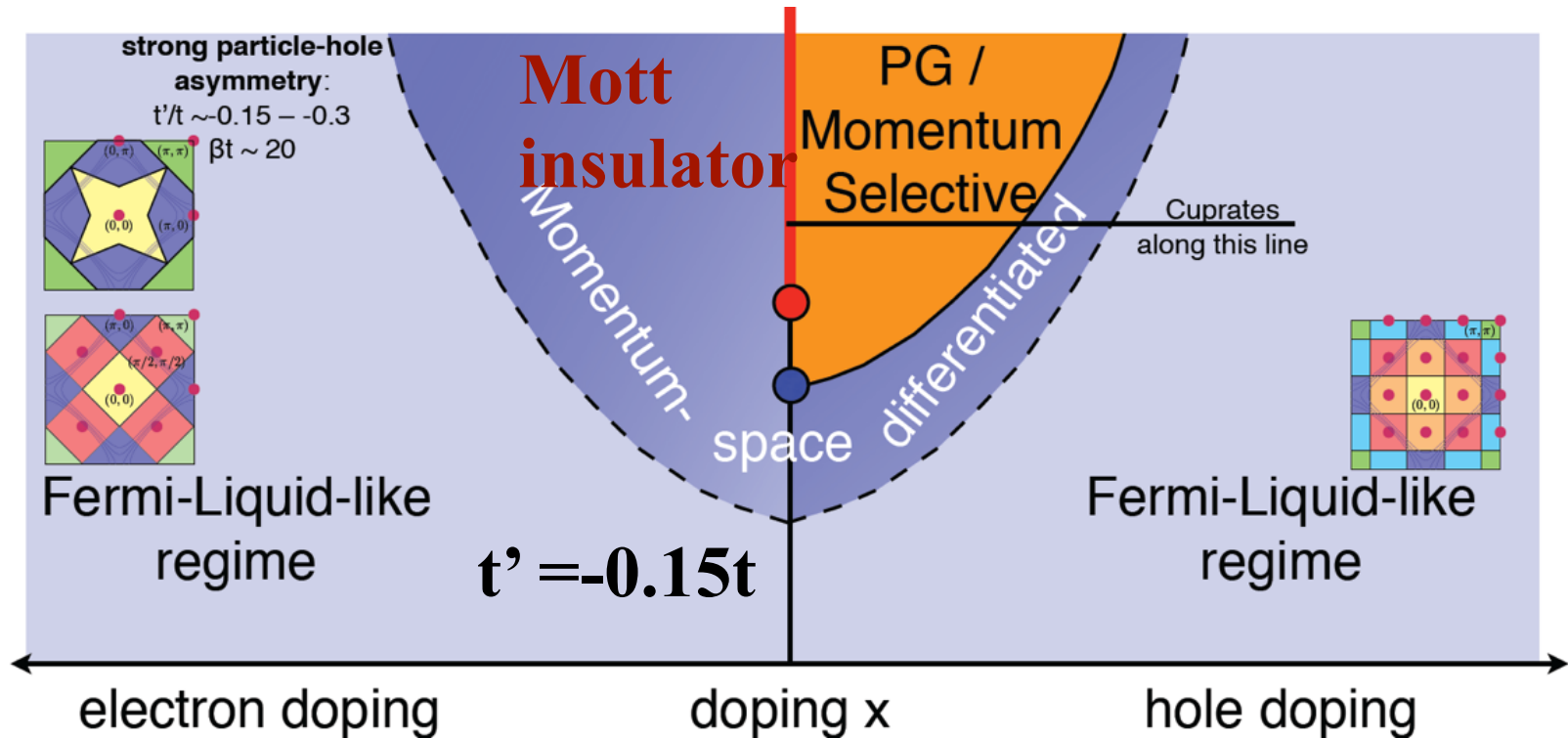
My view: interpolations that might smooth out the discontinuities are dangerous: potential for introducing new (and possibly wrong) physics.

**But see T. Maier and T. Schulthess: DCA+
Phys. Rev. B 88, 115101 (2013)**

**Alternative (CDMFT) approach:
break translational invariance.
'Periodization' needed. Kotliar,
Tremblay. Civelli. Sakai**



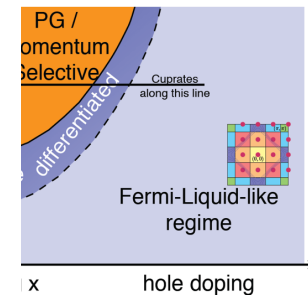
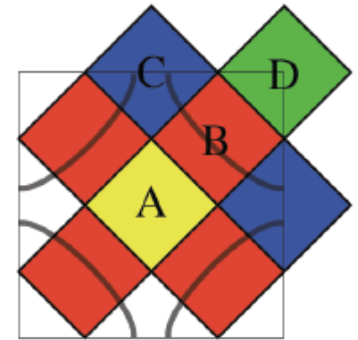
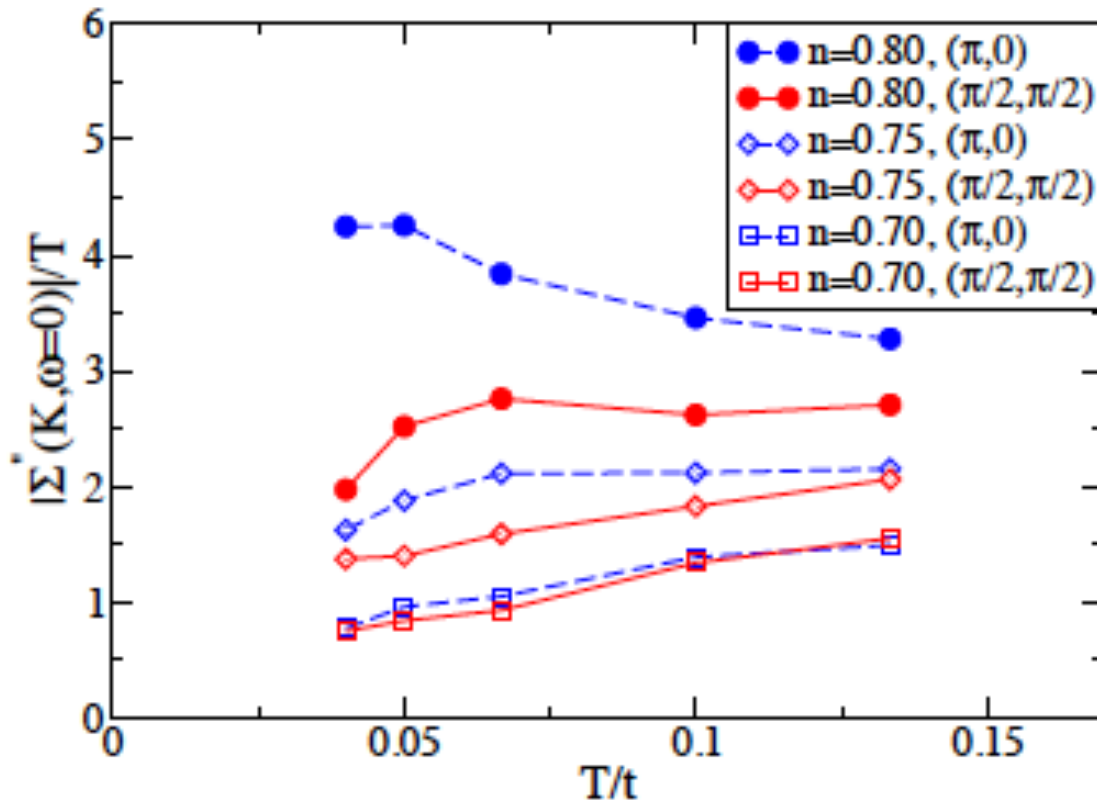
Phase diagram: Mott insulator separated from fermi liquid by pseudogap for hole but not electron doping



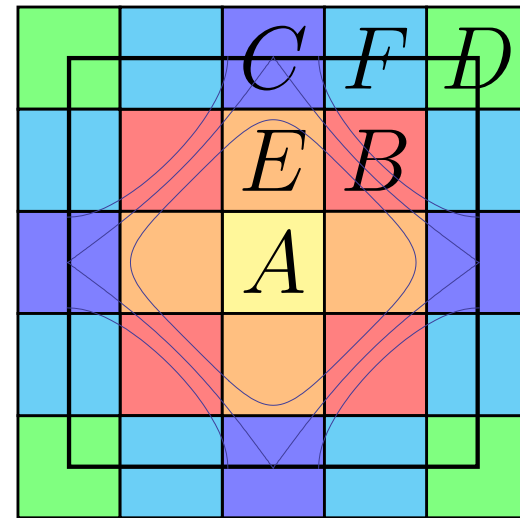
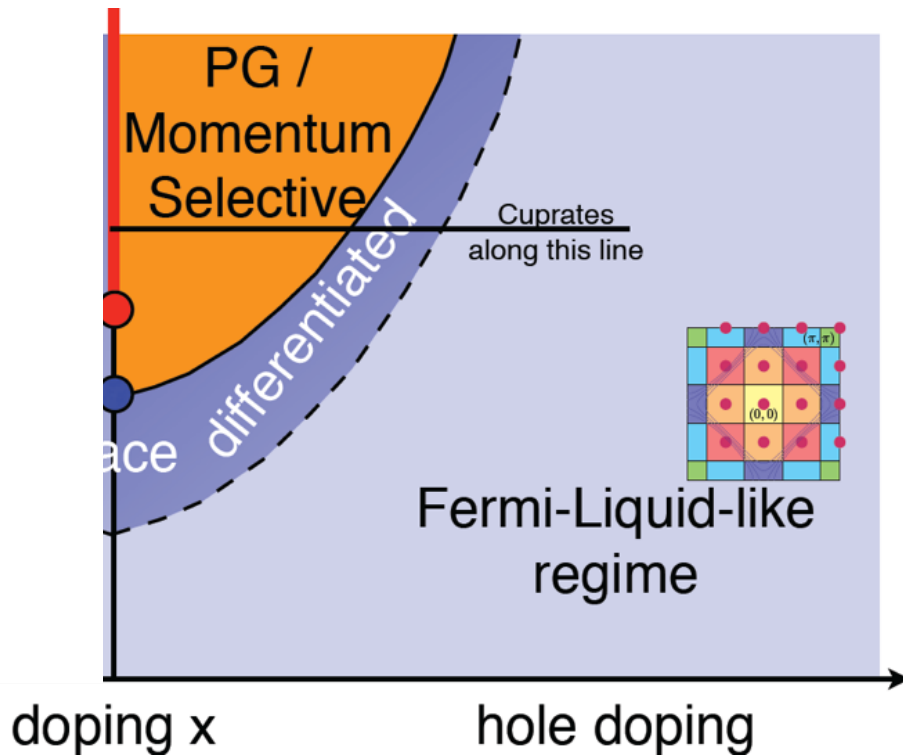
P. Werner, E. Gull, O. Parcollet and A. J. Millis, Phys. Rev. B 79, 045120 (2009)
 E. Gull, O. Parcollet, P. Werner, and A. J. Millis, Phys. Rev. B 80, 045120 (2009).
 Emanuel Gull, Michel Ferrero, Olivier Parcollet, Antoine Georges, Andrew J. Millis,
 Phys. Rev. B82 155101 (2010).

Electron scattering rate by sector

differentiated phase

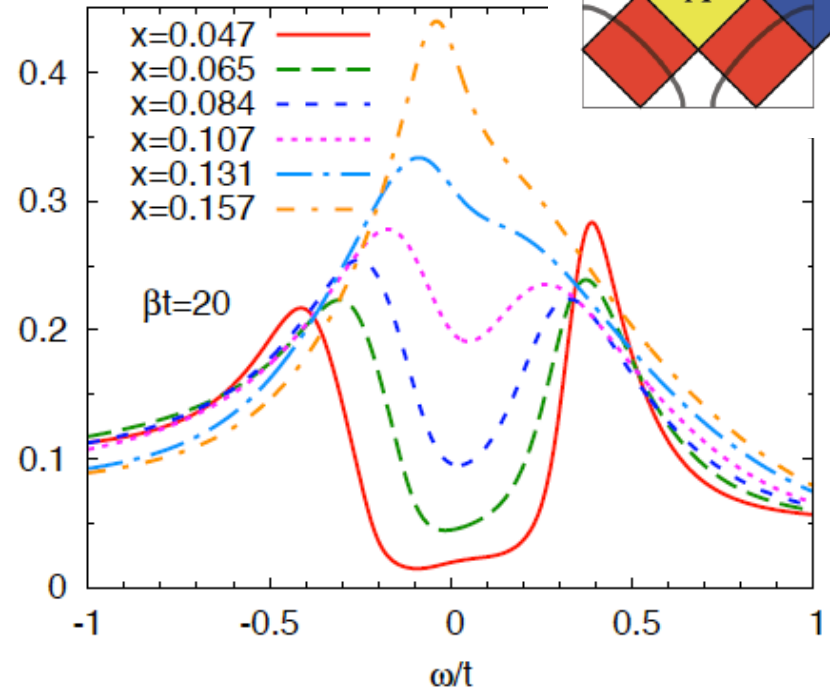
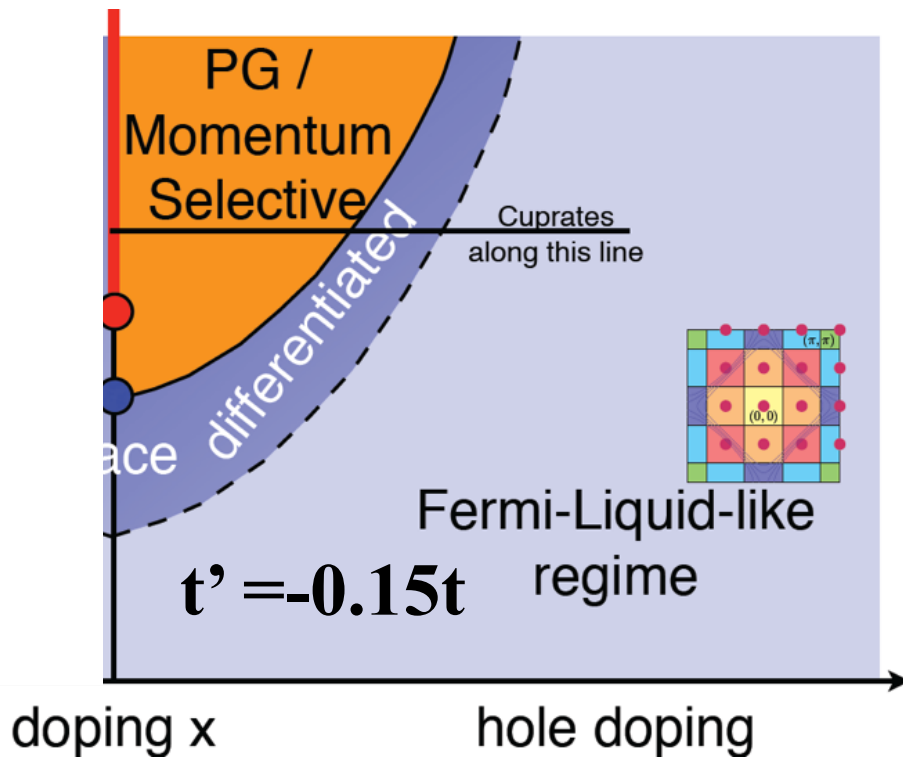
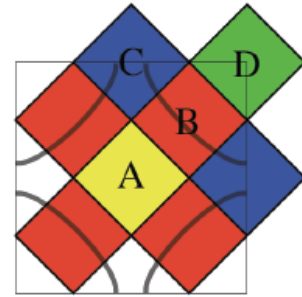


Momentum selective phase: partially gapped fermi surface



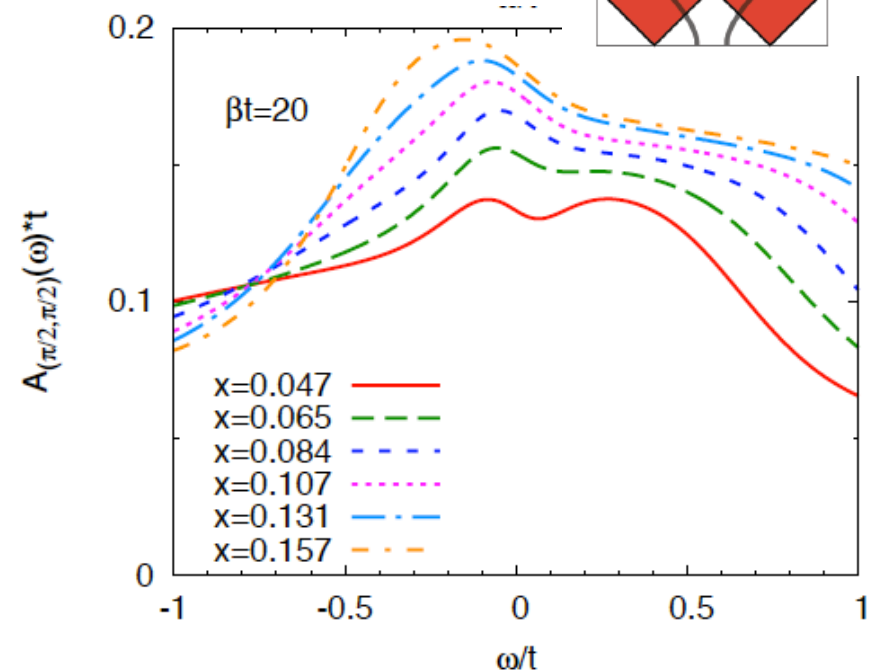
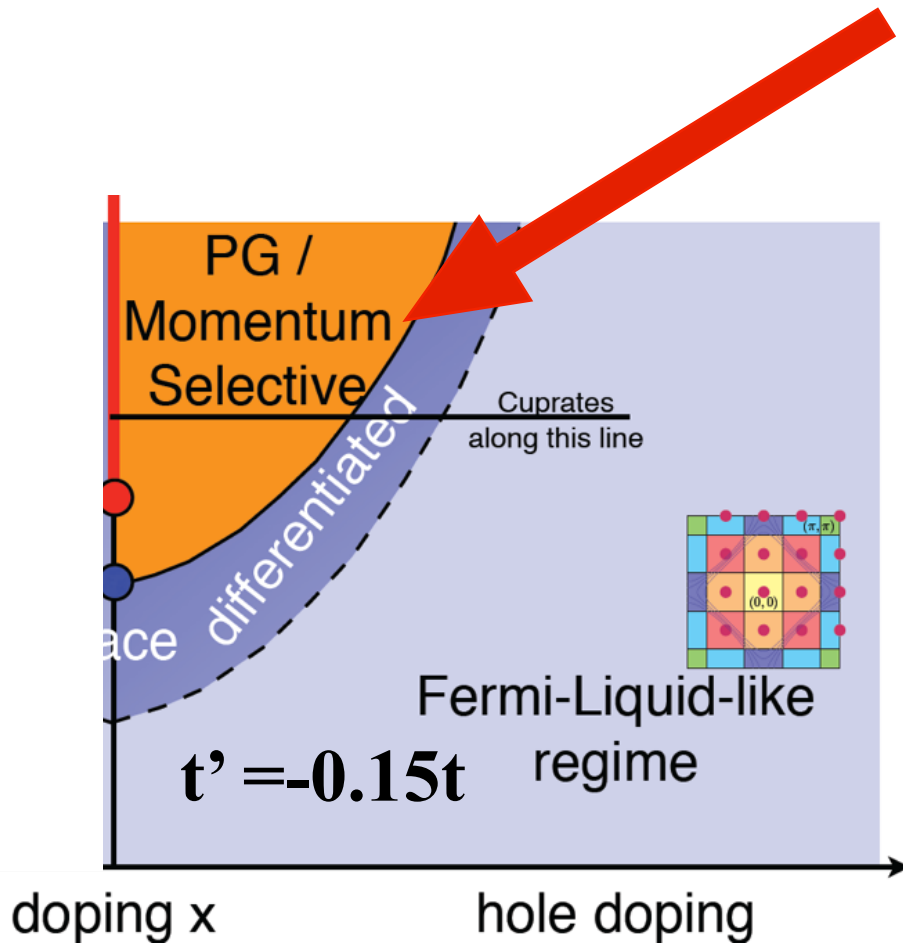
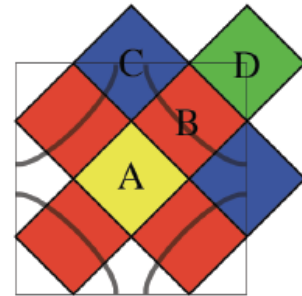
We see: gap opens in momentum sector C but not in B

Momentum selective phase: partially gapped fermi surface



gap opens around zone face

Intermediate phase: partially gapped fermi surface



but not along zone diagonal

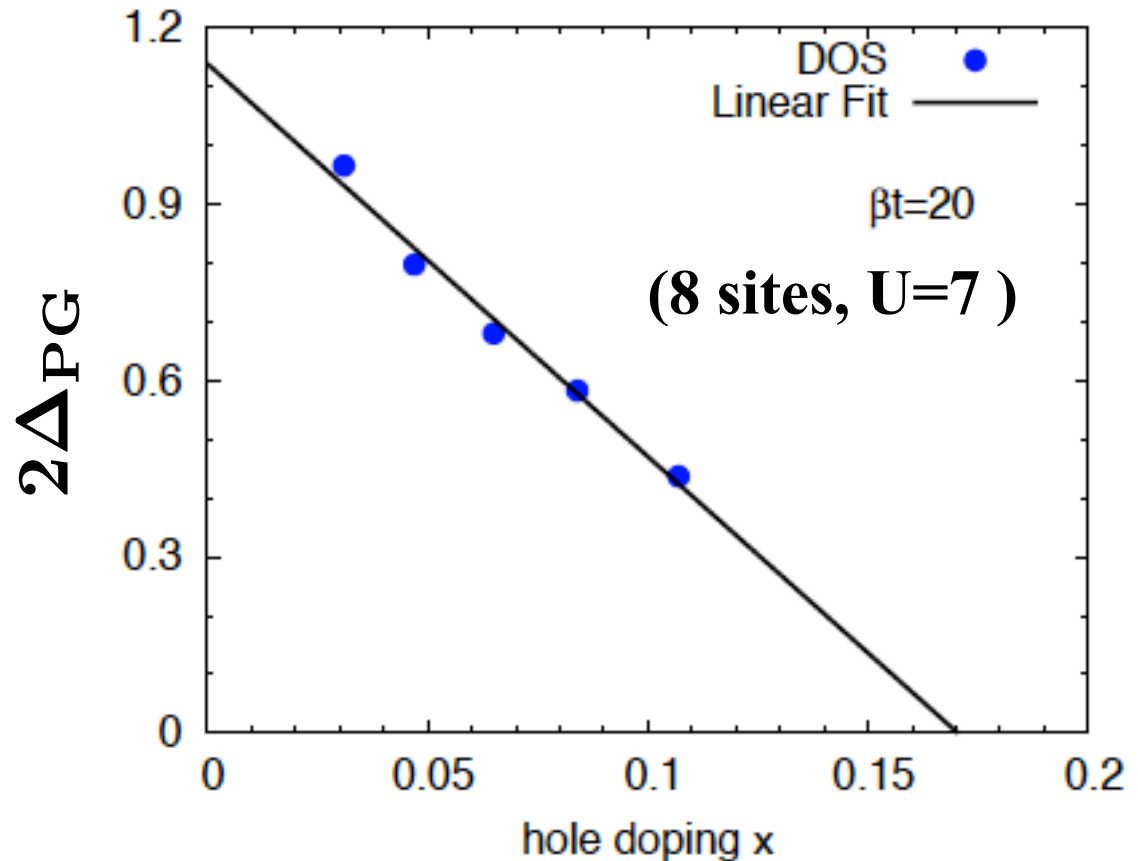
The pseudogap does NOT connect smoothly to the Mott gap

as $x \rightarrow 0$

$$2\Delta_{\text{PG}} \rightarrow 1$$

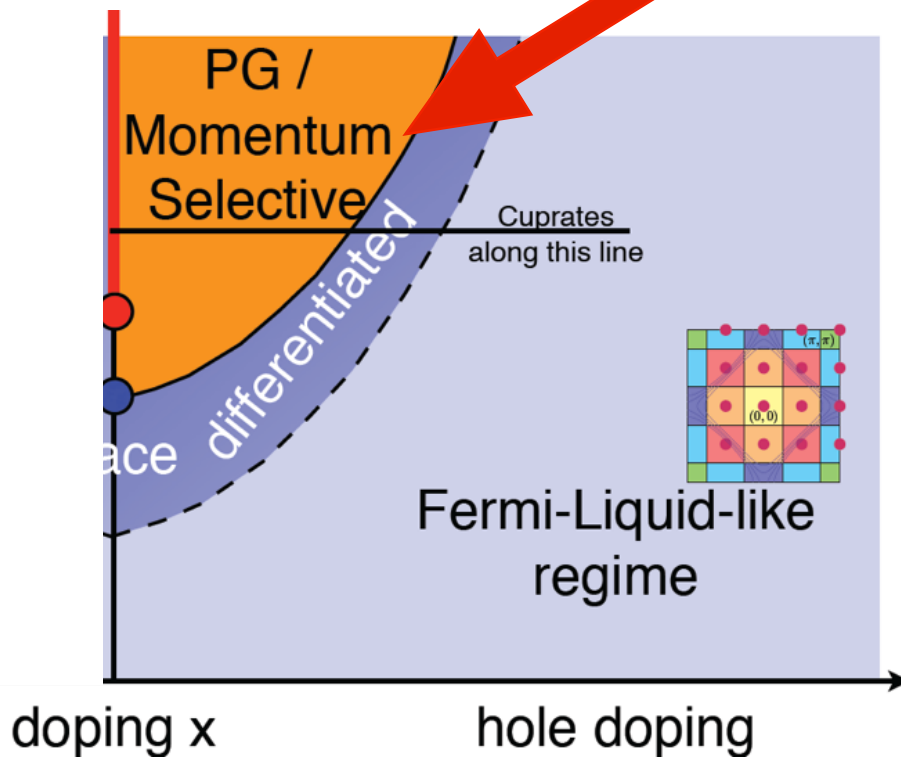
but

$$2\Delta_{\text{Mott}} \approx 2$$



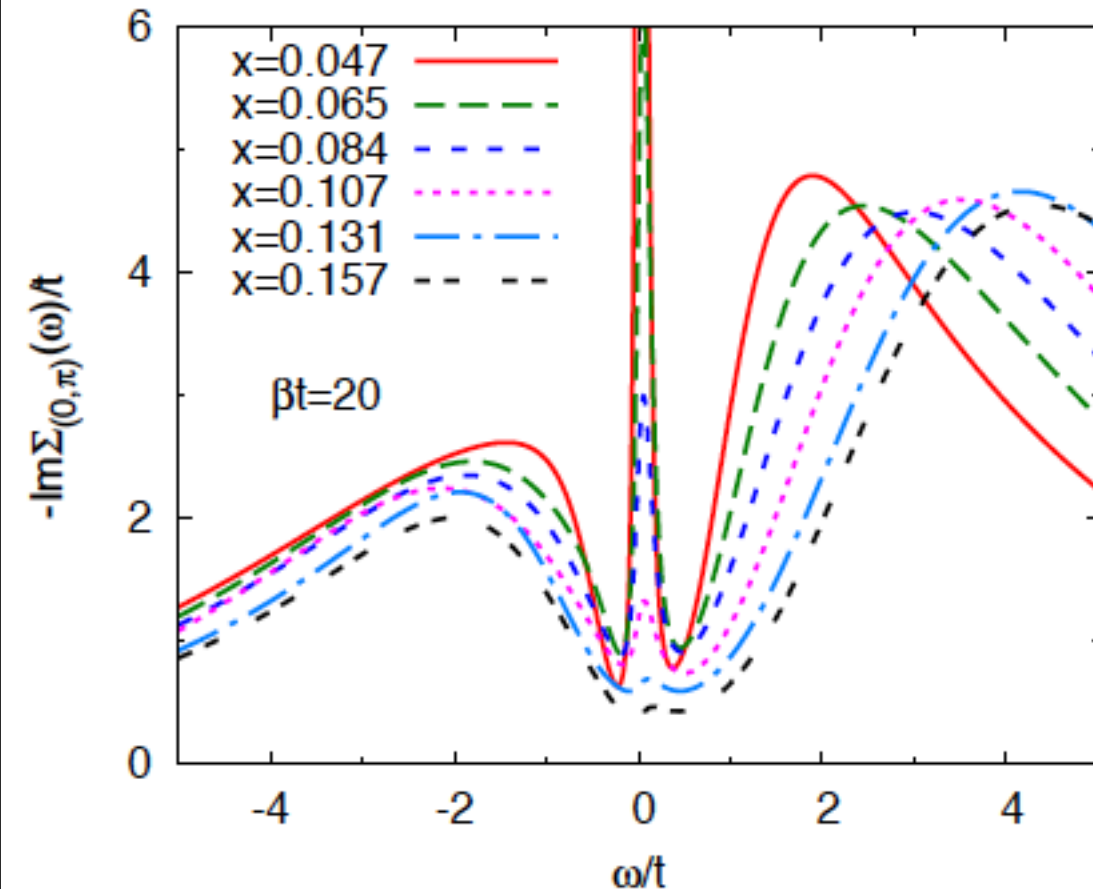
Momentum selective phase: partially gapped fermi surface

critical point:



Clear separation of PG transition from Mott transition (except in 4 site CDMFT)

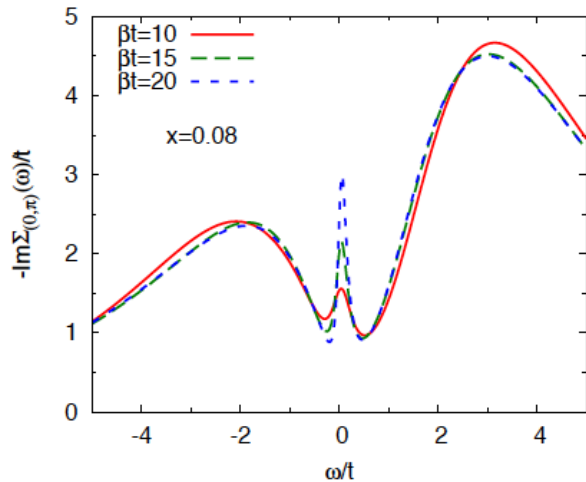
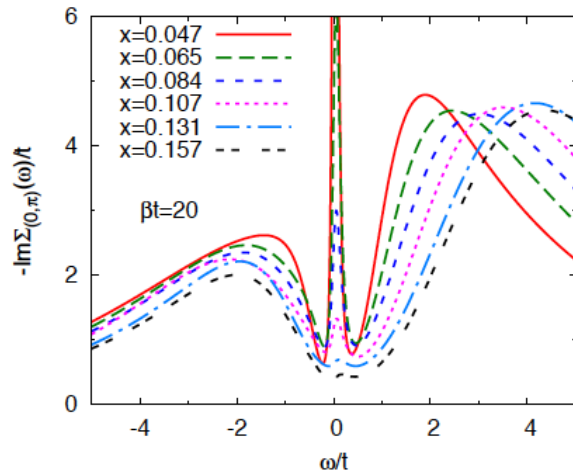
Pseudogap: marked by appearance of pole in self energy ($N > 4$)



Phys. Rev. B 82, 045104 (2010)

As far as we can tell (have looked down to $T=T/80$ at half filling, $N=8$) transition to PG state is smooth (second order) for 8 and 16 site clusters. First order transition found in 4-site CDMFT is peculiar to that approach.

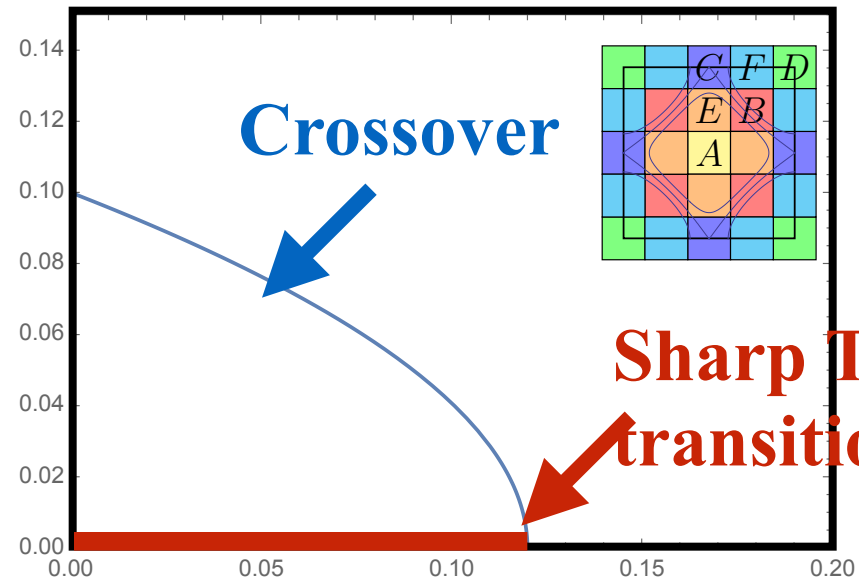
Order parameter: gap in (0, π) sector sharply defined only at $T=0$



$T=0$: sharp transition

$T>0$: crossover

T



x

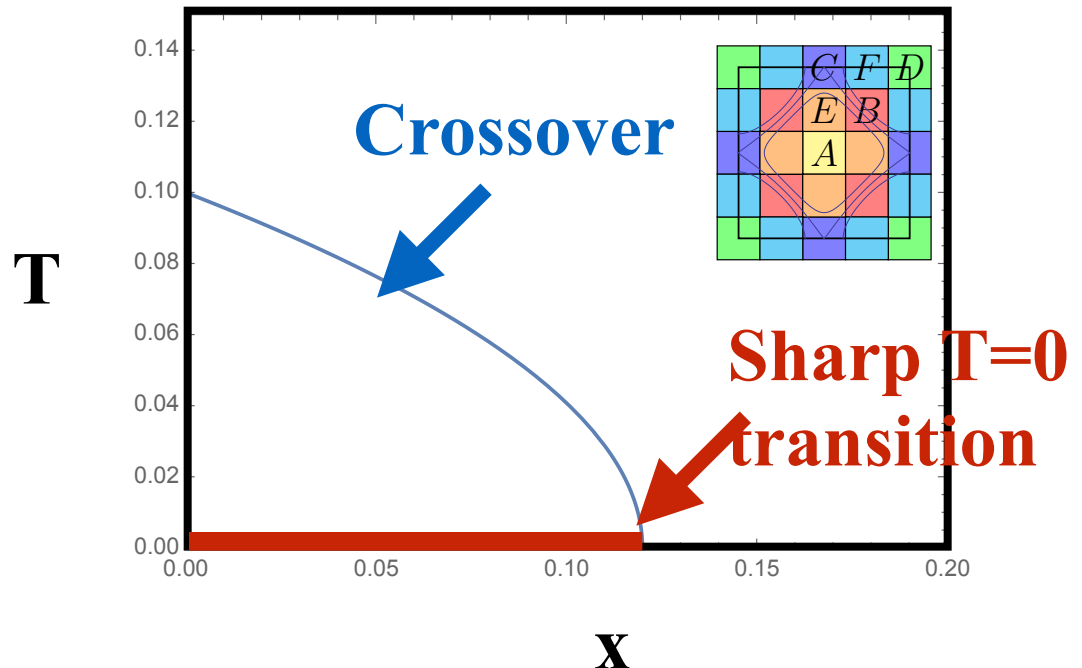
Order parameter: gap in $(0,\pi)$ sector sharply defined only at $T=0$

Crossover: reasonably sharp change in physical properties.

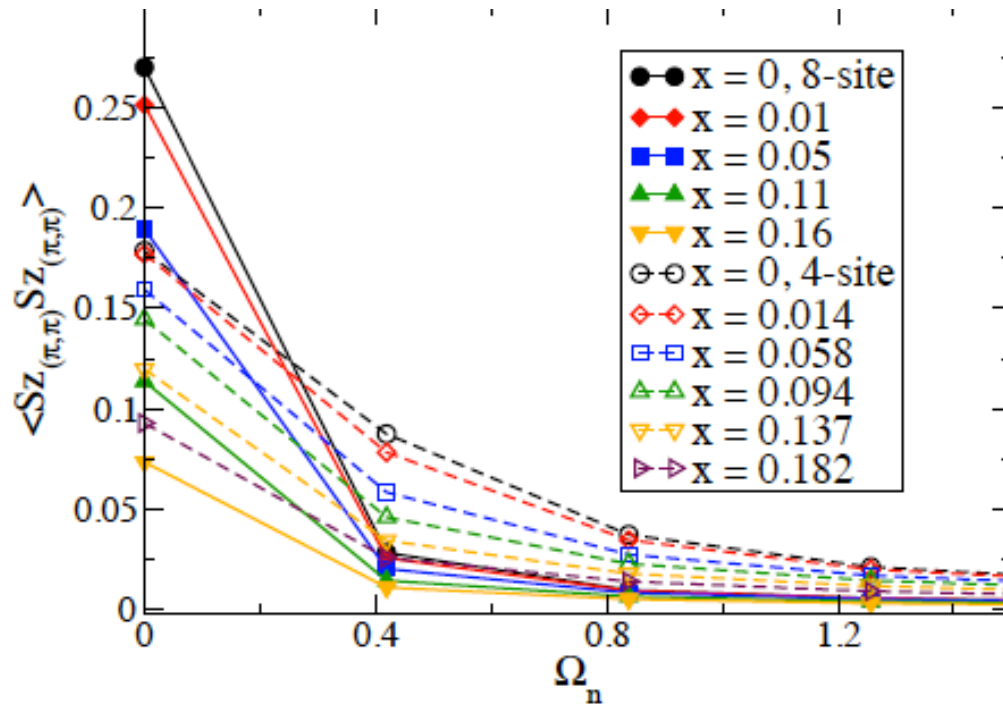
$T=0$: sharp transition

$T>0$: crossover

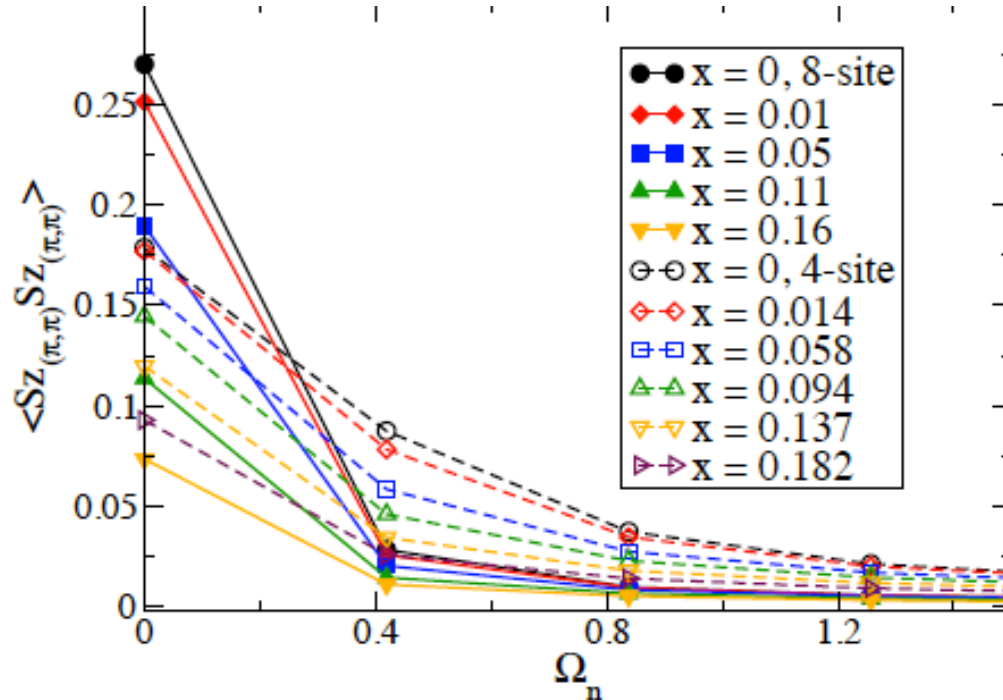
We find that the $T=0$ transition is second order for $N>4$. If it were weakly first order, nothing important would change



Pseudogap associated with enhanced antiferromagnetic spin correlations

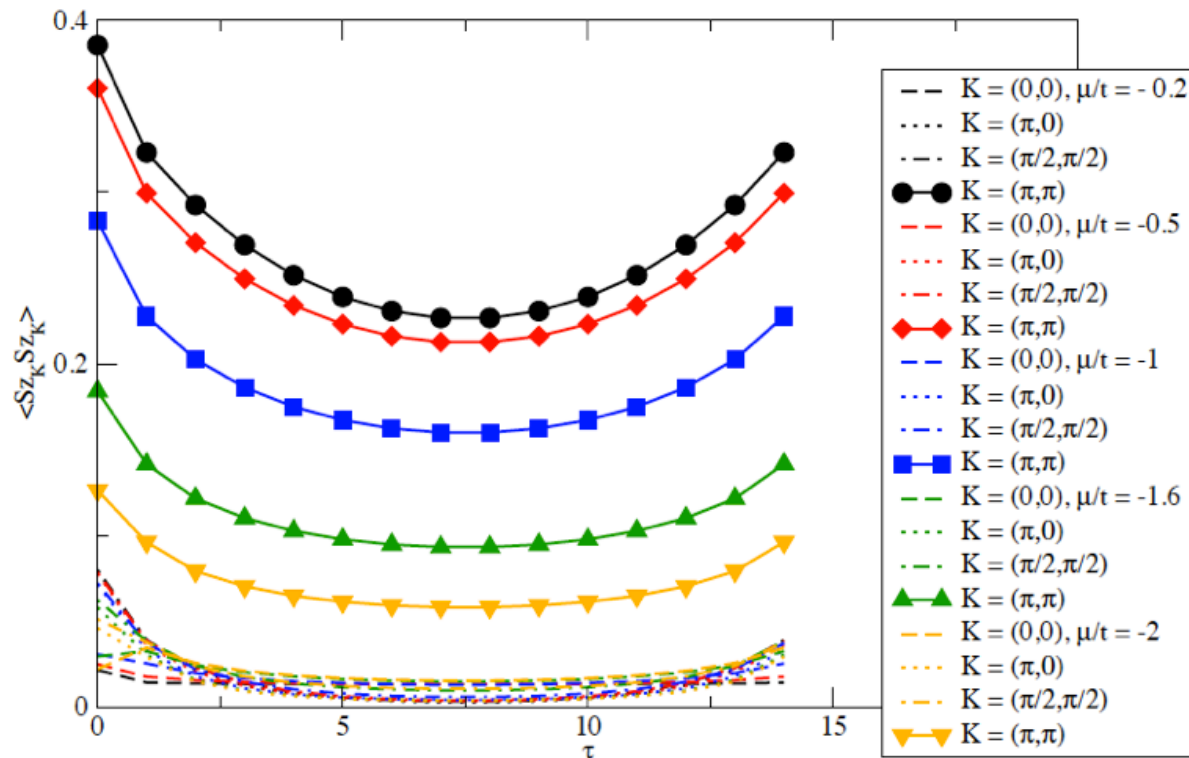


Pseudogap associated with enhanced antiferromagnetic spin correlations

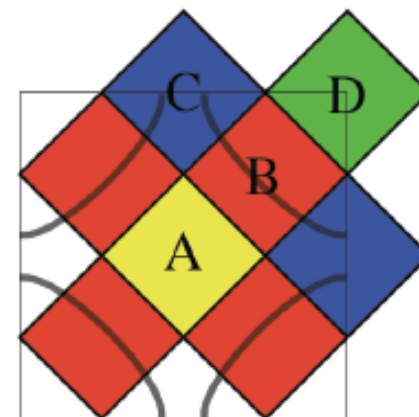
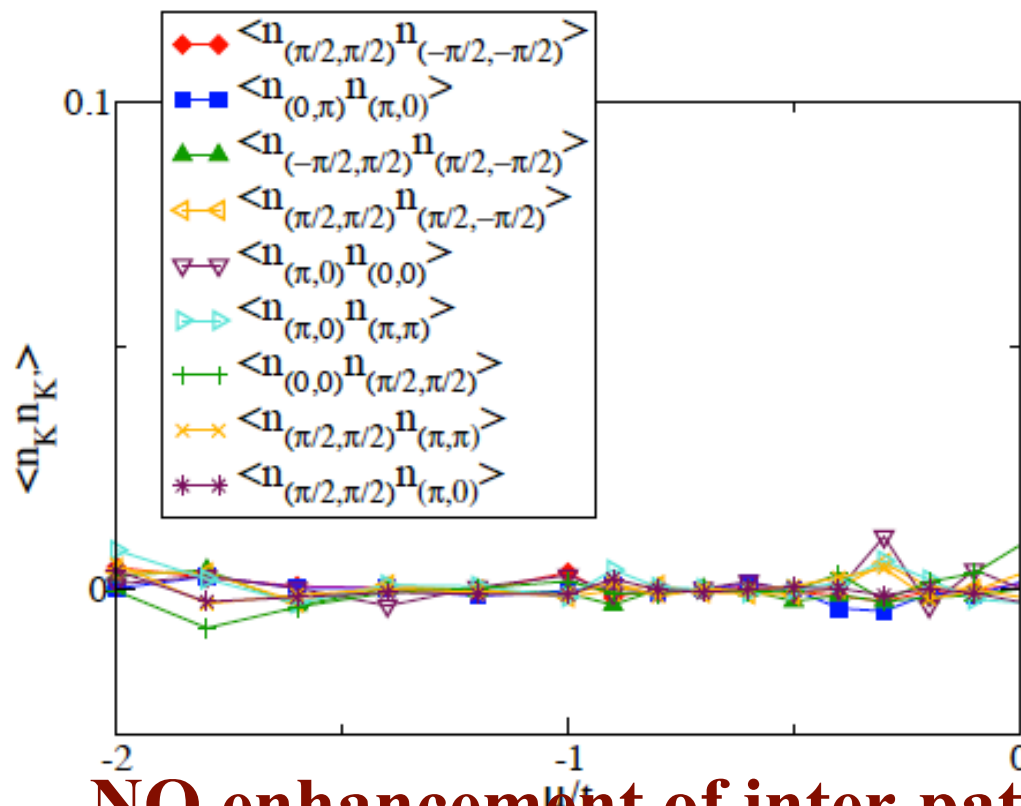


**These are equal-time impurity-model correlations.
We are working on getting the real dynamical
correlations (vertex corrections needed).**

Spin correlations at other wavevectors not enhanced

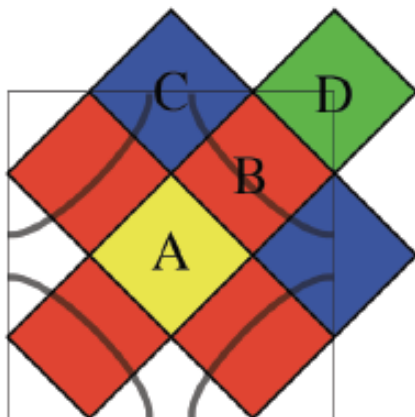


No charge 'nematicity'

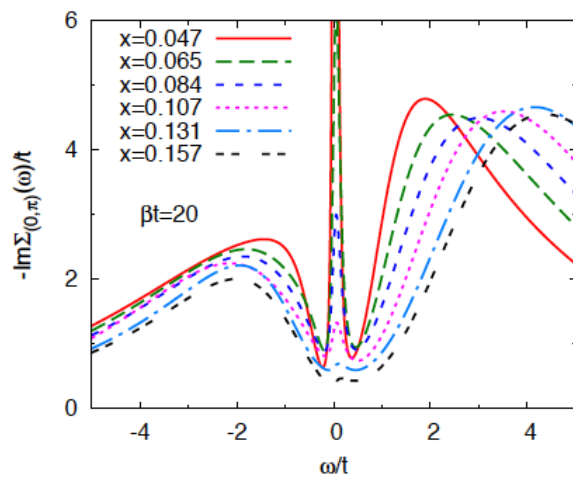


NO enhancement of inter-patch density fluctuations is visible

However, sensitivity to breaking C4 symmetry



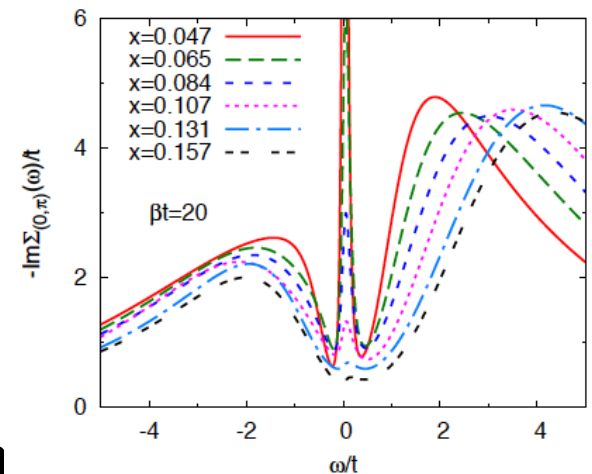
Transition can happen in one node before the other=>large anisotropy



See Okamoto, Senechal, Civelli and Tremblay, arXiv:1008.5118

Pseudogap Summary

- **Intrinsic property of 2D Hubbard model**
- **Not directly connected to Mott gap**
- **Within 'DCA' theory**
 - **Phase transition at $T=0$**
 - **Crossover at $T>0$**
 - **Associated with pole in self energy**
 - **Associated with spin correlation**
 - **No obvious charge nematic fluctuations; coupling to hopping anisotropy**



Now: superconductivity



SC in DMFT

Pioneers: (2x2 cluster)

- Lichtenstein, Katsnelson PRB 62, R9283 (2000).
- Maier, Jarrell, Pruschke, Keller, PRL 85, 1524 (2000).

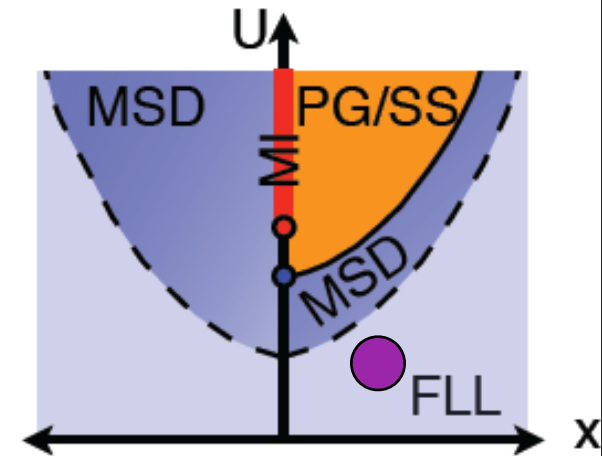
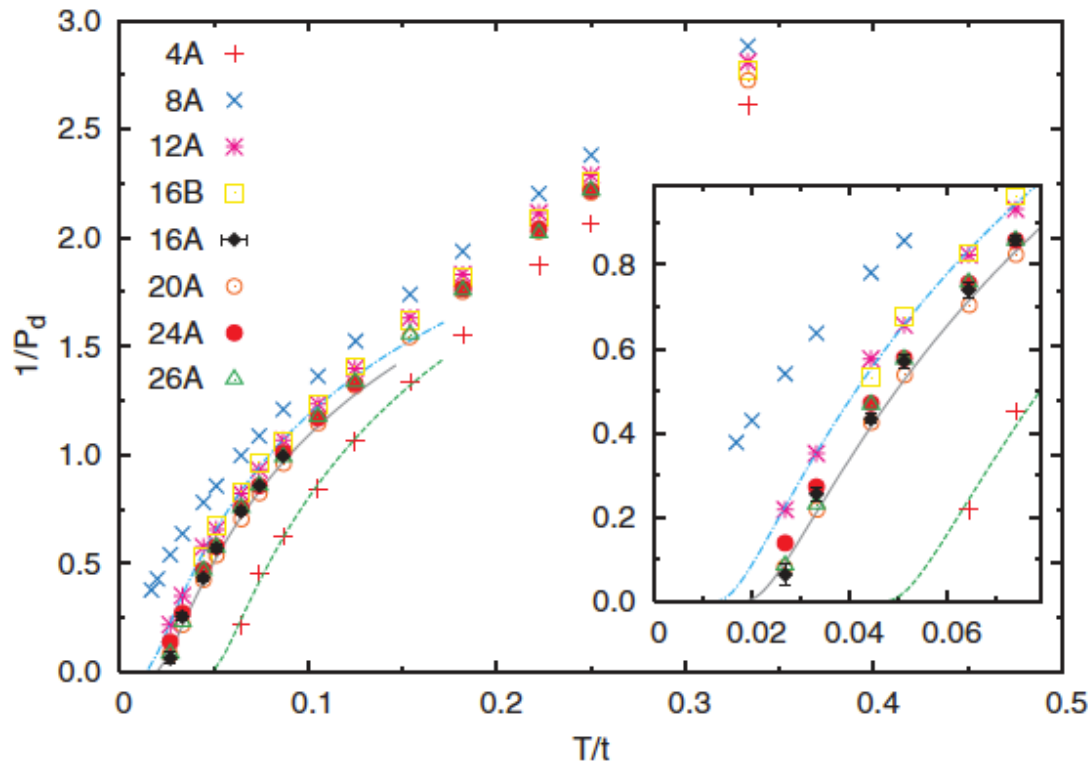
Lots of subsequent work (mainly 2x2 clusters):

- S. S. Kancharla, B. Kyung, D. Sénéchal, M. Civelli, M. Capone, G. Kotliar and A.-M. S. Tremblay, Phys. Rev. B 77, 184516 (2008).
- T. A. Maier, D. Poilblanc, and D. J. Scalapino, Phys. Rev. Lett. 100, 237001 (2008).
- M. Civelli, M. Capone, A. Georges, K. Haule, O. Parcollet, T. D. Stanescu, and G. Kotliar, Phys. Rev. Lett. 100, 046402 (2008).
- M. Civelli, Phys. Rev. Lett. 103, 136402 (2009).
- G. Sordi, P. Sémon, K. Haule, and A.-M. S. Tremblay, Phys. Rev. Lett. 108, 216401 (2012).

Large clusters: Superconductivity established

T. A. Maier, M. Jarrell, T. Schultheiss, P. Kent and J. White, Phys. Rev. Lett. 95, 237001 (2005)

High T susceptibility: clusters up to $N=26$ at $x=0.1$ $U=4t$ (too small for Mott phase)



● MJSKW point

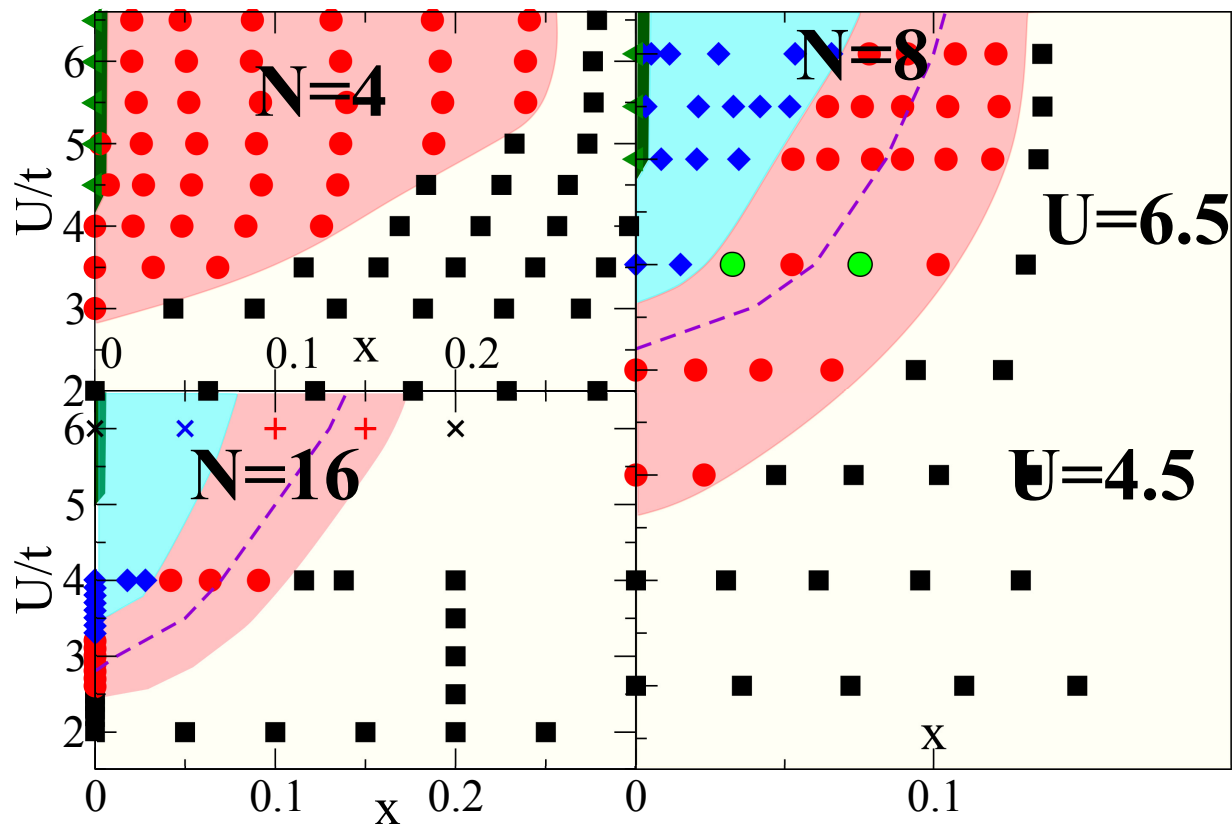
Our work: construct the sc state and determine some properties

- **E. Gull, O. Parcollet and A. J. Millis, Phys. Rev. Lett. 110 216406 (2013)**
- **E. Gull and A. J. Millis, Physical Review B86 241106 (2012).**
- **Emanuel Gull, Andrew J. Millis. Phys. Rev. B88, 075127 (2013).**
- **E. Gull and A. J. Millis, Phys. Rev. B90, 041110 (2014).**
- **E. Gull and A. J. Millis, Physical Review B91, 085116 (2015).**



Phase diagram, different clusters

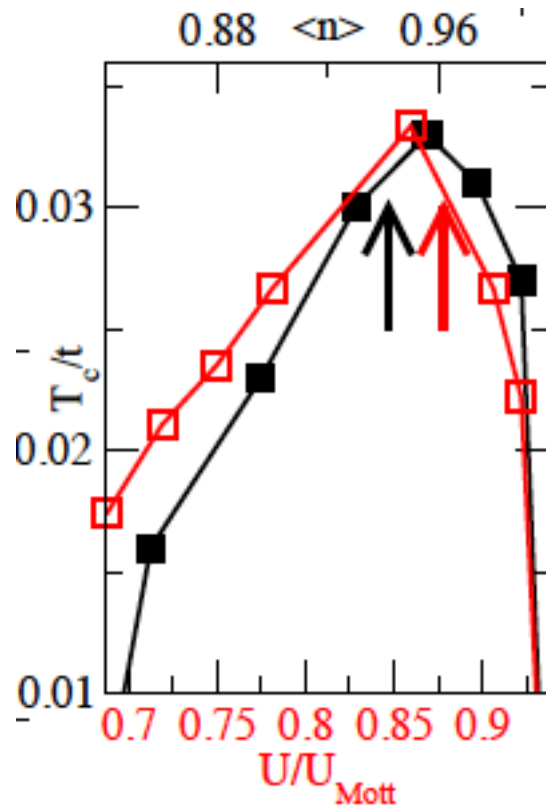
$$T=t/40$$



superconductivity
only near
insulator, cut off
by pseudogap

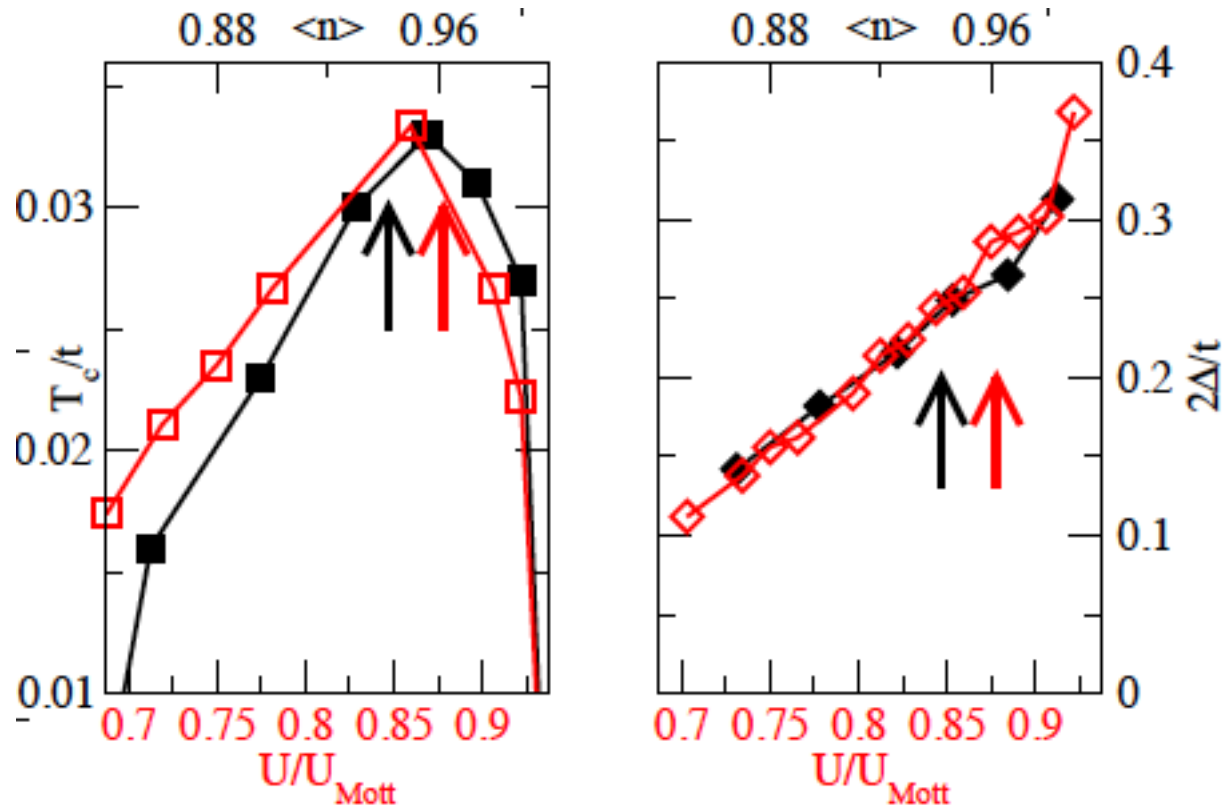
**$N=4$ a bit of an outlier. 8 and 16 differ at small U
but have similar doping dependence at larger U**

Transition temperature and gap

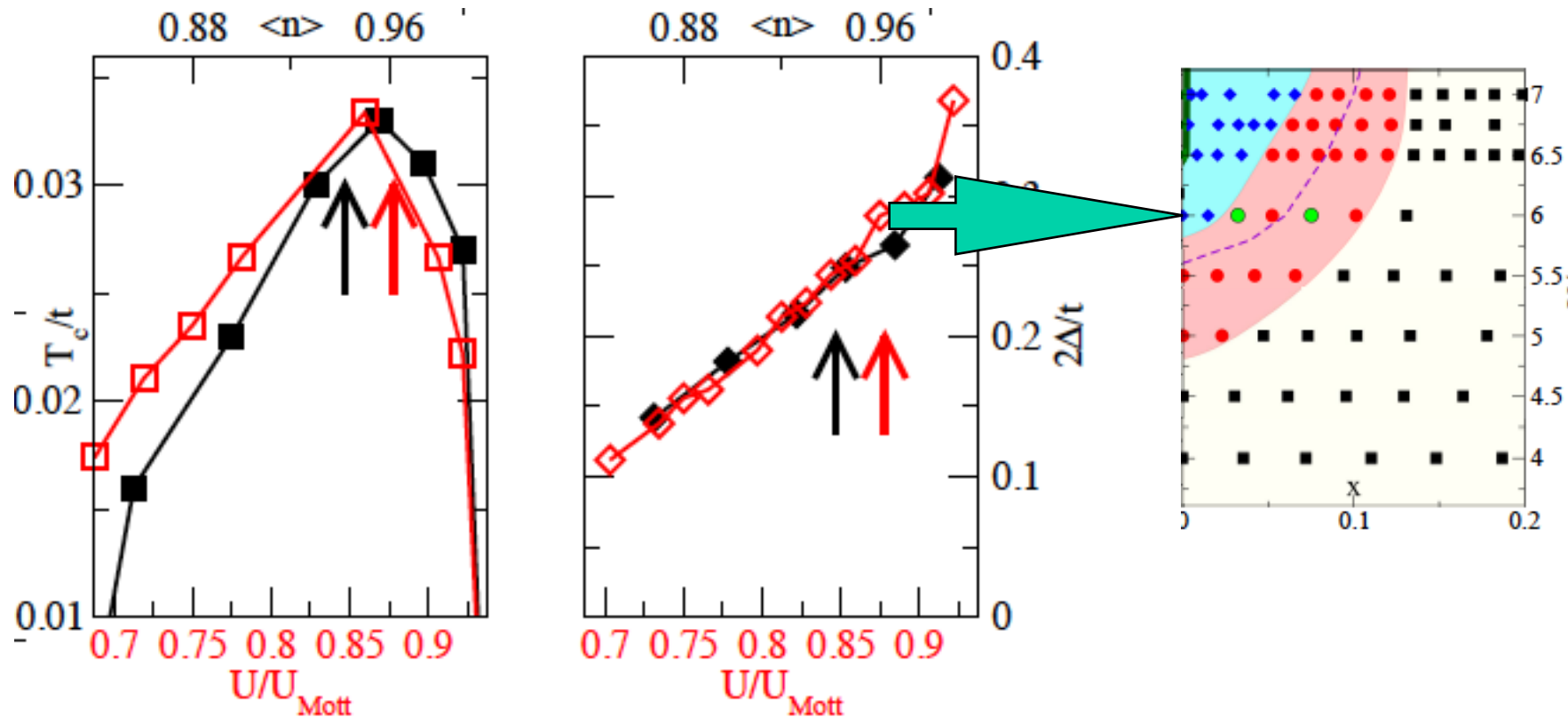


$t \sim 0.3 \text{ eV} \Rightarrow T_c^{\text{max}} \sim 170 \text{ K}$

Transition temperature and gap



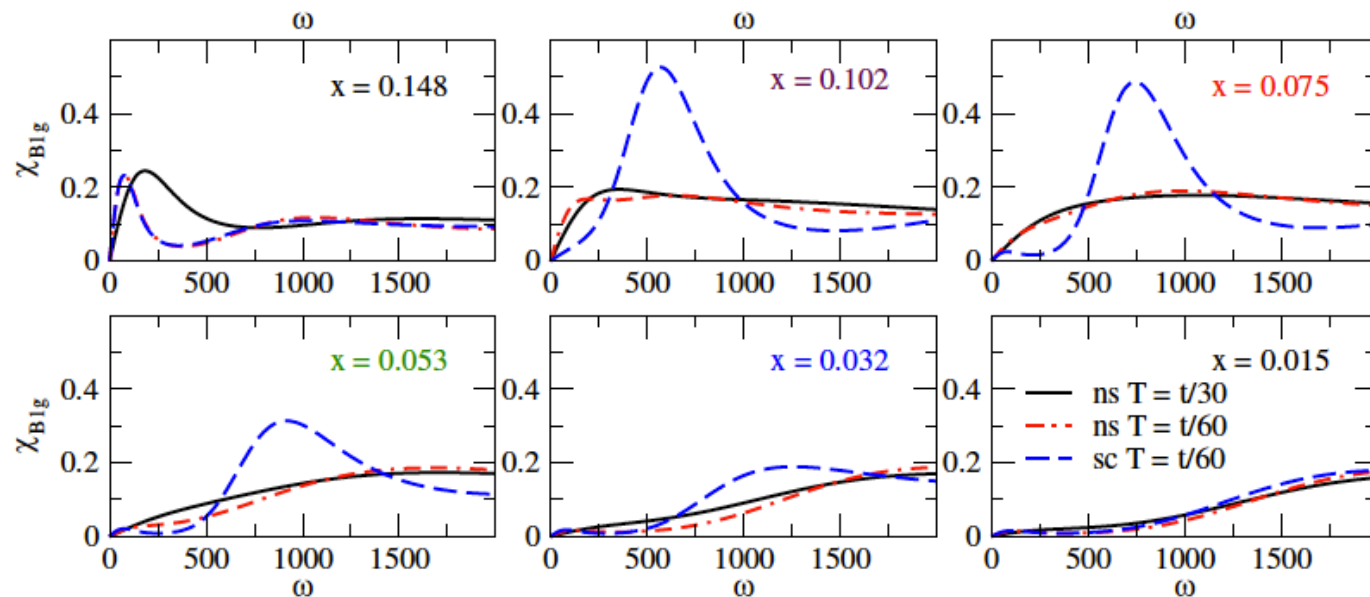
Transition temperature and gap



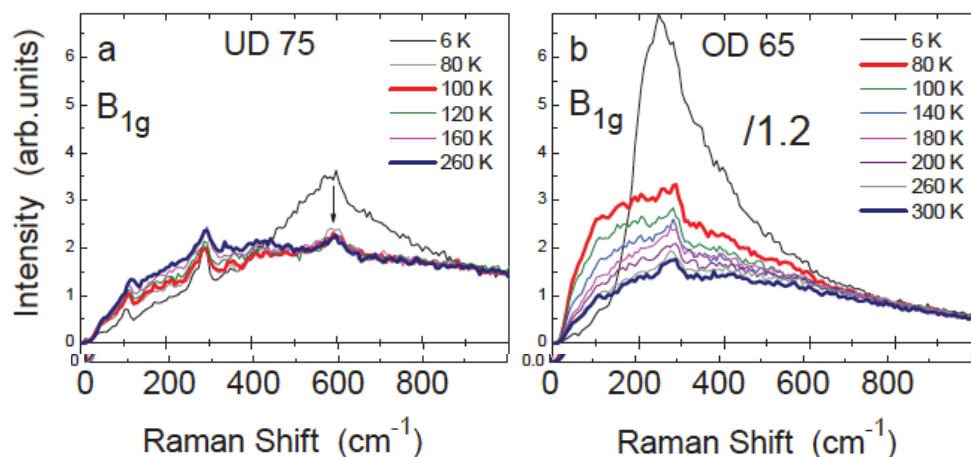
**Superconductivity cut off by pseudogap.
Transition is strongly first order**

Some other physical properties

Raman scattering



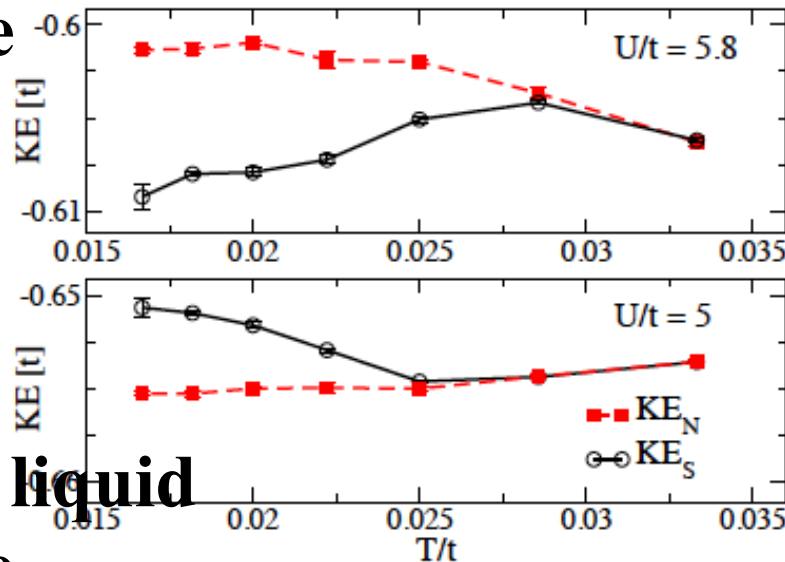
Calculation



Data
Sacuto et al

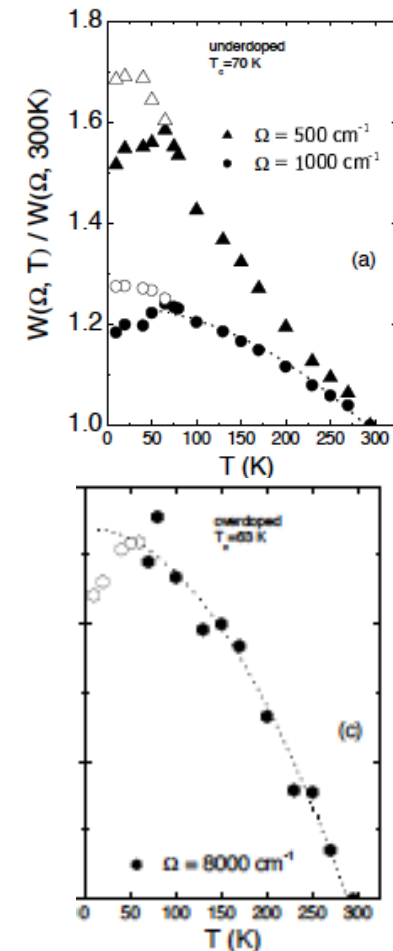
Temperature dependence of kinetic energy

Pseudogapped regime



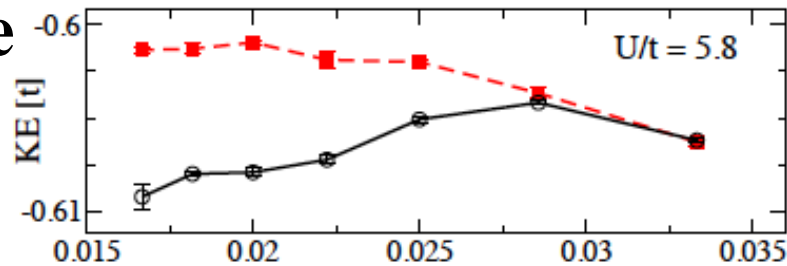
Fermi liquid regime

This trend in KE change is observed in optics: Santander-Syro, Lobo, Bontemps arXiv:0404.2901

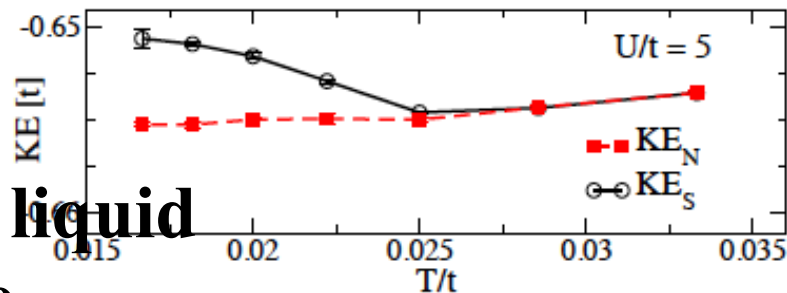


Temperature dependence of kinetic energy

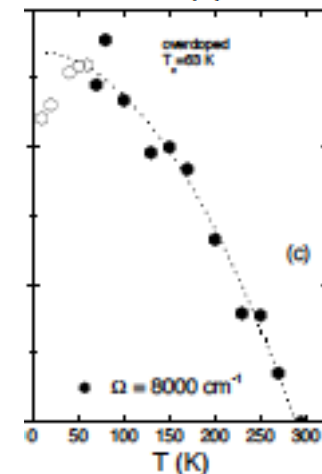
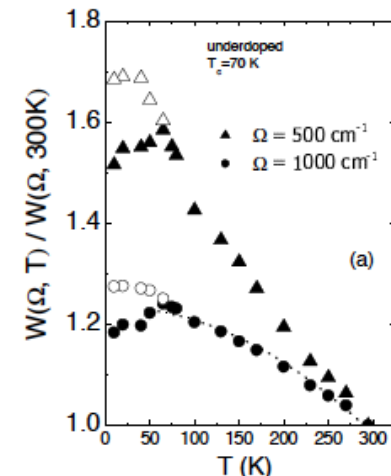
Pseudogapped regime



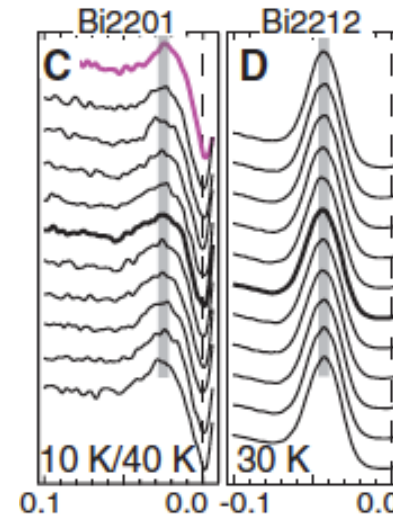
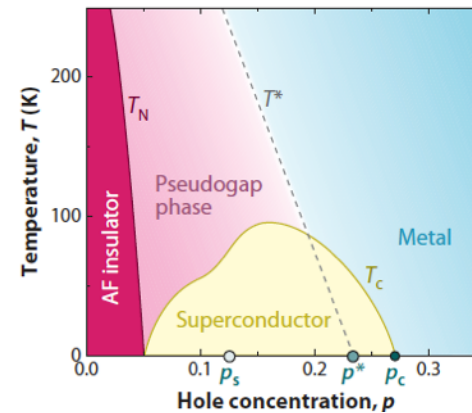
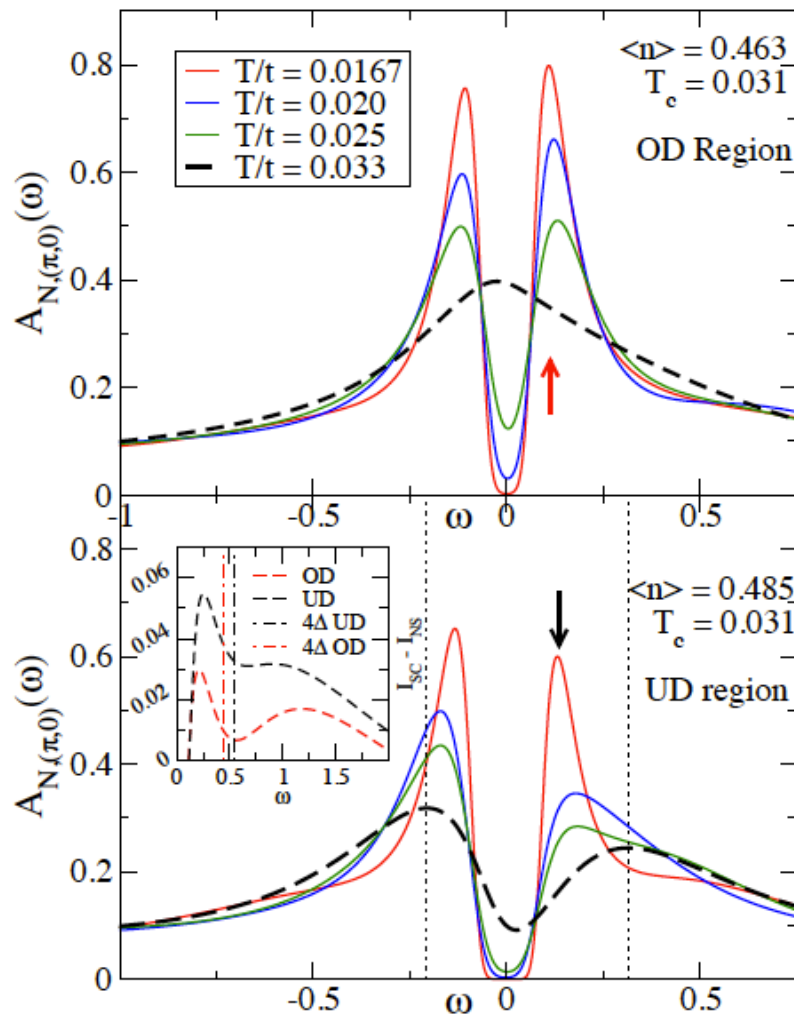
Fermi liquid regime



SC and PG are competing "phases".
 PG reduces kinetic energy: SC weakens PG, allows KE magnitude to rise



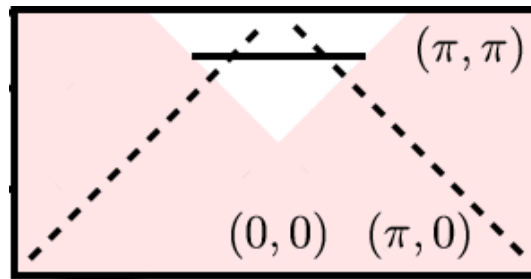
Superconductivity and the pseudogap



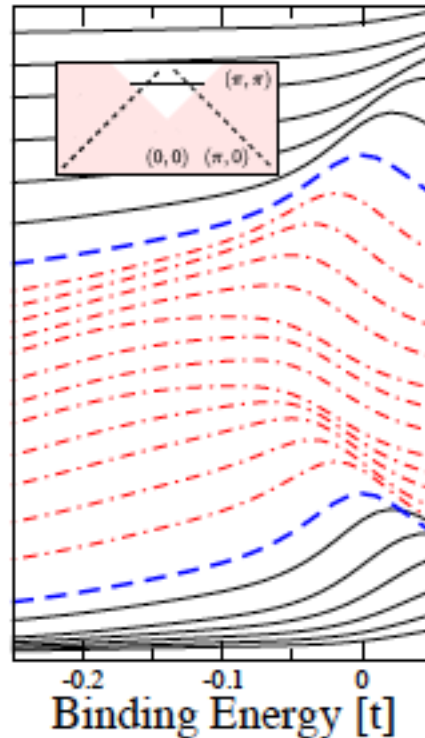
SC Gap smaller than PG

Simulated ARPES Spectra

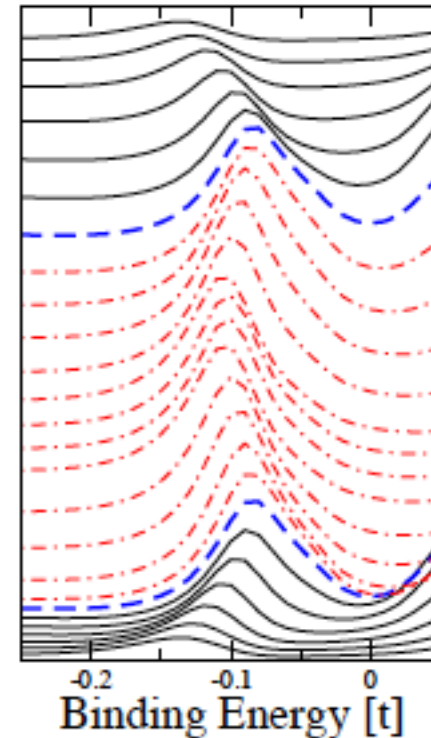
Fermi Liquid (no pseudogap)



$\beta=30$



$\beta=60$

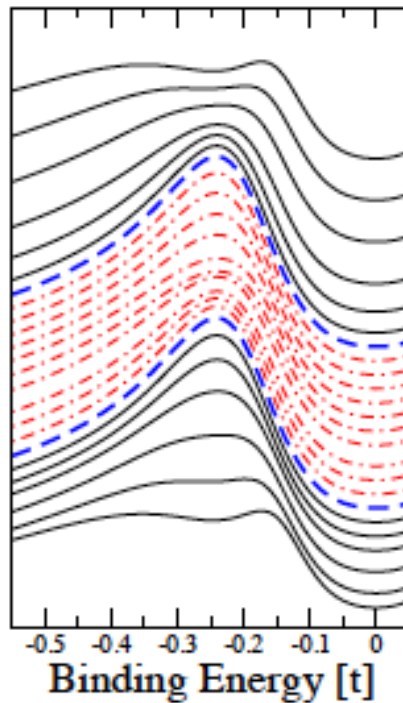


Simulated ARPES Spectra

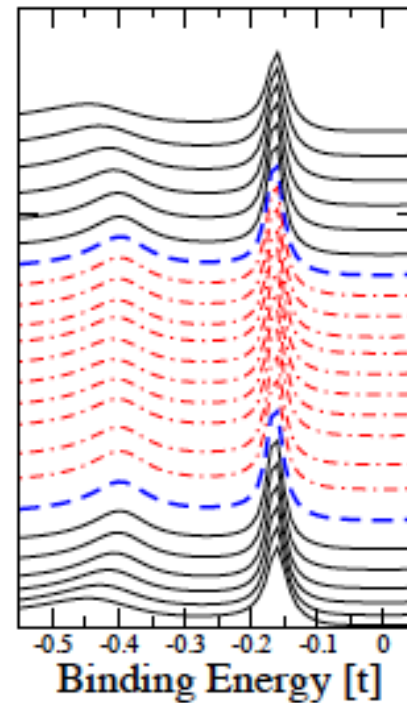
PG regime

New states created inside gap. Existing peak moved up in energy

$\beta=30$

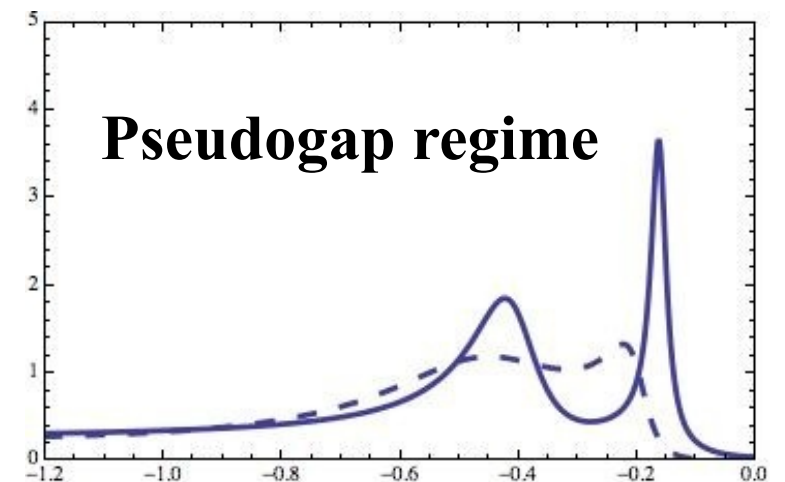
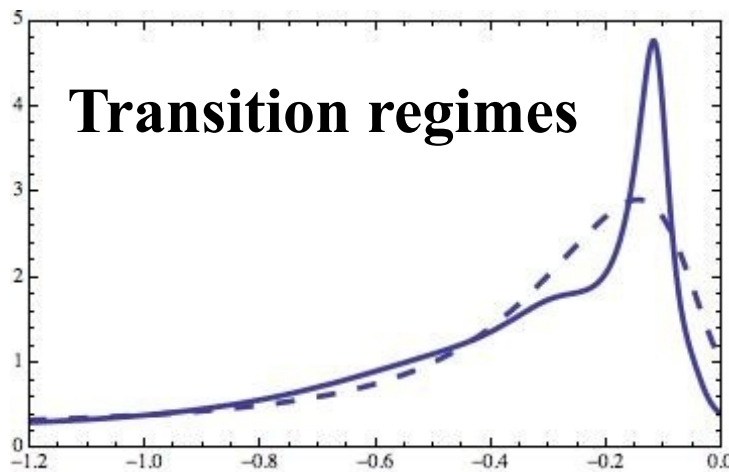
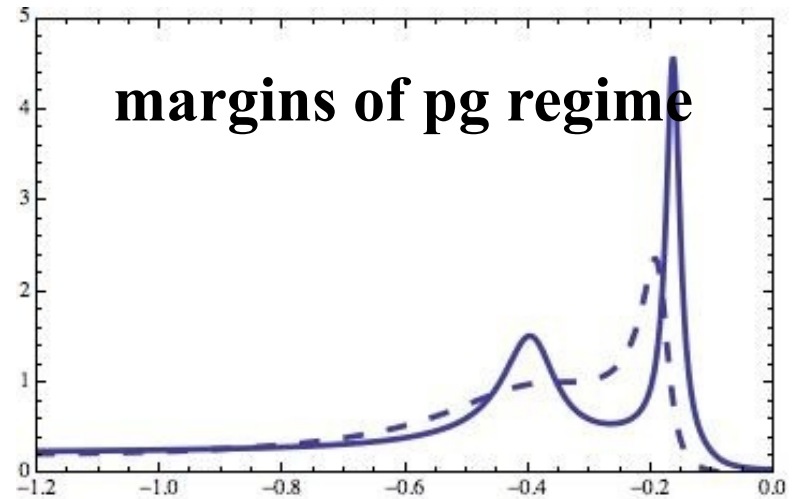
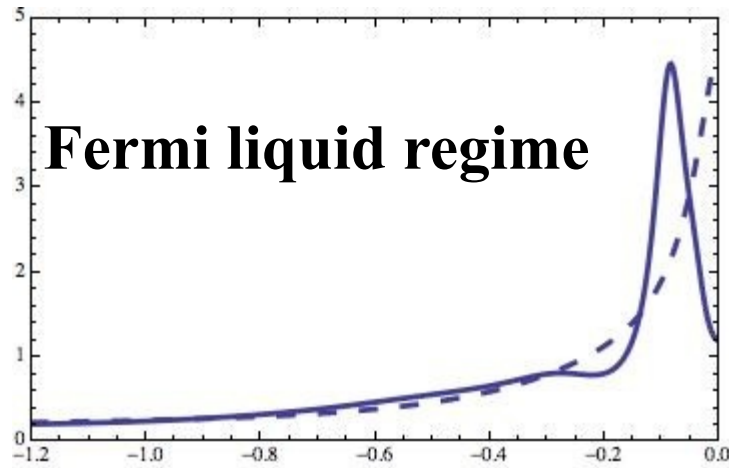


$\beta=60$

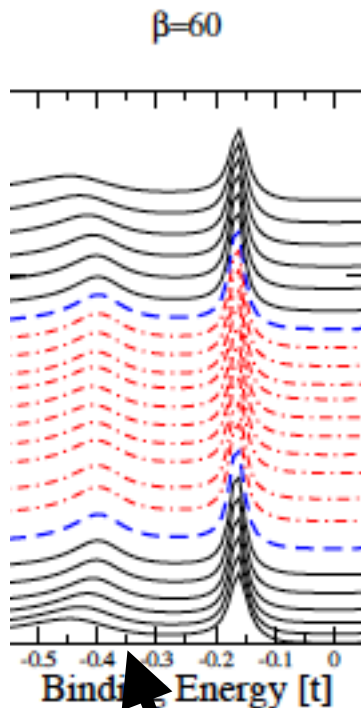


Note: 'peak-dip-hump' structure

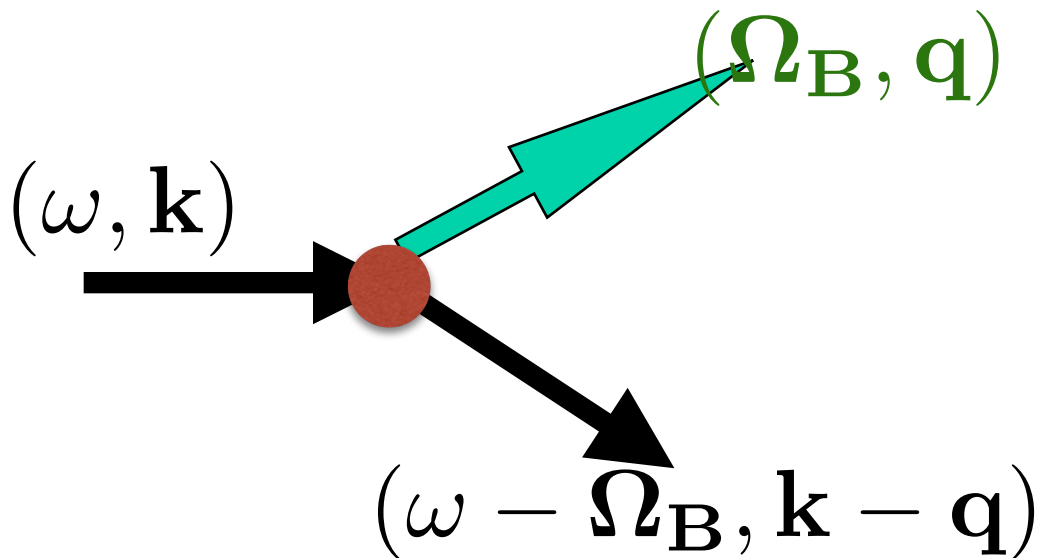
Evolution of photoemission Spectra



Physics of the “hump”



Standard interpretation:
“shakeoff”



Leading edge of “hump” is
interpreted as a threshold for
creating an excitation

Mathematically

$$A(\mathbf{k}, \omega) = \text{Im} [G(\mathbf{k}, \omega)] = \frac{\Sigma^{(2)}(\omega)}{(\omega - \varepsilon_{\mathbf{k}} - \Sigma^{(1)}(\omega))^2 + (\Sigma^{(2)}(\omega))^2}$$

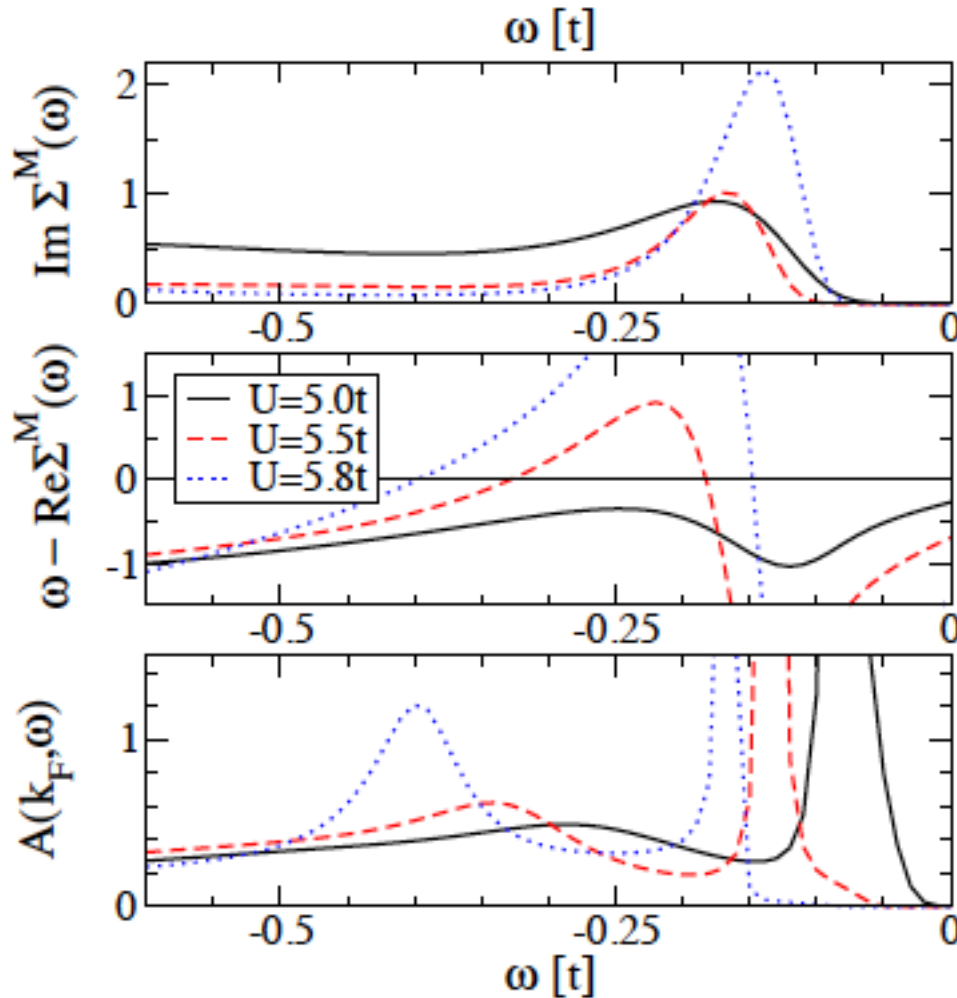
Two sources of peak in A

Shakeoff: onset of structure in imaginary part with real part non-zero (off resonance)

**THIS IS NOT WHAT HAPPENS
IN THE HUBBARD MODEL**

Alternative: resonance

We find: ‘hump’ is a resonance coming from a zero crossing of the real part of the self



$$\frac{\Sigma^{(2)}(\omega)}{(\omega - \varepsilon_{\mathbf{k}} - \Sigma^{(1)}(\omega))^2 + (\Sigma^{(2)}(\omega))^2}$$

**‘hump’ when
 $\omega - \text{Re}\Sigma(\omega) = 0$**

Pairing mechanism

Distinguish normal (N) and anomalous (A) components of self energy. Split normal part into Matsubara-frequency odd and even parts

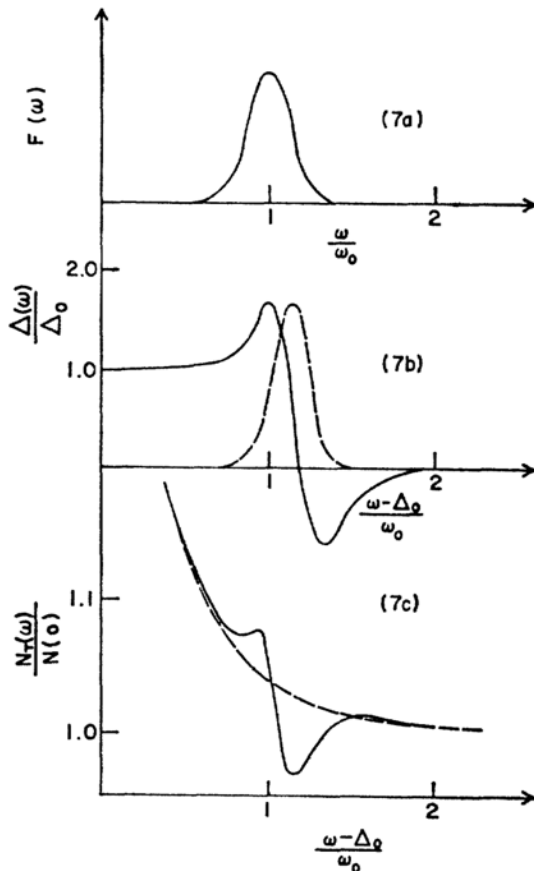
$$\Sigma_{o,e}^N = \frac{\Sigma^N(k, \omega_n) \mp \Sigma^N(k, -\omega_n)}{2}$$

Define gap function

$$\Delta(i\omega_n) = \frac{\Sigma^A(i\omega_n)}{1 - \frac{\Sigma_o^N(i\omega_n)}{\omega_n}} = \int \frac{d\mathbf{x}}{\pi} \frac{\Delta^{(2)}(\mathbf{x})}{i\omega_n - \mathbf{x}}$$

In conventional superconductors

Lead

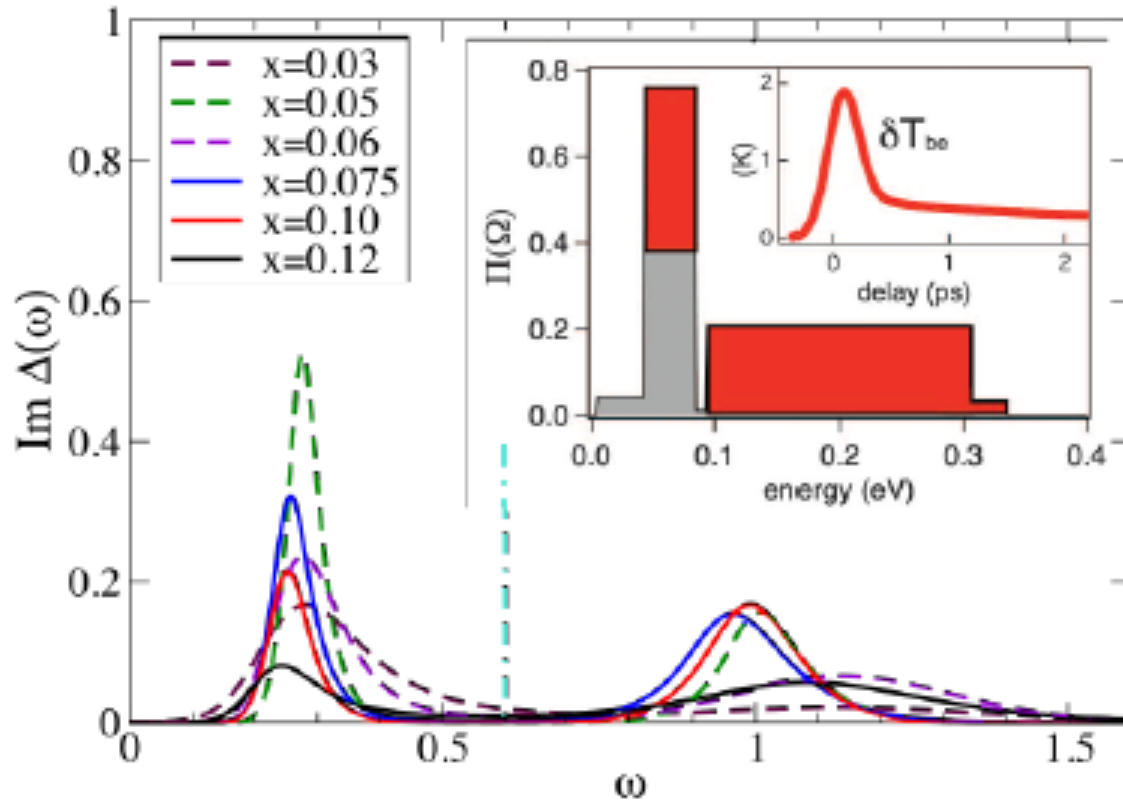


Imaginary part of gap function peaked at frequencies of pairing phonons

Scalapino, Schrieffer,
Wilkins, PBR 148 263 1966

In the Hubbard model

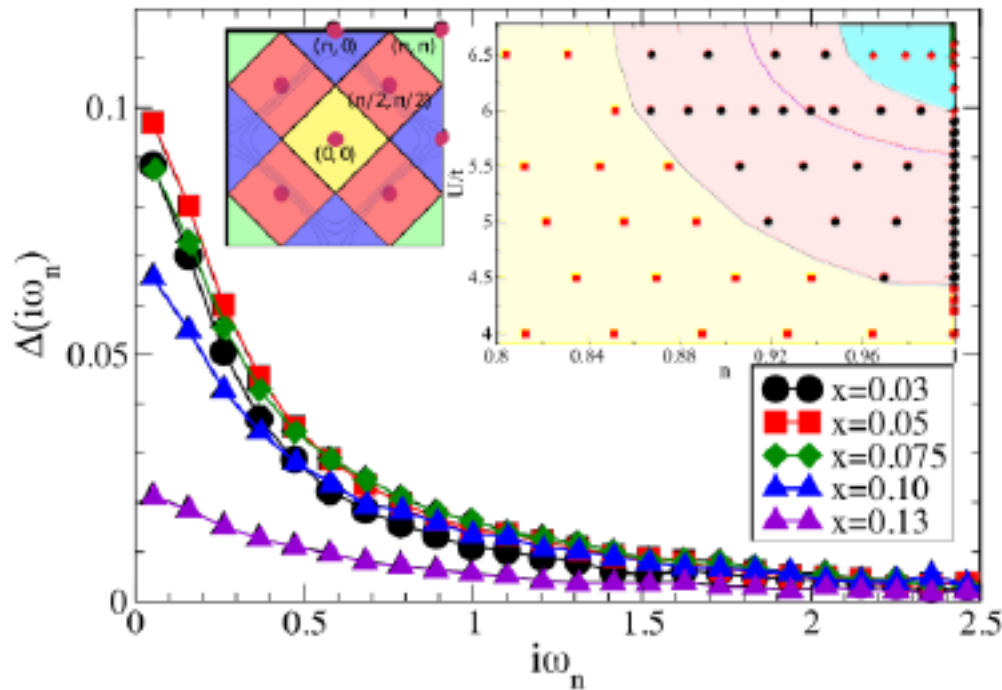
Inset: experimental estimate of pairing boson spectral function Dal Conte et al, Science, 335 6067 (2012)



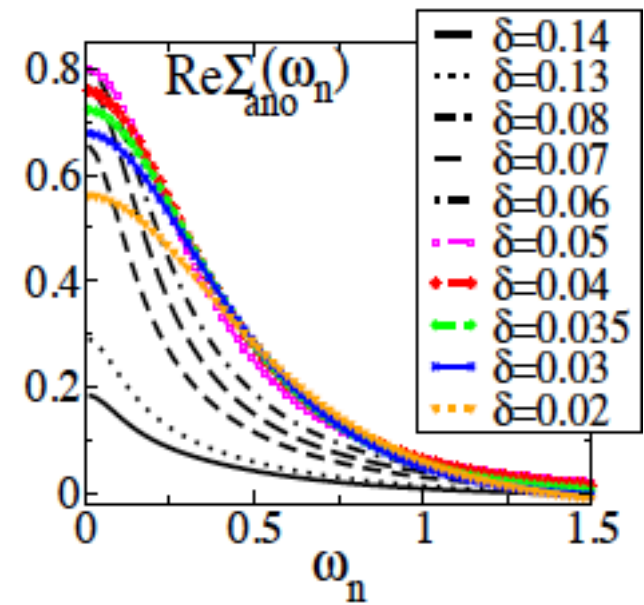
**All the pairing comes from low frequencies;
most from very low frequencies**

Can see this in raw data

Our results

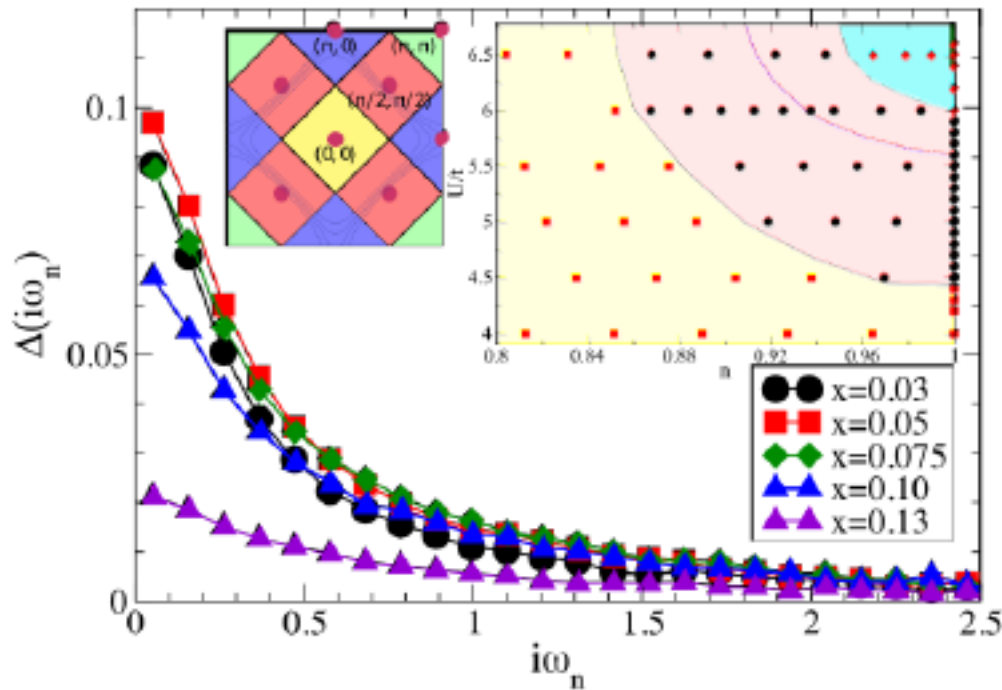


Civelli 09 4 site CDMFT. ED solver

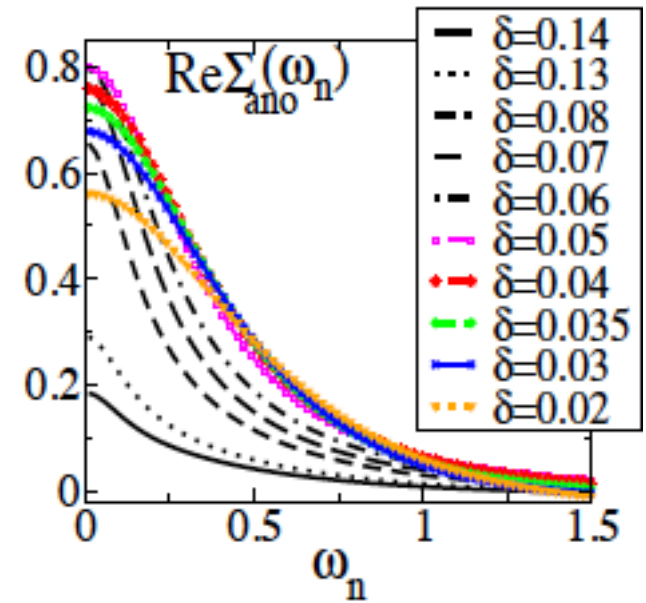


Can see this in raw data

Our results



Civelli 09 4 site CDMFT. ED solver

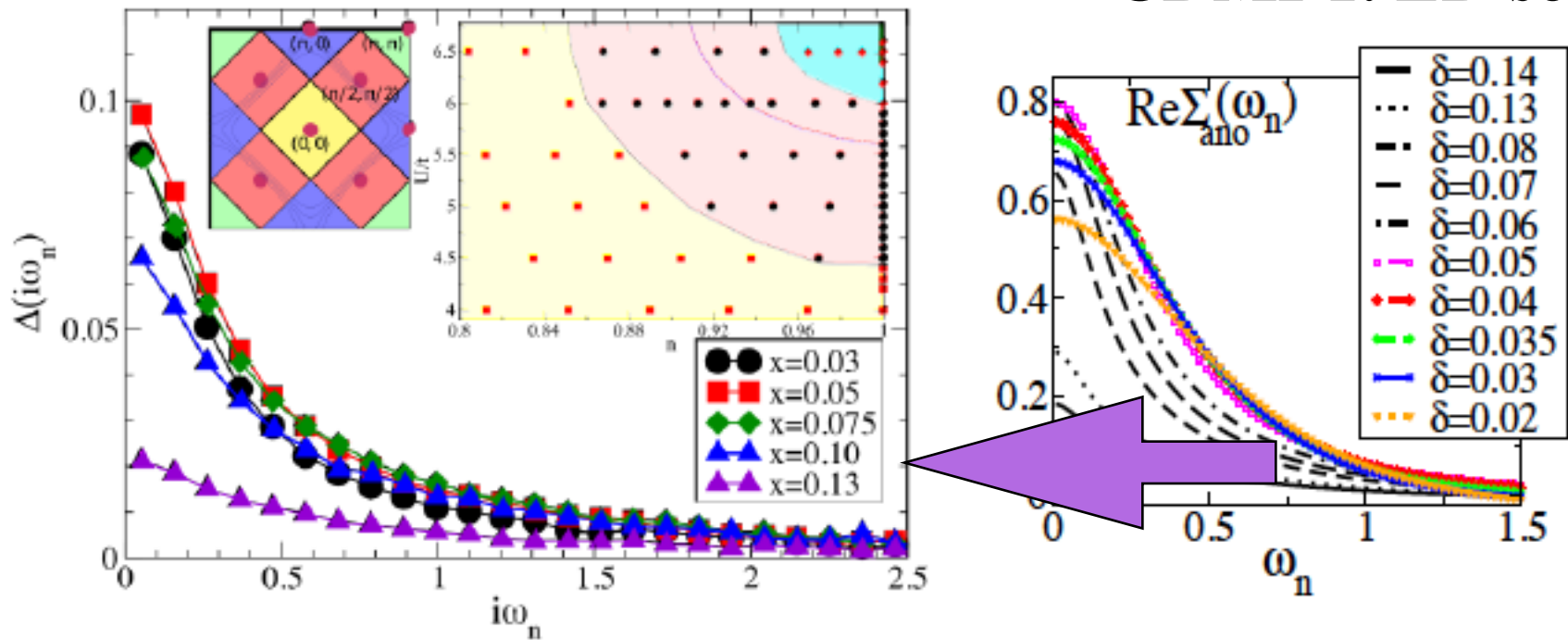


Note: Maier, Poilblanc, Scalapino: 20% or more of pairing comes from high frequencies $\sim U$.

$$\Delta(i\omega_n) = \frac{\Sigma^A(i\omega_n)}{1 - \frac{\Sigma_o^N(i\omega_n)}{\omega_n}} = \int \frac{dx \Delta^{(2)}(x)}{\pi i\omega_n - x}$$

Our results

**Civelli 09 4 site
CDMFT. ED solver**



Note: Maier, Poilblanc, Scalapino: 20% or more of pairing comes from high frequencies $\sim U$. If this were right, would see it in our raw data

**Many years ago,
D. J. Scalapino asked:**



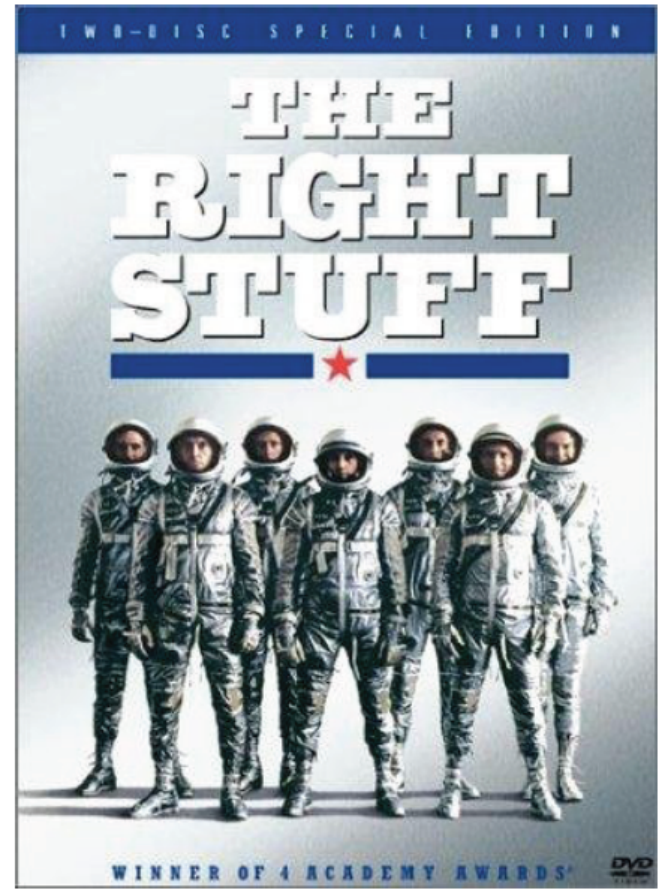
**Many years ago,
D. J. Scalapino asked:**

**Does the two
dimensional Hubbard
model have**



**Many years ago,
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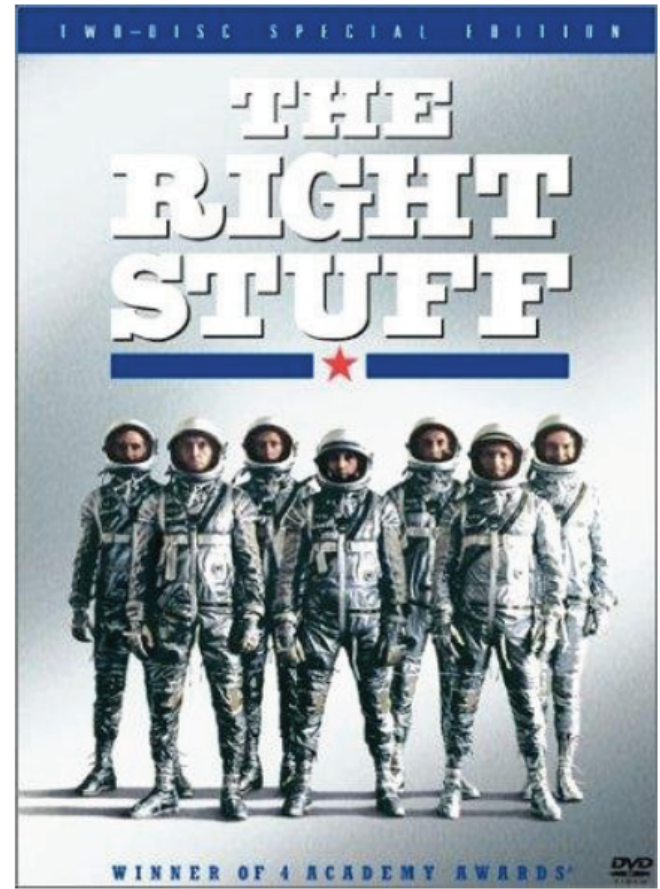
**Does the two
dimensional Hubbard
model have**



**Many years ago,
D. J. Scalapino asked:**

**Does the two
dimensional Hubbard
model have**

**(i.e. can it account for the
essential aspects of the low
energy physics of the high-Tc
cuprates)**



I hope that this talk has persuaded you that

The answer is YES

Implication: the pseudogap and superconductivity are basic phenomena of strong correlation physics.

Charge order, nematicity, etc are extra things ``along for the ride'



Issues

- 1. We have nice numerical results—
but what is the physics??**
- 2. Limit: $U \sim 7$, $N \sim 16$ is not as good as
we would like.**
 - A. Stronger coupling?**
 - B. Larger N —are the $N=8, 16$
results really representative of
Hubbard physics**
- 3. Stripes and other ordered states are
(presently) beyond the reach of this
approach. What is their
importance?**

Coda: SDW order and fluctuations in Iron Arsenide Superconductors.

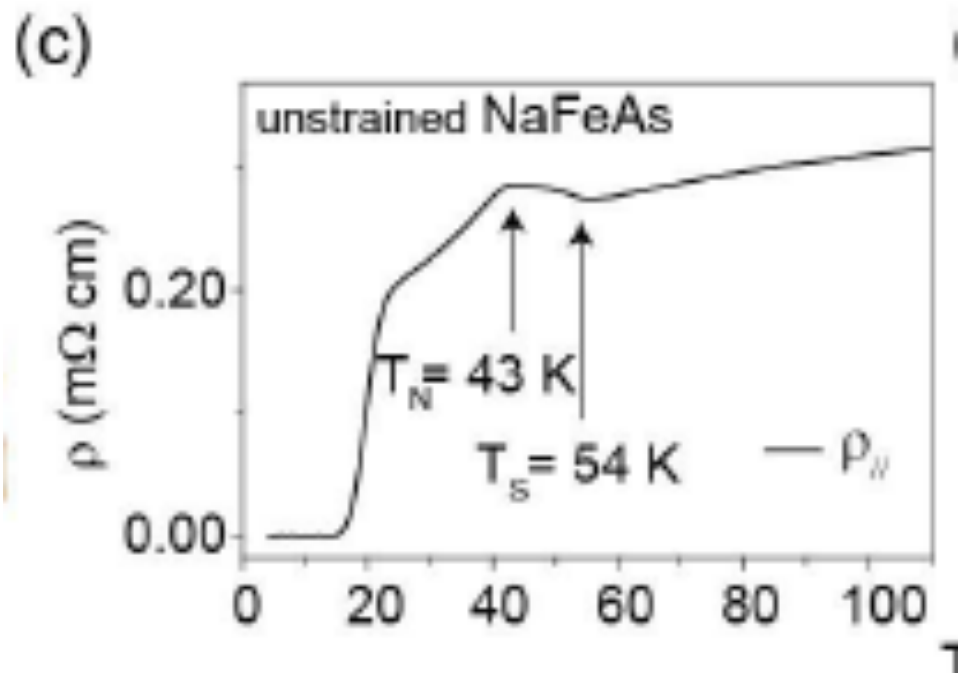
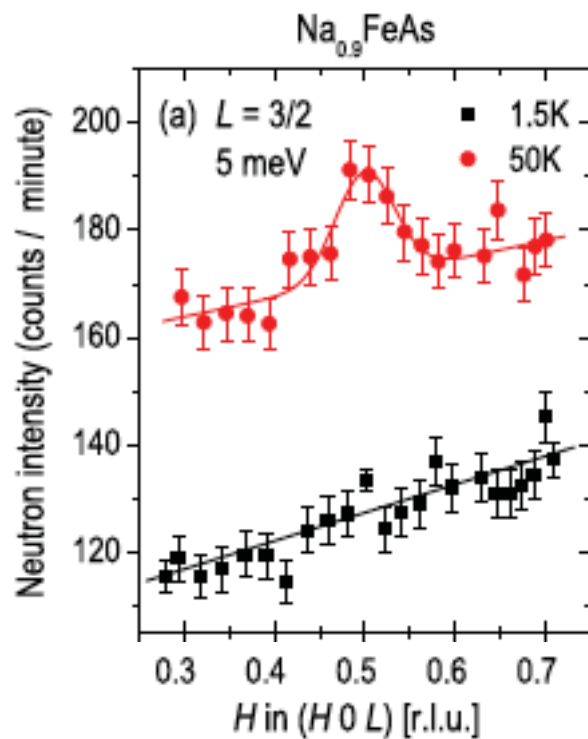
With Abhay Pasupathy & Rafael Fernandes

NaFeAs: 'stripe' (0, π) order below 43K

'nematic' order below 54K

Park...Inosov PRB 86 024437

Y. Zhang,¹ C. He,¹ Z. R. Ye,¹ J. Jiang,¹ F. Chen,¹ M. Xu,¹ Q. Q. Ge,¹ B. P. Xi,¹ J. Wei,² M. Aeschlimann,² X. Y. Cui,³ M. Shi,³ J. P. Hu,⁴ and D. L. Feng^{1,*}

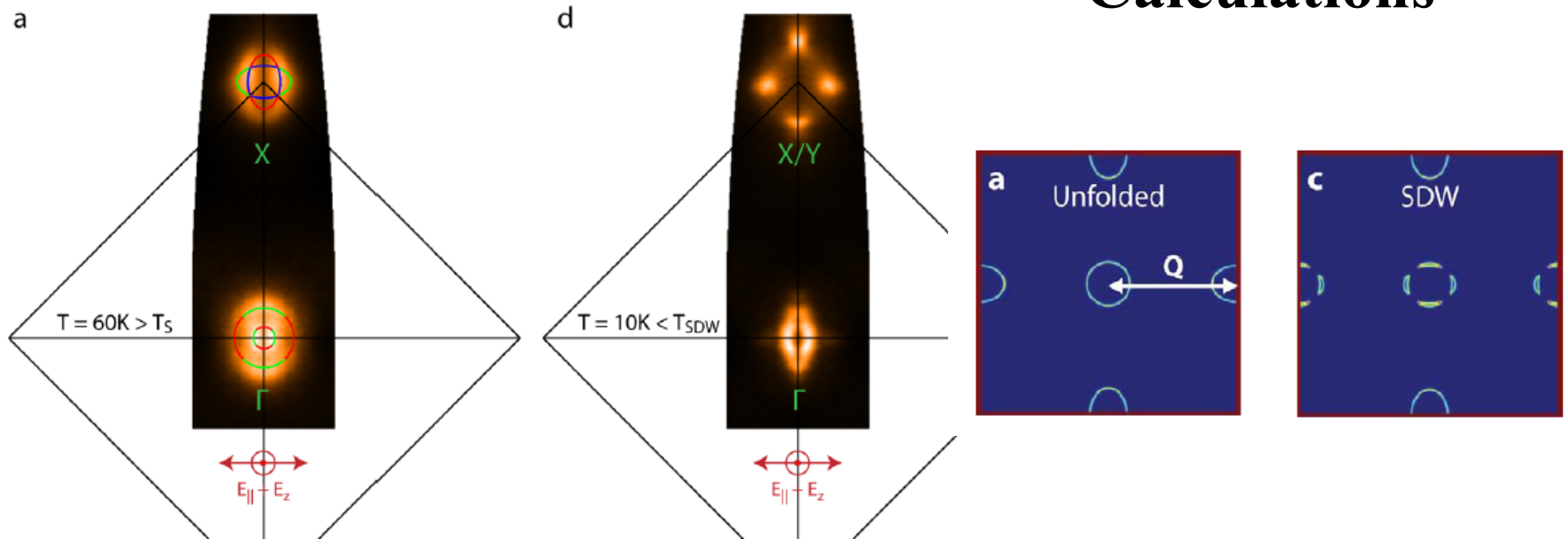


SDW rearranges the Fermi surface

Data:

M Yi^{1,2}, D H Lu³, R G Moore¹, K Kihou^{4,5}, C-H Lee^{4,5}, A Iyo^{4,5}, H Eisaki^{4,5}, T Yoshida^{5,6}, A Fujimori^{5,6}, Z-X Shen^{1,2*}

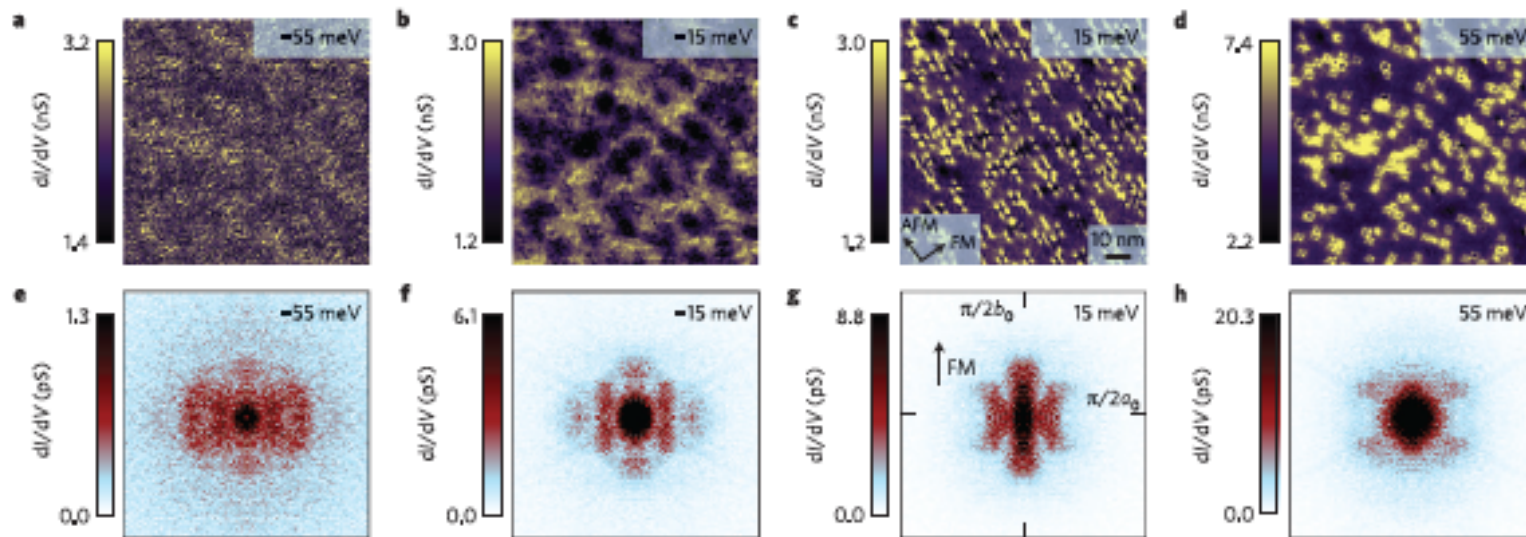
Calculations



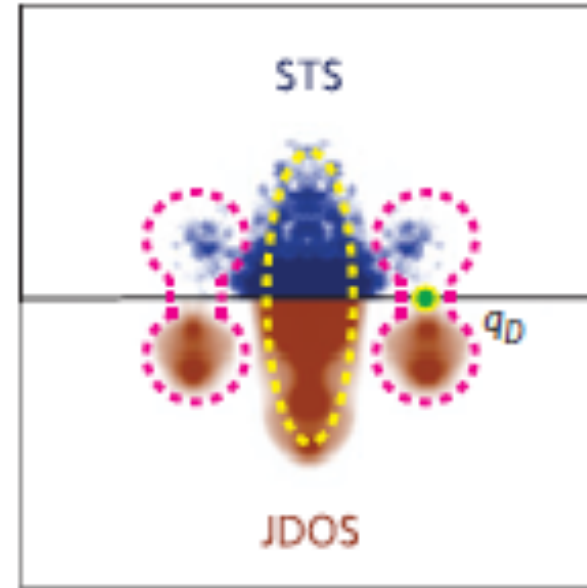
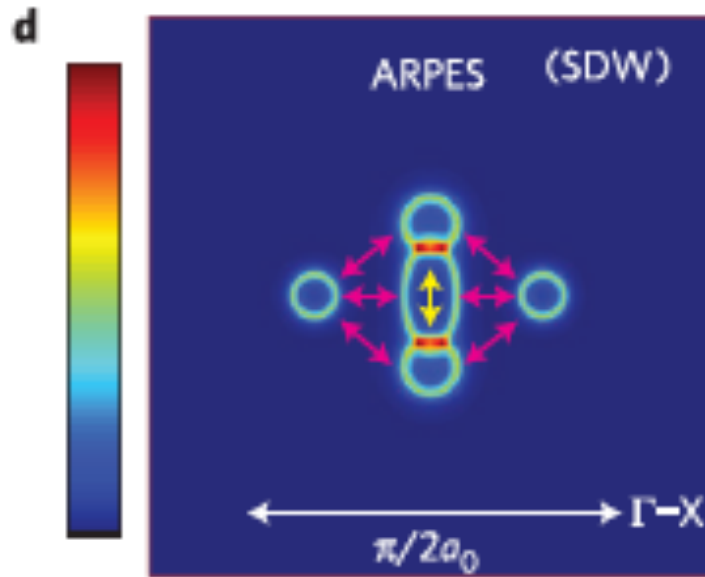
Quasiparticle Interference Reveals the Reconstruction

E. P. Rosenthal¹, E. F. Andrade¹, C. J. Arguello¹, R. M. Fernandes², L. Y. Xing³, X. C. Wang³, C. Q. Jin³, A. J. Millis¹ and A. N. Pasupathy^{1*}

N. Phys. 10 225 (2014)

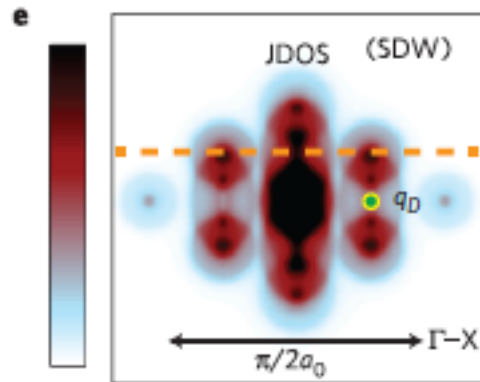


Structure in QPI reveals fermi surface

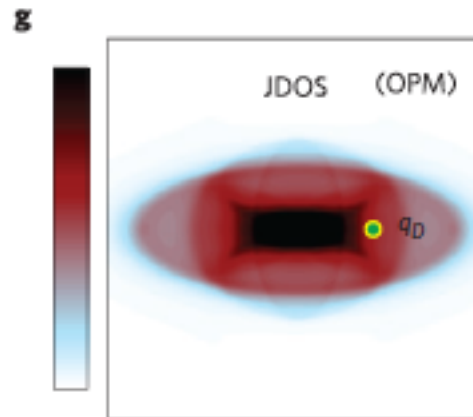


As you raise the temperature, expect the SDW-derived features to go away

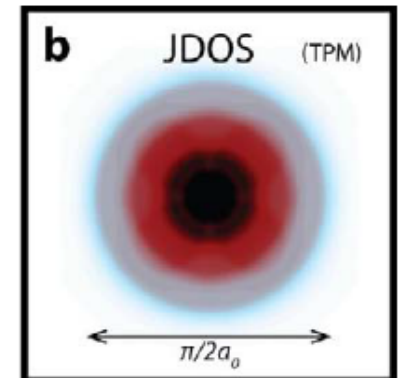
$T < T_N \sim 43\text{K}$



$T_N < T < T_{\text{nematic}}$

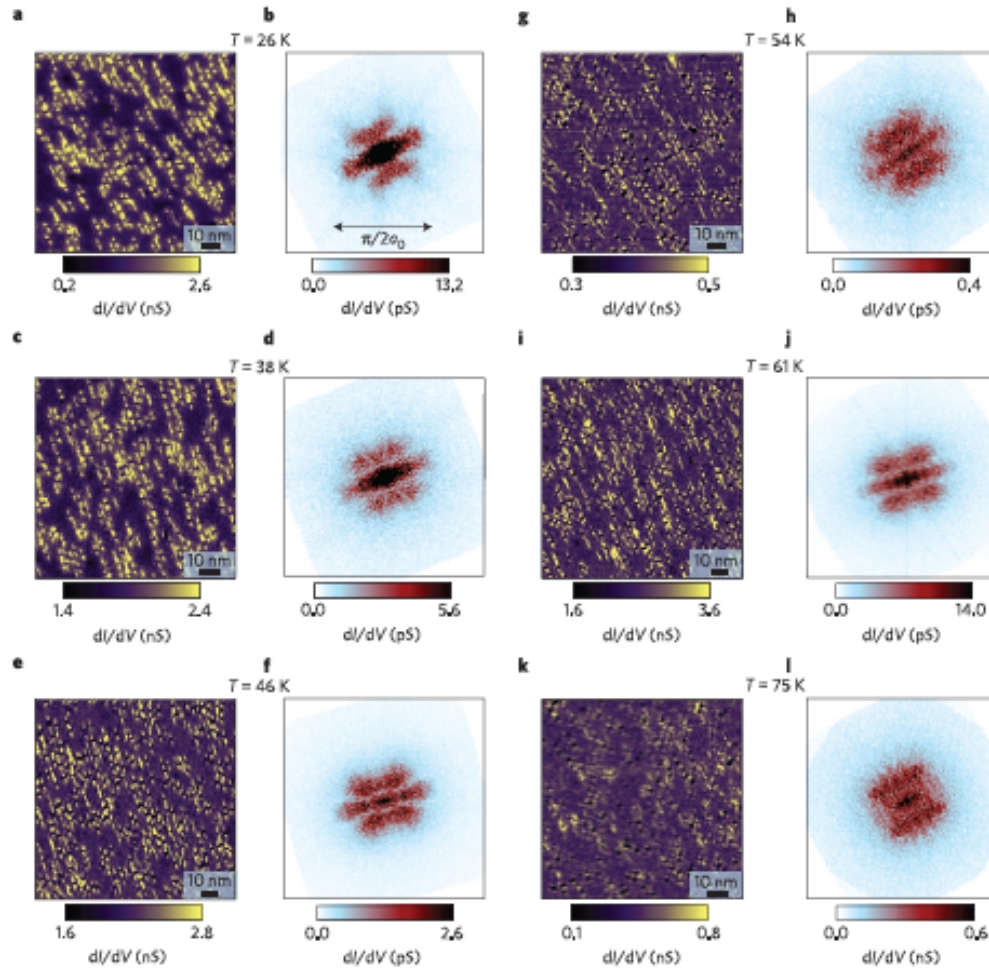


$T_{\text{nematic}} < T$

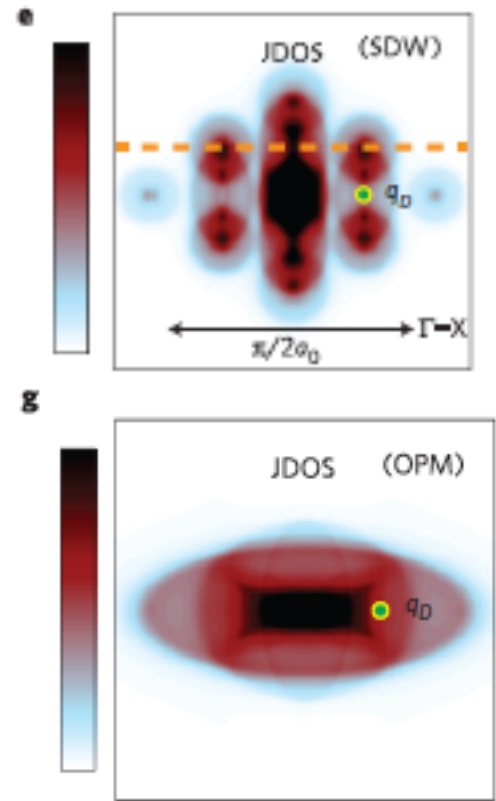
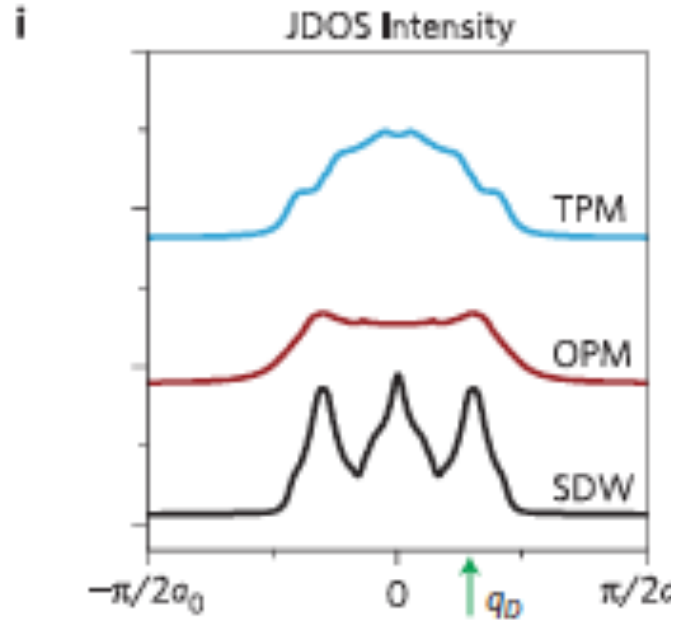
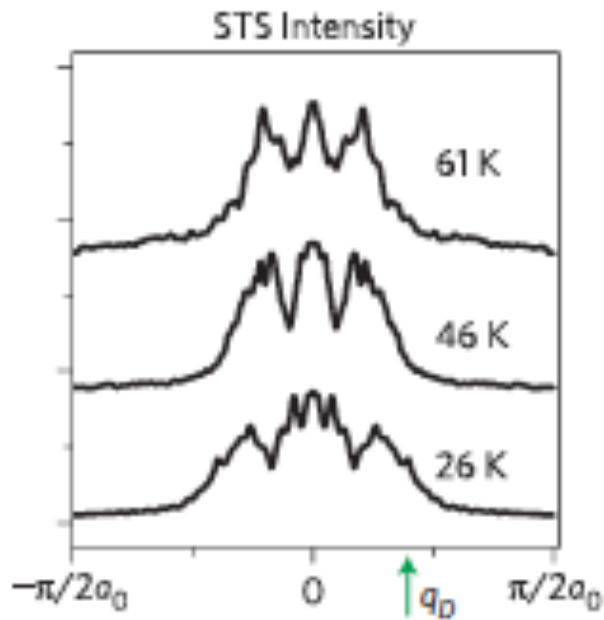


**Here I show you joint DOS for simplicity.
Full QPI calculations give the same physics**

This is not what happens



More easily visualized as a line cut



Key Result: SDW-like features persist to high T, in fact up to $T=2T_N$

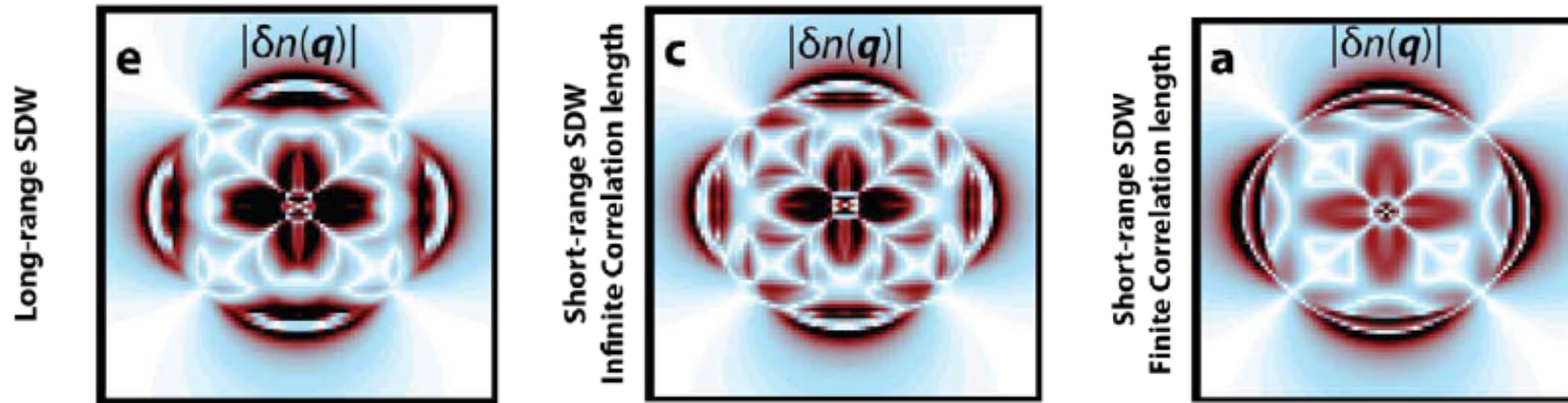
Model

'Lee-Rice-Anderson' ansatz

$$\Sigma(\omega, \mathbf{k}) = \frac{\Delta^2}{\omega - \varepsilon_{\mathbf{k}+\mathbf{Q}} - \frac{i}{\xi}}$$

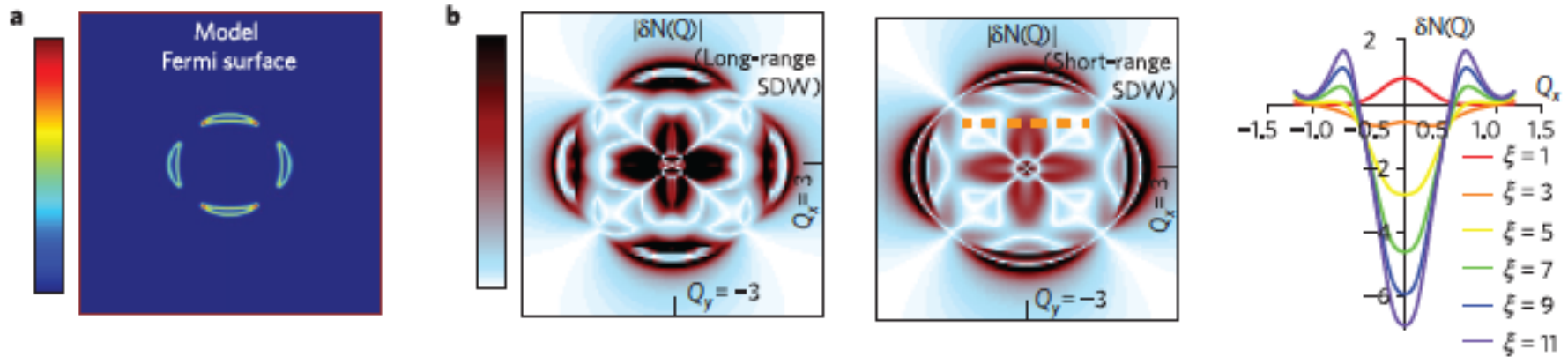
**This is broadened backscattering (no
`coherence factors in normal state)**

Model QPI Calculations



The short ranged SDW calculations use the standard QPI formula but with the Lee-Rice-Anderson G in a simplified 3-band approximation to the pnictide bands

Vary correlation length

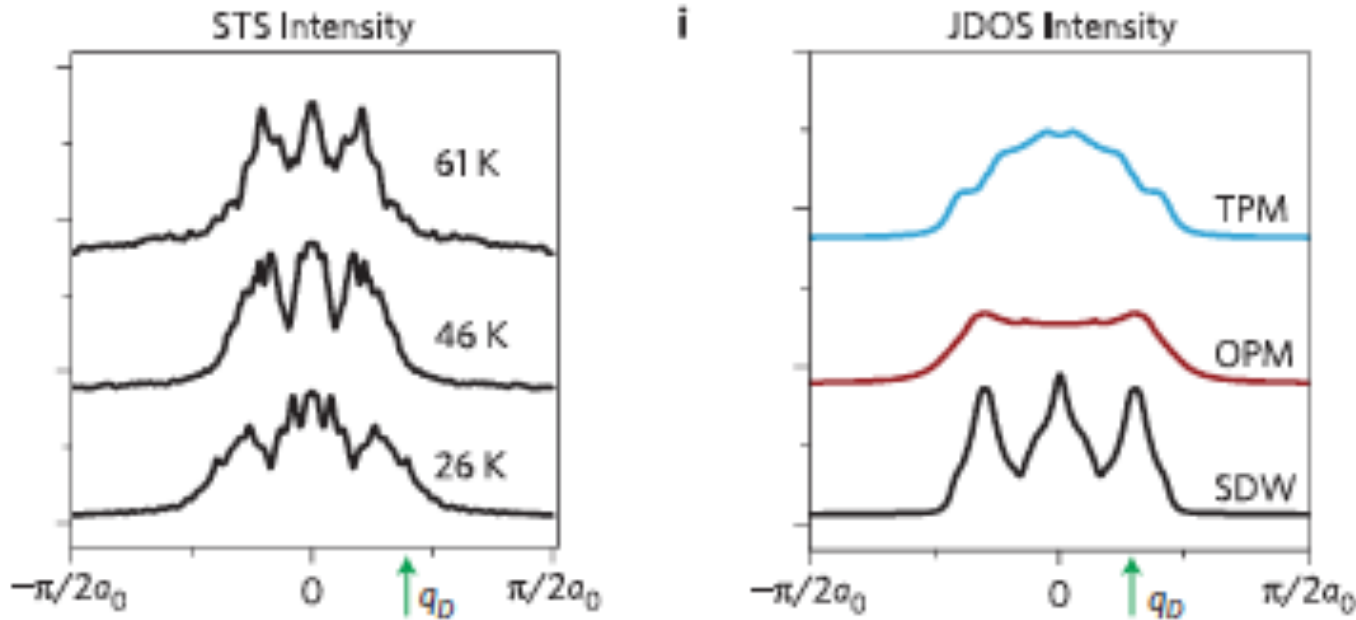


$$\Sigma(\omega, \mathbf{k}) = \frac{\Delta^2}{\omega - \varepsilon_{\mathbf{k}+\mathbf{Q}} - \frac{i}{\xi}}$$

To get peaks in line cuts need to keep Delta at approximately the T=0 value., have correlation length not too short.

In other words

Data imply large amplitude, slow fluctuations of density wave order, persisting up to $\sim 2x$ observed transition temperature



Paramagnetic phase has hidden structure

Poetically

Poetically

**We used to think of the fermi sea as a
(relatively) placid lake with modest ripples
(RPA fluctuations).**



Poetically

We used to think of the fermi sea as a (relatively) placid lake with modest ripples (RPA fluctuations).



Poetically

Pasupathy's results suggest an alternative picture



Poetically

Pasupathy's results suggest an alternative picture



Poetically

Pasupathy's results suggest an alternative picture: A stormy sea with giant amplitude, slowly moving waves.



Poetically

Pasupathy's results suggest an alternative picture: A stormy sea with giant amplitude, slowly moving waves.



Which picture is correct. If large amplitude density wave fluctuations are present, how do we surf on them