Quenched Disorder and Vestigial Nematicity

Laimei Nie and Gilles Tarjus,

Sam Lederer, Yoni Shatner, Erez Berg

Eduardo Fradkin,

Also

Ian Fisher, Jiun-Haw Chu,

S. Sachdev, L. Sierens, Roger Melko,

Akash Maharaj, John Tranquada

Rafael Fernandez, Dung-Hai Lee, Fa Wang, Vadim Oganesyan, Andy Mackenzie, Erica Carlson, Karin Dahmen,Yi Zhang, Sri Raghu, Eun-Ah Kim, Michael Lawler, Hong Yao, Jiaping Hu, John Robertson, Ted Geballe

Paris 2015

Effective Field Theory Approach

- Emphasis on broken symmetries,
- Sharp distinctions between phases of matter (mostly based on **long-range order**).
- Long-distance low energy emergent properties.

Vestigial Order

When there is a sequence of transitions separating an ordered (broken symmetry) state from a disordered (symmetric) state, intermediate phases that restore some but not all of the symmetries broken in the fully ordered state can be said to have "vestigial order."

Electron Nematic Order

Spontaneous breaking of rotational (point group) symmetry as an emergent property of the electron fluid (in a crystal).

Role of Quenched Disorder

When spontaneous breaking of spatial symmetries occurs in solids, quenched disorder is always qualitatively important.

Theory of Vestigial Nematic Order

We will consider a concrete model problem
(which may, however, be directly applicable
to the charge-ordering phenomena
found in the cuprate high temperature
superconductors and spin and charge ordering
in the Fe-based superconductors)

Incommensurate CDW Order

$$\rho(\vec{r}) = \bar{\rho} + \left[\psi_x(\vec{r})e^{i\vec{Q}_x\cdot r} + \psi_y(\vec{r})e^{i\vec{Q}_y\cdot r} + \text{c.c.}\right] + \dots$$

$$\text{Stripe ordered phase} : |\langle\psi_x\rangle| \neq 0 \text{ or } |\langle\psi_y\rangle| \neq 0$$

$$\text{so } \langle\left[|\psi_x|^2 - |\psi_y|^2\right]\rangle \neq 0$$

$$\text{Checkerboard ordered phase} : |\langle\psi_x\rangle| = |\langle\psi_y\rangle| \neq 0$$

$$\text{with } \langle\left[|\psi_x|^2 - |\psi_y|^2\right]\rangle = 0$$

$$\text{Nematic phase} : |\langle\psi_x\rangle| = 0 \text{ and } |\langle\psi_y\rangle| = 0$$

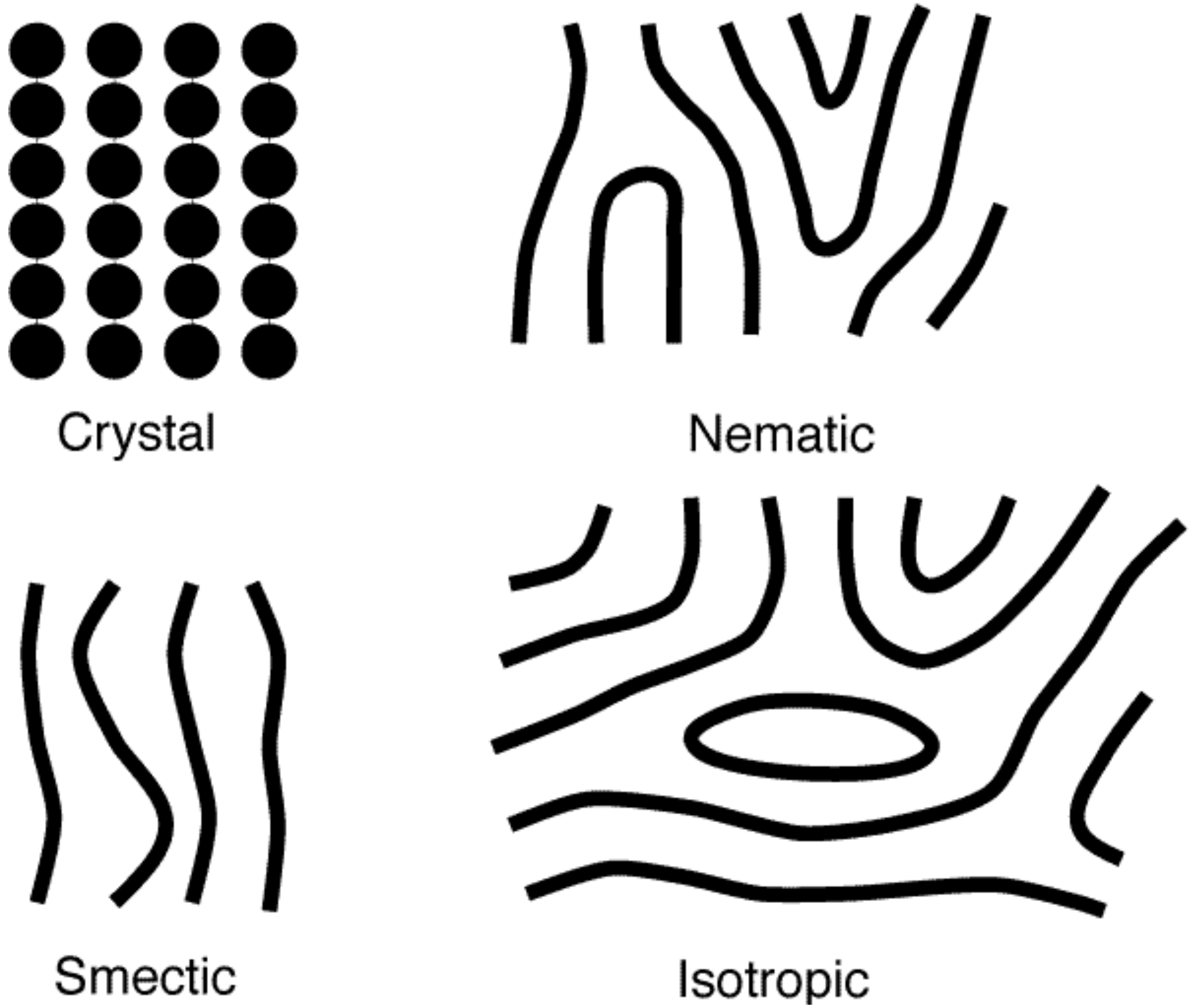
$$\text{but } \langle\left[|\psi_x|^2 - |\psi_y|^2\right]\rangle \equiv \mathcal{N} \neq 0$$

L. Nie, G. Tarjus, and SAK PNAS (2013)

Snapshot of a fluctuating stripe nematic

- Stripes are one of those revolving trends, frequently fluctuating in and out of style. Well, stripes are officially "in" again, thanks to ...

Fluctuating stripe nematic in a Mott insulator

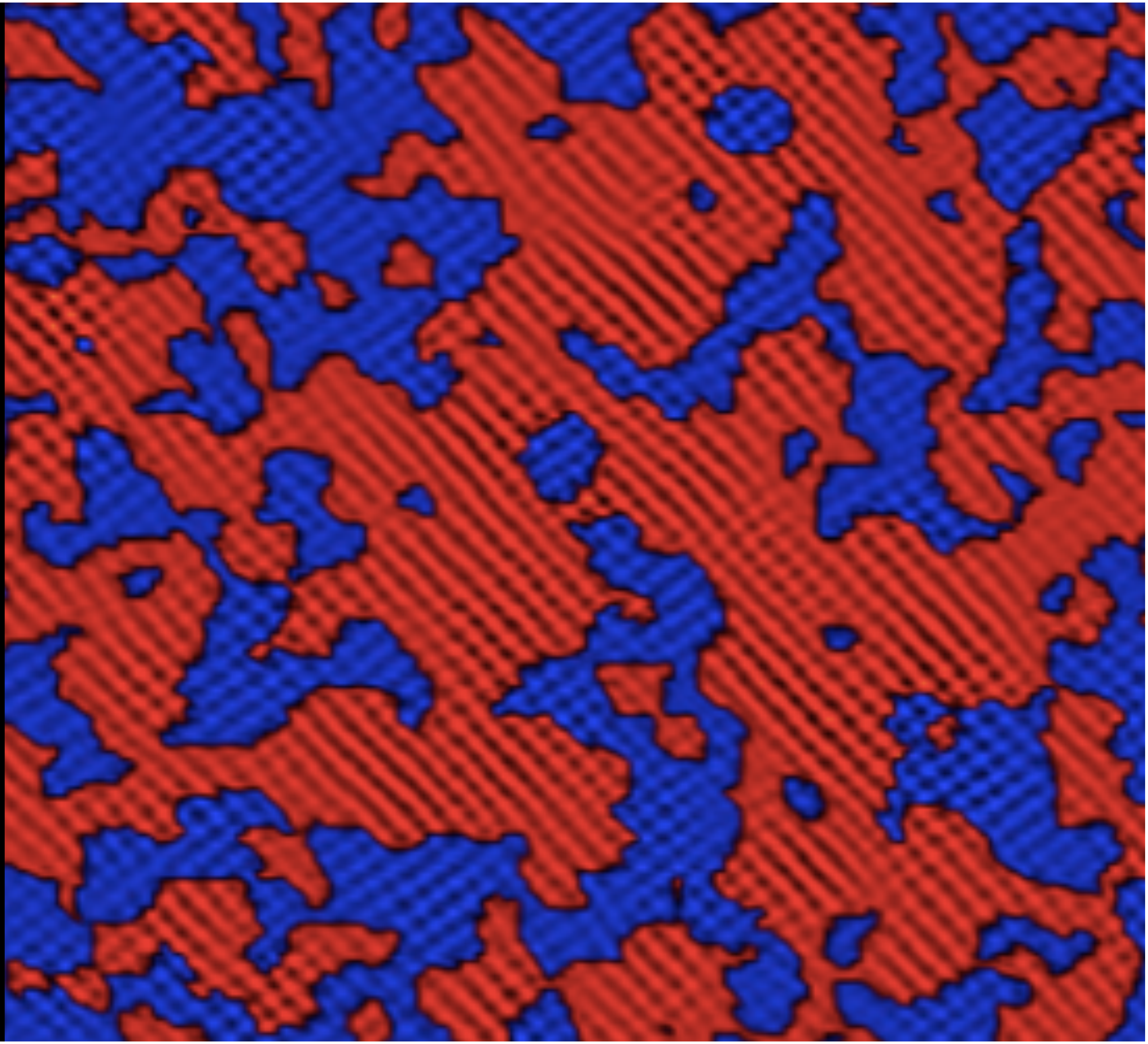


S. A. Kivelson, E. Fradkin and V. J. Emery, *Nature* **393**, 550-553(11 June 1998)

“Snapshot of a state with “vestigial” nematic order

Red region has $N=[\ \langle|\psi_x|^2\rangle - \langle|\psi_y|^2\rangle\ ] > 0$

Courtesy J.E. Hoffman, E. Hudson, C-L Song E.W.Carlson and co.



Incommensurate CDW Order

$$\rho(\vec{r}) = \bar{\rho} + \left[\psi_x(\vec{r})e^{i\vec{Q}_x \cdot r} + \psi_y(\vec{r})e^{i\vec{Q}_y \cdot r} + \text{c.c.}\right] + \dots$$

$$\rho(\vec{r}) \to \rho(\vec{r} + \vec{R}) \quad \equiv \quad \psi_a(\vec{r}) \to \psi_a(\vec{r})e^{i\theta_a} \quad \text{where} \quad \theta_a = \vec{Q}_a \cdot \vec{R}$$

$$\text{O}(2) \times \text{O}(2) \times Z_2 \text{ symmetry}$$

L. Nie, G. Tarjus, and SAK PNAS (2013)

Incommensurate CDW Order

$$\rho(\vec{r}) = \bar{\rho} + \left[\psi_x(\vec{r})e^{i\vec{Q}_x\cdot r} + \psi_y(\vec{r})e^{i\vec{Q}_y\cdot r} + \text{c.c.}\right] + \ldots$$

$$\rho(\vec{r}) \to \rho(\vec{r}+\vec{R}) \quad \equiv \quad \psi_a(\vec{r}) \to \psi_a(\vec{r})e^{i\theta_a} \quad \text{where} \quad \theta_a = \vec{Q}_a \cdot \vec{R}$$

$$\text{O}(2) \times \text{O}(2) \times Z_2 \text{ symmetry}$$

Commensurate SDW

$$\mathbf{S}(\vec{r}) = \left[\mathbf{\Psi}_x(\vec{r})e^{i\vec{Q}_x\cdot\vec{r}} + \mathbf{\Psi}_y(\vec{r})e^{i\vec{Q}_y\cdot\vec{r}} + \text{H.C.}\right] + \ldots$$

$$\vec{Q}_x = (\pi, 0); \quad \vec{Q}_y = (0, \pi) \qquad \text{SO}(3) \times Z_2$$

Fang, Yao, Tsai, Hu, and Kivelson, PRB **77**, 224509 (2008)
Xu, Muller, and Sachdev , PRB **78**, 020501 (2008).

Incommensurate CDW Order

$$\rho(\vec{r}) = \bar{\rho} + \left[\psi_x(\vec{r})e^{i\vec{Q}_x\cdot r} + \psi_y(\vec{r})e^{i\vec{Q}_y\cdot r} + \text{c.c.}\right] + \dots$$

$$\rho(\vec{r}) \to \rho(\vec{r}+\vec{R}) \quad \equiv \quad \psi_a(\vec{r}) \to \psi_a(\vec{r})e^{i\theta_a} \quad \text{where} \quad \theta_a = \vec{Q}_a\cdot\vec{R}$$

$$\text{O}(2)\times\text{O}(2)\times Z_2 \text{ symmetry}$$

Commensurate SDW

$$\mathbf{S}(\vec{r}) = \left[\mathbf{\Psi}_x(\vec{r})e^{i\vec{Q}_x\cdot\vec{r}} + \mathbf{\Psi}_y(\vec{r})e^{i\vec{Q}_y\cdot\vec{r}} + \text{H.C.}\right] + \dots$$

$$\vec{Q}_x = (\pi, 0); \quad \vec{Q}_y = (0,\pi) \qquad \text{SO}(3)\times Z_2$$

Incommensurate SDW

$$\text{SO}(3)\times\text{O}(2)\times\text{O}(2)\times Z_2$$

Incommensurate CDW Order

$$\rho(\vec{r}) = \bar{\rho} + \left[\psi_x(\vec{r})e^{i\vec{Q}_x\cdot r} + \psi_y(\vec{r})e^{i\vec{Q}_y\cdot r} + \text{c.c.}\right] + \dots$$

$$\mathcal{H} = \frac{\kappa_\parallel}{2}\left[|\partial_x\psi_x(j)|^2 + |\partial_y\psi_y(j)|^2\right] + \frac{\kappa_\perp}{2}\left[|\partial_y\psi_x(j)|^2 + |\partial_x\psi_y(j)|^2\right]$$
$$-\frac{m}{2}\left[|\psi_x(j)|^2 + |\psi_x(j)|^2\right] + \frac{u}{4}\left[|\psi_x(j)|^2 + |\psi_x(j)|^2\right]^2$$
$$+\frac{\gamma}{2}|\psi_x(j)|^2|\psi_y(j)|^2 + J_\perp\left[\psi_x^*(j+1)\psi_x(j) + \psi_y^*(j+1)\psi_y(j) + \text{c.c.}\right]$$

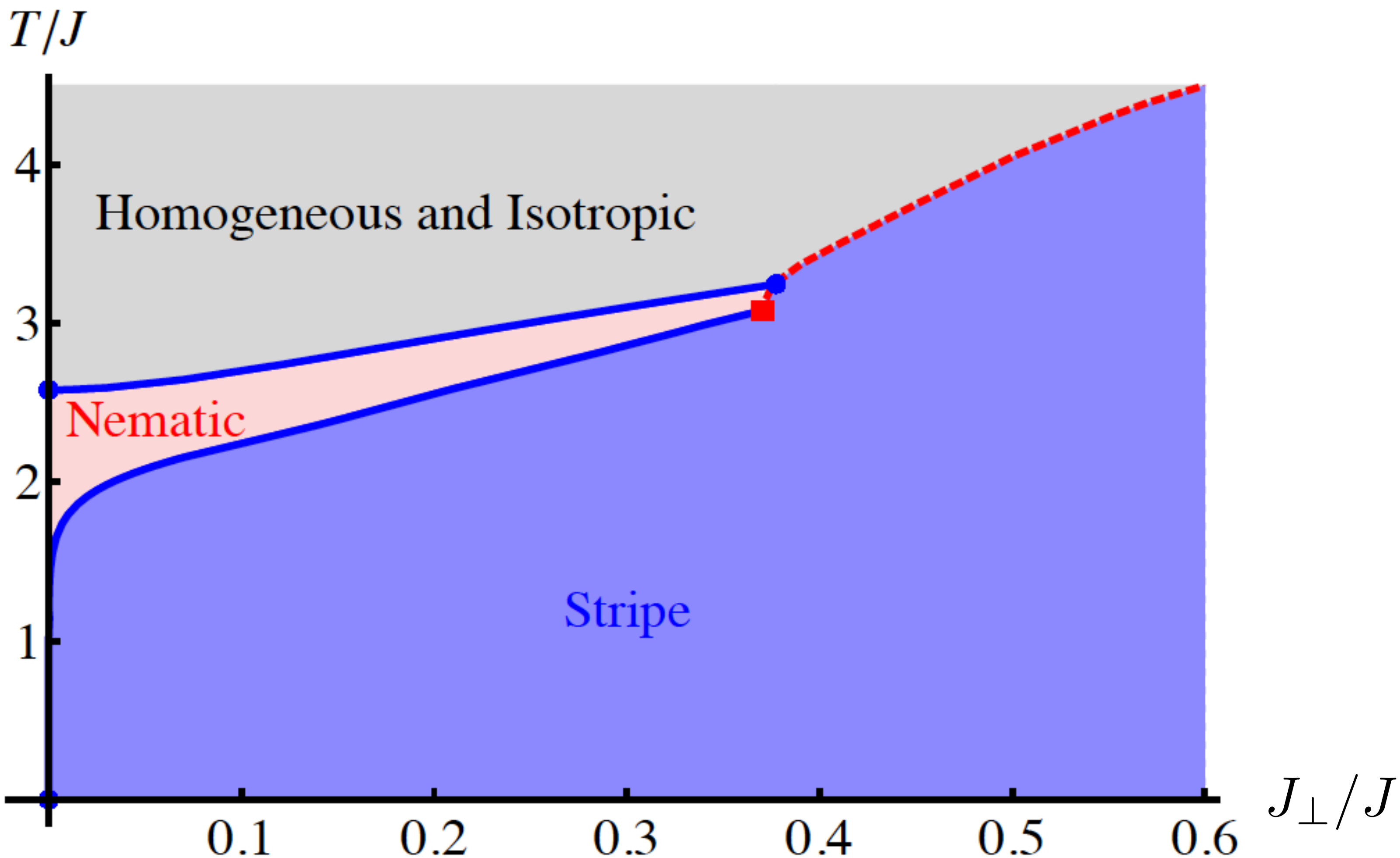
Can promote ψ_a from a $U(1)$ to $O(N)$ field

Exactly solvable in $N \to \infty$ limit

Same solution provides good mean $-$ field description for $N = 2$

"Stripes" for $\gamma > 0$; "checkerboards" for $\gamma < 0$

L. Nie, G. Tarjus, and SAK PNAS (2013)



L. Nie, G. Tarjus, and SAK PNAS (2013)

Electron Nematic Phases

Traceless symmetric order parameter
Relation to spatial symmetries – makes it more like gravity
than like internal symmetries
Nematic as a (vestigial) composite order parameter

$$e.g.\ \ \mathcal{N}_{ab} \propto \frac{1}{2}[\epsilon_{ab} + \epsilon_{ba}] - \frac{\delta_{ab}}{D}\ \epsilon$$

$$\epsilon = \sum_a \epsilon_{aa}$$

(Quadrapolar order)

Ising nematic in a tetragonal crystal: $\mathcal{N} = \frac{[\epsilon_{xx} - \epsilon_{yy}]}{[\epsilon_{xx} + \epsilon_{yy}]}$

Electron Nematic Phases

Traceless symmetric order parameter
Relation to spatial symmetries – makes it more like gravity
than like internal symmetries
Nematic as a (vestigial) composite order parameter

Examples of "interesting" systems for which compelling evidence of electron nematic phases has been discovered "recently:

Nematic Quantum Hall Metal and Nematic Quantized Hall Fluid

Field induced nematic phase in $Sr_3Ru_2O_7$

Nematic phases in Fe-based superconductors, e.g. $BaFe_{2-x}Co_xAs_2$

and various cuprate high temperature superconductors.

Quadrapolar Ordered phase of $YbRu_2Ge_2$
Colossal magneto-resistive Managanites – e.g. $La_{1-x}Ca_xMnO_3$.

…..

Electron Nematic Phases

Traceless symmetric order parameter
Relation to spatial symmetries – makes it more like gravity
than like internal symmetries
Nematic as a (vestigial) composite order parameter

Examples of "interesting" systems for which compelling evidence of electron nematic phases has been discovered "recently:

Nematic Quantum Hall Metal and Nematic Quantized Hall Fluid

Field induced nematic phase in $Sr_3Ru_2O_7$

Nematic phases in Fe-based superconductors, e.g. $BaFe_{2-x}Co_xAs_2$

and various cuprate high temperature superconductors.

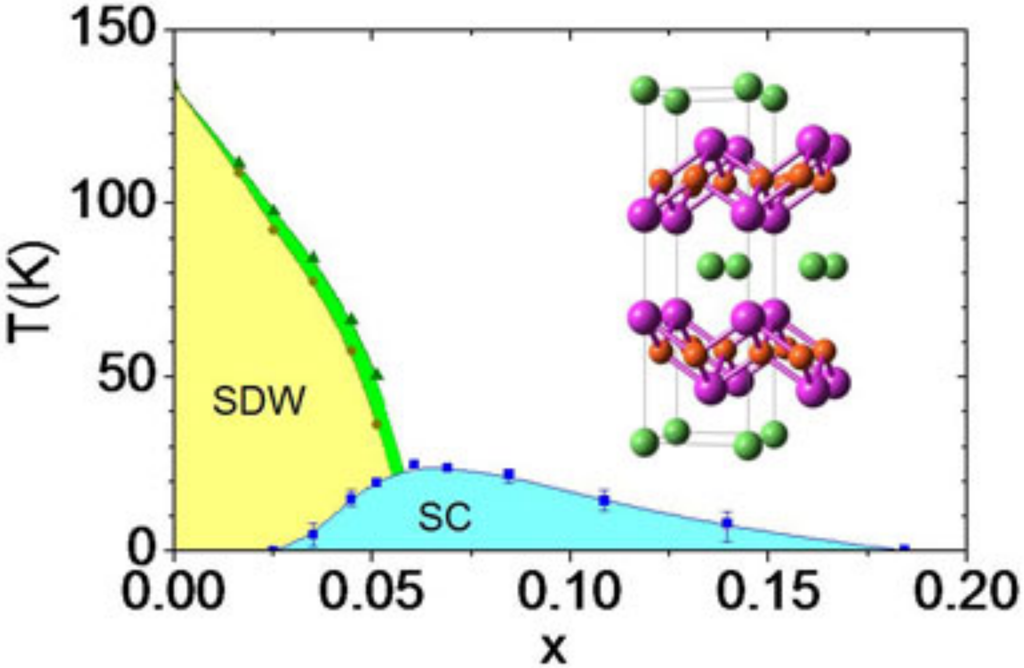
Quadrapolar Ordered phase of $YbRu_2Ge_2$

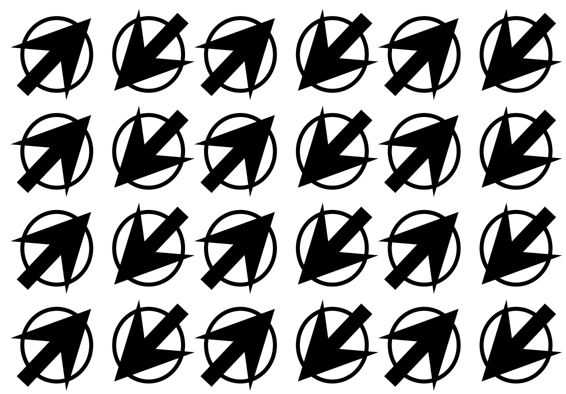
Colossal magneto-resistive Managanites – e.g. $La_{1-x}Ca_xMnO_3$.

.....

Let's take a quick look at the Fe-based superconductors:

$BaFe_{2-x}Co_xAs_2$ studies by I.R. Fisher group

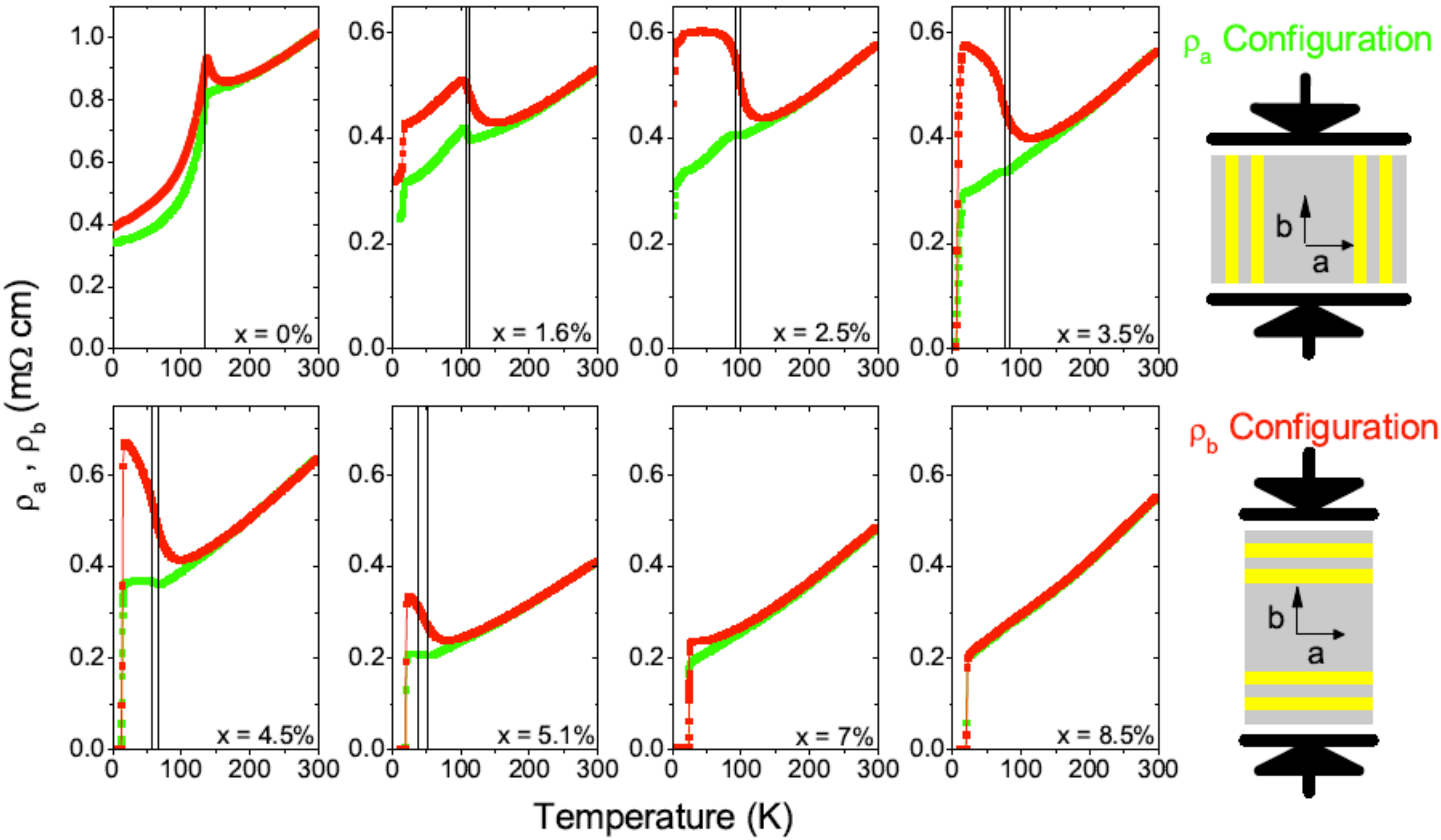




The AF "striped" phase breaks the C_4 symmetry of the host crystal

The orthorhombic transition sometimes occurs simultaneously and sometimes slightly before the magnetic transition.

It is possible to measure an "electron nematic" order parameter



Resistive anisotropy in crystals of Fe-pnictide superconductor
J-H. Chu *et al*, Science **329**, 824 (2010)

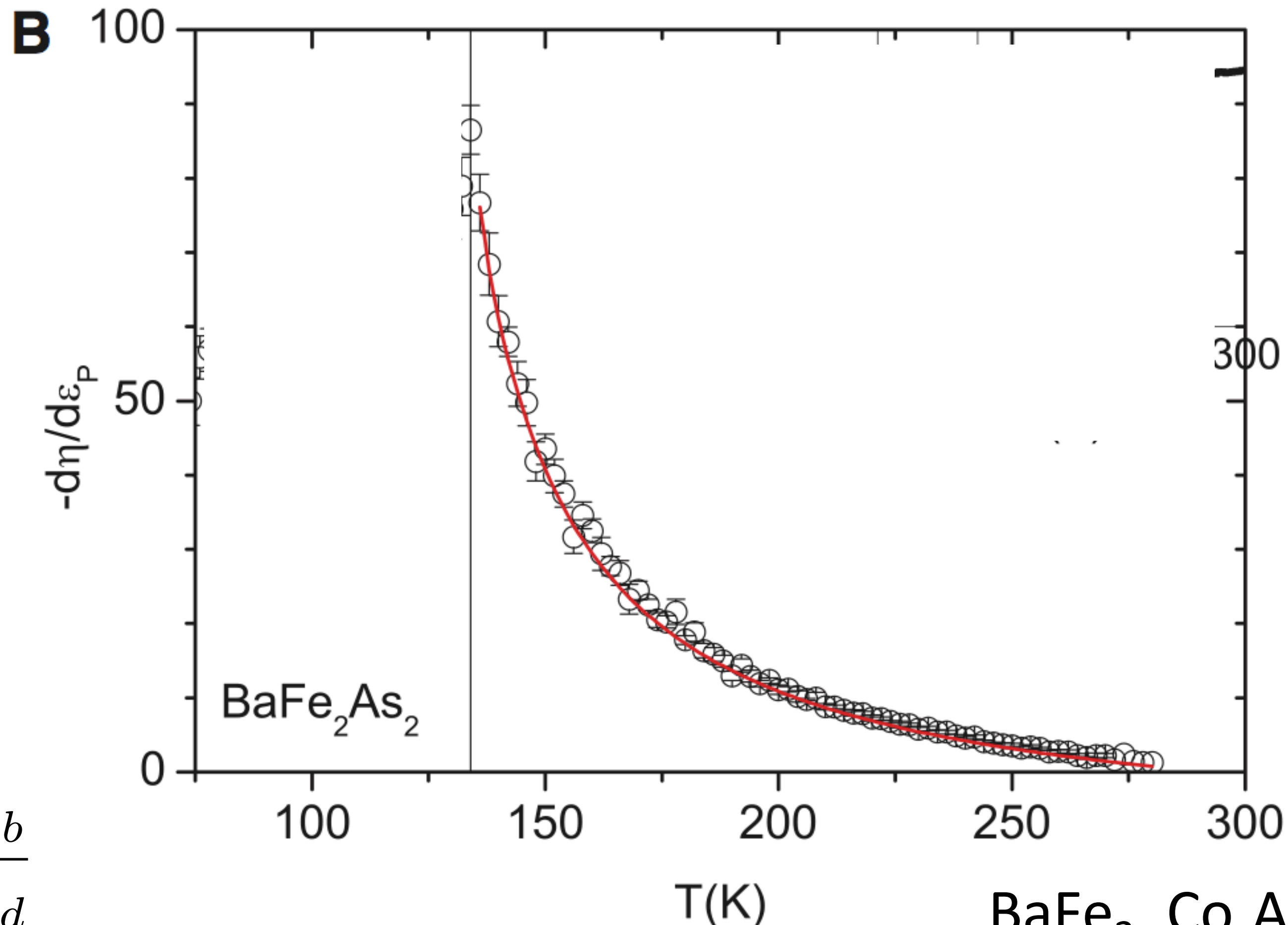
$$\eta = (\rho_{xx} - \rho_{yy})/(\rho_{xx}+\rho_{yy})$$

$$\varepsilon = \text{strain} = u_{xx} - u_{yy}$$

$$d\eta/d\varepsilon = \text{nematic susceptibility}$$

J.W. Chu *et al*, Science **337**, 710 (2012)

One of several new probes of electron nematicity



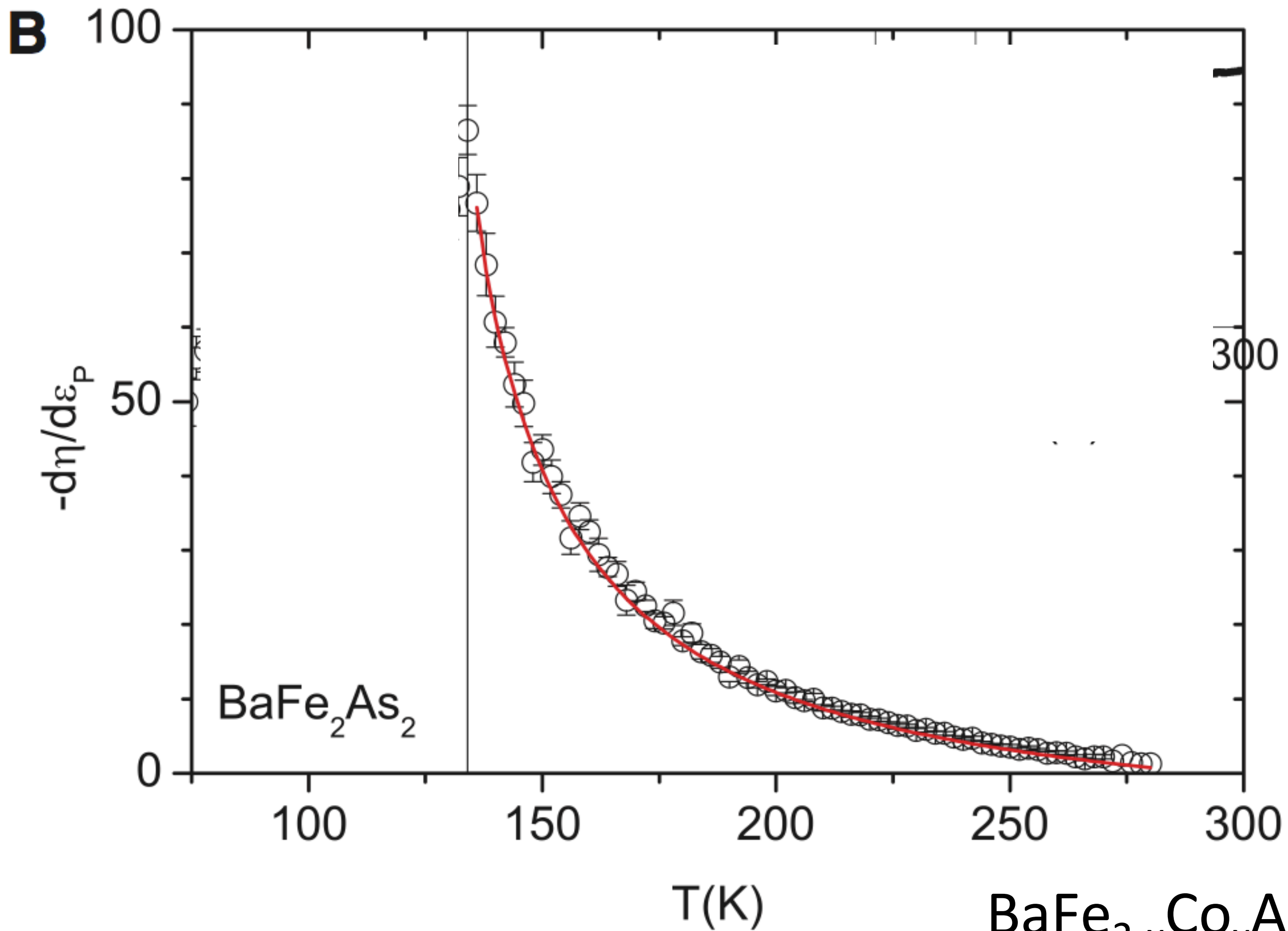
$BaFe_{2-x}Co_xAs_2$

$$R_{abcd} = \frac{d\rho_{ab}}{du_{cd}}$$

Resistoelastic tensor

$$\text{Currie Weiss fit :} \quad -\frac{d\eta}{d\varepsilon} = \frac{\chi_0}{T - T^\star}$$

J.W. Chu *et al*, Science **337**, 710 (2012)



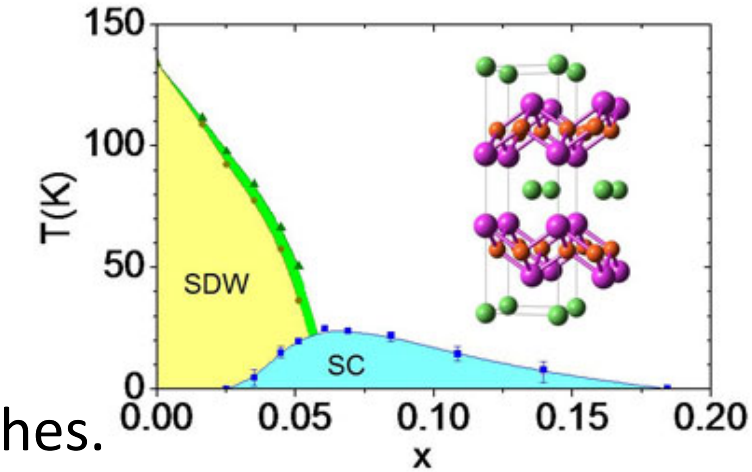
$BaFe_{2-x}Co_xAs_2$

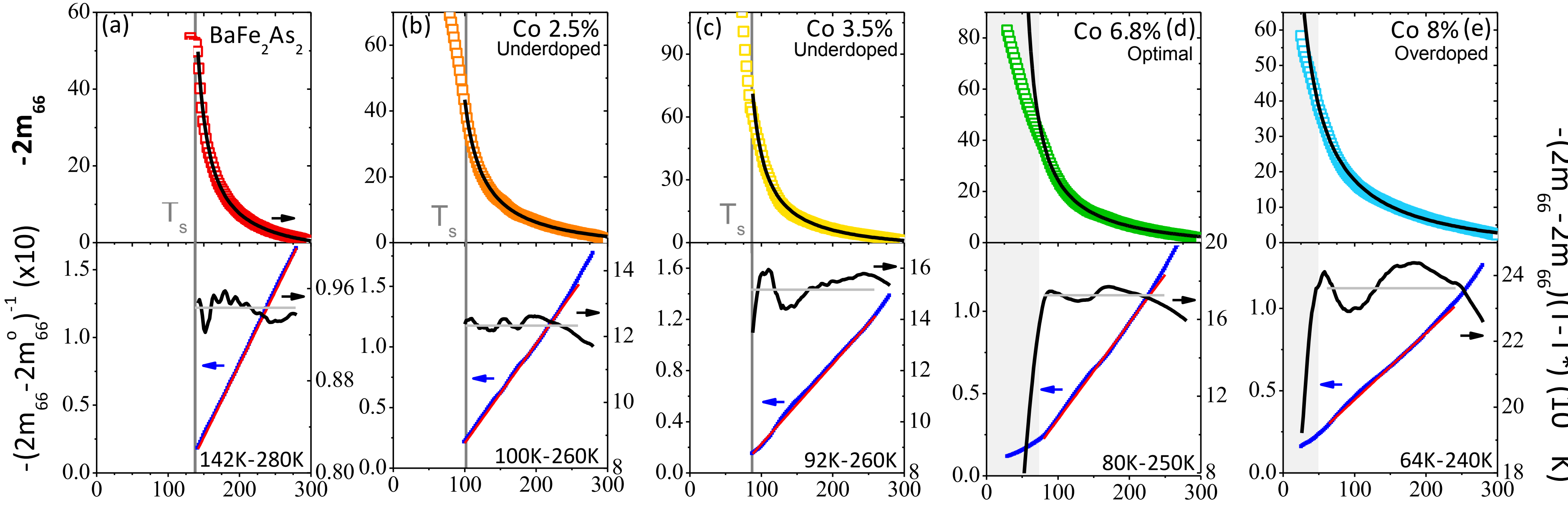
How generic is this behavior?

$$\text{Currie Weiss fit : } \quad -\frac{d\eta}{d\varepsilon} = \frac{\chi_0}{T - T^\star}$$

T^* is only ~ 10% below T_s for small x

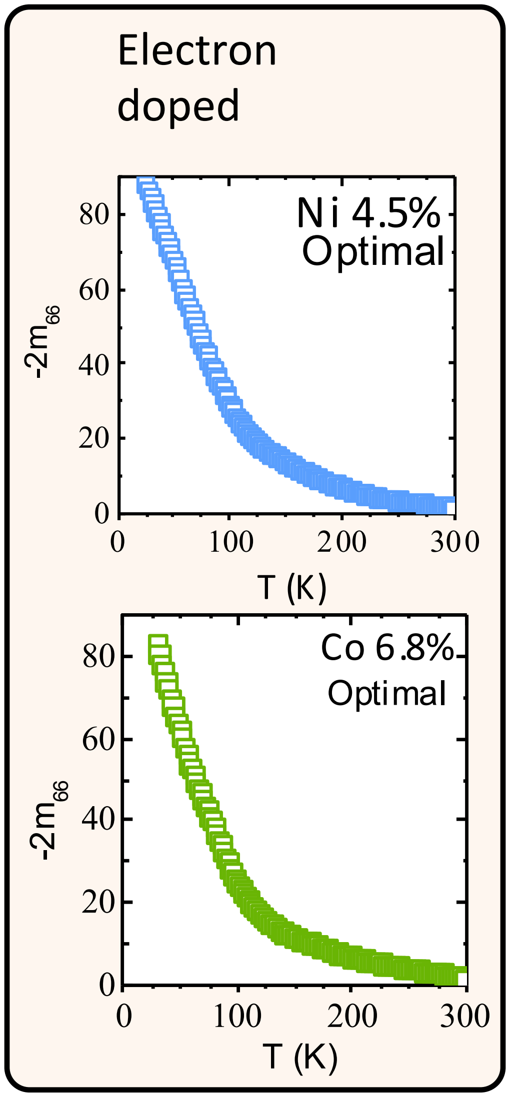
Optimal superconducting T_c occurs at roughly same x at which T^* vanishes.

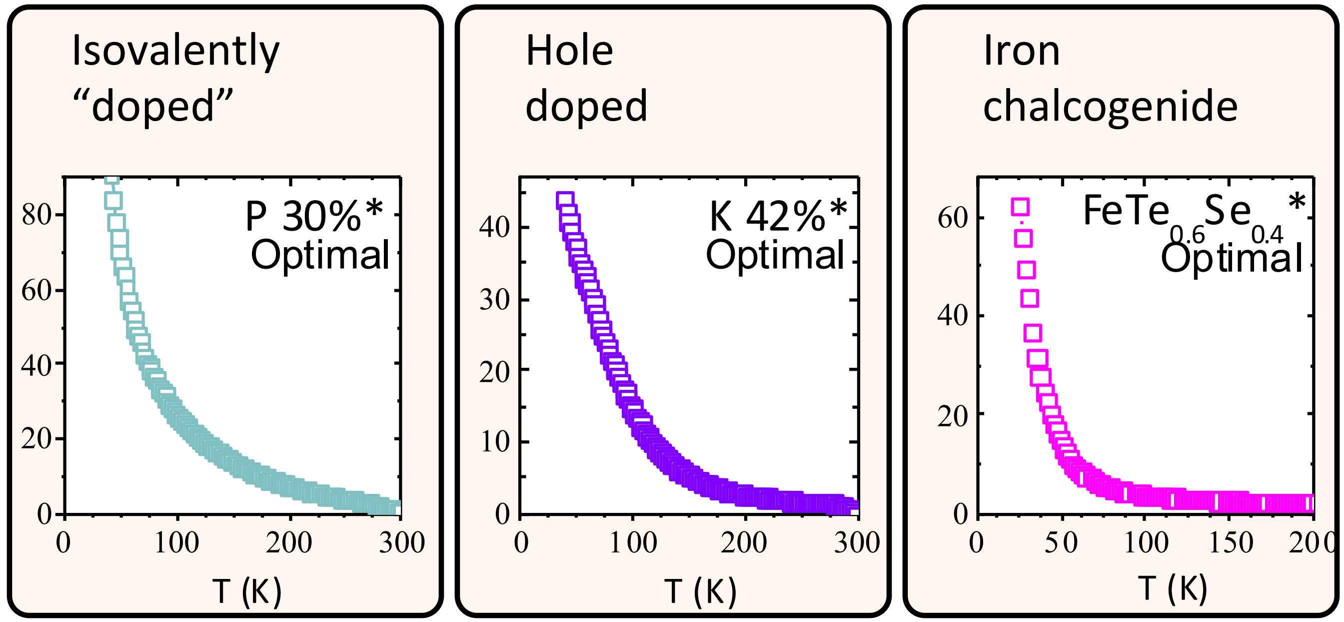




H-H. Kuo, J-H. Chu, S.A. K., I.R. Fisher, arXiv:1503.00402

How generic is this behavior?





All data taken above T_c –

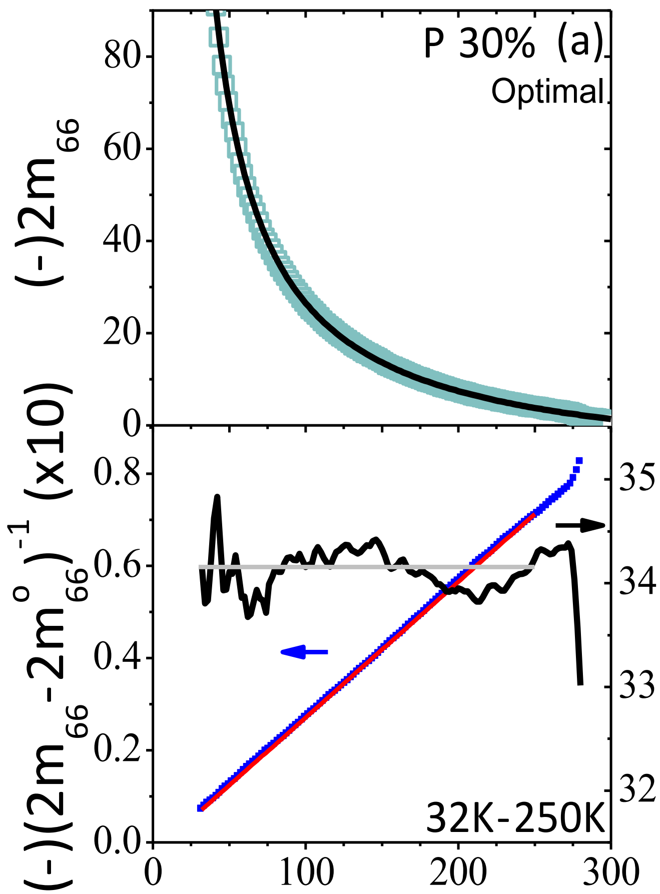
but fits to a Currie-Weiss law with T*
which vanishes as a function of x approximately
at optimal doping.

H-H. Kuo, J-H. Chu, S.A. K., I.R. Fisher, arXiv:1503.00402

Curie-Weiss fits

$\chi \sim A/T$

$A \sim 90\ T_c$



H-H. Kuo, J-H. Chu, S.A. K., I.R. Fisher, arXiv:1503.00402

Relation between nematic QCP and optimal T_c in Fe-based SCs

- Nematic susceptibility is universally large in neighborhood of optimal doping
- The extrapolated nematic critical temperature, $T^*=T_{nem}$, is universally well below T_c for optimally doped materials.
- This is highly suggestive of a causal relation.

Electron Nematic Phases

Traceless symmetric order parameter
Relation to spatial symmetries – makes it more like gravity than like internal symmetries
Nematic as a (vestigial) composite order parameter

Examples of "interesting" systems for which compelling evidence of electron nematic phases has been discovered "recently:

Nematic Quantum Hall Metal and Nematic Quantized Hall Fluid

Field induced nematic phase in $Sr_3Ru_2O_7$

Nematic phases in Fe-based superconductors, e.g. $BaFe_{2-x}Co_xAs_2$

and **various cuprate high temperature superconductors.**

Hidden Order phase of URu_2Si_2

Colossal magneto-resistive Managanites – e.g. $La_{1-x}Ca_xMnO_3$.

…..

Effect of Quenched Randomness

The random field problem is relevant to problems involving the breaking of pure spatial symmetries.

"Stripes," i.e. incommensurate charge density wave (CDW) order was discovered in one family of cuprate HTC in 1994 by Tranquada et al. (also "bidirectional")

Short-range CDW order is now widely observed in hole-doped cuprates

Incommensurate CDW Order

$$\rho(\vec{r}) = \bar{\rho} + \left[\psi_x(\vec{r})e^{i\vec{Q}_x\cdot r} + \psi_y(\vec{r})e^{i\vec{Q}_y\cdot r} + \text{c.c.}\right] + \dots$$

$$\text{Stripe ordered phase}: |\langle\psi_x\rangle| \neq 0 \text{ or } |\langle\psi_y\rangle| \neq 0$$

$$\text{so } \langle\left[|\psi_x|^2 - |\psi_y|^2\right]\rangle \neq 0$$

$$\text{Checkerboard ordered phase}: |\langle\psi_x\rangle| = |\langle\psi_y\rangle| \neq 0$$

$$\text{with } \langle\left[|\psi_x|^2 - |\psi_y|^2\right]\rangle = 0$$

$$\text{Nematic phase}: |\langle\psi_x\rangle| = 0 \text{ and } |\langle\psi_y\rangle| = 0$$

$$\text{but } \langle\left[|\psi_x|^2 - |\psi_y|^2\right]\rangle \equiv \mathcal{N} \neq 0$$

L. Nie, G. Tarjus, and SAK (2013)

Effect of Quenched Randomness

Random Field problem in Statistical Mechanics

$$H[\mathbf{S}] = H_0[\mathbf{S}] + \sum_j \mathbf{h}_j \cdot \mathbf{S}_j$$

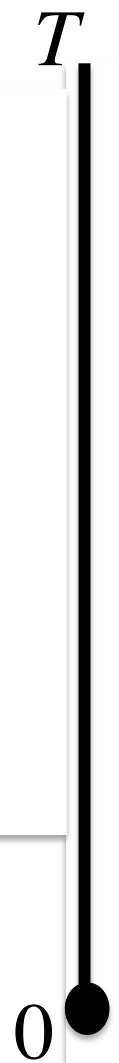
$$\overline{\mathbf{h}_j} = 0 \qquad \overline{\mathbf{h}_j \mathbf{h}_i} = \sigma^2 \delta_{ij}$$

D=2 is lower critical dimension for Ising (Z_2) model
(and presumably other models with discrete
broken symmetries)

D=4 is the lower critical dimension for Heisenberg (SO(3))
model and most models with continuous broken symmetries.

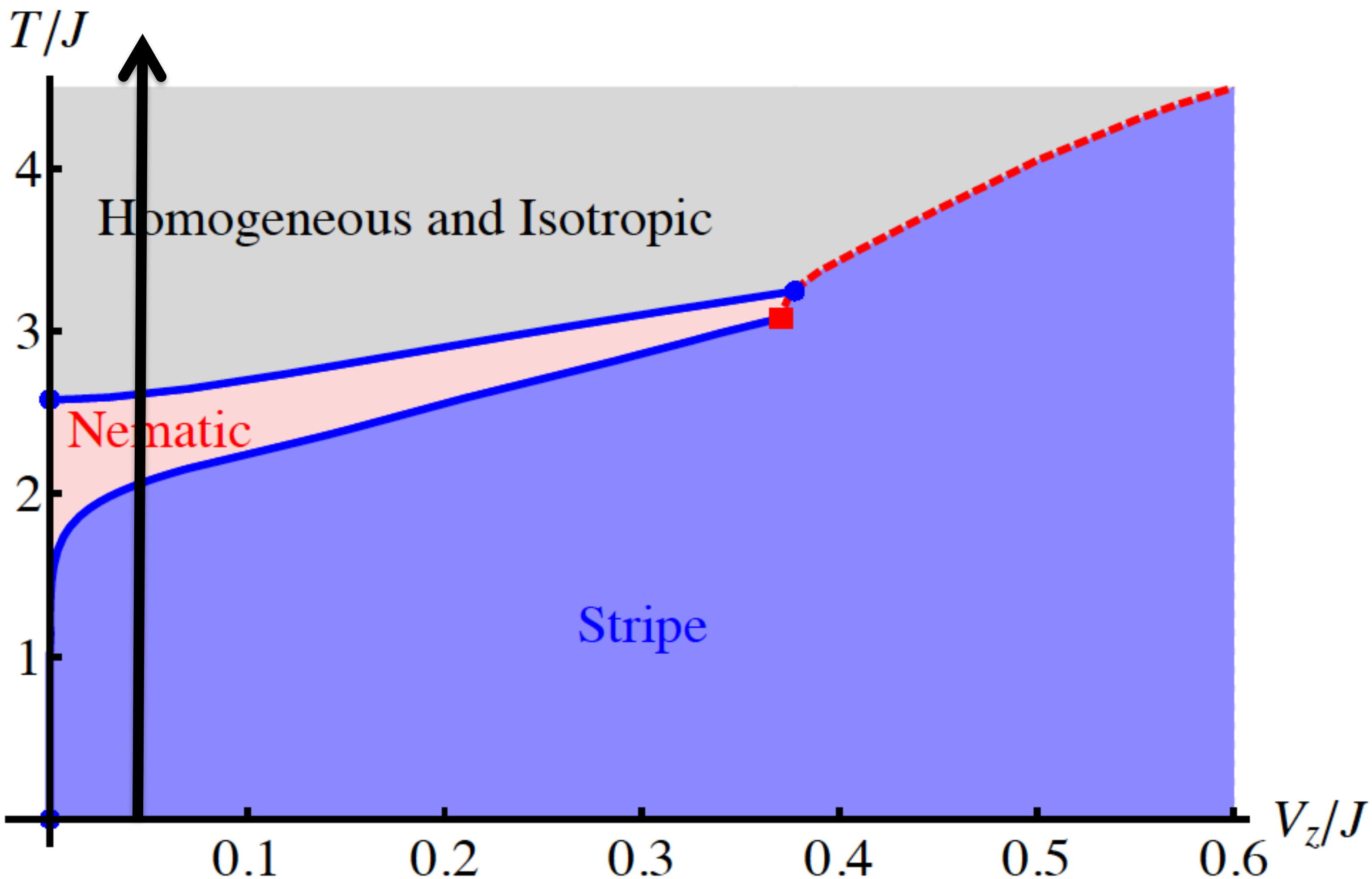
(There is the possible subtlety of a "Bragg glass" phase
for D=3 and XY (SO(2)) symmetry.)

Incommensurate Stripe Order



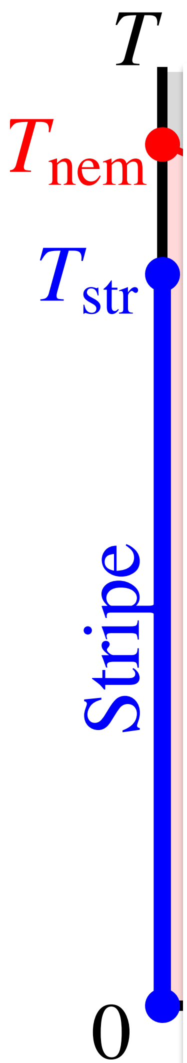
Assume a stripe-ordered ground-state and $V_z << J$

L. Nie, G. Tarjus, and SAK (2013)



T/J
4
3
2
1
Homogeneous and Isotropic
Nematic
Stripe
0.1
0.2
0.3
0.4
0.5
0.6
V_z/J

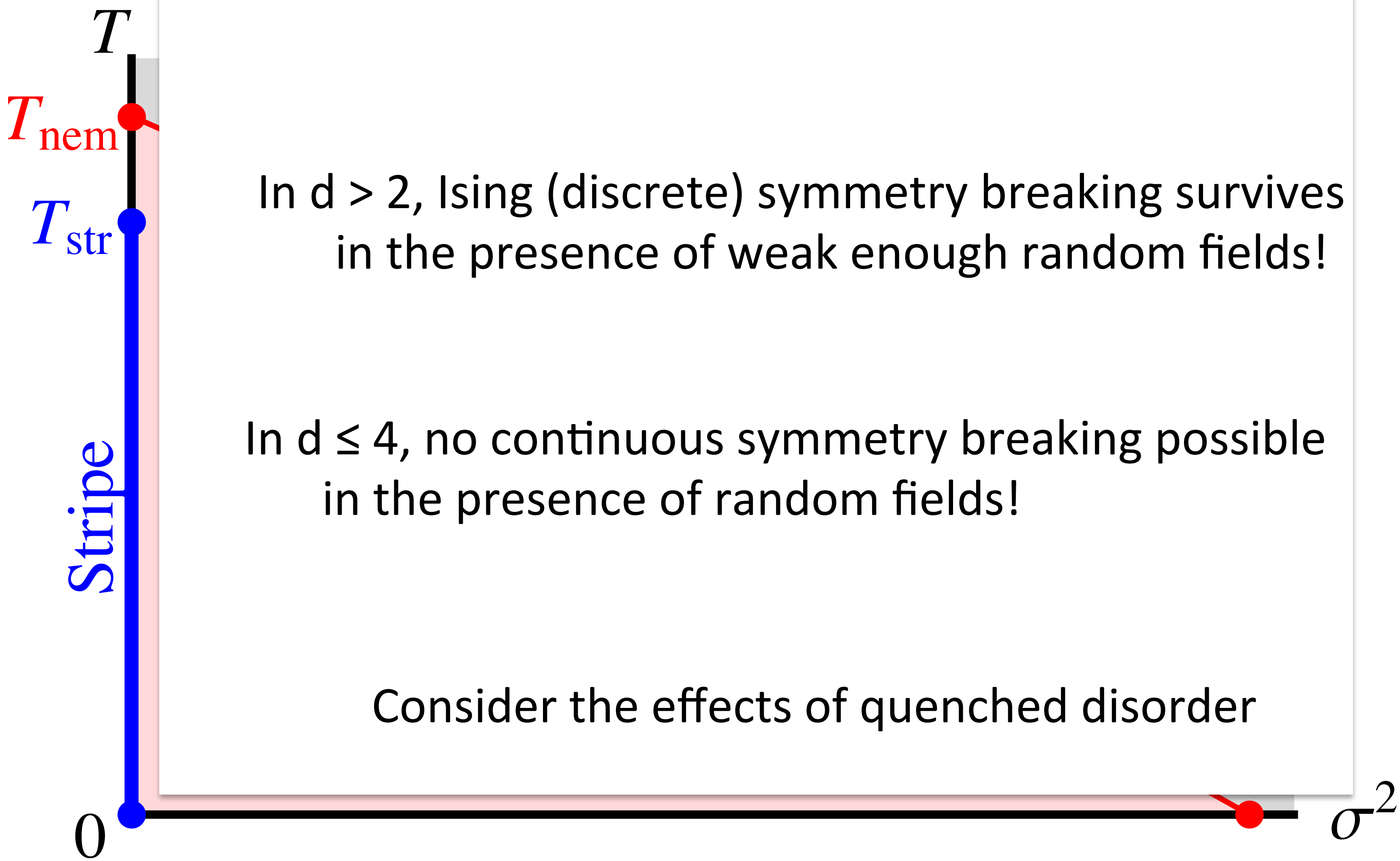
"Vestigial" Nematic Order



Assume a stripe-ordered ground-state and V_z << J

L. Nie, G. Tarjus, and SAK (2013)

"Vestigial" Nematic Order



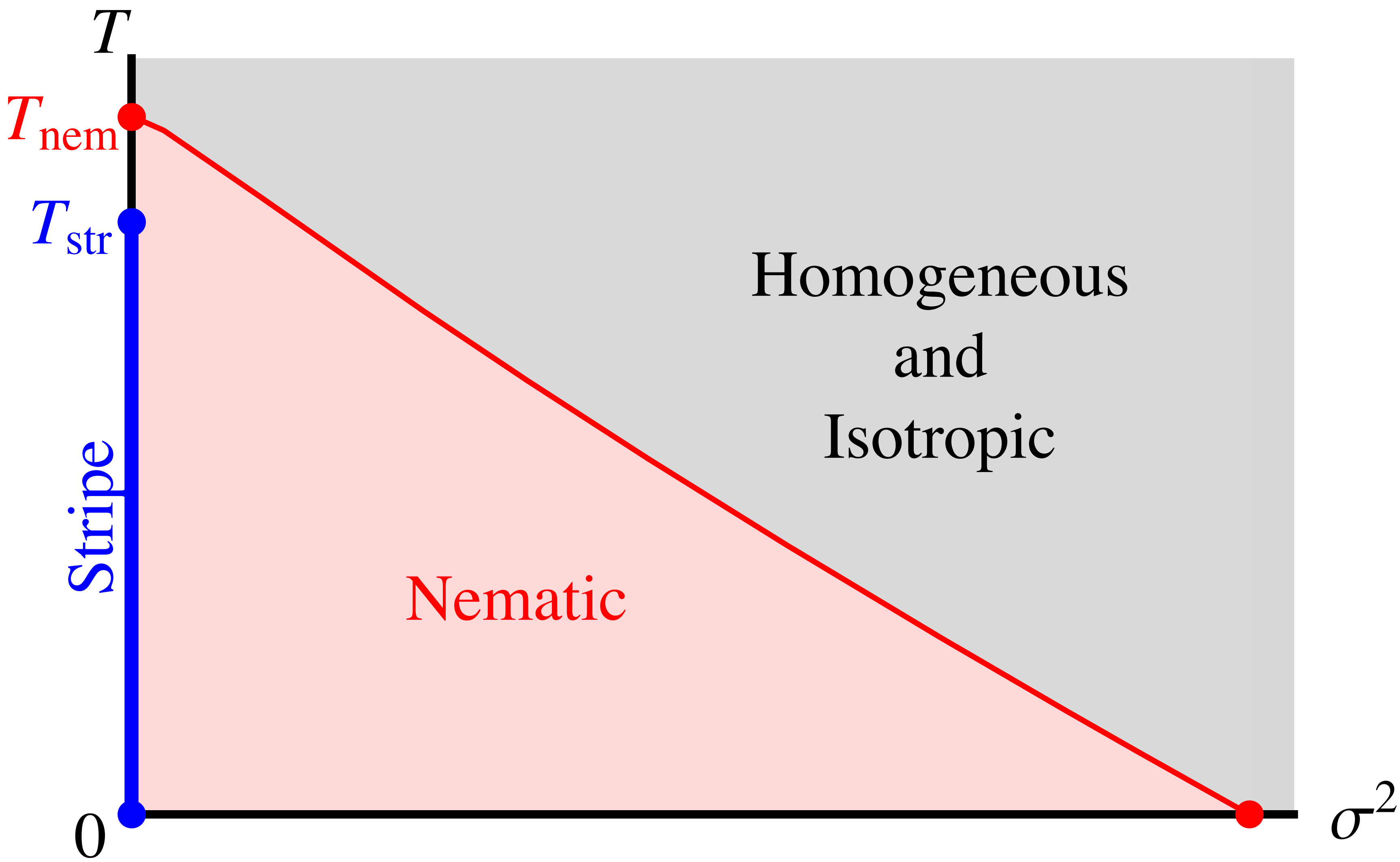
In d > 2, Ising (discrete) symmetry breaking survives in the presence of weak enough random fields!

In d ≤ 4, no continuous symmetry breaking possible in the presence of random fields!

Consider the effects of quenched disorder

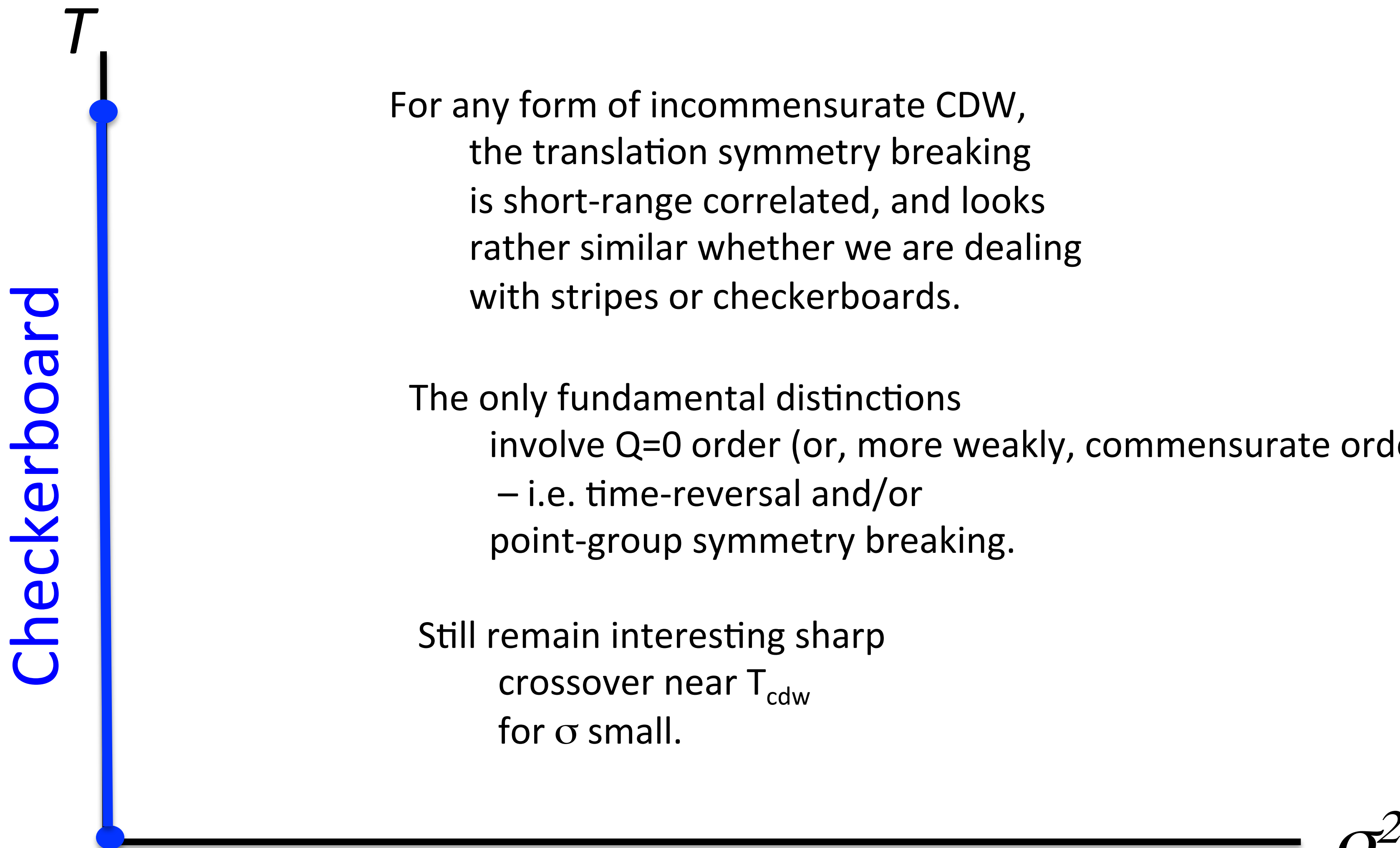
L. Nie, G. Tarjus, and SAK (2013)

"Vestigial" Nematic Order



L. Nie, G. Tarjus, and SAK (2013)

For Checkerboard Order



For any form of incommensurate CDW,
the translation symmetry breaking
is short-range correlated, and looks
rather similar whether we are dealing
with stripes or checkerboards.

The only fundamental distinctions
involve Q=0 order (or, more weakly, commensurate order)
– i.e. time-reversal and/or
point-group symmetry breaking.

Still remain interesting sharp
crossover near T_{cdw}
for σ small.

L. Nie, G. Tarjus, and SAK (2013)

Visualizing vestigial nematicity when a unidirectional (stripe) ordered CDW is disrupted by quenched randomness

Courtesy Jenny Hoffman

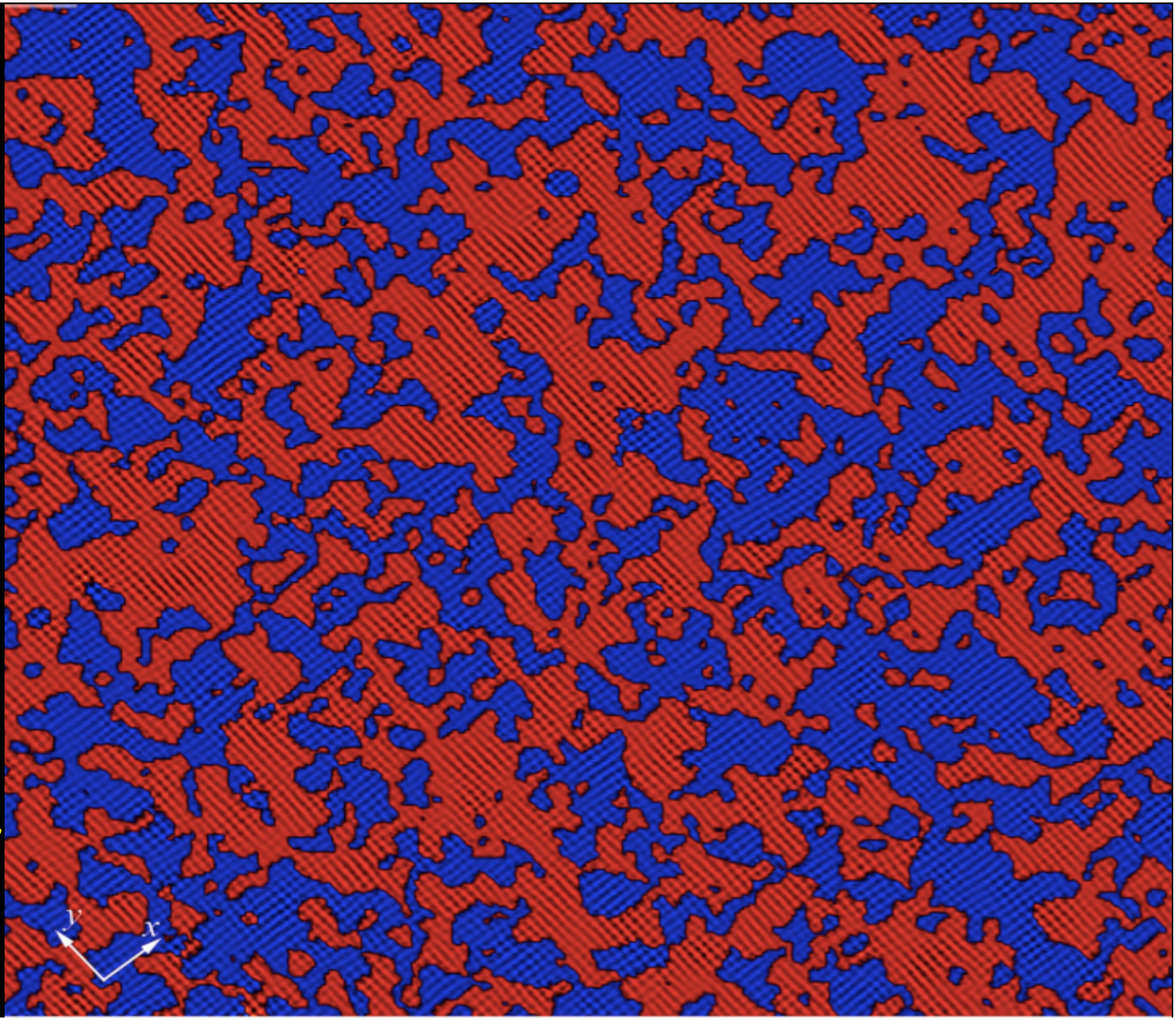
Bi 2201

650 x 650 A

Z –maps Fourier filtered data near $Q^{**}=(3/4,0)$ and (0,3/4)

Patterns correspond to 2d surface of 3d RFIM near σ_c

Courtesy J.E. Hoffman, E. Hudson, C-L Song E.W.Carlson and co.



A few things that are particularly interesting about electron nematic order

Nematic QCP is possible even in the presence of quenched disorder

Nematic fluctuations enhance superconductivity in any channel.

Quantum critical nematic fluctuations destroy FL on entire FS.

and, absent crystal field effects, in the entire nematic phase

S. Lederer, Y.Schattner, E. Berg, and SAK, arXiv:1406.1193
T. A. Maier and D. J. Scalapino, arXiv:1405.5238

V. Oganesyan, SAK, and E. Fradkin, PRB **64**, 195109 (2001)

H. Watanabe and A. Vishwanath, PNAS accepted.