DQMP Department of Quantum Matter Physics

# 1D Hubbard model

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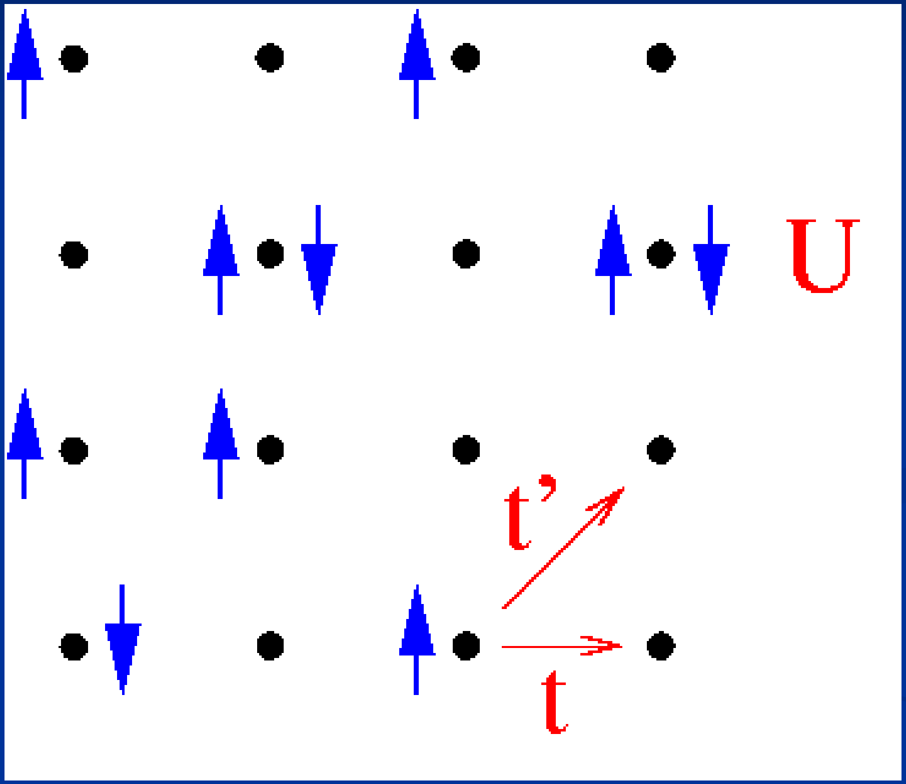
(S. Greshner, C. Berthod, P. Ruggiero, F. Hartmeier, G. Morpurgo, T. Jin, N. Caballero, A.M. Visuri, M. Filippone, J. Ferreira, C. Bardyn)

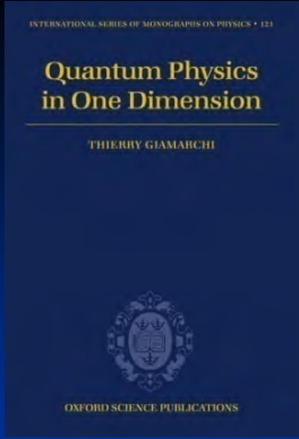
Theory: H.J. Schulz, E. Orignac, R. Citro, B.S. Shastry, A. Georges, S. Biermann, C. Berthod, C. Kollath, U. Schollwöck, P. Bouillot, A. Kantian, …

# Hubbard model (1963)




$$H = -t \sum_{<i,j>,\sigma} (c^{\dagger}_{i,\sigma} c_{j,\sigma} + h.c.) + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

# References




TG, Quantum physics in one dimension, Oxford (2004)

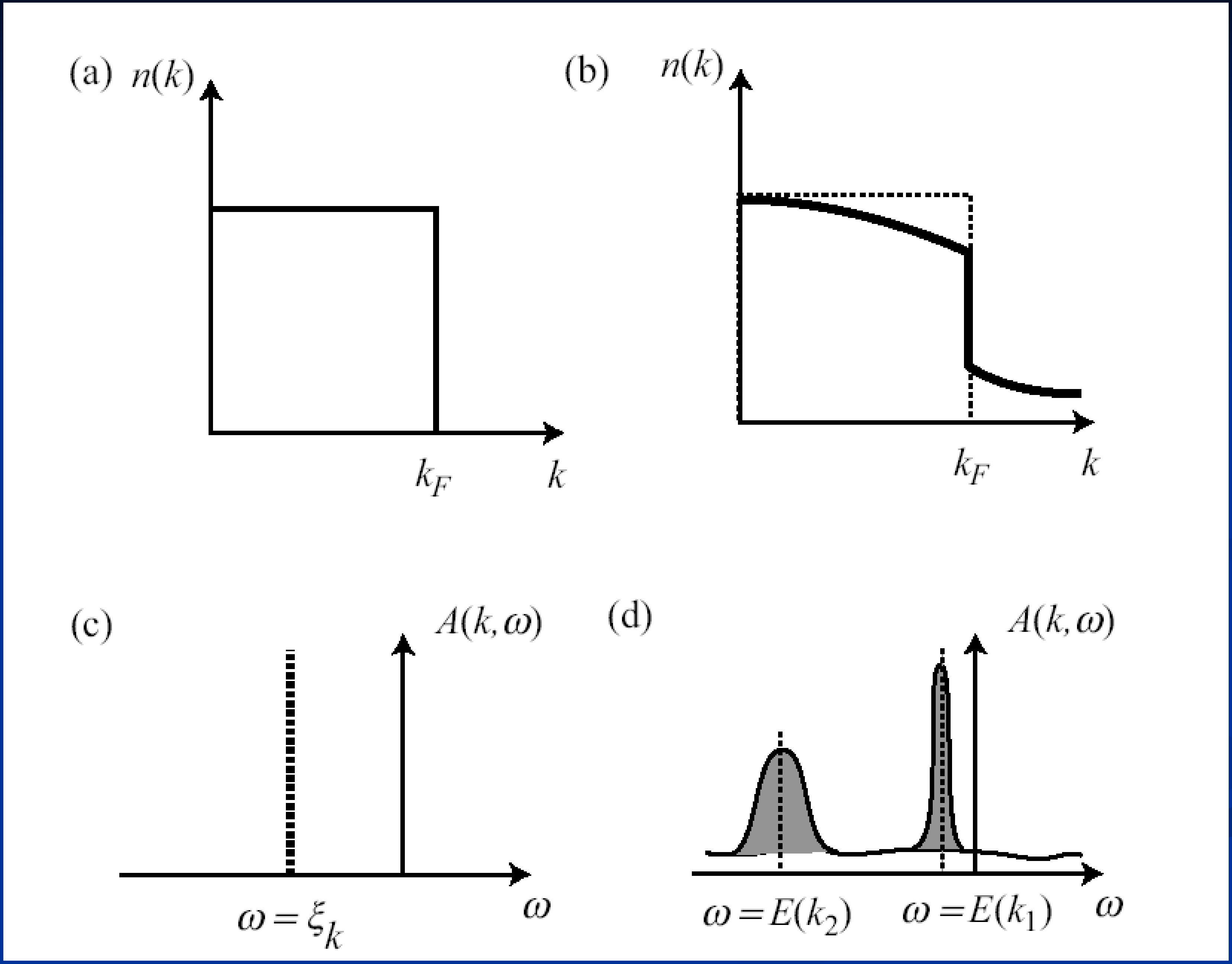
F. Essler, The 1D Hubbard model, Cambridge (2005)

M. Cazalilla et al.,
Rev. Mod. Phys.83 1405 (2011)

TG, Int J. Mod. Phys. B 26 1244004 (2012)

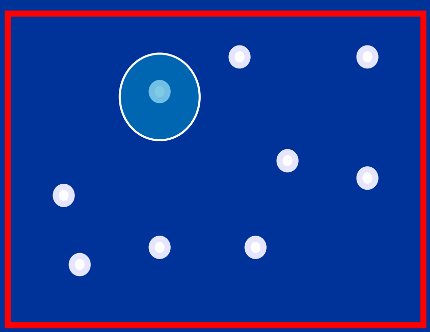
TG, C. R. Acad. Sci. 17 322 (2016)

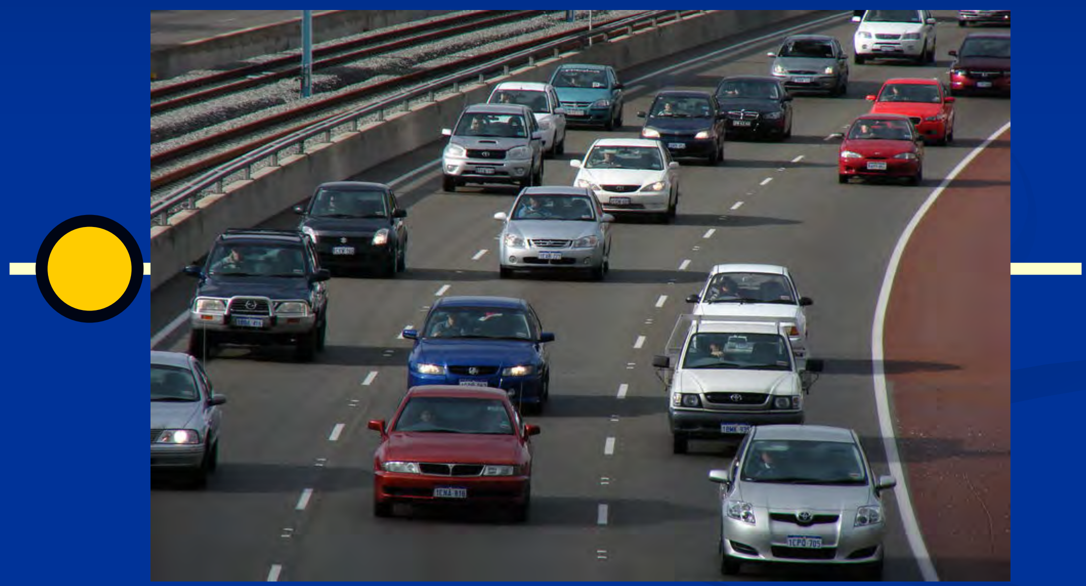
# Free electrons/Fermi liquid


(a)
n(k)
k_F
k
(b)
n(k)
k_F
k
(c)
A(k,ω)
ω = ξ_k
ω
(d)
A(k,ω)
ω = E(k_2)
ω = E(k_1)
ω

# What is special to 1D

- No individual excitation can exist (only collective ones)





- Strong quantum fluctuations

$$\psi = |\psi| e^{i\theta}$$

Difficult to order

# Drastic evolution of the 1d world

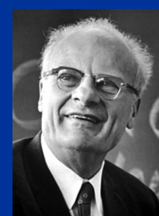
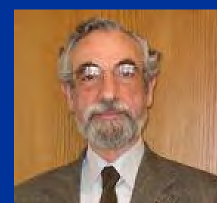
- New methods (DMRG, correlations from BA, etc.)
- New systems (cold atoms, magnetic insulators, etc.)
- New questions (strong SOC, out of equilibrium, etc)

# How to treat ?

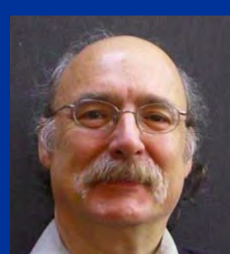
- ``Standard" many body theory

- Exact Solutions (Bethe ansatz)

- Field theories (bosonization, CFT)

- Numerics (DMRG, MC, etc.)

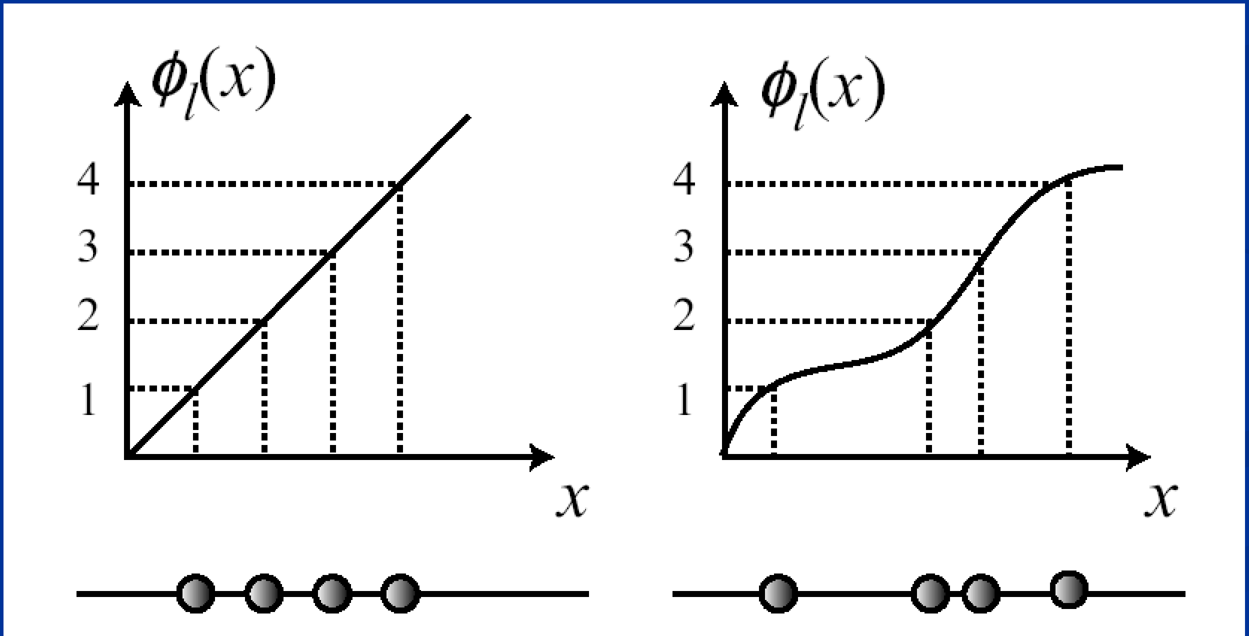
 

# And now we start….

# Labelling the particles

$$\rho(x) = \sum_i \delta(x - x_i)$$
$$= \sum_n |\nabla \phi_l(x)| \delta(\phi_l(x) - 2\pi n)$$

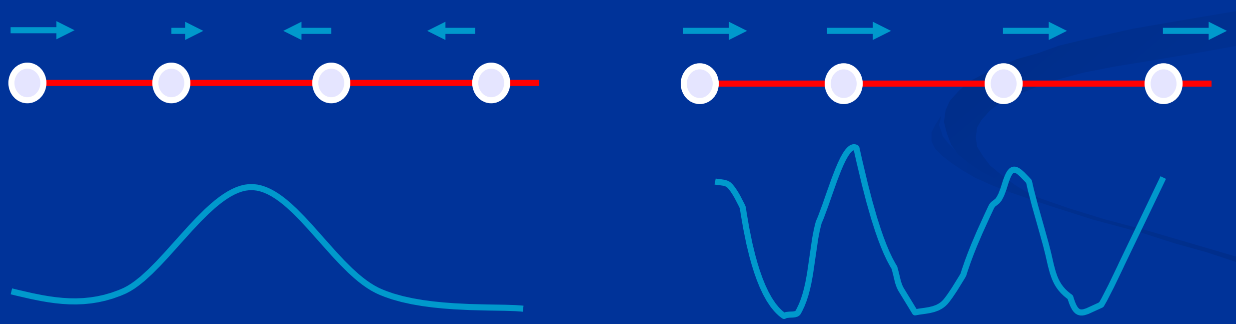
1D: unique way of labelling



$$\phi_l(x) = 2\pi\rho_0 x - 2\phi(x)$$

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right]\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

$\phi(x)$ varies slowly



CDW

$q \sim 0$ $\quad$ $q \sim 2\pi\rho_0$

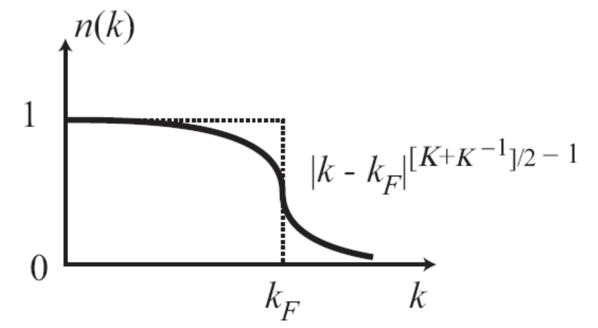
# Fermions

$$\psi_F^\dagger(x) = \psi_B^\dagger(x) e^{i\frac{1}{2}\phi_l(x)}$$

$$\psi_F^\dagger(x) = [\rho_0 - \frac{1}{\pi}\nabla\phi(x)]^{1/2} \sum_p e^{i(2p+1)(\pi\rho_0 x - \phi(x))} e^{-i\theta(x)}$$

## Right ($+k_F$) and left ($-k_F$) particles

$$\langle\rho(x,\tau)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(x^2+y_\alpha^2)^2} + \rho_0^2 A_2 \cos(2\pi\rho_0 x)\left(\frac{\alpha}{r}\right)^{2K} + \rho_0^2 A_4 \cos(4\pi\rho_0 x)\left(\frac{\alpha}{r}\right)^{8K}$$



# Hubbard model

$$H = H_\rho + H_\sigma$$

$$H_\rho = \frac{1}{2\pi}\int dx\,[u_\rho K_\rho(\pi\Pi_\rho)^2 + \frac{u_\rho}{K_\rho}(\nabla\phi_\rho)^2] + g_3\cos(\sqrt{8}\phi_\rho) - \mu\nabla\phi_\rho$$

$$H_\sigma = \frac{1}{2\pi}\int dx[u_\sigma K_\sigma(\pi\Pi_\sigma)^2 + \frac{u_\sigma}{K_\sigma}(\nabla\phi_\sigma)^2] + g_1\cos(\sqrt{8}\phi_\sigma) - h\nabla\phi_\sigma$$

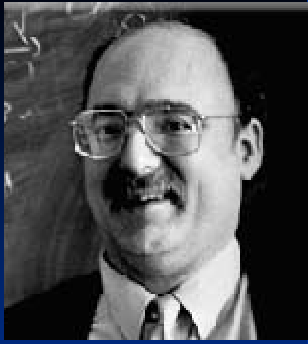
$$u_\rho K_\rho = u_\sigma K_\sigma = v_F$$

$$u_\rho / K_\rho = v_F(1 + \frac{U}{\pi v_F})$$

$$u_\sigma / K_\sigma = v_F(1 - \frac{U}{\pi v_F})$$

$$g_{1\perp} = U$$

# Luttinger liquid concept



- How much is perturbative ?

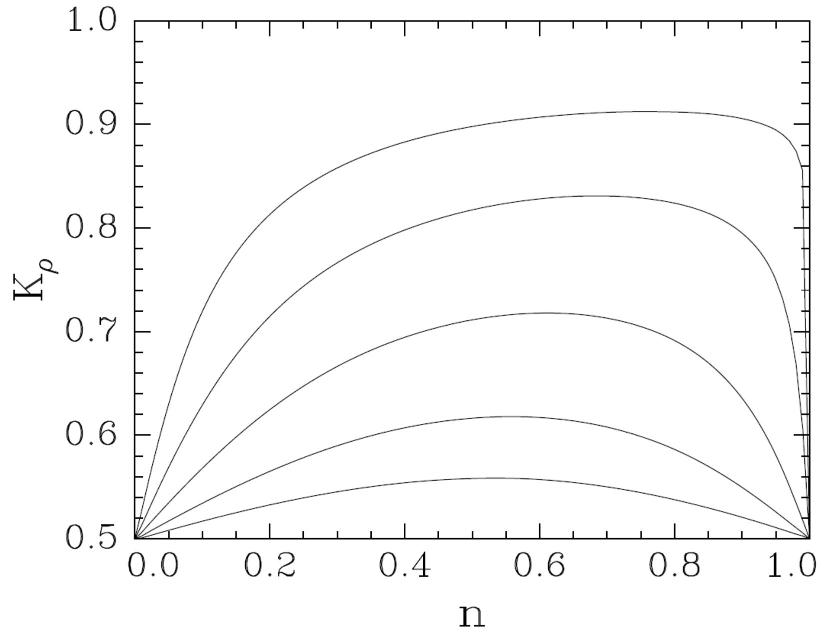
- Nothing (Haldane):
provided the correct u,K are used

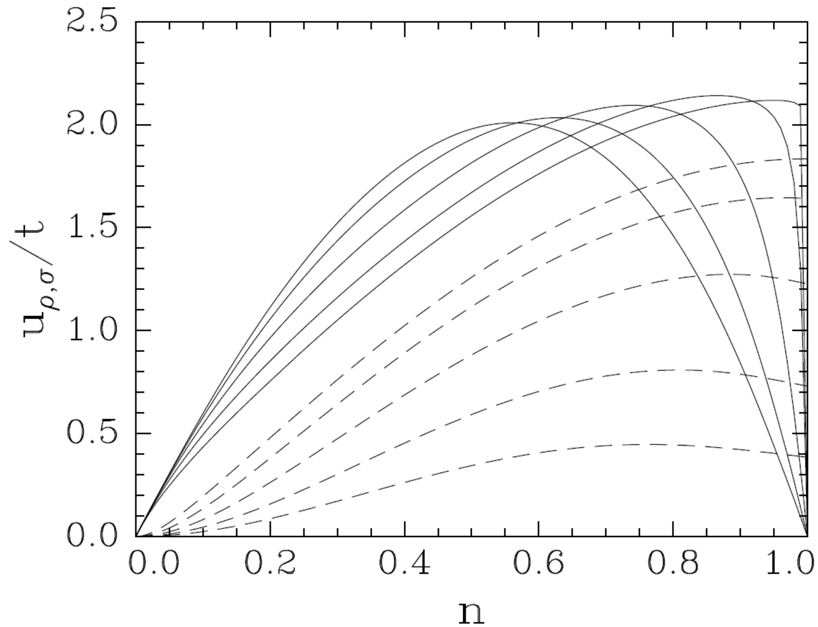
- Low energy properties: Luttinger liquid
(fermions, bosons, spins…)

VOLUME 64, NUMBER 23 PHYSICAL REVIEW LETTERS 4 JUNE 1990

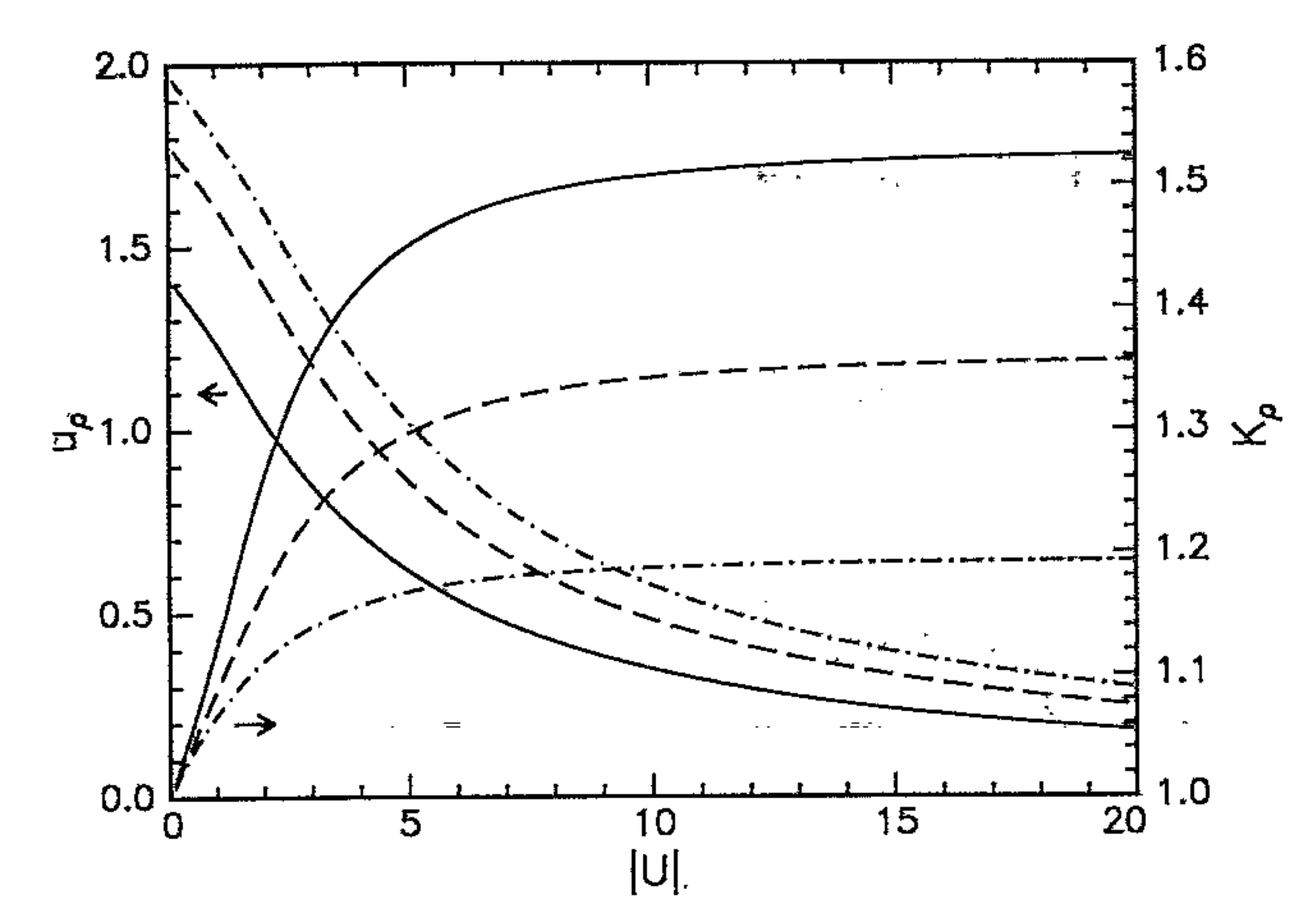
**Correlation Exponents and the Metal-Insulator Transition in the One-Dimensional Hubbard Model**

H. J. Schulz



# Attractive Hubbard model

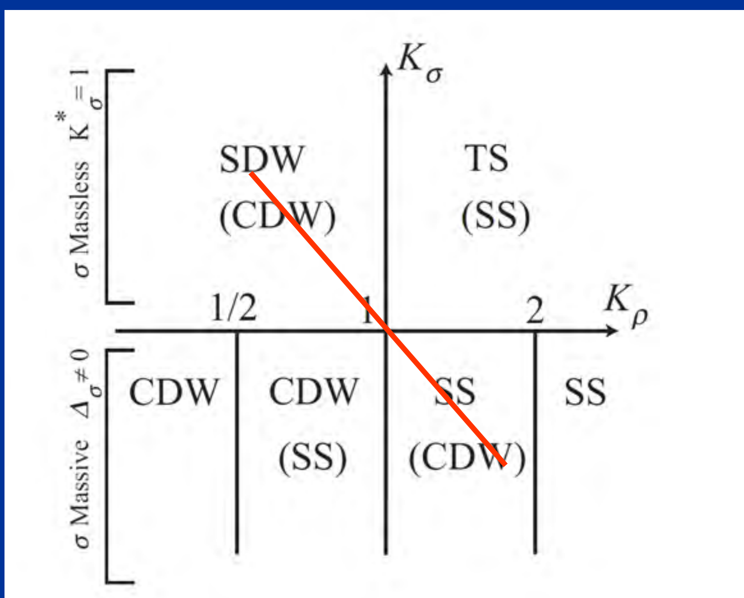



TG + B. S. Shastry PRB 51 10915 (1995)

# Consequences

# Powerlaw correlations

$$\langle \delta\rho(x)\delta\rho(0)\rangle = \frac{K}{\pi^2}\frac{y_\alpha^2 - x^2}{(x^2+y_\alpha^2)^2} + \rho_0^2 A_2 \cos(2\pi\rho_0 x)\left(\frac{\alpha}{r}\right)^{K_\rho+1}\log^{-3/2}(\alpha/r)$$
$$+ \rho_0^2 A_4 \cos(4\pi\rho_0 x)\left(\frac{\alpha}{r}\right)^{4K_\rho} + \cdots$$
$$\langle S_\mu(x,\tau)S_\mu(0)\rangle = \frac{K}{4\pi^2}\frac{y_\alpha^2 - x^2}{(x^2+y_\alpha^2)^2}$$
$$+ A_2' \cos(2\pi\rho_0 x)\left(\frac{\alpha}{r}\right)^{K_\rho+1}\log^{1/2}(\alpha/r) + \cdots \qquad (7.15)$$



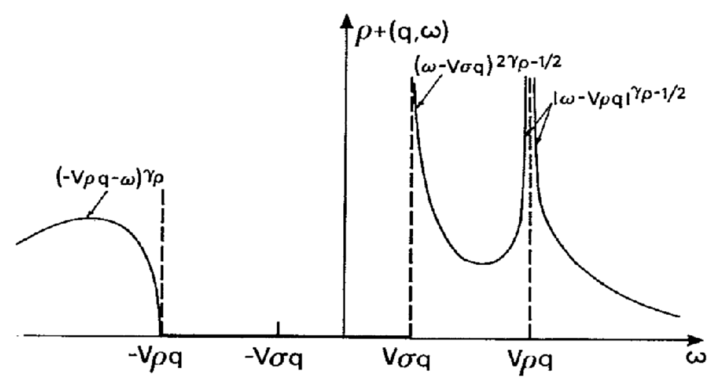
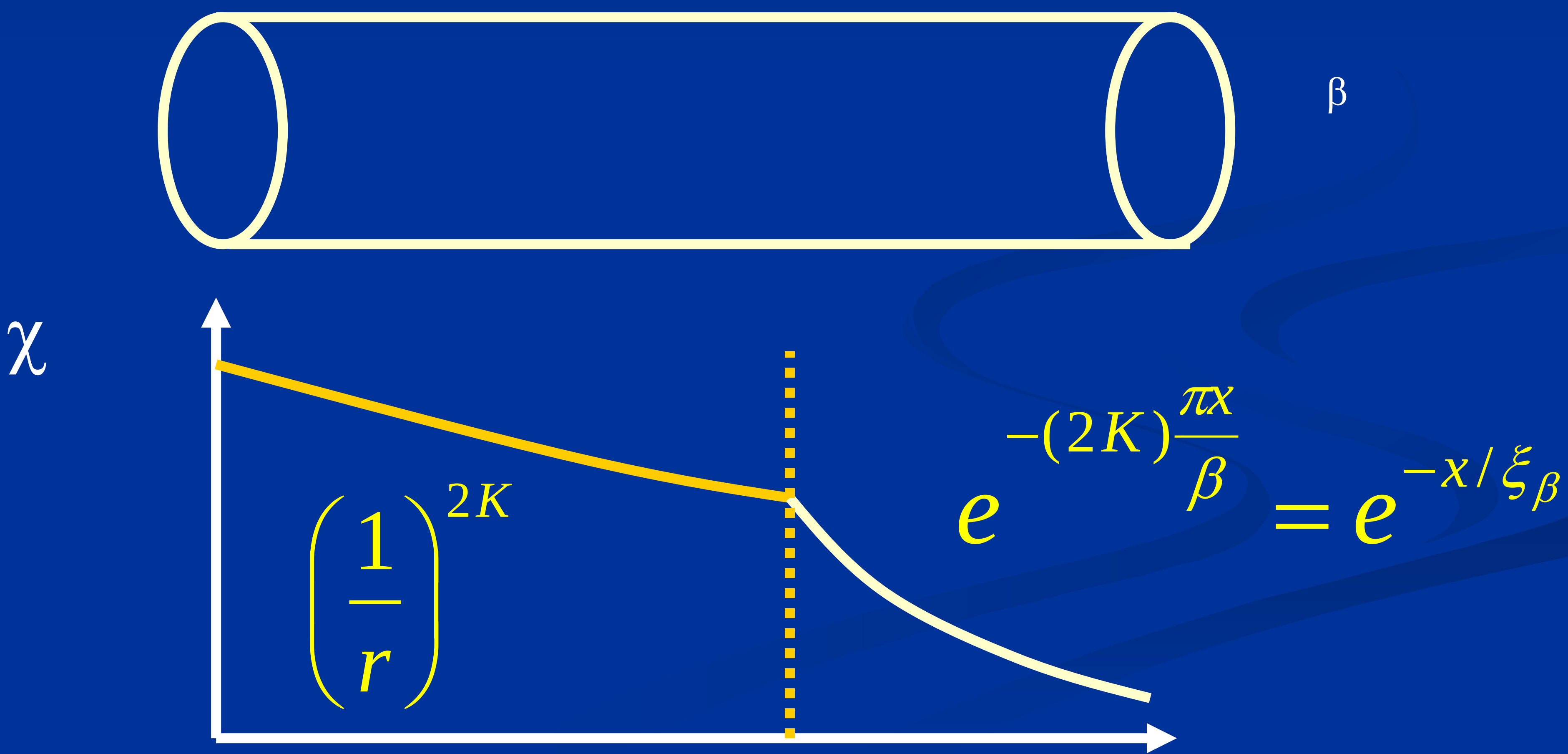




FIG. 3. Spectral function $\rho_+(q,\omega)$ for the spin-$\frac{1}{2}$ Luttinger liquid for $q>0$.

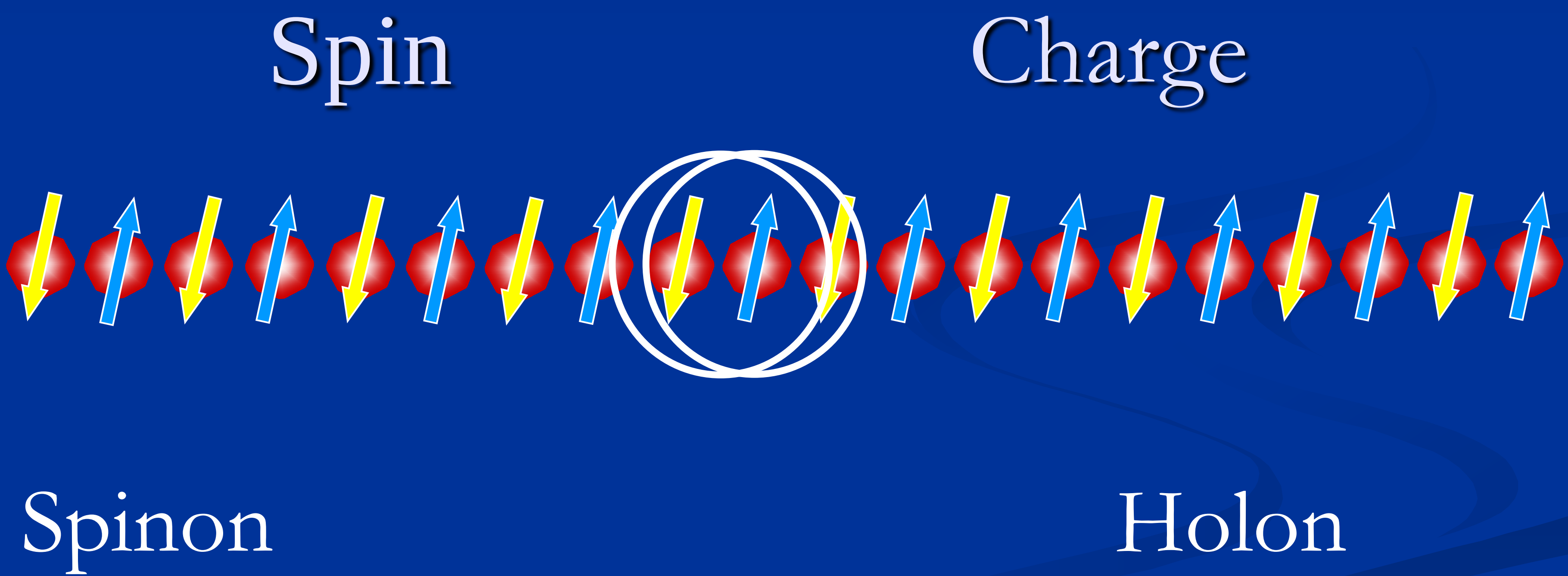
J. Voit

# Finite temperature

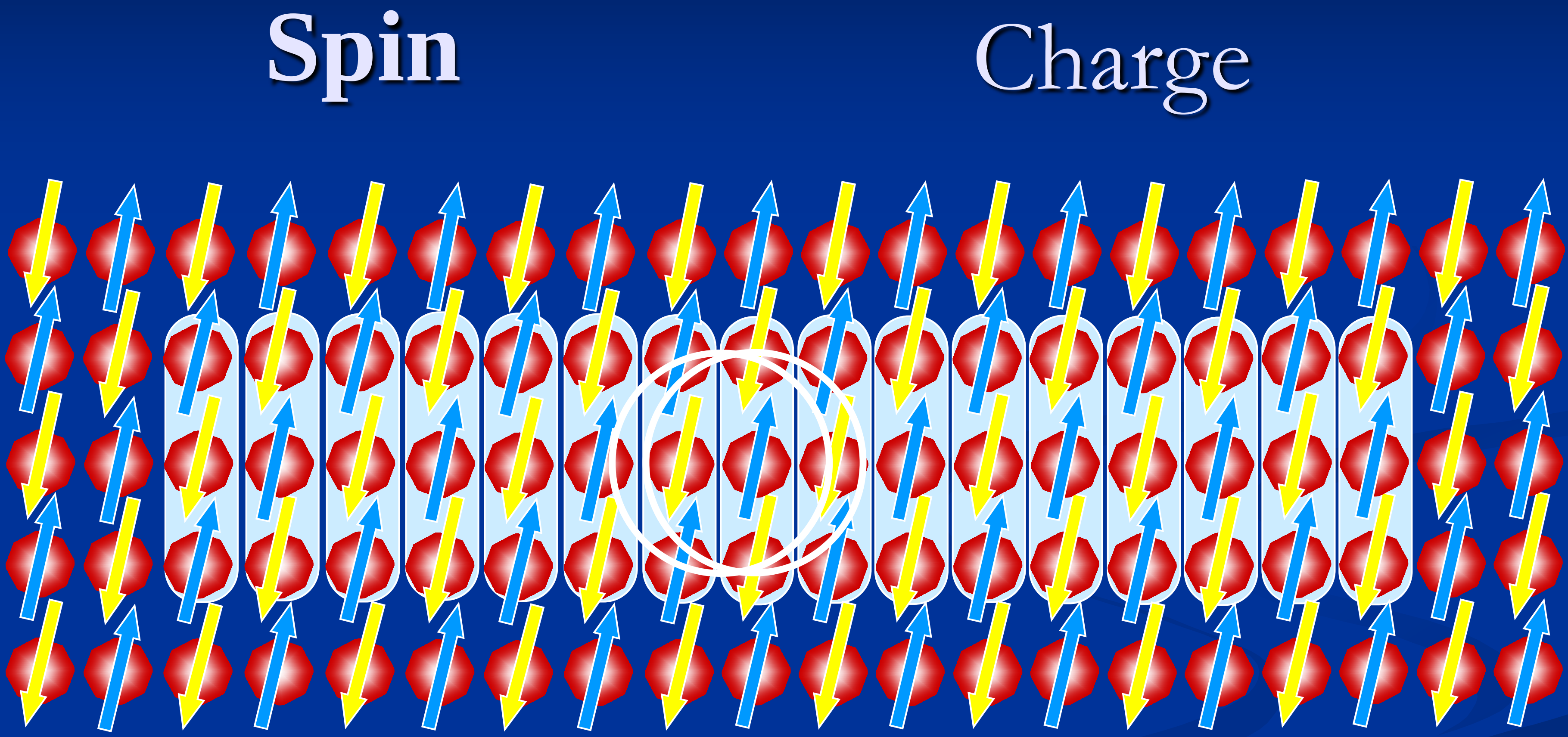
Conformal theory



# Deconstruction of the electron: spin-charge separation



# Spin-Charge Separation higher D ?




Energy increases with spin-charge separation

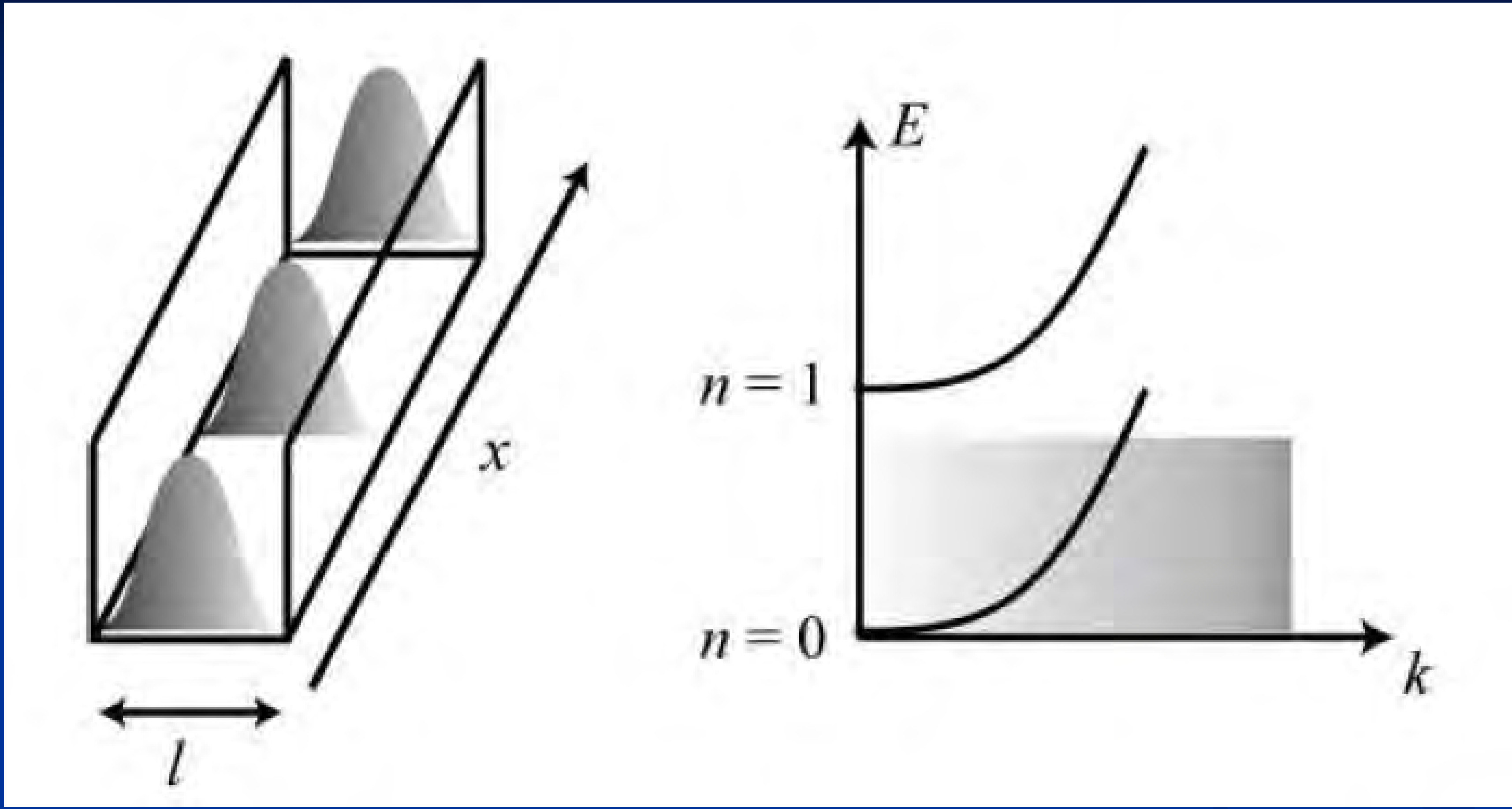
Confinement of spin-charge: « quasiparticle »

# Topological excitations is the norm in 1D


Topological Phase Transitions and New Developments, pp. 147-164 (2018)

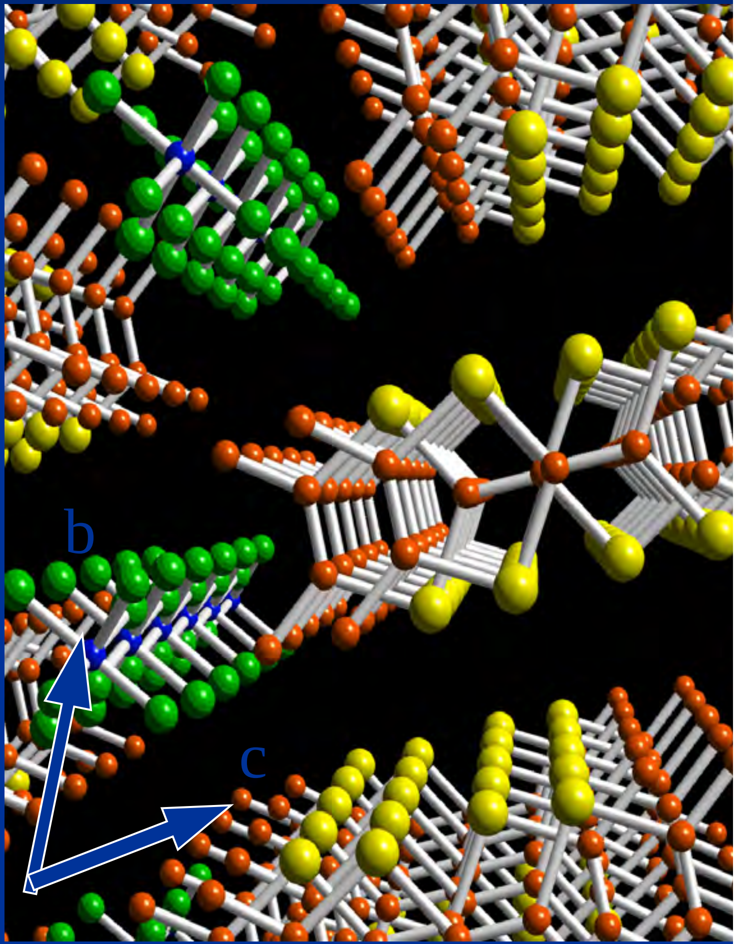
**Clean and dirty bosons in 1D lattices**

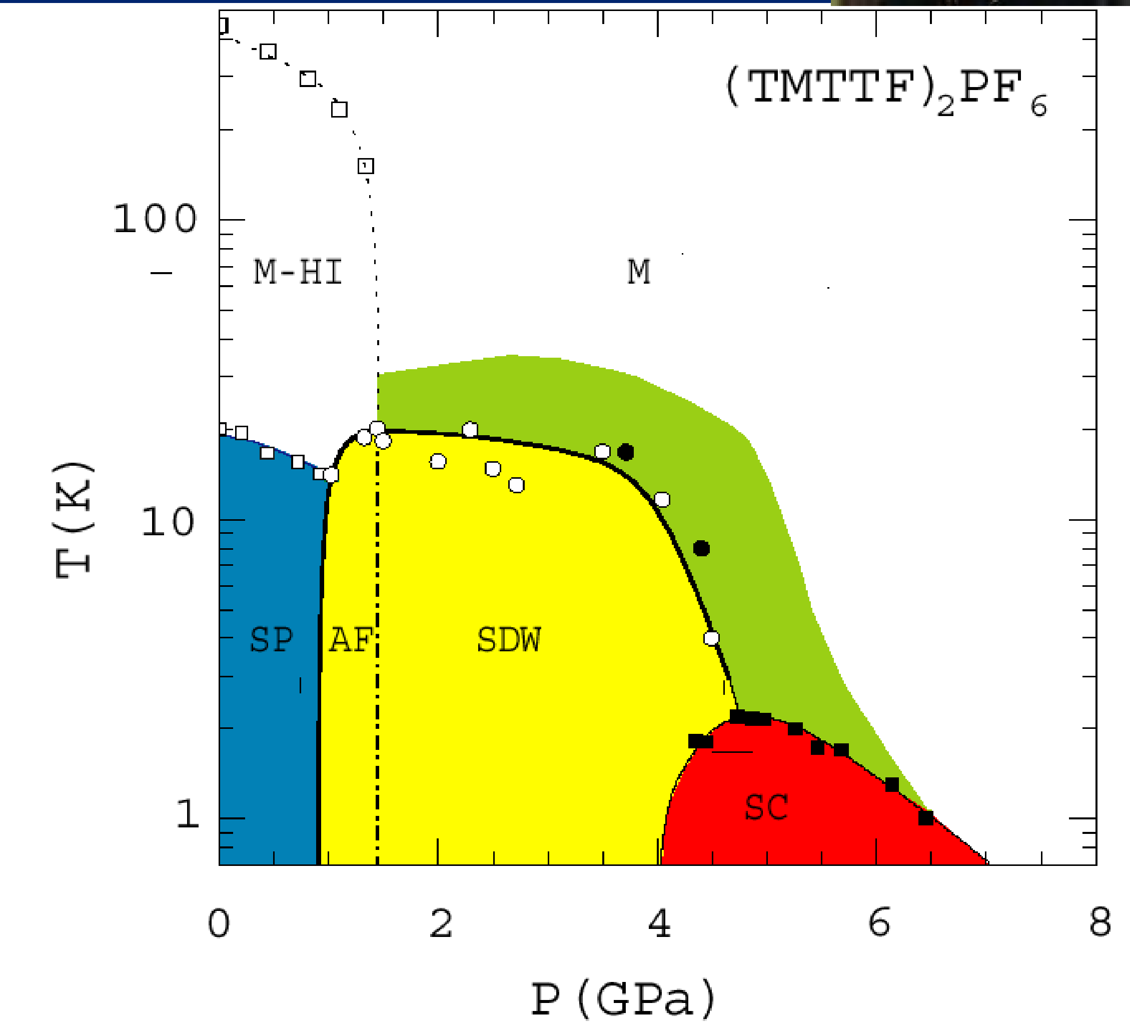
# Experiments



x
l
E
n = 1
n = 0
k

# Organic conductors

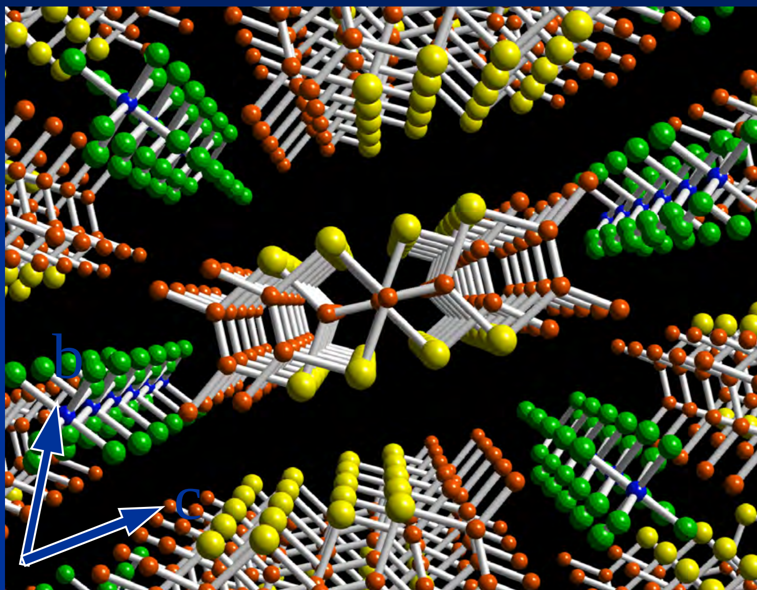


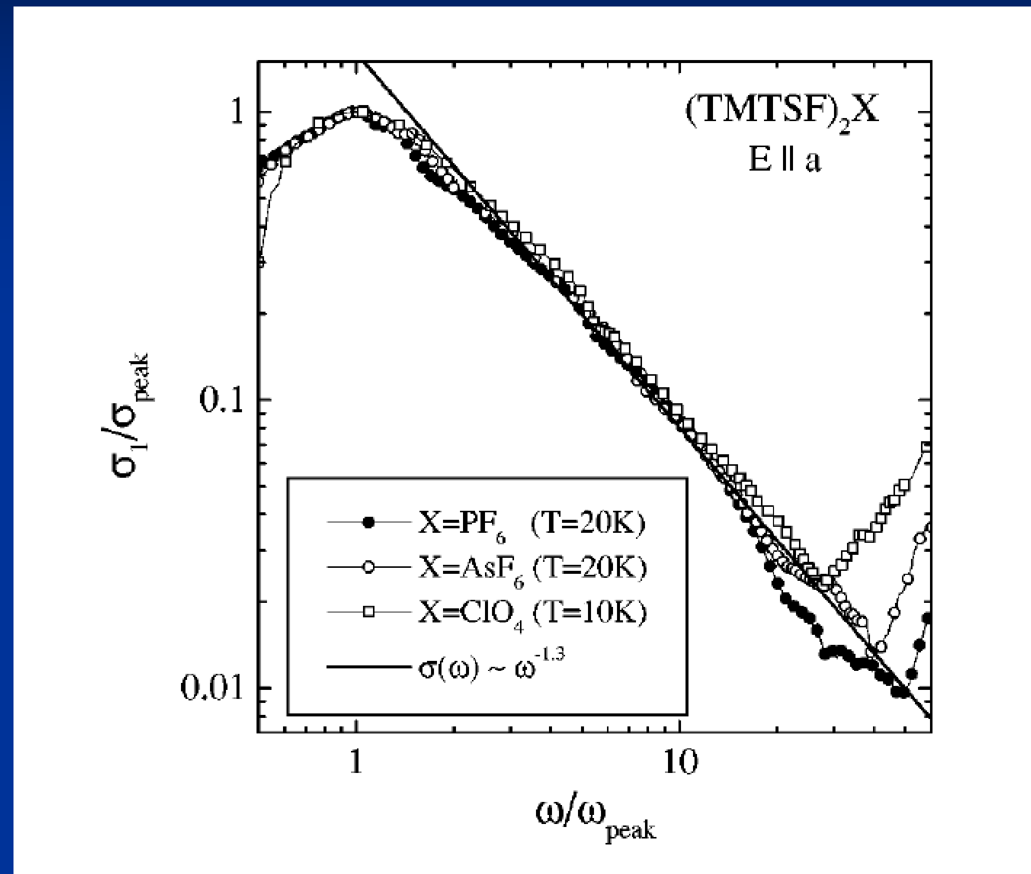

D. Jaccard et al., J. Phys. C, 13 L89 (2001)

# Power laws

# Organic conductors



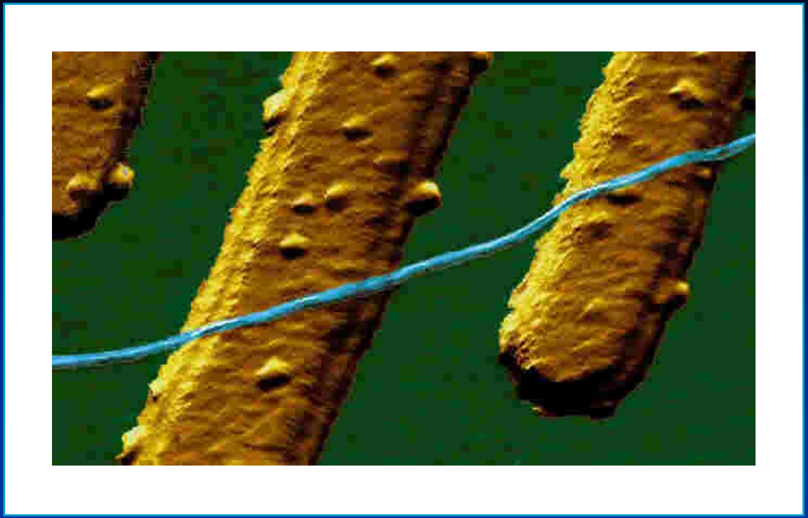

$$\sigma(\omega) \sim \omega^{-\nu}$$



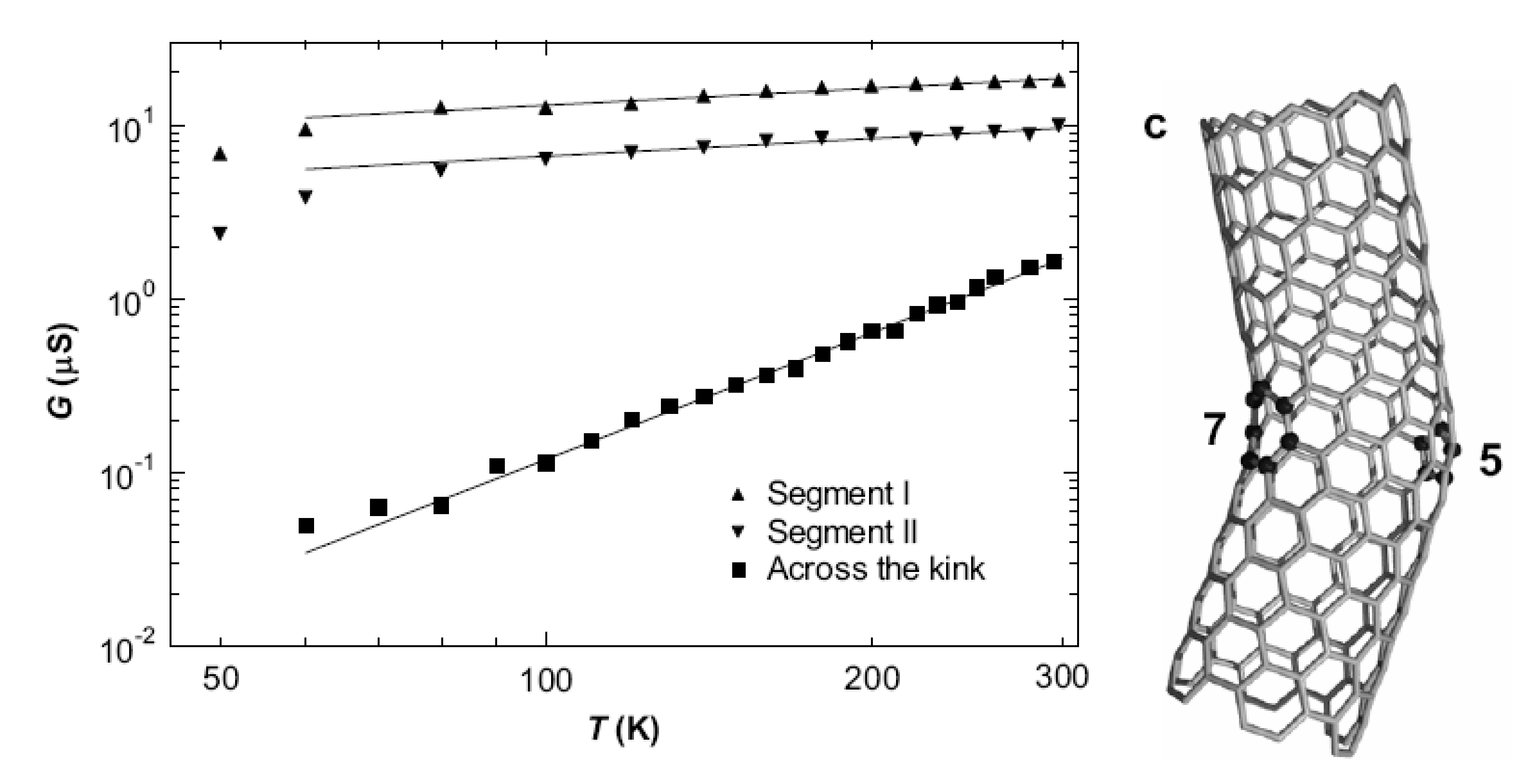

TG PRB (91) :
Physica B 230 (1996)

A. Schwartz et al. PRB 58 1261 (1998)

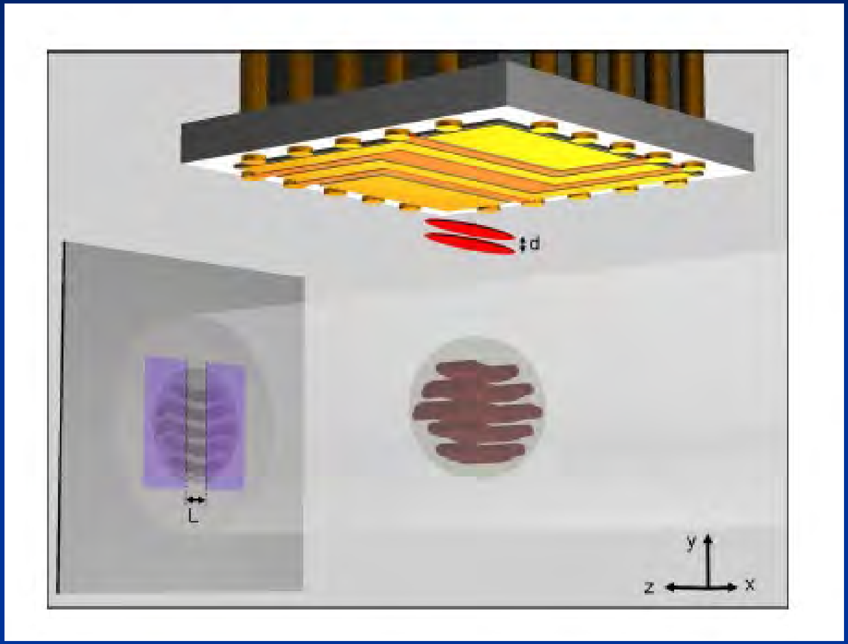
First observation of LL/powerlaw !!

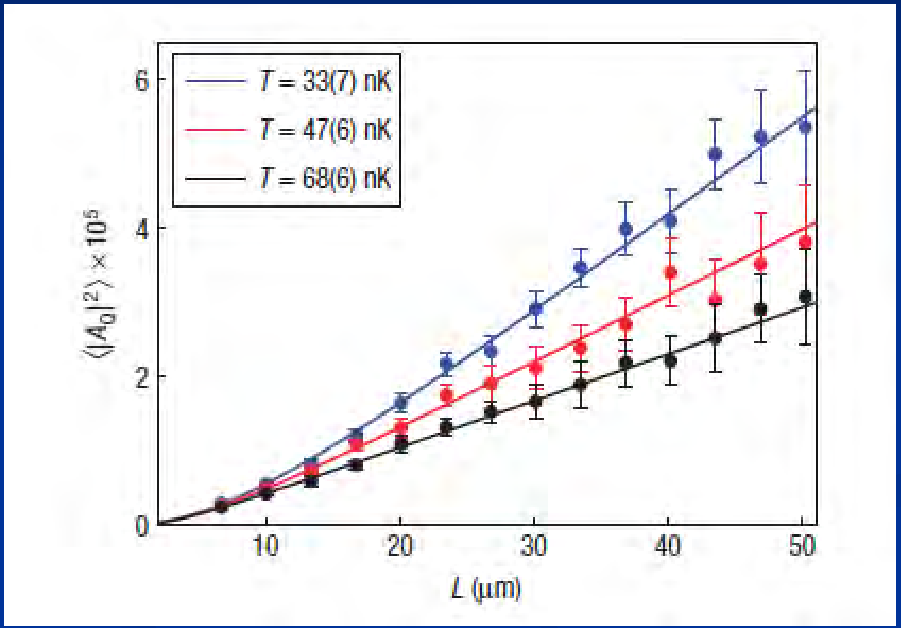


Z. Yao et al. Nature 402 273 (1999)



# Atom chips

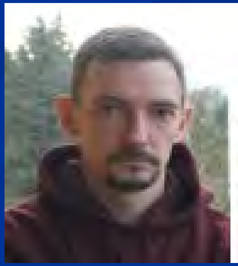



$$\int_0^L dr \langle \psi(r) \psi^\dagger(0) \rangle$$

S. Hofferberth et al. Nat. Phys 4 489 (2008)

K large (42)



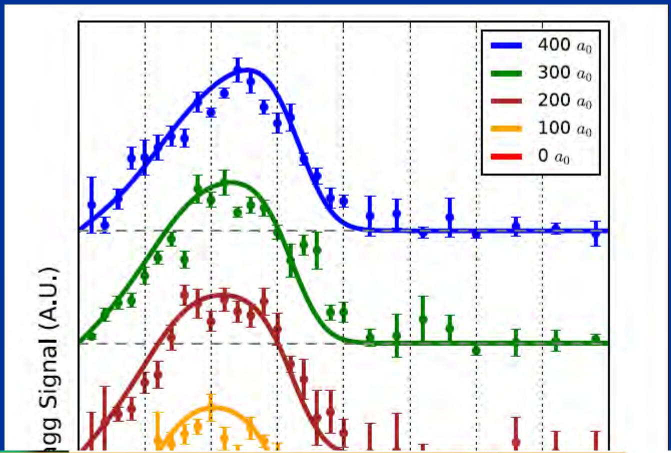
# Charge velocity

PHYSICAL REVIEW LETTERS **121,** 103001 (2018)

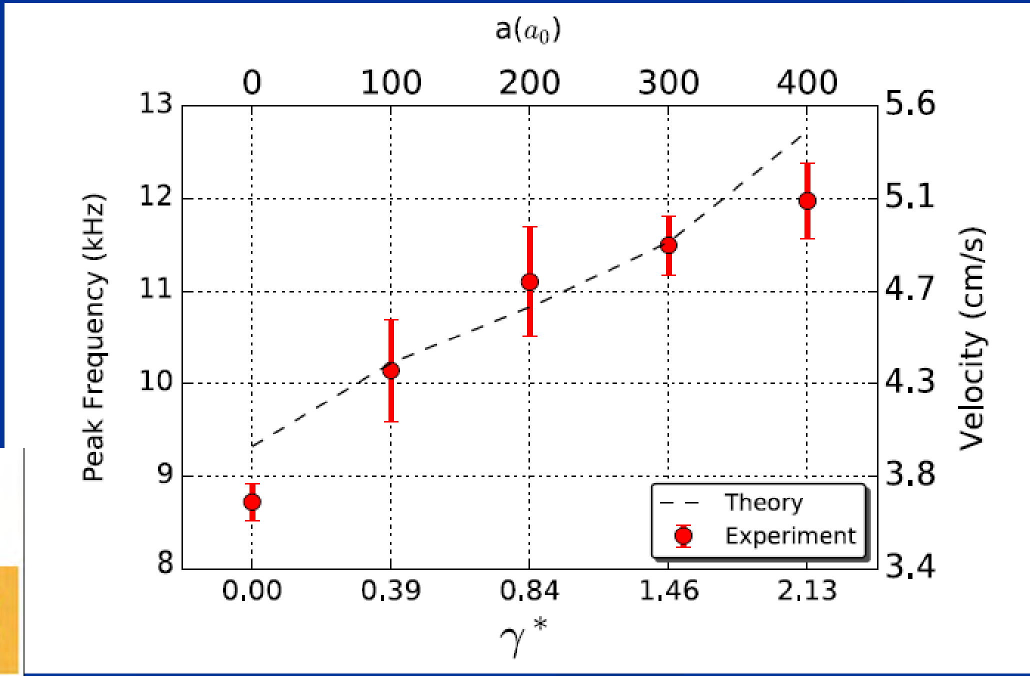
Editors' Suggestion

**Measurement of the Dynamical Structure Factor of a 1D Interacting Fermi Gas**

T. L. Yang,[1] P. Grišins,[2] Y. T. Chang,[1] Z. H. Zhao,[1] C. Y. Shih,[1] T. Giamarchi,[2] and R. G. Hulet[1]

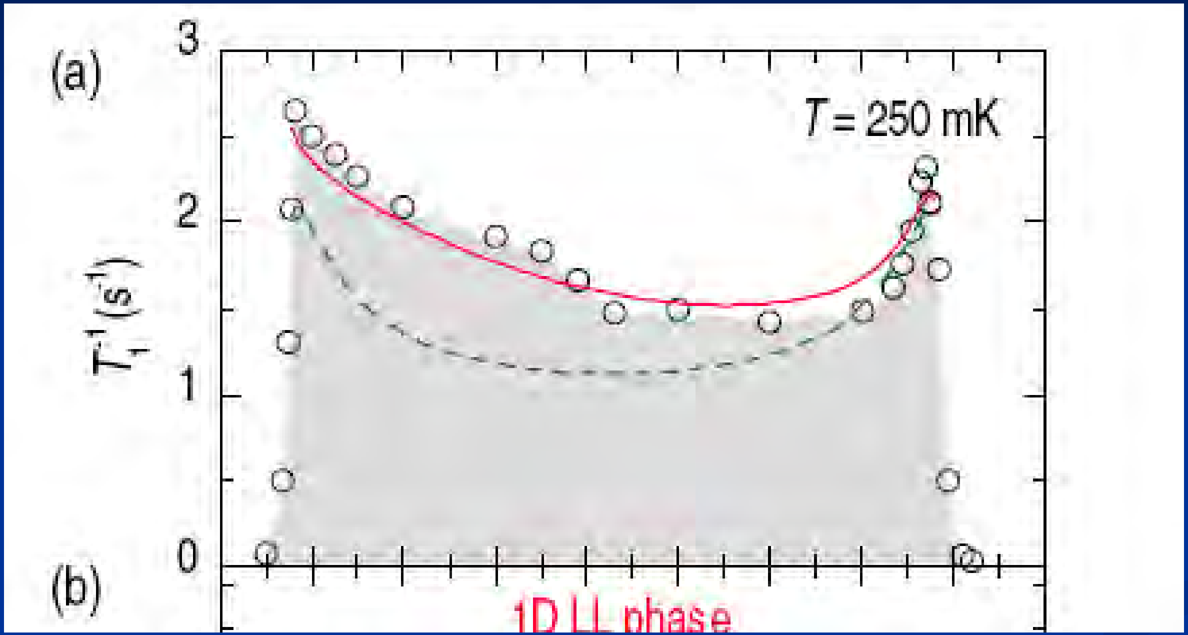


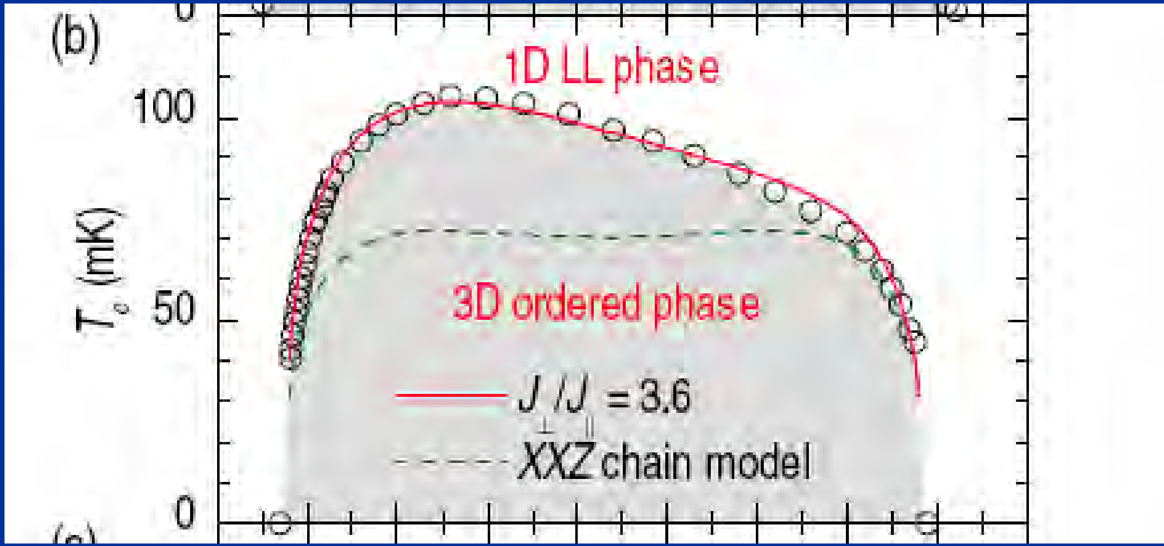

**ULTRACOLD ATOMS**
**CONFIRM 55-YEAR-OLD**
**PHYSICS THEORY**




https://www.futurity.org/one-dimensional-electrons-physics-1858622/


# Quantitative test of TLL


M. Klanjsek et al.,

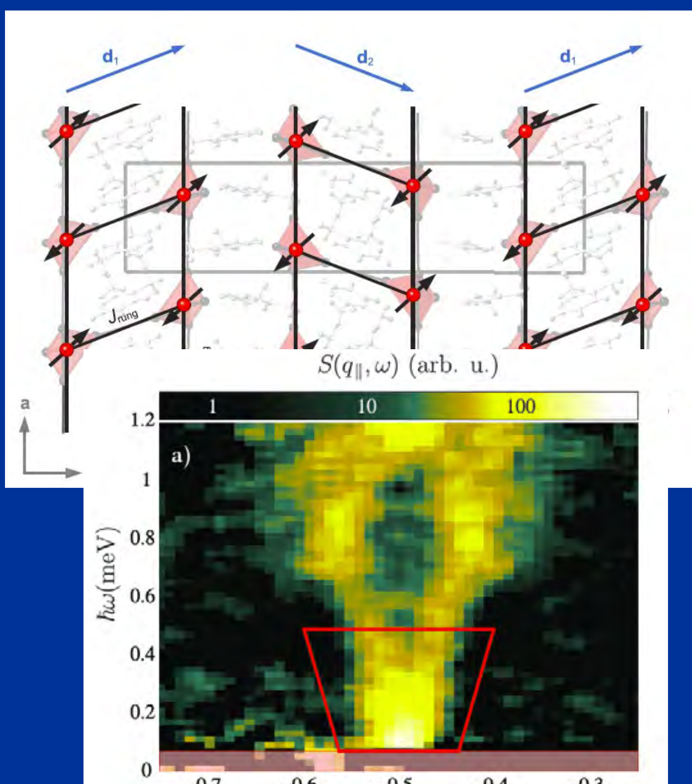
PRL 101 137207 (2008)

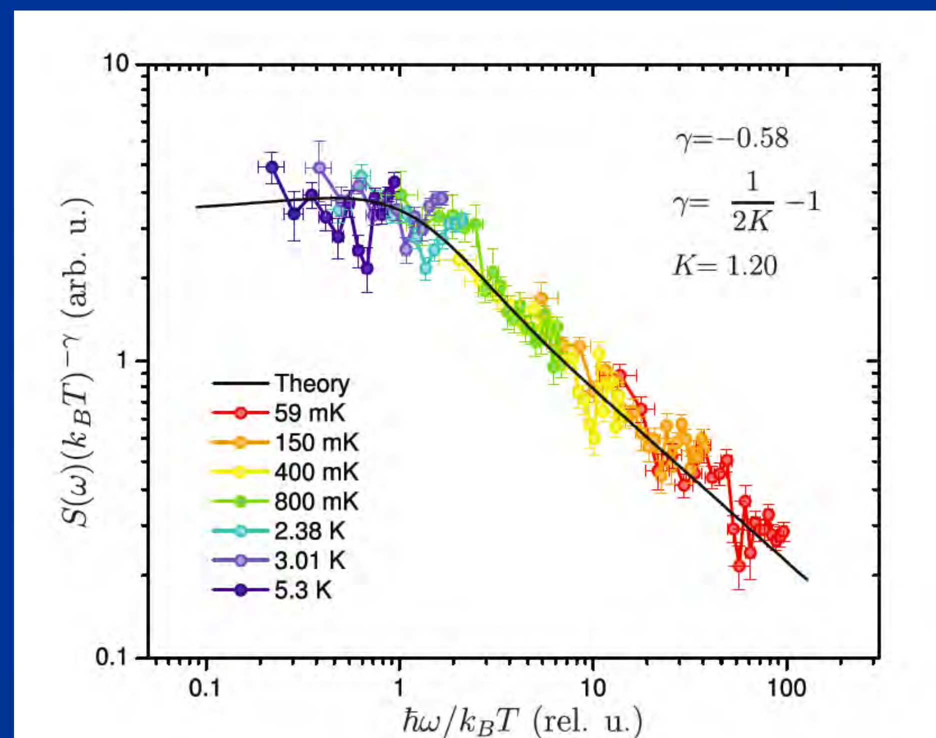
# Spin ladders

D. Schmidiger et al. PRL 108 167201 (2012):

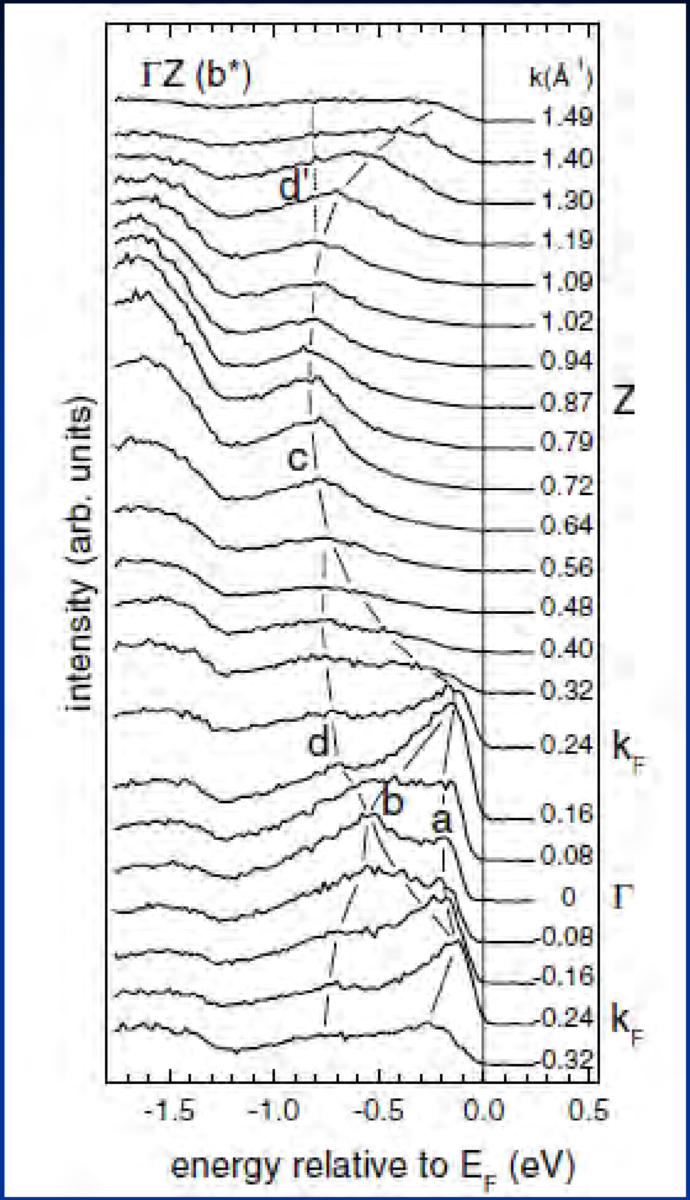
K. Yu et al. PRB 91 020406(R) (2015)

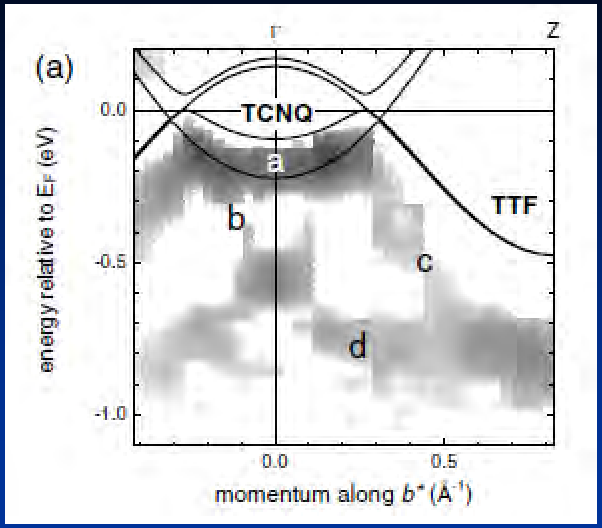
$$\langle S^- S^+ \rangle_{q,\omega} = \langle \psi \psi^\dagger \rangle_{q,\omega}$$

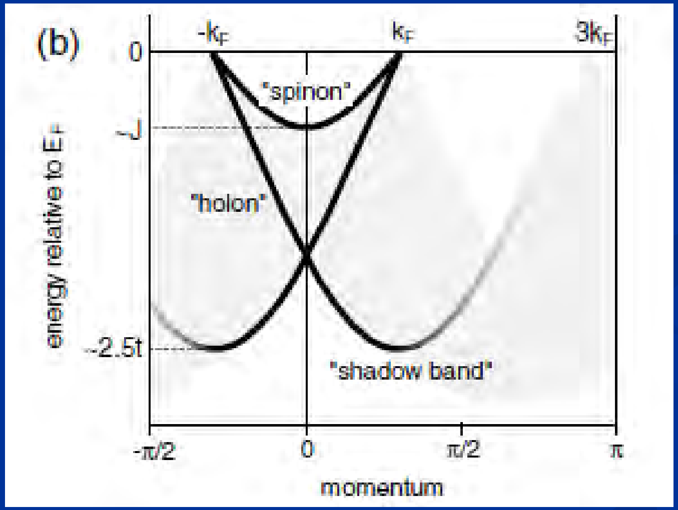


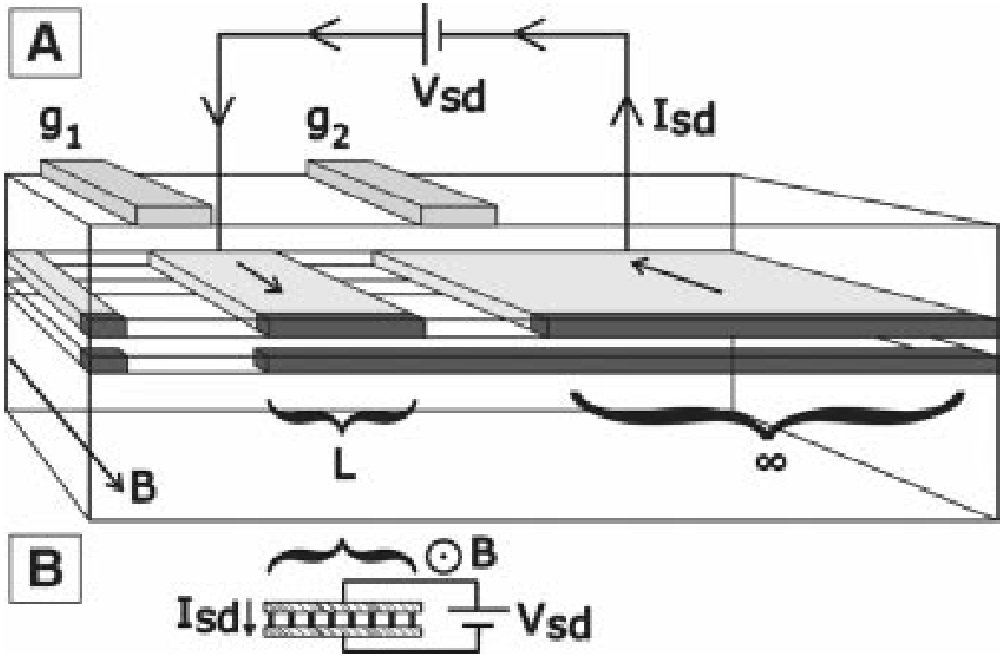
# Spin-charge separation

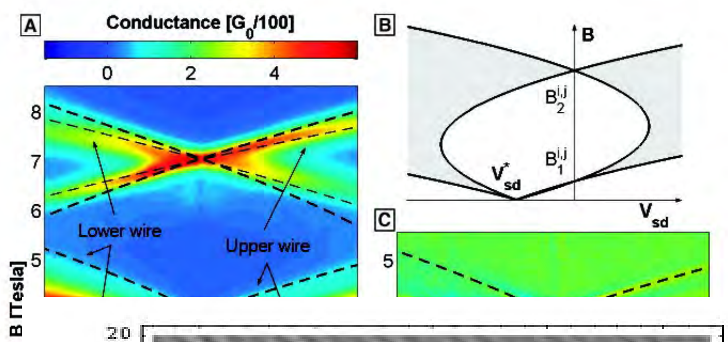


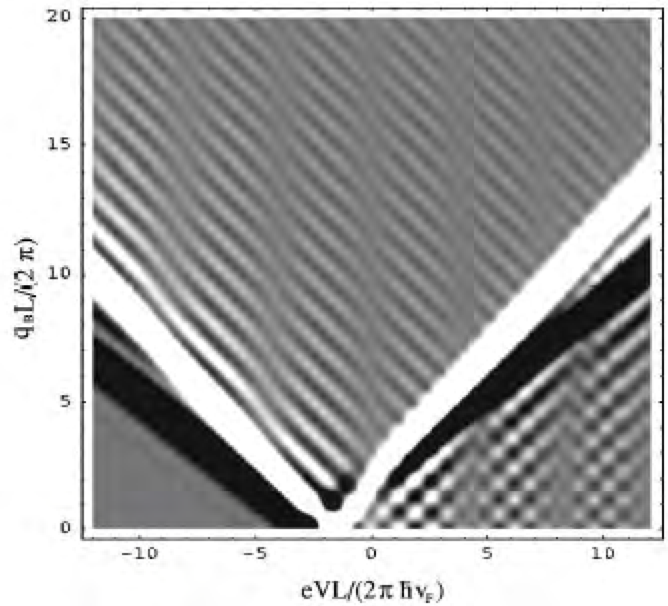




R. Claessens et al. PRL 88 096402 (2002)




O.M Ausslander et al., Science **298** 1354 (2001)

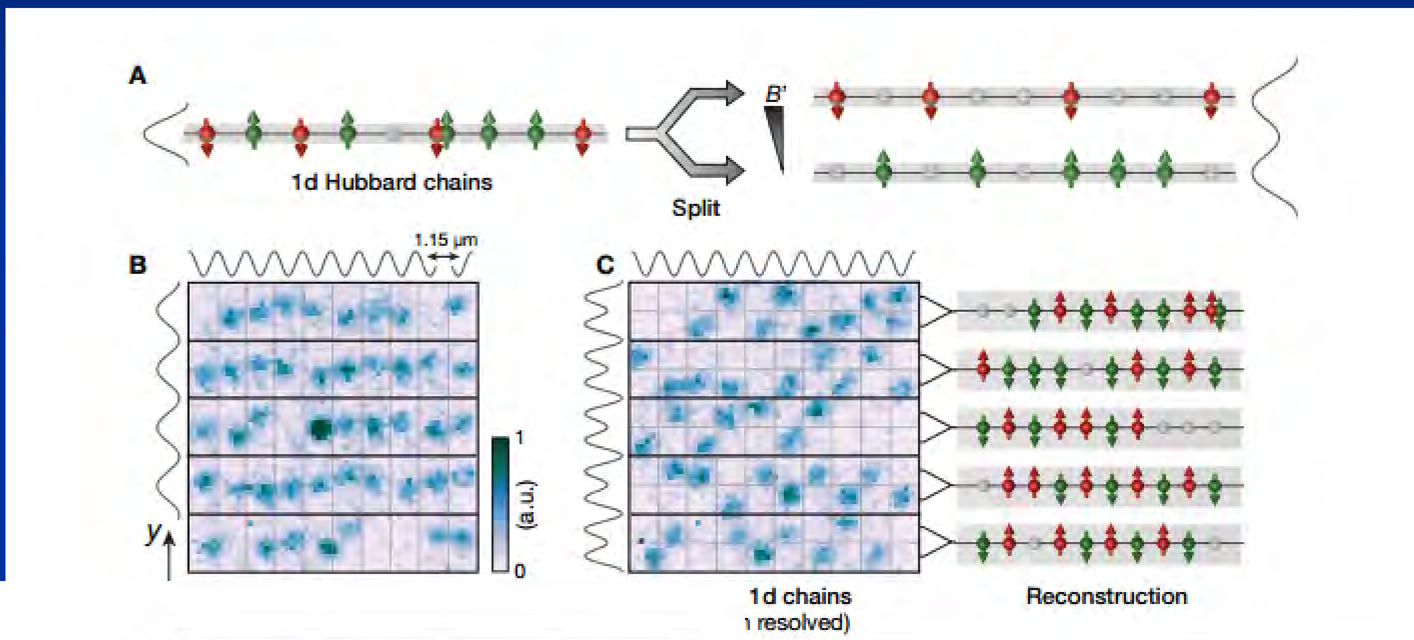
Y. Tserkovnyak et al., PRL **89** 136805 (2002)

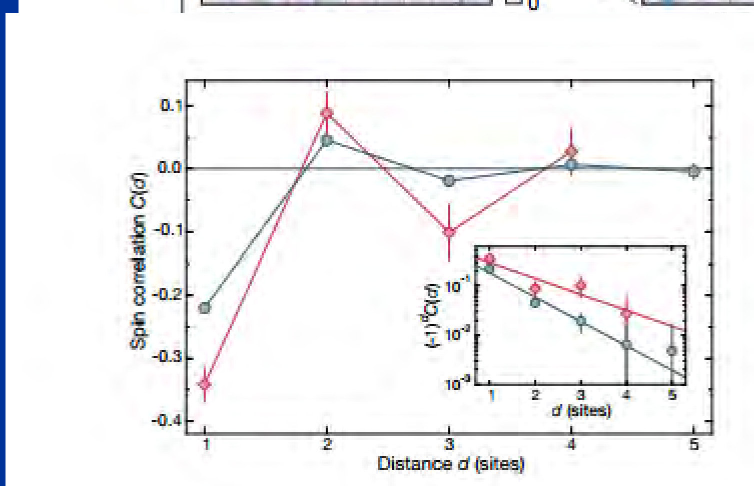
Y. Tserkovnyak et al., PRB **68** 125312 (2003)

**Spin and Charge Resolved Quantum Gas Microscopy of Antiferromagnetic Order in Hubbard Chains**

Martin Boll[1*], Timon A. Hilker[1*], Guillaume Salomon[1*], Ahmed Omran[1], Immanuel Bloch[1,2], and Christian Gross[1†]

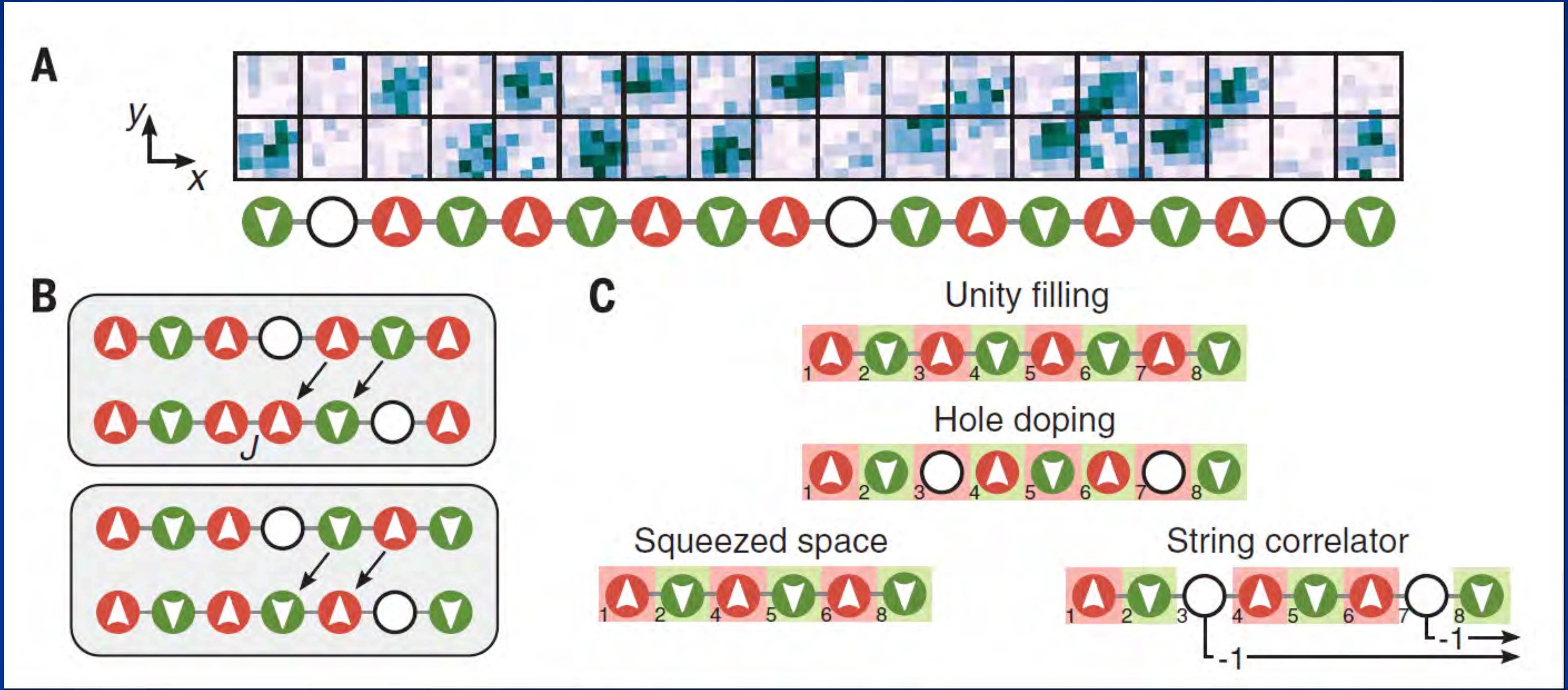
arXiv:1605.05661v2



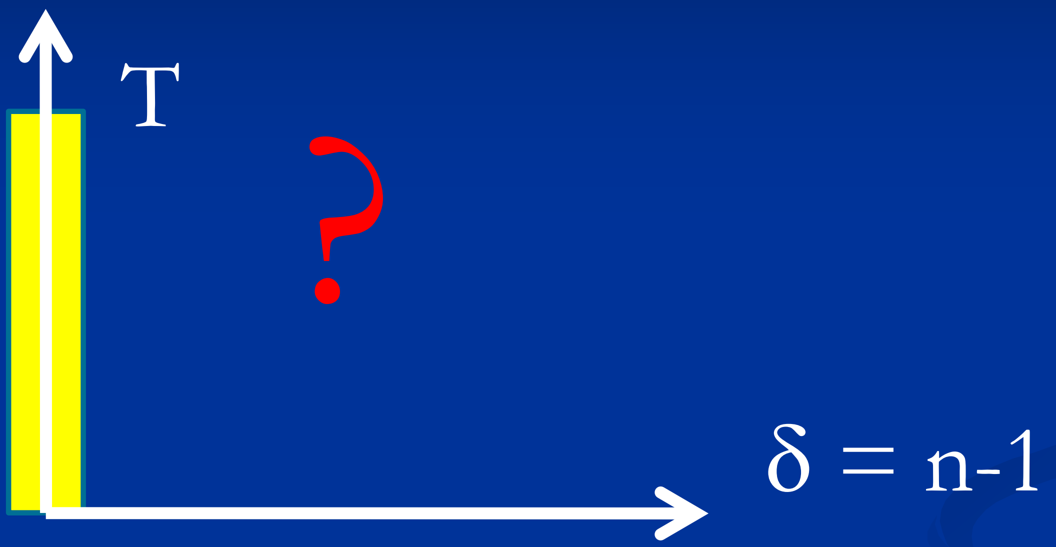

The inset shows the decay of the rectified spin correlations $(-1)^d C(d)$ in a logarithmic plot together with an exponential fit $C(d) \propto \exp(-d/\xi)$, which reveals an average correlation length of $\xi = 0.9(1)$ sites and $\xi = 1.4(4)$ sites for the lowest entropy tube. All error bars represent one s.e.m.

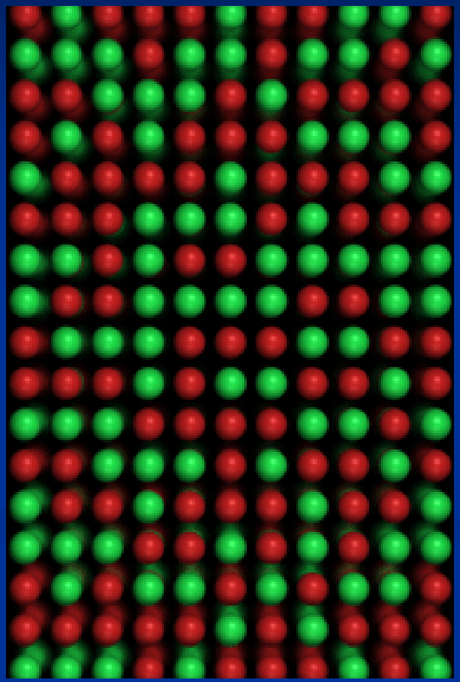
# Doped Hubbard model




Hilker *et al.*, *Science* **357**, 484–487 (2017)

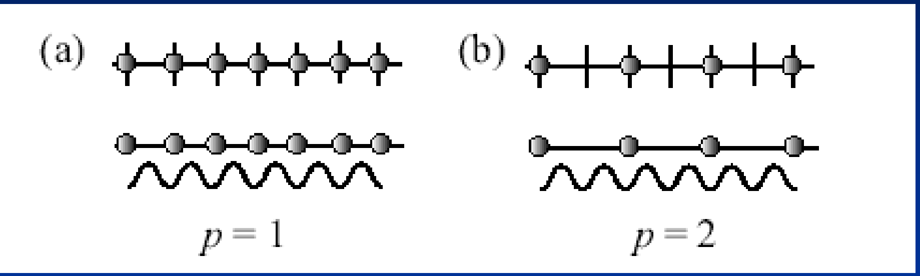
# Mott transition



- Mott insulator (n=1)
- T < T_N : antiferromagnetic phase

# Periodic lattice




$$H=\int dx V_0 \cos(Qx)\rho(x)$$

$$H=\int dx V_0 \cos(Qx)\rho_0 e^{i(2\pi\rho_0 x-2\phi(x))}$$

- Incommensurate: $Q \neq 2\pi\rho_0$

$$H=\int dx \cos(2\phi(x)+\delta x)$$

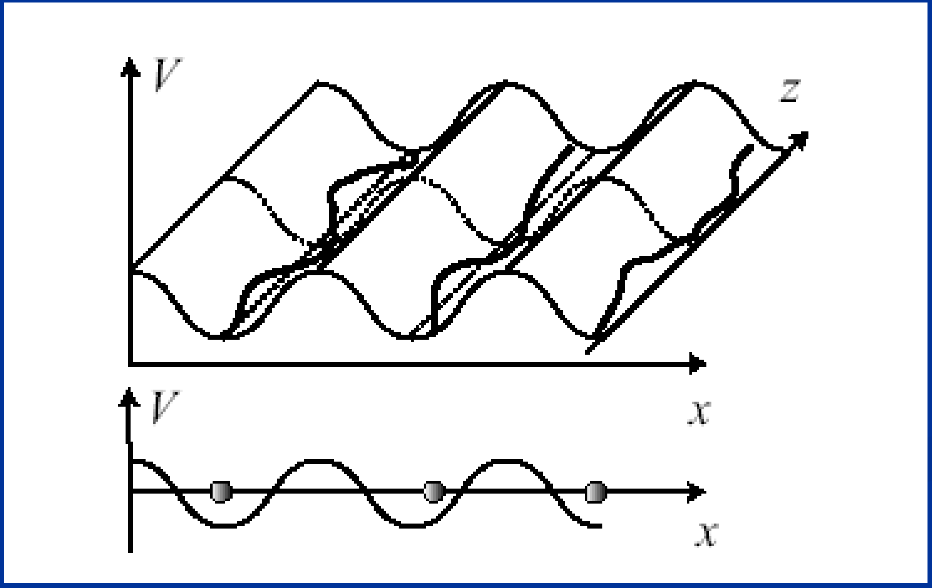
- Commensutate: $Q = 2\pi\rho_0$

$$H=\int dx \cos(2\phi(x))$$

# Competition

$$S_0 = \int \frac{dx d\tau}{2\pi K}[\tfrac{1}{u}(\partial_\tau \varphi(x,\tau))^2 + u(\partial_x \varphi(x,\tau))^2]$$

$$S_L = -V_0 \rho_0 \int dx d\tau \cos(2\phi(x))$$

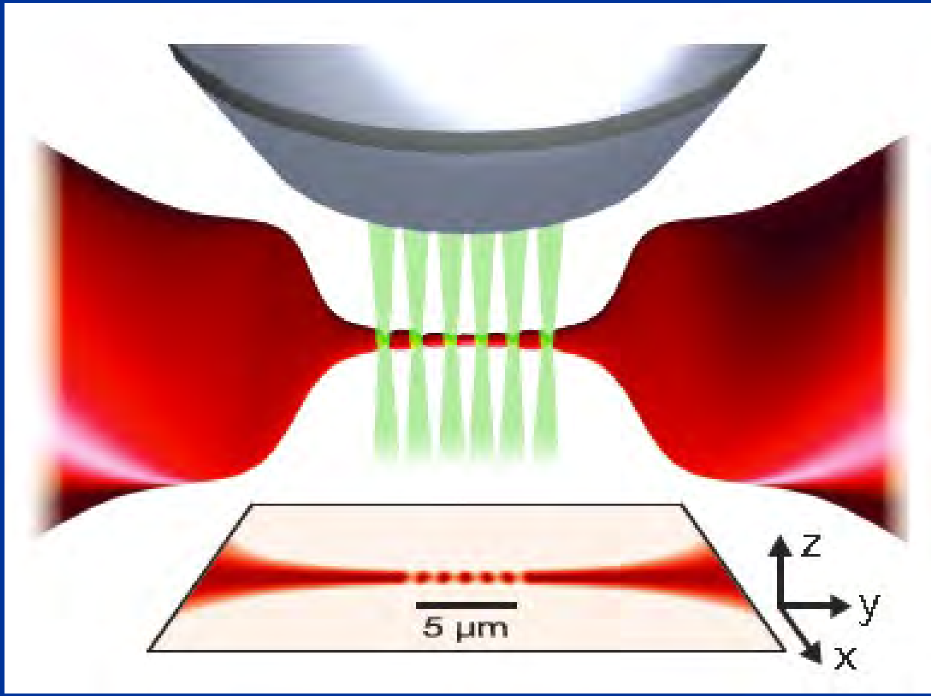


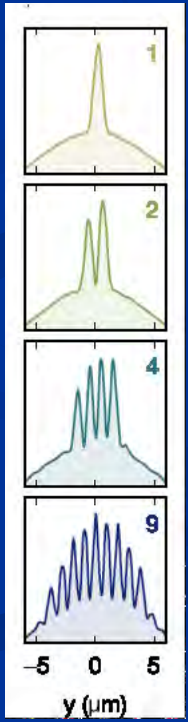

Beresinskii-Kosterlitz-Thouless transition at K=2

String order parameter

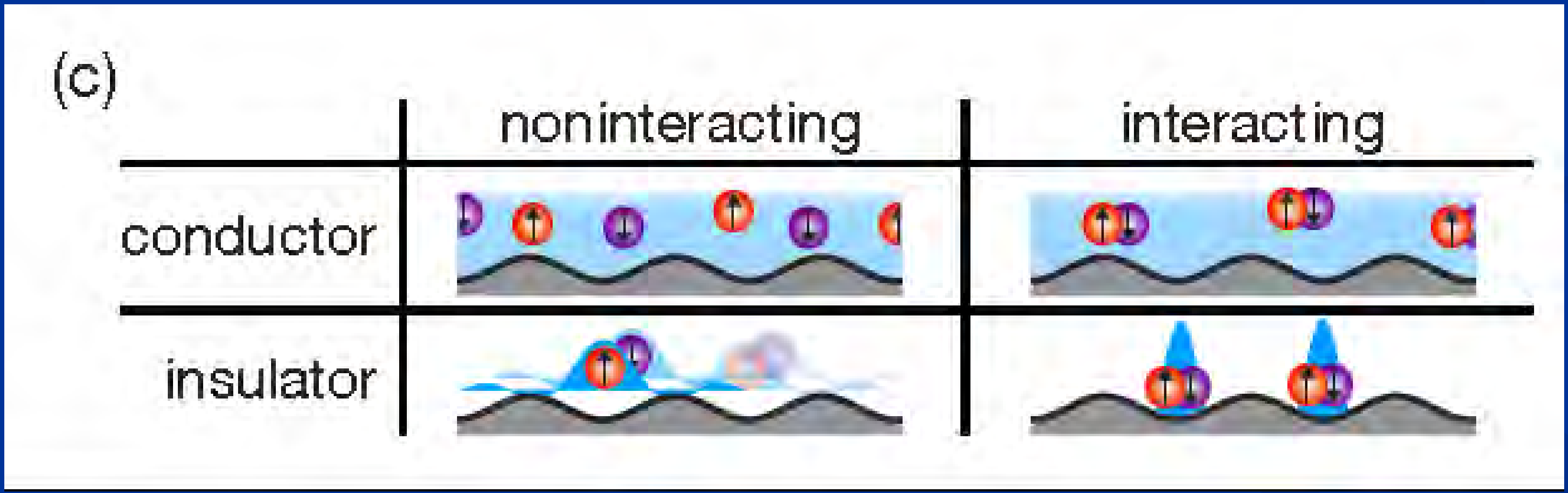
# Atomtronic

M. Lebrat, P. Grisins et al., PRX 8 011053 (18)

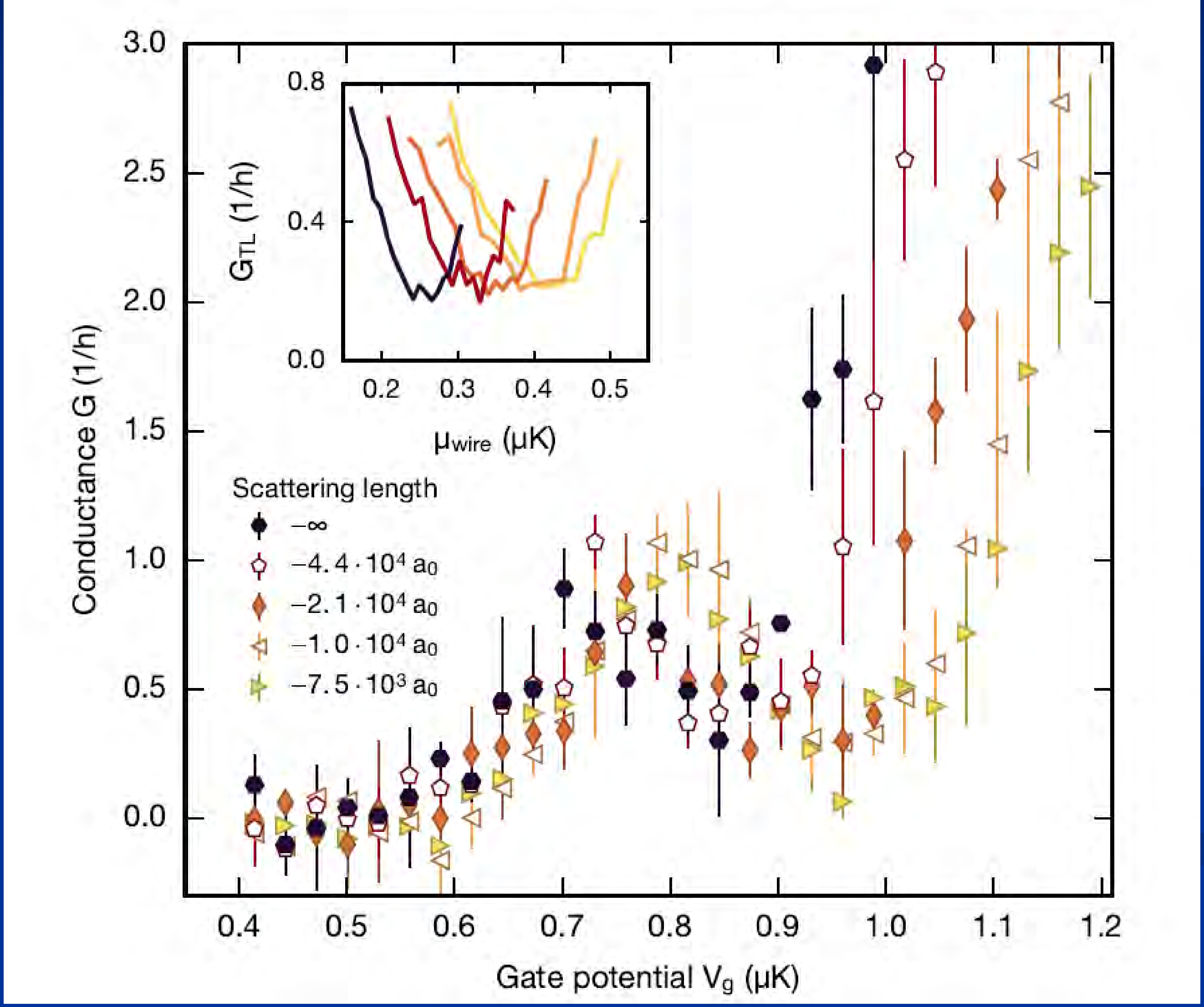


# Many-body insulator ``pinned" L.E. liquid

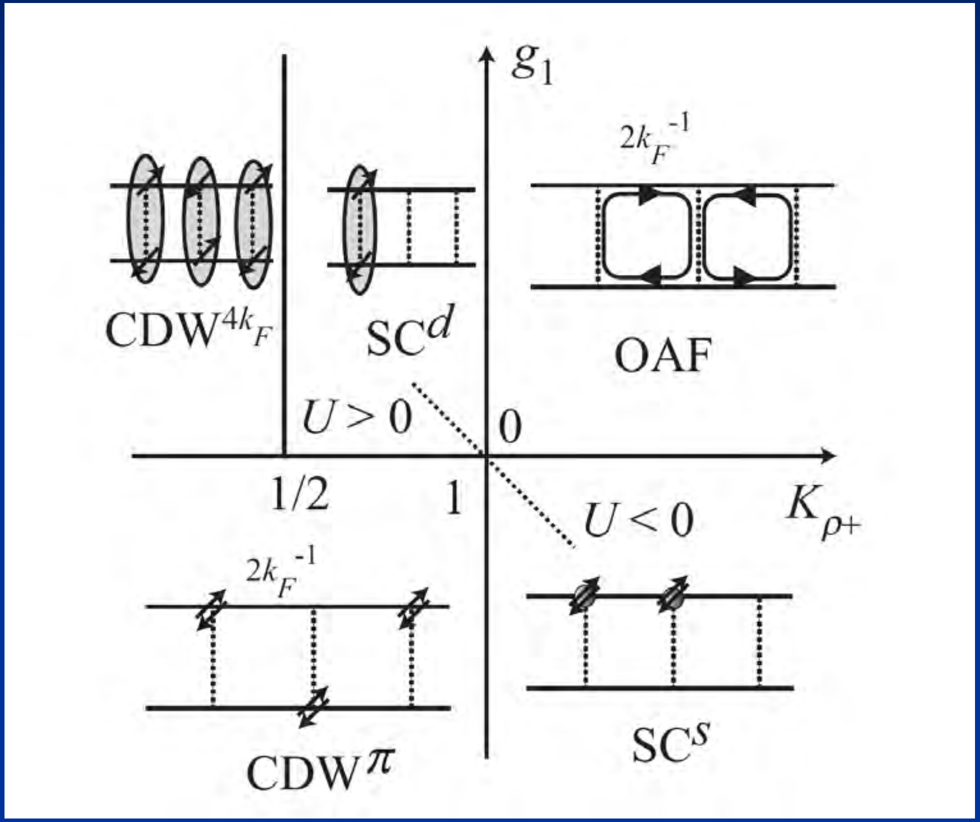


# Experimental evidence for L.E. liquid



# Beyond 1D

# Ladders



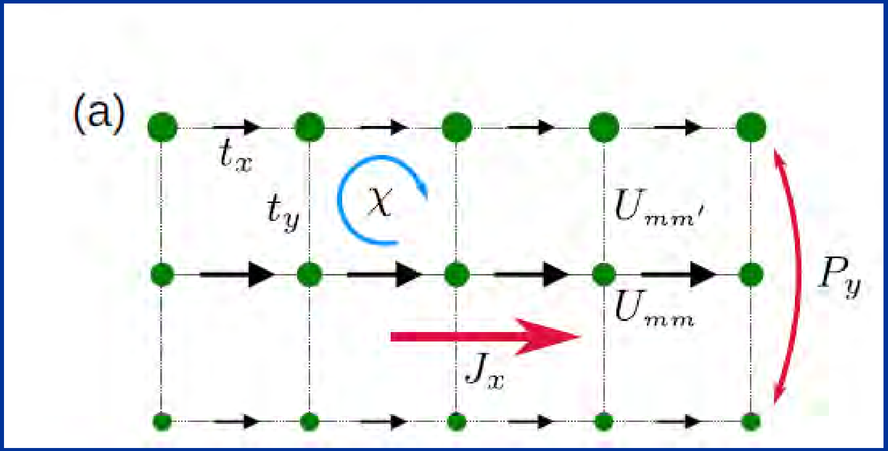

D-wave superconductivity !

Pair correlations in doped Hubbard ladders

Michele Dolfi, Bela Bauer, Sebastian Keller, and Matthias Troyer
Phys. Rev. B **92**, 195139 – Published 19 November 2015

# Hall effect on ladders

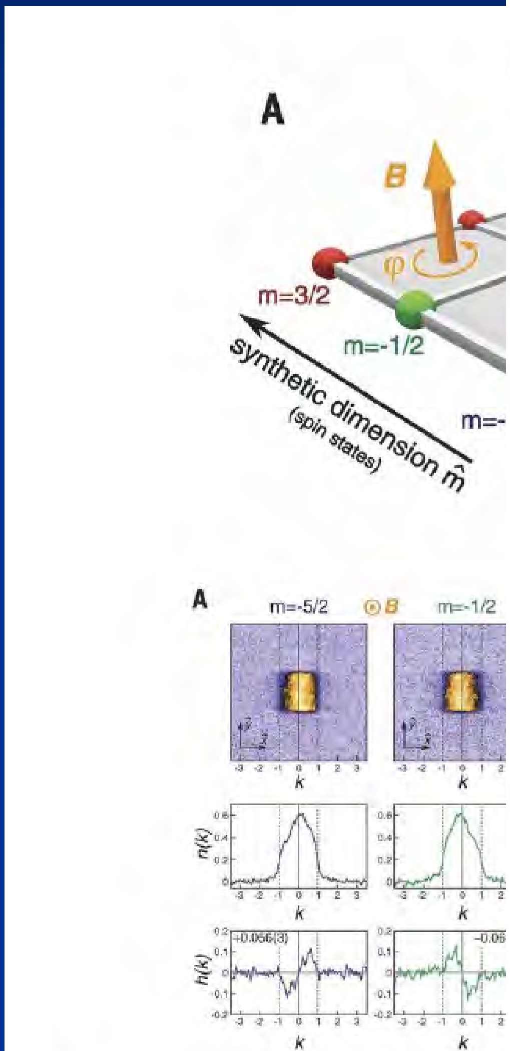
S. Greshner, M. Filippone, TG, PRL 122, 083402 (19)



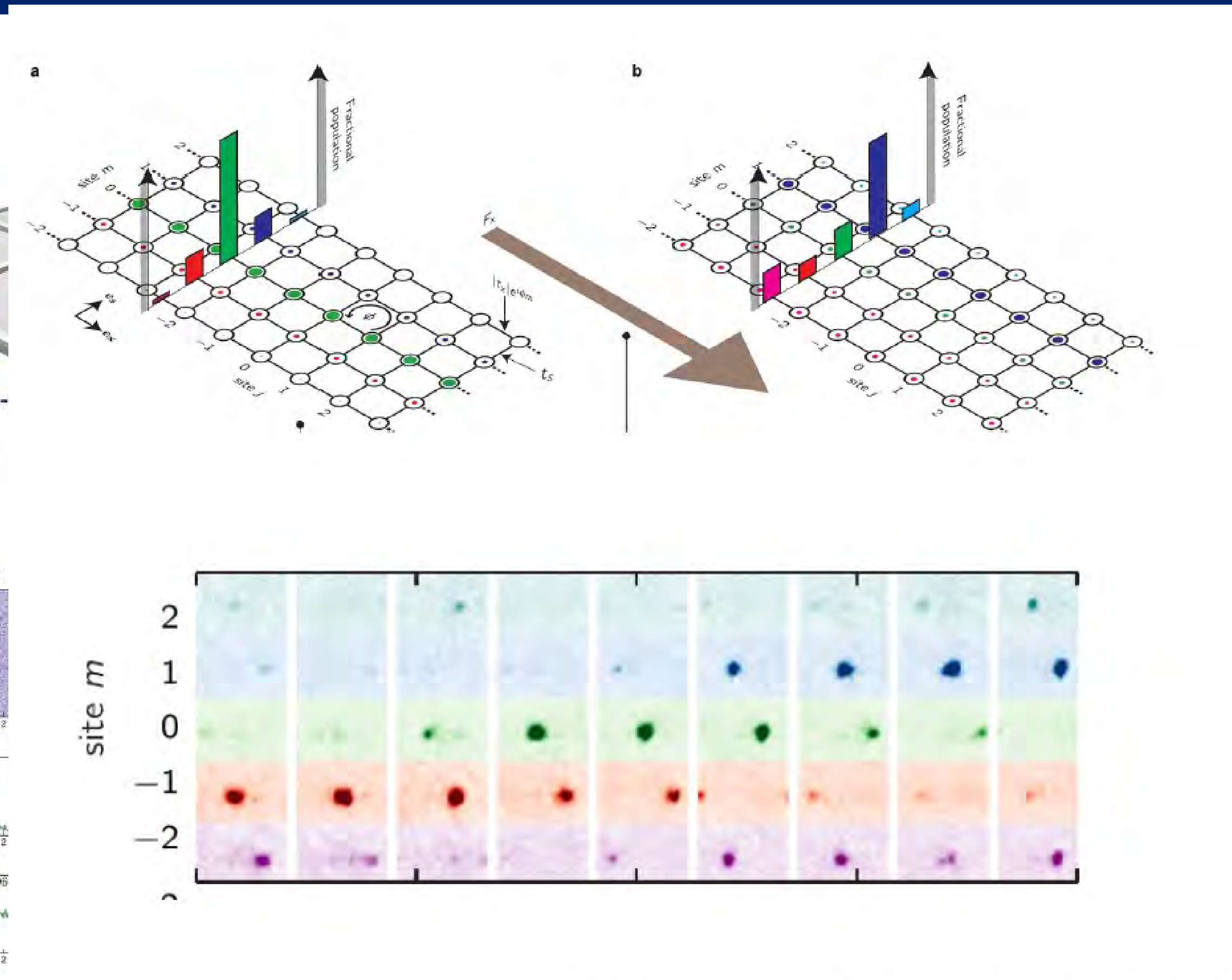

Analytic: calculation on a ring

Experiments: out of equilibrium

# Cold atoms (balistic !)



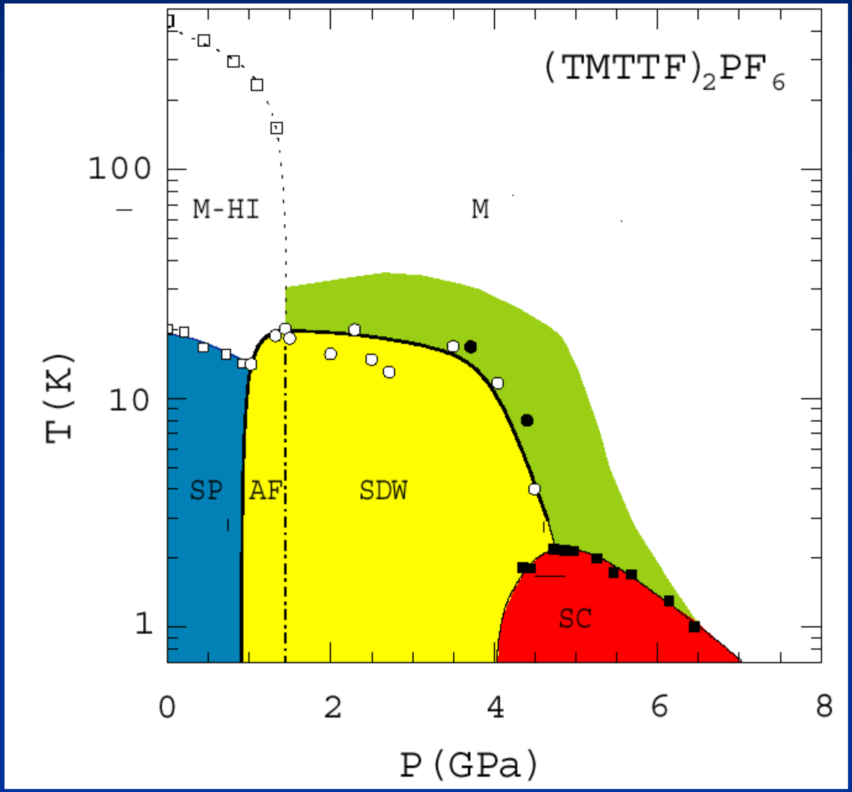

Fallani's group (Florence) **Scien**

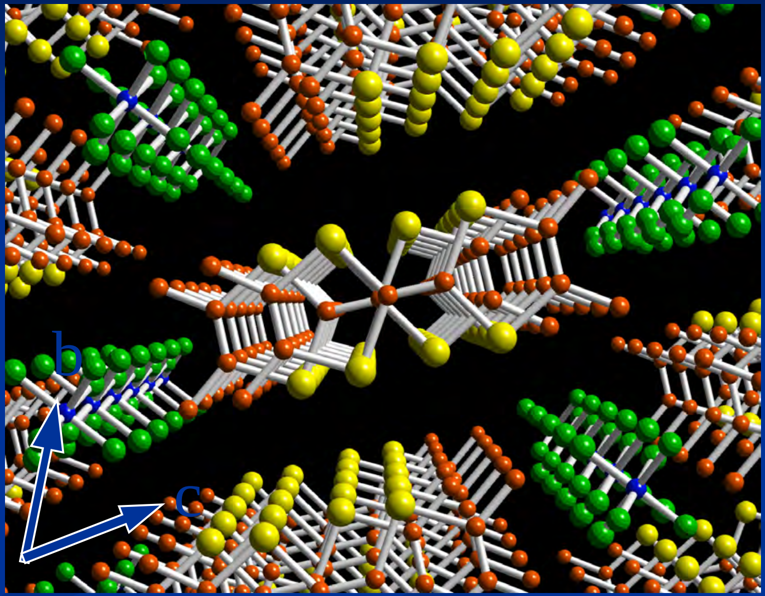



Spielman's group (Maryland) **arXiv:1804.06345**

# Many chains

# Deconfinement

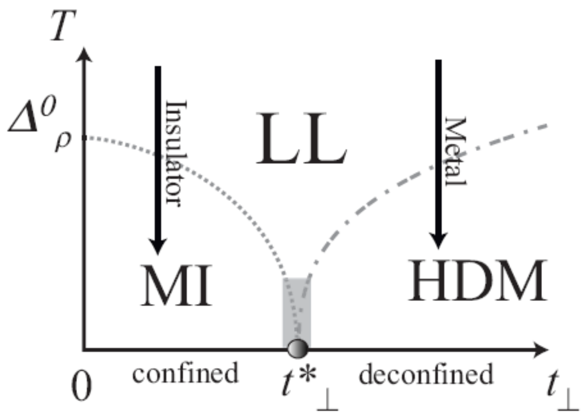


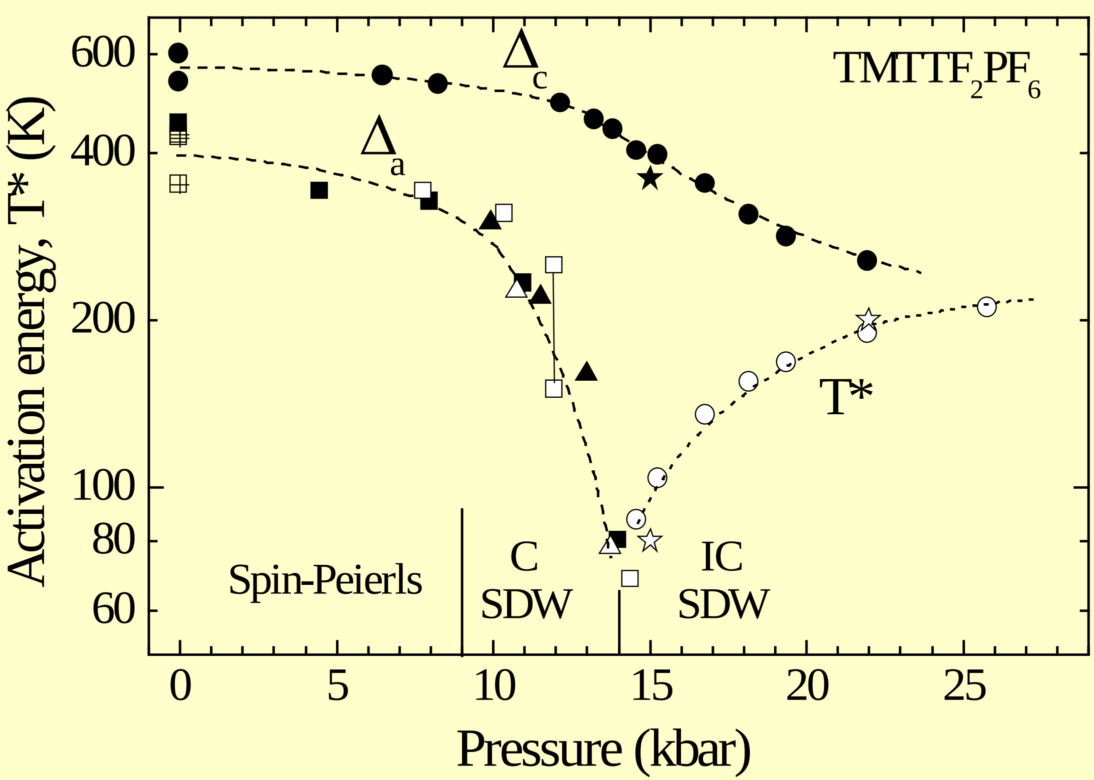




TG Chemical Review 104 5037 (2004)

D. Jaccard et al., J. Phys. C, 13 L89 (2001)

# Deconfinement



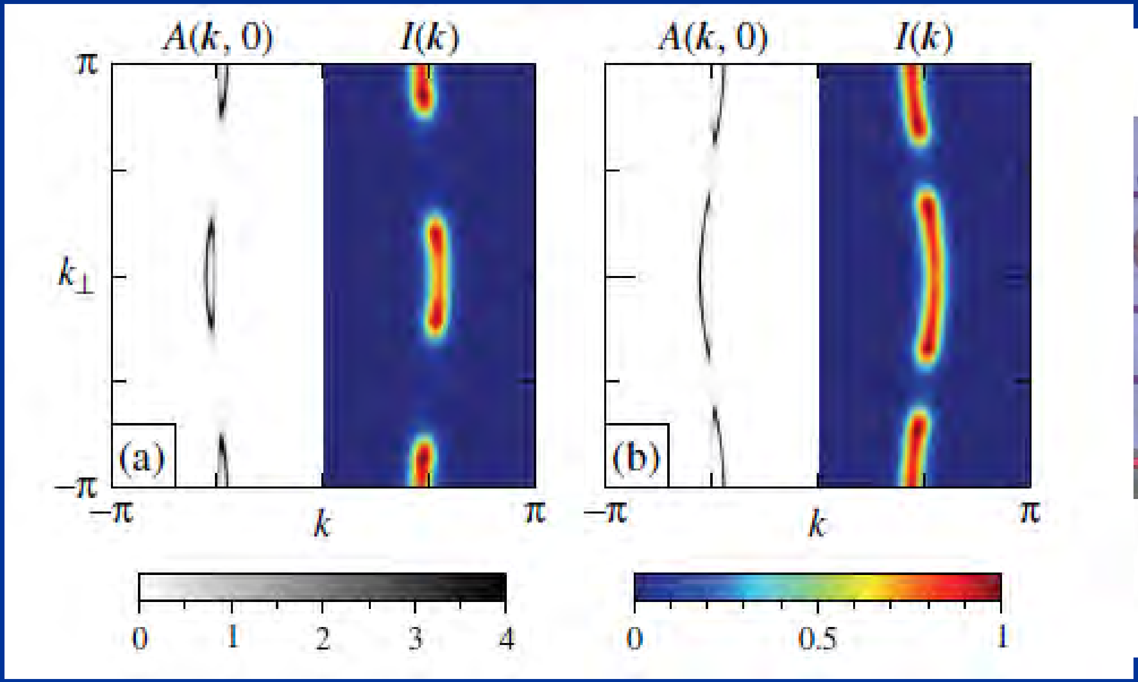



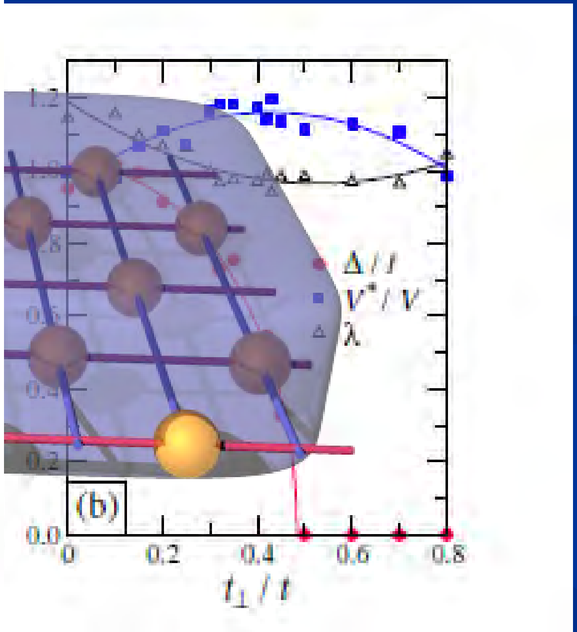

TG Chemical Review 104 5037 (2004)

P. Auban-Senzier, D. Jérome, C. Carcel and J.M. Fabre J de Physique IV, (2004)
A. Pashkin, M. Dressel, M. Hanfland, C. A. Kuntscher, PRB 81 125109 (2010)

# Difficult to study

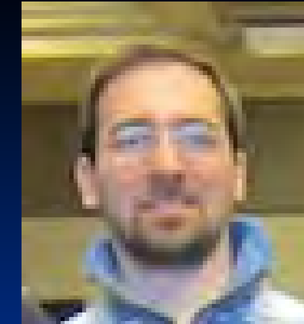
S. Biermann, A. Georges, A. Lichtenstein, TG, PRL 87 276405 (2001)
C. Berthod et al. PRL 97, 136401 (2006)




Two transitions ?　Nature of the phases ?　Ch-DMFT

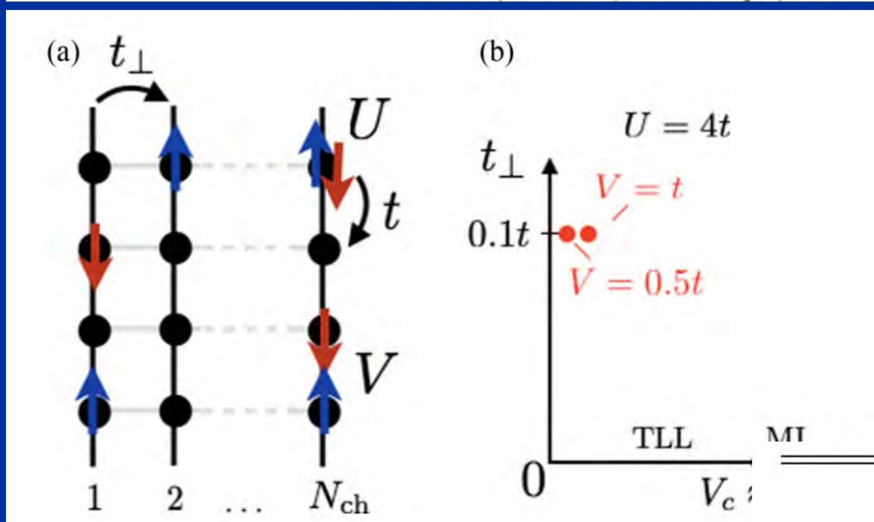
# DMRG



PHYSICAL REVIEW B **100**, 075138 (2019)

**Understanding repulsively mediated superconductivity of correlated electrons via massively parallel density matrix renormalization group**

A. Kantian,[1] M. Dolfi,[2,3,*] M. Troyer,[2] and T. Giamarchi[4]




| | $\overline{\Delta}_s[N_{ch}]$ $[t \times 10^{-3}]$ | | | |
|---|---|---|---|---|
| | Straight extrapolation | | Extr. + estimates | |
| $N_{ch}$ | $V/t = 0.5$ | $V/t = 1$ | $V/t = 0.5$ | $V/t = 1$ |
| 2 | 5.59 | 7.59 | 5.36 | 7.31 |
| 4 | 9.68 | 10.2 | 10.77(10)[a] | 10.268(11) |
| 6 | 9.37 | 11.4 | 4.3(1.4)[b] | 11.20(97) |
| 8 | 9.45 | | 7.6(1.3)[c] | |

# Conclusions

- Tour of Hubbard / one dimensional physics
- Luttinger liquid theory provides a framework to study this physics, and to go beyond
- Beautiful and challenging questions going beyond the Luttinger liquids
- Requires interplay of analytical and numerical techniques (and new ideas!) to make progress
- Many experimental realizations both in condensed matter and in cold atoms