Particules Élémentaires, Gravitation et Cosmologie
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Le Modèle Standard et ses extensions

Cours V: 22 février 2008

Standard Model Higgs and one-family Lagrangian

Reminder of previous week

Starting from Fermi's 1934 model and using experimental and theoretical inputs, we arrived at the conclusion that the simplest extension of the SU(3)xU(1) gauge theory of strong and EM interactions that could include the weak interactions needs a gauge group SU(3)xSU(2)xU(1) with the l.h. (or r.h.) fundamental fermions filling a well-defined complex representation.

Such a theory cannot contain any gauge boson or fermionic mass term.

We then discussed the mechanism of SSB for global and local symmetries, arguing that the latter could provide a promising way for overcoming the above « mass generation » problem.

The issue

How can we use the Higgs mechanism in order to solve the abovementioned problems? In particular, which is the gauge structure of the scalar fields that is needed?

Since we are fully happy with QCD we should keep SU(3) unbroken, i.e. the Higgs fields should be SU(3) singlets.

On the other hand, since we want to break spontaneously $SU(2)_L xU(1)_Y$, the Higgs fields should transform non-trivially under $SU(2)_L xU(1)_Y$.

Also, there must be at least 3 of them with charge 0, ±1 so that they can be «eaten-up» by the $W^{\pm}$ and the W^3 (Z_0).

Finally, it should be possible to write down Yukawa interactions which can eventually generate masses for the quarks and the charged leptons (later also for the neutrinos..).

Let us recall the table we arrived at last time for one family of quarks and leptons

l.h. ferms	SU(3)	SU(2)	$U(1)_Y$
(u,d)	3	2	1/6
(ν, e)	1	2	-1/2
u^c	3*	1	-2/3
d^c	3*	1	+1/3
e^c	1	1	+1

Since the lh fermions are either in doublets or in singlets of $SU(2)_L$ the simplest (only?) way to have a gauge invariant Yukawa coupling is for the scalars themselves to be doublets.

In order to have enough of them one real doublet is not enough, but two doublets (or one complex doublet) can possibly do the job. It is then easy to show that the minimal scalar-field structure needed is the one shown in the next table:

Quantum numbers of one SM family

	SU(3)	SU(2)	$U(1)_Y$
$(u,d) = Q$	3	2	1/6
$(\nu, e) = L$	1	2	-1/2
u^c	3*	1	-2/3
d^c	3*	1	+1/3
e^c	1	1	+1
$(\phi^+, \phi^0) = \Phi$	1	2	1/2

+ the c.c. fields, including $\Phi^* = (\phi^{0*}, \phi^-)$

The following gauge-invariant Yukawa interactions are possible:

$$L^{Yukawa}_{quarks} = -\lambda^u \Phi Q u^c - \lambda^d \Phi^* Q d^c$$
$$= -\lambda^u(\phi^0 u u^c + \phi^+ d u^c) - \lambda^d(\phi^- u d^c + \phi^{0*} d d^c)$$

$$L^{Yukawa}_{leptons} = -\lambda^e \Phi^* L e^c = -\lambda^e(\phi^- \nu e^c + \phi^{0*} e e^c)$$

We shall see in a moment how these couplings generate masses for the fermions as a consequence of SSB.

For the rest it's enough to follow the general procedure. Gauge invariance (once more) restricts the Higgs potential to consist of a mass term (possibly with the «wrong» sign) and a quartic interaction with the right sign.

$$V^{Higgs} = \mu^2 \Phi^* \Phi + \lambda (\Phi^* \Phi)^2 \; ; \; \mu^2 < 0 \; ; \; \lambda > 0$$

Such a potential induces an expectation value for Φ which, without lack of generality, can be taken as:

$$\langle (\phi^+, \phi^0) \rangle = (0, v) \; , \; \langle (\phi^{0*}, \phi^-) \rangle = (v, 0) \; , \; v = \sqrt{\frac{-\mu^2}{2\lambda}}$$

Inserting these VEV's into the Yukawa interaction terms generated mass terms for the u and d quark and for the electron:

$$L_{quarks}^{Yukawa} = -\lambda^u \Phi Q u^c - \lambda^d \Phi^* Q d^c$$
$$= -\lambda^u(\phi^0 u u^c + \phi^+ d u^c) - \lambda^d(\phi^- u d^c + \phi^{0*} d d^c)$$

$$L_{leptons}^{Yukawa} = -\lambda^e \Phi^* L e^c = -\lambda^e(\phi^- \nu e^c + \phi^{0*} e e^c)$$

$$\langle(\phi^+, \phi^0)\rangle = (0, v) \ , \ \langle(\phi^{0*}, \phi^-)\rangle = (v, 0)$$

generates: $$m_u = \lambda^u v \ , \ m_d = \lambda^d v \ , \ m_e = \lambda^e v$$

The full one-family Lagrangian

We can finally put all the pieces together and write down the full SM Lagrangian for one family of quarks and leptons. The philosophy here is that everything that is not forbidden should be allowed.

$$L_{SM} = L_{Gauge} + L_{Kinetic} + L_{Yukawa} + L_{Higgs}$$

$$L_{Gauge} = -\frac{1}{4}\sum_a F^a_{\mu\nu}F^a_{\mu\nu}$$

$$L_{Kinetic} = i\bar{\Psi}\gamma^\mu D_\mu \Psi + \frac{1}{2}D_\mu \Phi^* D^\mu \Phi$$

$$L_{Yukawa} = \lambda_Y \left(\Phi \Psi_\alpha \Psi_\beta \varepsilon_{\alpha\beta}\right) + c.c.$$

$$L_{Higgs} = -\mu^2 \Phi^* \Phi - \lambda(\Phi^*\Phi)^2$$

Let us comment on each term separately

The gauge kinetic term

$$L_{Gauge} = -\frac{1}{4}\sum_a F^a_{\mu\nu}F^a_{\mu\nu}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f^a_{bc} A^b_\mu A^c_\nu = -F^a_{\nu\mu}$$

Note that the sum over the generators of the gauge group runs over 12 values: 8 for SU(3), 3 for SU(2), and 1 for U(1).

There will be, correspondingly, three gauge couplings g_3, g_2, g_1, appearing in the covariant derivatives.

(It is possible -and actually nicer- to redefine the gauge fields by A --> g A in such a way that the coupling constant disappears from the covariant derivative and appears instead as a factor g^{-2} in front of the above gauge kinetic terms)

The fermionic and bosonic kinetic terms

$$L_{Kinetic} = i\bar{\Psi}\gamma^{\mu}D_{\mu}\Psi + \frac{1}{2}D_{\mu}\Phi^{*}D^{\mu}\Phi$$

Again, each covariant derivative contains a sum over all the generators of the gauge group. These drop out for fermions or bosons which are neutral wrt a particular subgroup: e.g. the covariant derivative of u^c contains only the SU(3) and U(1) gauge fields and that of e^c only the latter.

The above terms generate trilinear couplings of bosons and fermions to a single gauge boson and a quartilinear coupling involving two scalars and two gauge fields

When we replace Φ by its VEV this last piece of the lagrangian gives rise to a mass matrix for the original gauge bosons. The physical gauge bosons are the eigenstates of that mass matrix.

The Yukawa couplings

$$L_{Yukawa} = \lambda_Y \Phi \Psi_\alpha \Psi_\beta \varepsilon_{\alpha\beta} + c.c.$$

This writing is a bit symbolic. Φ stands for either Φ or Φ^* and the two fermions for pairs of l.h. fermions (the c.c. contains only r.h. fermions) consisting of one doublet and one singlet of SU(2).

When we replace Φ by its VEV this piece of the lagrangian gives, in general, a big fermionic mass matrix. The physical fermions are the linear combinations of the original ones that diagonalize that mass matrix.

The scalar potential

$$L_{Higgs} = -\mu^2 \Phi^* \Phi - \lambda (\Phi^* \Phi)^2$$

This is the most general (renormalizable) Higgs potential that we can write. For $\mu^2 < 0$ it generates a VEV for Φ whose detailed consequences will be analyzed momentarily

So far the neutrino has remained massless. There is actually a striking asymmetry between the Yukawa interactions of quarks and leptons:

$$L^{Yukawa}_{quarks} = -\lambda^u \Phi Q u^c - \lambda^d \Phi^* Q d^c$$
$$= -\lambda^u(\phi^0 u u^c + \phi^+ d u^c) - \lambda^d(\phi^- u d^c + \phi^{0*} d d^c)$$

$$L^{Yukawa}_{leptons} = -\lambda^e \Phi^* L e^c = -\lambda^e(\phi^- \nu e^c + \phi^{0*} e e^c)$$

Why? The asymmetry is due to the absence of a ν^c. So, why did we not include such a fermion? The answer was/is: it does not seem to matter since ν^c, being a singlet wrt the whole gauge group, would be completely non-interacting.

Let's then have another look at the table!

L and Φ can be combined into a gauge singlet but this, being a fermion, cannot appear as part of the Lagrangian

	SU(3)	SU(2)	$U(1)_Y$
$(u,d) = Q$	3	2	1/6
$(\nu, e) = L$	1	2	-1/2
u^c	3*	1	-2/3
d^c	3*	1	+1/3
e^c	1	1	+1
$(\phi^+, \phi^0) = \Phi$	1	2	1/2

+ the c.c. fields, including $\Phi^* = (\phi^{0*}, \phi^-)$

Let's then add another row for ν^c!

	SU(3)	SU(2)	$U(1)_Y$
(u,d) = Q	3	2	1/6
(ν, e) = L	1	2	-1/2
u^c	3*	1	-2/3
d^c	3*	1	+1/3
e^c	1	1	+1
ν^c	1	1	0
$(\phi^+, \phi^0) = \Phi$	1	2	1/2

Now we can add a $\Phi L \nu^c$ Yukawa interaction! ν^c is no longer free.
Not only: even a straight mass term $M\nu^c\nu^c$ is now possible!

A more symmetric Yukawa structure in the presence of a «r.h. neutrino» (better: a total singlet fermion)

$$L^{Yukawa}_{quarks} = -\lambda^u \Phi Q u^c - \lambda^d \Phi^* Q d^c$$
$$= -\lambda^u (\phi^0 u u^c + \phi^+ d u^c) - \lambda^d (\phi^- u d^c + \phi^{0*} d d^c)$$

$$L^{Yukawa}_{leptons} = -\lambda^\nu \Phi L \nu^c - \lambda^e \Phi^* L e^c$$
$$= -\lambda^\nu (\phi^0 \nu \nu^c + \phi^+ e \nu^c) - \lambda^e (\phi^- \nu e^c + \phi^{0*} e e^c)$$

to which we add

$$L^{quad}_{\nu^c} = i\bar{\nu}^c \gamma^\mu \partial_\mu \nu_c - M\, \nu^c_\alpha \nu^c_\beta \varepsilon_{\alpha\beta} + c.c$$

Nature apparently makes use of this opportunity. Neutrinos have mass and mixing of different neutrino species has been observed (march 14 lecture/seminar)