Particules Élémentaires, Gravitation et Cosmologie
Année 2007-'08

Le Modèle Standard et ses extensions

Cours VIII: 29 février 2008

Further consequences of the Standard Model

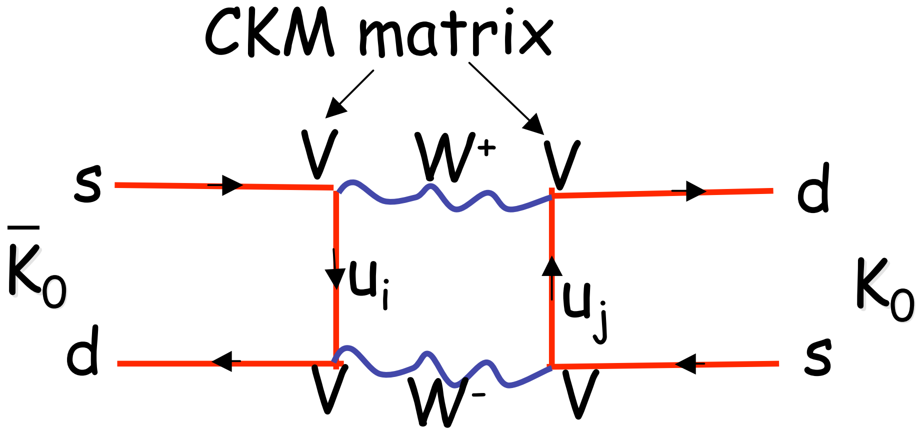
List of topics

1. The K_0-$\bar{K}_0$ system
2. Mixing in the leptonic sector
3. Neutrino oscillations
4. Accidental symmetries of the SM
5. Renormalizability of the SM
6. An example of radiative corrections: the ρ parameter

The K_0-$\bar{K}_0$ system

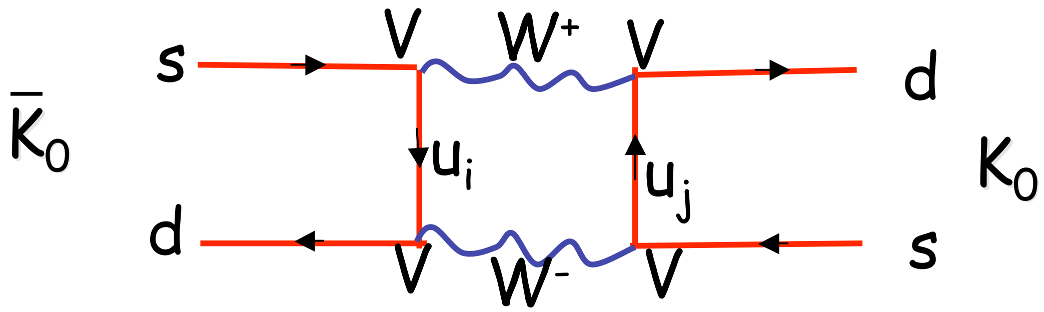
The K_0-$\bar{K}_0$ system is very interesting. In the absence of weak interactions K_0 and $\bar{K}_0$ are degenerate eigenstates of the Hamiltonian ($m \sim 497.7$ MeV) and cannot mix since strangeness is a good quantum number (an $s\bar{d} = \bar{K}_0$ cannot become an $\bar{s}d = K_0$)

The weak interactions, at tree level, do not induce any such transition either (no FCNC!) but, at one-loop, they do (of course at a very low level). Here is one of the two Feynman diagrams responsible for the mixing



The actual calculation involves an "easy" EW part and a difficult QCD part (basically done numerically)

The K_0-$\bar{K}_0$ system (cont.d)



Neglecting CP violation, the eigenstates are just the CP eigenstates $K_{1,2} \sim K_0 \pm \bar{K}_0$. K_1 is CP even and can decay in 2 pions while K_2 is CP odd and decays in 3 pions. The former has a shorter lifetime ($\tau \sim 10^{-10}$ s) and became known as K_S, the latter ($\tau \sim 5 \times 10^{-8}$ s) as K_L. Until it was found that, occasionally, also K_L decays into 2 pions, violating CP. Because of the possible phase in V, our diagram can give the observed amount of CP violation. The eigenstates, K_S and K_L, are not quite the CP eigenstates, K_1 and K_2 (see next week's lectures/seminars)

K_0-$\bar{K}_0$ oscillations (cont.d)

Consider a strong interaction process like $\pi^- p \to K\Lambda$. Because of strangeness conservation by the strong interactions the produced kaon is a pure K_0. How does this state evolve in time? Let us neglect CP violation. Then, at t=0, we start with:

$$|K_0;t=0\rangle = \frac{1}{\sqrt{2}}\left(|K_1;t=0\rangle + |K_2;t=0\rangle\right)$$

At a later time, t, the energy eigenstates $K_{1,2}$ will have picked a phase $\exp(-iE_{1,2}t)$ so that:

$$|K_0;t\rangle = \frac{1}{\sqrt{2}}\left(e^{-iE_1 t}|K_1;t=0\rangle + e^{-iE_2 t}|K_2;t=0\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\, e^{-iEt}\left(e^{-\frac{i}{2}\Delta E t}|K_1;t=0\rangle + e^{+\frac{i}{2}\Delta E t}|K_2;t=0\rangle\right)$$

$$\textit{where}\quad E = \frac{E_1+E_2}{2}\,,\ \Delta E = E_1 - E_2$$

K_0-$\bar{K}_0$ oscillations (cont.d)

$$|K_0;t\rangle = \frac{1}{\sqrt{2}}\left(e^{-iE_1t}|K_1;t=0\rangle + e^{-iE_2t}|K_2;t=0\rangle\right)$$
$$= \frac{1}{\sqrt{2}}\, e^{-iEt}\left(e^{-\frac{i}{2}\Delta Et}|K_1;t=0\rangle + e^{+\frac{i}{2}\Delta Et}|K_2;t=0\rangle\right)$$
$$where \quad E = \frac{E_1+E_2}{2}\,,\; \Delta E = E_1 - E_2$$

We now re-express the state at time t in terms of the original strangeness eigenstates and find that (up to an overall irrelevant phase) the original K_0 after a time t has evolved into a coherent superposition of K_0 and $\bar{K}_0$:

$$|K_0;t=0\rangle \rightarrow cos\left(\frac{\Delta Et}{2}\right)|K_0\rangle - i\, sin\left(\frac{\Delta Et}{2}\right)|\bar{K}_0\rangle$$

K_0-$\bar{K}_0$ oscillations (cont.d)

$$|K_0; t=0\rangle \to cos\left(\frac{\Delta E t}{2}\right)|K_0\rangle - i\, sin\left(\frac{\Delta E t}{2}\right)|\bar{K}_0\rangle$$

This means that, after a time t, there is a probability $\sin^2(\Delta E\, t/2)$ for the K_0 to have turned into a $\bar{K}_0$.

How large is the time τ_{osc} for which this probability is 1?

Clearly $\tau_{osc} = \pi/\Delta E$.

ΔE depends on the mass difference Δm between the two eigenstates (actually $\Delta E \sim \Delta m\, m/E$). Experimentally (and, to some approximation, even theoretically):

$\Delta m \sim 3.5\times10^{-6}$ eV ($\sim 5\times10^{9}$ Hz in natural units). This gives $\tau_{osc} \sim 2\times10^{-10}$ s (a few centimeters if $v/c = O(1)$)

Oscillations and CP violation have now been observed in other systems, in particular in B_d = bd and B_s = bs mesons with $\tau_{osc} \sim 1$ ps & 10^{-1} ps, respectively.

Mixing in the leptonic sector?

If neutrinos are massless there is no analog of the CKM matrix in the leptonic sector since there is no way to distinguish neutrino current and mass eigenstates.

But, if there is a non-trivial neutrino mass matrix, the leptonic sector is not qualitatively different from the quark sector.

It turns out however that, quantitatively, the situation is quite different.

The 3-family neutrino mass matrix
(« see-saw » case)

The relevant mass terms in the lagrangian are just

$$L_{\nu-mass} = -\sum_{i,j=1}^{3}\left(v\,\nu_i\lambda_{ij}^{(\nu)}\nu_j^c + \frac{1}{2}\nu_i^c M_{ij}\nu_j^c\right) + c.c.$$

For large M we can "integrate out" the ν^c neutrino and get:

$$L_{\nu-mass} = \frac{1}{2}\sum_{i,j=1}^{3}\nu_i M_{ij}^{(\nu)}\nu_j + c.c.$$

$$M^{(\nu)} = v^2\lambda^{(\nu)}M^{-1}\lambda^{(\nu)T} = V^{(\nu)T}M_{diag}^{(\nu)}V^{(\nu)}$$

The physical neutrinos correspond to the mass eigenstates:

$\tilde{\nu}_i = V_{ij}^{(\nu)}\nu_j$ with masses given by ${}^{(\nu)}M_{diag}$

Mixing in the charged leptonic currents

Recalling that the physical leptons are given by:

$$\tilde{\nu}_i = V_{ij}^{(\nu)} \nu_j \qquad M_{ij}^{(e)} = v\lambda_{ij}^{(e)} = (V_L^{(e)T} M_{diag}^{(e)} V_R^{(e)})_{ij} \; ; \; \tilde{e}_i = (V_L^{(e)})_{ij} e_j$$

we see that, again, there is no flavour change in the neutral leptonic currents, while, in the charged currents we find:

$$\begin{aligned} L_{Ch.\ lep.\ Curr.} &= ig\bar{\tilde{\nu}} V^{(\nu)} \gamma^\mu W_\mu^+ V_L^{(e)\dagger} \tilde{e} + c.c. \\ &= ig\bar{\tilde{\nu}} V_l \gamma^\mu W_\mu^+ \tilde{e} \\ V_l = V^{(\nu)} V_L^{(e)\dagger} &, \; V_l V_l^\dagger = 1 \end{aligned}$$

V_l is the leptonic analog of the CKM matrix for the quarks

Neutrino oscillations
(details in two weeks!)

In a typical weak process (say $\pi \to \mu\nu_\mu$ decay) one produces, to begin with, a current eigenstate, say a ν_μ. How does such a state evolve in time? Let us proceed like we did for the kaons and write the initial state as

$$\nu_i = V_{ij}^{(\nu)\dagger}\tilde{\nu}_j \qquad |\nu_i, t=0\rangle = V_{ij}^{(\nu)\dagger}|\tilde{\nu}_j, t=0\rangle$$

with i=2 in our example

After some time the state will have evolved into

$$|\nu_i, t\rangle = \sum_j V_{ij}^{(\nu)\dagger} e^{-iE_j t}|\tilde{\nu}_j, t=0\rangle$$

As for the neutral kaons we can project this state back into the current eigenstates and find the oscillation amplitudes

Neutrino oscillations (cont.d)

In general the result is messy, particularly if all 3 neutrino species mix in an appreciable way. For the case of a two-neutrino mixing problem (probably a good approximation for both the solar and the atmospheric neutrinos) one finds e.g.:

$$A(\nu_\mu(t=0) \to \nu_\mu(t)) = cos\left(\frac{\Delta E t}{2}\right) + i\, cos(2\theta)\, sin\left(\frac{\Delta E t}{2}\right)$$
$$A(\nu_\mu(t=0) \to \nu_e(t)) = i\, sin(2\theta)\, sin\left(\frac{\Delta E t}{2}\right)$$

where θ is the mixing angle (for the kaons we had $\theta = \pi/4$).

For neutrinos we can use the ultra-relativistic limit, $\Delta E \sim \Delta(m^2)/2E$. The oscillation phase becomes

$$\Delta(m^2)t/4E = 1.267\ (ct/Km)(GeV/E)\ \Delta(m^2)/(eV)^2$$

or x/L with $L \sim 0.8Km\ (E/GeV)\ (eV)^2/\Delta(m^2)$

Neutrino oscillations (cont.d)

$$A(\nu_\mu(t=0) \to \nu_\mu(t)) = cos\left(\frac{\Delta E t}{2}\right) + i\, cos(2\theta)\, sin\left(\frac{\Delta E t}{2}\right)$$

$$A(\nu_\mu(t=0) \to \nu_e(t)) = i\, sin(2\theta)\, sin\left(\frac{\Delta E t}{2}\right)$$

Note that there is no mixing without masses, but also that the oscillations depend only on mass^2 differences. We need other data to pin down absolute values for (or limits on) the masses themselves (to be discussed later in the course).

We also need, of course, non-vanishing mixing angles. In the quark sector the CKM mass matrix is almost diagonal. The surprise was that in the leptonic sector mixing angles are large, almost maximal, pointing at a very different origin (see-saw?) of the neutrino mass matrix. Again, more later…

Accidental symmetries of the Standard Model

The SM Lagrangian has automatically (accidentally) some global symmetries due to the structure of all possible terms that we can write down (in the low energy limit).

For instance, in the absence of neutrino masses, one can easily check that lepton flavour (i.e. separate e, μ and τ lepton number) is conserved. The small neutrino masses should induce very small lepton flavour violations while preserving total lepton number L. Lepton-flavour violating transitions are being looked for (e.g. in $\mu \to e\, \gamma$) so far unsuccessfully (see next week for limits)

In the quark sector, only the overall baryon number B (# of quarks - # of antiquarks) is conserved.

Note: When radiative corrections are added one finds that B and L are no longer strictly conserved individually (weak instantons break it but very mildly) while the combination B-L is still OK.

Renormalizability of the Standard Model

The GSW electroweak theory started to be taken seriously only when, around 1973, 't Hooft (based on previous work with Veltman, shared 1999 Nobel prize) showed that, like QED and QCD, also the EW theory a la GSW is renormalizable.
Let us recall the definition of renormalizability. Loop/radiative corrections are usually infinite or, if we introduce a suitable UV cutoff, depend sensitively on it.
A theory is called renormalizable if all these UV-sensitive contributions can be reabsorbed in a redefinition of the original (tree-level) parameters of the theory. The above parameters are not calculable, even if they had been given at tree level, have been taken from the data, but everything else is in principle predicted. Yet another reason for allowing in the classical Lagrangian everything that is not forbidden.

Renormalizability of the SM (cont.d)

Fermi's old model is not renormalizable

The proof of renormalizability is quite complicated but it basically amounts to showing that the phenomenon of SSB does not spoil renormalizability (SSB is what distinguishes the EW theory from its predecessors).

In Sid Coleman's words (Erice school lectures) SSB is a "secret symmetry" while the renormalizability of the EW theory amounts to a "secret renormalizability "

By now there are many explicit calculations of radiative correction that fully confirm the renormalizability of the EW theory and allow to compute finite corrections to many observables (see later lecture on precision tests)

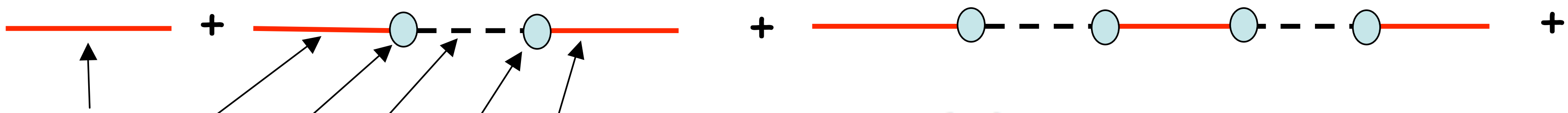
As an example of a radiative correction let us consider the one that concerns the already mentioned ρ parameter.

Radiative corrections: an example

Recall its definition:

$$\rho \equiv \frac{m_W^2}{\cos^2\theta_W m_Z^2} = 1 \text{ , } \textit{at tree level}$$

A simple way to see how the masses of W and Z are generated consists in summing diagrams like:

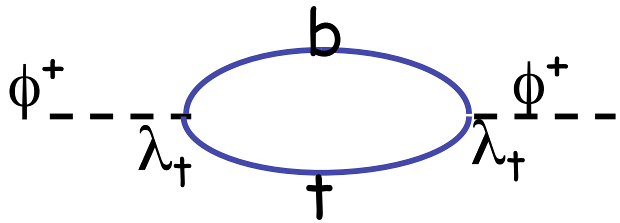


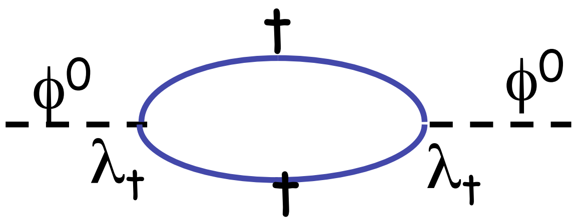
$$\frac{1}{q^2} + \frac{1}{q^2}\frac{gvq_\mu}{\sqrt{2}}\frac{1}{q^2}\frac{gvq_\mu}{\sqrt{2}}\frac{1}{q^2} + \cdots = \frac{1}{q^2}\left(1 + \frac{g^2v^2}{2q^2} + \ldots\right) = \frac{1}{q^2 - g^2v^2/2}$$

This gives the correct tree-level (no-loop) result for the W mass. Similarly, one obtains the Z mass and $\rho = 1$.

Radiative corrections: an example

At one loop the scalar propagator gets a correction due to a fermionic loop where the most important contribution comes from the heaviest, b and t, quarks (recall: $\lambda_i \sim m_i$)





The charged Higgs couples to a tb pair while the neutral one couples to tt. The Higgses remain massless but the residues of the poles at $q^2=0$ start to differ at one loop order. As a result, the corrections are different in the neutral and charged channels. A rather easy calculation gives, for $m_b/m_t \ll 1$,

$$\rho = 1 + \frac{3\lambda_t^2}{32\pi^2} = 1 + \frac{3m_t^2}{32\pi^2 v^2} \sim 1.01$$

A small but observable effect that led to an estimate of m_t before its discovery...

La suite…

- 7 march:
  - Professor R. Barbieri (Scuola Normale Superiore, Pisa) on « Flavour Dynamics and CP Violation »
- 14 march:
  - Professor F. Feruglio (University of Padua) on « Neutrino Masses, Mixing, and Oscillations »
- 21 march: no course (Paques)
- 28 march: courses resumes (last on 11 April)