Particules Élémentaires, Gravitation et Cosmologie
Année 2007-'08

Le Modèle Standard et ses extensions

Cours XI: 11 avril 2008

MSSM (2nd part) & Grand Unified Theories

Plan

1. The MSSM (part 2)
    - Getting rid of undesired SUSY terms
    - Adding soft SUSY breaking
    - Pluses and minuses of the MSSM
    - The NMSSM

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2. Grand-Unified Theories: general considerations
3. SU(5)
4. SU(5) phenomenology:
    - Relations among couplings and masses
    - Proton decay
5. GUT vs. SUSY-GUT

Chiral matter superfields in the MSSM
(NB: vector superfields fixed by gauge group)

	SU(3)	SU(2)	$U(1)_Y$
$(u, d)_i = Q_i$	3	2	1/6
$(\nu, e)_i = L_i$	1	2	-1/2
$u^c{}_i$	3*	1	-2/3
$d^c{}_i$	3*	1	+1/3
$e^c{}_i$	1	1	+1
$\nu^c{}_i$	1	1	0
$(\phi_u{}^+, \phi_u{}^0) = \Phi_u$	1	2	1/2
$(\phi_d{}^0, \phi_d{}^-) = \Phi_d$	1	2	-1/2

+ the c.c. antichiral superfields

General form of a SUSY gauge theory

$$\begin{aligned}
L^{(SUSY\ gauge\ theory)} &= L^{gauge-kin} + L^{matt-kin.} + L^{Potential} \\
L^{gauge-kin} &= L^{gauge} + L^{gaugino} \\
L^{matt-kinetic} &= L^{kin} + L^{matt-gaug-Yukawa} - V^{D} \\
L^{Potential} &= L^{mass} + L^{matt-Yukawa} - V^{F}
\end{aligned}$$

There is no freedom in choosing the first two lines (D-terms)
There is a priori some freedom in choosing the third (F-terms)

Getting rid of undesired terms

In the SM gauge invariance automatically excludes unwanted terms in the Lagrangian. The same is not true for the MSSM.

Here are the undesired terms

a. Recall that L and Φ (now Φ_u) can be combined into a gauge singlet. However, this was not a Lorentz invariant object. With superfields nothing seems to forbid an $L\Phi_u$ mixing in an F-term and consequently violation of lepton number

b. A $\mu\Phi_u\Phi_d$ bilinear (so-called μ–term) is allowed. It gives a supersymmetric (positive definite) mass^2 to the Higgs field and cannot be too large (if not there is no SSB!).

c. At the level of trilinear (Yukawa etc.) terms we find very dangerous couplings (LQd^c, LLe^c, $u^cd^cd^c$) that violate lepton and baryon number and would give rise to fast proton decay.

An way out is to introduce, from the outside, a discrete symmetry, called R-parity (R = ± 1).

The two members of a supermultiplet have opposite R-parity. While the particles of the SM have all R=+1, their partners have R=-1. Imposing such a symmetry has two interesting effects:

1. Eliminates all the dangerous terms leaving only the possibility of a $\mu\Phi_u\Phi_d$ (μ can be naturally small being protected from loop corrections by a chiral symmetry)
2. Makes the lightest SUSY partner (LSP) stable. Such an LSP is an excellent candidate for the dark matter that represents about 22% of the total energy of the Universe (see RB's seminars)

To discuss the phenomenology associated with the MSSM we have to introduce some form of (spontaneous or explicit) SUSY breaking.

A spontaneous breaking of SUSY may be problematic because it implies the existence of a massless goldstino. In the case of SB of local SUSY (i.e. of Supergravity) the goldstino is "eaten up" by the gravitino and gives it a mass (2+2=4). At low energy this kind of spontaneous SUSY breaking manifests itself, effectively, as an explicit soft breaking.

We will discuss SUSY breaking in the language of explicit soft breaking keeping in mind that a deeper understanding of SUSY breaking can strongly constrain its detailed form.

Soft SUSY breaking

As we have emphasized, in exact SUSY there are no quadratically divergent correction to bosonic masses (since there cannot be for their degenerate fermionic partners)

Q: Can we add SUSY breaking terms and still preserve this nice feature? A: Yes, they defines softly-broken SUSY. There are 4 kinds of interesting soft SUSY breaking terms:

1. Gaugino masses:

$$\Delta L_{gaugino} = -M_1\lambda^{(1)}\lambda^{(1)} - M_2\lambda^{(2)}\lambda^{(2)} - M_3\lambda^{(3)}\lambda^{(3)}$$

2. Scalar masses:

$$\Delta L_{scalars} = -\tilde{m}_u^2\phi_u^*\phi_u - \tilde{m}_d^2\phi_d^*\phi_d - (\tilde{m}_Q^2)_{ij}\tilde{Q}_i^*\tilde{Q}_j - \ldots$$

3. A so-called B term: $\Delta L_B = -B\mu\phi_u\phi_d$
4. So-called A-terms:

$$\Delta L_A = -A_{ij}\phi_u\tilde{Q}_i\tilde{u}_j^c + \ldots$$

MSSM phenomenology

Successes

1. All the SUSY partners (bosonic & fermionic) can be made sufficiently heavy
2. The LSP is a good candidate for dark matter
3. The (lightest) Higgs boson is naturally light
4. Grand-Unification works much better (see below)

Problems

1. The (lightest) Higgs boson is naturally too light
2. Large increase in the number of parameters
3. Flavour conservation is not at all automatic

The lightest Higgs and SUSY's fine-tuning problem

Consider the Higgs potential in the MSSM:

$$\begin{aligned} V_{Higgs} &= \left(m_u^2+|\mu|^2\right)|\Phi_u|^2+\left(m_d^2+|\mu|^2\right)|\Phi_d|^2-B\mu\left(\Phi_u\Phi_d+h.c.\right) \\ &+ \frac{1}{8}(g_1^2+g_2^2)\left(|\Phi_u|^2-|\Phi_d|^2\right)^2+\frac{1}{2}g_2^2|\Phi_u|^2|\Phi_d|^2 \end{aligned}$$

Thus in the MSSM the quartic Higgs coupling λ is of order g^2
Recalling that v is fixed by G_F , that $m_{W,Z} \sim g\, v$, and that $m_H \sim \lambda^{1/2}v$, we arrive at the conclusion that $m_H \sim m_{W,Z}$
A detailed calculation actually gives an interesting upper bound on the tree-level mass of the lightest MSSM Higgs particle:

$$M_{lightest\ Higgs} \leq M_Z|cos\ 2\beta|\ ,\ tan\beta = \frac{v_u}{v_d}$$

The lightest Higgs and SUSY's fine-tuning problem

This is already excluded by LEP2. Paradoxically, in the MSSM the Higgs from too heavy became too light!
Can loop corrections save the MSSM? Yes, provided the "stop" is heavy enough (~ 1 TeV). The one loop contribution to the mass^2 of the lightest Higgs scales like:

$$\Delta M^2_{lightest\ Higgs} \sim m_t^2 log(\frac{m_{\tilde{t}}^2}{m_t^2})$$

Unfortunately, such a heavy stop mass implies some fine-tuning in order to keep the Higgs mass parameter small enough (loop contributions ~ m^2_{stop}, not unexpectedly).
Disappointing, in view of our motivations for SUSY....

The flavour problem

There is no reason in general of why, after SUSY breaking, the squark mass matrix should be aligned with the quark mass matrix. FCNC may thus appear at a problematic level.

The solution?

A large, universal bosonic mass term with small flavour-dependent corrections (justified in some Supergravity scenarios). Yet, one might have expected some of these effects to have shown up in the data (see again RB's seminars)

The NMSSM

In order to overcome the fine-tuning problems one can add a total singlet superfield S (to be distinguished from the one containing the r.h. neutrino) and replace the μ-term by two trilinear coupling (we are talking about the superpotential!):

$$\mu \Phi_u \Phi_d \dashrightarrow c\, S \Phi_u \Phi_d + c'\, S^3$$

This gives an extra contribution to the quartic coupling λ_{eff} in the Higgs potential and therefore to the Higgs mass…

My personal opinion on low-energy SUSY:

Taking into account the present experimental situation, the SUSY solution to the hierarchy problem looks more contrived than elegant. Furthermore, low-energy SUSY appears to bring up more questions than provide new answers. Too pessimistic?

Grand Unification

Motivations for Grand Unified Theories (GUT) are quite obvious when one considers the following aspects of the SM:

- G_{SM} is the direct product of 3 factors => 3 gauge couplings
- Fermions are in a highly (5 or 6 x) reducible representation
- The number of free parameters is large

The GUT idea: look for a simple gauge group (not a direct product) G_{GUT} that contains G_{SM} as a subgroup; use a Higgs-mechanism to break spontaneously G_{GUT} to G_{SM} at some very high energy scale M_{GUT}.
The physics will be that of the SM at scales $E \ll M_{GUT}$ but the higher symmetry above M_{GUT} will imply some constraints among the SM parameters.

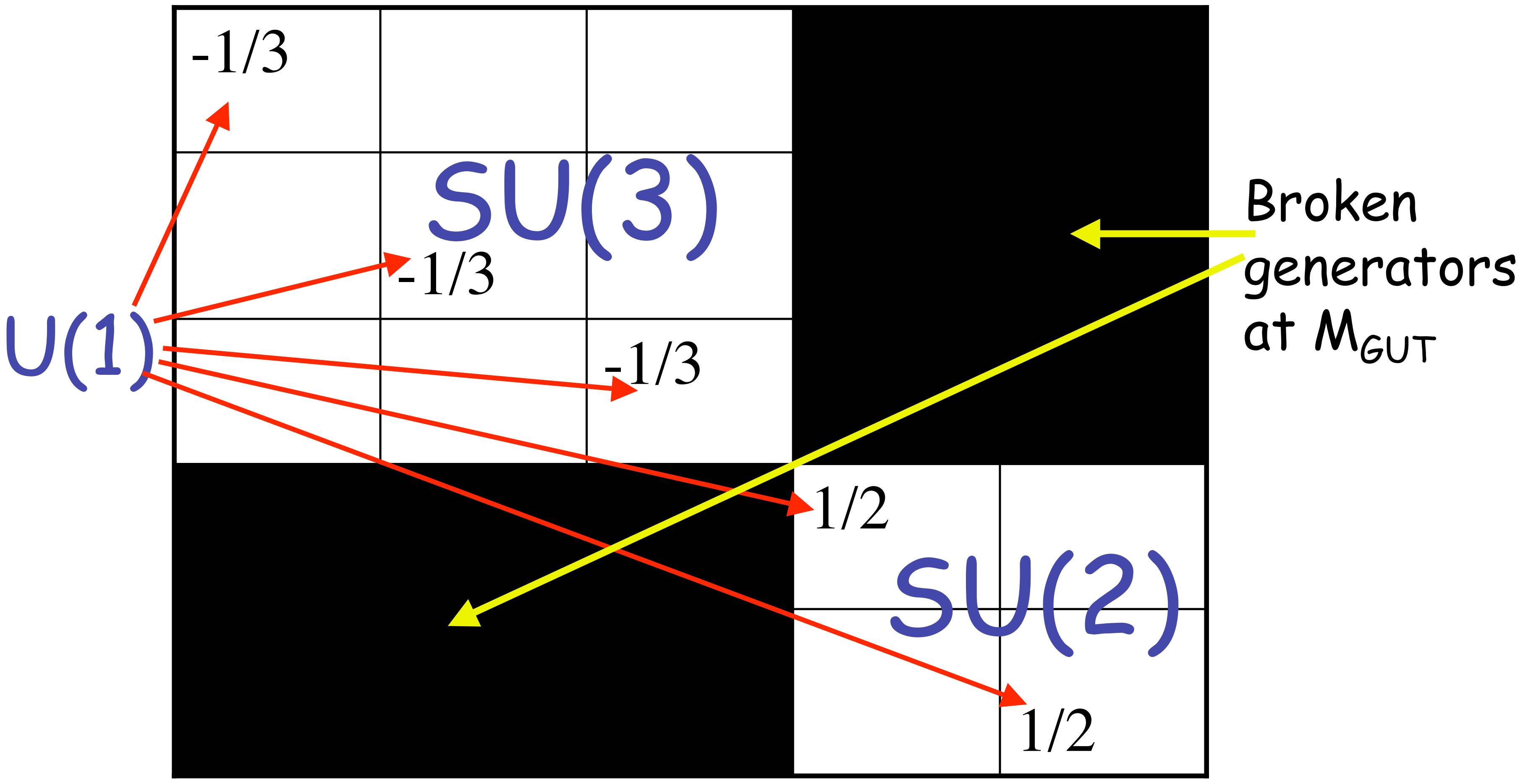
SU(5)

The first and simplest GUT goes back to work by Georgi and Glashow in 1974. It identifies G_{GUT} with the group SU(5).

SU(5) is the only group that has the same rank (maximal number of commuting generators) as G_{SM} (it cannot be smaller) and has complex representations (otherwise it cannot give the complex rep. of the SM).
The rank of SU(5) is 4 (N-1 for SU(N), 1 for U(1) => 4 for SM)

There is an obvious way to embed the SM in SU(5). Identify the $SU(3)_c$ and the $SU(2)_L$ generators with the first 3 and last 2 row and columns of the 5x5 generators of SU(5). The $U(1)_Y$ generator is then automatically identified (up to an important normalization factor).

Embedding SU(3)xSU(2)xU(1) in SU(5)



SU(5) representations

Simplest representations of SU(5) are the 5, the 10, the 15, their c.c. (5*, 10*, 15*) and the adjoint, 24 =24*. Forgetting the r.h. neutrino there are 15 fermions in the SM. Naive guess, use the 15, does not work (get fermions in the 6 of SU(3)!)

Consider instead the 5*. Under the SM it decomposes as (3*, 1, 1/3) +(1, 2, -1/2). These are just the q.n. of our d^c and L! The 5 will give the corresponding r.h. antifermions…

Take now the 10 = $(5\mathrm{x}5)_a$ (a = antisymmetrized) and use:

[(3, 1, -1/3)+(1, 2, 1/2)]x [(3, 1, -1/3)+(1, 2, 1/2)] =

(3*, 1, -2/3) +(3, 2, 1/6) + (1,1,1). These are u^c, Q and e^c !

A r.h. neutrino can be added as an SU(5) singlet.

The SSB od SU(5) to the SM is done through a new Higgs field in the 24 while the “low-energy” SM breaking needs the SM Higgs to be part of a 5+5*…

Matter fields in the one-family SM

	SU(3)	SU(2)	$U(1)_Y$
$(u, d) = Q$	3	2	1/6
$(\nu, e) = L$	1	2	-1/2
u^c	3*	1	-2/3
d^c	3*	1	+1/3
e^c	1	1	+1
ν^c	1	1	0
$(\phi^+, \phi^0) = \Phi$	1	2	1/2

+ the c.c. fields, including $\Phi^* = (\phi^{0*}, \phi^-)$

SU(5) phenomenology

1. Unification of couplings:

There is a single gauge coupling, g_5 . For the non abelian couplings this gives $g_2 = g_3$. For U(1) we have to normalize the $U(1)_Y$ generator wrt the SU(5) generators and we get $g_1^2 = 3/5\ g_2^2$ (easily converted into $\sin^2\theta_W = 3/8 = 0.375$). Experimentally $\sin^2\theta_W \sim 0.22$.

Are these problems? Not necessarily! These are the ratios of the couplings at M_{GUT}. Since couplings “run” we have to fix M_{GUT} and then compute the “low-energy” couplings and check with the data.

At the time (1974) this looked very promising with $M_{GUT} \sim 10^{15}$ GeV. With the present precision data there are significant discrepancies (see below)

2. Unification of masses

There are only two possible Yukawas (5*x5*x10 and 5x10x10) instead of the SM's three. One gets relations between d-type quarks and charged lepton masses like $m_e = m_d$, etc. These too are supposedly valid at M_{GUT} and one has to evolve them to low-energy before deciding whether there is a problem. Well, there is…

3. Proton decay

The broken generators of SU(5) change leptons into quarks and vice versa. They violate baryon number. Since the corresponding gauge bosons get a very large mass O(M_{GUT}) these baryon number violating processes are suppressed but, at some level, they induce a very "dangerous" process, proton decay.

In SU(5) the dominant decay mode would be $p \to e^+ \pi^0$. The lifetime τ_p grows with M_{GUT} but for $M_{GUT} \sim 10^{15}$ GeV it is still too short.

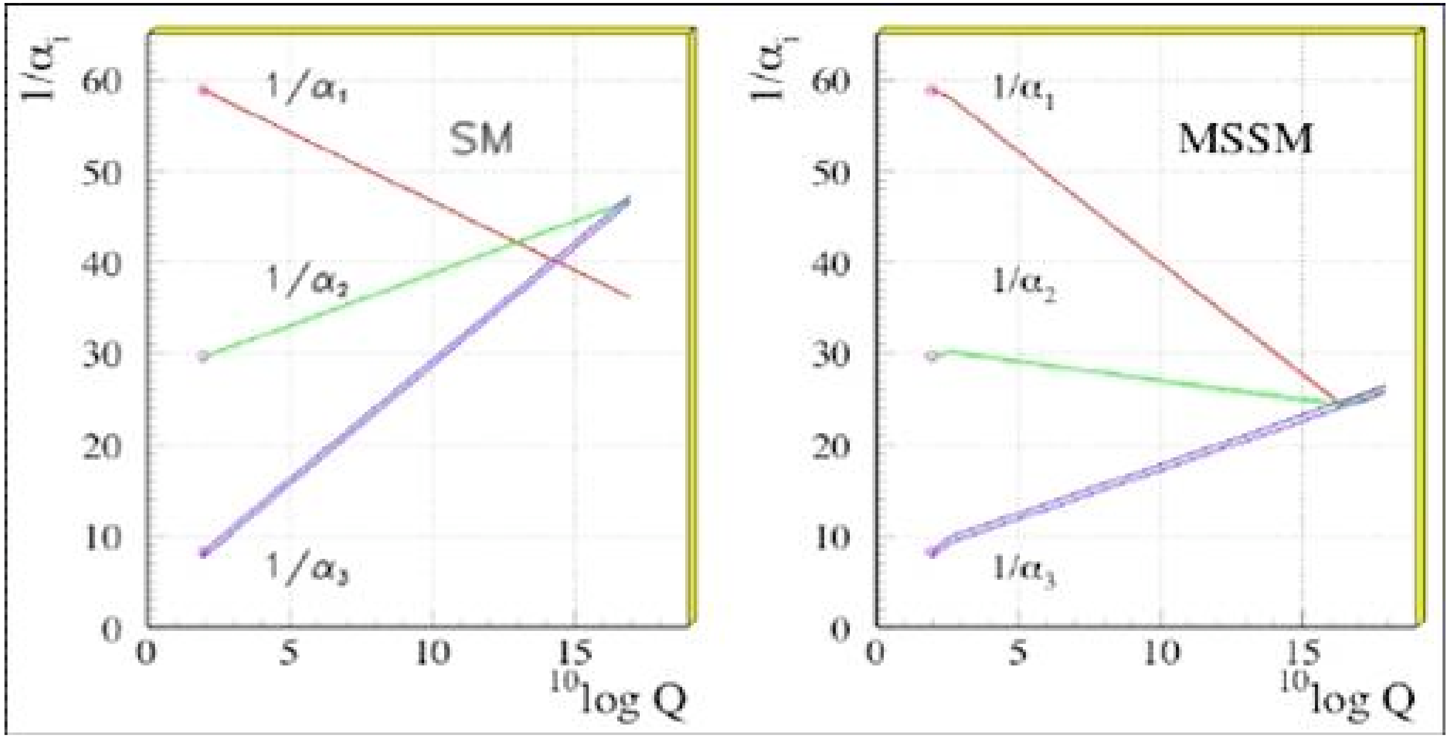
Present lower bound on τ_p is ~10^{33} years (Cf. age of Universe ~ 10^{10} years) and might be improved by almost 2 orders of magnitude soon (see RB's seminar)

Other GUTs?

Since SU(5), in spite of its theoretical appeal, does not seem to work well, people turned to other (larger) groups like O(10) (with fermions in the 16 of SO(10) neatly giving 5*+10 +1 of SU(5)!), E6, ..
The phenomenological problems can be alleviated but not completely eliminated

All this was done without SUSY. How do things change if we consider SUSY GUTs? One clear improvement is in the unification of gauge couplings.

SM and MSSM gauge coupling unification



Clearly, with the present precise knowledge of the gauge couplings MSSM unification is favoured. This and the possibility of the LSP as dark matter are the strongest arguments for SUSY…while waiting for the LHC verdict…

Conclusion on the whole course (plus the two given on QCD)

The SM of elementary particles is one of the greatest achievements of theoretical and experimental physics.
It tells us that 3 (out of the 4) fundamental forces, the EM, the strong and the weak force have a deep common denominator, they are all described by gauge theories.
Their obvious phenomenological differences can be simply attributed to how the gauge symmetry is "realized":

- In a Coulomb phase for EM
- In a Higgs phase for the Weak
- In a Confining phase for the Strong

But the SM is sufficiently "baroque" to suggest that it cannot be the end of the story. Can we find something better/nicer? Theorists do not lack imagination, but the SM is hard to beat!