E. RABINOVICI

II

HAMILTONIAN

FIXED TIME

TASK

$$H\psi_E = E\psi_E$$

FIND E AND ψ_E.

* $$\mathcal{L} = \frac{m\dot{x}^2}{2} - V(x)$$

$$p_x = \frac{\delta \mathcal{L}}{\delta \dot{x}} = m\dot{x}$$

$$H = p\dot{x} - \mathcal{L}(x, \dot{x}) \qquad \dot{x} = \dot{x}(p_x, x)$$

$$H = \frac{p_x^2}{2m} + V(x), \quad \psi = \psi(x)$$

$$\psi(p)$$

THAT WAS ONE PARTICLE CLASS.

QUANTUM

$$[p, x] = -i\hbar$$

$$p = -i\hbar \frac{d}{dx}$$

$$H\psi = E\psi \qquad \psi(x), \psi(p)$$

(**) 2 PARTICLES

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V(x_1) + U(x_2) + U(x_1, x_2)$$

$$\psi(x_1, x_2) \qquad [p_i, x_j] = -i\delta_{ij}$$

$$\psi(x_1, \cdots x_n)$$

(***) N PARTICLES

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \sum_i V(x_i) + \sum_{i,j} V(x_i, x_j) + \cdots$$

$$\psi(x_1, x_2, \cdots x_N) \equiv [p_j, x_i] = -i\hbar\delta_{ij}$$

(*) FINE TUNE TO GET RELATIVISTIC SPECTRUM

$$H = \sum_i \frac{p_i^2}{2} + V(x_i) + \sum_i (x_i - x_{i-1})^2$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$H = \sum_i \frac{\pi_i^2}{2} + \frac{1}{2}(\phi_i - \phi_{i-1})^2 + V(\phi_i)$$

$p \to \pi$ $\qquad [\pi_i, \phi_j] = -i\hbar\delta_{ij}$

$x \to \phi$ $\qquad \pi_i = -i\hbar \frac{\delta}{\delta\phi_j}$

$$\psi(x_1, \cdots x_N) = \psi(\phi(x_i))$$

FUNCTIONAL

(*) Q.E.D.

D.O.F $\quad A_\mu(x) \quad$ (TOO MUCH) 4

ONLY 2 POLARIZATIONS FOR PHOTON

$$A_\mu \equiv A_\mu + \partial_\mu \alpha(x)$$

$A_0 = 0 \qquad A_i \qquad$ 3 D.O.F

(TOO MUCH)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\Pi_{A_i(x)} = \frac{\delta \mathcal{L}}{\delta \dot{A}_i(x)} = F_{0i} = E_i(x)$$

$$\psi(A_i(x))$$

$$[E_i(x), A_j(y)] = -i\hbar\, \delta_{ij}\, \delta^{(3)}(x-y)$$

$$H = \frac{1}{2}\int d^3x\, (E^2 + B^2) =$$

$$= \frac{1}{2}\int d^3x \left(E^2(x) + (\nabla \times A)^2(x) \right)$$

$$\pi^2 \qquad (\phi_i - \phi_j)^2$$

$$p^2 \qquad V(x_i, x_{i-1})$$

$\underline{\nabla}\,\underline{E}$ GENERATES RESIDUAL SYMMETRIES

$$A_0 = 0 \rightarrow A_0 = 0 \quad A_i \rightarrow A_i + \partial_i \alpha$$

$$\frac{\partial \alpha}{\partial t} = 0$$

$[H, \underline{\nabla}\,\underline{E}(x)] = 0$ FOR ANY x

$$[\underline{\nabla}\underline{E}(x), \underline{\nabla}\underline{E}(y)] = 0$$ (ANOMALIES*)

$$\underline{\nabla} \cdot \underline{E}\,\psi(A_i) = 0$$ 2 D.O.F ! PER X

O.K.

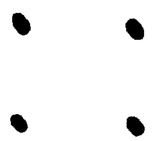
$$H\,\psi(A_i) = E\,\psi(A_i)$$

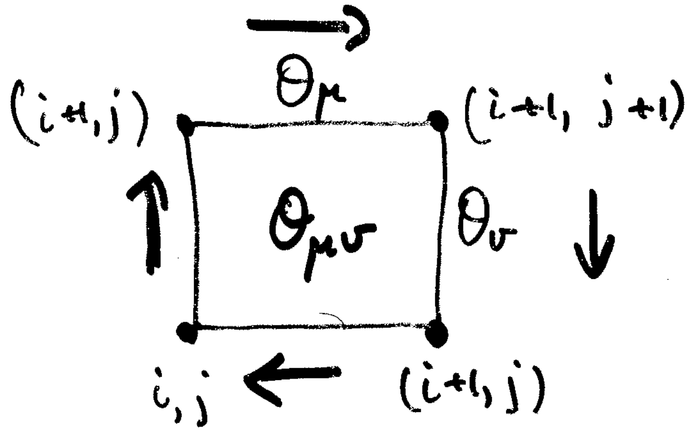
$$\underline{\nabla} \cdot \underline{E}(x)\,\psi(x) = 0$$

$(\underline{\nabla}\underline{E} \to \underline{\nabla}\underline{E} - \rho$ WITH MATTER$)$

LATTICE FOR $U(1)$

$$\mathcal{L} = -\frac{1}{e^2} \sum_{\underline{i},\mu,\nu} (\cos\theta_{\mu\nu}(\underline{i}) - 1)$$





$$\theta_{\mu\nu} = \theta_\mu(i+1, j) - \theta_\mu(i, j) + \theta_\nu(i, j) - \theta_\nu(i+1, j+1)$$

$$U(i, j; i+1, j) = \exp(i\theta_\nu(i, j))$$

$$H = \frac{1}{2}\sum_{\underline{i}, k} E_k^2(\underline{i}) + \frac{1}{g^2}\sum_{\underline{i}} (\cos\theta_{\mu\nu}(\underline{i}) - 1)$$

$k = x, y, z$ $\quad$ $\mu, \nu = 1, 2, 3$ NOT 0 !

$$[E_k(\underline{i}), \theta_\mu(\underline{j})]$$

$$= -i\hbar\, \delta_{k\mu}\, \delta_{\underline{i}, \underline{j}}$$

NOTE θ_μ COMPACT

$$H = \sum_{\underline{i}} \left(E_k^2(\underline{i}) - \frac{1}{g^2} \left(\cos \theta_{ij}(\underline{i}) - 1 \right) \right)$$

$$\psi(\theta_i(\underline{i}))$$

$$\longrightarrow \quad H_{g \to \infty} = \sum_{i} E_k^2(\underline{i})$$

$$\frac{p^2}{2m} \qquad p \text{ QUANTIZED}$$

(*) SYMMETRY "RESTORED"

$$\psi(x) \to \psi(p)$$

$$\psi(\theta_i(\underline{i})) \to \psi(E_k(\underline{i}))$$

$$\bullet\ \underline{n_1}\ \bullet\ \underline{n_2}\ \bullet\ \underline{n_3}\ \bullet\ \underline{n_4}\ \bullet\ \underline{n_5}\ \bullet$$

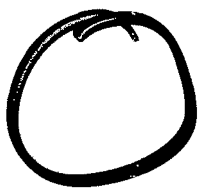
BUT $\underline{\nabla}\,\underline{E}\, \psi(E) = 0$!

$$n_1 = n_2 = \cdots = n_N = n$$

$(\cos\theta_{\mu\nu} - 1)$ NOT $\theta_{\mu\nu}^2$

$U(1)$ NOT R^1

QM ON A CIRCLE



p QUANTISED!

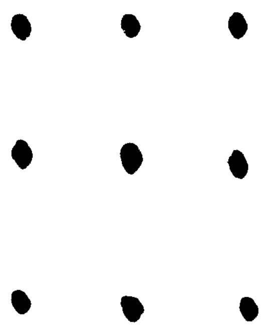
& SO IS $E_p(\underline{i})$.

A LIMIT –

STRONG COUPLING $g \to \infty$

$(\cos\theta_{\mu\nu} - 1)$ BOUNDED!

NO SOURCES SO STATES ARE



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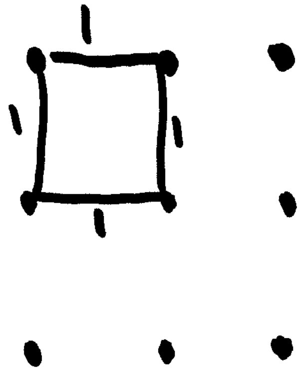

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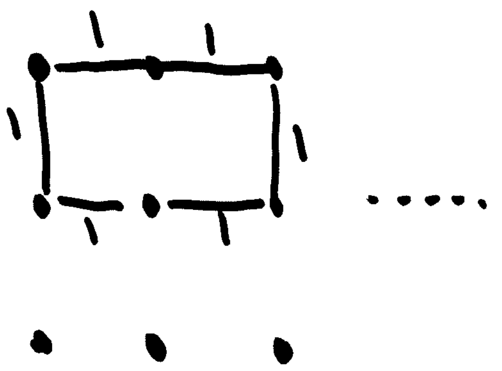

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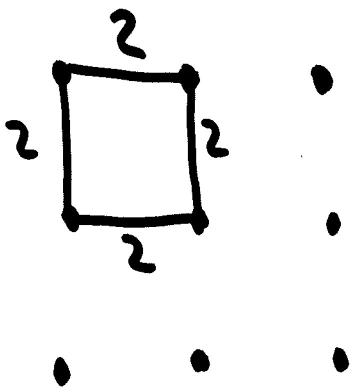

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STATES CLOSED LOOPS OF LENGTH L, n FLUX

ENERGY $L \cdot n^2$

GROUND STATE $|0, 0, 0\rangle$

$E = 0$

INSERT TWO CHARGES $m = \infty$

SUM ZERO CHARGE

DISTANCE L (LATTICE UNITS)

CALCULATE

$$E_{g.s.}\Big(\overset{-1}{\cdot}\;\overbrace{\cdot \quad \cdot \quad \cdot \quad \cdot}^{L}\;\overset{1}{\cdot}\Big) - E_{g.s.}(\text{EMPTY}) =$$

$$\underline{\nabla}\,\underline{E} = 0 \rightarrow$$

$$\underline{\nabla}\,\underline{E} = \rho$$

$$-1 \qquad\qquad\qquad \cdot 1$$

$$-1 \xrightarrow{1} \cdot \qquad\qquad \cdot 1$$

$$-1 \xrightarrow{1} \cdot \xrightarrow{1} \qquad \cdot 1$$

$\Downarrow$

DIV E=0

$$-1 \;\overset{1}{—}\cdot\overset{1}{—}\cdot\overset{1}{—}\cdot\overset{1}{—}\cdot\overset{1}{—}\; 1$$

$$= L \cdot 1 - 0 = L \quad \text{CONFINEMENT!}$$

+ * QUANTIZED FLUX IS THE SOURCE

(+) * $g = \infty$ NOT POINCARE INVARIANT

WILL IT SURVIVE

$g \neq \infty$, CONTINUUM LIMIT?

(+) * ALSO SU(N) IS COMPACT THUS QUANTIZED

(+) * WORKS FOR <u>ANY</u> DIMENSION!

P.T. $d=2$ $V(r) = r$ ✓

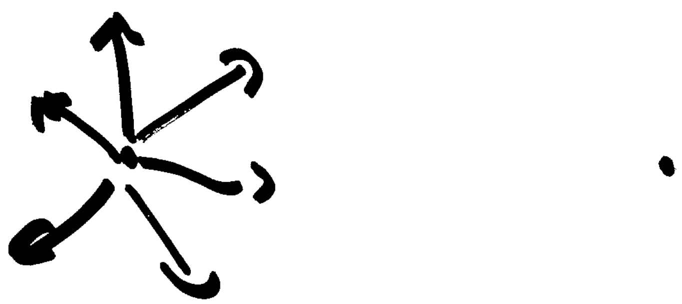
$d=3$ $\ln r \to r$ ✓

$d=4$ $\frac{1}{r} \to r$ ✓

$d=5$ $\frac{1}{r^2} \to$?

FLUX

AND NOT SPREADING



REMINDS OF DUAL HIGGS.

BACK TO THE LAGRANGIAN

$$\mathcal{L} = -\frac{1}{e^2}\sum_{i}\left(\cos\theta_{\mu\nu}(i) - 1\right)$$

$$Z = \int_0^{2\pi} D\theta_\mu \exp\left(-\int \mathcal{L}\, dt\, dx\right)$$

UNIVERSALITY - SAME SYMMETRY

$$\int_0^{2\pi} \prod D\theta_\mu \exp\left(-\frac{1}{e^2} \sum_i \left(\cos\theta_{\mu\nu}^{(i)} - 1\right)\right)$$

$$= \sum_{l_{\mu\nu}} \exp\left(-\tfrac{1}{2} e^2 \sum l_{\mu\nu}^2\right) \delta\left(\Delta_\nu l_{\mu\nu}\right) \}$$

↑

"MONOPOLE"

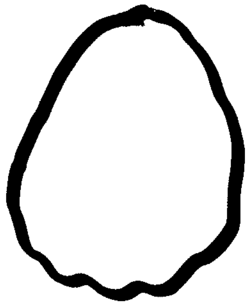
SOURCES.

$$m_\mu = \frac{1}{2} \varepsilon_{\mu\nu\lambda\sigma} \Delta_\nu l_{\lambda\sigma}$$

CURRENT

CAN BE "MAPPED" TO AN

EFFECTIVE PICTURE.



LOOPS HAVE ENERGY

$\exp(-L\, f(g))$

STRONGLY SUPPRESSED!

BUT MANY CONFIGURATIONS

$\exp(L *)$ ONES

(NON BACK TRACKING
RANDOM WALKS)

SO LIKE IN STAT. MECH

COMPETITION

$F = E - TS$

$T \ll$ "1" SMALL LOOPS...

$T \gg$ "1" LOOPS CONDENSE!!

$$\mathcal{L} = |\partial_\mu \phi|^2 - \underbrace{(\# - g^2 \#)}_{m^2} \phi^* \phi$$

COMPLEX FIELD (STRING THEORY)

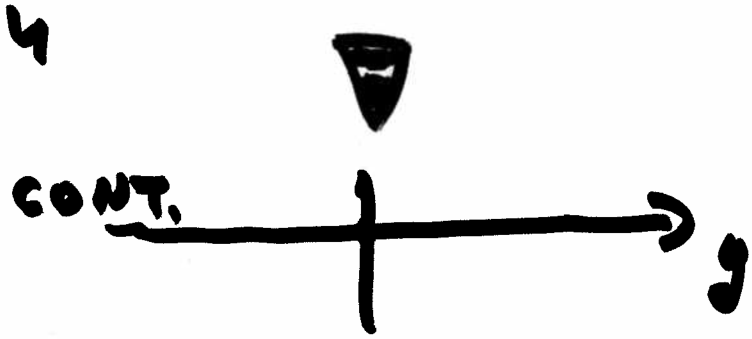
SMALL g COULOMB LIKE R

LARGE g CONDENSES

CONFINEMENT!

$d = 4$

U(1)



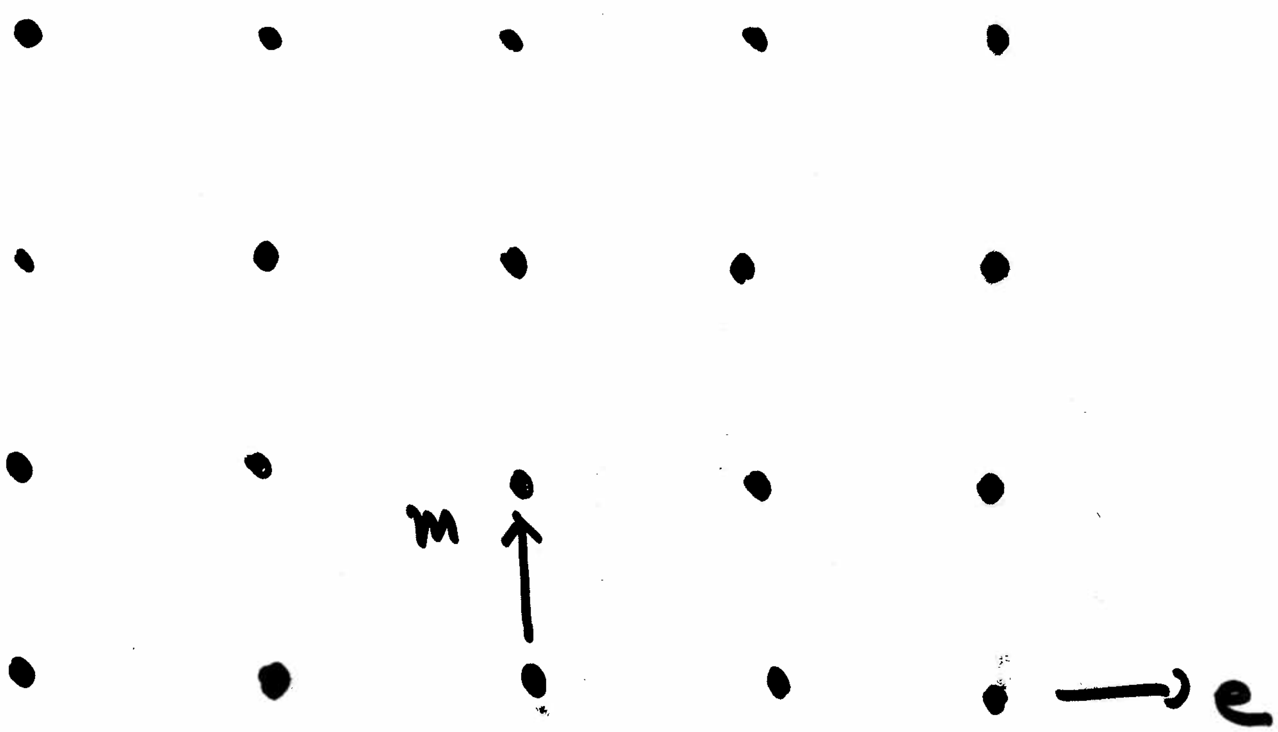
NO STOP SIGN FOR

ION-ABELIAN GAUGE THEORIES!

WITH MATTER CHARGED

LATTICE OF CHARGES OF

OPERATORS.



$$\frac{m^2}{T} + n^2 T$$

ELLIPSE IN (m,n)

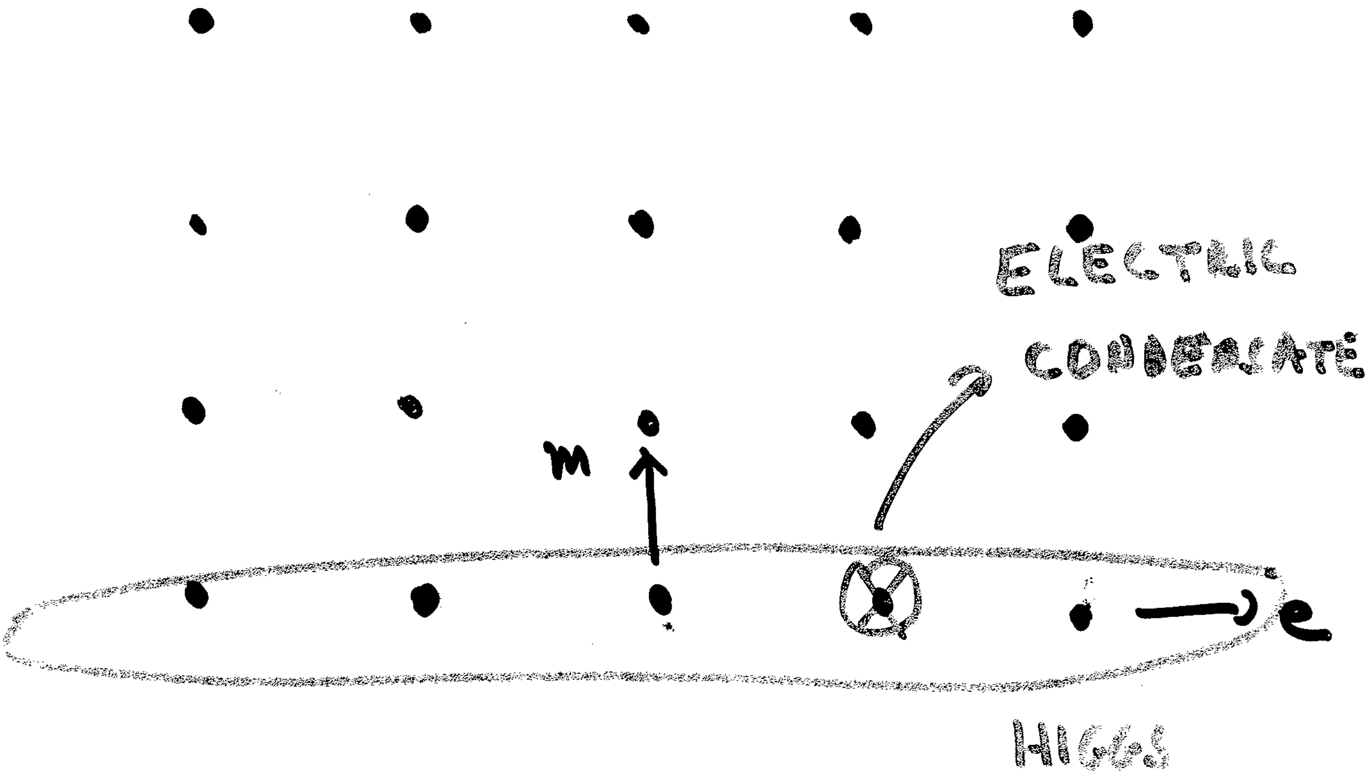
T COUPLING

TEMPERATURE

WITH MATTER CHARGED

LATTICE OF CHARGES OF

OPERATORS.



LOW T

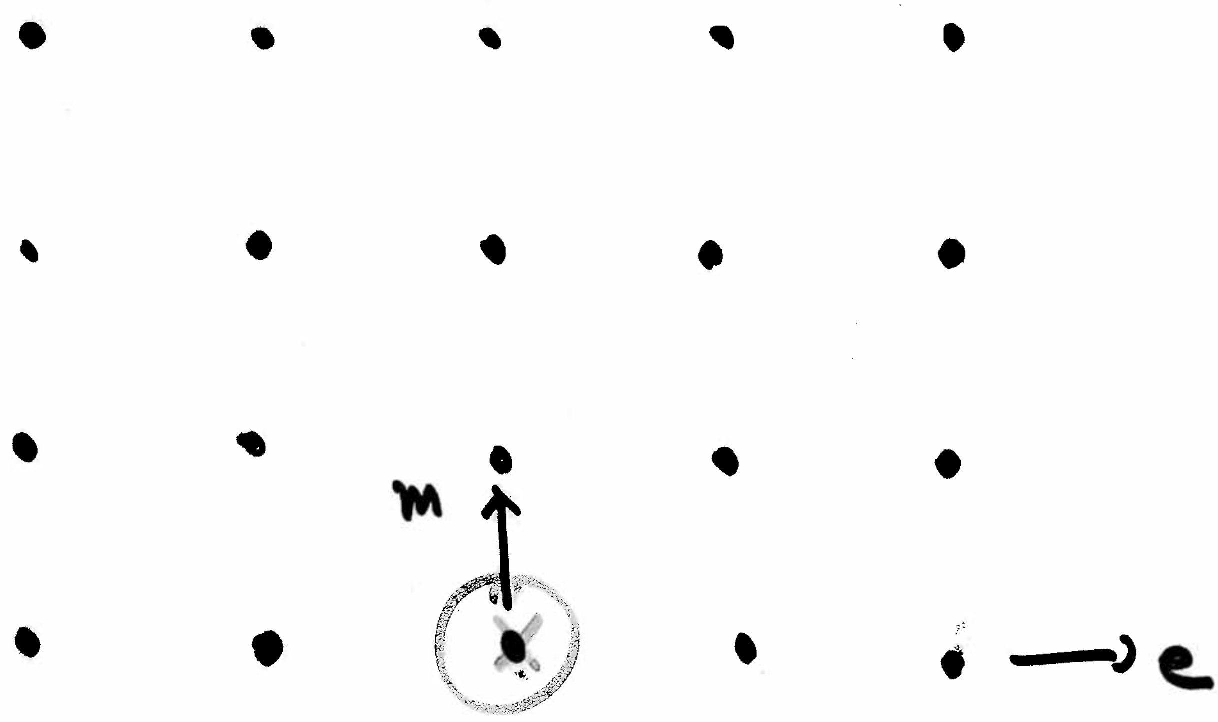
ELECTRIC CHARGE SCREENED

MAGNETIC " CONFINED

WITH MATTER CHARGED

LATTICE OF CHARGES OF

OPERATORS.



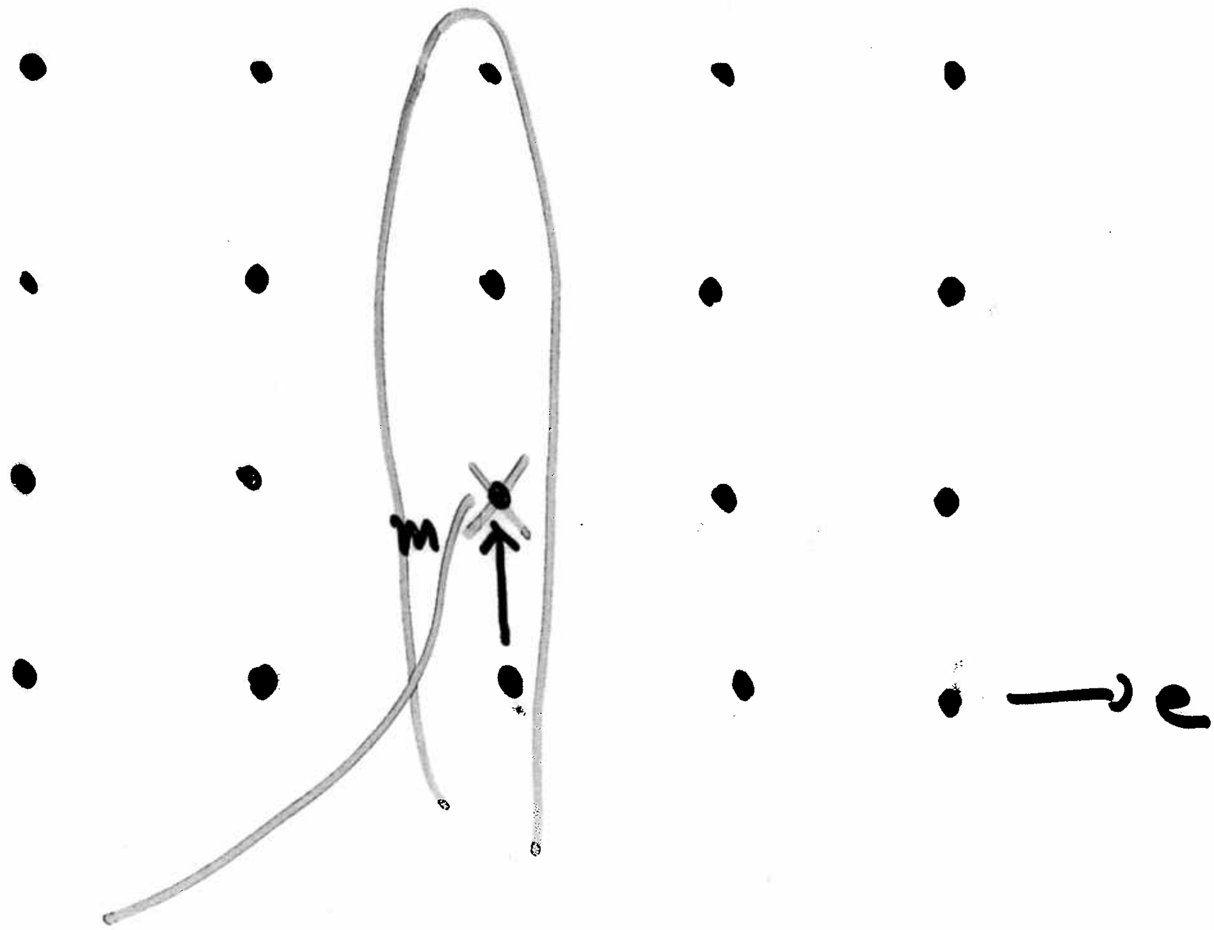
NOTHING CONDENSE!

COULOMB

WITH MATTER CHARGED

LATTICE OF CHARGES OF

OPERATORS.



MAGNETIC CONDENSATE

MAGNETIC CHARGE - SCREENED

ELECTRIC CHARGE - CONFINED

MORE STRUCTURE:

WITH $\theta F\tilde{F}$

$n = n_0 + \frac{\theta m}{2\pi}$ OBLIQUE

ELECTRIC

$n_0 = 0$ $m \neq 0$ IS E.CHARGED!

DYONS CAN CONDENSE.

NEW PHASES – NATURE?

HALL EFFECT.

LESS STRUCTURE

MATTER IN THE GROUP CENTR.

HIGGS – QUARKS

GAS – LIQUID

SUSY
CONFORMAL

PHASES OF GRAVITY