Particules Élémentaires, Gravitation et Cosmologie
Année 2007-'08

Le Modèle Standard et ses extensions

Cours VII: 29 février 2008

Adding families: GIM mechanism and CKM matrix

Reminder of last week

We started discussing how to use the Higgs mechanism in the specific context of the standard model and we argued in favour of introducing two $SU(2)_L$ Higgs doublets corresponding to four scalar fields.

We showed that three of them are «eaten up» by the $SU(2)_L \times U(1)_Y$ gauge fields thereby giving mass to the $W^\pm$ and to a linear combination of W^3 and B, the Z_0-vector boson, and leaving just the photon as the only surviving massless gauge boson (besides the QCD gluons!).

We computed, at tree-level, G_F, the $W^\pm$ and Z_0 masses, as well as the mass of the surviving Higgs-particle, H, in terms of the $SU(2)_L \times U(1)_Y$ gauge couplings (equivalently of α and θ_W), and of the two parameters, μ and λ, appearing in the Higgs potential (in particular, we related v to G_F and μ to m_H)

Example of tree-level prediction

4 parameters, 5 observables!

Parameter	Observable
g	e (α)
θ_W ($g'/g = tg\ \theta_W$)	m_W
μ	m_Z
v (λ)	G_F
	m_H

$$e = \sqrt{4\pi\alpha} = g\ sin\theta_W$$

$$m_W = \frac{gv}{\sqrt{2}}$$

$$m_z = \frac{m_W}{cos\theta_W}$$

$$G_F^{-1} = 2\sqrt{2}v^2$$

$$m_H = \sqrt{-2\mu^2}$$

Unfortunately the prediction is NOT m_H! It is instead a relation among 4 already measured quantities.

The relation reads:

$$\alpha = \frac{\sqrt{2}}{\pi} G_F m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right)$$

Inserting the presently measured values:
$G_F = 1.16637\ 10^{-5}\ \mathrm{GeV}^{-2}$
$m_W = 80.403$ GeV
$m_Z = 91.1876(21)$ GeV
gives $\alpha \sim 1/132$ (instead of $\sim 1/137$)

Not bad, but already showing that the tree-level predictions do not match experimental precision

Reminder of last week (cont.d)

We also computed the gauge couplings of the fermions (identifying a precise structure for the neutral weak currents), the fermion masses in terms of the Yukawa couplings and the corresponding (proportional) Higgs-fermion couplings.

We finally mentioned how to introduce masses for the neutrinos by adding a completely neutral fermion (the r.h. neutrino <=> l.h. antineutrino = ν^c but often denoted by N)

Let us list once more the fundamental fields

	SU(3)	SU(2)	$U(1)_Y$
(u,d) = Q	3	2	1/6
(ν, e) = L	1	2	-1/2
u^c	3*	1	-2/3
d^c	3*	1	+1/3
e^c	1	1	+1
ν^c	1	1	0
$(\phi^+, \phi^0) = \Phi$	1	2	1/2

plus the r.h. antifermions + Φ^*

For totally mysterious reasons (I. Rabi about the muon..) Nature was not happy with one family: She decided to replicate it three times (except possibly for ν^c)! In other words, the full SM list is actually (here i=1,2,3 is the so-called "family" index):

	SU(3)	SU(2)	$U(1)_Y$
$(u_i,d_i) = Q_i$	3	2	1/6
$(\nu_i, e_i) = L_i$	1	2	-1/2
u_i^c	3*	1	-2/3
d_i^c	3*	1	+1/3
e_i^c	1	1	+1
ν_i^c	1	1	0
$(\phi^+, \phi^0) = \Phi$	1	2	1/2

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e_i^c	1	1	+1
ν_i^c	1	1	0
$(\phi^+, \phi^0) = \Phi$	1	2	1/2

Particles of different families carry different letters and names: $(u_i; d_i) = (u, c, t\ ; d, s, b)$; $(\nu_i\ ; e_i) = (\nu_e, \nu_\mu, \nu_\tau\ ; e, \mu, \tau)$. Most of them have been detected and many of their properties have been measured. They seem to be just heavier copies of the first family.
There is evidence, from the Z_0 width and for astrophysics, that there are only three active light neutrinos => only three families? We do not know!

The real-world Lagrangian?

$$L_{SM}^{(3fam)} = L_{Gauge} + L_{Kinetic} + L_{Yukawa} + L_{H\,pot} + L_{mass}$$

$$L_{Gauge} = -\frac{1}{4}\sum_a F^a_{\mu\nu}F^a_{\mu\nu}$$

$$L_{Kinetic} = \sum_{i=1}^{3} i\bar{\Psi}_i\gamma^\mu D_\mu \Psi_i + D_\mu\Phi^* D^\mu\Phi$$

$$L_{Yukawa} = -\sum_{i,j=1}^{3} \lambda^{(Y)}_{ij}\Phi\Psi_{\alpha i}\Psi^c_{\beta j}\varepsilon_{\alpha\beta} + c.c.$$

$$L_{H\,pot} = -\mu^2\Phi^*\Phi - \lambda(\Phi^*\Phi)^2$$

$$L_{mass} = -\frac{1}{2}\sum_{i,j=1}^{3} M_{ij}\,\nu^c_{\alpha i}\nu^c_{\beta j}\varepsilon_{\alpha\beta} + c.c$$

Where is the news (wrt one-family)?

The most important changes occur in the Yukawa interactions and in the resulting fermionic mass matrix. For the former we now have the following structure:

$$L_{Yukawa} = -\sum_{i,j=1}^{3} \left(\lambda_{ij}^{(u)} \Phi Q_i u_j^c + \lambda_{ij}^{(d)} \Phi^* Q_i d_j^c + \lambda_{ij}^{(\nu)} \Phi L_i \nu_j^c + \lambda_{ij}^{(e)} \Phi^* L_i e_j^c \right)$$

Inserting the VEV of Φ induces the following fermionic mass terms:

$$L_{F.mass} = -v \sum_{ij=1}^{3} \left(u_i \lambda_{ij}^{(u)} u_j^c + d_i \lambda_{ij}^{(d)} d_j^c + \nu_i \lambda_{ij}^{(\nu)} \nu_j^c + e_i \lambda_{ij}^{(e)} e_j^c \right)$$

The fermionic mass matrix

We will neglect for a moment the neutrino sector and concentrate on the other three. We can always write:

$$M_{ij}^{(u)} = v\lambda_{ij}^{(u)} = (V_L^{(u)T} M_{diag}^{(u)} V_R^{(u)})_{ij}$$
$$M_{ij}^{(d)} = v\lambda_{ij}^{(d)} = (V_L^{(d)T} M_{diag}^{(d)} V_R^{(d)})_{ij}$$
$$M_{ij}^{(e)} = v\lambda_{ij}^{(e)} = (V_L^{(e)T} M_{diag}^{(e)} V_R^{(e)})_{ij}$$

where V_L, V_R are unitary 3x3 matrices and the three M_{diag} are diagonal 3x3 matrices with real positive entries.

A parenthetical remark: V_L, V_R include, in general, a $U(1)_A$ factor (i.e. $V_L = V_R = \exp(i\gamma) \times$(unit matrix)). This operation is not innocent for the quarks: it entails an anomaly, so that an overall phase of the quark masses cannot be eliminated: this is related to the strong-CP problem discussed in 2006.

The fermionic mass matrix (cont.d)

We thus conclude that the physical fermions, being the mass eigenstates, correspond the following linear combinations of the original fermions.

$$\tilde{u}_i = (V_L^{(u)})_{ij} u_j \ , \ \tilde{u^c}_i = (V_R^{(u)})_{ij} u^c_j$$

Then, ${}^{(u)}M_{diag}$, ${}^{(d)}M_{diag}$, ${}^{(e)}M_{diag}$ provide the masses of the 6 « physical » quarks and of the 3 « physical » charged leptons (in the case of quarks, masses are only measured indirectly).

The GIM mechanism

We now rewrite the gauge couplings of the original fermions ("current eigenstates") in terms of the physical ones (= mass eigenstates) using:

$$u = V_L^{(u)\dagger}\tilde{u}\ ,\ d = V_L^{(d)\dagger}\tilde{d}\ ;\ V_L V_L^\dagger = 1$$ and similarly for u^c, d^c.

We also recall: $$L_{Kinetic} = \sum_{i=1}^{3} i\bar{\Psi}_i\gamma^\mu D_\mu \Psi_i$$

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^- + W_\mu^- T^+) - i\frac{g}{cos\theta_W} Z_\mu (T^3 - sin^2\theta_W Q) - ieA_\mu Q$$

(to which we have to add the SU(3)/QCD piece of the covariant derivative).

We see that the replacement is trivial for all gauge couplings that do not mix u- and d-type quarks since, in that case,

The GIM mechanism (cont.d)

$$L_{Kinetic} = \sum_{i=1}^{3} i\bar{\Psi}_i\gamma^\mu D_\mu \Psi_i = i\bar{\tilde{\Psi}} V_L \gamma^\mu D_\mu V_L^\dagger \tilde{\Psi} = i\bar{\tilde{\Psi}}\gamma^\mu D_\mu \tilde{\Psi}$$

provided that V and V^+ carry the same label.

This is the case for the QCD interactions since gluons only couple quarks and antiquarks with the same flavour.

It is also true for the neutral part of the SU(2)xU(1) gauge fields, hence for electromagnetism and for the neutral weak currents.

In other words, QCD, QED and the neutral weak currents conserve flavour at tree level.

In particular there are no tree-level flavour changing neutral currents (FCNC)

The GIM mechanism (cont.d)

This is the famous Glashow, Iliopoulos, Maiani (GIM, 1969) mechanism, which led to the prediction of a 4th quark (discovered in 1974 and called c for charm). There was no way to avoid FCNC with just the u, d and s quarks!

A similar result is found in the couplings of the Higgs particle to the fermions. Again there are no tree-level flavour-changing couplings!

The CKM matrix

Consider finally the charged weak currents, corresponding to the exchange of the $W^{\pm}$ vector bosons. In that case

$$L_{Ch.\ q.Curr.} = -i\frac{g}{\sqrt{2}}\bar{\tilde{u}}V_L^{(u)}\gamma^\mu W_\mu^+ V_L^{(d)\dagger}\tilde{d} + c.c. = -i\frac{g}{\sqrt{2}}\bar{\tilde{u}}V\gamma^\mu W_\mu^+\tilde{d}$$

$$V = V_L^{(u)}V_L^{(d)\dagger}\ ,\ VV^\dagger = 1$$

V is the (equally famous) Cabibbo-Kobayashi-Maskawa (CKM) 3x3 unitary matrix telling us that the weak charged currents do mix different flavours. The same happens for the couplings of fermions to the (eaten-up) charged Higgses

The CKM matrix (cont.d)

$$L_{Ch.\ q.Curr.} = -i\frac{g}{\sqrt{2}}\bar{\tilde{u}}V_L^{(u)}\gamma^\mu W_\mu^+ V_L^{(d)\dagger}\tilde{d} + c.c. = -i\frac{g}{\sqrt{2}}\bar{\tilde{u}}V\gamma^\mu W_\mu^+\tilde{d}$$

$$V = V_L^{(u)}V_L^{(d)\dagger}\ ,\ VV^\dagger = 1$$

How many physical parameters do appear in the CKM matrix? A priori an NxN unitary matrix contains N^2 parameters. But are they all physically meaningful?

It is possible, for instance, to multiply each of the 2N fermionic fields by a phase. An overall phase does not change V but the remaining 2N-1 do. Hence only N^2-2N +1 parameters are physical.

These naturally split in two sets: N(N-1)/2 mixing angles (of, say, the d-type quarks) and (N-1)(N-2)/2 "phases".

The CKM matrix (cont.d)

Let us first consider the case of 2 families, $N=2$, corresponding to the two doublets (u, d), (c,s,). There is just one physical parameter, an angle: it is nothing but the famous Cabibbo angle (1967), telling us that the u quark couples, through a W, to the linear combination ($d' = \cos\theta_c d + \sin\theta_c s$).

By unitarity of V, in the absence of a third family, the c-quark couples to ($s' = \cos\theta_c s - \sin\theta_c d$).

The physical case, $N=3$, is particularly interesting. There are now a total of 4 physical parameters, 3 angles and a phase: the angles are simply a generalization of the Cabibbo angle corresponding to a rotation in (d,s,b) space. The phase, if non-vanishing, corresponds to having a complex V and thus a CP-violating interaction. Since CP-violation has been observed, it is most interesting that it can occur in the SM only if there are at least 3 families.

The actual CKM matrix
(a part from the phase)

At present, many entries of the CKM matrix have been measured and partial tests of its unitarity have been made (see later talks).

Presently estimated values are given below

V_{ij} =

	d	s	b
u	0.97383 (~cosθ_c)	0.2272 (~ sinθ_c)	0.00396
c	0.2271	0.97296	0.04221
t	0.00814	0.0416	0.99910