Particules Élémentaires, Gravitation et Cosmologie
Année 2009-'10

Théorie des Cordes: une Introduction
Cours IX: 12 mars 2010

Résumé des cours et séminaires précédents

Date	9h45-10h45	11h-12h
29/01	Strong interactions in the 60s	Strong interactions in the 60s
05/02	DHS duality and a bootstrap	A simple, exact solution

- I gave a brief historical introduction to the birth of the DRM as a way to get out of the impasse in which the theory of strong interaction was in the mid sixties.
- QFT methods looked completely inadequate.
- Regge-Chew-Mandelstam theory, coupled to DHS duality (1967), gave rise to a bootstrap program that ended in 1968 with the construction of an explicit closed-form solution (the B-function anzatz) for $\pi\pi \to \pi\,\omega$, soon extended to $\pi\pi \to \pi\pi$ scattering.

- We then discussed some properties of the 4-point function, its singularities (corresponding to zero-width resonances in its "planar" channels), its high-energy fixed-t (Regge) behaviour, and its high-energy fixed-angle behaviour.
- The duality properties are easily visualized by drawing duality diagrams. These also suggest the way to add internal quantum numbers for the external particles (via Chan-Paton factors).

Date	9h45-10h45	11h-12h
12/02	DRM: counting states, ghosts, operators, algebras	The no-ghost theorem, loops, D=26

- In order to understand the true spectrum "hidden" below the singularities (poles) of the Beta-function we need to generalize that amplitude to a process involving N external particles, the so-called N-point function, and then use a fundamental property of single-particle intermediate states: factorization of the corresponding pole's residue.
- We first discussed the expected singularities and dualities of the N-point function for each cyclic ordering of the external legs.

We then gave the N-point-function formula in the convenient form due to Koba and Nielsen:

$$B_N = \int_{-\infty}^{+\infty} \frac{\prod dz_i \theta(z_i - z_{i+1})}{dV_{abc}} \prod_{j>i} (z_i - z_j)^{2\alpha' p_i \cdot p_j}$$

Factorization of the N-point function is facilitated by introducing an "operator" formalism.

$$[q_\mu \ , \ p_\nu] = i\eta_{\mu\nu} \ , \quad [a_{n,\mu} \ , \ a^\dagger_{m,\nu}] = \delta_{n,m}\eta_{\mu\nu} \ , \quad \eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$$

$$(n = 1, 2, \ldots \ ; \ \mu = 0, 1, 2, \ldots D - 1)$$

$$[q_\mu \ , \ p_\nu] = i\eta_{\mu\nu} \ , \quad [a_{n,\mu} \ , \ a^\dagger_{m,\nu}] = \delta_{n,m}\eta_{\mu\nu} \ , \quad \eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$$

- The operator formalism clearly points to the danger that, in order to achieve factorization, one would need to introduce states of negative norm, so-called ghosts, which would then be produced in a scattering process with negative probability.
- Ghost states are obtained by applying the time component of the harmonic oscillator creation operators on the Fock vacuum.
- Fortunately, the full set of harmonic oscillator states is sufficient, but not necessary, to achieve full factorization. Some of the states decouple from the external states, others have zero-norm and thus are also unnecessary.

The "ghost hunting" project was a "tour de force" that culminated in the proof of a "no-ghost theorem" by Brower and by Goddard & Thorn.

At the basis of the theorem was the discovery of the Virasoro operators (needed to construct the spurious/ physical states) and of their algebra, the construction of the vertex operators and of an underlying Conformal Field Theory, and, finally, the explict construction of an infinite set of positive-norm physical (DDF) states.

There was a price to pay for the absence of ghosts: the Regge intercept, α_0, had to be exactly 1 (implying a massless spin one particle and a spin zero tachyon) and the dimensionality of spacetime had to be less than (or equal to) 26.

At exactly D=26 the physical Hilbert space would be completely spanned by the DDF states corresponding to oscillators in (D-2)=24 dimensions.

Meanwhile, C. Lovelace had shown that loops were consistent with unitarity only if D=26.

For D=26 and α_0=1 the model looked consistent except for the tachyon ($M^2 = -1/\alpha'$).

Date	9h45-10h45	11h-12h
19/02	Birth of string theory: NG action, LC quantization	Polyakov's CFT approach

After recalling how several hints of a string underlying DRM were missed, we finally came to define string theory.

We started from the construction of a geometric action (the Nambu-Goto action) that generalizes the action of a relativistic particle of mass m. In string theory, m is replaced by the string tension T, a quantity with dimensions of energy/length. Its inverse α' has dimensions of J/M^2.

$$S_{NG} = -T \int d^2\xi \sqrt{-det\gamma_{\alpha\beta}}$$

$$\gamma_{\alpha\beta} \equiv \frac{\partial X^\mu(\xi)}{\partial \xi^\alpha}\frac{\partial X^\nu(\xi)}{\partial \xi^\beta} G_{\mu\nu}(X(\xi)) \ , \quad \alpha, \beta = 0, 1 \quad , \quad \xi^0 = \tau, \ \xi^1 = \sigma$$

We compared the constraints for points and strings:

$$p_\mu(\tau)p_\nu(\tau)g^{\mu\nu}(x(\tau)) = -m^2$$

$$P_\mu(\xi)X'^\mu(\xi) = 0$$

$$P_\mu(\xi)P_\nu(\xi)G^{\mu\nu}(X(\xi)) + T^2 X'^\mu(\xi)X'^\nu(\xi)G_{\mu\nu}(X(\xi)) = 0$$

$$\dot{X}^\mu(\xi) \equiv \frac{\partial X^\mu(\xi)}{\partial \xi^0} \,, \quad X'^\mu(\xi) \equiv \frac{\partial X^\mu(\xi)}{\partial \xi^1}$$

They both come from the invariance of the action under reparametrization of the world line/sheet. From there on we worked in Minkowski spacetime and in the so-called orthonormal (or conformal) gauge in which the eom of the string becomes the d'Alembert equation:

$$\ddot{X}_\mu = X''_\mu$$ w/ solution: $$X_\mu(\sigma,\tau) = F_\mu(\tau-\sigma) + G_\mu(\tau+\sigma)$$

and the constraints read: $$(\dot{X} \pm X')^2 = 0$$

Next we discussed boundary conditions, particularly for open strings. They gave rise to two options (for each direction of space): Neumann (N) or Dirichlet (D)

$$\text{N} \qquad X'_\mu = 0 \quad , \quad \sigma = 0, \pi$$

$$\text{D} \qquad \dot{X}_\mu = 0 \quad , \quad \sigma = 0, \pi$$

At this point (concentrating on N-strings) we wrote down solutions to the eom and to the boundary conditions (but not yet to the constraints).

We also discussed a particular classical solution (which includes the constraints), the rigid, rotating rod which maximizes, classically, the ratio J/M^2 .

And, finally, we proceeded to the quantum theory.

In the lecture we did that by going to the light-cone gauge a further specification of the conformal gauge (the CG still leaves a residual gauge freedom under separate reparametrizations of $\tau \pm \sigma$ = conformal transformations).

$$X^+(\sigma,\tau) \equiv \frac{X^0 + X^{D-1}}{\sqrt{2}} = 2\alpha' p^+ \tau \qquad (\dot{X} \pm X')^2 = 0$$

In the L.C. gauge we can easily express all the oscillators in terms of the transverse ones (with D-2 components). The theory has a positive norm Hilbert space but manifest Lorentz invariance is lost. Imposing the Lorentz algebra gives back the two DRM conditions: D=26 and α_0=1.

In the seminar PDV reformulated string theory starting from the so-called Polyakov action and discussed a more elegant method of quantization, which keeps manifest Lorentz invariance at the price of introducing Fadeev-Popov ghosts (these are good ghosts not to be confused with those we were trying to kill:they are anticommuting fields and are supposed to cancel against the bad ghosts).

In order to proceed, one introduces a BRST (Grassmann) symmetry and charge Q, satisfying $Q^2 = 0$. Physical states are those annihilated by Q. Physical operators commute with Q.

At the end of some non-trivial calculations one arrives at a the construction of a finite Q operator (and at defining a positive norm space of physical states) but only, again, iff $D=26$ and $\alpha_0=1$.

The conclusion is that, at least at the level of the spectrum, DRM and string theory (independently of the gauge used) are equivalent provided D=26 and α_0=1.

This is not surprising since the physical dof of a string are transverse to it and this was only a property of DRM (completeness of DDF states) for D=26.

We also counted the number of physical string states and introduced the concept of a limiting (Hagedorn) temperature ~ due to the exponential growth of the spectrum.

Full equivalence, including the scattering amplitudes, can be established with quite some extra work (and a bit of guessing?).

Date	9h45-10h45	11h-12h
26/02	Neveu-Schwarz and Ramond generalizations: WS-SUSY	GSO projection: Target-space SUSY

We introduced fermionic dof (anticommuting coordinates) i.e. 2-component spinors on the world sheet, yet D-component vectors in spacetime. In Polyakov-style form the action has local supersymmetry and is quite complicated:

$$S = S^{\rm b} + S^{\rm f} \; ; \; S^{\rm b} = -\frac{T}{2}\int d^2\xi \sqrt{-g}\; g^{\alpha\beta}\; \partial_\alpha X^\mu \cdot \partial_\beta X_\mu$$

$$S^{\rm f} = -\frac{T}{2}\int d^2\xi \sqrt{-g}\left[i\bar{\psi}^\mu \gamma^\alpha \cdot \partial_\alpha \psi_\mu - i\bar{\chi}_\alpha \gamma^\beta \partial_\beta X^\mu \cdot \gamma^\alpha \psi_\mu \right.$$

$$\left. - \; 1/4(\bar{\chi}_\alpha \gamma^\beta \gamma^\alpha \psi^\mu) \cdot (\bar{\chi}_\beta \psi_\mu)\right]$$

(NB:to avoid confusion, we denoted by $g_{\alpha\beta}$ the 2D-metric $\gamma_{\alpha\beta}$)

In a "superconformal" gauge S simplifies drastically:

$$
\begin{aligned}
S^{SCG} &= -\frac{T}{2}\int d^2\xi \left[\, \partial_\alpha X^\mu \cdot \partial^\alpha X_\mu + i\bar{\psi}^\mu \gamma^\alpha \cdot \partial_\alpha \psi_\mu \right] \\
&= T\int d^2\xi \left[\, 2\partial_+ X^\mu \cdot \partial_- X_\mu + \psi^\mu_- \partial_+ \psi_{-\mu} + \psi^\mu_+ \partial_- \psi_{+\mu} \right]
\end{aligned}
$$

leading to very simple solutions

$$
\begin{aligned}
X_\mu(\sigma,\tau) &= F_\mu(\tau-\sigma) + G_\mu(\tau+\sigma) \\
\psi^\mu_-(\sigma,\tau) &= \psi^\mu_-(\tau-\sigma) \ ; \ \psi^\mu_+(\sigma,\tau) = \psi^\mu_+(\tau+\sigma)
\end{aligned}
$$

The boundary conditions for the fermions are

$$
\left[\psi^\mu_+ \delta\psi^\mu_+ - \psi^\mu_- \delta\psi^\mu_-\right](\sigma=0) = \left[\psi^\mu_+ \delta\psi^\mu_+ - \psi^\mu_- \delta\psi^\mu_-\right](\sigma=\pi)
$$

and give, for open strings,

$$
\begin{aligned}
\psi^\mu_+(\sigma=0) &= \pm\psi^\mu_-(\sigma=0) \\
\psi^\mu_+(\sigma=\pi) &= \pm\psi^\mu_-(\sigma=\pi)
\end{aligned}
$$

R: $\psi_+^\mu(\sigma = 0) = +\psi_-^\mu(\sigma = 0)$ and $\psi_+^\mu(\sigma = \pi) = +\psi_-^\mu(\sigma = \pi)$

NS: $\psi_+^\mu(\sigma = 0) = +\psi_-^\mu(\sigma = 0)$ and $\psi_+^\mu(\sigma = \pi) = -\psi_-^\mu(\sigma = \pi)$

For closed strings

$$\begin{aligned} \psi_+^\mu(\sigma) &= \pm\psi_+^\mu(\sigma+\pi) \\ \psi_-^\mu(\sigma) &= \pm\psi_-^\mu(\sigma+\pi) \end{aligned}$$

and we get four kinds of states:

NS-NS: $\psi_+^\mu(\sigma) = -\psi_+^\mu(\sigma+\pi)\ ,\ \psi_-^\mu(\sigma) = -\psi_-^\mu(\sigma+\pi)$

NS-R: $\psi_+^\mu(\sigma) = -\psi_+^\mu(\sigma+\pi)\ ,\ \psi_-^\mu(\sigma) = +\psi_-^\mu(\sigma+\pi)$

R-NS: $\psi_+^\mu(\sigma) = +\psi_+^\mu(\sigma+\pi)\ ,\ \psi_-^\mu(\sigma) = -\psi_-^\mu(\sigma+\pi)$

R-R: $\psi_+^\mu(\sigma) = +\psi_+^\mu(\sigma+\pi)\ ,\ \psi_-^\mu(\sigma) = +\psi_-^\mu(\sigma+\pi)$

For open strings we get the mode expansions:

$$\psi_{\pm}^{\mu}(\sigma,\tau) = \sqrt{\alpha'} \sum_{n \in Z} d_n^{\mu} e^{-in(\tau \pm \sigma)}$$ R

$$\psi_{\pm}^{\mu}(\sigma,\tau) = \sqrt{\alpha'} \sum_{r \in Z+1/2} b_r^{\mu} e^{-ir(\tau \pm \sigma)}$$ NS

For closed strings we get the 4 possible combinations of

$$\psi_{\pm}^{\mu}(\sigma,\tau) = \sqrt{2\alpha'} \sum_{n \in Z} d_{\pm,n}^{\mu} e^{-2in(\tau \pm \sigma)}$$ R

$$\psi_{\pm}^{\mu}(\sigma,\tau) = \sqrt{2\alpha'} \sum_{r \in Z+1/2} b_{\pm,r}^{\mu} e^{-2ir(\tau \pm \sigma)}$$ NS

The canonical anticommutation relations for $\psi^{\mu}{}_{a}(\xi)$ lead to

$$\{b_r^\mu, b_s^\nu\} = \eta^{\mu\nu}\delta_{r+s} \; ; \; \{d_n^\mu, d_m^\nu\} = \eta^{\mu\nu}\delta_{n+m}$$

In the R sector, the vacuum cannot satisfy: $d_0^\mu|0\rangle = 0$

since $\{d_0^\mu, d_0^\nu\} = \eta^{\mu\nu}$

In the R sector the vacuum state is a representation of the Dirac algebra and is, indeed, a spacetime fermion.

Since the other states of the R sector are obtained by applying spacetime vector creation operators, all the states of the R-sector are fermions.

Similarly, all the states in the NS sector are bosons.

For closed strings NS-NS and R-R are bosons while NS-R and R-NS are fermions.

Proceeding as in the bosonic case with quantization in the light cone gauge:

$$X^+(\sigma,\tau) \equiv \frac{X^0 + X^{D-1}}{\sqrt{2}} = 2\alpha' p^+ \tau \; ; \; \psi^+(\sigma,\tau) \equiv \frac{\psi^0 + \psi^{D-1}}{\sqrt{2}} = 0$$

and imposing the constraints

$$\begin{aligned} \partial_\pm X \cdot \partial_\pm X \quad &+ \quad \frac{i}{2}\psi \cdot \partial_\pm \psi = 0 \\ \psi \cdot \partial_\pm X \quad &= \quad 0 \end{aligned}$$

we can solve for X^- and ψ^- in terms of the transverse

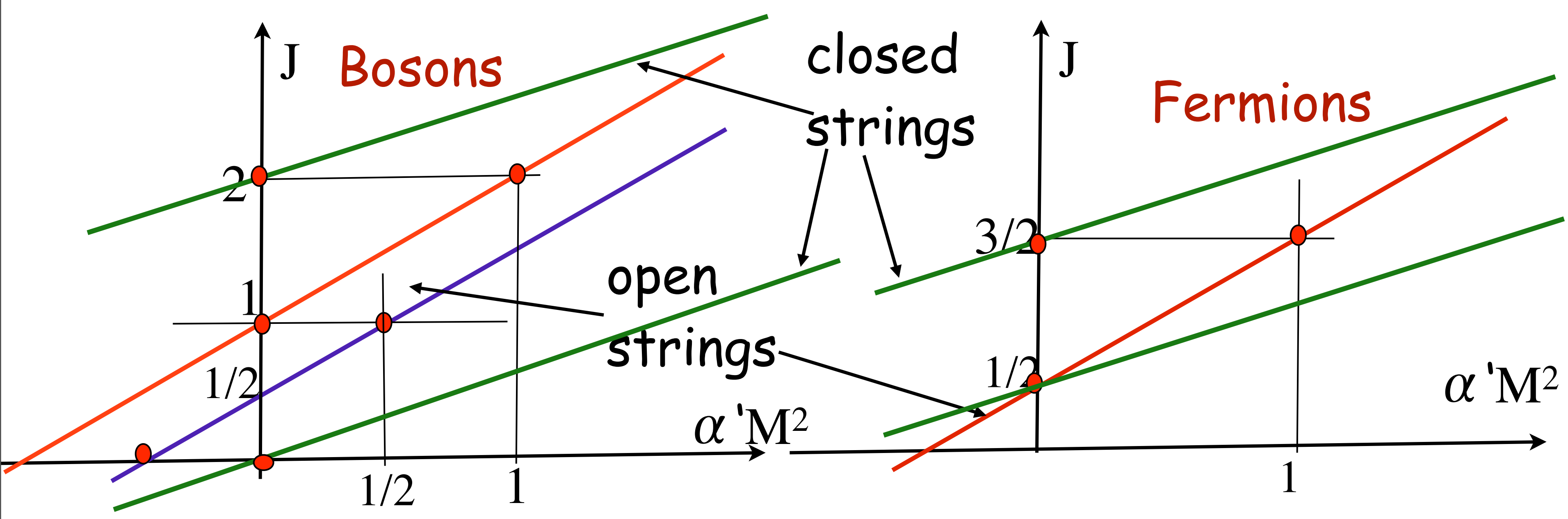
operators. Imposing the validity of the Lorentz algebra forces the conditions D=10 and $\alpha_0 = 1/2$.

These results are confirmed by the covariant (BRST) quantization procedure.

Conclusions on NSR string

The NSR model is much richer than the bosonic string: it has also fermions (with no tachyon) and a bosonic trajectory with intercept 1/2 (with a tachyon on it).

It also has an amusing (though only partial) degeneracy between the bosonic and fermionic spectra.



GSO projection and spacetime SUSY

The NSR model had still a tachyon. In 1976, Gliozzi, Scherk and Olive found a smart way to eliminate the tachyon. They introduced, in the NSR model, a WS fermion "parity" P_F.

This is defined in different ways for the NS and R sectors.

For NS one defines:

$$P_F^{NS} = (-1)^{F_{NS}+1} \; ; \; F_{NS} = \sum_{r=+1/2}^{\infty} b_{-r} b_r$$

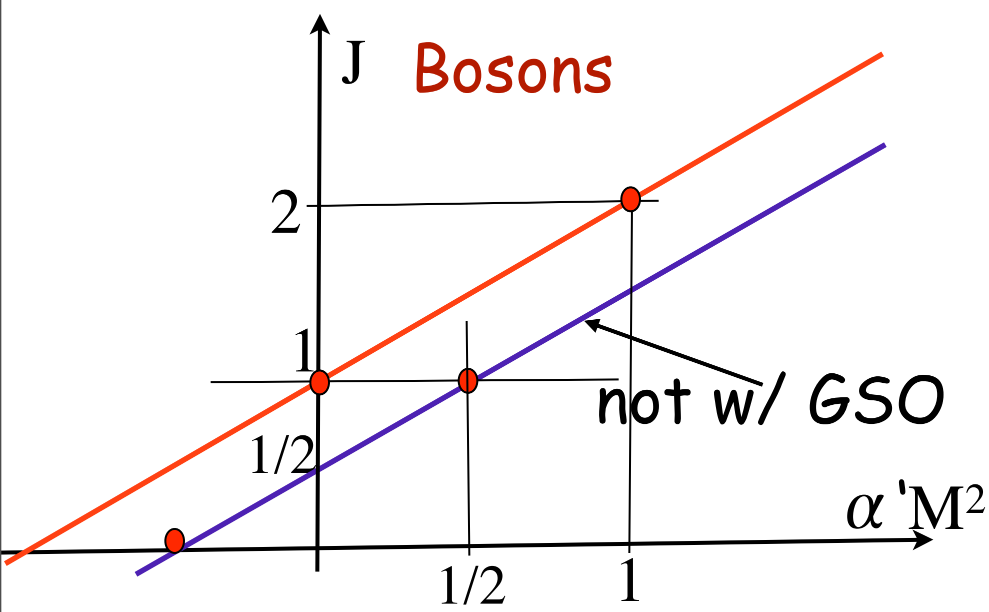
while for R:

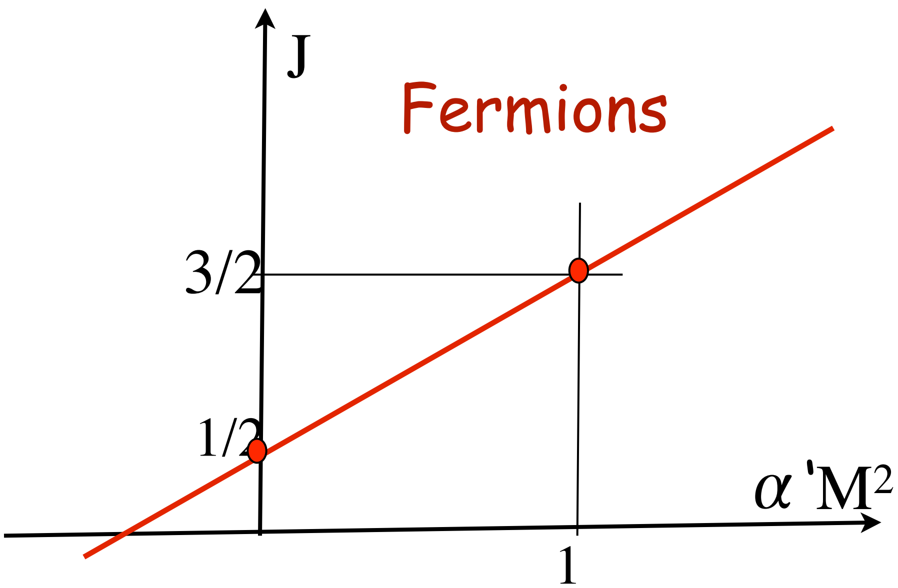
$$P_F^{R} = \gamma_{11} (-1)^{F_R} \; ; \; F_R = \sum_{n=+1}^{\infty} d_{-n} d_n$$

NB: the tachyon has $P_F=-1$, the massless vector $P_F=+1$

GSO then proved that P_F is conserved in the NSR model so that a projection on external states with $P_F=+1$ is consistent with factorization ($P_F=-1$ states do not appear as intermediate states). The tachyon is eliminated. Also, half of the fermions in the fermionic sectors are projected out.

The fermionic ground state is a Majorana-Weyl spinor in D=10. It has 8 components (just like a massless vector) and is chiral in the 10-dimensional sense.





A counting of states shows that, not only the massless spectrum, but also the excited states contain the same number of bosons and fermions.

This follows from an "aequatio identica satis abstrusa" among Jacobi functions (Jacobi 1829, see seminar).

The (spacetime) supersymmetry of the spectrum can be generalized to interactions and is a true symmetry of string theory after the GSO projection.

This is the so-called Type I superstring, a theory of open strings. It automatically generates also a closed string sector at the non-planar-loop level.

At tree level its massless states are a vector (in the adjoint representation of an SO(N) or Sp(N) group) and a Weyl-Majorana spinor. It is a chiral theory in D=10 as a consequence of the GSO projection.

Changing γ_{11} into $-\gamma_{11}$ does not lead to a different theory.

For closed strings one applies GSO separately to left- and right-movers. However, we can use either the same or an opposite GSO projection for left and right-movers. In the former case we have a chiral theory, called Type IIB, in the latter a non-chiral theory, called Type IIA.

The 4 massless sectors of Type IIA look as follows (the numbers correspond to reps. of an SO(8) subgroup of SO(9,1), its "little group" for massless states).

SO(8) has 3 inequivalent 8-dimensional representations, 8_v, 8_c,8_s: they are related to one another by an interesting "triality"...

$(8_v+8_c)\times(8_v+8_s) = 8_v\times 8_v + 8_c\times 8_s + 8_v\times 8_s + 8_c\times 8_v =$

$(1+35_v+28)_{NS\text{-}NS} + (8_v+56_v)_{R\text{-}R} +(8_c+56_c)_{NS\text{-}R} + (8_s+56_s)_{R\text{-}NS}$

In words: the combination of two NS vectors leads to a scalar (the dilaton) , a symmetric 2-index tensor (the graviton) and an antisymmetric 2-index tensor ($B_{\mu\nu}$).

Two R-spinors give a vector C_1 and a 3-form C_3 (with $8\times 7\times 6/3! = 56$ components). The NS-R & R-NS give 2 gravitinos and 2 dilatinos of opposite chirality.

The 4 massless sectors of Type IIB look as follows:

$(8_v+8_c)\times(8_v+8_c) = 8_v\times 8_v + 8_c\times 8_c + 8_v\times 8_c + 8_c\times 8_v =$

$(1+35_v+28)_{NS-NS} + (1+28+35_c)_{R-R} + (8_s+56_s)_{NS-R} + (8_s+56_s)_{R-NS}$

In words: NS-NS as in Type IIA. Two R-spinors give a scalar C_0, a 2-form C_2 (with 8x7/2! = 28 components) and a self-dual(*) 4-form C_4 (with 8x7x6x5/2x4! = 35 components). The NS-R & R-NS give 2 gravitinos and 2 dilatinos of the same chirality.

Finally, the closed string sector of Type I theory.

It is chiral and coincides with a particular subsector of Type IIB: $(1+35_v)_{NS-NS} + (28)_{R-R} + (8_s+56_s)_{NS-R+R-NS}$

dilaton, graviton, C_2 (of GS anomaly cancellation!) and chiral fermions.

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(*) Meaning $F_5 = dC_4 = {}^*F_5$

Date	9h45-10h45	11h-12h
05/03	Zero-slope limit, QCD: end of a dream, the SS proposal	The GS breakthrough: a theory of everything?

We discussed the phenomenological shortcomings of string theory (in particular its softness) and how it could not resist the competition of QCD.

We then considered the zero-slope (or low-energy) limit of string theory and argued that it corresponds to QFT. Gauge and gravitational interactions as described by gauge theories and GR emerge as effective low-energy approximations to string theory.

These properties of string theory motivated the 1974 suggestion by Scherk and Schwarz that string theory should rather be considered as an extension of the SM and GR for the description of the elementary particles appearing in those theories, the gauge bosons, the fermions, the graviton and fundamental scalars.

This proposal did not find a resonance in the scientific community for about 10 years.

One problem with it was that it looked impossible to have a string theory with chiral fermions in D=4 (as demanded by the SM) and yet unaffected by gauge and/or gravitational anomalies.

The situation changed drastically in 1984 when Green and Schwarz found that all those anomalies cancel in Type I string theory for the particular choice of SO(32) as gauge group.

Rather than summarizing the GS anomaly cancellation argument (see 4th seminar by Di Vecchia) we will approach the anomaly question from a different angle later today.

Soon after the GS paper, two more consistent superstring theories were constructed by Gross, Harvey, Martinec and Rohm. They are called heterotic and have gauge groups SO(32) and $E_8 \times E_8$ (to be discussed later in the course).

Tentative revised program for the rest of the course

Old

Date	9h45-10h45	11h-12h
12/03	Strings in non-trivial backgrounds, effective action	Field and String-theoretic symmetries: N&D-strings
19/03	D-branes & SUGRA solutions	D-branes & gauge theories
26/03	The AdS/CFT correspondence	Unification of string theories

New

Date	9h45-10h45	11h-12h
12/03	Recap of previous lectures and seminars	Loops in QFT and QST
19/03	Strings in non-trivial backgrounds: effective action	Field and String-theoretic symmetries. T-duality
26/03	D-branes & gauge theories	M theory & unification