Particules Élémentaires, Gravitation et Cosmologie

Année 2004-2005

Interactions fortes et chromodynamique quantique I: Aspects perturbatifs

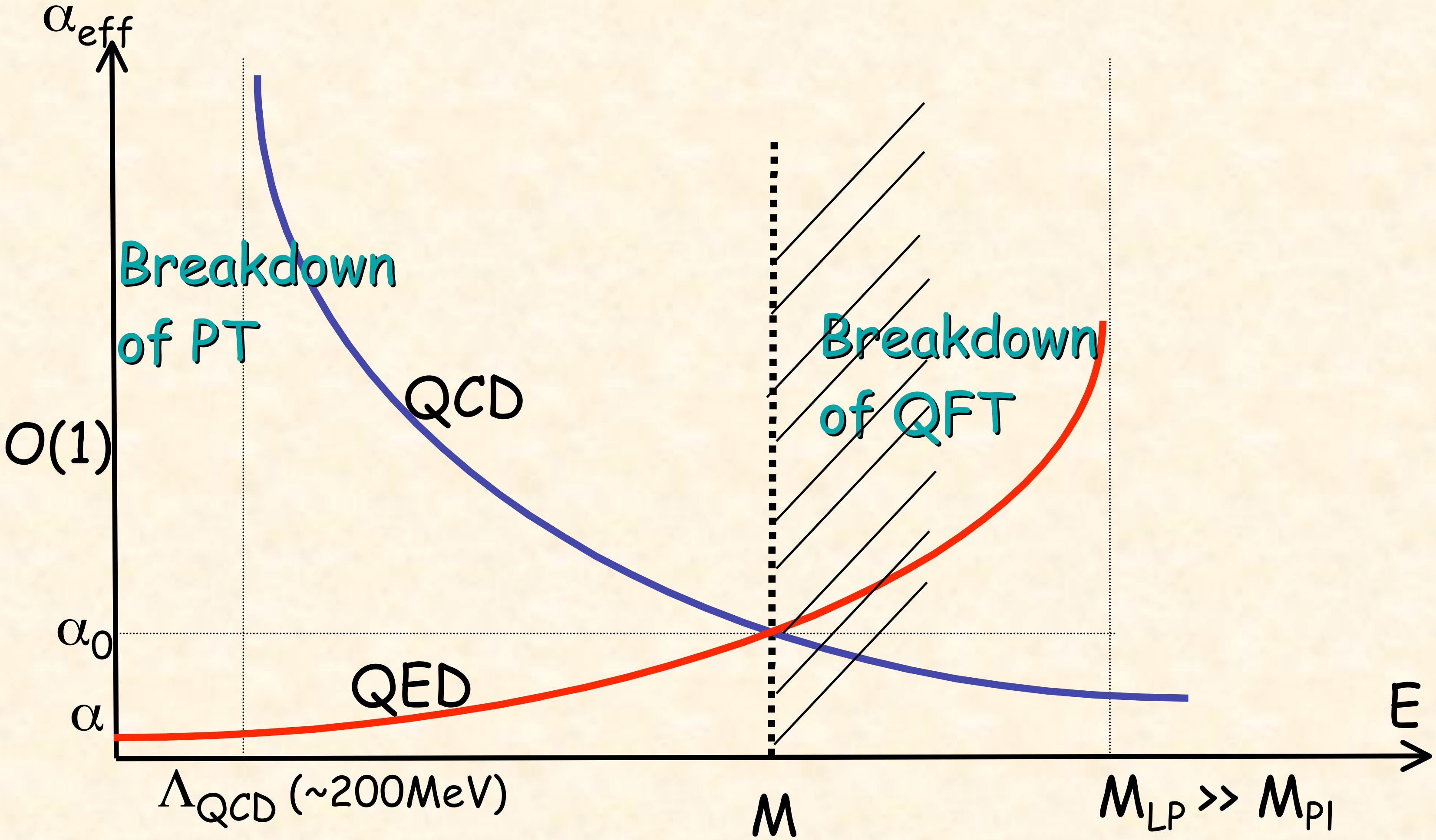
Cours III: 15 mars 2005

1. Summary of previous course
2. Renormalization and RG equations.
3. The limited power of AF
4. Classification of hard processes
5. IR safe and unsafe quantities in e^+e^- --> hadrons

1. Short summary of course no. 2..

- Dirac fermions: (r) = (1/2,0)+(0,1/2) i.e. made out of one l.h. + one r.h. Weyl fermion in the same rep. of *G* (for this we need a vector-like theory)
- Classical actions, their symmetries under *P x G*

  => Classical QED, QCD
- Semi-classical approximation & loop corrections; effective gauge couplings in QED, QCD
- IR triviality vs. asymptotic freedom: picture of «running» QED, QCD couplings



α_{eff}
Breakdown of PT
QCD
Breakdown of QFT
O(1)
α_0
QED
α
E
Λ_{QCD} (~200MeV)
M
$M_{LP} >> M_{Pl}$

2. Renormalization and RG equations

- The detailed construction of the effective action depends on the gauge used.
- It is particularly useful to adopt the so-called background gauge (see S.W. 17.4) since, in this case, S_{eff} obeys the same symmetries as S_{cl}. As a result, in this gauge our previous discussion of gauge coupling renormalization generalizes to other terms in the action (even to higher orders in the loop expansion).
- Let us recall from last week:

$$S_{Cl} = -\frac{1}{16\pi\alpha_0}\int d^4x\, \hat{F}^a_{\mu\nu}\hat{F}^{a,\mu\nu} + \int d^4x\, \bar{\Psi}(i\gamma^\mu \hat{D}_\mu - m_0)\Psi$$

$$\hat{A}_\mu = e_0 A_\mu\,,\ \hat{A}^a_\mu = g_0 A^a_\mu \qquad \alpha_0 \equiv \frac{e_0^2}{4\pi}\,,\ \frac{g_0^2}{4\pi}$$

$$S^{1-loop}_{eff} = -\frac{1}{16\pi}\int d^4x\, \left(\frac{1}{\alpha_0} + L_3(-\frac{\Box}{M^2}, \frac{m}{M})\right)\hat{F}^a_{\mu\nu}\hat{F}^{a,\mu\nu} + \ldots$$

Including higher orders and filling up the dots:

$$S_{eff} = -\frac{1}{16\pi}\int d^4x\, \left(\frac{1}{\alpha_0} + L_3(-\frac{\Box}{M^2}, \frac{m}{M}, \alpha_0)\right)\hat{F}^a_{\mu\nu}\hat{F}^{a,\mu\nu}$$

$$+ \int d^4x\, \left(1 + L_2(-\frac{\Box}{M^2}, \frac{m}{M}, \alpha_0)\right)\bar{\Psi} i\gamma^\mu \hat{D}_\mu \Psi$$

$$- \int d^4x \left(1 + L_m(-\frac{\Box}{M^2}, \frac{m}{M}, \alpha_0)\right) m_0 \bar{\Psi}\Psi + S'_{eff}$$

where S'_{eff} contains terms with more fields.

Since the L_i (i= 3, 2, m) are both UV-divergent (sensitive) and non-local, let us handle one problem at the time. To this purpose, we introduce an arbitrary energy scale μ and write:

$$L_i(-\frac{\Box}{M^2}, \frac{m}{M}, \alpha_0) = L_i(\frac{\mu^2}{M^2}, \frac{m}{M}, \alpha_0) + L_i'$$

where $L_i(\mu^2/M^2, \ldots)$ are « divergent » but local, while the L'_i are non-local but finite. Let us then keep the former and add the finite non-local pieces to S'_{eff} (which was already non-local)

We thus end up with

$$S_{eff} = -\frac{1}{16\pi\alpha_\mu}\int d^4x\, \hat{F}^a_{\mu\nu}\hat{F}^{a,\mu\nu} + \int d^4x\, \bar{\Psi}_\mu(i\gamma^\nu\hat{D}_\nu - m_\mu)\Psi_\mu + \tilde{S}_{eff}$$

where:

$$\alpha_\mu^{-1} = \alpha_0^{-1} + L_3(\mu,\dots)\,,\ \Psi_\mu = (1+L_2(\mu,\dots))^{1/2}\Psi\,,\ m_\mu = \frac{1+L_m(\mu,\dots)}{1+L_2(\mu,\dots)}m_0$$

Note that the contribution to the mass term is proportional to m_0 and not to M. This is a consequence of the so-called chiral symmetry, a peculiarity of massless fermions and a well-known problem for keeping bosonic masses stable under radiative corrections.

A central result in renormalization theory is the proof, to all orders in PT, that the effective action, as a function of its new (μ-dependent) arguments, has a smooth limit as M goes to infinity. At the same time

$$S_{eff}(A, \Psi_\mu; \alpha_\mu, m_\mu, \mu, M)$$

only knows about μ through the fact that we decided to express it in terms of μ-dependent arguments but is actually μ-independent. This leads to the famous RG-equations expressing the fact that the explicit dependence on μ should be cancelled by the implicit one due to the μ-dependent arguments that we have introduced. Therefore:

$$\mu^2 \frac{dS_{eff}(\hat{A}, \Psi_\mu; \alpha_\mu, m_\mu, \mu, M)}{d\mu^2} = \mu^2 \frac{\partial S_{eff}}{\partial \mu^2} + \sum_i \mu^2 \frac{\partial x_i}{\partial \mu^2} \frac{\partial S_{eff}}{\partial x_i} = 0$$

where it is customary to take i = α_μ, log Ψ_μ, log m_μ and to write:

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta \frac{\partial}{\partial \alpha_\mu} - \gamma_m m_\mu \frac{\partial}{\partial m_\mu} - \gamma_\Psi \Psi_\mu \frac{\partial}{\partial \Psi_\mu} \right) S_{eff} = 0$$

This is usually called the Callan-Symanzik Eq., where, for instance,

$$\beta(\alpha_\mu, m_\mu/\mu) \equiv \mu^2 \frac{\partial \alpha_\mu}{\partial \mu^2} = -\alpha_\mu^2 \, \mu^2 \frac{\partial \alpha_\mu^{-1}}{\partial \mu^2}$$

Recalling def. of α_μ , L_3 from previous course (@ $\mu^2 = q^2 >> m^2$)

$$\alpha_\mu^{-1} = \alpha_0^{-1} + L_3(\mu, \ldots) \sim \alpha_0^{-1} + \beta_0 \log(M^2/\mu^2)$$

we get the standard result: $\beta(\alpha_\mu, m_\mu/\mu) = \beta_0 \alpha_\mu^2 + O(\alpha_\mu^3)$

with $\beta_0 > 0$ (< 0) in QED (QCD)

3. The limited power of asymptotic freedom

- Consider now a certain physical probability such as a decay rate, or a cross section, $\Gamma = \Gamma\,(E, \theta_i; \mu, \alpha_\mu, m_\mu)$
- In this case, the rules to construct Γ out of S_{eff} are such that the « anomalous dimension » term (prop. to γ_Ψ) drops out and one gets the simpler RG equation:

$$\left(\mu^2\frac{\partial}{\partial\mu^2}+\beta\frac{\partial}{\partial\alpha_\mu}-\gamma_m m_\mu\frac{\partial}{\partial m_\mu}\right)\Gamma=0$$

The physical meaning of this equation is clear: μ being an arbitrary scale that we have introduced, a change of μ can be compensated by a change in α_μ and in m_μ so that physical quantities remain unchanged. Physics should only depend on two (rather than three!) parameters!

At this point one can argue as follows:

- Since μ is arbitrary, let us take it to be of O(E) and write $\Gamma = \Gamma\,(E, \theta_i,..; E, \alpha_E, m_E)$ with $\alpha_E \sim (2|\beta_0|\log E)^{-1}$ --> 0 @ large E

Can we now claim that:

$$\Gamma = \Gamma\,(E, \theta_i,..; E, \alpha_E, m_E) \dashrightarrow \Gamma\,(E, \theta_i,..; E, 0, m_E)\ ?$$

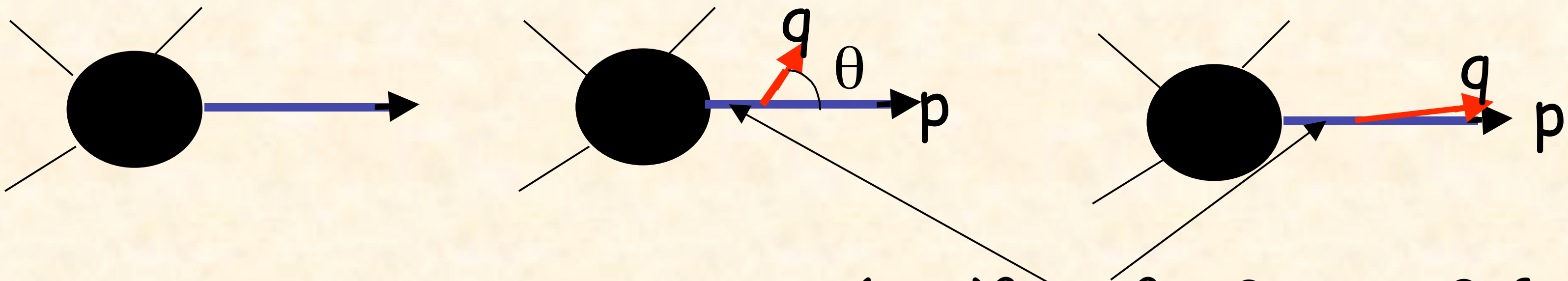
(in general, that we take the lowest non-trivial order for Γ)

Obviously, it cannot be as simple as that! Indeed we could have taken $\mu = 10^6$ E and, by this naive argument, say that we can put $\alpha_\mu = 0$ to an even better precision. The problem is that, on purely dimensional grounds, Γ depends on ratios of energies that involve also μ. If we take $\mu \gg E$, some large $\log(\mu/E)$ will pop up, compensate for the smallness of α_μ, and spoil the naive argument. This we can avoid by taking $\mu \sim E$.

Unfortunately (actually fortunately, as we shall see) that's not all. Even if we take $\mu \sim E$:

- logs of (E/m_E) can spoil the naive argument whenever the limit m_E --> 0 is not smooth (mass singularity)
- If we take the large-E limit while keeping some momenta fixed (can be the momentum of one of the many particles in the final state, or a momentum transfer,so that some θ_i-->0) there can also be large-logs (or even worse if we approach some Coulomb-like singularity)

- Finally, another source of large logs is the famous infrared (IR) problem, already well-known from QED. Cross-sections for processes involving charged particles, but not allowing for an arbitrary number of soft photons in the final state, ar IR divergent (actually vanish!).
- And when m-->0 a similar divergence is found if we do not allow emission of massless particles by massless particles in the same direction (so-called collinear singularities)
- Both of them are due to a propagator that ends up being very close to its mass-shell (where it blows up):



$(p+q)^2 + m^2 \sim 2p.q$ -->0 for q -->0 and/or for θ--> 0 if m=0

4. Classification of hard processes

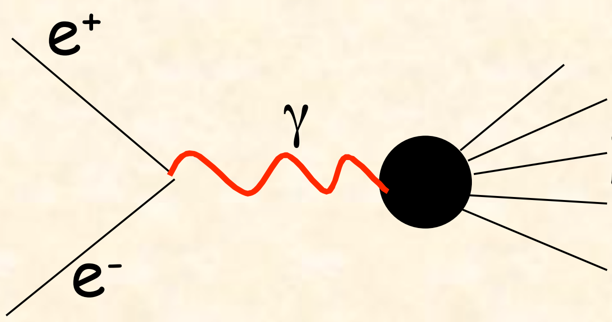
Because of the above discussion, we can classify the various high-energy QCD processes in three broad categories:

1. IR&CO-safe processes
   - In this case Γ can be expanded as a power-series in $\alpha_E \sim (2|\beta_0|\log E)^{-1}$ with no E-dependent enhancements in the coefficients of the expansion. At large E we can trust the leading term (not always of $O(\alpha_E{}^0)$) and have a good estimate of the error
2. IR-safe processes with collinear singularities
   - In this case the expansion parameter is O(1) and one has to find ways to resum the contribution of collinear divergences to all orders in order to see what kind of predictivity is left
3. IR-unsafe processes
   - In this case the expansion parameter is typically >> 1 ($\alpha_E (\log E)^2 \sim \log E$) and one has to hope that some resummation makes sense. This is the regime that borders on truly non-perturbative QCD and is the hardest

Let us see some examples in the case of the reaction e^+e^---> hadrons

5. Safe and unsafe quantities in e^+e^- --> hadrons

- One of the golden reactions in QCD is the famous process e^+e^---> hadrons. To lowest order in α_{em} it can be depicted as follows:



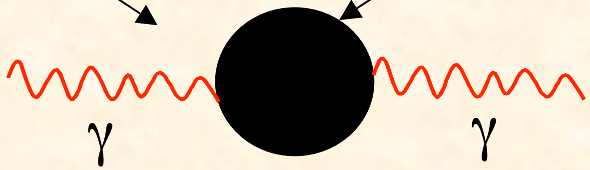
hadrons => $A(e^+e^- \text{-->} \text{hadrons})$

$$\Gamma = |A|^2 = \Gamma\,(E, \theta_i, ..; E, \alpha_E, m_E)$$

- However, if we use PT, what we find as final states are not hadrons but quarks and gluons. This seems to contradict the statement that AF allows us to use PT!
- Fortunately, our previous analysis has shown that PT is only applicable if we consider IR&CO-safe quantities…

- The simplest one is just $\sigma_T(e^+e^- \to \text{hadrons})$. It is related by the optical theorem to

any QCD diagram can be in black box

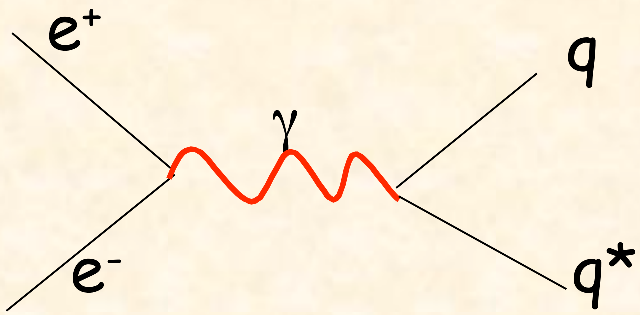
Im 

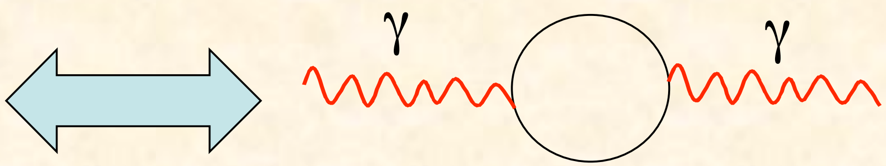
$Q^2 = E^2$

One can easily show that such an object is free from IR&CO problems. For this it is important that the initial particles (e^+e^- or the virtual photon) are colour singlets. Therefore:

$$\sigma_T(e^+e^- \to \text{hadrons}) = \sigma_{tree}(e^+e^- \to qq^*) + O(1/\log E)$$

since the lowest order final state consists of just a qq* pair





NB: the black box has become white!

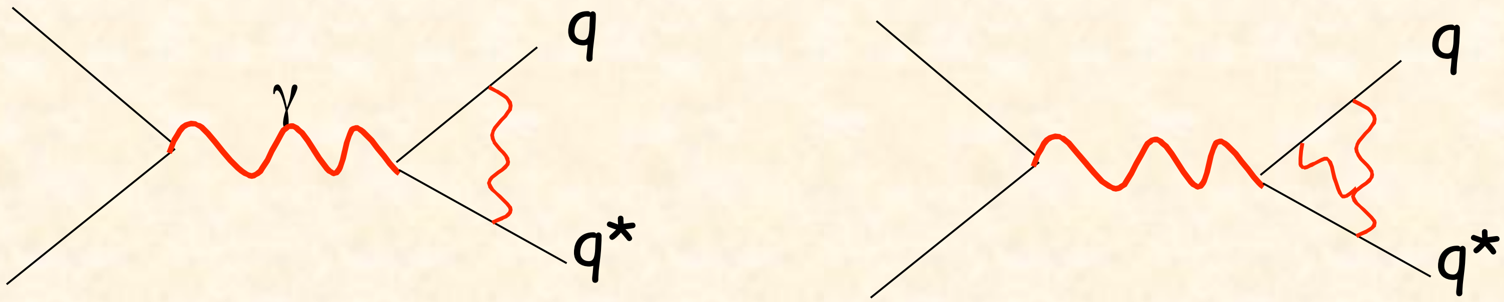
Q&A

Q: But then why does not the final state at large E consists of just a qq* pair? A: because what we computed was $\sigma_T(e^+e^- \to$ quarks & gluons) and not $\sigma(e^+e^- \to qq^*)$ ...Q: But did we not say that $\sigma_T(e^+e^- \to$ quarks & gluons) = $\sigma_{tree}(e^+e^- \to qq^*)$?

A:Yes we did, but we should not forget the «tree »!

Indeed, let us try to compute $\sigma(e^+e^- \to qq^*)$ using AF.

At tree level we get the same as the (final) result for σ_T of course. But, as we go on, we get diagrams such as



which are all IR divergent!! Thus $\sigma(e^+e^- \to qq^*)$ is not σ_T!! The IR divergences only cancel if we add other channels..

To summarize:

- Thanks to AF and to IR-CO-safety we can compute σ_T in terms of the simplest lowest order diagram with just qq* in the final state. However, this is by no means the correct description of the final state.
- Thus, in checking our QCD prediction against the data, we should not match the final state but just compare data and prediction for σ_T . A nice way (also the historical one) to do this is to consider the ratio:

$$R = \sigma_T(e^+e^- \text{-->hadrons})/\sigma(e^+e^- \text{-->}\ \mu^+\mu^-) \sim$$

$$\sigma_{tree}(e^+e^- \text{-->} qq^*)/\sigma(e^+e^- \text{-->}\ \mu^+\mu^-) = 3\text{x}(4/9 + 1/9 + 1/9) = 2$$

which should hold above the strange quark threshold but below that of charm. The overall factor 3 comes from colour, while the other numbers are the squares of the quark charges.

Calculation gives

$$R = 2\left(1 + \frac{\alpha(E^2)}{\pi} + \ldots\right)$$

Comparison with the data is very satisfactory particularly for the average (see graph and note that the colour factor 3 is crucial for the agreement!). Historically, this was one of the very first indications (~1970) of a point-like structure inside the hadrons. In the absence of such a structure the expectation is that R should fall fast with energy…

http://van.hep.uiuc.edu/van/qa/section/New_and_Exciting_Physics/Antimatter/20031005144616.htm

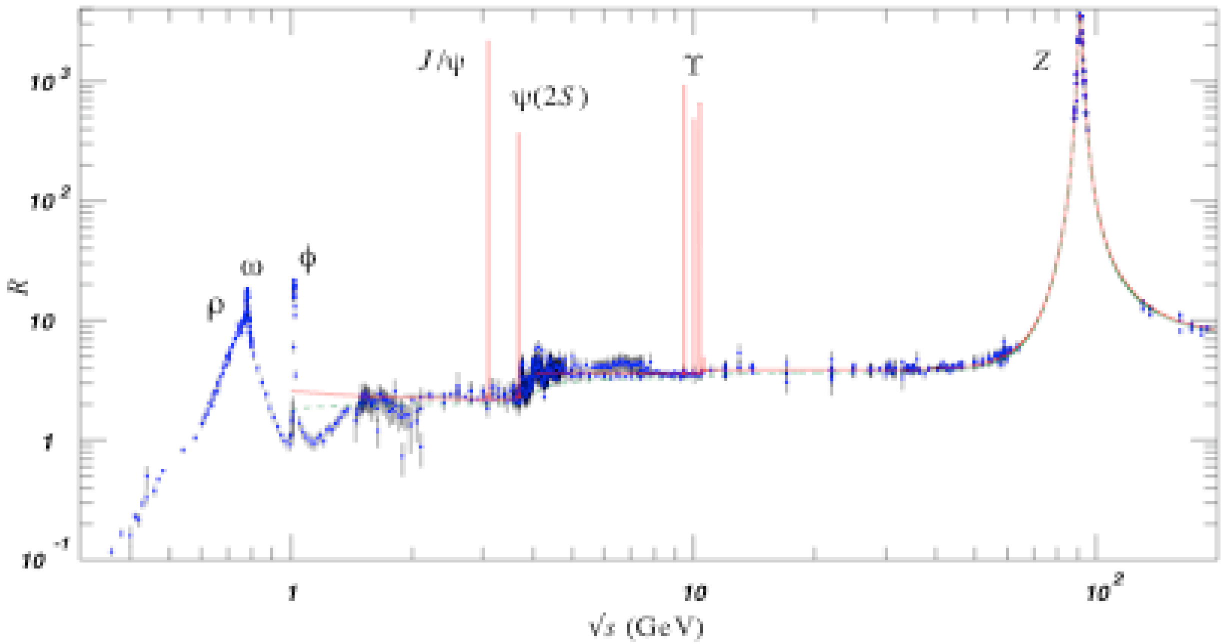


Figure 39.6, Figure 39.7: World data on the total cross section of $e^+e^- \to hadrons$ and the ratio $R = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-$, QED simple pole). The curves are an educative guide. The solid curves are the 3-loop pQCD predictions for $\sigma(e^+e^- \to hadrons)$ and the R ratio, respectively [see our Review on Quantum chromodynamics, Eq. (9.12)] or, for more details, K.G. Chetyrkin *et al.*, Nucl. Phys. **B586**, 56 (2000), Eqs. (1)–(3)). Breit-Wigner parameterizations of J/ψ, $\psi(2S)$, and $\Upsilon(nS), n = 1..4$ are also shown. **Note:** The experimental shapes of these resonances are dominated by the machine energy spread and are not shown. The dashed curves are the naive quark parton model predictions for σ and R. The full list of references, as well as the details of R ratio extraction from the original data, can be found in O.V. Zenin *et al.*, `hep-ph/0110176` (to be published in J. Phys. **G**). Corresponding computer-readable data files are available at `http://wwwppds.ihep.su/~zenin o/contents plots.html`. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, November 2001.)

σ_T is not the only thing we can compute. We can compute, for instance, the two-jet x-section defined as the x-section for all but at most a fraction ε of the total energy goes into two back-to-back cones of angle δ. Here corrections to the lowest order calculations are of order α_E $(\log \varepsilon)(\log \delta)$ hence small if E is large enough and ε, δ are not too small.

The rule of the thumb for IR-CO-safety of a certain x-section is that a soft or collinear emission should not take us outside the channel for which the x-section is computed. This is the case for the two jet X-section but it is not, for instance, if we decide to fix the number and/or the species of the final quanta since soft and collinear emissions change the number and, in general, the species of the final particles

This will be discussed in much greater detail in YD's seminar