Particules Élémentaires, Gravitation et Cosmologie
Année 2007-'08

Le Modèle Standard et ses extensions

Cours IV: 15 février 2008

Spontaneous symmetry breaking

General Considerations

- Symmetries play an essential role in our present understanding of elementary particles and fundamental interactions
- In QFT we have to make the important distinction between global symmetries (under x-independent transformations) and local symmetries(x-dependent tr.)
- Example of former: Lorentz (Poincaré) invariance
- Example of latter: gauge symmetries, general covariance (Cf. general relativity)
- Symmetries can be exact or (slightly) broken
- In latter case the breaking can be either explicit or spontaneous

- Recall: local symmetries mean that some d.o.f. are redundant, unphysical.
- Local symmetries are sacred, have to be broken «very carefully» , spontaneously as we shall see.
- Global symmetries are not so sacred: we are allowed to break them even explicitly. Sometimes this is actually needed in order to have realistic theories
- Occasionally, symmetries that are exact at the classical level are explicitly broken at the quantum level: one talks about «anomalies»
- Anomalies are acceptable/useful in global symmetries
- Anomalies are unacceptable in local symmetries
- Let's illustrate the casistics in a Table

Symmetries and their breaking

Symmetry / Breaking	Global	Local
Unbroken	Conservation law: B-L? Multiplets	massless gauge bosons, confinement
Explicitly broken	Broken multiplets, absence of NGB	Inconsistency, need anomaly cancellation
Spontaneously broken	Nambu-Goldstone bosons	Higgs et al. mechanism
Spont.ly & expl/ly broken	Pseudo (or no) NG bosons	Inconsistency

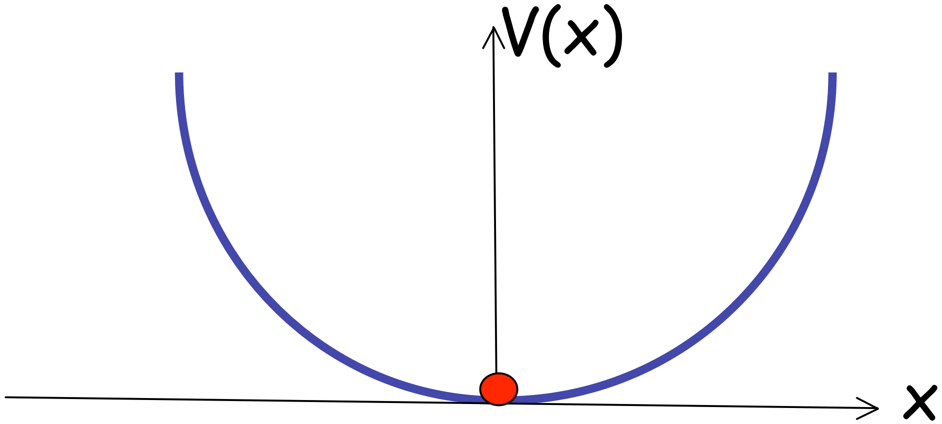
Spontaneous symmetry breaking (SSB)

- A very basic distinction has to be made between what one calls explicit breaking and spontaneous breaking of a symmetry

- The former case is the most familiar one. The gravitational field, for instance, breaks rotation invariance in this room by distinguishing the vertical direction. O(3) is broken to O(2)
- In the case of SSB the symmetry is not really broken: it is simply hidden. The apparent «breaking» is due to the non-invariance of the ground state (the so-called vacuum) under the symmetry's transformations.

Some illustrative examples

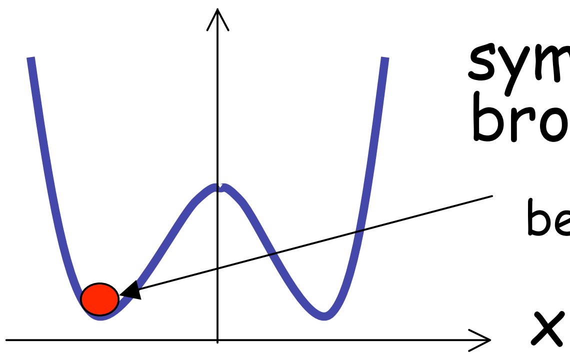
1. Symmetric potential with one global minimum



symmetry is x -> -x
(discrete = Z_2)

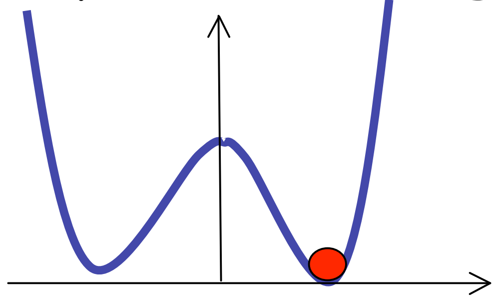
Neither explicit nor spontaneous SB

2. A double-well potential in Classical and Quantum Mechanics

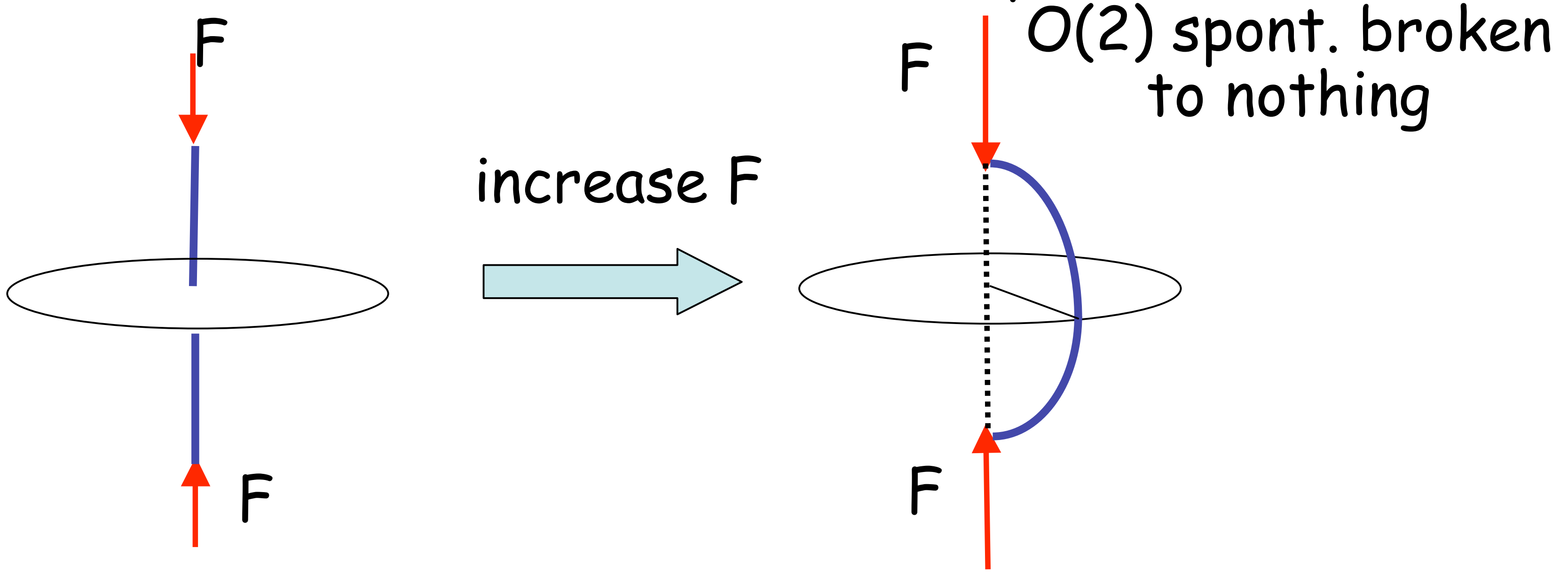


symmetry is x -> -x
broken spont. in CM
(unbroken in QM because of tunneling)

explicit breaking



3. Continuous symmetry: O(2) ~ U(1)



4. A larger continuous symmetry: SO(3) ~ SU(2)

A ferromagnet: the rotation symmetry is broken by the direction along which the magnet's magnetic field aligns. An O(2) survives = rotations around the axis of that (arbitrarily) chosen direction

Spontaneous Symmetry Breaking <=> Degenerate Ground State

In a QM framework states are vectors in a Hilbert space.
The ground state , |0> , is no exception;
A symmetry transformation g (g is a particular element of the symmetry group) becomes a unitary operator U(g) acting on these states. Furthermore, [H, U(g)]=0.

If we apply a symmetry transformation to |0> we get U(g)|0> and there are two possibilities:

1. U(g) |0> = |0,g> = |0> => no SSB
2. U(g) |0> = |0,g> ≠ |0> => SSB w/ <0|H|0> = <0,g|H|0,g>

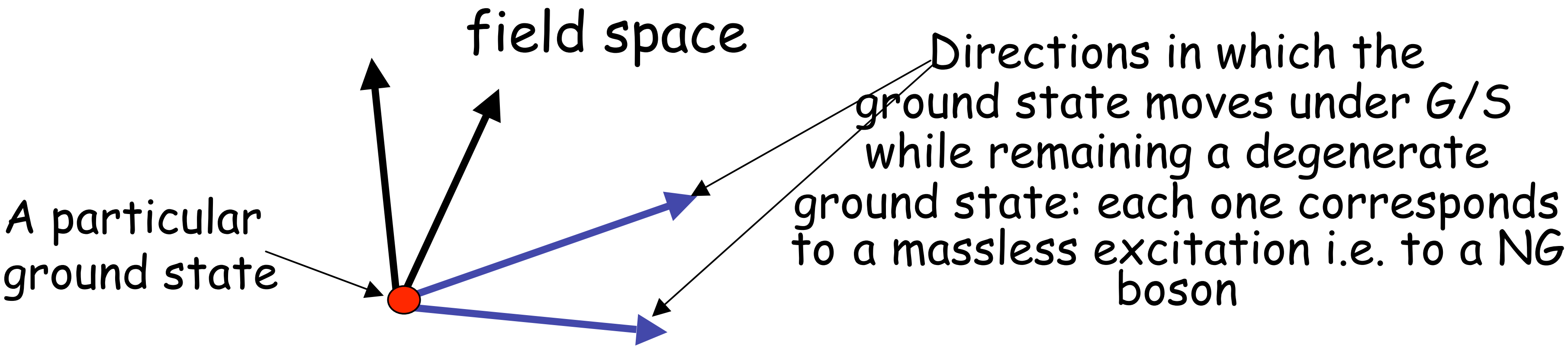
In latter case the subspace |0,g>, the so-called vacuum manifold, provides a non-trivial rep. of G

SSB and Nambu-Goldstone(NG) bosons

Goldstone's theorem: If a continuous global symmetry G is spontaneously broken down to its subgroup S (meaning that S|0> =|0>) there must one massless NG boson for each generators of G that is not a generator of S.

In formulae: (number of NG-bosons) = dimG - dimS

The formal proof is a bit techical, but the physics behind is quite clear:



SSB in massless QCD

QCD with N_f massless quark flavours has a large global symmetry group:

$$G_{global}^{QCD} \equiv G_F = U(N_f)_F \otimes U(N_f)_{\bar{F}}$$

more traditionally called

$$U(N_f)_L \otimes U(N_f)_R$$

Under which the lh quarks (and the corresponding rh antiquarks) undergo a unitary transformation while the rh quarks (and the corresponding lh antiquarks) undergo a different/independent one

SSB in massless QCD

There is both theoretical and experimental evidence that, at zero temperature, the ground state of massless QCD is degenerate as a result of a Bose-Einstein condensate (BEC):

$$\langle \bar{\Psi}_f \, \psi_{f'} \rangle = c \, \delta_{ff'} \, \Lambda^3_{QCD}$$

What is S? We easily find S= $U(N_f)_V$, the «diagonal» subgroup of $U(N_f)_F \times U(N_f)_{F^*}$.

By the Goldstone theorem we would expect $2N_f^2 - N_f^2 = N_f^2$ massless pseudoscalar (J=0, P=-1) bosons. This is NOT what we observe (even after taking quark masses into account). Fortunately, an anomaly breaks explicitly a U(1) factor in G removing the corresponding NG boson (this is the resolution of the U(1)-problem in QCD, see 2006 course)

Spontaneous breaking of a gauge symmetry

Goldstone's theorem is evaded if the broken symmetry is local. This is actually the only known consistent way to break a gauge symmetry and to give mass to gauge bosons & chiral fermions Let us see how SSB works in the simplest case of a U(1) gauge symmetry, i.e. let us consider the lagrangian of scalar QED:

$$L^{SQED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\bar{\phi}D^\mu\phi - \mu^2\bar{\phi}\phi - \lambda(\bar{\phi}\phi)^2$$

Note that, unlike in QED, we were able to add a matter self interaction term without breaking the U(1) gauge symmetry

$$U(1) : \phi(x) \to e^{i\alpha(x)}\phi(x)\ ,\ \bar{\phi}(x) \to e^{-i\alpha(x)}\bar{\phi}(x)$$

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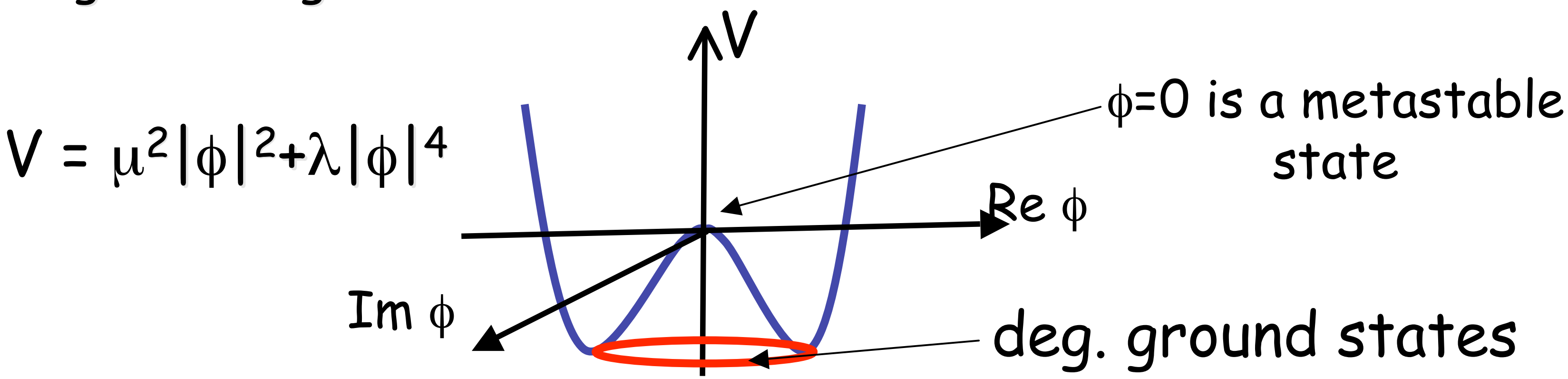
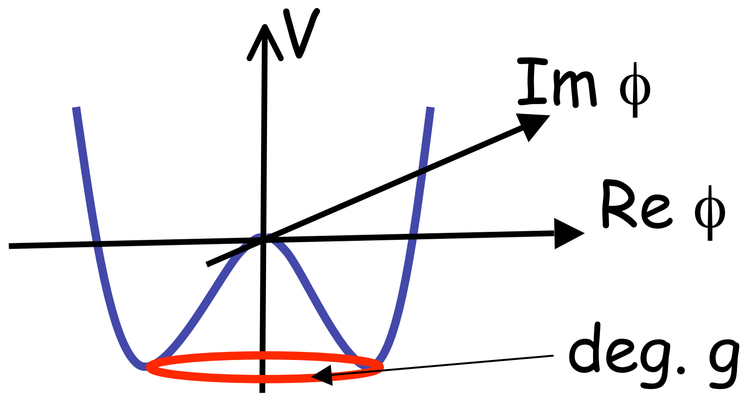
$$U(1) : \phi(x) \to e^{i\alpha(x)}\phi(x)\ ,\ \bar{\phi}(x) \to e^{-i\alpha(x)}\bar{\phi}(x)$$

Although we wrote it as $-\mu^2 |\phi|^2$, the bosonic mass term can have either sign, while λ must be positive if we want the energy (V~ -L) to be bounded from below.
Let us then take $\mu^2 < 0$. The scalar potential will look like a «mexican hat» with a degenerate ground state



In the absence of a gauge coupling this model, by Goldstone's theorem, has one massless NG boson (associated with the phase of ϕ while $|\phi|$ corresponds to a massive scalar).

$V = \mu^2|\phi|^2+\lambda|\phi|^4$



At non-zero gauge coupling something qualitatively new happens: the NG boson is «eaten up» by the gauge boson. In turn, the latter becomes massive. The total number of dof has not changed since a massive J=1 particle has 3 dof and 3=2+1! One can guess this to be what happens by noticing that the Lagrangian contains a term $g^2 |\phi|^2 A_\mu A^\mu$. Since in all the degenerate ground states we have $2|\phi|^2 = - \mu^2/\lambda = v^2$ in these vacua the gauge field acquires a mass term $- g^2 (\mu^2/2 \lambda) A_\mu A^\mu$ corresponding to a mass term with $m = g |\mu|/\lambda = g v$

A similar phenomenon is known to occur in a plasma: Debye screening is equivalent to photon getting a mass in a medium

A neat example: the non-linear U(1) model

Consider SQED in the large-λ limit: the VEV becomes a constraint

$$L^{NLSQED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\bar{\phi}D^\mu\phi \ , \ |\phi|^2 = v^2$$

$$U(1) : \phi(x) \to e^{i\alpha(x)}\phi(x) \ , \ \bar{\phi}(x) \to e^{-i\alpha(x)}\bar{\phi}(x)$$

Define nov $B_\mu = A_\mu + \frac{1}{v^2}\,\bar{\phi}\,\partial_\mu\phi \ , \ B_\mu \to B_\mu$ under a gauge tr.

The following identity holds:

$$L^{NLSQED} = -\frac{1}{4}F_{\mu\nu}(B)F^{\mu\nu}(B) - \frac{1}{2}g^2v^2B_\mu B^\mu$$

ϕ has disappeared. We get instead the lagrangian of a free vector boson of mass m = gv!