Particules Élémentaires, Gravitation et Cosmologie
Année 2007-'08

Le Modèle Standard et ses extensions

Cours VI: 22 février 2008

Tree-level predictions: one family

Let us recall: I. The fundamental fields

	SU(3)	SU(2)	$U(1)_Y$
(u,d) = Q	3	2	1/6
(ν, e) = L	1	2	-1/2
u^c	3*	1	-2/3
d^c	3*	1	+1/3
e^c	1	1	+1
ν^c	1	1	0
$(\phi^+, \phi^0) = \Phi$	1	2	1/2

II. The full one-family Lagrangian

$$L_{SM} = L_{Gauge} + L_{Kinetic} + L_{Yukawa} + L_{Higgs}$$

$$L_{Gauge} = -\frac{1}{4}\sum_a F^a_{\mu\nu} F^a_{\mu\nu}$$

$$L_{Kinetic} = i\bar{\Psi}\gamma^\mu D_\mu \Psi + \frac{1}{2} D_\mu \Phi^* D^\mu \Phi$$

$$L_{Yukawa} = \lambda_Y \left(\Phi \Psi_\alpha \Psi_\beta \varepsilon_{\alpha\beta}\right) + c.c.$$

$$L_{Higgs} = -\mu^2 \Phi^* \Phi - \lambda (\Phi^* \Phi)^2$$

to which we have added

$$L^{quad}_{\nu^c} = i\bar{\nu}^c \gamma^\mu \partial_\mu \nu_c - M\, \nu^c_\alpha \nu^c_\beta \varepsilon_{\alpha\beta} + c.c$$

How do we work out the predictions?

We will consider various pieces in the one-family SM lagrangian and look at their physical implications.

At tree-level different pieces of L do not «talk» too much to each other*).

I will denote by some dots the pieces of the Lagrangian that are irrelevant (at tree level) for each individual question

*) *As we shall see this is no longer true when radiative corrections are added.*

The VEV of the Higgs

The minima of the SM Higgs potential

$$V^{Higgs} = \mu^2 \Phi^* \Phi + \lambda (\Phi^* \Phi)^2 \; ; \; \mu^2 < 0 \; ; \; \lambda > 0$$

satisfy $\Phi^*\Phi = -\mu^2/2\lambda > 0$. This implies an expectation value for Φ which, without lack of generality, can be taken to point in a certain direction (in group space) e.g. to be of the form:

$$\langle(\phi^+, \phi^0)\rangle = (0, v) \; , \; \langle(\phi^{0*}, \phi^-)\rangle = (v, 0) \; , \; v = \sqrt{\frac{-\mu^2}{2\lambda}}$$

The original Higgs multiplet is then written as its VEV plus fluctuations around it. The VEV generates mass terms.

The gauge boson sector

$$L = -\frac{1}{4}\sum_a F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2}D_\mu\Phi^* D^\mu\Phi + \ldots$$

$$(D_\mu\Phi)^i \equiv (D_\mu)^i_j\Phi^j = (\partial_\mu\delta^i_j - igA^a_\mu(T^a)^i_j)\Phi^j$$

What are the effects of SSB in the gauge boson sector?

First of all the group $SU(2)\times U(1)_Y$ is spontaneously broken to $U(1)_Q$, since the non-vanishing VEV is that of a neutral (Q=0) field i.e. it's left invariant under the $U(1)_Q$ subgroup. As a consequence, we expect 3 of the 4 gauge bosons to get a mass (by eating up as many NG-bosons) and one, the photon, to remain strictly massless. To check this, we insert the above VEV in the quartic term $g^2\Phi\Phi^* A_\mu A_\mu$ and get a mass-matrix for the gauge bosons. The result is (g, g' = SU(2), $U(1)_Y$ couplings)

The gauge boson sector (cont.d)

$$L = -\frac{v^2}{4}\left[2g^2 W_\mu^+ W_\mu^- + (gW_\mu^3 - g'B_\mu)^2\right]$$

This immediately implies that the charged intermediate vector bosons $W^\pm$ have acquired a mass:

$$m_W = \frac{gv}{\sqrt{2}}$$

Inserting this value in the relation between G_F, the W-mass, and its coupling g to fermions (see below), we also get:

$$G_F^{-1} = \frac{4\sqrt{2}m_W^2}{g^2} = 2\sqrt{2}v^2$$

Since $G_F^{-1/2} \sim 293$ GeV we find: $v \sim 174$ GeV

The gauge boson sector (cont.d)

$$L = -\frac{v^2}{4}\left[2g^2 W_\mu^+ W_\mu^- + (gW_\mu^3 - g' B_\mu)^2\right]$$

In the neutral-vector-boson sector the situation is a bit more complicated, but it's clear that only one combination of W^3 and B gets a mass while the orthogonal combination, the photon, does not. Let us then introduce the (now-famous) "electro-weak angle" θ_W (W for weak) by:

$$tan\ \theta_W \equiv \frac{g'}{g}$$

and introduce the eigenvectors

The gauge boson sector (cont.d)

$$Z_\mu = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2+g'^2}} = cos\theta_W W_\mu^3 - sin\theta_W B_\mu$$

$$A_\mu = \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2+g'^2}} = cos\theta_W B_\mu + sin\theta_W W_\mu^3$$

We easily find m_γ =0 and:

$$m_z = \sqrt{\frac{g^2+g'^2}{2}}v = \frac{m_W}{cos\theta_W} \text{ , } \rho \equiv \frac{m_W^2}{cos^2\theta_W m_Z^2} = 1$$

Since the weak angle appears in many other observables this can be seen as a prediction for m_Z . The data are now so precise that these tree-level predictions are off by many σ's…

The fermion (except for neutrino) masses

$$L_{quarks}^{Yukawa} = -\lambda^u \Phi Q u^c - \lambda^d \Phi^* Q d^c$$
$$= -\lambda^u (\phi^0 u u^c + \phi^+ d u^c) - \lambda^d (\phi^- u d^c + \phi^{0*} d d^c)$$

$$L_{leptons}^{Yukawa} = -\lambda^\nu \Phi L \nu^c - \lambda^e \Phi^* L e^c$$
$$= -\lambda^\nu (\phi^0 \nu \nu^c + \phi^+ e \nu^c) - \lambda^e (\phi^- \nu e^c + \phi^{0*} e e^c)$$

Inserting

$$\langle(\phi^+, \phi^0)\rangle = (0, v) \ , \ \langle(\phi^{0*}, \phi^-)\rangle = (v, 0)$$

we get $m_u = \lambda^u v \ ; \ m_d = \lambda^d v \ ; \ m_e = \lambda^e v \ ; \ (m_\nu = \lambda^\nu v \ ??)$

There is no prediction so far: just trading masses for Yukawa's

The fermionic gauge couplings

$$L = i\bar{\Psi}\gamma^\mu D_\mu \Psi + \dots$$

$$(D_\mu \Psi)^i \equiv (D_\mu)^i_j \Psi^j = (\partial_\mu \delta^i_j - igA^a_\mu (T^a)^i_j)\Psi^j$$

Again, each covariant derivative contains a sum over all the generators of the gauge group. These drop out for fermions or bosons which are neutral wrt a particular subgroup: e.g. the covariant derivative of u^c contains only the SU(3) and U(1) gauge fields and that of e^c only the latter. Finally, for ν^c the covariant derivative is just the normal partial derivative.

The fermionic gauge couplings (cont.d)

$$L = i\bar{\Psi}\gamma^\mu D_\mu \Psi + \dots$$

Let us rewrite in detail the covariant derivative (leaving out as usual the QCD part)

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^- + W_\mu^- T^+) - igW_\mu^3 T^3 - ig'YB_\mu$$

Inserting the inverse of the previous relations

$$W_\mu^3 = cos\theta_W Z_\mu + sin\theta_W A_\mu \; ; \; B_\mu = cos\theta_W A_\mu - sin\theta_W Z_\mu$$

we get, after a longish but easy calculation (recall Q = T^3+Y)

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^- + W_\mu^- T^+) - i\frac{g}{cos\theta_W} Z_\mu (T^3 - sin^2\theta_W Q) - ieA_\mu Q$$

where e = g′cos θ_W = g sin θ_W is related to the fine-structure constant α ~1/137

A precise non-trivial structure of neutral weak currents

The Higgs sector

$$L = \frac{1}{2} D_\mu \Phi^* D^\mu \Phi - \mu^2 \Phi^* \Phi - \lambda (\Phi^* \Phi)^2 + \ldots$$

The left-over Higgs boson is neutral and turns out to have $m_H^2 = -2\mu^2 > 0$ (for a potential like ours the second derivative of V at the minimum is minus twice the second derivative at the maximum)

We have already seen that v is related to G_F . Hence the two parameters of the Higgs potential are both related to experimentally measurable quantities…except that the latter has not been measured yet. μ^2 enters in other observables, via radiative corrections, but only through log's of μ (this is why we only have rough estimates of its value)

Higgs-fermion couplings

$$L_{quarks}^{Yukawa} = -\lambda^u \Phi Q u^c - \lambda^d \Phi^+ Q d^c$$
$$= -\lambda^u(\phi^0 u u^c + \phi^+ d u^c) - \lambda^d(\phi^- u d^c + \phi^{0*} d d^c)$$

$$L_{leptons}^{Yukawa} = -\lambda^e \Phi^+ L e^c = -\lambda^e(\phi^- \nu e^c + \phi^{0*} e e^c)$$

Since fermion masses are proportional to the Yukawa couplings, not surprisingly the couplings of fermions to the physical Higgs boson are proportional to their masses. This is an interesting prediction that can be verified once the Higgs boson is produced. Not only: it is relevant for finding which are the dominant H-production mechanisms (e.g at LEP or at the LHC).

Neutrino masses (an introduction)

$$L = \lambda^\nu \phi^0 \nu \nu^c - \frac{1}{2} M \nu^c \nu^c + c.c. + \dots$$

gives rise to a 2x2 neutrino mass matrix

$$M_\nu = \begin{pmatrix} 0 & \lambda^\nu v \\ \lambda^\nu v & M \end{pmatrix}$$

Such a matrix can be easily diagonalized. Two limiting cases:

1. If $M \ll \lambda^\nu v$ this is like any other fermion mass. The neutrino is very light because its Yukawa coupling λ^ν is much smaller than those of the quarks and the electron.
2. If, instead, $M \gg \lambda^\nu v$ the two eigenvalues are:

$m_1 \sim M$ and $m_2 \sim (\lambda^\nu v)^2/M$

Neutrino masses (cont.d)

$$M_\nu = \begin{pmatrix} 0 & \lambda^\nu v \\ \lambda^\nu v & M \end{pmatrix}$$

If $M \gg \lambda^\nu v$ $\quad m_1 \sim M$ and $m_\nu = m_2 \sim (\lambda^\nu v)^2/M \sim m_e^2/M$?

Theorists like very much this second, "see-saw", scenario. The idea is that M is naturally large since it has nothing to do with the SSB scale. Consequently the neutrino is very light. Anticipating the existence of heavier families, let's write:

$$m_\nu = m_2 = 10^{-1} eV \left(\frac{\lambda^\nu v}{1 GeV}\right)^2 \frac{10^{10} GeV}{M}$$

Suggests neutrinos as a window on very high energy physics!