Particules Élémentaires, Gravitation et Cosmologie
Année 2007-'08

Le Modèle Standard et ses extensions

Cours I: 8 février 2008

Théories de jauge : un rappel

- Programme du cours 2008
- Théories de jauge: un rappel à partir de mon cours*) 2005

*) http://www.college-de-france.fr/site/par_ele/p1089183508965.htm

Le modèle Standard et ses extensions

Date	9h45-10h45	11h-12h
08/02	Gauge theories, a reminder	QED/QCD, a reminder
15/02	Weak interactions: early days	Spont. symmetry breaking
22/02	SM Higgs	SM Lagrangian
29/02	Adding families, CKM	Accidental symmetries of SM
07/03	Flavour dynamics & CPX	Flavour dynamics & CPX
14/03	Neutrino masses/mixing	Neutrino masses/mixing
28/03	Higgs sector: fine tuning?	Status of EW precision tests
04/04	Composite-Higgs models	Higgs-less models
11/04	Supersymmetry	Where can new physics hide?

Le Modèle Standard (MS) et ses extensions

Date	9h45-10h45	11h-12h
08/02	Théories de jauge, un rappel	QED/QCD, un rappel
15/02	Interactions faibles: le début	Brisure spontanée de symétrie
22/02	Boson de Higgs du MS	Lagrangien du MS
29/02	Familles, matrice CKM	Symétries accidentelles du MS
07/03	Dynamique du saveur, XCP	Dynamique du saveur, XCP
14/03	Mass/mélange des neutrinos	Mass/mélange des neutrinos
28/03	Secteur de Higgs, réglage fin	Les tests de précision EF
04/04	Modèles à Higgs composé	Modèles sans Higgs
11/04	Supersymetrie	La nouvelle physique: où pourrait-elle se cacher?

Non-gravitational interactions are described by a gauge theory!

A theory of elementary particles should combine the principles of Special Relativity (SR) and those of Quantum Mechanics (QM) (RR + MQ = TRC, see my L.I. 2005)

It has been shown (see S. Weinberg's book on QFT) that, at sufficiently low energies, any theory that combines SR and QM must look like a « renormalizable » Quantum Field Theory (RQFT)

For reasons that are still not fully understood, Nature appears to have chosen a very special class of RQFT's, those known as Gauge Theories

Let us remind ourselves of SR, QM and of how one can define a generic gauge theory

Special Relativity

All objects we deal with (particles, fields, equations, …) must transform nicely under the

P = Poincaré group = Lorentz x Translations = *L* x *T*

$$x^\mu \Rightarrow x'^\mu = \Lambda^\mu_\nu \, x^\nu + a^\mu \qquad x' = \Lambda x + a$$

repeated index convention etc. etc.

$$\Lambda \, \eta \, \Lambda^T = \eta$$

$$\eta = diag(-1,1,1,1)$$

$x^T \eta x = \vec{x} \cdot \vec{x} - c^2 t^2$ is invariant

raises and lowers indices

6+4 = 10-parameter group

In above eqns $x^0 = ct$. We will use units in which c=1

Quantum Mechanics

States of quantum-systems are vectors in a Hilbert space

The vectors corresponding to free single-particle states should provide unitary representations of ***P***

Two operators commute with all 10 generators of ***P*** and label the reps. One of them is (remember: c=1!):

$$p^2 = p_\mu \, p^\mu = -E^2 + \mathbf{p}^2 = - m^2 \leq 0$$

This leads to two distinct cases:

a) $m \neq 0$, b) $m=0$

We will use units in which $\hbar = h/2\pi = 1$

a) $m \neq 0$

We can go to the rest frame of the particle: things become exactly like in NR-QM.

The second label becomes the spin $J = 0, 1/2, 1, 3/2, 2, ..$

For spin J there are ($2J+1$) states ($J_z = -J, -J+1, \ldots +J$)

b) $m=0$

We can go to a frame in which the momentum is along a particular axis. The second label now becomes the helicity h of the particle (= projection of J along that axis)

One can show that $h = 0, \pm 1/2, \pm 1, \pm 3/2, \pm 2, ..$

A single h gives an ir.rep. for the proper L ($\det\Lambda = +1$). If space-inversion is included, we need to put together two states ($\pm$ h), except if h=0:

Ex: photon ($h = \pm 1$), graviton ($h = \pm 2$), neutrino ($h = \pm 1/2$)

Quantum-Relativistic Fields

Like the corresponding particles, the fields of a QFT are characterized by their transformation properties under the ***P*** group. In general:

$$\phi_s^{(r)}(x) \to D_{ss'}^{(r)}(\Lambda)\phi_{s'}^{(r)}(\Lambda x + a)$$

The matrices *D* provide a finite-dimensional (in general a non-unitary) representation of *L*.

Since *L* = O(3,1) ~ O(3)xO(3), the irr. reps can be classified in terms of two «spins» j_- and j_+ i.e. (r) = (j_- , j_+):

$$\dim (j_- , j_+) = (2j_- +1)(2j_+ +1)$$

Let us give some simplest examples:

1. J=0 « scalar field »: $\phi(x)$, with $(j_-, j_+) = (0, 0)$
It describes a J=0 particle and has a trivial $D^{(r)} = 1$

2. J=1/2 « fermions » <u>can be of two kinds</u>:
Left-handed spinor: $\psi_\alpha(x)$, with $(j_-, j_+) = (1/2, 0)$
Right-handed spinor: $\chi_\alpha(x)$, with $(j_-, j_+) = (0, 1/2)$
Each one has two components (α=1,2)

3. J=1 « vector field »: $A_\mu(x)$, with $(j_-, j_+) = (1/2, 1/2)$
It has four components, (μ=0,1,2,3).

This is all we need as basic ingredients to define the SM or even its supersymmetric/grand-unified extensions!!

Only if we introduce (super) gravity we need some more « actors » with J>1.

Actually, if we neglect weak interactions (2005, 2006 courses), we do not even need J=0 fields!

Three forces, one principle!

The principle of gauge invariance is quite old: goes back to Maxwell's Classical Electrodynamics. There are many ways of introducing it.

In our context, the way I like best is the following: if we want to describe massless J=1 fields in a manifestly covariant way we need a vector field $A_\mu(x)$. However, such a field is too rich to describe just the two physical d.o.f. of a massless J=1 particle.

Gauge invariance is what allows us to get rid of (« gauge away ») the extra d.o.f. These are not only redundant but, more often than not, also pathological.

This is why gauge invariance must be there and has to be preserved after quantum corrections have been fully included (absence of anomalies in gauge symmetries)

Therefore if we ask:

Why does Nature like Gauge Theories?

we may answer:

Because it likes massless J=1 particles!

But we still don't know why Nature likes massless J=1 (and J=2 for gravity) particles;

Without them our world would lack the long-range forces that form the structures we see in the sky, but also the atoms, the molecules, and life…

The only (partial) answer I know to this second question is: quantum string theory necessarily gives such kinds of particles (next year?)

Construction of a generic Gauge Theory:
a) Without scalars (QED, QCD, ...)
b) With scalars (adding EW-interactions)

The recipe for defining a generic (renormalizable) gauge theory is simple. Solving it's another story...

For case a) it consists of the following 3 steps:

1. Choose a gauge group G;
2. Assign the l.h. fermions to a representation of G;
3. Add a generic mass term for the fermions

For case b) a few more steps are needed, as we will discuss in a forthcoming lecture

1. Gauge group and gauge bosons

Choose a gauge group, G (e.g. U(1), SU(3), SU(2)xU(1)) and introduce as many gauge fields $A^a_\mu(x)$ as there are generators in G.

Construct the field-strength tensors $F^a_{\mu\nu}(x)$ associated with $A^a_\mu(x)$ (generalization of $F^a_{\mu\nu}(x)$ of QED, see below)

If there are no «matter» fields, the action is completely determined up to assigning the gauge couplings (one for each factor in G, a single one if G is simple)

2. Fermions

Add l.h. fermions by specifying the representation (r) of *G* they belong to.

There will also be, automatically, the corresponding r.h. (anti)fermions belonging to the c.c. rep. (r*) of *G* .

The way the fermions appear in the theory, including their coupling to the gauge bosons, is completely fixed (see below) up to mass terms

Examples:

a) QED

G = U(1) => 1 gauge boson: the photon.

Add electrons and positrons:

e, representing the lh electron with charge -1,

e^c representing the lh positron with charge +1

The corresponding r.h. antiparticles have charge +1 and -1

b) QCD (with 1 quark flavour)

G = SU(3) => 8 gauge bosons called gluons. Add quarks:

u^i = l.h. up quark in **3** of SU(3) and

u^c_i = l.h. up antiquark in **3***

The corresponding r.h. antiparticles fill a **3*** and a **3**, resp.

Table of prototype model

G

Left-handed fermion	U(1)	SU(3)
electron	-1	1
positron	+1	1
up quark	2/3	3
up antiquark	-2/3	3*

A SM without weak interactions

l.h. fermion	U(1)	SU(3)
electron	-1	1
positron	+1	1
neutrino	0	1
u quark	2/3	3
u antiquark	-2/3	3*
d quark	-1/3	3
d antiquark	1/3	3*

In this fake world the quarks, the electron and the neutrino can all have masses, p=(uud), n=(ddu) but n does NOT decay!

3. Fermionic masses

- Mass terms are non-derivative bilinear terms in the Action. For J=0 «scalars» it is always possible to write down a gauge-invariant mass term (as we shall see)
- Instead, femionic mass terms must contain two l.h. fermions (+ c.c. term with two r.h. ones).
- This is because (1/2, 0)x(1/2, 0) = (0,0) +(1,0) while (1/2, 0)x(0, 1/2) = (1/2, 1/2)
- As such, fermionic mass terms can be made gauge-invariant iff the product of their reps. contains the singlet. One calls such theories vectorlike. Otherwise they are called «chiral»

Two important classes of vector-like teories

A) The two fermions belong to complex-conjugate reps (case of a « Dirac mass », e.g. QED, QCD, see table) and we write mass terms like:

$$L_{mass} \sim -m_u \, u^i_\alpha \, u^c_{i\,\beta} \, \varepsilon_{\alpha\beta} + c.c.$$

B) A fermion belongs to a real irr. rep. (r=r*): the two fermions can be one and the same: (example of neutrino « Majorana mass » ~)

$$L_{mass} \sim -m_\nu \, \nu_\alpha \, \nu_\beta \, \varepsilon_{\alpha\beta} + c.c.$$