Fonctions de distribution partonique

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Introduction

Proton, we're told, is make of *2 up quarks, 1 down quark.*

The picture seems consistent: up-charge $= +\frac{2}{3}$; down charge $= -\frac{1}{3}$

$$2 \times \frac{2}{3} - 1 \times \frac{1}{3} = +1$$

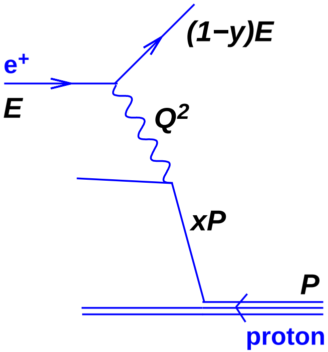
But is this *right?*

Formalism discussed by Prof. Veneziano allows us to look inside the proton and *find out for sure.*

Deep Inelastic Scattering: kinematics

We will be discussing Deep Inelastic Scattering (DIS) for 3/4 of seminar.

Recall what the process is and the main kinematic variables:



- x = momentum fraction of struck parton in proton
- Q^2 = photon virtuality $\leftrightarrow$ transverse resolution at which it probes proton structure
- y = momentum fraction lost by photon (in proton rest frame)

Kinematic relation:

$$Q^2 = xys$$

$\sqrt{s}$ = c.o.m. energy

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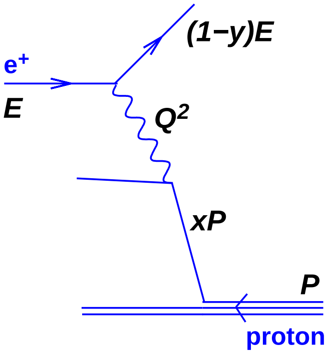

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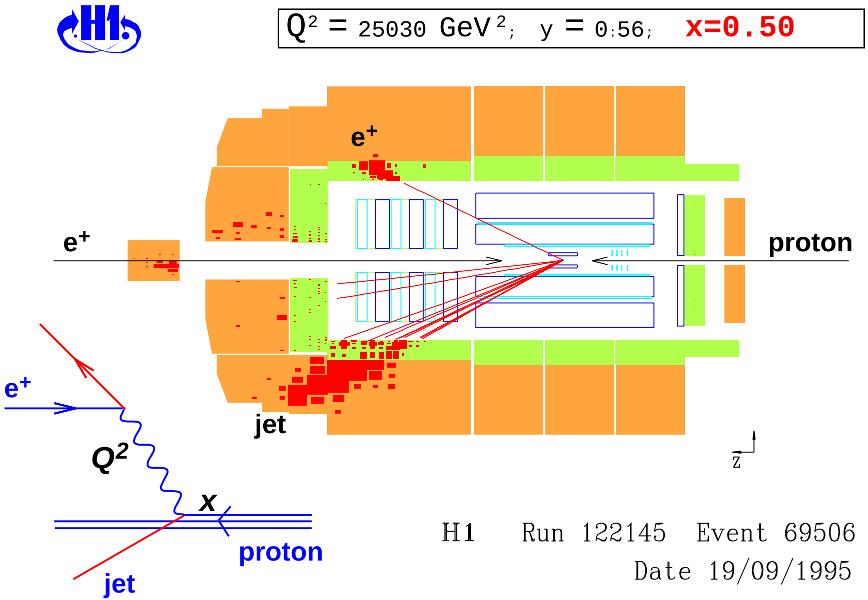
Deep Inelastic scattering (DIS): example



DIS: photon exchange

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

$\propto F_2^{em}$ [structure function]

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

[$u(x)$, $d(x)$: parton distribution functions (PDF)]

NB:

- use perturbative language for interactions of up and down quarks
- but distributions themselves have a *non-perturbative* origin.

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F_2 gives us *combination* of u and d.
How can we extract them separately?

Using neutrons

Assumption ($SU(2)$ isospin): neutron is just proton with $u \Leftrightarrow d$:
proton = uud; neutron = ddu $[-2 \times \frac{1}{3} + 2 \times \frac{1}{3} = 0]$

Isospin: $u_n(x) = d_p(x)\,, \qquad d_n(x) = u_p(x)$

$$F_2^p = \frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)$$
$$F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x) = \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

Linear combinations of F_2^p and F_2^n give separately $u_p(x)$ and $d_p(x)$.

Experimentally, get F_2^n from deuterons: $F_2^d = \frac{1}{2}(F_2^p + F_2^n)$

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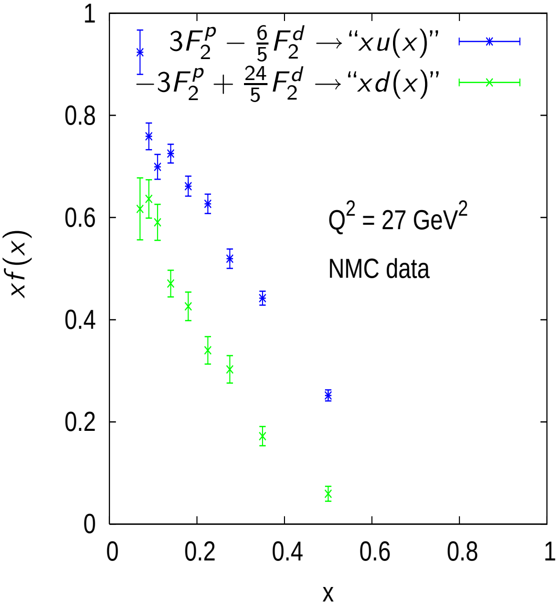

Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

How much u and d?

- Total $U = \int dx\ u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

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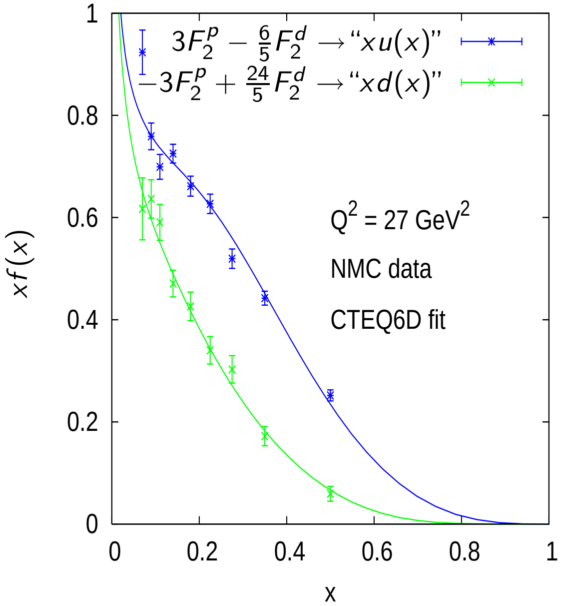

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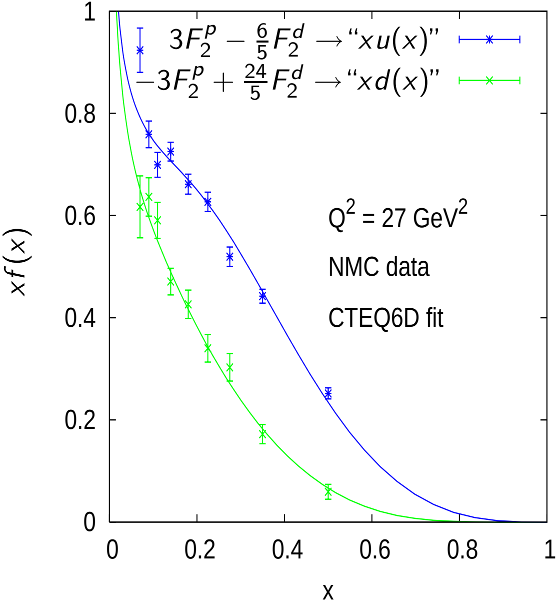

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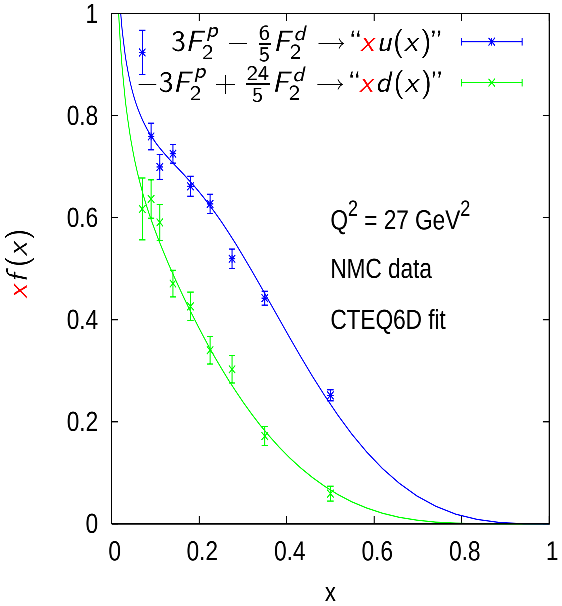

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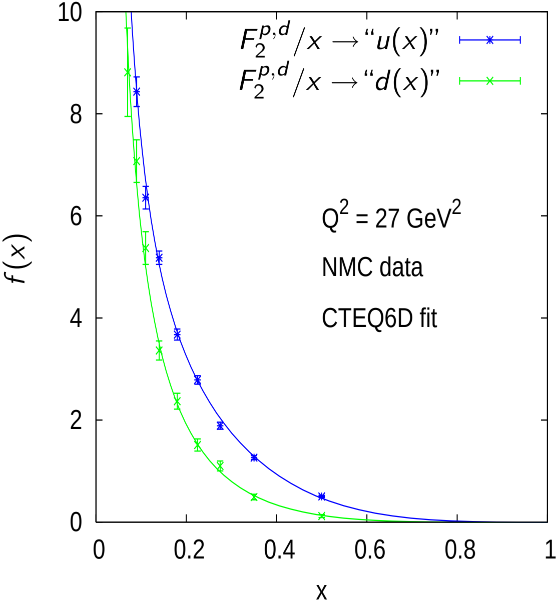

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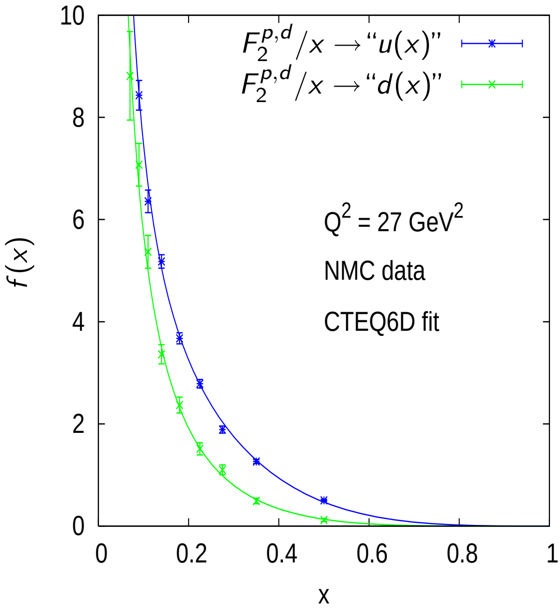

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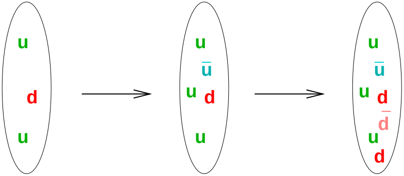

How can there be infinite number of quarks in proton?

Proton wavefunction *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

Antiquarks also have distributions, $\bar{u}(x)$, $\bar{d}(x)$

$$F_2 = \frac{4}{9}(xu(x) + x\bar{u}(x)) + \frac{1}{9}(xd(x) + x\bar{d}(x))$$

NB: photon interaction $\sim$ square of charge $\rightarrow$ +ve

- Previous transparency: we were actually looking at $\sim u + \bar{u}$, $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite*, so

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as long as they carry little momentum (mostly at low x)

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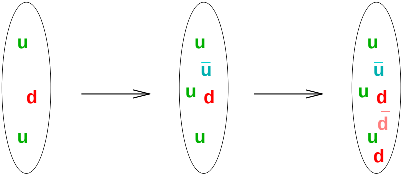

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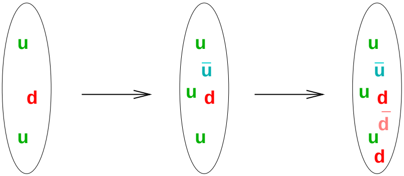

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"Valence" quarks

When we say proton has 2 up quarks & 1 down quark we mean

$$\int dx\,(u(x) - \bar{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \bar{d}(x)) = 1$$

$u - \bar{u} = u_V$ is known as a *valence* distribution.

How do we measure *difference* between u and $\bar{u}$? Photon interacts identically with both → no good...

Question: what interacts differently with particle & antiparticle?

Answer: *W^+ or W^-*

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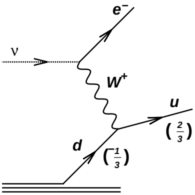

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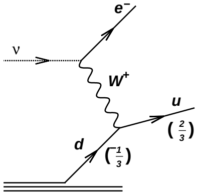

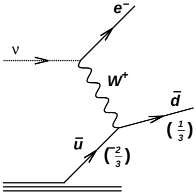

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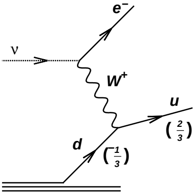

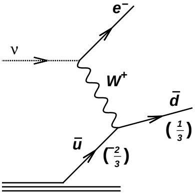

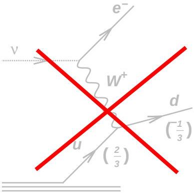

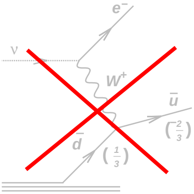

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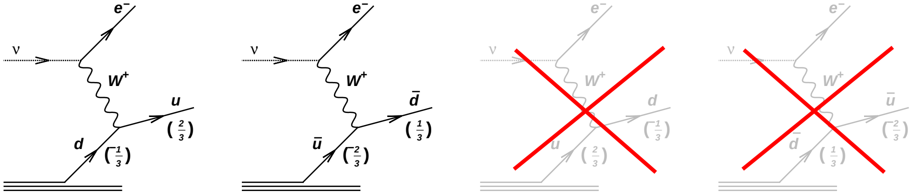

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Combination of νp and $\bar{\nu} p$ scattering in principle provides all necessary information for getting separately u, d, $\bar{u}$ and $\bar{d}$.

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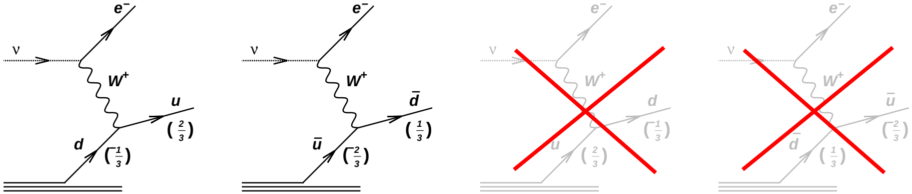

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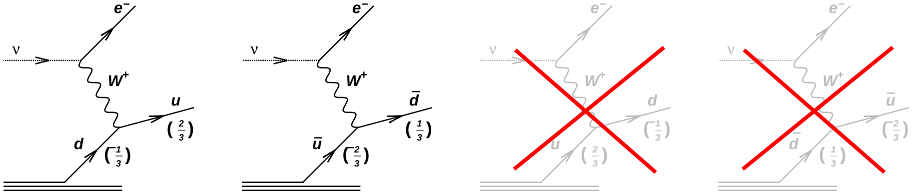

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Valence quarks

Problem: experiments with neutrinos are *difficult* (small cross sections).

Look at collisions on *nuclei* (e.g. Fe) to increase cross section, and use *isospin symmetry* ($d_n = u_p$) to relate $F_3^{W^+p}$, $F_3^{W^+n}$

$$F_3^{W^+N} = \frac{1}{2}(F_3^{W^+p} + F_3^{W^+n}) = d_p(x) - \bar{u}_p(x) + d_n(x) - \bar{u}_n(x)$$
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E.g.: use this to check total number of *valence quarks is 3:*

$$\int dx\, F_3^{W^+N}(x) = \int dx\, (\, d_V(x) + u_V(x)\,) = 3$$

Gross LLewellyn Smith sum rule

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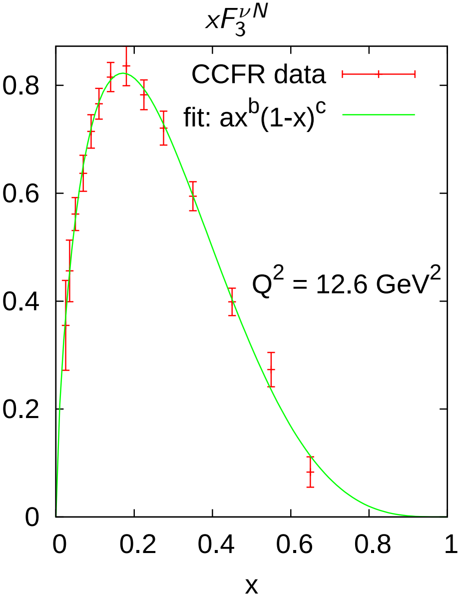

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  Regge theory: $xu_V, xd_V \sim x^{0.5}$

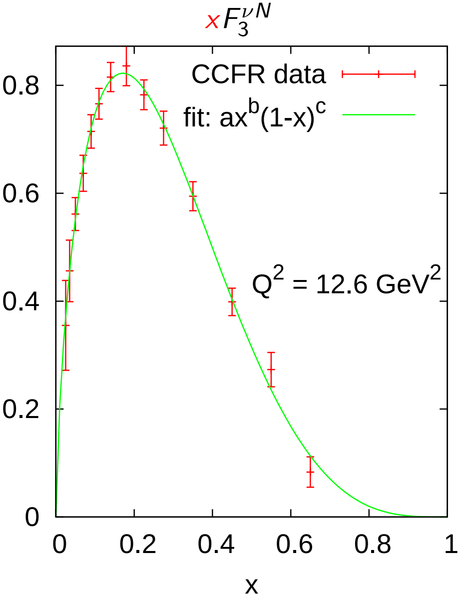
- $F_3^{\nu N} \simeq u_V + d_V$ should be *integrable*

  $\hookrightarrow \int dx\, F_3^{\nu N} = 2.50 \pm 0.08$

  CCFR, $Q^2 = 3$ GeV^2

  We expected 3 (uud)...

νN data



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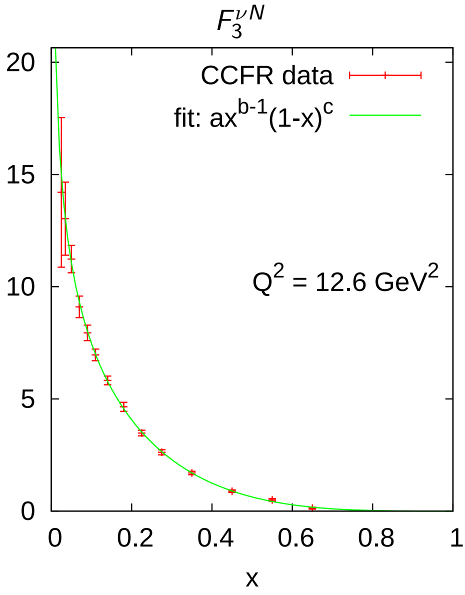

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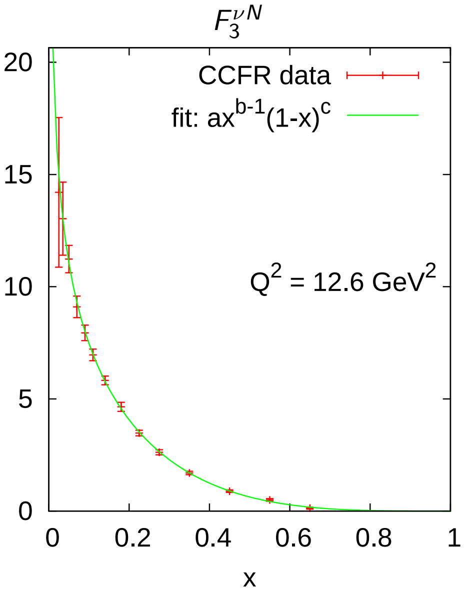

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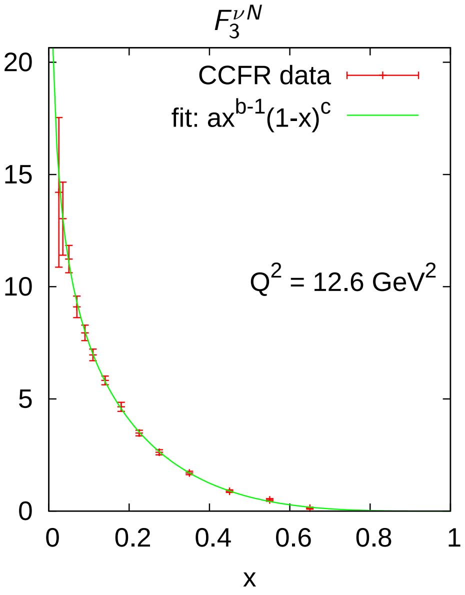

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But interaction with W^+ receives *higher order QCD corrections:*

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$$\simeq 2.52 \qquad [\alpha_s(3\ \mathrm{GeV}^2) \simeq 0.34]$$

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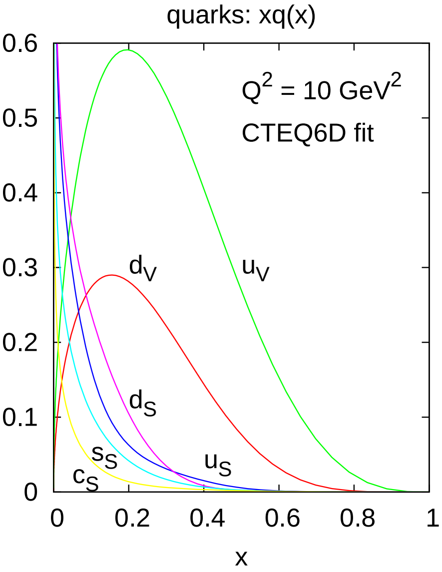

These & other methods $\longrightarrow$ whole set of quarks & antiquarks

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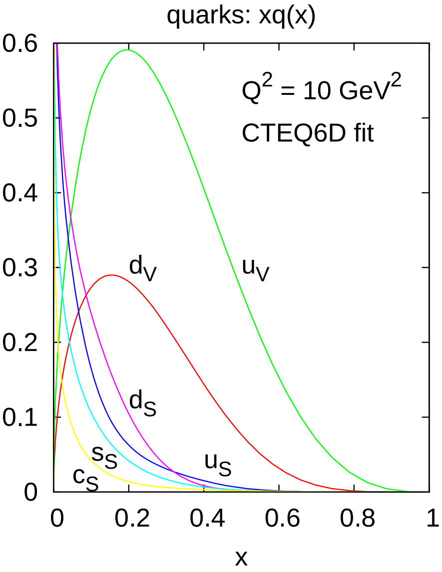

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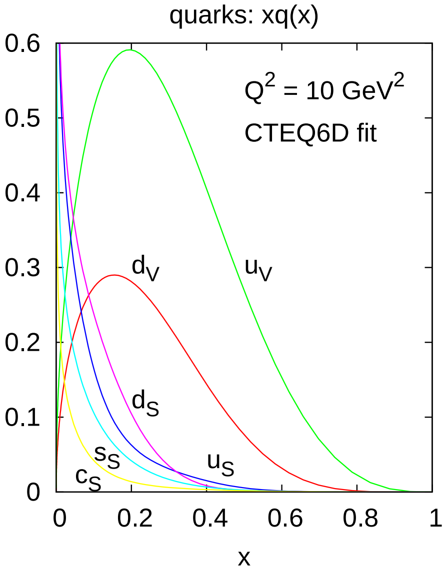

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Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx \, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
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s_S	0.033
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total	0.546

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Only parton type we've neglected so far is the

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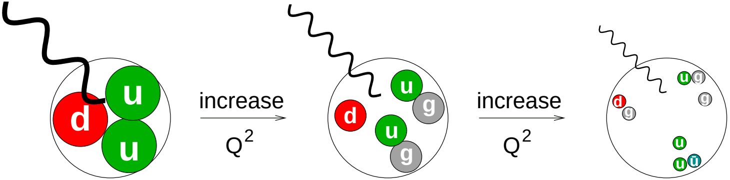

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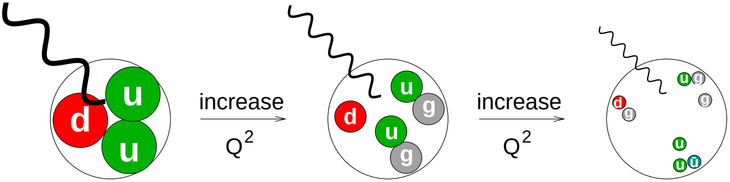

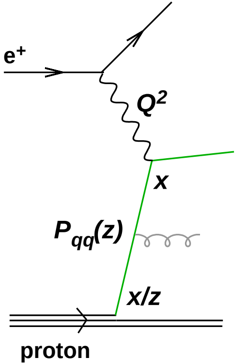

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Quark, gluon distributions are functions of x *and* Q^2, $q(x, Q^2)$, $g(x, Q^2)$. They evolve with Q^2:

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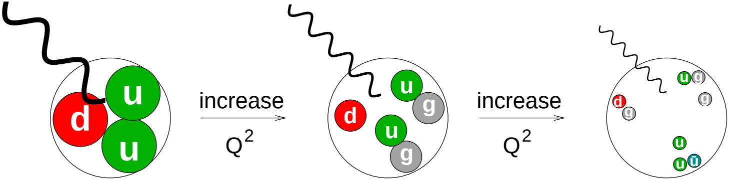

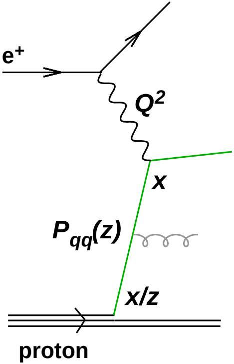

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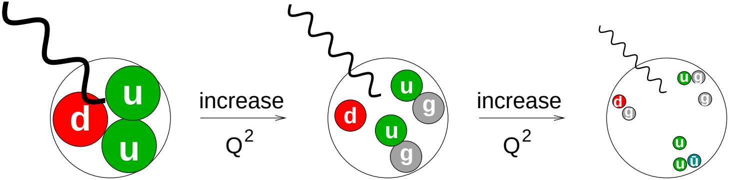

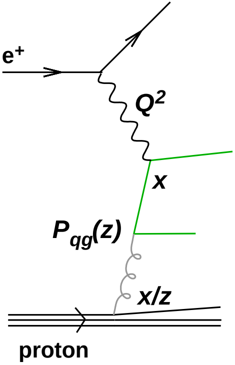

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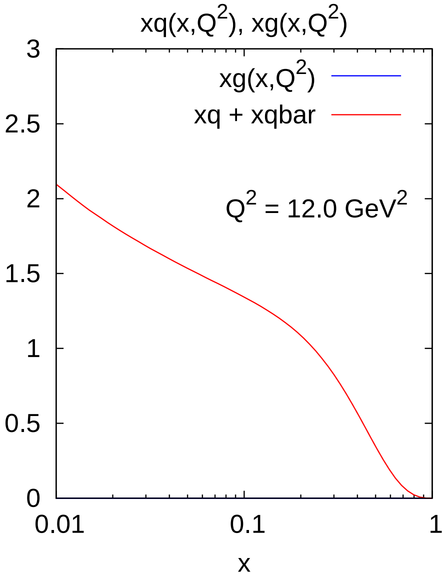

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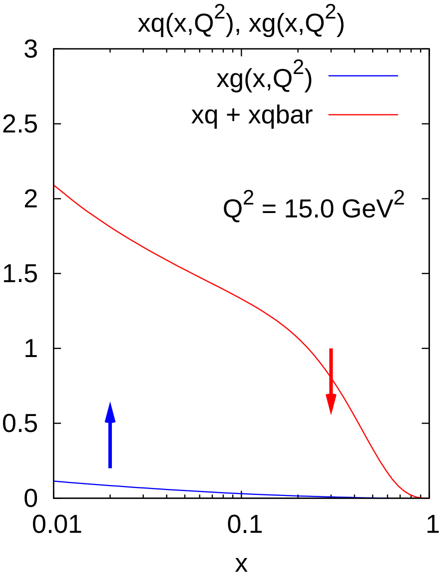

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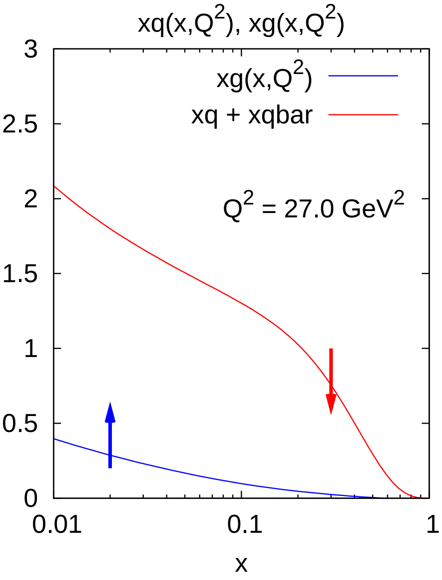

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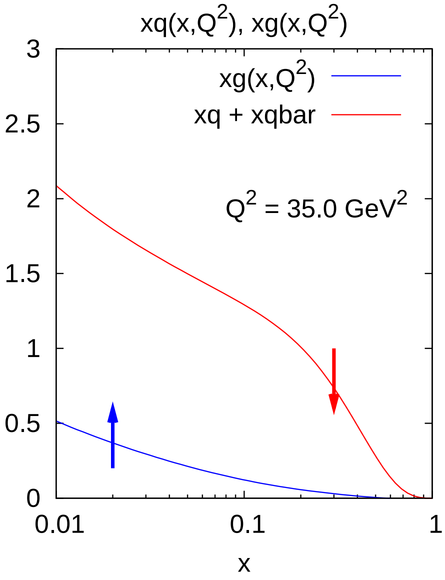

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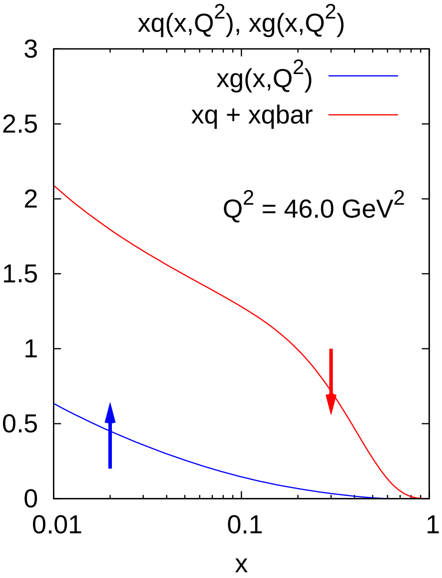

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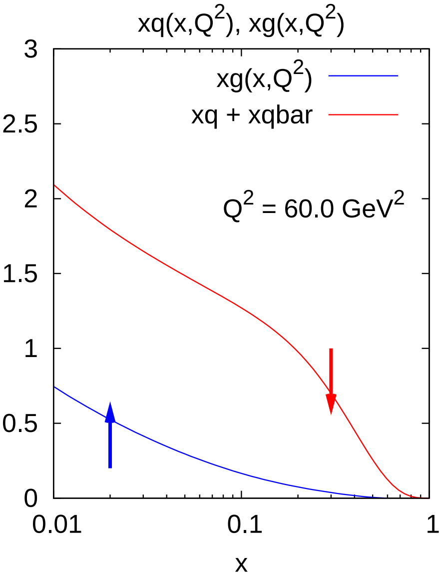

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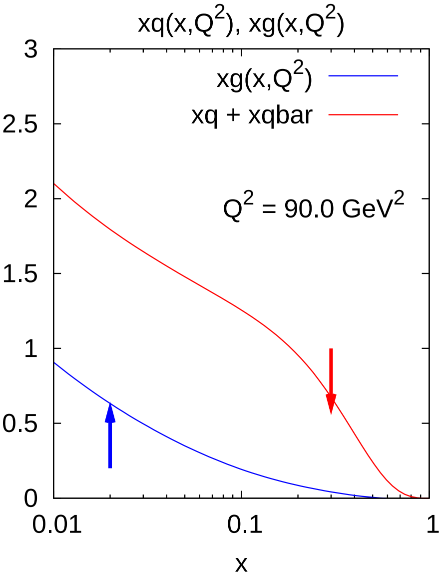

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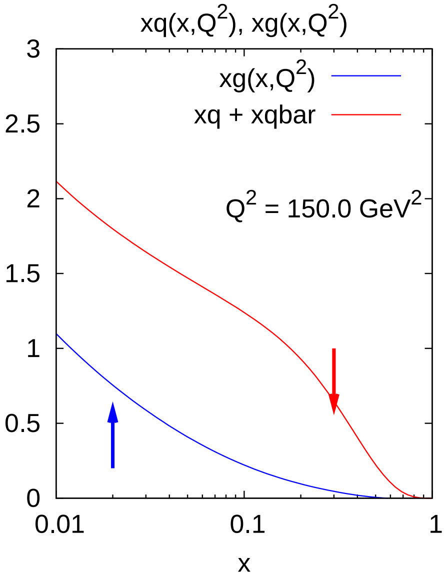

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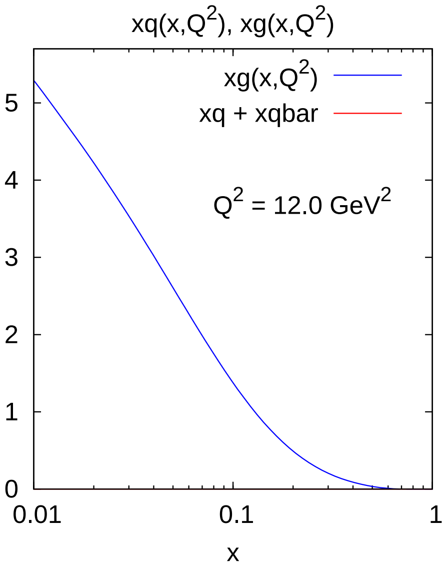

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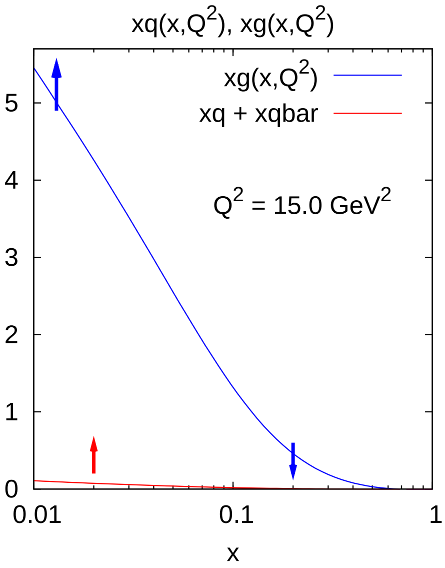

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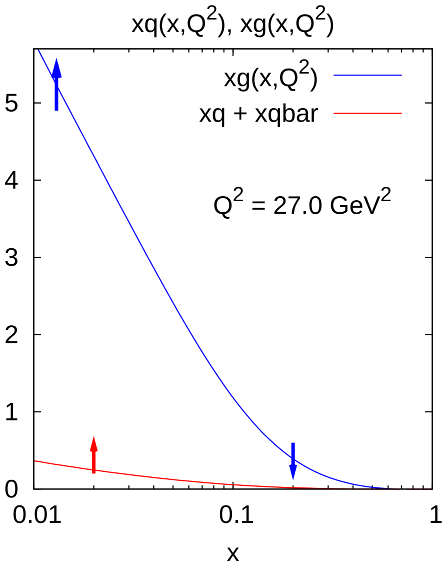

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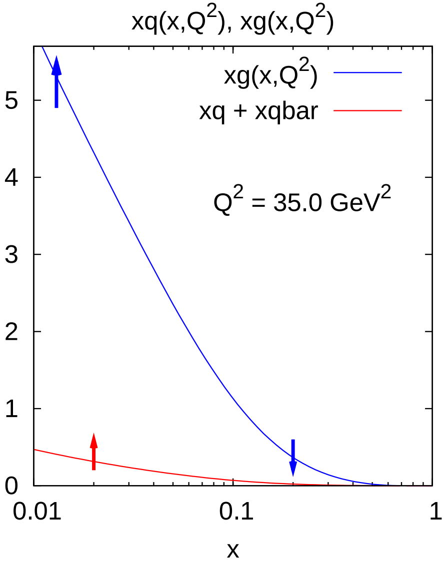

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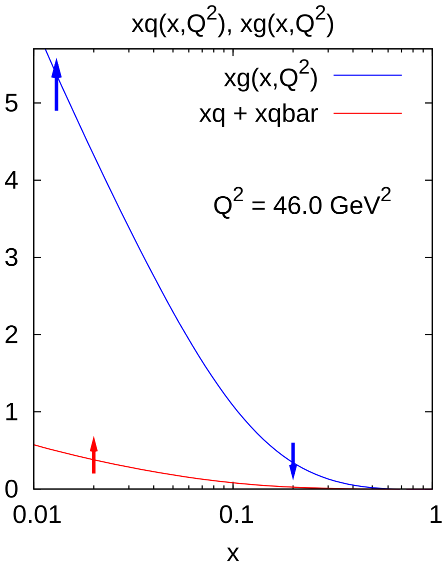

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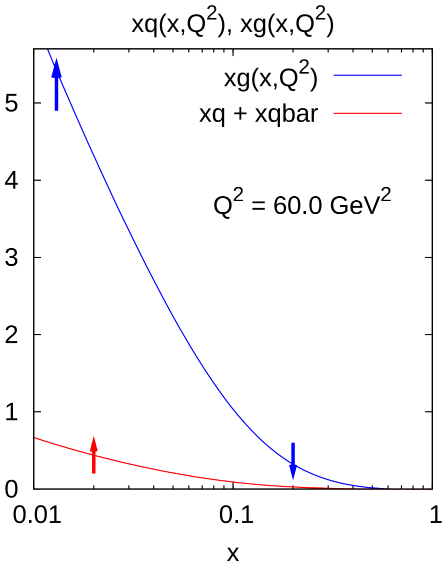

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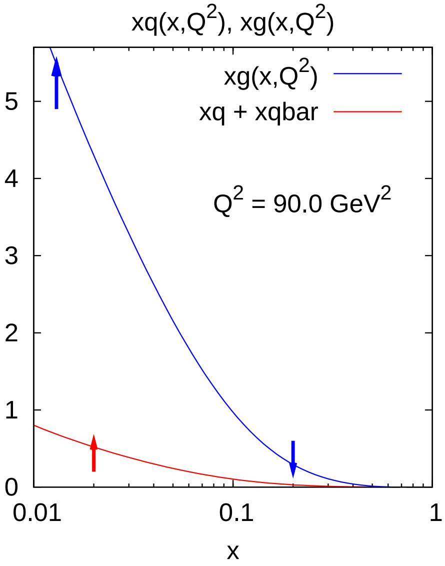

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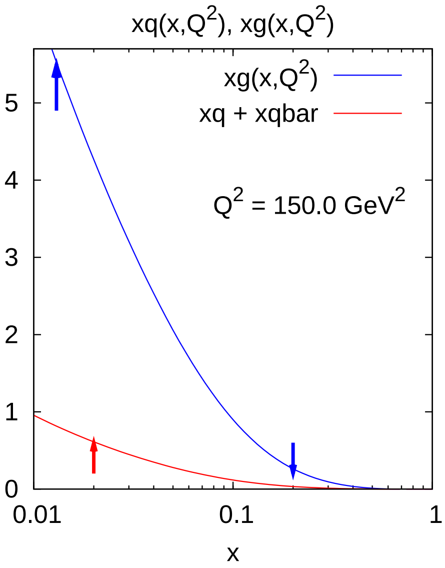
$$\partial_{\ln Q^2} g = P_{g \leftarrow g} \otimes g$$
$$\partial_{\ln Q^2} q = P_{q \leftarrow g} \otimes g$$

- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

Effect of DGLAP (initial gluons)



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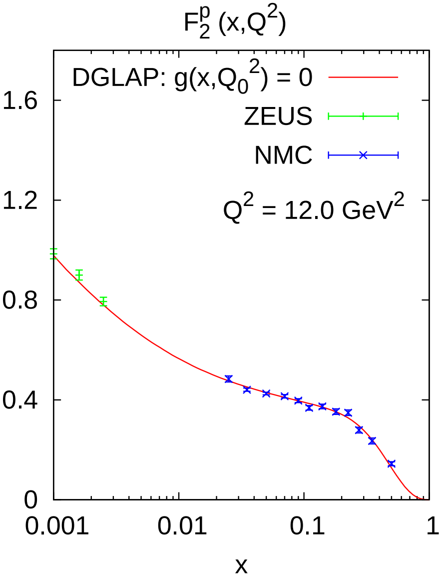
DGLAP evolution

- As Q^2 increases, partons lose longitudinal momentum; distributions all shift to lower x.
- gluons can be seen because they help drive the quark evolution.

Now consider data

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

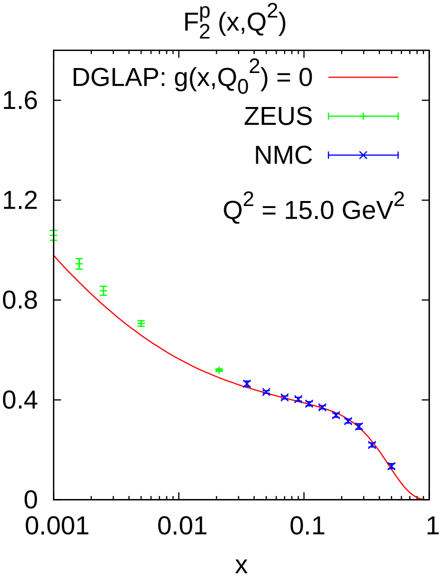
NB: Q_0 often chosen lower

Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

DGLAP with initial gluon $= 0$



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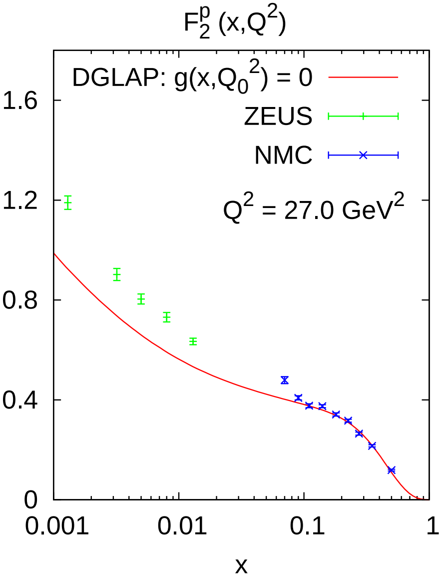

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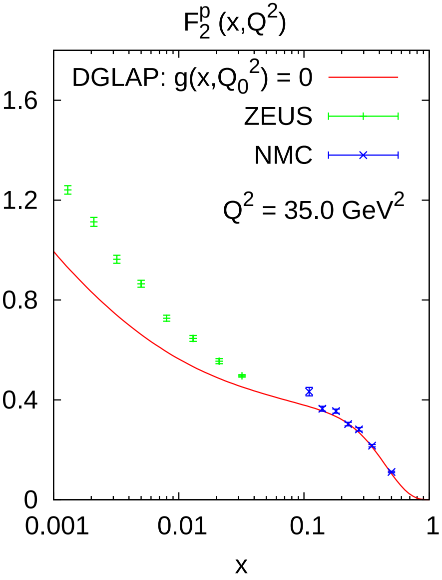

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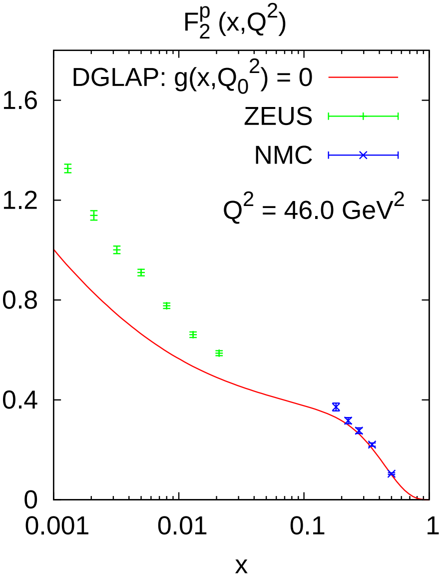

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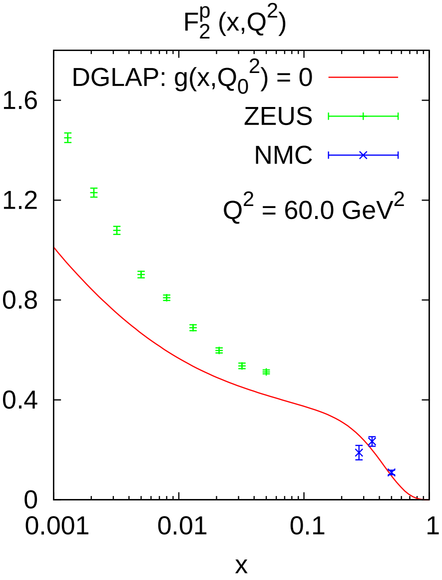

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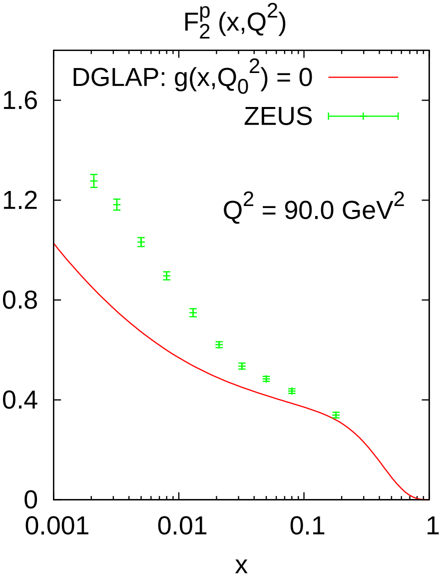

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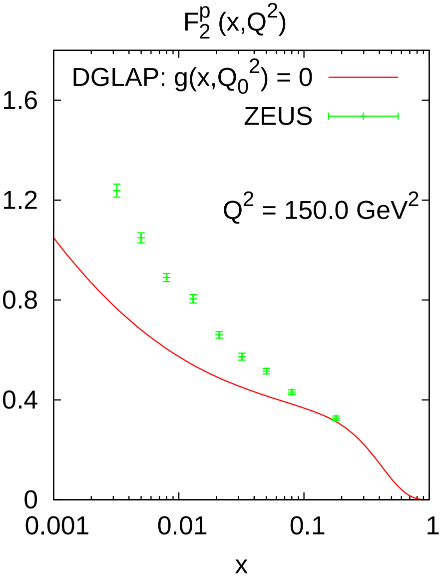

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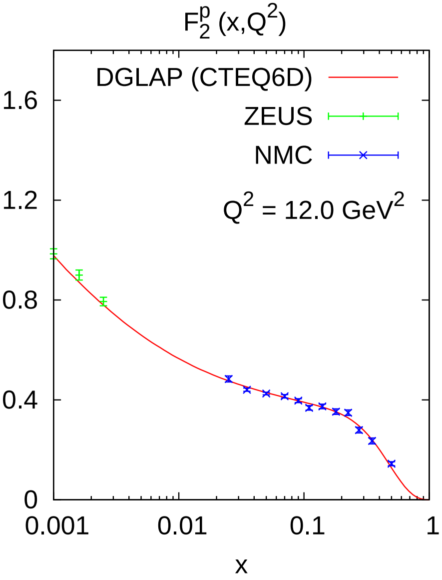

If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...

PDF fitting collaborations.

Success!

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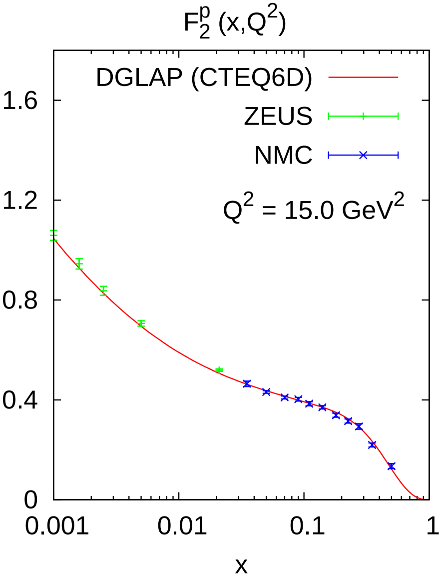

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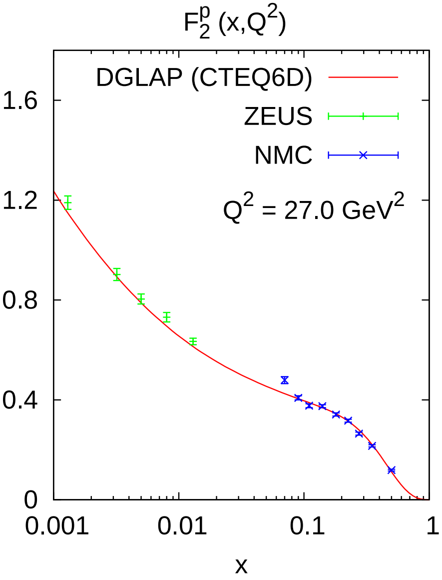

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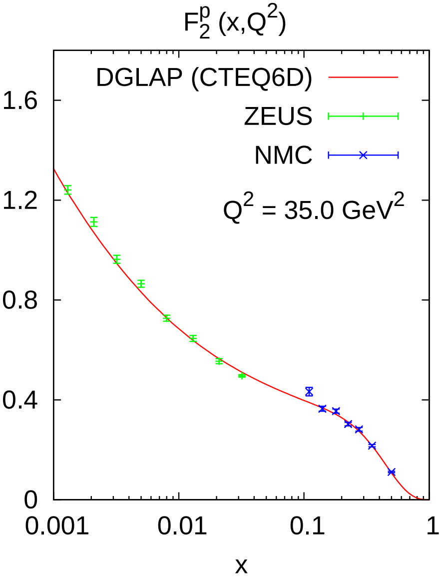

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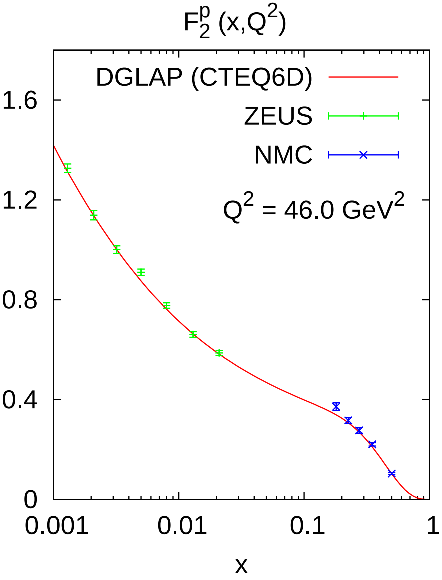

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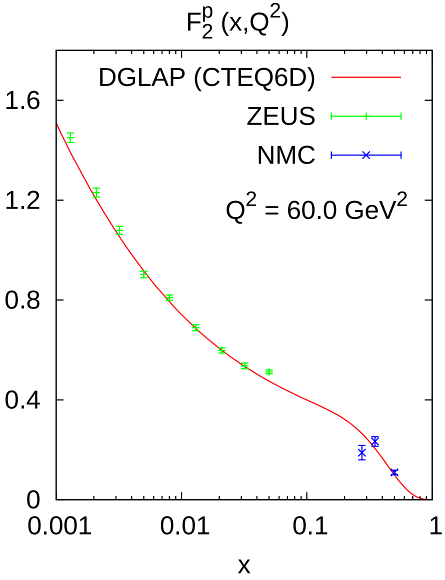

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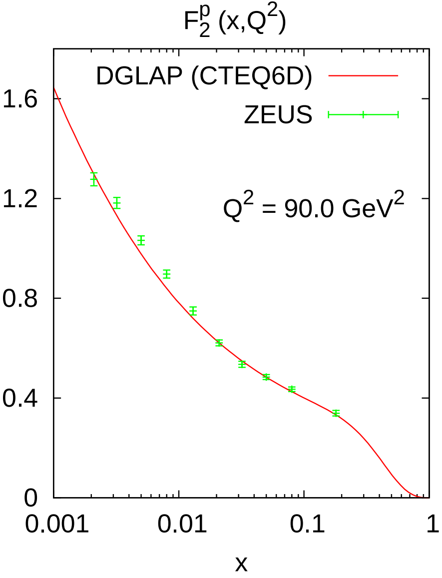

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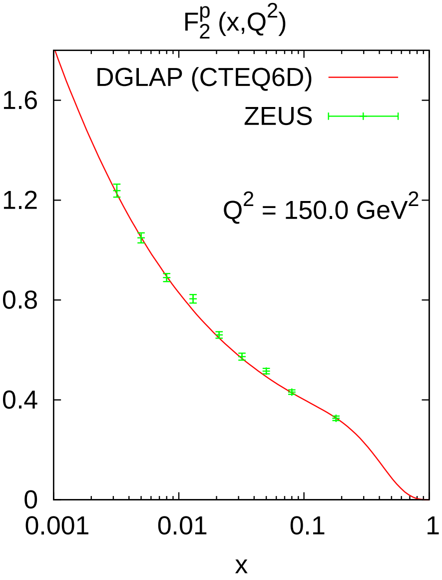

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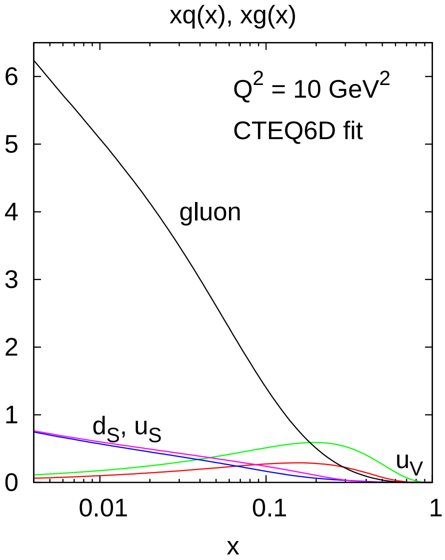
Gluon distribution



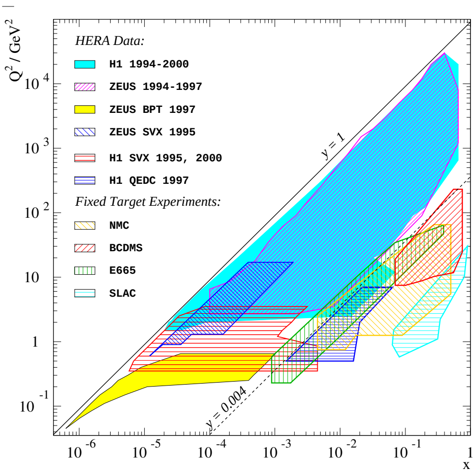
Gluon distribution is **HUGE!**

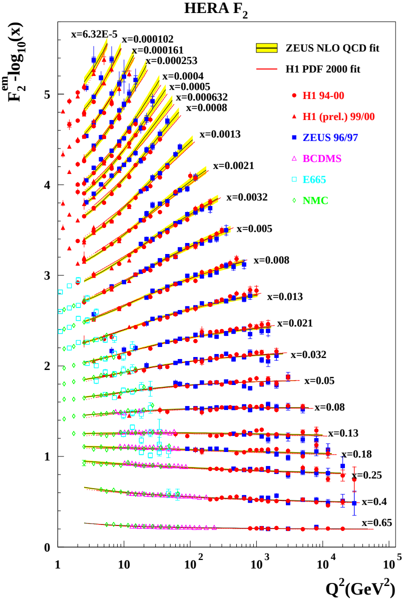
Can we really trust it?

- Consistency: momentum sum-rule is now *satisfied*.

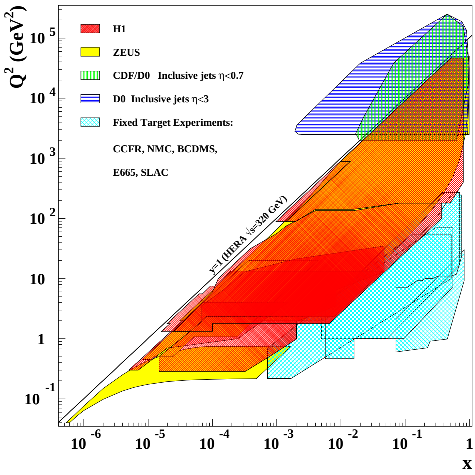
  NB: gluon mostly at small x
- Agrees with vast range of data

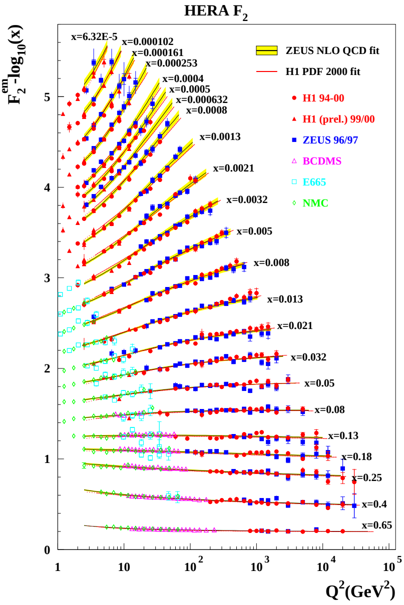
DIS data


HERA Data:
H1 1994-2000
ZEUS 1994-1997
ZEUS BPT 1997
ZEUS SVX 1995
H1 SVX 1995, 2000
H1 QEDC 1997
Fixed Target Experiments:
NMC
BCDMS
E665
SLAC
y = 1
y = 0.004
Q^2 / GeV^2
x


HERA F_2
F_2^{em}-log$_{10}$(x)
Q^2(GeV^2)
ZEUS NLO QCD fit
H1 PDF 2000 fit
H1 94-00
H1 (prel.) 99/00
ZEUS 96/97
BCDMS
E665
NMC
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65

DIS data


H1
ZEUS
CDF/D0 Inclusive jets η<0.7
D0 Inclusive jets η<3
Fixed Target Experiments:
CCFR, NMC, BCDMS,
E665, SLAC
y=1 (HERA √s=320 GeV)
Q² (GeV²)
x


HERA F₂
F₂^em-log₁₀(x)
ZEUS NLO QCD fit
H1 PDF 2000 fit
H1 94-00
H1 (prel.) 99/00
ZEUS 96/97
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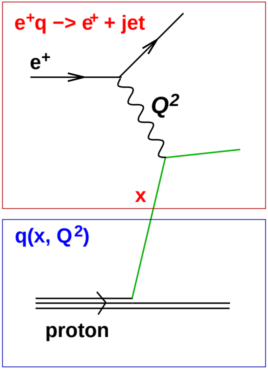
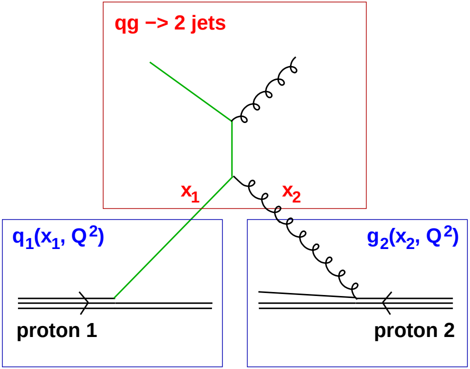
Q²(GeV²)

Crucial check: other processes

Factorization of QCD cross-sections into convolution of:

- hard (perturbative) process-dependent **partonic subprocess**
- non-perturbative, process-independent **parton distribution functions**





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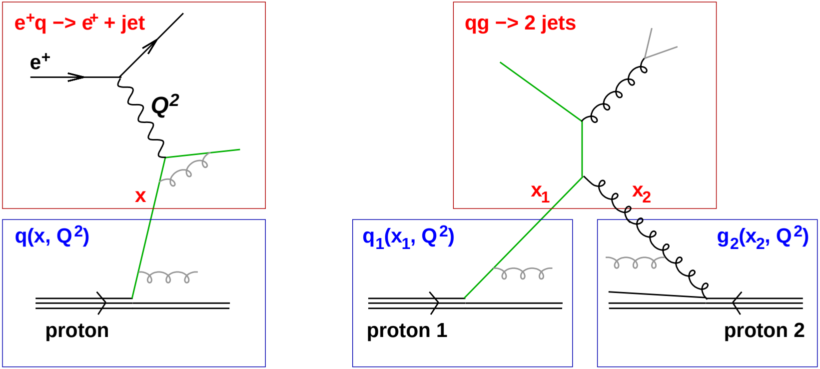

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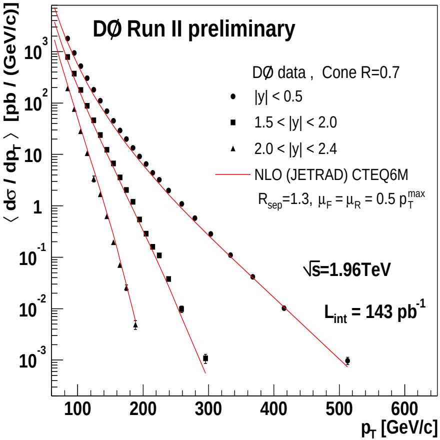
Jet production in $p\bar{p}$



Jet production in proton-antiproton collisions is *good test of large gluon distribution*, since there are large direct contributions from

$$gg \to gg\,, \quad qg \to qg$$

NB: more complicated to interpret than DIS, since many channels, and x_1, x_2 dependence.

$p_T \sim \sqrt{x_1 x_2 s}$ jet transverse mom.

$\sim Q$

$$y \sim \frac{1}{2}\log\frac{x_1}{x_2} \qquad y = \log\tan\frac{\theta}{2}$$

jet angle wrt $p\bar{p}$ beams

Good agreement confirms factorization

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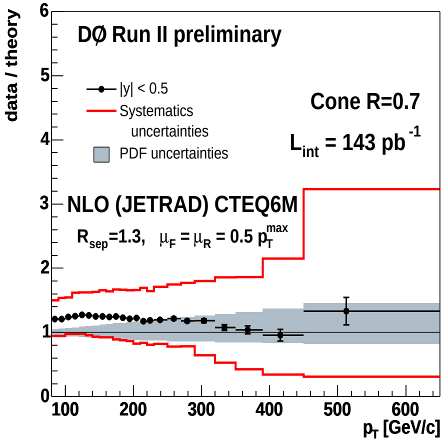

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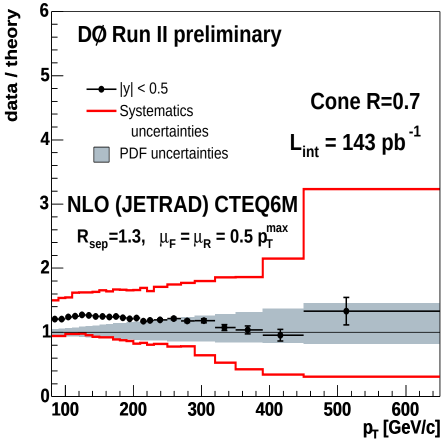

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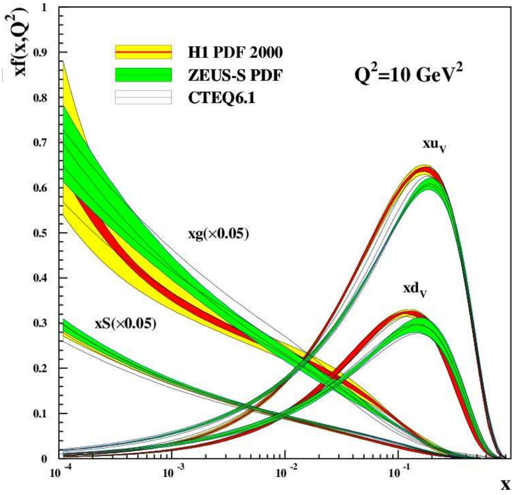
PDF uncertainties



Major recent activity is translation of experimental errors (and theory uncertainties) into *uncertainty bands on extracted PDFs.*

PDFs with uncertainties allow one to estimate *degree of reliability* of future predictions

Higher-order calculations

Earlier, we saw leading order (LO) DGLAP splitting functions, $P_{ab} = \frac{\alpha_s}{2\pi} P^{(0)}_{ab}$:

$$P^{(0)}_{qq}(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right] ,$$

$$P^{(0)}_{qg}(x) = T_R \left[x^2 + (1-x)^2 \right] ,$$

$$P^{(0)}_{gq}(x) = C_F \left[\frac{1+(1-x)^2}{x} \right] ,$$

$$P^{(0)}_{gg}(x) = 2C_A \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \delta(1-x)\frac{(11C_A - 4n_f T_R)}{6} .$$

Higher-order calculations

NLO:

$$P_{ab} = \frac{\alpha_s}{2\pi} P^{(0)} + \frac{\alpha_s^2}{16\pi^2} P^{(1)}$$

Curci, Furmanski & Petronzio '80

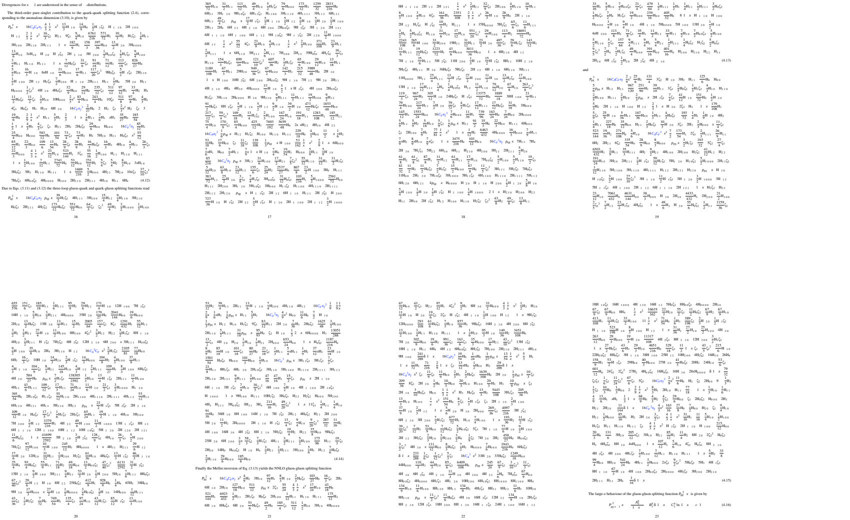
$$P^{(1)}_{\rm ps}(x) = 4\, C_F n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4\mathrm{H}_0 + x^2\left[\frac{8}{3}\mathrm{H}_0 - \frac{56}{9}\right] + (1+x)\left[5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\right]\right)$$

$$P^{(1)}_{\rm qg}(x) = 4\, C_A n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\rm qg}(-x)\mathrm{H}_{-1,0} - 2p_{\rm qg}(x)\mathrm{H}_{1,1} + x^2\left[\frac{44}{3}\mathrm{H}_0 - \frac{218}{9}\right] + 4(1-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_0 + x\mathrm{H}_1\right] - 4\zeta_2 x - 6\mathrm{H}_{0,0} + 9\mathrm{H}_0\right) + 4\, C_F n_f \left(2p_{\rm qg}(x)\left[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 - \zeta_2\right] + 4x^2\left[\mathrm{H}_0 + \mathrm{H}_{0,0} + \frac{5}{2}\right] + 2(1-x)\left[\mathrm{H}_0 + \mathrm{H}_{0,0} - 2x\mathrm{H}_1 + \frac{29}{4}\right] - \frac{15}{2} - \mathrm{H}_{0,0} - \frac{1}{2}\mathrm{H}_0\right)$$

$$P^{(1)}_{\rm gq}(x) = 4\, C_A C_F \left(\frac{1}{x} + 2p_{\rm gq}(x)\left[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 - \frac{11}{6}\mathrm{H}_1\right] - x^2\left[\frac{8}{3}\mathrm{H}_0 - \frac{44}{9}\right] + 4\zeta_2 - 2 - 7\mathrm{H}_0 + 2\mathrm{H}_{0,0} - 2\mathrm{H}_1 x + (1+x)\left[2\mathrm{H}_{0,0} - 5\mathrm{H}_0 + \frac{37}{9}\right] - 2p_{\rm gq}(-x)\mathrm{H}_{-1,0}\right) - 4\, C_F n_f \left(\frac{2}{3}x - p_{\rm gq}(x)\left[\frac{2}{3}\mathrm{H}_1 - \frac{10}{9}\right]\right) + 4\, C_F^{\,2} \left(p_{\rm gq}(x)\left[3\mathrm{H}_1 - 2\mathrm{H}_{1,1}\right] + (1+x)\left[\mathrm{H}_{0,0} - \frac{7}{2} + \frac{7}{2}\mathrm{H}_0\right] - 3\mathrm{H}_{0,0} + 1 - \frac{3}{2}\mathrm{H}_0 + 2\mathrm{H}_1 x\right)$$

$$P^{(1)}_{\rm gg}(x) = 4\, C_A n_f \left(1 - x - \frac{10}{9}p_{\rm gg}(x) - \frac{13}{9}\left(\frac{1}{x} - x^2\right) - \frac{2}{3}(1+x)\mathrm{H}_0 - \frac{2}{3}\delta(1-x)\right) + 4\, C_A^{\,2} \left(27 + (1+x)\left[\frac{11}{3}\mathrm{H}_0 + 8\mathrm{H}_{0,0} - \frac{27}{2}\right] + 2p_{\rm gg}(-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_{-1,0} - \zeta_2\right] - \frac{67}{9}\left(\frac{1}{x} - x^2\right) - 12\mathrm{H}_0 - \frac{44}{3}x^2\mathrm{H}_0 + 2p_{\rm gg}(x)\left[\frac{67}{18} - \zeta_2 + \mathrm{H}_{0,0} + 2\mathrm{H}_{1,0} + 2\mathrm{H}_2\right] + \delta(1-x)\left[\frac{8}{3} + 3\zeta_3\right]\right) + 4\, C_F n_f \left(2\mathrm{H}_0 + \frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\left[4 - 5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\right] - \frac{1}{2}\delta(1-x)\right) .$$

Higher-order calculations



NNLO, $P_{ab}^{(2)}$: Moch, Vermaseren & Vogt '04

Conclusions

- Experiments tell us that proton really is what we expected (*uud*)
- Plus lots more: large number of 'sea quarks' ($q\bar{q}$), gluons (50% of momentum)
- We see *factorization:* parton distributions extracted in electron-proton collisions can be used to *predict* characteristics of proton-(anti)proton collisions
  - jet cross sections
  - top-quark cross section
  - Drell-Yan cross section
  - ...
- *Precision* of data & QCD calculations steadily increasing.
- Crucial for understanding future signals of *new particles,* e.g. Higgs Boson production at LHC *pp* collider (CERN, 2007-8)