# Particules Élémentaires, Gravitation et Cosmologie
# Année 2010-'11

# Théorie des cordes: quelques applications

# Cours VI: 25 février 2011

## Black holes and fundamental strings

# Cold vs hot black holes

Last week we have seen how, for special kinds of black holes, string theory is able to give a microscopic explanation of their entropy.

Those BHs had the property of being extremal (and therefore had vanishing temperature).

Also, they were constructed as supersymmetry-preserving field configurations. This allowed to extend a small-string-coupling calculation to the strong coupling regime where black holes supposedly do exist.

Today we shall see what we can say about the more realistic (but also technically harder) case of non-extremal (and thus finite-temperature) BHs.

# Density of free string states

Density of physical (DDF) states at tree level (i.e. in the limit of vanishing string coupling):

$$d(N) = N^{-p} e^{2\pi\sqrt{\frac{N(D-2)}{6}}} = M^{-2p} e^{2\pi\sqrt{\frac{\alpha'(D-2)}{6}}M}$$

Neglecting numerical factors this gives, at large M,

$$S_{st} \sim \frac{\alpha' M}{l_s} = \frac{M}{M_s} \;;\; l_s = \sqrt{2\alpha'\hbar} \;;\; M_s = \sqrt{\frac{\hbar}{2\alpha'}}$$

A nice physical interpretation of $S_{st}$: the number of "string bits" contained in the total length of the string, $L = \alpha' M$.

# Relating string and GR parameters

Compare the tree-level effective action of string theory and the Einstein Hilbert action of GR:

$$\Gamma_{st} = -l_s^{-(D-2)} \int d^D x \sqrt{-G} e^{-2\Phi} \left[ R(G) + \ldots \right]$$

$$\Gamma_{EH} = -l_D^{-(D-2)} \int d^D x \sqrt{-g} \left[ R(g) + \ldots \right]$$

for a constant $\Phi$ we find $l_D^{D-2} = e^{2\Phi} l_s^{D-2} = g_s^2 l_s^{D-2}$

In QST Planck's length (G) emerges as an effective scale (coupling) determined by $l_s$ and $g_s$ (Cf. $G_F$ vs $M_W$ in the SM).

We will assume $g_s^2 < (\ll) 1$ throughout.

# Comparing free string and BH entropies

The Bekenstein-Hawking formula for BH entropy can be easily extended to arbitrary D ( $l_D^{D-2} = G_D \hbar$)

$$S_{BH} = \frac{\mathcal{A}}{4 l_D^{D-2}} \; ; \quad \mathcal{A} \sim R_S^{D-2} \sim (G_D M)^{\frac{D-2}{D-3}} \Rightarrow S_{BH} = \frac{M R_S}{\hbar}$$

which can be compared with:

$$S_{st} \sim \frac{\alpha' M}{l_s} = \frac{M}{M_s} \; ; \; l_s = \sqrt{2\alpha' \hbar} \; ; \; M_s = \sqrt{\frac{\hbar}{2\alpha'}}$$

The two entropies look very different... but in this case we cannot trust the zero-coupling result of string theory!

# The correspondence curve

$S_{BH}$ grows faster with M than $S_{st}$ but the latter starts higher at small M. Hence, the two entropies must meet at some value of M. Indeed:

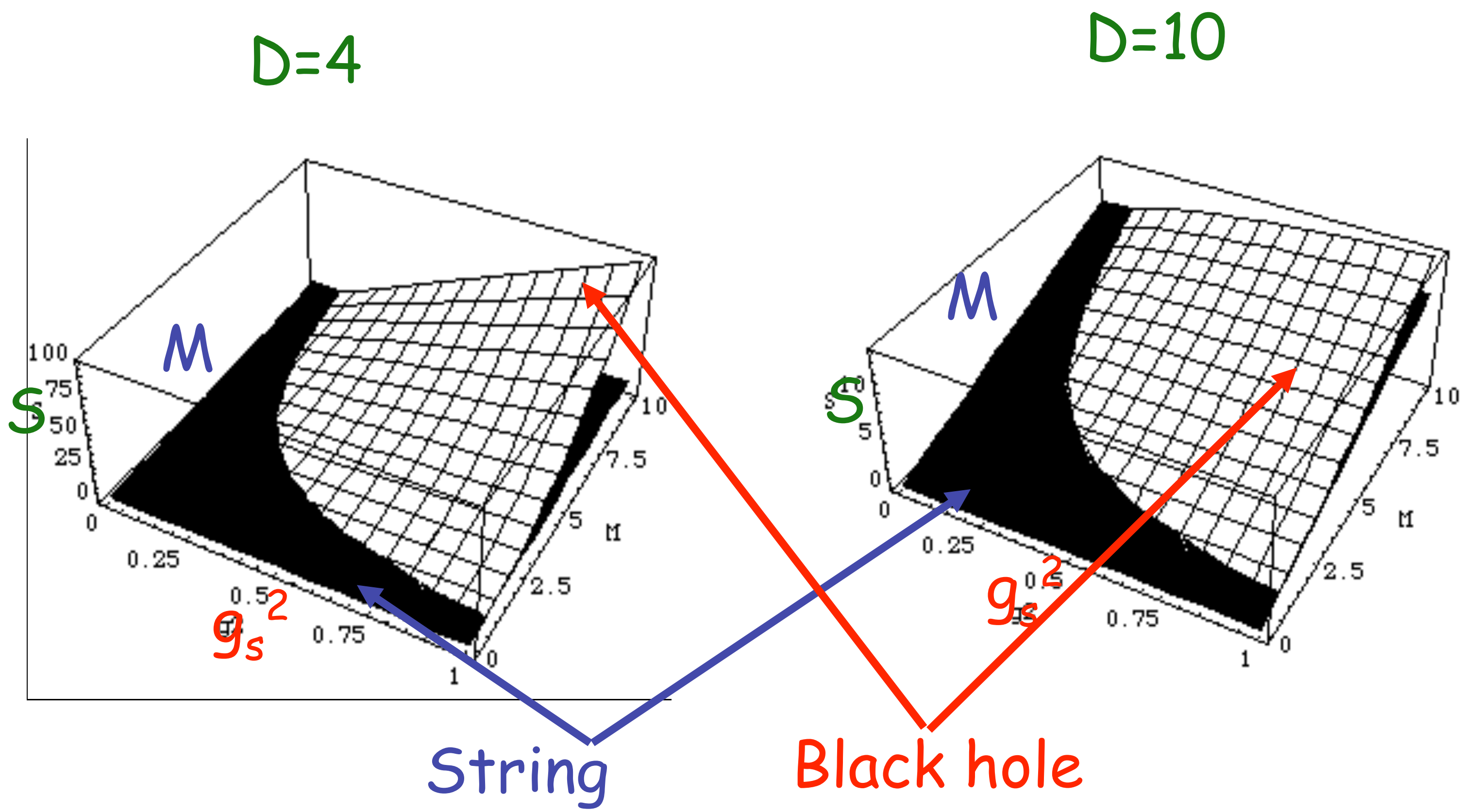
$$\frac{S_{BH}}{S_{st}} = \frac{MR/\hbar}{M/M_s} = \frac{M_s R}{\hbar} = R/l_s$$

$S_{BH}$ wins over $S_{st}$ for $R > l_s$, the opposite is true for $R < l_s$. They coincide at $R = l_s$ and take the common value:

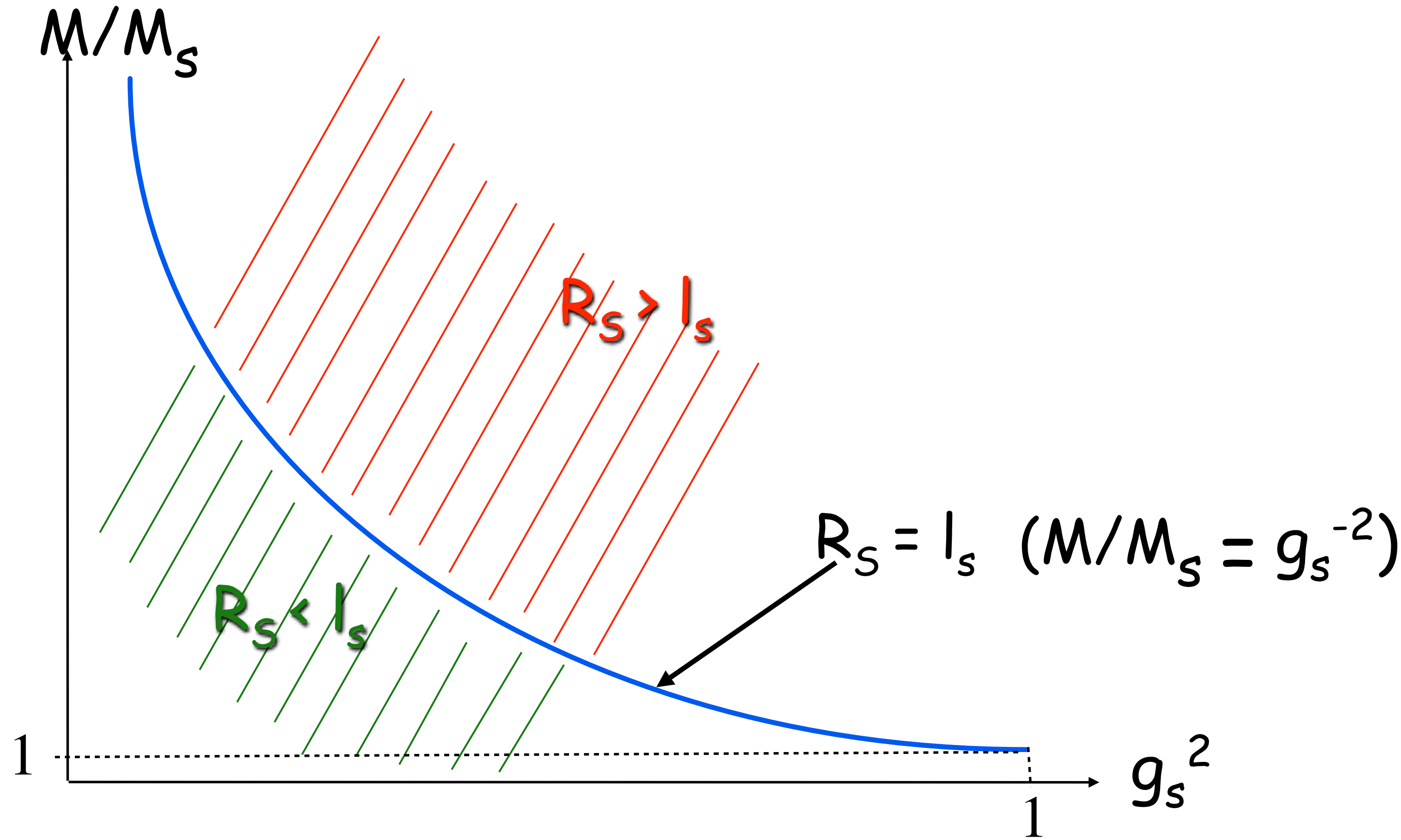
$$S_{BH} = S_{st} = \frac{l_s^{D-2}}{l_D^{D-2}} = g_s^{-2} \gg 1 \Rightarrow M = M_* \equiv g_s^{-2} M_s$$

If we regards both M and $g_s$ as variables we see that $S_{BH} = S_{st}$ defines a hyperbola in the $(g_s, M)$ plane. We shall call it the correspondence curve.

# Comparing entropies in D=4, 10



# The correspondence curve



# Below the correspondence curve

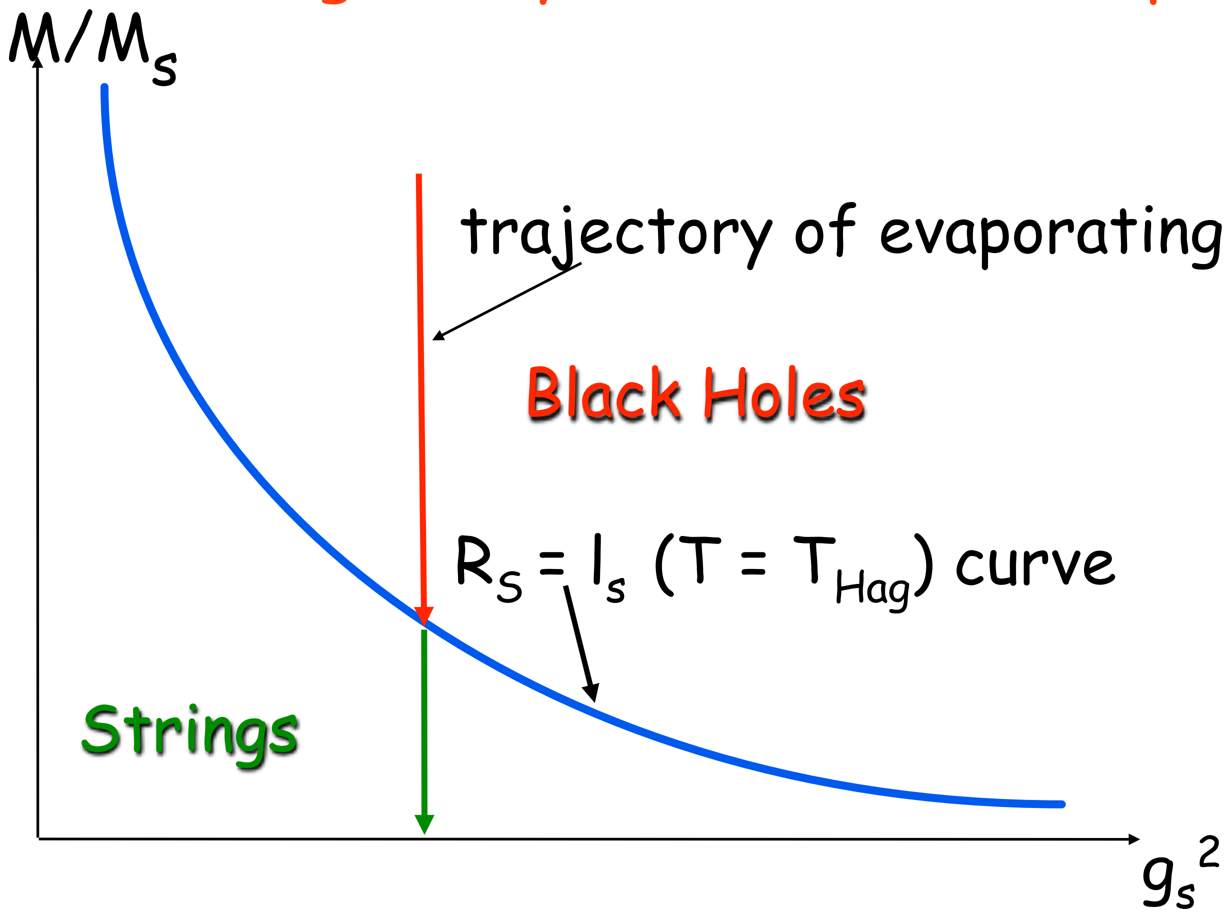
Below the correspondence curve (CC) the Schwarzschild radius of the string is smaller than the string length scale. However, the latter is believed to be the minimal size of any string (a consequence of QM!).

An object whose physical size is larger than its Schwarzschild radius (like the earth, the sun and most objects in the sky) is simply NOT a BH.

Interpretation: in QST there are no BHs whose SR is smaller than $l_s$, i.e. whose Hawking temperature is higher than $M_s$. Actually, $T = M_s$ is nothing but the so-called Hagedorn temperature of QST, believed to be a maximal temperature. Thus, so far, everything looks consistent!

# Evaporation of a BH at fixed $g_s$ (Bowick et al. 1987)

Singularity at the end of evaporation avoided?



# Approaching the correspondence curve: the random-walk puzzle

Below the CC strings are not BHs. And above?

If we want to identify BH with FS above the CC, their properties should match as we approach the curve.

By definition the two entropies match (up to O(1) factors) but there is still a "random-walk puzzle".

$S_{st}$ can be understood in terms of a "random walk" but then a string on the CC being much longer (heavier) than $l_s$ ($M_s$), will have a typical size much bigger than its Schwarzschild radius $l_s$.

But then it has nothing to do with a BH!

# Size distribution of free strings

The resolution of the RW puzzle is actually quite simple. One has to compute the distribution of the string size for a given M (NB: M fixes length not size!).

This was done in a paper by T. Damour & GV (2000). The log of the number of strings of given M and size R is given by ($c_1$, $c_2$ are pure numbers):

$$S(M,R) \equiv \log d(M,R) = a_0 \frac{M}{M_s} f\left(\frac{R}{l_s}, \frac{\alpha' M}{l_s}\right);$$

$$a_0 = 2\pi\sqrt{\frac{D-2}{6}}\ ;\ f\left(\frac{R}{l_s}, \frac{\alpha' M}{l_s}\right) = \left(1 - \frac{c_1 l_s^2}{R^2}\right)\left(1 - \frac{c_2 R^2}{(\alpha' M)^2}\right)$$

Entropy is maximized for: $\frac{R}{l_s} \sim \sqrt{\frac{M}{M_s}} = \text{random walk value}$

However there are still many strings of size $O(l_s)$!

# Solution of RW puzzle ?

The entropy in (free) strings whose size is of order $l_s$ is of order $M/M_s$ and this ratio approaches the BH entropy $g_s^{-2}$ as one approaches the correspondence line.

One finds that other properties, like the decay rate or the radiated power, agree as well:

$$\Gamma \sim g_s^2 M \rightarrow M_s = l_s^{-1} \; ; \; P \sim g_s^2 M M_s \rightarrow M_s^2 = l_s^{-2}$$

the expected values for a BH of radius $l_s$. Thus things seem to be working semi-quantitatively at least up to the CC. But did we have any reason to trust the calculation?

# Effects due to interactions

We have neglected corrections of order $g_s^2$: these were absent in the SUSY case but are present in ours.
We took $g_s \ll 1$, but can we really neglect interactions?
The answer is that we can... except if the strings are very heavy (or very energetic, see next lecture).

For a heavy string the most important effect is self-gravity. Bits of string interact with each other gravitationally. This "binding energy" will shift the masses of our free strings by an amount we can easily estimate (and even compute):

$$\delta M \sim -G_D \frac{M^2}{R^{D-3}} \Rightarrow \frac{\delta M}{M} = -\left(\frac{R_s}{R}\right)^{D-3}$$

Negligible below the CC and of O(1) just on the CC!

# Above the correspondence curve

It is reassuring that the string-coupling corrections become of O(1) just when we can reproduce BH properties up to factors O(1).

As we go farther and farther above the CC the discrepancy between free-string and BH entropy becomes larger and larger.

In order to see whether we can have agreement there we would have to compute the effect of interactions when they become very large.

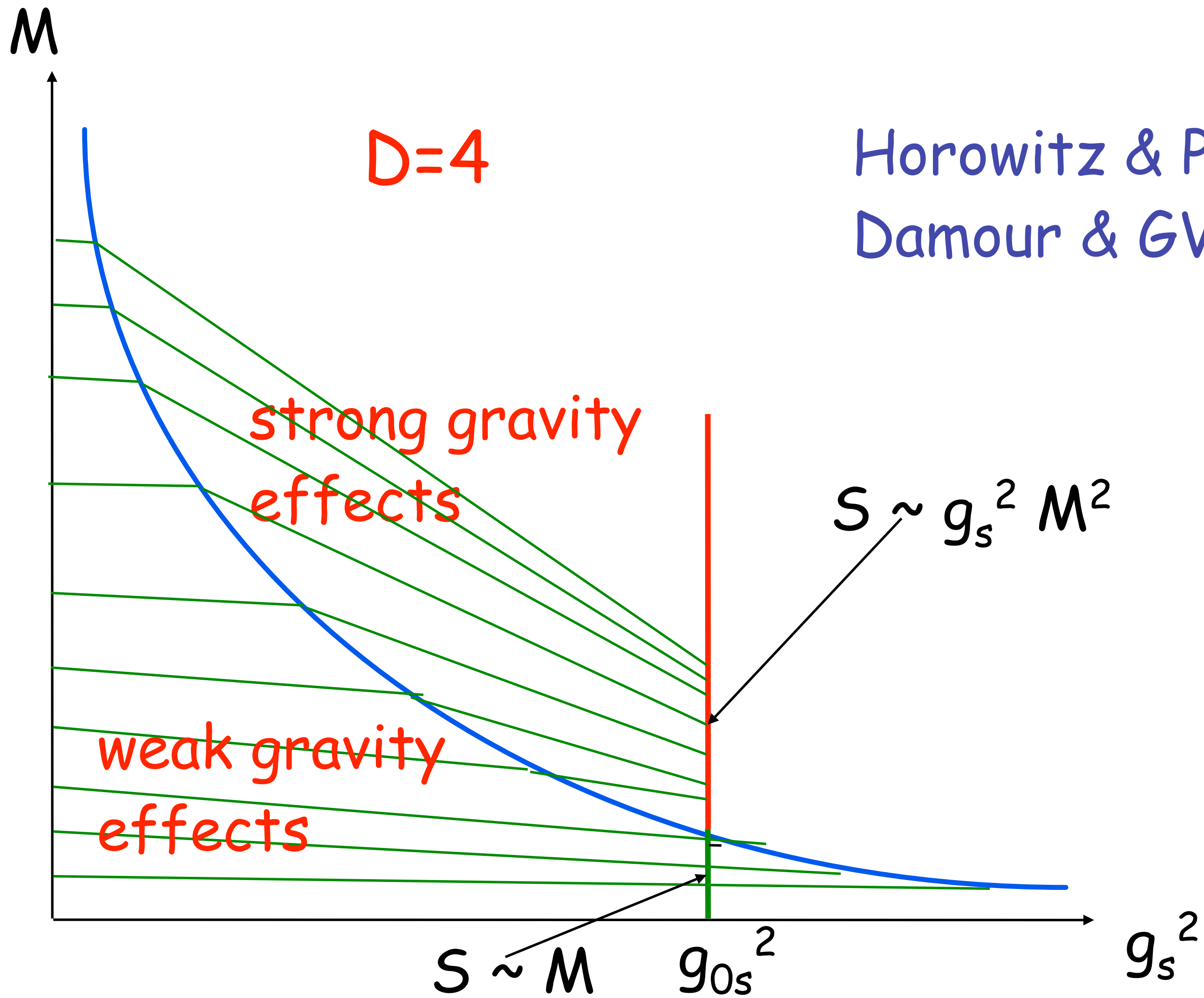
This is a hard & unsolved problem. Yet we have a qualitative understanding of what could possibly do the job.

The basic idea is that, as we increase the coupling, the mass of our heavy strings will generically decrease as a result of gravitational binding energy.

For a given bare ($g_s$ = 0) mass, $M_0$, the more compact strings (the BH candidates) will be more affected than the bigger ones. Degeneracy will be broken and states will start to "migrate" towards the low-mass region.

Above the CC free-string entropy is smaller than $S_{BH}$. This effect goes in the right direction but, in order to see whether it can restore full agreement, we would have to compute it when it becomes very large.

# Gravity-induced increase in density of states



# A toy model

Let's assume that the effects of gravity can be resummed by expressing the mass $M_0$ (at $g_s = 0$) in terms of the mass $M_g$ at a generic finite $g_s$ as:

$$M_0 = M_g + \frac{G_D M_g^2}{R^{D-3}} = M_g \left[ 1 + \left( \frac{R_S}{R} \right)^{D-3} \right]$$

The + sign corresponds to attraction. On the CC we recover the claim that $M_0$ and $M_g$ are of the same order. Assuming that the states at $g_s = 0$ simply "migrate" but their number is conserved, the number of states of a given $M_g$ will be obtained simply by replacing M in our previous formulae by $M_0(M_g)$.

Recalling the previous formulae:

$$S(M,R) \equiv \log d(M,R) = a_0 \frac{M}{M_s} f\left(\frac{R}{l_s}, \frac{\alpha' M}{l_s}\right);$$

$$a_0 = 2\pi\sqrt{\frac{D-2}{6}}\ ;\ f\left(\frac{R}{l_s}, \frac{\alpha' M}{l_s}\right) = \left(1 - \frac{c_1 l_s^2}{R^2}\right)\left(1 - \frac{c_2 R^2}{(\alpha' M)^2}\right)$$

$$M_0 = M_g + \frac{G_D M_g^2}{R^{D-3}} = M_g \left[1 + \left(\frac{R_S}{R}\right)^{D-3}\right]$$

we obtain (up to a possible renormalization of R):

$$S(M,R) = a_0 \frac{M}{M_s}\left[1 + \left(\frac{R_S}{R}\right)^{D-3}\right]\left(1 - \frac{c_1 l_s^2}{R^2}\right)\left(1 - \frac{c_2 R^2}{(\alpha' M)^2}\right)$$

We can look now at the most favoured R for different M, $g_s$ and D (see DV, 2000).

# It works in D=4!

$$S(M,R) = a_0 \frac{M}{M_s}\left[1 + \left(\frac{R_S}{R}\right)^{D-3}\right]\left(1 - \frac{c_1 l_s^2}{R^2}\right)\left(1 - \frac{c_2 R^2}{(\alpha' M)^2}\right)$$

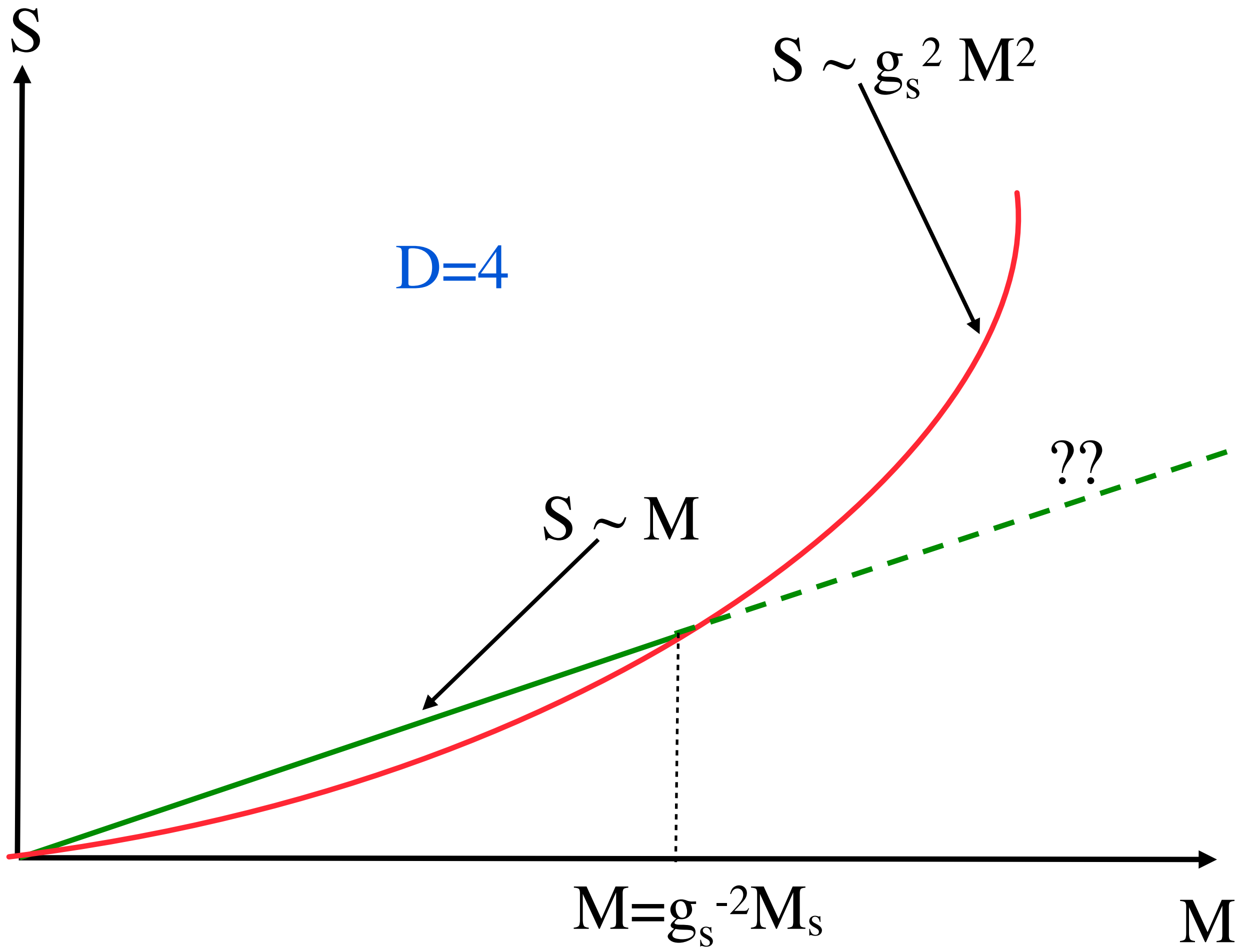
This formula favours strings that are as small as possible.

For D=4 let us take R = $l_s$. We get:

$$S(M,R) \sim \frac{M}{M_s}\left(1 + \frac{R_S}{l_s}\right) = \frac{M}{M_s} + \frac{GM^2}{\hbar} \sim S_{st} + S_{BH}$$

a formula that nicely interpolates between string entropy below the CC and BH entropy above it!

# Smooth transition from $S_{st}$ to $S_{BH}$



$$S(M,R) = a_0 \frac{M}{M_s}\left[1+\left(\frac{R_S}{R}\right)^{D-3}\right]\left(1-\frac{c_1 l_s^2}{R^2}\right)\left(1-\frac{c_2 R^2}{(\alpha' M)^2}\right)$$

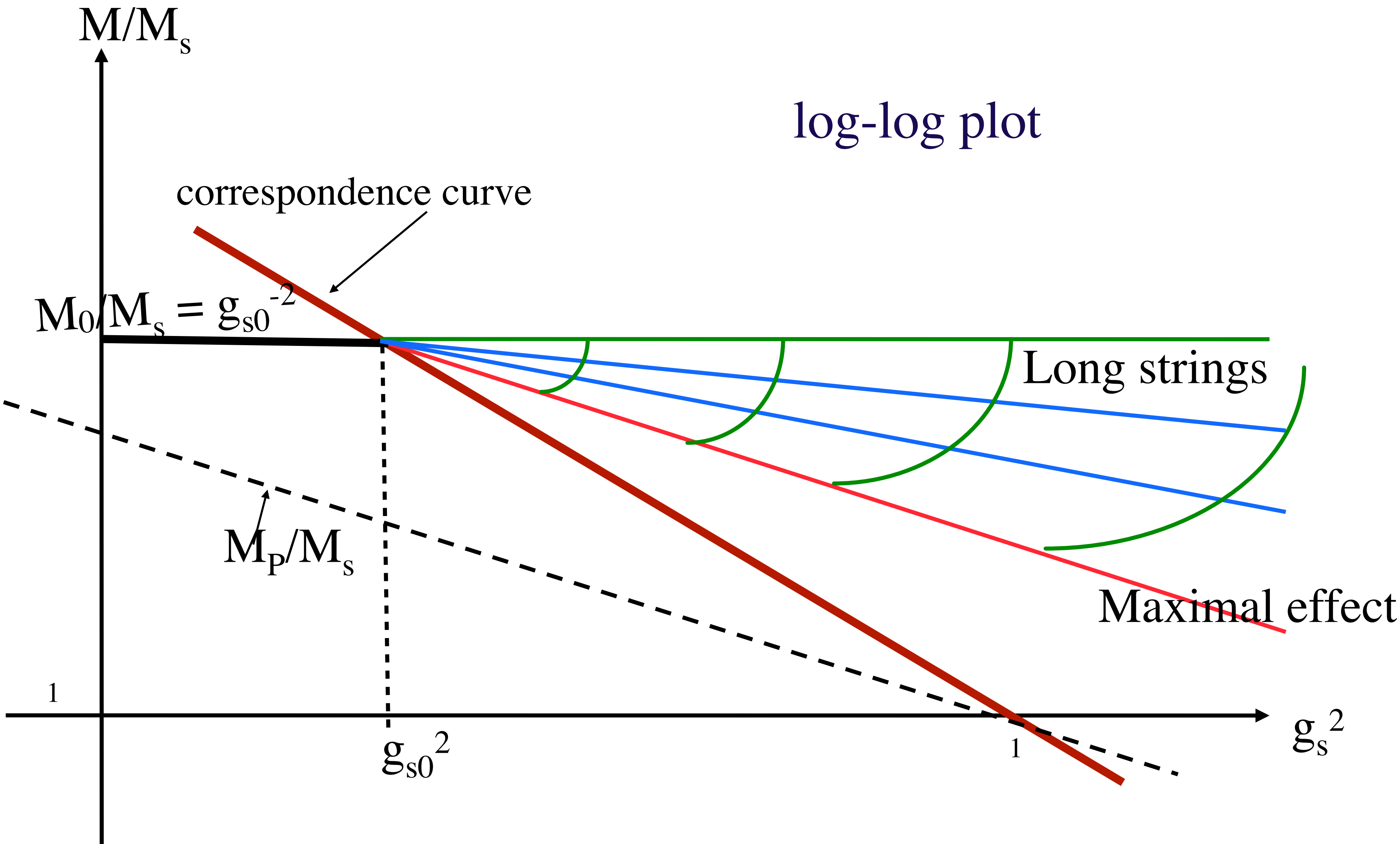
However, if we take R = $l_s$ at D > 4, we find a result that overshoots: string entropy exceeds BH entropy!

$$S(M,R) \sim= \frac{M}{M_s} + \frac{R_S M}{\hbar}\ \frac{R_S^{D-4} l_s}{R^{D-3}} = S_{st} + S_{BH} \frac{R_S^{D-4} l_s}{R^{D-3}}$$

In order not to exceed BH entropy, the ratio $M/M_0$ cannot decrease faster than $M_P/M_s$ above the CC.

$$\frac{M_P}{M_s} \sim g_s^{-\frac{2}{D-2}}\ ;\ \frac{M}{M_0} \geq \left(\frac{g_{s0}}{g_s}\right)^{\frac{2}{D-2}}\ ;\ \frac{M}{M_s} \geq g_{s0}^{-2}\left(\frac{g_{s0}}{g_s}\right)^{\frac{2}{D-2}}\ ;\ (g_s > g_{s0})$$

# Bound on self-gravity effects (D > 4)



# To summarize

1. Black hole properties appear to agree with those of (a sufficient number of) fundamental strings on a "Correspondence Curve" defined by $M = g_s^{-2} M_s$.

2. Below this CC we can trust perturbative calculations and string entropy exceeds BH entropy. Actually, we expect no BH to exist in this mass region (neatly solving the problem of the end-point of BH evaporation?).

3. Near the CC perturbative calculations should be reliable modulo O(1) correction factors.

4. It is still a challenge to show that (some) fundamental strings match BHs above the CC, but one can identify non-perturbative effects that can possibly make this happen.