Particules Élémentaires, Gravitation et Cosmologie
Année 2009-'10

Théorie des cordes: une introduction
Cours I&II: 29 janvier 2010

Les interactions fortes dans les années '60

- Introduction et programme du cours '09-'10
- Interactions fortes dans les années '60

Introduction

During 4 academic years ('04-'05, '05-'06, '07-'08, '08-'09) we have discussed our present theory of the four fundamental interactions (electromagnetic, weak, strong and gravitational).

That theory consists of two separate pillars:

1. The Standard Model of elementary particles (SMEP);
2. The Standard Model of gravity (SMG), General Relativity.

The SMEP belongs to a class of theories known as QFTs, a framework combining the principles of Quantum Mechanics with those of Special Relativity. Actually, it belongs to a special class of QFTs, called gauge theories: these appear naturally as the way to describe, in a Lorentz-invariant way, spin-1 massless particles (with 2 physical polarizations).

The SMEP thus contains 8+3+1 = 12 such "gauge bosons" one for each generator of the SM group:
G = SU(3)xSU(2)xU(1).

It also contains 3 gauge couplings, one for each "factor" in G (traded for α, $\sin\theta_W$, Λ_{QCD}).

The other "actors" in the SMEP are the matter fields. With the exception of the yet-to-be-discovered Higgs boson, a spin zero particle, the matter fields are spin-1/2 Weyl fermions, the smallest non-trivial reps. of the Lorentz group: (1/2,0) left-handed & (0,1/2) right-handed fermions.

These fermions belong to a highly reducible and somewhat baroque representation of G.

Furthermore, this rep. is repeated 3 times, giving the well-know 3 families of quarks and leptons.

Group-theory assignments for one family of quarks and leptons (from 2007-'08 course)

l.h. ferm's	SU(3)	SU(2)	$U(1)_Y$
(u, d)	3	2	1/6
(ν, e)	1	2	-1/2
u^c	3*	1	-2/3
d^c	3*	1	+1/3
e^c	1	1	+1

The representation to which the l.h. fermions belong is "chiral" meaning, in physical terms, that we cannot write down gauge-invariant mass terms (which are bilinear in l.h. or r.h. fermions). Fermion (quark and lepton) masses can only appear as a consequence of the spontaneous breaking of the gauge symmetry à la Brout-Englert-Higgs. The same mechanism gives mass to the gauge bosons of SU(2)xU(1) leaving just a massless photon (and 3 massive "intermediate bosons" the $W^{\pm}$ and the Z^0).

For the SU(3) part a different (non-perturbative) mechanism (confinement) prevents the existence of free quarks and gluons which, instead, bind into SU(3)-singlets, the hadrons (mesons and baryons) whose mass-scale is controlled by Λ_{QCD} (see '04-'05 and '05-'06 courses)

The SMG, General Relativity, is also based on a local symmetry, general covariance, which implements the equivalence principle and is naturally associated with a tensor field, the gravitational field. Semiclassically, such a field describes a massless particle of spin 2, the graviton, the analogue of the photon for the electromagnetic field.

General covariance, like gauge invariance, removes the unphysical degrees of freedom, in this case of a massless spin-2 particle.

The analogy unfortunately stops here.

So far, theorists have been unable to extend to gravity the fully quantum framework that led them to the SMEP: for quantum gravity the UV divergences are too strong!

There are strong indications that, in order to arrive at a fully consistent quantum theory of gravity, one needs to go beyond the framework of local QFT.

At present, string theory (which, as we shall see, predicts the existence of massless J=1 and J=2 particles) is the most promising avenue we have to combine the principles of QM, Gauge Invariance and General Covariance and to arrive at a fully unified quantum theory of all forces and of all elementary particles (but we are not there yet!).

This, however, was not the way historically string theory came about. It came from an attempt, in the sixties, to describe in an unconventional way the strong interactions. That attempt, as such, failed. However, in the process, a beautifully consistent theoretical framework was constructed which, instead, looked perfectly capable of addressing the deeper question of how to reconcile gravity and Quantum Mechanics.

Most courses in string theory start directly from this end (a top-down approach) arriving at the model that historically led to string theory after many pages of non-trivial calculations.

I thought it would be better, for this audience, to use the opposite, bottom-up approach.

In the first part of the course we will retrace the birth of the so-called Dual Resonance Model (DRM) -and of its interpretation as a string theory- as a candidate theory of strong interactions.

We will then discuss some basic (and apparently unavoidable!) properties of quantum strings and why these properties led to abandoning the original goal when QCD came about. Moreover, QCD even explained, a posteriori, why string theory was invented in an apparently "serendipitous" way and why it even had remarkable success in explaining some strong interaction phenomenology.

The string theory of the sixties & seventies was so predictive to be easily falsifiable! Will it be the same for its new incarnation?

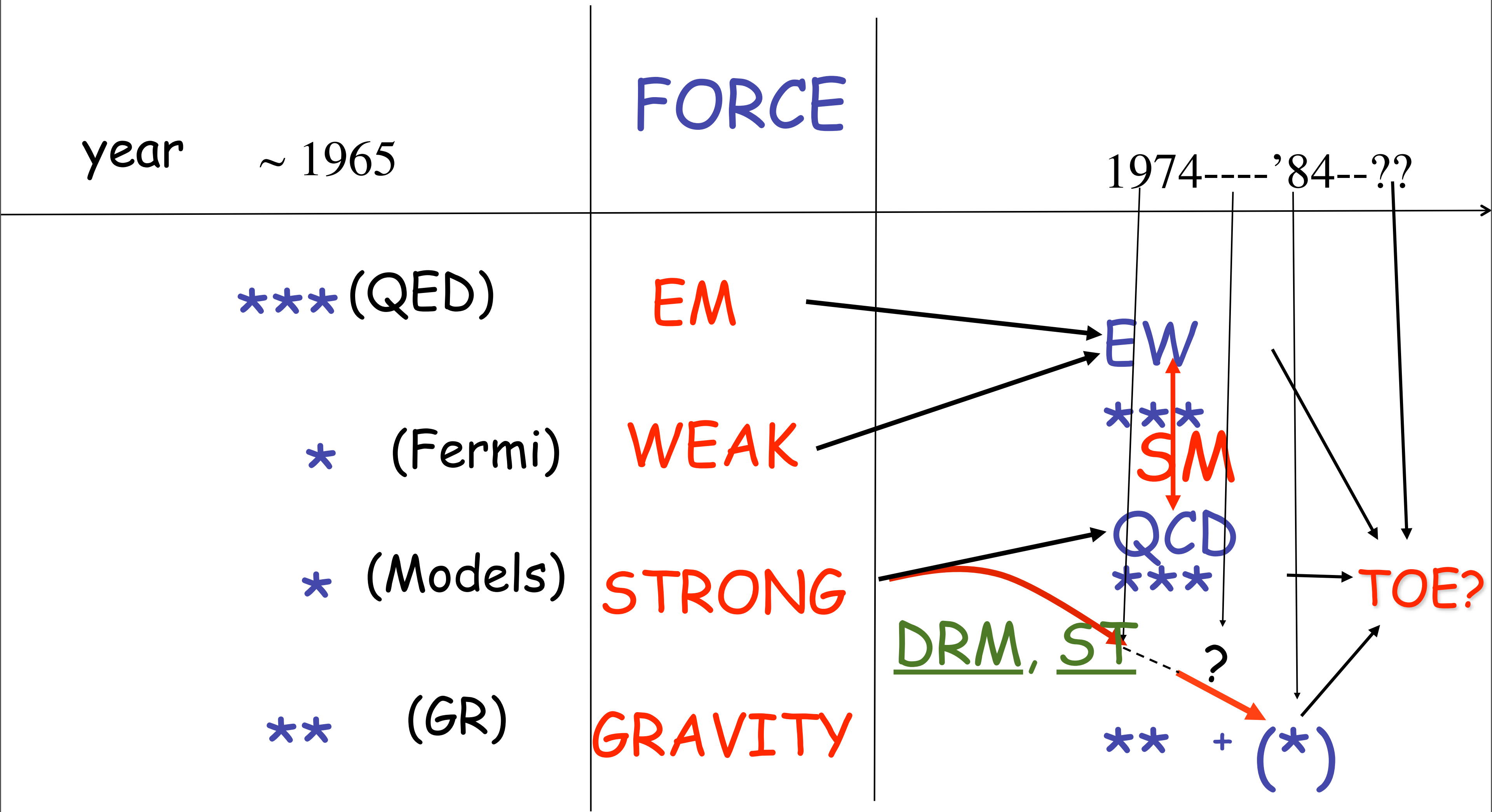
In the second part of the course we will discuss the modern formulation and reinterpretation of string theory as a unified quantum theory of all interactions, including gravity.

For lack of time we shall only discuss general features of string theory leaving most of its applications (to black holes, to cosmology, to strongly coupled gauge theories) to future courses.

Here is an outline of this year's course (in red are the seminars, to be given by Professor P. Di Vecchia from Nordita)

Date	9h45-10h45	11h-12h
29/01	Strong interactions in the 60s	Strong interactions in the 60s
05/02	DHS duality and a bootstrap	A simple, exact solution
12/02	DRM: counting states, ghosts, operators, algebras	The no-ghost theorem, loops, D=26
19/02	Birth of string theory: NG action, LC quantization	Polyakov's CFT approach
26/02	Neveu-Schwarz and Ramond generalizations: WS-SUSY	GSO projection: Target-space SUSY
05/03	Zero-slope limit, QCD: end of a dream, the SS proposal	The GS breakthrough: a theory of everything?
12/03	Strings in non-trivial backgrounds, effective action	Field and String-theoretic symmetries: N&D-strings
19/03	D-branes & SUGRA solutions	D-branes & gauge theories
26/03	The AdS/CFT correspondence	Unification of string theories

An evolving "Michelin guide"



STRONG INTERACTIONS in the 60s

No Theory, rather:

A handful of models capturing one or another aspect of hadronic physics e.g.

- Short range i.e. no massless particles
- Symmetries, conservation laws (P, C, T, I, SU(3),…)
- Many metastable states (resonances) extending to large J: an ever increasing zoo?

Why did we take the (a posteriori) wrong way?

A QFT approach looked hopeless:

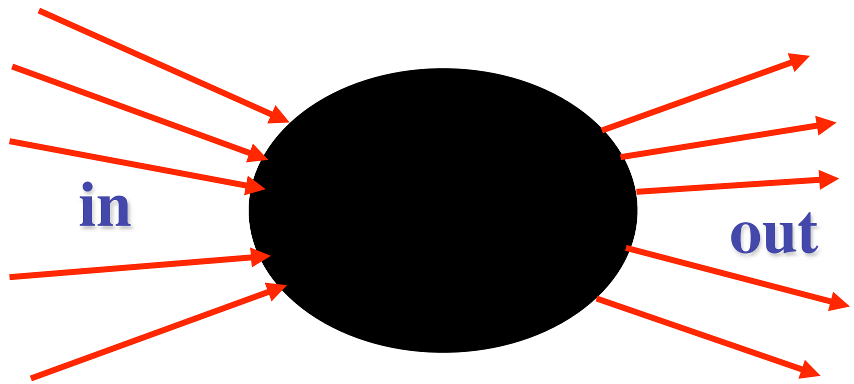
1. Too many d.o.f. => too many fields
2. High-J QFT's are pathological (J=2 is already bad-enough!)

An S-matrix approach looked more promising:

The S-Matrix (Heisenberg 1943)

$$\langle out|S|in\rangle = S(in \to out) = complex\ number$$

$$|S(in \to out)|^2 = Prob.\ for : in \to out$$



- Symmetries: easy to implement on S
- Causality => analyticity, dispersion relations
- Conservation of Prob => Unitarity constraint: $SS^\dagger = 1$

Organizing the hadronic zoo

A) Group theory:

- $SU(2)_I$, $SU(3)_{F,}$ same-J particles
- SU(4), SU(6) ... combining $\Delta J \leq 1$ particles (not rigorous)

B) Regge theory of complex J

- For combining different-J particles (Regge)
- For describing high-energy scattering (Chew-Mandelstam)

Sketch of Regge's theory of complex J

- Consider non-relativistic potential scattering. Expand the scattering amplitude (~ the S-matrix) in partial waves:

$$A(E,\theta) = \sum_{J=0}^{\infty} A_J(E) P_J(cos\theta)$$

- In 1959 Tullio Regge had the bold idea of looking at $A_J(E)$ as an analytic function of complex J. He found that, quite generically, there were poles in J at $J = \alpha(E)$:

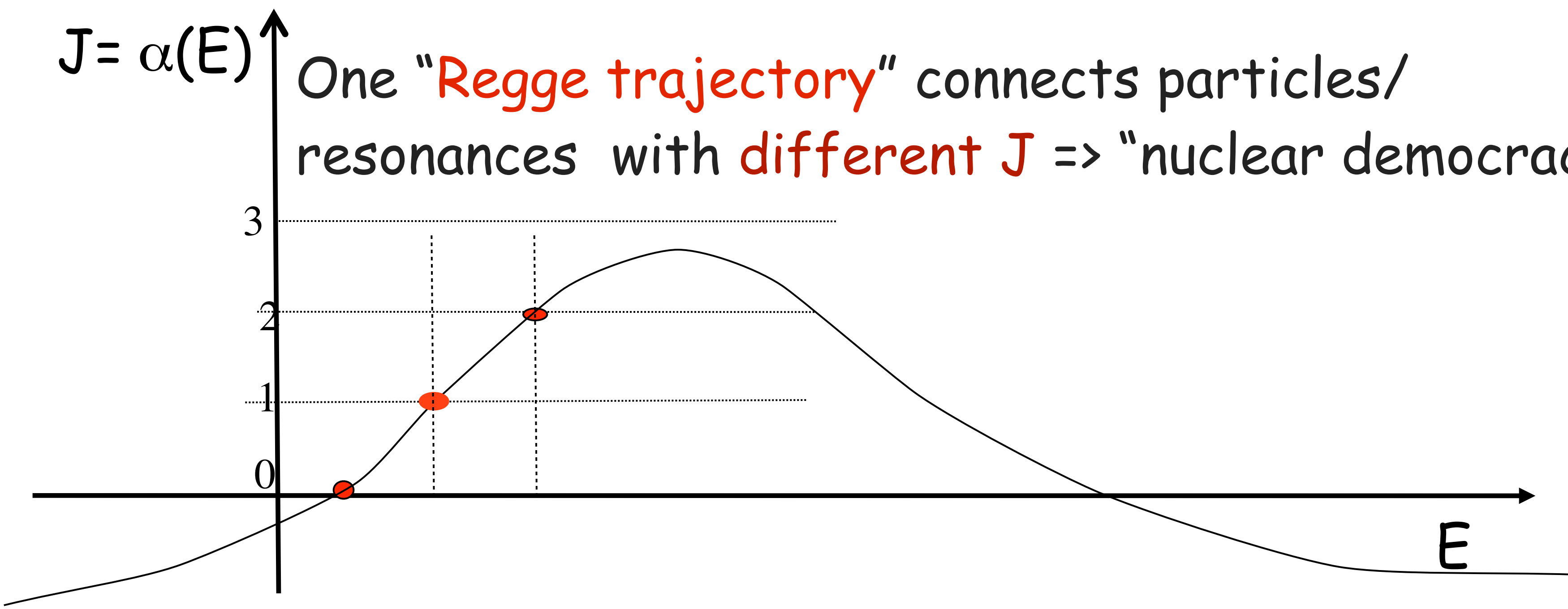
$$A_J(E) \sim \frac{\beta(E)}{J - \alpha(E)}$$

$$A_J(E) \sim \frac{\beta(E)}{J-\alpha(E)}$$

$$\alpha(E_n) = n \Rightarrow A(E,\theta) = \frac{\beta(E_n)}{n-\alpha(E)} P_n(cos\theta) \sim -\frac{\beta(E_n)}{\alpha'(E-E_n)} P_n(cos\theta)$$

This is just the contribution to the scattering amplitude of a single resonance of energy E_n.

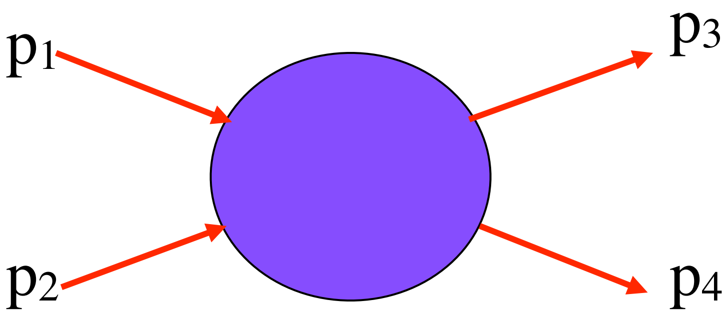
One "Regge trajectory" connects particles/ resonances with different J => "nuclear democracy".



Chew-Mandelstam application of Regge theory in relativistic scattering

Relativistic 2-body scattering amplitude A(s,t) expanded in t-channel partial waves:

$$
\begin{aligned}
s = -(p_1 + p_2)^2 &= -(p_3 + p_4)^2 \\
t = -(p_1 - p_3)^2 &= -(p_2 - p_4)^2 \\
u = -(p_1 - p_4)^2 &= -(p_2 - p_3)^2 \\
s + t + u &= \sum m_i^2
\end{aligned}
$$



s,t, u are the so-called Mandelstam variables

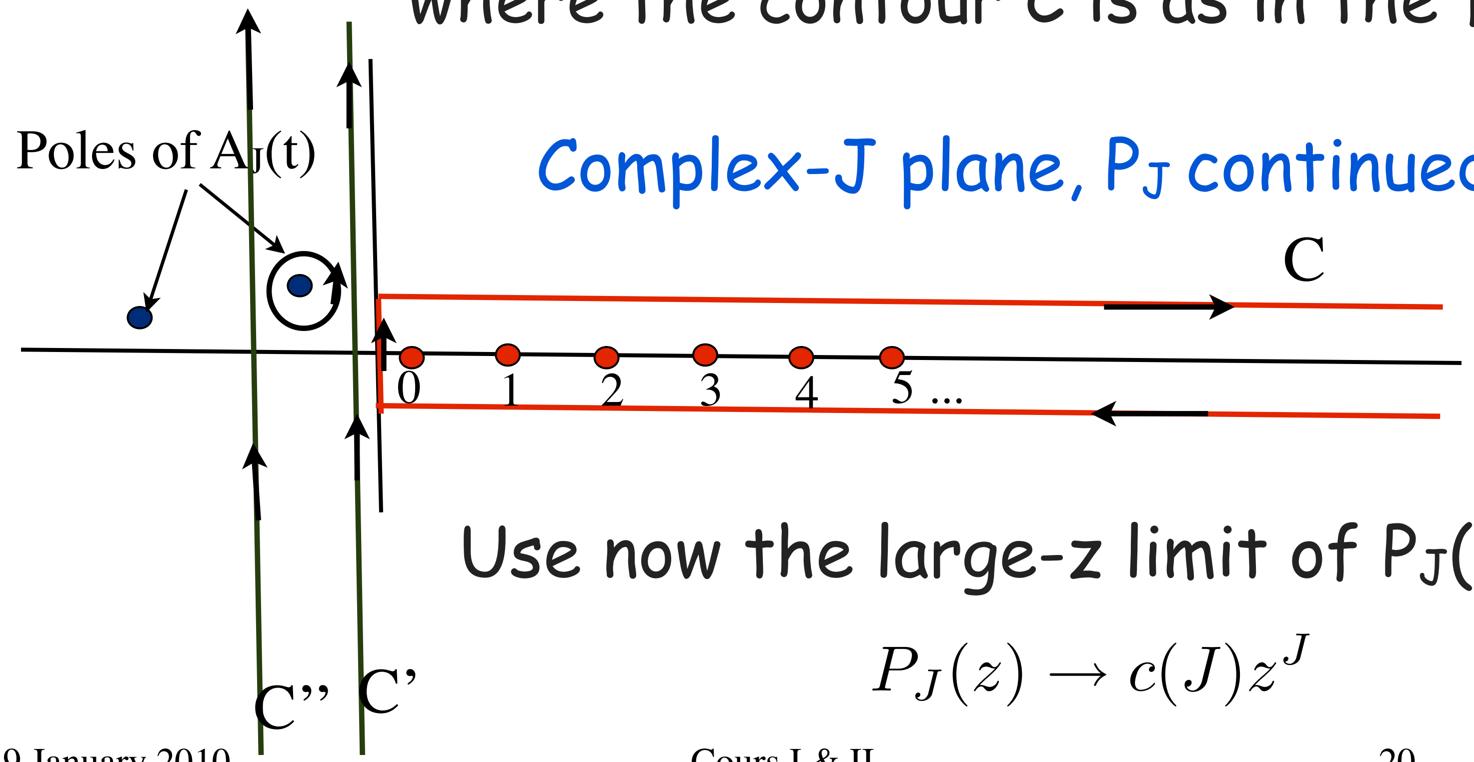
$$A(s,t) = \sum_{J=0}^{\infty} A_J(t) P_J(cos\theta_t) \quad ; \quad cos\theta_t = 1 + 2s/t$$

considered in the "unphysical" region: s large and positive, t < 0 fixed. $cos\theta_s = 1 + 2t/s \rightarrow 1$

The sum diverges but can be analytically continued using a trick due to Froissart & Gribov

$$A(s,t) = \sum_{J=0}^{\infty} A_J(t) P_J(cos\theta_t) = \frac{1}{2i}\int_C dJ \frac{e^{i\pi J}}{sin(\pi J)} A_J(t) P_J(cos\theta_t)$$

where the contour *C* is as in the figure.



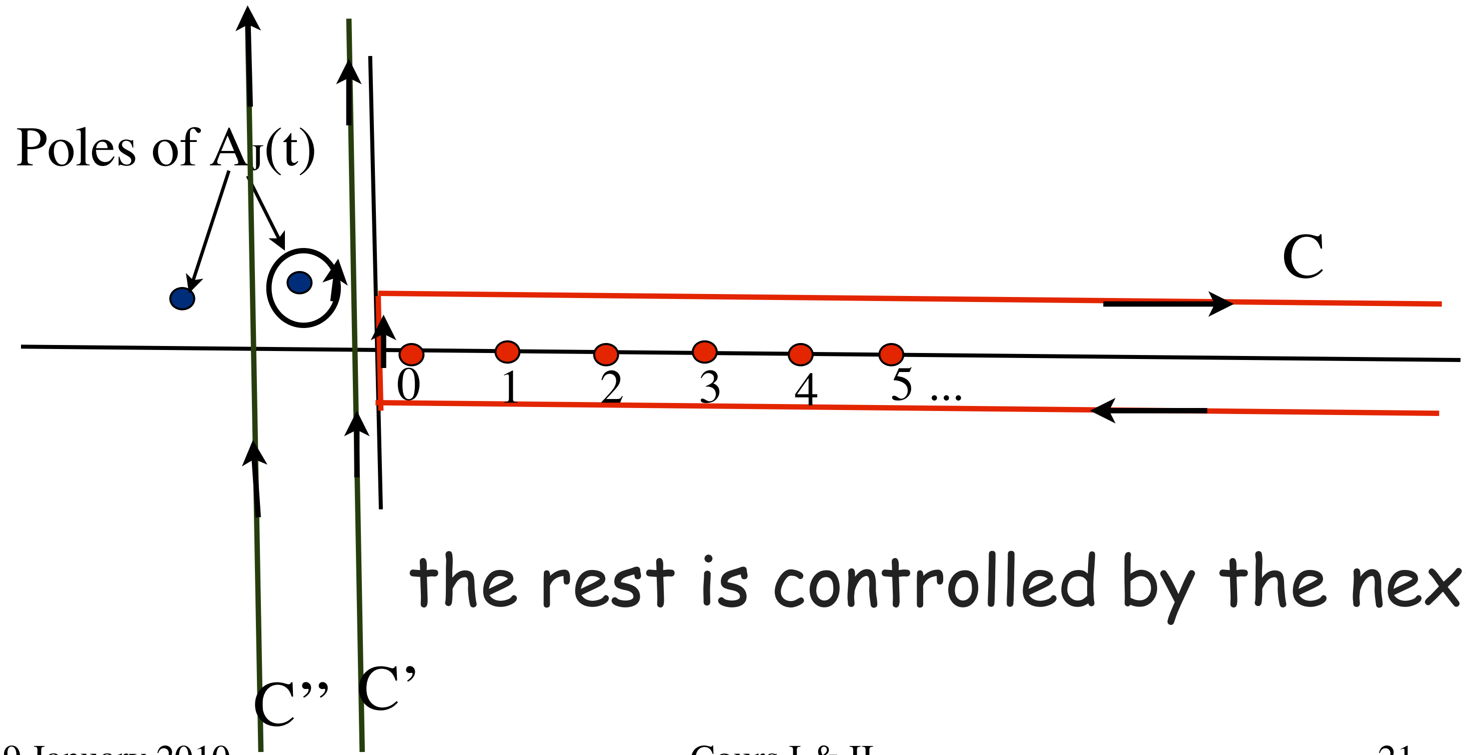
Complex-J plane, P_J continued as well

Use now the large-z limit of $P_J(z)$:

$$P_J(z) \to c(J) z^J$$

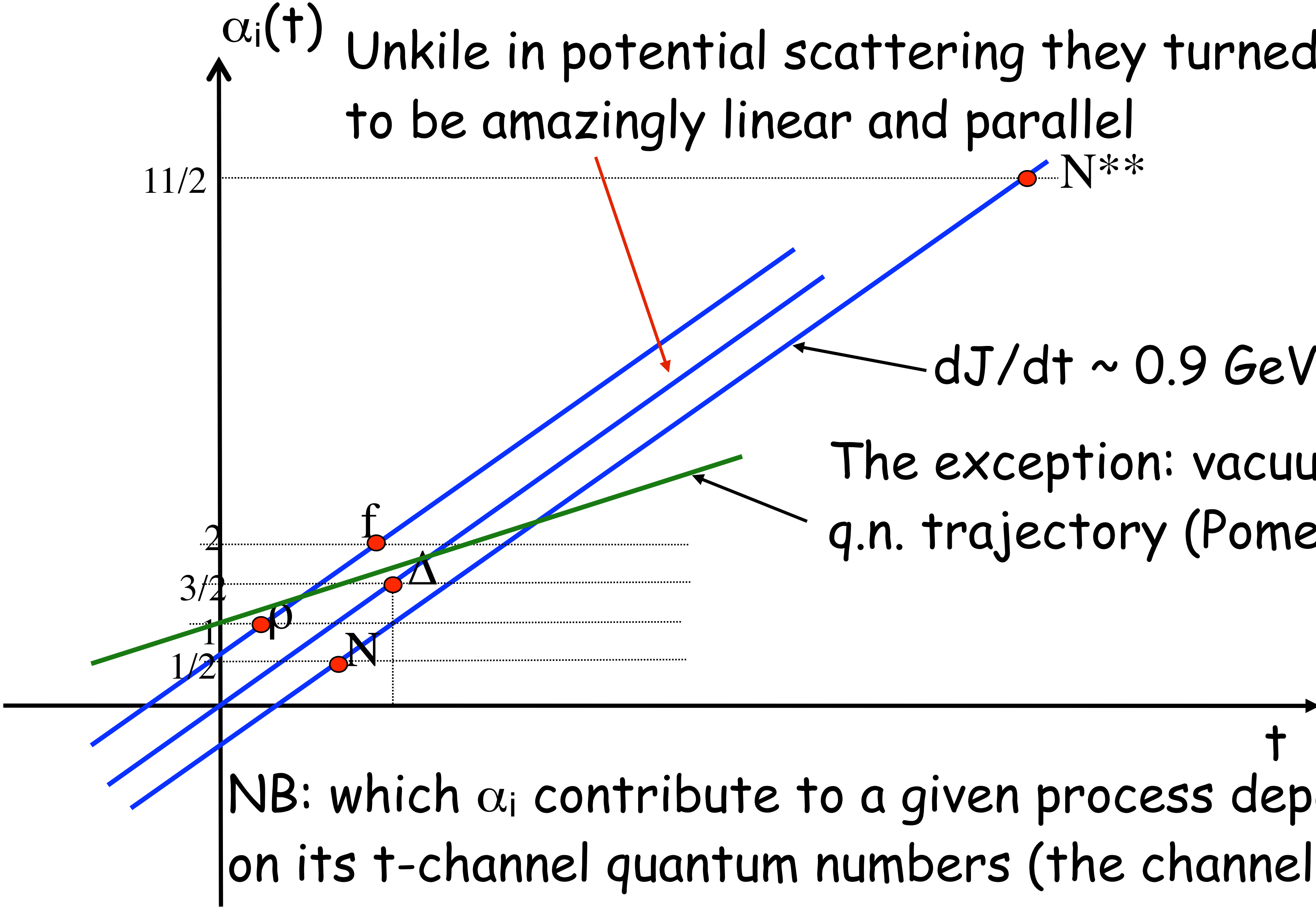
Deforming the contour from C to C' to C'' (which includes the little circle around the rightmost Regge pole) we get, from the latter:

$$A(s,t) \sim \frac{\beta(t)}{sin(\pi\alpha(t))}\left[(-s)^{\alpha(t)} \pm (-u)^{\alpha(t)}\right] \sim \frac{\beta(t)[e^{i\pi\alpha} \pm 1]}{sin(\pi\alpha(t))} s^{\alpha(t)}$$



the rest is controlled by the next pole..

Unkile in potential scattering they turned out to be amazingly linear and parallel



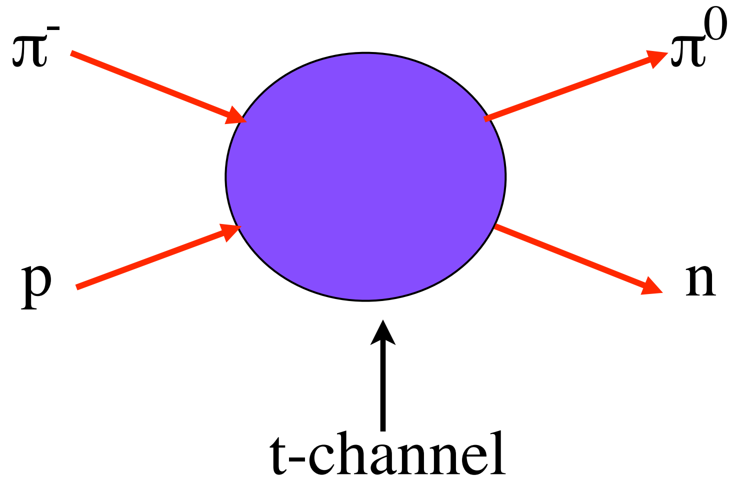
$dJ/dt \sim 0.9\ GeV^{-2}$

The exception: vacuum q.n. trajectory (Pomeron)

NB: which α_i contribute to a given process depends on its t-channel quantum numbers (the channel whose Mandelstam variable is kept fixed)

Examples

1. Pion-nucleon charge exchange



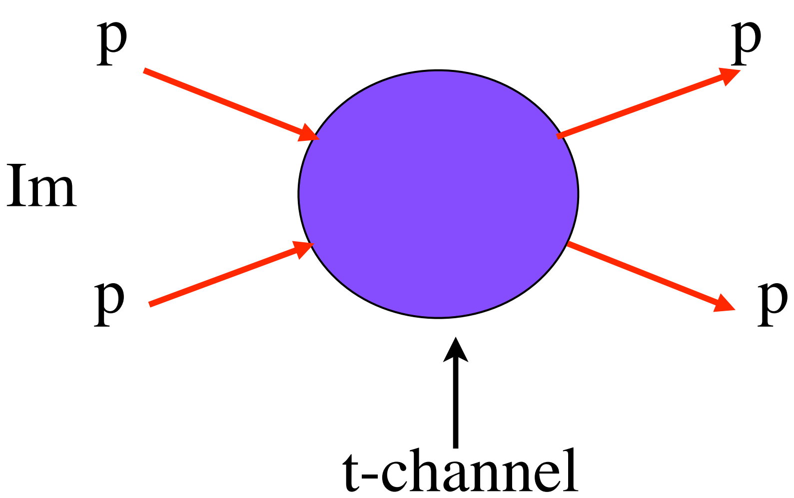
I=1 trajectories of both signatures can contribute

$$A(s,t) \sim \frac{\beta_\rho(t)[e^{i\pi\alpha_\rho} - 1]}{sin(\pi\alpha_\rho(t))} s^{\alpha_\rho(t)} + \frac{\beta_{A2}(t)[e^{i\pi\alpha_{A2}} + 1]}{sin(\pi\alpha_{A2}(t))} s^{\alpha_{A2}(t)}$$

Fitting data gives $\alpha_\rho(0) \sim \alpha_{A2}(0) \sim 0.57$ explaining quite well the scattering data above a few GeV.

Examples

2. Proton-proton total cross section (LHC)



I=0, 1 trajectories of both signatures can contribute

$$\sigma_T = \frac{1}{s} Im A(s,0) \sim \frac{1}{s} Im \frac{\beta_{\mathcal{P}}(0)[e^{i\pi\alpha_{\mathcal{P}}} + 1]}{sin(\pi\alpha_{\mathcal{P}}(0))} s^{\alpha_{\mathcal{P}}(0)} + \dots = \beta_{\mathcal{P}(0)} s^{\alpha_{\mathcal{P}}(0)-1} + \dots$$

Fitting data gives $\alpha_P(0) \sim 1.07$ violating the Froissart bound ($\log^2 s$): the story must be more complicated!

Chew's "expensive" bootstrap…

Add to the general constraints of symmetry, causality, unitarity that of Nuclear Democracy

"All hadrons lie on Regge trajectories @ $M^2>0$;

All asymptotics fixed by same trajectories @ $M^2<0$"

Will this give a unique S-matrix?

A posteriori we can say that Chew's program was too optimistic. We now believe the answer to the question to be negative.

String theory is a perfect example of Nuclear democracy and satisfies the other constraints as well…but adds to them a crucial new dynamical input: strings!