Particules Élémentaires, Gravitation et Cosmologie

Année 2005-2006

Interactions fortes et chromodynamique quantique

II: aspects non-perturbatifs

Cours VIII: 4 avril 2006

Fermions chiraux et température finie

- Fermion-doubling problem and Wilson fermions
- Ginsparg-Wilson fermions: an exact chiral symmetry on the lattice?
- QCD at finite-temperature: some history
- Deconfinement transition and the quark-gluon plasma

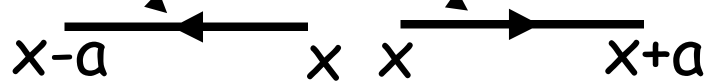
Fermion-doubling problem and Wilson fermions

- We have seen in LG's 1st seminar that the simplest definition of massless fermions on the lattice leads to an unwanted multiplication of the fermionic species (the doubling problem)

$$\frac{1}{2}a^4 \sum_x \bar{\psi}(x)\gamma_\mu(\nabla_\mu + \nabla^*_\mu)\psi(x)$$ where:

$$\bar{\psi}(x)\nabla_\mu\psi(x) = \frac{1}{a}\bar{\psi}(x)[U_\mu(x)\psi(x+a\vec{\mu}) - \psi(x)]$$

$$\bar{\psi}(x)\nabla^*_\mu\psi(x) = -\frac{1}{a}\bar{\psi}(x)[U^\dagger_\mu(x-a\vec{\mu})\psi(x-a\vec{\mu}) - \psi(x)]$$



$$\bar{\psi}(x)\gamma_\mu(\nabla_\mu + \nabla^*_\mu)\psi(x) = \frac{1}{a}\bar{\psi}(x)\gamma_\mu[U_\mu(x)\psi(x+a\vec{\mu}) - U^\dagger_\mu(x-a\vec{\mu})\psi(x-a\vec{\mu})]$$

Taking the free-theory limit, U --> 1

$$\frac{1}{a}\bar{\psi}(x)\gamma_\mu[\psi(x+a\vec{\mu}) - \psi(x-a\vec{\mu})]$$ couples LL & RR

We see that the lagrangian vanishes not only if ψ is constant but also when ψ has periodicity 2a (p = $\pm\,\pi/a$) in any of the 4 directions. This is the origin of the doubling

Wilson's recipe to solve this problem was to introduce an extra term which, unfortunately, breaks the chiral symmetry:

$$\delta L_{Wilson} \sim a\,\bar{\psi}(x)\nabla^*_\mu\nabla_\mu\psi(x)$$ couples LR & RL!

$$\delta L_{Wilson} \sim a\, \bar{\psi}(x)\nabla^*_\mu \nabla_\mu \psi(x)$$

Altough naively this term vanishes for a-->0, UV divergences (~ 1/a) make it contribute to the renormalized quark mass (another chirality-breaking term)

This is not a killer: it simply means that the mass parameter one puts in the Lagrangian is not the physical mass. The latter has to be fixed a posteriori in terms of some calculated observable. At the same time, operators that would not mix because of chiral-symmetry do mix because of the breaking.

LG showed how to deal with this and still keep predictivity. Can one do any better? For many years people answered: No!

Chiral fermions in the continuum

- In the continuum we define L and R-handed fermions through chiral projections ($\gamma_5^2 = 1 \Rightarrow P_\pm^2 = P_\pm$, $P_+ + P_- = 1$)

$$\psi_{R,L} = P_\pm \psi \; ; \; \bar{\psi}_{R,L} = \bar{\psi} P_\mp \; ; \; P_\pm \equiv \frac{1 \pm \gamma_5}{2}$$

Reminder: ψ_L and ψ^*_R are in (1/2,0) rep. of Lorentz (ψ_R and ψ^*_L are in (0,1/2)) . A chirality-preserving operator couples ψ^*_L to ψ_L and/or ψ^*_R to ψ_R while a chirality-violating operator couples ψ^*_L to ψ_R and/or ψ^*_R to ψ_L.
Therefore $\psi^* D \psi$ preserves chirality if $\{D, \gamma_5\}=0$ (this is why $D \sim \gamma_\mu$ and $\gamma_\mu \gamma_5$ are OK, while $D \sim 1$ and γ_5 are not).

Ginsparg-Wilson fermions

- In the continuum we need $\{D, \gamma_5\} = 0$ but this cannot be achieved (NN thrm, see LG). GW fermions (1982) replace that condition by a weaker one:

$$\{D, \gamma_5\} = \frac{a}{\rho} D\, \gamma_5\, D\ ,\ 0 < \rho < 2$$

Taking hereafter (for simplicity) $\rho=1$, the GW relation can be rewritten as

$$\gamma_5\, D = -D\, \hat{\gamma}_5\ ;\ \hat{\gamma}_5 = \gamma_5\, (1 - aD)$$

If we then define:

$$\bar{\psi}_{R,L} = \bar{\psi} P_{\mp}\ ;\ \hat{\psi}_{R,L} = \hat{P}_{\pm} \psi\ ;\ \hat{P}_{\pm} \equiv \frac{1 \pm \hat{\gamma}_5}{2}$$

$$\bar{\psi}_{R,L} = \bar{\psi} P_{\mp} \; ; \; \hat{\psi}_{R,L} = \hat{P}_{\pm} \psi \; ; \; \hat{P}_{\pm} \equiv \frac{1 \pm \hat{\gamma}_5}{2}$$

we find that, since $P_{\pm} \, D \, \hat{P}_{\pm} = 0$

$$\bar{\psi} \, D \, \psi = \bar{\psi}_L \, D \, \hat{\psi}_L + \bar{\psi}_R \, D \, \hat{\psi}_R$$

Therefore such a term is invariant under independent (global) $U(N_f)$ transformations of ψ^*_L and ψ^*_R (and of the corresponding fields with hats). The point/hope is that this symmetry should become the continuum $U(N_f) \times U(N_f)$ symmetry when a-->0

This old GW idea was resurrected by the explicit construction of an operator D satisfying the GW relation (Neuberger 1998). Its existence was first guessed using a higher-dimensional argument (domain walls in 5D), but D can be defined directly at D=4. We will not need its explicit form..

Full fermionic action including q-masses

$$S_F^{GW} = a^4 \sum_x \left(\bar{\psi}_L D \hat{\psi}_L + \bar{\psi}_R D \hat{\psi}_R + \bar{\psi}_L M \hat{\psi}_R + \bar{\psi}_R M^\dagger \hat{\psi}_L \right)$$

This action has the same (classical) symmetry and (explicit) symmetry-breaking as continuum QCD.

Q: Does it also have the same anomalies at the quantum level? In particular, is the $U(1)_A$ subgroup of $U(N_f) \times U(N_f)$ broken explicitly? The answer is yes and the anomaly can be obtained directly from the functional-integral over the fermions à la Fujikawa (see KK seminar1). This has an extra bonus: it provides us with a lattice definition of the topological-charge density Q(x), the quantity appearing in the anomalous WTIs

In principle one can study chiral gauge theories on the lattice!

Lattice definition of the anomaly

Proceeding à la Fujikawa one finds:

$$a^4 Q^{Latt.}(x) = \frac{1}{2} Tr[\gamma_5 D(x,x)]$$

and also the analogue of the index thrm

$$a^4 \sum_x Q^{Latt.}(x) = index(D) \equiv n_- - n_+$$

Recall: the index of the Dirac operator in a gauge field with topological charge ν is ν

At this point all the continuum WTIs can be rewritten essentially unchanged on the lattice. This will be the starting point of LG's use of GW fermions for physical applications

QCD at finite-temperature: some history

- In the early days (beg. sixties) the ever increasing number of strongly-interacting particles led to the so-called Hagedorn model in which the number of distinct hadrons of mass M grew like $\exp(M/T_H)$. Part. fnct. diverges at $T>T_H$

➡ Existence of a limiting temperature, T_H

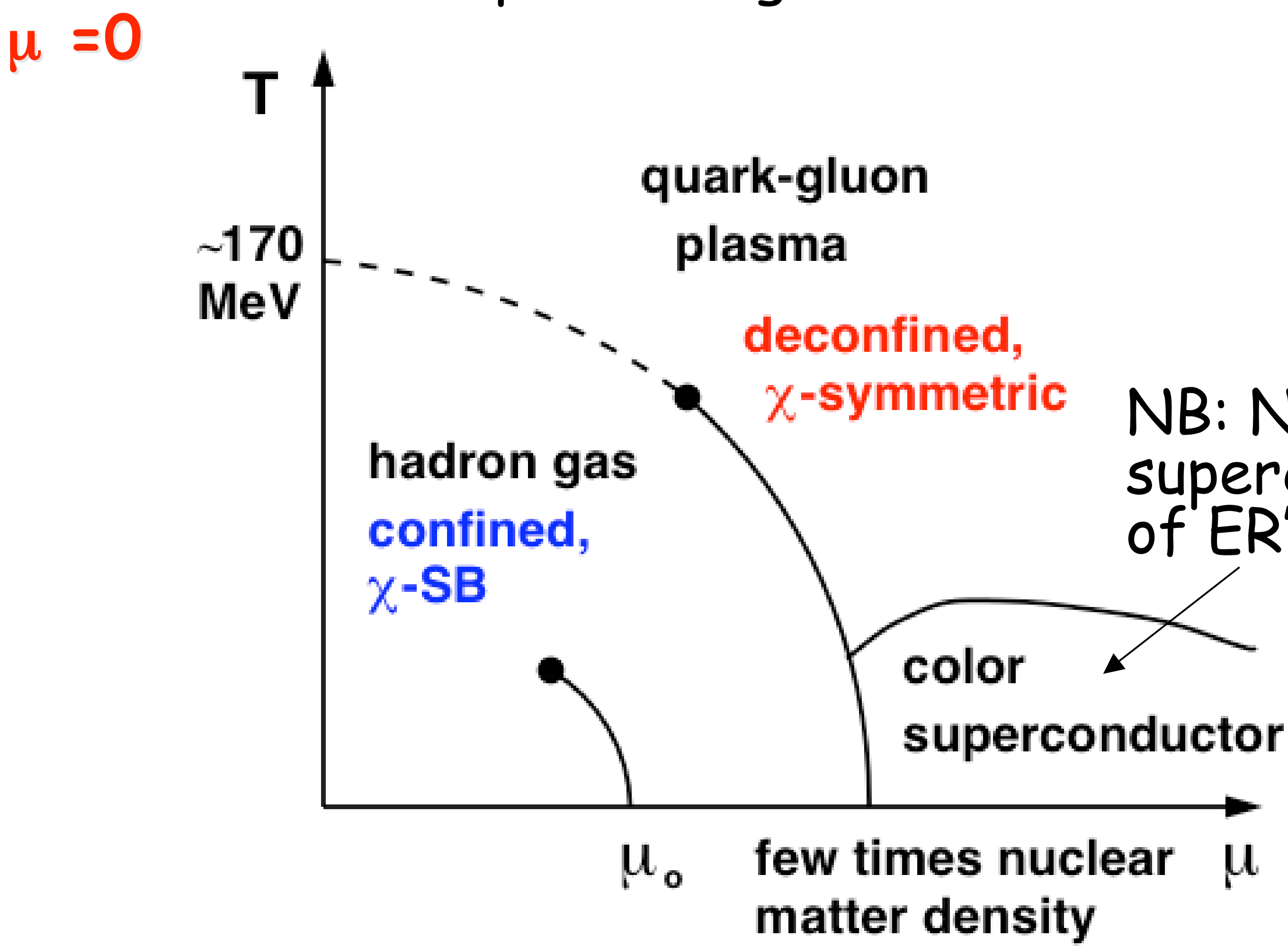
- The dual resonance model (later string theory) came to the same conclusion in 1969, and related the Hagedorn temperature to the string tension
- For a while this possibility was taken seriously (cf. Weingerg's book on GR and Cosmology)

- With the advent of QCD (early seventies) people started to suspect that the correct interpretation of T_H was not that of a maximal temperature, but rather of a phase transition above which confinement disappears.
- Above T_H one would enter a so-called quark-gluon plasma (QGP) phase in which the linear confining potential is screened (like Debye screening in a standard QED plasma)
- Such a phase transition should have occurred when the temperature of the Universe fell below T_H , or about 200MeV, i.e. very early in its history ($T_0 \sim 10^{-4}$ eV).
- The idea also came about that similar situations could be created in the laboratory by colliding heavy nuclei (ions in practice: Pb, Au..) after accelerating them to energies of hundreds of GeV/nucleon
- Of course energy is not enough: T is the crucial quantity

- Experiments have been performed, in particular at CERN's SPS and are running right now at RHIC (BNL). One of the four experiments at the LHC (ALICE) will be a heavy-ion experiment for the discovery/study of the QGP
- One peculiarity of these experiments is that the system has also net baryon number (unlike in a pp* collision, for instance). This means that QCD will be probed in HIC not just at finite T but also at finite chemical potential μ (actually less and less so as we more to higher and higher energies and stay in the so-called central region).

- Several «signals» of a deconfining transition have been proposed theoretically and (perhaps) seen experimentally:
  1. Suppression of J/ψ production
  2. Enhanced strange-particle production
  3. Other features in leptonic channels (photons from the plasma)
- A warning: the theory of HIC is complicated (a finite system, out of thermal equilibrium, living for a short time…)
- In the rest of the lecture I will briefly summarize what is known, via lattice calculations, about the idealized situation of QCD in thermal equilibrium at finite T and μ (main source, F. Karsch, hep-lat/0601013)
- Another warning: finite T OK, finite μ more tricky

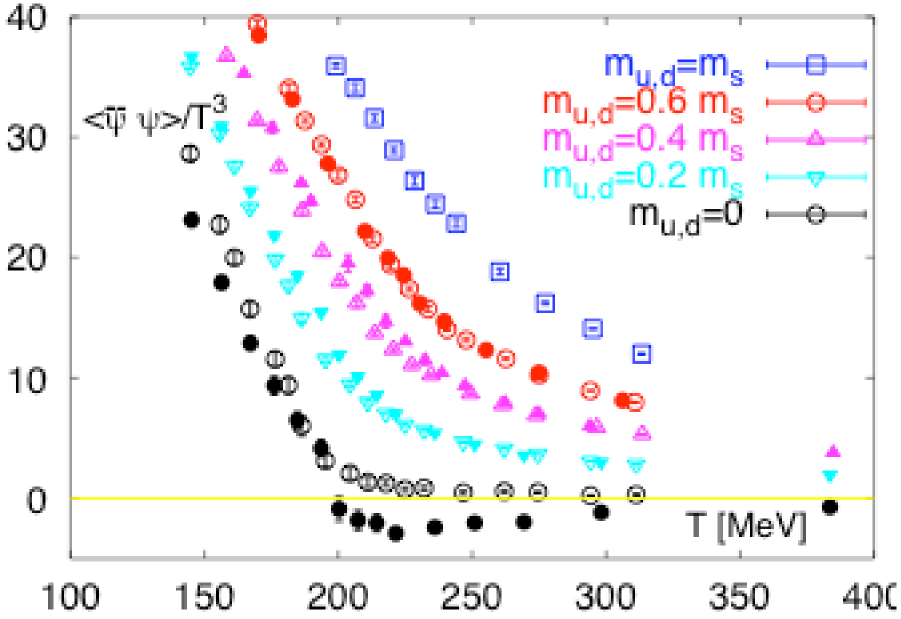
Sketch of QCD phase diagram



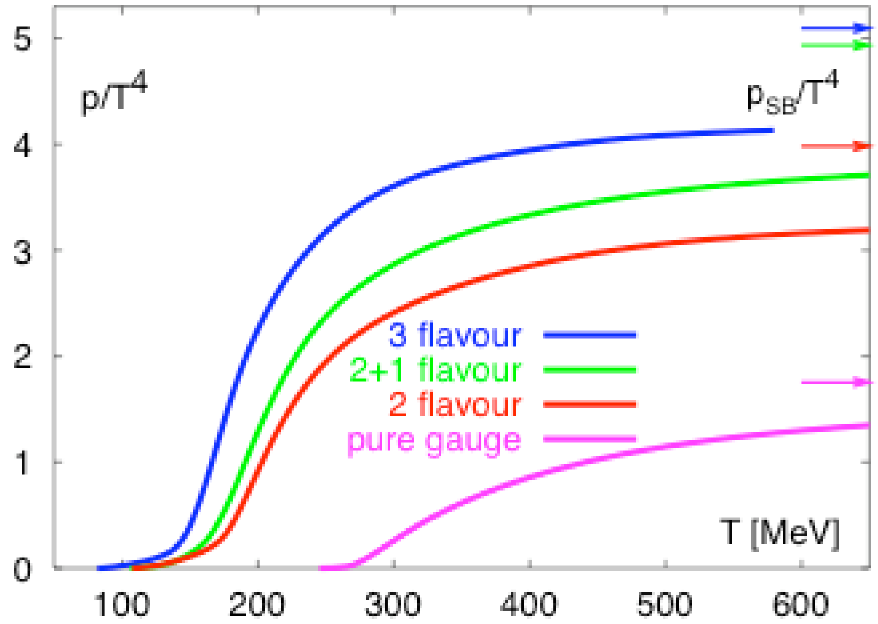
NB: Not the dual superconductor of ER's seminar!

μ =0

The chiral condensate as a function of T for various values of quark masses



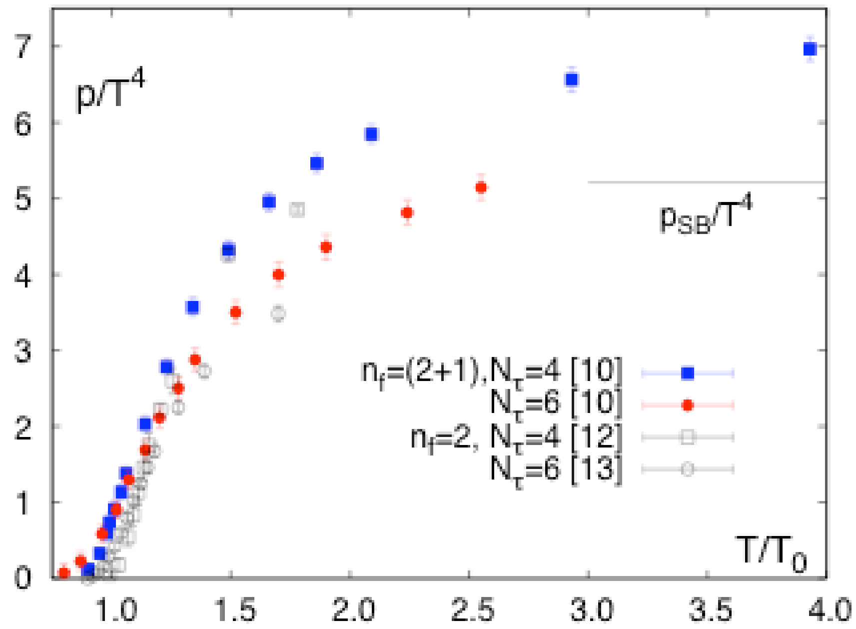
Expected behaviour of p/T^4 at various values of N_f and quark masses

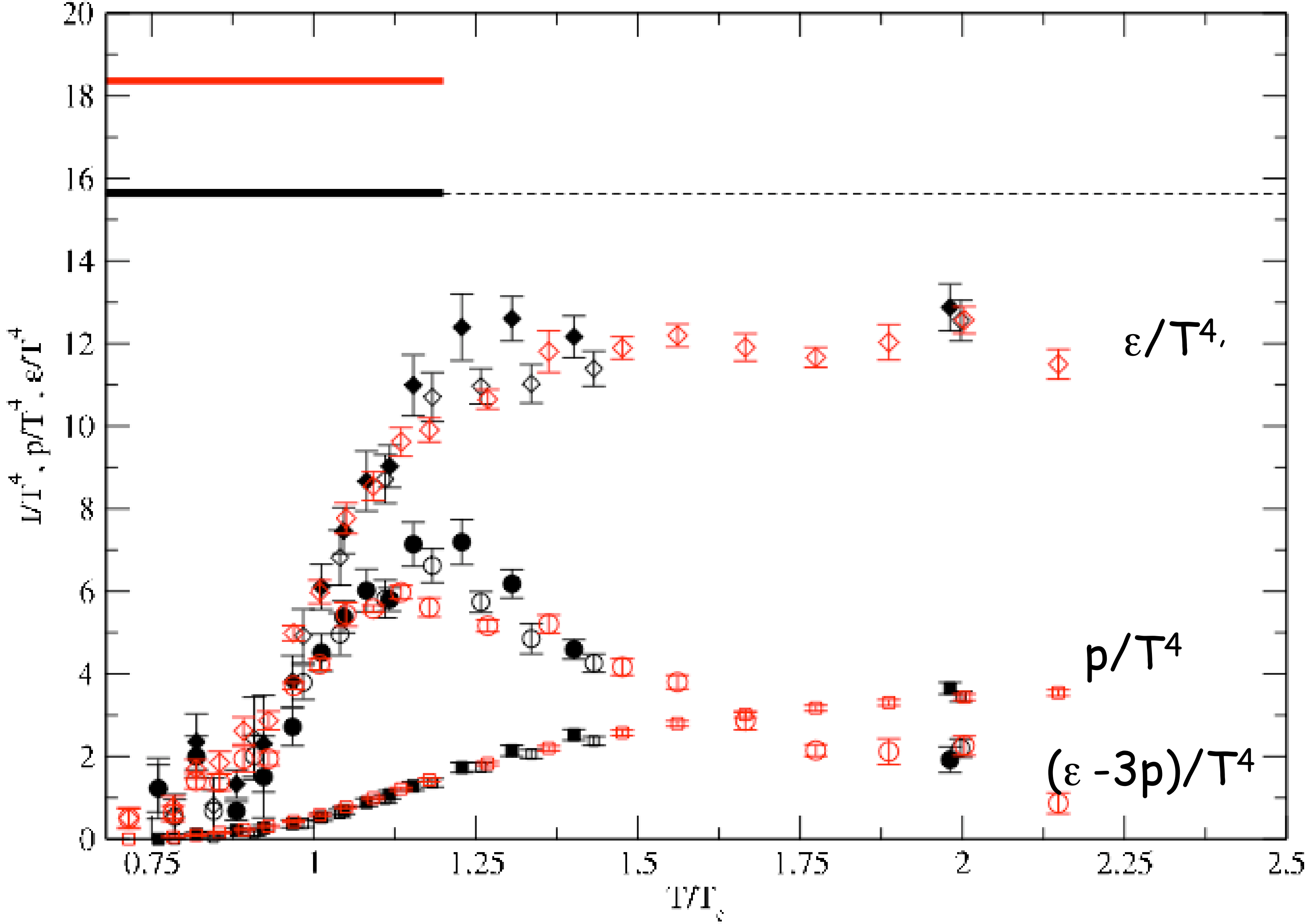


$$\frac{p_{SB}}{T^4} = \frac{\pi^2}{90}\left(16 + \frac{21}{2} g_f\right)$$

μ =0

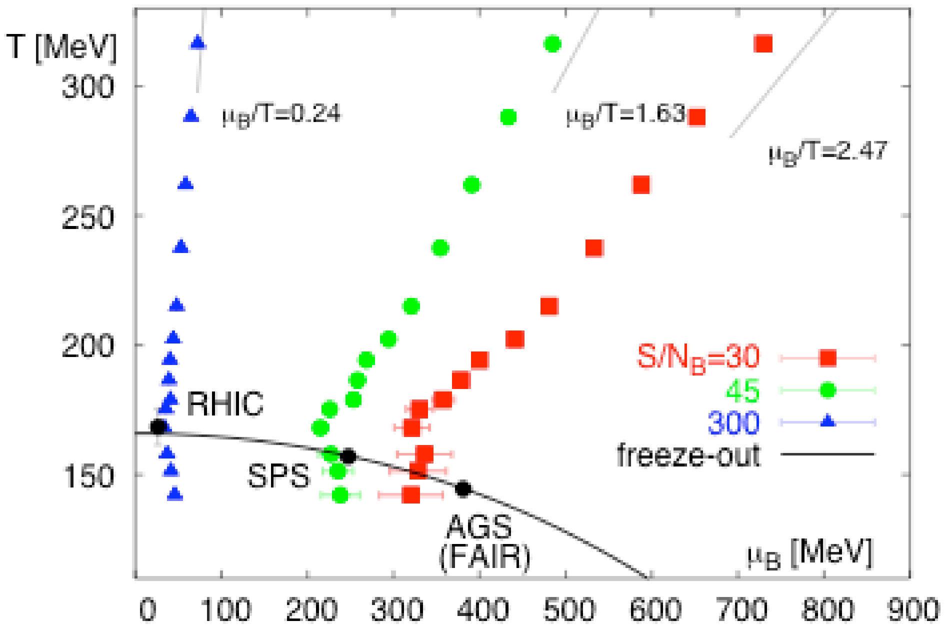
p/T^4 at various values of N_f and quark masses



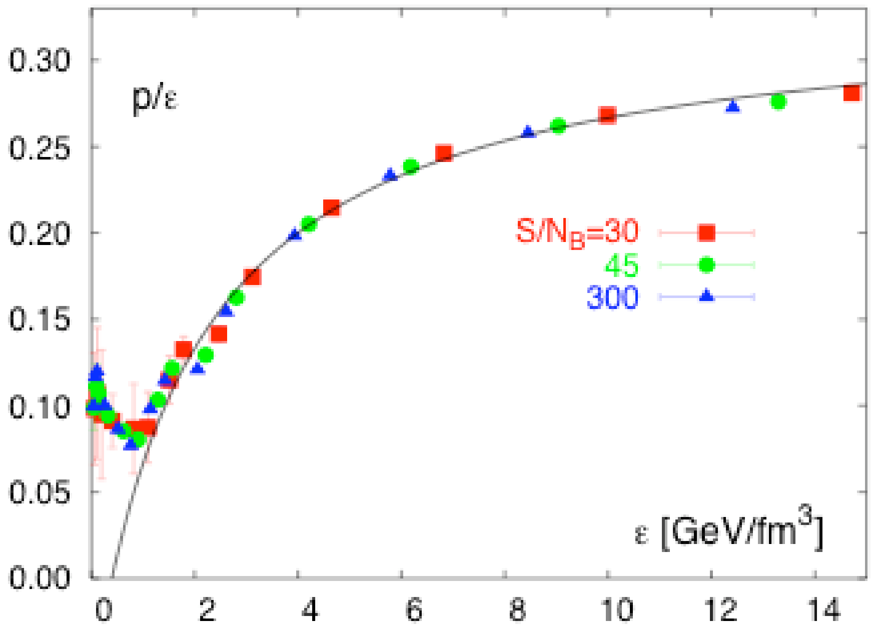


20
18
16
14
12
10
8
6
4
2
0
0.75
1
1.25
1.5
1.75
2
2.25
2.5
T/T_c
I/T^4, p/T^4, ε/T^4
ε/T^4,
p/T^4
(ε -3p)/T^4

Curves of constant S/N_B in a μ-T plot

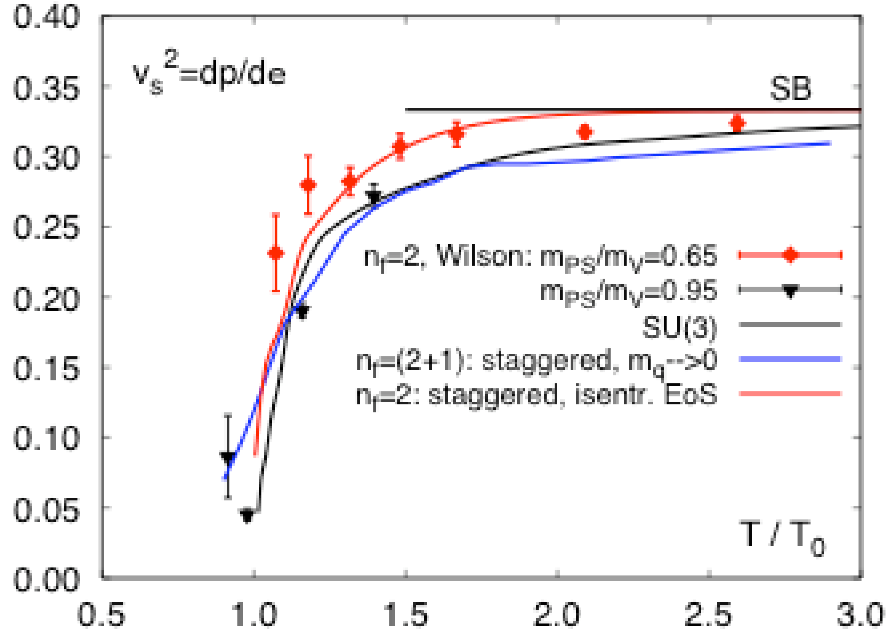


p/ε vs ε for various values of S/N_B



$$\frac{p}{\varepsilon} \sim \frac{1}{3}\left(1 - \frac{1.2}{1 + 0.5\varepsilon fm^3/GeV}\right)$$

$v_s^2 = dp/d\varepsilon$
versus T/T_0



To conclude this (and last) year's course(s)

QCD is a conceptually simple, but highly non-trivial and extremely rich theory. Its successes in the perturbative regime are beyond doubt. In the non-perturbative regime QCD possesses qualitatively all the features needed for it to agree with the experiments, but quantitative tests are more difficult, even if they begin to be available.

There are no real QCD crises so far. There are puzzles about the QCD parameters (why $m_u/m_t \sim 10^{-5}$, $\theta < 10^{-9}$?).

The answer to these mysteries must be looked for elsewhere, either in the EW sector and the mechanism of mass generation in the SM, or beyond (neutrino masses).

Perhaps the subject of next year's course!