Particules Élémentaires, Gravitation et Cosmologie
Année 2007-'08

Le Modèle Standard et ses extensions

Cours IX: 28 mars 2008

The Higgs sector: fine-tuning issues

Old plan

Date	9h45-10h45	11h-12h
08/02	Gauge theories, a reminder	QED/QCD, a reminder
15/02	Weak interactions: early days	Spont. symmetry breaking
22/02	SM Higgs	SM Lagrangian
29/02	Adding families, CKM	Accidental symmetries of SM
07/03	Flavour dynamics & CPX	Flavour dynamics & CPX
14/03	Neutrino masses/mixing	Neutrino masses/mixing
28/03	Higgs sector: fine tuning?	Status of EW precision tests
04/04	Composite-Higgs models	Higgs-less models
11/04	Supersymmetry	Where can new physics hide?

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Un cours et un séminaire de transition

Cours:

Summary of previous lectures/seminars
The fine-tuning issue
Outline of two possible solutions
(details next week)

Séminaire (RB)

The precision tests

Summary of previous lectures/seminars

In order to describe the phenomenology of all three (non-gravitational) interactions we were led to consider a gauge theory based on the gauge group $G = SU(3)_c X SU(2)_L x U(1)_Y$ with the following matter content (i=1, 2, 3 = family label)

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$(u_i,d_i) = Q_i$	3	2	1/6
$(\nu_i, e_i) = L_i$	1	2	-1/2
u_i^c	3*	1	-2/3
d_i^c	3*	1	+1/3
e_i^c	1	1	+1
ν_i^c	1	1	0
$(\phi^+, \phi^0) = \Phi$	1	2	1/2

plus the r.h. antifermions + Φ^*

The most general (renormalizable) Lagrangian associated with that structure is (R.B.'s grouping):

$$L_{SM}^{(3fam)} = L_{Gauge} + L_{Yukawa} + L_{Hpot} + L_{mass}$$

$$L_{Gauge} = -\frac{1}{4}\sum_a F^a_{\mu\nu}F^a_{\mu\nu} + \sum_{i=1}^{3} i\bar{\Psi}_i\gamma^\mu D_\mu \Psi_i + D_\mu\Phi^* D^\mu\Phi$$

$$L_{Yukawa} = -\sum_{i,j=1}^{3} \lambda_{ij}^{(Y)} \Phi \Psi_{\alpha i} \Psi^c_{\beta j} \epsilon_{\alpha\beta} + c.c.$$

$$L_{Hpot} = -\mu^2 \Phi^*\Phi - \lambda(\Phi^*\Phi)^2$$

$$L_{mass} = -\frac{1}{2}\sum_{i,j=1}^{3} M_{ij}\ \nu^c_{\alpha i}\nu^c_{\beta j}\epsilon_{\alpha\beta} + c.c.$$

There are two qualitatively new features of L_{SM} with respect to a theory of just the strong and electromagnetic interactions:

1. The fermions belong to a complex representation of *G* with the single exception of ν_i^c (N_i in RB's notation). Only these « right-handed » neutrinos can be given an explicit gauge-invariant mass term
2. We have added a scalar (so-called Higgs) field, a complex SU(2) doublet, in order to be able to:

a. Break spontaneously the gauge symmetry
b. Allow gauge-invariant Yukawa interactions

We took the «wrong sign» for the quadratic term in the Higgs potential . This gave rise to the Spontaneous breaking of G down to $SU(3)_c x U(1)_Q$ and produced, as a result,

a. Masses for the W and the Z (while the photon and the gluons remain strictly massless)
b. Masses for all the fermions (including the light neutrinos) and for the single surviving Higgs boson (4-3=1!) .

With 3 families (generations) the fermionic masses are actually mass matrices and the physical fermions (those of well-defined mass) are the eigenvectors of this mass matrix.

When we express the gauge interactions in terms of the physical (rather than the original) fermions we find that:

a. Nothing happens in the neutral weak currents (no FCNC)

b. Flavour mixing occurs in the charged weak currents.

For quarks this mixing is fully contained in the unitary Cabibbo-Kobayashi-Maskava matrix V_{CKM}.

For 3 families V_{CKM} contains 3 angles and a CP-violating phase.

The quark Yukawas thus give a total of 6 masses, 3 angles and a CP-violating phase (actually 2, because of the strong-CP problem!)

==> R.B.'s seminars

If neutrinos have mass there is an analogous mixing matrix U_{PMNS} for the leptons. This is not very relevant for the charged leptons
but implies striking neutrino oscillations

==> F.F.'s seminars

In the last 4 seminars we have seen how all this works both in the quark-flavour sector and in the lepton/neutrino sector. Later today we will hear about further precision tests of the standard model and about how impressive the agreement between theory and experiments is.

As we have mentioned and will see in detail later today, radiative (loop) corrections are essential in order to ensure such a good agreement.

In the rest of this lecture we will turn to a more theoretical issue related to such corrections: it goes under the name of the fine-tuning (or hierarchy or naturalness) problem

The fine-tuning issue: A false problem?

The effect of radiative corrections can be encoded in the replacement (see my 2005 course) :

$$L^{(Class.)} = L^{(Tree)} \to L_{eff.} = L^{(Class.)} + L^{(Loops)} + L^{(non-pert.)}$$

In basically all D=4 QFT, $L^{(Loops)}$ is UV-divergent. Two attitudes are possible:

Old: we introduce an UV cutoff (Λ_{UV}) and proceed in a well-defined mathematical (physically uncomfortable?) way and eventually send Λ_{UV} to infinity. A finite number of parameters cannot be predicted, have to be taken from experiments

New: we admit ignorance about physics above a certain scale $\Lambda_{n.p.}$ and check sensitivity to $\Lambda_{n.p}$ and to our ignorance

Example of QED

Recall (again from my 2005 course) that in QED:

$$L_{eff.}^{(QED)} = L^{(Class.)}(\alpha \to \bar{\alpha}(\Lambda); m \to \bar{m}(\Lambda)) + L_{eff.}^{finite}$$

A particular case of a renormalizable theory: sensitivity to $\Lambda = \Lambda_{UV}$ fully contained in a redefinition of the parameters of the classical Lagrangian (here α and m). Furthermore, one finds that $\bar{m} = m\, Z(\Lambda)$ because of the chiral symmetry that appears for m=0 (loops preserve the symmetry).

Q: How can the dimensionless α and Z depend on a dimensionful scale such as Λ? The answer is twofold:

A_1: the dependence is logarithmic;

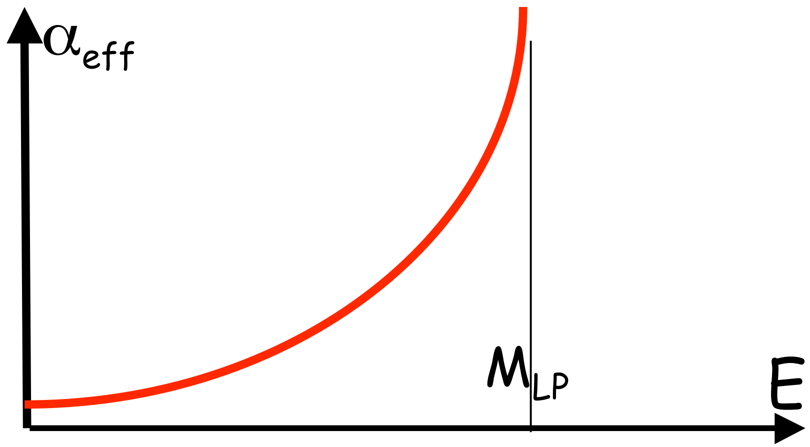
A_2: the scale of the log is E/Λ where E (provided by the finite part of L_{eff}) is some characteristic energy scale of the process at hand (=> so called running of coupling constants)

Example of QED

In QED the effective coupling constant α grows (logarithmically) with energy

$$\alpha_{eff}^{-1}(E) = \alpha_0^{-1} - 2\beta_0 \, log\left(\frac{E}{M}\right) \; ; \; \beta_0^{QED} = \frac{1}{3\pi}(n_{ch.\ lep.}) > 0$$

It is believed that α blows up at a finite (although very high) energy scale, the so-called Landau-pole M_{LP}



Within the old attitude we would conclude that a theory like QED does not make sense: we cannot send Λ to infinity without making QED trivial (triviality problem)

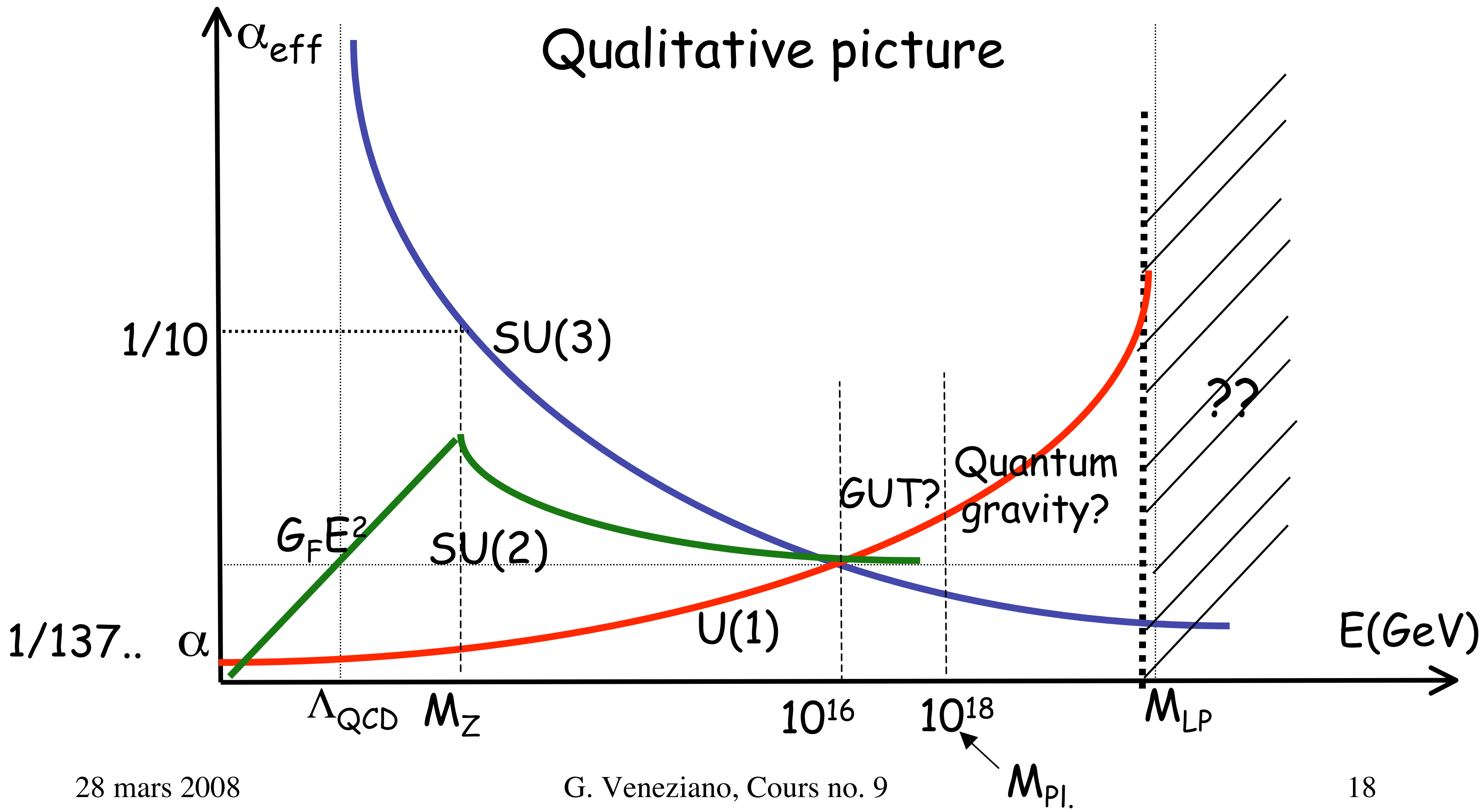
Within the new attitude we simply say that QED has to be modified above a scale $E_{n.p.} \ll M_{LP}$. Fortunately, the testable predictions of QED are very insensitive to where exactly we put the scale of new physics

What happens in the SM?

$$
\begin{aligned}
L_{eff.}^{(SM)} &= L^{(Class.)}(\alpha \to \bar{\alpha}(\Lambda); \lambda^Y \to \bar{\lambda}^Y(\Lambda); \lambda \to \bar{\lambda}(\Lambda); \mu^2 \to \bar{\mu}^2(\Lambda)) \\
&+ L_{eff.}^{finite}
\end{aligned}
$$

How do the various terms in the SM Lagrangian get affected by the radiative corrections? The answer is quite simple:

L_{gauge} suffers a renormalization similar that of QED. The only difference is that while the $U(1)_Y$ coupling grows with energy those of $SU(3)_c$ and of $SU(2)_L$ decrease



$$L_{eff.}^{(SM)} = L^{(Class.)}(\alpha \to \bar{\alpha}(\Lambda); \lambda^Y \to \bar{\lambda}^Y(\Lambda); \lambda \to \bar{\lambda}(\Lambda); \mu^2 \to \bar{\mu}^2(\Lambda)) + L_{eff.}^{finite}$$

L_{Yukawa} and L_{mass} behave like the electron mass term in QED (after all that's how the electron gets its mass!) and thus λ^Y and $M^{(\nu)}$ get just a logarithmic dependence on E/Λ

So far so good: how about V_{Higgs}?

In the Higgs potential the quartic coupling λ acquires a logarithmic dependence on E/Λ (basically for dimensional reasons) while the Higgs mass term gets a radiative correction which is quadratic in Λ:

$$\begin{aligned}\bar{\mu}^2 &= \mu^2 + g^2(\Lambda^2 + c\mu^2 log(E/\Lambda)) + \cdots \sim \mu^2 + g^2\Lambda^2 \\ \bar{\lambda} &= \lambda(1 + g^2 log(E/\Lambda)) \sim \lambda\end{aligned}$$

$$G_F^{-1/2} = \bar{v} = \sqrt{\frac{-\bar{\mu}^2}{2\bar{\lambda}}} \Rightarrow \bar{\lambda} = \frac{1}{2} G_F \bar{\mu}^2$$

This means that λ_{eff} becomes large if μ_{eff} exceeds the Fermi scale of a few hundred GeV. However, all the checks of the SM assume λ_{eff} to be perturbative (need a not-too-large μ_{eff})

$$\bar{\mu}^2 = \mu^2 + g^2(\Lambda^2 + c\mu^2 log(E/\Lambda)) + \cdots \sim \mu^2 + g^2\Lambda^2$$
$$\bar{\lambda} = \lambda(1 + g^2 log(E/\Lambda)) \sim \lambda$$

We are in some kind of impasse:

Either some new physics appears at the 100 GeV scale

...but there is no sign of this!

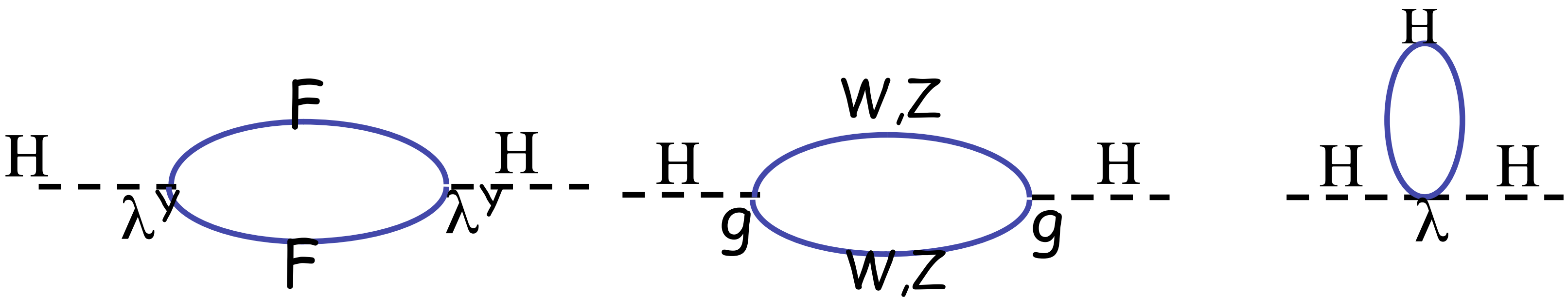
Or there is a cancellation making μ_{eff} small enough

Two types of cancellation are in principle possible:

a. Between μ^2 and $g^2 \Lambda^2$ (old attitude, ignoring fine tuning)
b. Between $g^2\Lambda^2$ and the new physics (the dots...)

Even in the latter case a certain amount of fine-tuning is necessary in order to push the scale of new physics sufficiently high...

The actual calculation (one-loop)



It is similar to the calculation of the correction to the ρ parameter but the external legs are those of the physical Higgs-particle (not of the eaten-up NG bosons) and, instead of looking at the renormalization of the kinetic terms, we look at the renormalization of the mass itself

$$\delta\mu^2 = -\frac{3\Lambda^2}{32\pi^2 v^2}\left(4m_t^2 - 2m_W^2 - m_Z^2 - m_H^2 + \dots\right) \sim -\frac{3\Lambda^2}{8\pi^2 v^2} m_t^2$$

having already assumed $m_H < 350$ GeV. Let us then write:

$$m_H^2 = -2\mu^2 = \epsilon^2 \frac{3\Lambda^2}{4\pi^2 v^2} m_t^2$$ where ε^2 is the allowed fine-tuning

We finally rewrite this relation as:

$$\epsilon\Lambda_{n.p.} < \frac{2\pi}{\sqrt{3}}\frac{v}{m_t}\frac{1}{\epsilon}m_H = 400\ GeV\,\frac{m_H}{115\ GeV}$$

The so-called small hierarchy problem is simply the fact that the r.h.s. of this equation cannot be much larger than 400 GeV. On the other hand the absence of signals of new physics appears to set a lower bound on $\Lambda_{n.p.}$ of about 5 TeV (assuming couplings O(1)!) i.e. we may need a fine-tuning parameter $\varepsilon^2 \sim 10^{-2}$. This is not impossible (particularly if the new physics comes in with a small coupling) but is already somewhat worrisome for some extensions of the SM.

In any case setting $\Lambda_{n.p.}$ near the GUT or Planck scale (the "desert" scenario) would need a huge fine-tuning (second only to the one related to the cosmological constant)

Q: Could the fine-tuning be built in the new physics?

Examples of « ways out » I: Technicolour

This (pseudo?) solution is suggested by a simple observation. Consider a fake (toy) SM in which there is a single family of massless quarks and leptons and no Higgs.

Q: What is the low-energy physics of such a model?

A: Somewhat surprising. We know (2006 course) that the $SU(3)_c$ interactions break spontaneously the global symmetry $SU(2)_L \times SU(2)_R \times U(1)_V$ of m=0 QCD down to $SU(2)_V \times U(1)_V$ producing 3 massless NG bosons, the pions

$$\langle \bar{\Psi}_f \, \Psi_{f'} \rangle = c \, \delta_{ff'} \, \Lambda^3_{QCD}$$

The naïve answer is that the 3 pions, as well as the 3 gauge bosons of $SU(2)_L$, remain massless. This is wrong! The $SU(2)_L$ of the EW interactions is that same $SU(2)_L$ and is sp. broken

According to the general discussion of SSB of a local symmetry, the 3 pions would be "eaten up" by the 3 gauge bosons and the latter would acquire a mass.

The problem (besides the disappearence of the pions) is that the W, Z masses would be on the order of Λ_{QCD}. More precisely, G_F would be of order $1/F_\pi^2 \sim (100\ \mathrm{MeV})^{-2}$ instead of the experimental value ~ $(300\ \mathrm{GeV})^{-2}$

This toy model, however, suggests a better one: let's introduce, instead of the Higgs doublet, a new AF, QCD-like interaction ("technicolour") with a Λ_{tc} parameter a few thousands times larger than Λ_{QCD} and (at least) a doublet of "techniquarks"… can this work? See next week's seminar…

Examples of « ways out » II: Supersymmetry

An interesting property of the formula

$$\delta\mu^2 = -\frac{3\Lambda^2}{32\pi^2 v^2}\left(4m_t^2 - 2m_W^2 - m_Z^2 - m_H^2 + \ldots\right) \sim -\frac{3\Lambda^2}{8\pi^2 v^2} m_t^2$$

is that fermions and bosons contribute with opposite signs (BTW: the same is true for their contribution to the cosmological constant/dark energy!).

Supersymmetry (SUSY) is a special symmetry associating with each bosonic degree of freedom a fermionic one. In the limit in which SUSY is exact the radiative correction to the Higgs mass (and to the c.c.) is zero. But SUSY is not exact…see next week's lecture