Precision tests of QED and determination of fundamental constants

Collège de France
Seminar
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Outline

Description of the hydrogen atom: from Bohr to QED

Free and bound electron g-2

* fine structure constant
* electron mass

Lamb shift and the Rydberg constant

Fine structure constant from other systems

Tests of QED with positronium: lifetime and spectrum

Outlook

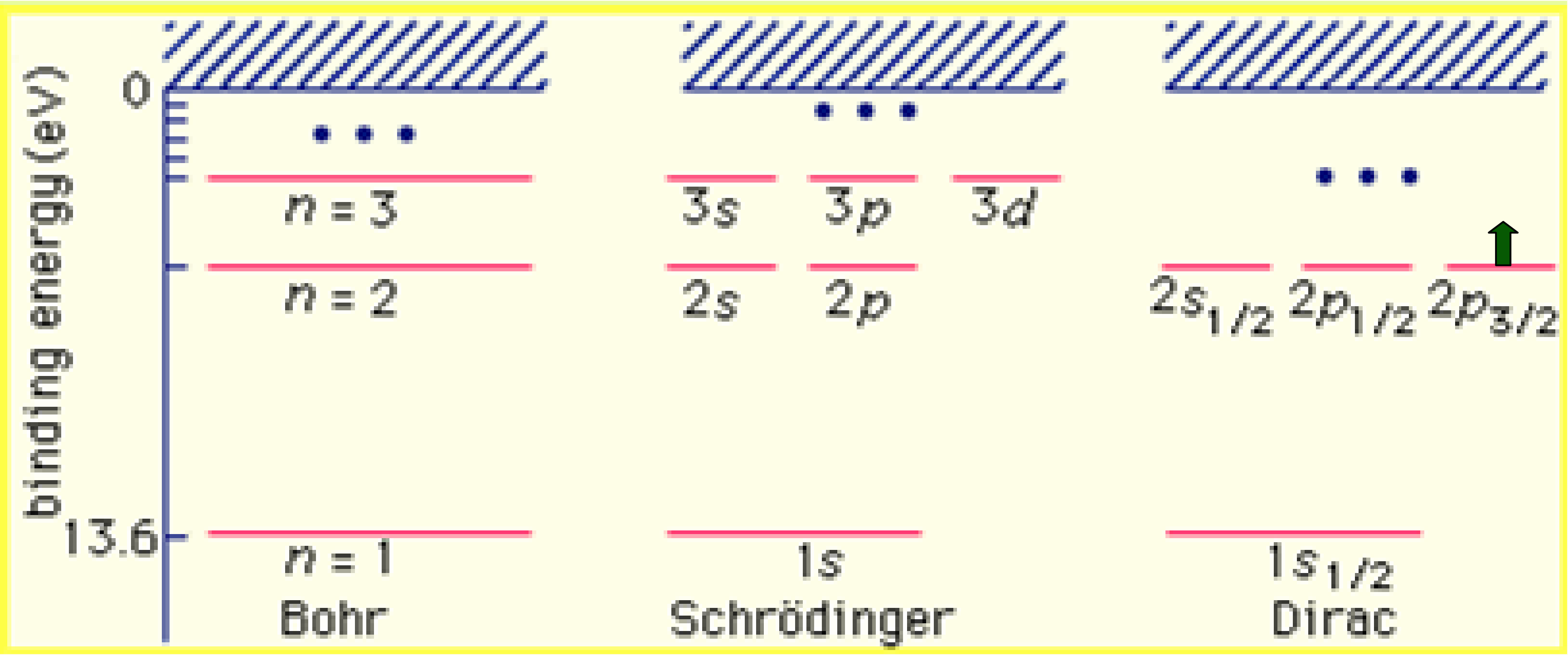
QED vs. QCD: historical perspective

QCD: Asymptotic freedom recognized
with the latest Nobel Prize in physics.

QED:
Experimental foundation:
Lamb shift and the anomalous magnetic moment "g-2"
Lamb and Kusch, Nobel 1955 (50 years ago)

Theoretical framework:
Tomonaga, Schwinger, Feynman, Nobel 1965 (40 years ago)

Hydrogen atom: early (quantum) history



From Encyclopaedia Britannica

(Model)

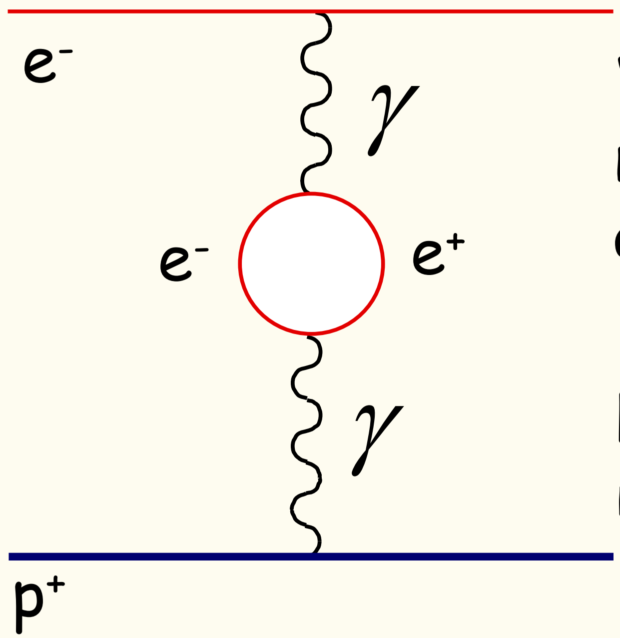
Theory with Coulomb potential

Relativistic theory, accounts for e^- spin. Spin-orbit partially lifts degeneracy: fine structure

Deviations from the Dirac equation predictions: experiments by Lamb…

Degeneracy of $2S_{1/2}$ & $2P_{1/2}$: only if exact Coulomb potential

Lamb shift: $2S_{1/2}$ lies higher (less strongly bound).
Connection with the running coupling constant:

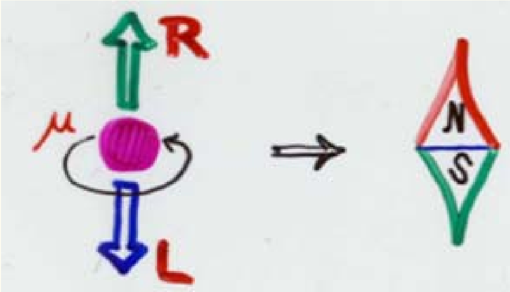


Virtual pairs screen the proton charge; near the proton attraction is stronger; affects mainly the S-state: -27 MHz

Full theory: electron self-interaction is crucial → Hans Bethe (+ 1000 MHz)

Deviations from the Dirac-equation predictions: experiments by Kusch: anomalous magnetic moment

Interaction of the electron's and nucleus' magnetic moments:
hyperfine structure (spin-dependent)
→ test of the electron magnetic moment.



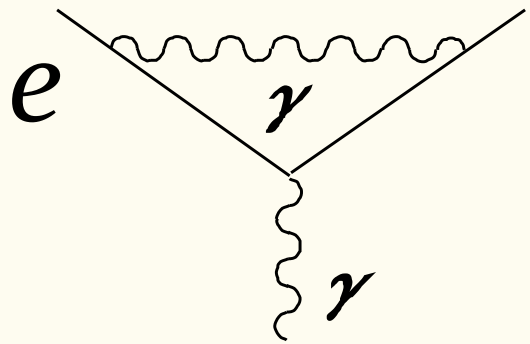
Spinning charged particle: magnetic dipole

$$\vec{\mu} = g \frac{q}{2m} \vec{s}$$

For elementary fermions (electron, muon):
Dirac equation predicts

$$g = 2$$

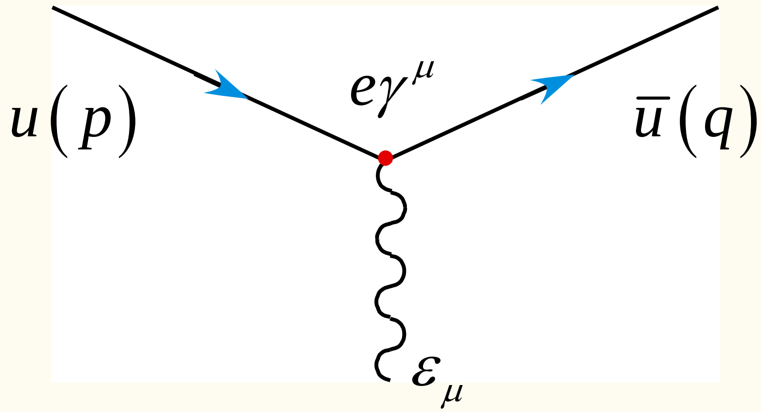
g-2: first results



Schwinger (1948) $$a(\text{theory}) \equiv \frac{g-2}{2} \simeq \frac{\alpha}{2\pi} = 0.00116$$

Kusch & Foley (1948) $$a(\exp) \simeq 0.001188(22)$$

The general picture after discoveries of the Lamb shift and g-2



First approximation for the hydrogen atom:
Schrödinger description.
Corrections: expand the QED Lagrangian in v/c

Quantum effects: loops.

Expansion parameter: $\alpha \equiv \frac{e^2}{4\pi} \simeq \frac{1}{137.036}$

Non-perturbative phenomena may be important for Z >> 1

For low-Z systems: very good convergence

Determination of fundamental constants

Fine structure constant (alpha) from *g*-2
m_e from the bound-electron *g*-2
Rydberg constant
Other determinations of alpha

Free-electron g-2

Electron:

$$a_e^{\rm QED} = \frac{\alpha}{2\pi} - 0.328478966\left(\frac{\alpha}{\pi}\right)^2 + 1.1812415\left(\frac{\alpha}{\pi}\right)^3 - 1.7260(50)\left(\frac{\alpha}{\pi}\right)^4 + \ldots$$

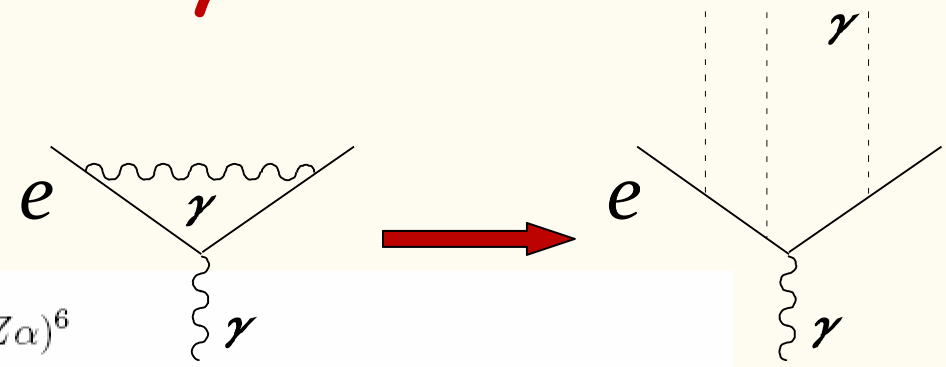
Schwinger   Sommerfield   Laporta+Remiddi   Kinoshita

Provides the most precise value of Fine Structure Constant

$$\alpha = 1/137.035\,998\,84(1.8)_{\rm th4}(2.4)_{\rm th5}(50)_{\rm exp}$$ (3.7 ppb)

Challenge: to estimate five-loop contribution "th5"

Bound-electron g-2: theory



$$g(nS) = \underbrace{2 - \frac{2\,(Z\alpha)^2}{3\,n^2} + \frac{(Z\alpha)^4}{n^3}\left(\frac{1}{2n} - \frac{2}{3}\right) + \mathcal{O}(Z\alpha)^6}_{\text{Breit (1928), Dirac theory}}$$

$$+ \underbrace{\frac{\alpha}{\pi}\left\{2 \times \frac{1}{2}\left(1 + \frac{(Z\alpha)^2}{6n^2}\right) + \frac{(Z\alpha)^4}{n^3}\left\{a_{41}\ln[(Z\alpha)^{-2}] + a_{40}\right\} + \mathcal{O}(Z\alpha)^5\right\}}_{\text{one-loop correction}}$$

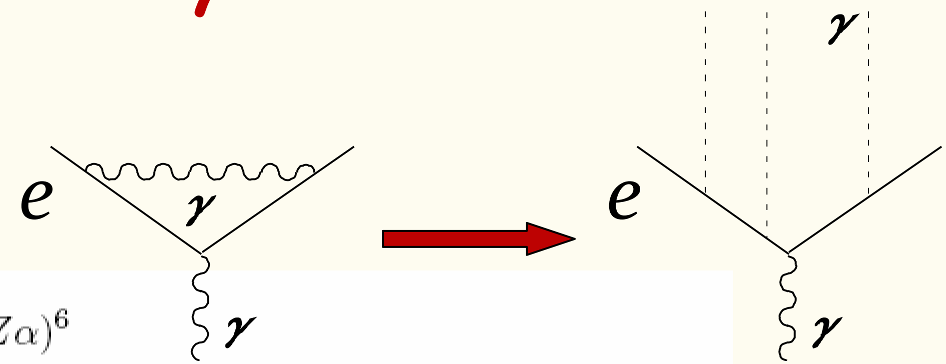
$$+ \underbrace{\left(\frac{\alpha}{\pi}\right)^2\left\{-0.656958\left(1 + \frac{(Z\alpha)^2}{6n^2}\right) + \frac{(Z\alpha)^4}{n^3}\left\{b_{41}\ln[(Z\alpha)^{-2}] + b_{40}\right\} + \mathcal{O}(Z\alpha)^5\right\}}_{\text{two-loop correction}}$$

$$+ \mathcal{O}(\alpha^3)\,.$$

From Jentschura and Evers

Note: Breit's calculation predates Schwinger's by 20 years

Bound-electron g-2: theory



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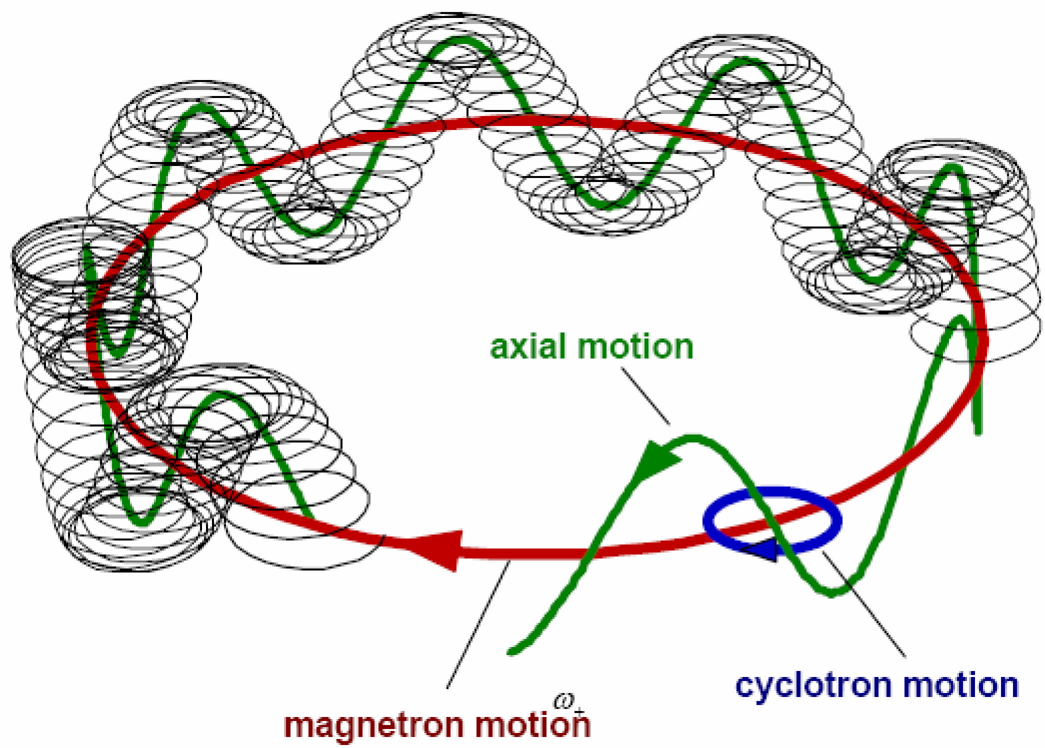
$$+ \mathcal{O}(\alpha^3)\,.$$

Main remaining uncertainty

From Jentschura and Evers

Bound-electron g-2: measurement



Spin precession (Larmor) frequency

$$h\nu_L = g \cdot \mu_B \cdot B$$

Cyclotron frequency:

$$h\nu_C = \frac{q}{M} B$$

$$g = 2 \frac{\nu_L}{\nu_C} \frac{q}{e} \frac{m}{M}$$

From Werth

Bound-electron g-2: result for m_e

VOLUME 93, NUMBER 15 PHYSICAL REVIEW LETTERS week ending 8 OCTOBER 2004

Nonrelativistic QED Approach to the Bound-Electron *g* Factor

Krzysztof Pachucki,[1] Ulrich D. Jentschura,[2] and Vladimir A. Yerokhin[3]

$$m(^{12}\mathrm{C}^{5+}) = 0.000\,548\,579\,909\,41(29)(3)\ \mathrm{u},$$

$$m(^{16}\mathrm{O}^{7+}) = 0.000\,548\,579\,909\,87(41)(10)\ \mathrm{u},$$

Hydrogen spectrum: Rydberg constant

$$R_\infty = \frac{m_e c^2 \alpha^2}{2} \frac{1}{hc} \simeq \frac{13.6\,\mathrm{eV}}{hc}$$

Energy level: Dirac part; Lamb shift; Hyperfine structure

T. Hänsch et al.

$$f_{1S2S} = R_\infty \left(e(2S) - e(1S) \right) + L_{2S} - L_{1S}$$

F. Biraben et al, ENS

$$f_{2S8D} = R_\infty \left(e(8D) - e(2S) \right) + L_{8D} - L_{2S}$$

$$f_{2S12D} = R_\infty \left(e(12D) - e(2S) \right) + L_{12D} - L_{1S}$$

$$f_{1S2S} = 2\,466\,061\,413\,187\,103\,(46)\ \mathrm{Hz}$$

$$R_\infty = 10\,973\,731,568\,550\,(84)\ \mathrm{m}^{-1}$$

Two orders of magnitude improvement
1989 -- 1999

Hydrogen spectrum: Lamb shift

Theory vs. experiment (Lamb shift of the ground state):

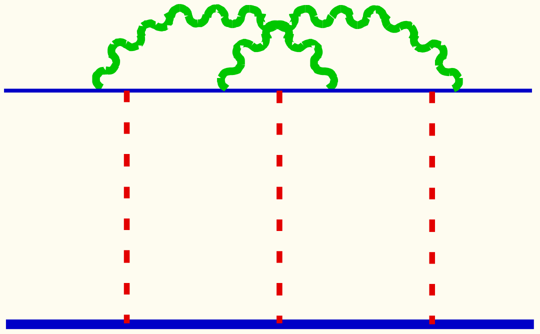
$$L\left(1\mathrm{S};\ \mathrm{theory}\right) = 8\ 172\ 804\,(32)(4)\,\mathrm{kHz}$$

$$L\left(1\mathrm{S};\ \mathrm{exp}\right) = 8\ 172\ 840\,(22)\,\mathrm{kHz}$$

Theoretical frontier: two-loops in external field

$$\Delta E = m\left(\frac{\alpha}{\pi}\right)^2 \frac{(Z\alpha)^4}{n^3} F(Z\alpha)\,,$$

$$F\left(Z\alpha\right) = B_{40} + Z\alpha B_{50}$$
$$+\left(Z\alpha\right)^2\left[L^3 B_{63} + L^2 B_{62} + L B_{61} + B_{60} + \mathrm{h.o.}\right]$$



Other determinations of the fine structure constant

Atom interferometry
Quantum Hall Effect
also: Helium fine structure
Josephson effect

The "kinematic" method of finding alpha

Rydberg constant is extremely well measured,

$$R_\infty = \frac{m_e c^2 \alpha^2}{2} \frac{1}{hc} \simeq \frac{13.6\,\text{eV}}{hc}$$

We can find alpha if we measure the quotient of the Planck constant and the "electron" mass,

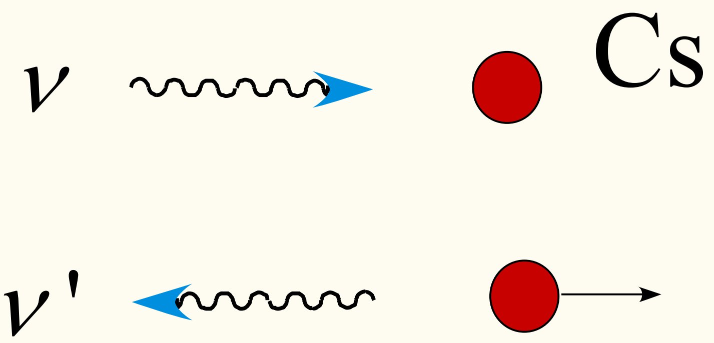
$$\alpha^2 = \frac{2hR_\infty}{m_e c}$$

$$= \frac{2R_\infty}{c} \frac{m_p}{m_e} \frac{m_{\text{Cs}}}{m_p} \frac{h}{m_{\text{Cs}}}$$

In practice heavier particles are better: neutrons or atoms.

Atom interferometry

Consider an elastic photon-atom collision:

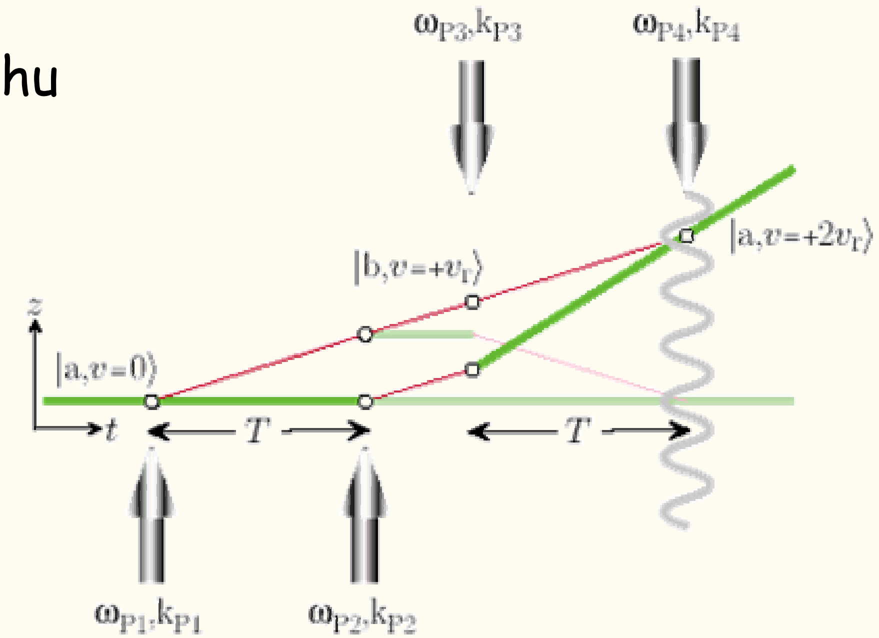


$$h \cdot (\nu - \nu') \simeq \frac{2h^2\nu^2}{m_{\mathrm{Cs}} c^2}$$

$$\frac{h}{m_{\mathrm{Cs}}} = \frac{\Delta \nu}{2\nu^2} c^2$$

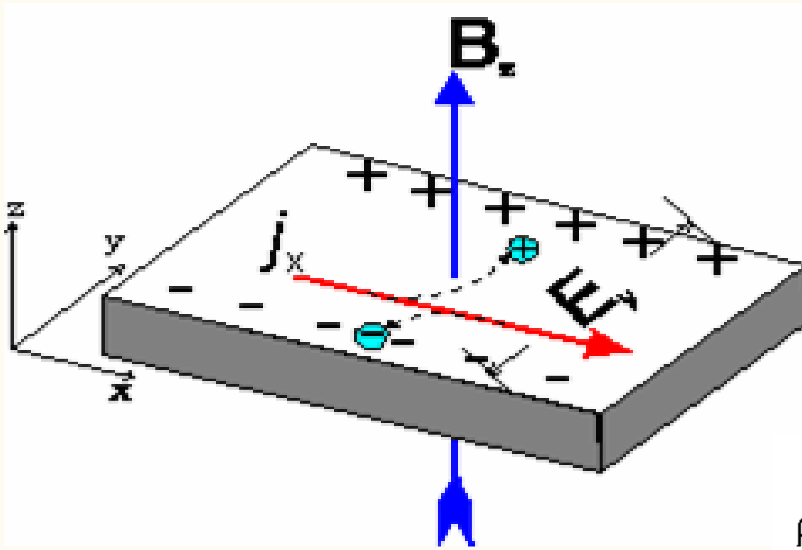
Atom interferometry

Wicht, Hensley, Sarajlic, Chu
(Stanford)



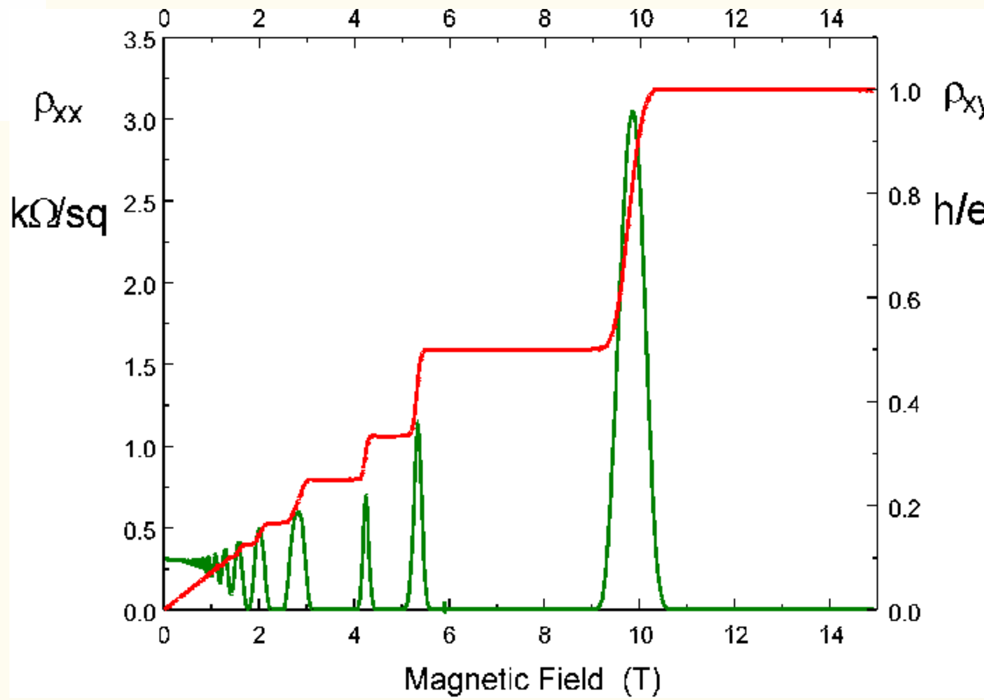
Preliminary Cesium result: 1 / 137. 035 999 71 (60) (4.4 ppb)

Quantum Hall Effect

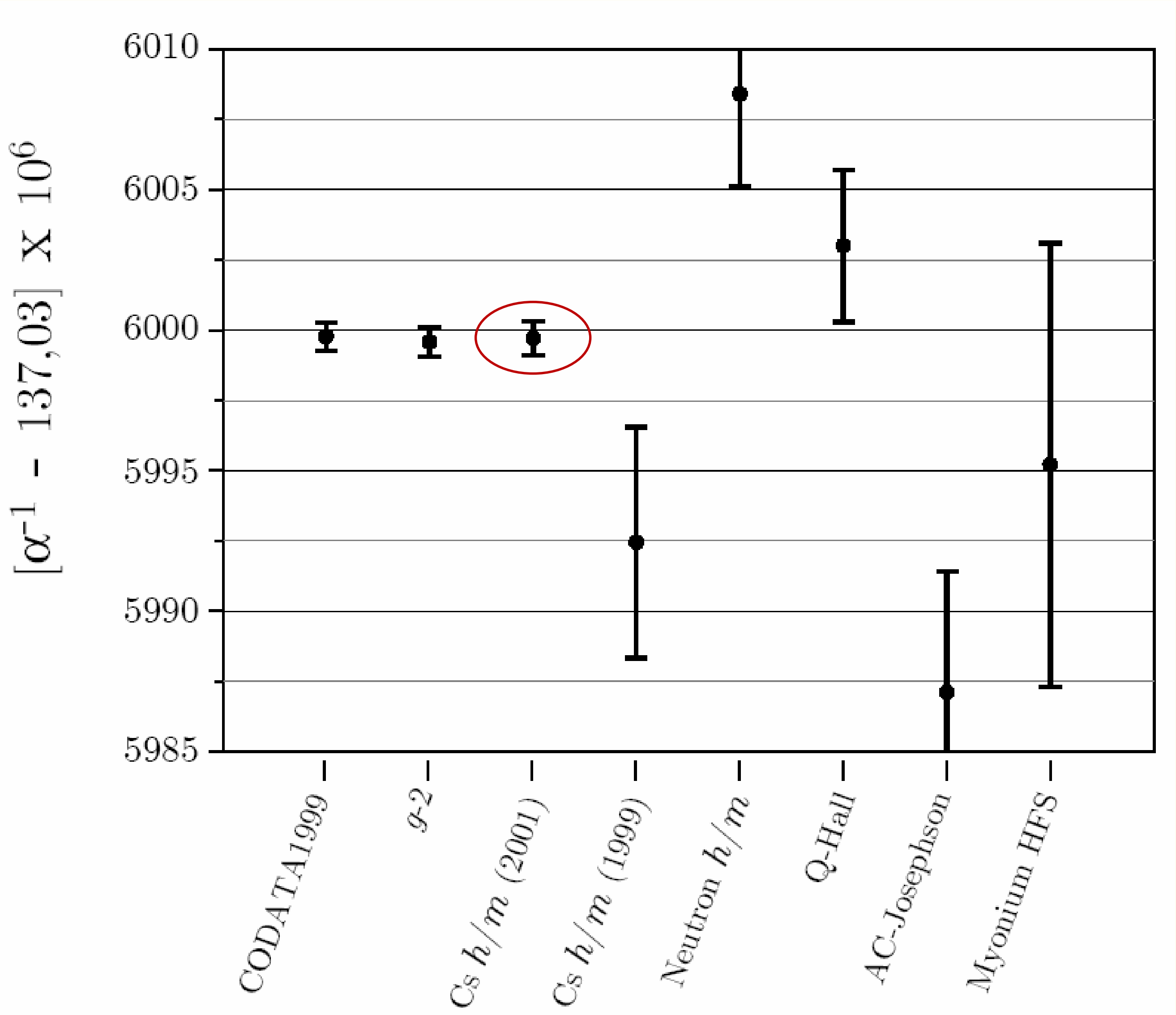


$$\rho_{xy} = \frac{1}{n}\frac{h}{e^2}$$

$$n = 1, 2, \ldots$$



Determinations of alpha: comparison



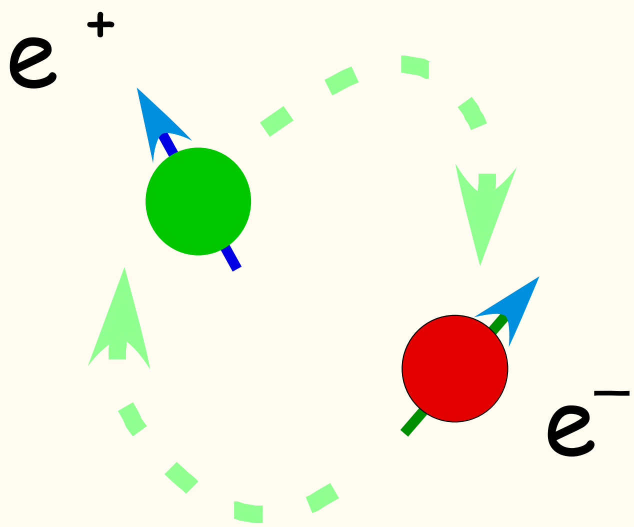
From Udem

Positronium and high-precision tests of QED

Lifetime

Hyperfine splitting

Positronium: the simplest atom



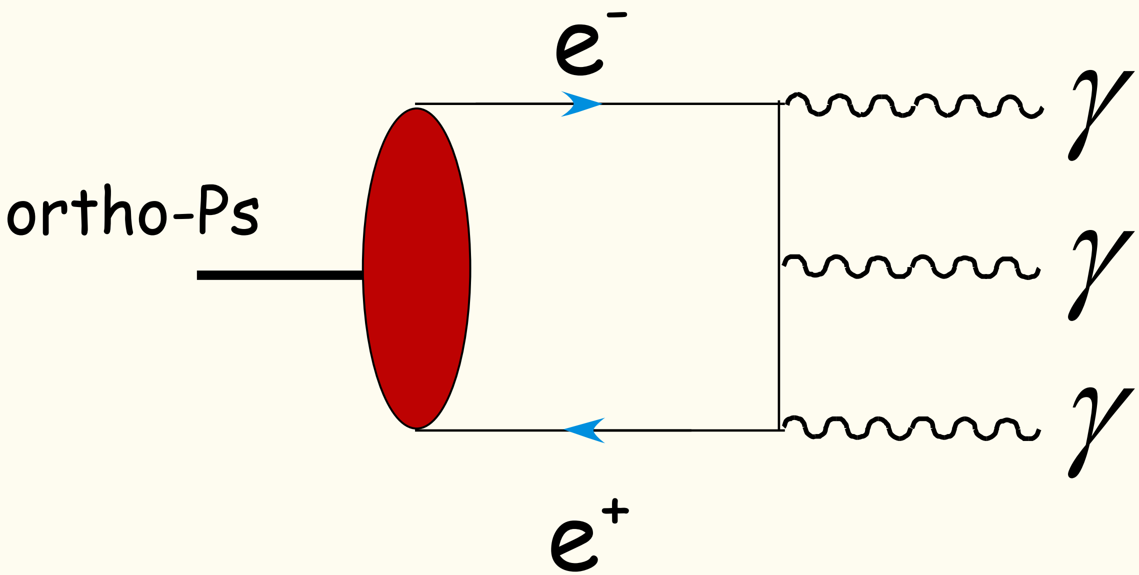
Two spin states:
singlet (para-Ps)
triplet (ortho-Ps)

Electron Compton wavelength $\lambda \sim \dfrac{1}{m_e} \sim 4 \cdot 10^{-13}\,\mathrm{m}$

Bohr radius $a \sim \dfrac{1}{m_e \alpha} \sim 5 \cdot 10^{-11}\,\mathrm{m}$

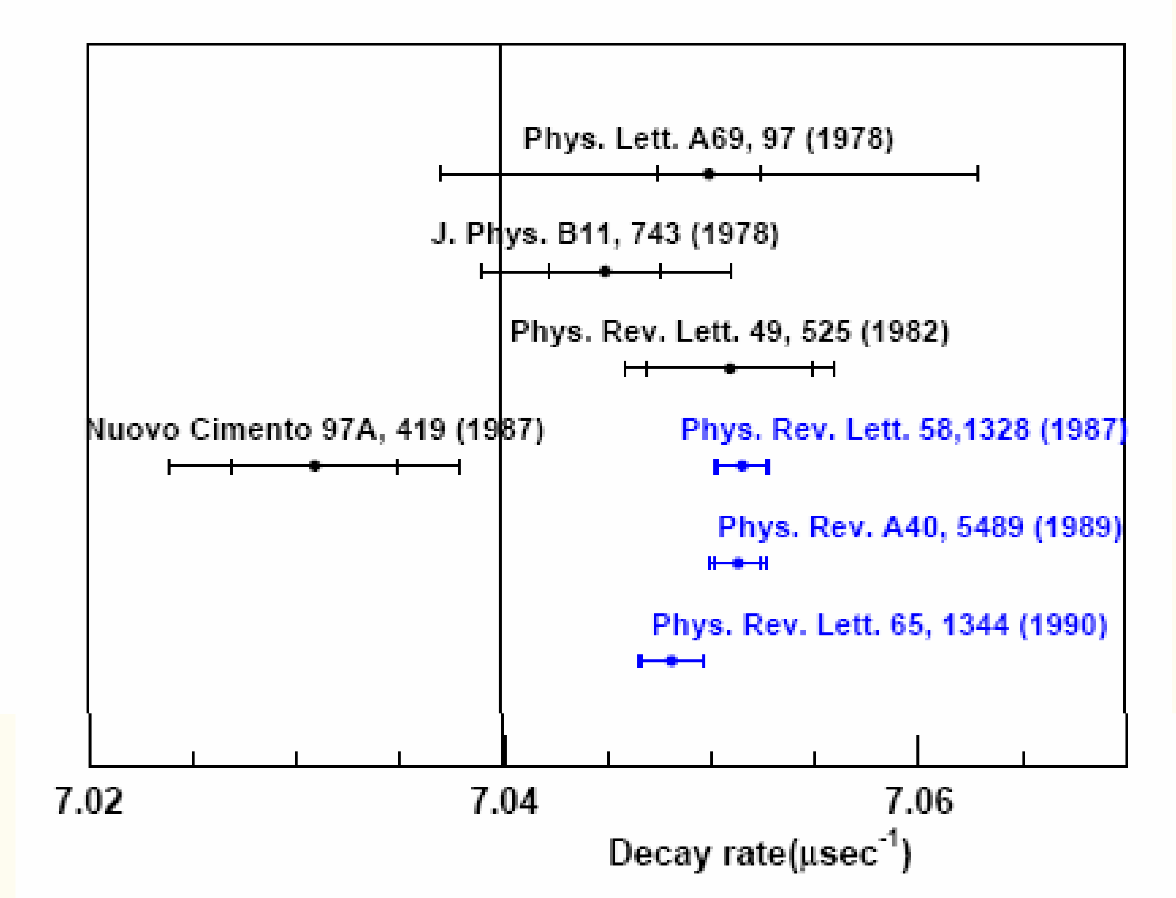
Difficult theory; limited experiments

Positronium decays



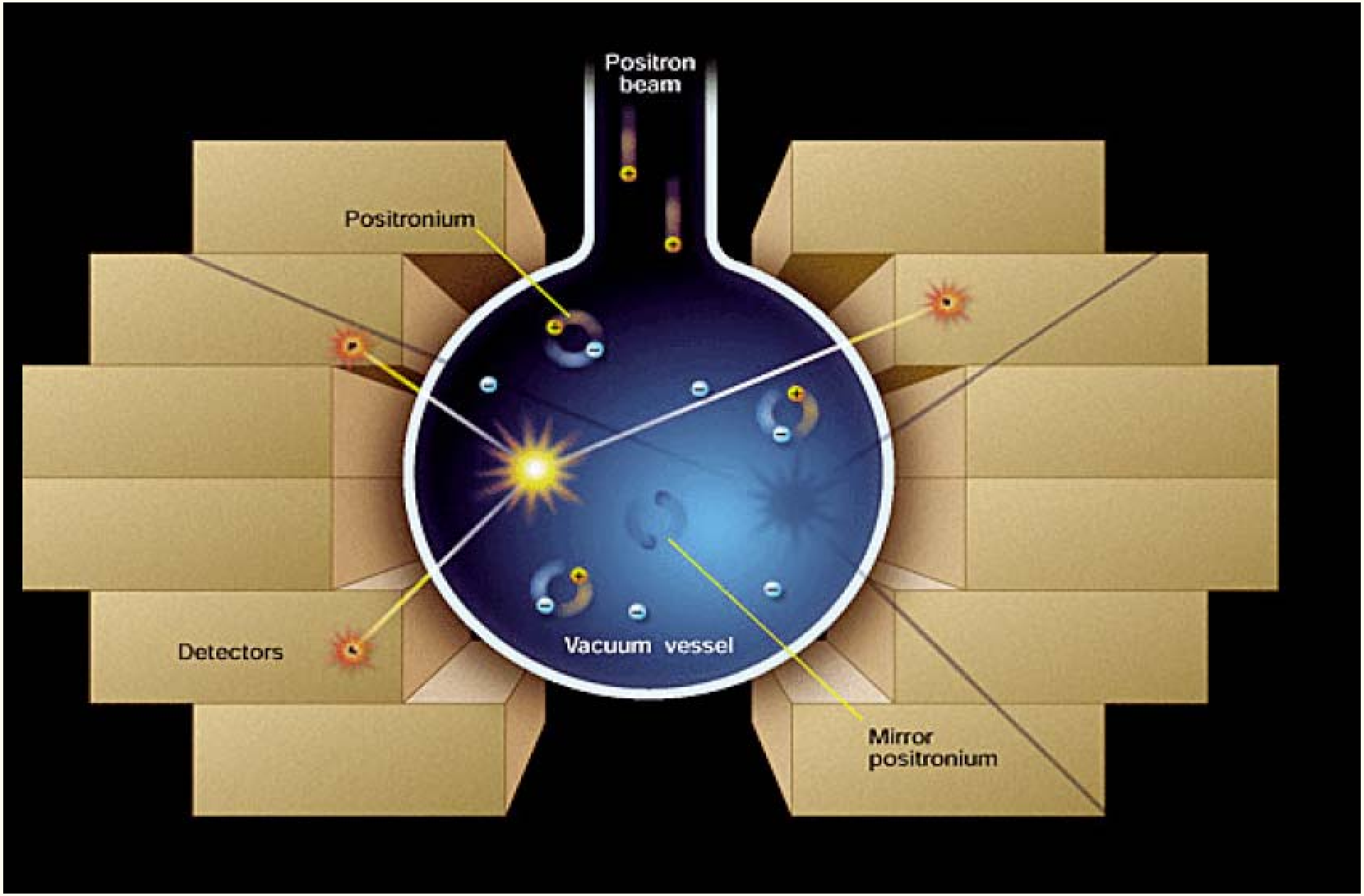
The decay rate is, in principle, calculable in QED, to any accuracy

History of positronium lifetime measurements: early and intermediate



Large discrepancy
5-9 sigma (crisis!)

One possible explanation: exotic decay channels



From C. Regenfus

The positronium lifetime puzzle: theory

$$\Gamma_{\text{o-Ps}} = \left| \int d^3k \; \phi(k) \; \mathfrak{M}\left(e^+ e^- \to \gamma\gamma\gamma\right) \right|^2$$

Bound state momentum distribution

Amplitude for free particles

Lowest order:

$$\left.\begin{aligned} \left|\mathfrak{M}\right|^2 &\sim \alpha^3 \\ \left|\psi(r=0)\right|^2 &\sim \alpha^3 \end{aligned}\right\} \quad \Gamma^0_{\text{o-Ps}} \sim \alpha^6$$

Theory of positronium decay: refinements

Corrections $O(\alpha)$

single hard photon loops

Corrections $O(\alpha^2)$

Para: AC, Melnikov, Yelkhovsky
Ortho: Adkins, Fell, Sapirstein

soft:
- $O(k^2)$ corrections to $\mathcal{M}$
- Breit hamiltonian → correction to $\psi(r=0)$

hard: Short-distance two-loop photon exchange

(Breit hamiltonian correction and short-distance two-loop exchange:) finite together, but give $\ln \frac{m}{m\alpha} = \ln \frac{1}{\alpha}$

Is the result huge? (Potential disaster for meson physics.)

Theory of positronium decay: refinements

Corrections $O(\alpha)$

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hard
- Short-distance two-loop photon exchange

finite together, but give

$$\ln\frac{m}{m\alpha} = \ln\frac{1}{\alpha}$$

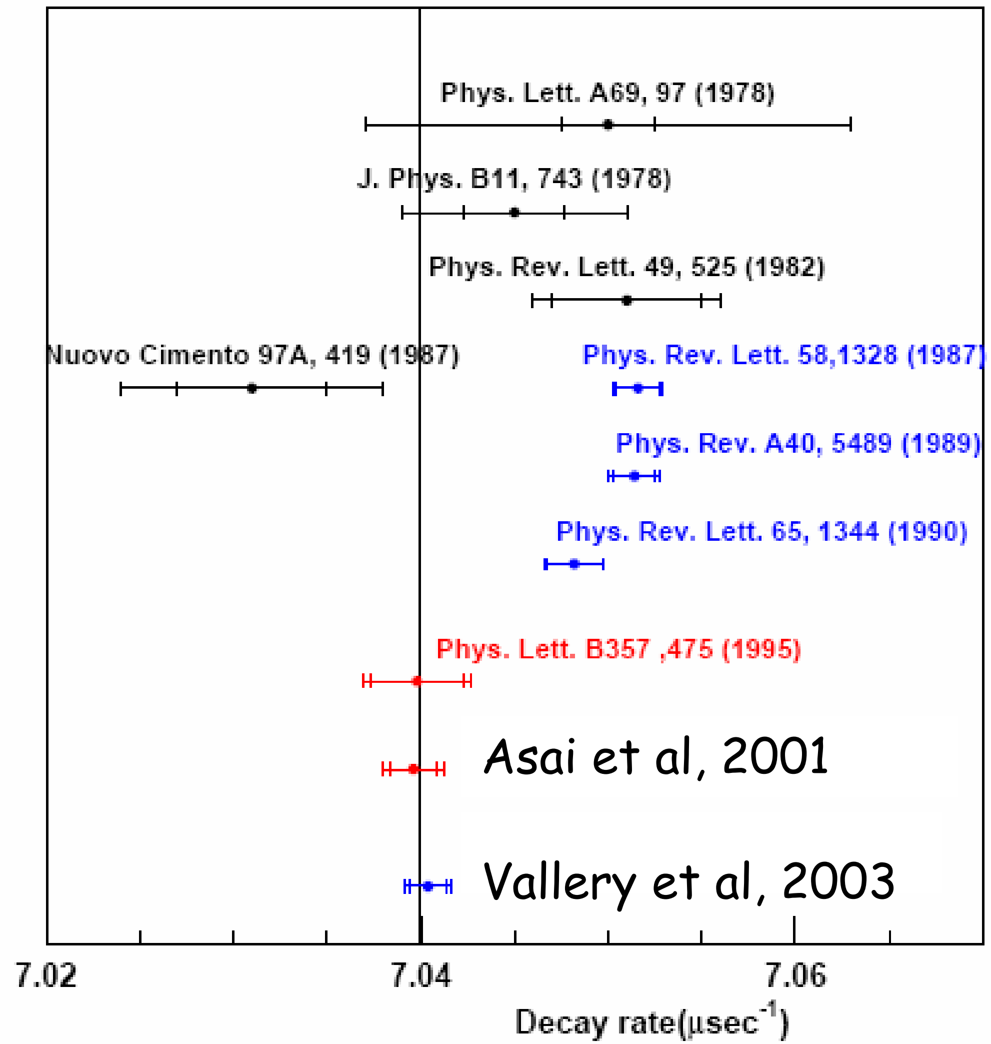
Is the result huge? (Potential disaster for meson physics.) No!

The problem was experimental…

History updated:

$\lambda_T = 7.0396(12)(11)$

$\lambda_T = 7.0404(10)(8)$



From Asai et al.

Summary

QED has been fantastically successful.

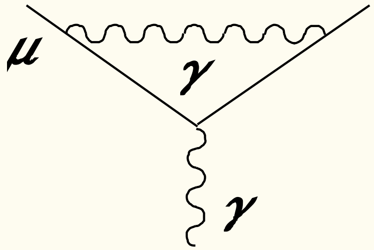
Description of quantum effects tested to 1 part in 10^8.

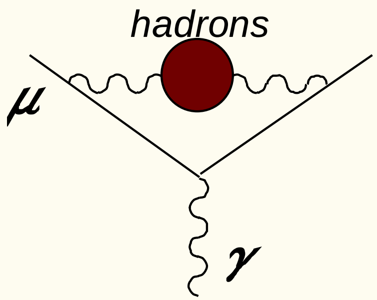
A model theory for developing QCD tools:
effective field theories;
description of bound states.

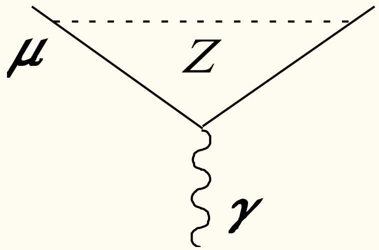
What about the rest of the Standard Model?
Muon *g*-2

Units: 10^{-11}

QED		116 584 718 (1)	hep-ph/0402206 & updates
Hadronic			
	LO	6 934 (64)	hep-ph/0308213 & updates
	NLO	– 98 (1)	hep-ph/0312250
	LBL	120 (40)	tentative, see hep-ph/0312226
Electroweak		154 (3)	hep-ph/0212229
Total SM		116 591 828 (75)	
Experiment – SM Theory =		252 (96)	(2.6σ deviation)







+SuSy?