Particules Élémentaires, Gravitation et Cosmologie

Année 2005-2006

Interactions fortes et chromodynamique quantique

II: aspects non-perturbatifs

Cours II: 14 fevrier 2006

Symétries et leur brisure

- Classical symmetries of massless QCD
- Spontaneous symmetry breaking and NG bosons
- Explicit symmetry breaking and PNG bosons:
  - Classical-level breaking
  - Quantum-level breaking (anomalies)

General Considerations

- Symmetries play a basic role in our present understanding of elementary particles and fundamental interactions
- In Quantum Field Theory we have to make the important distinction between symmetries under global (x-independent) and local (x-dependent) transformations
- Example of former: Lorentz (Poincaré) invariance
- Example of latter: the gauge symmetries

- Recall: local symmetries mean that some d.o.f. are redundant, unphysical (e.g. the redundant Lorentz-invariant description of a J=1, m=0 particle via the vector potential)

➡ They are sacred, have to be broken «very carefully» as in the GSW electro-weak theory

- Global symmetries are not sacred: we are allowed to break them explicitly and sometimes this is actually needed in order to have realistic theories
- Occasionally, symmetries that are exact at the classical level are broken at the quantum level: One talks about an «anomaly» (today's seminar)

- We have an example last year: Classical-CD contains no dimensionful parameter, it is scale-invariant. Instead, Quantum-CD, as we recalled last week, contains a scale, Λ_{QCD}
- One talks indeed about scale-invariance being broken by a so-called «trace anomaly» (since the quantum energy-momentum tensor is no longer traceless). Again, as for any explicit breaking:
- Anomalies are acceptable in global symmetries
- Anomalies are unacceptable in local symmetries (an important constraint on the standard model and its extensions, see again today's seminar)

1. Classical symmetries of massless QCD

- Let us consider, to start with, a more general gauge theory with just gauge fields and fermions (no scalars)
- Besides specifying the gauge group G we have to assign all (left-handed) fermions to some reps. of G (the right-handed antiparticles will be automatically in the c.c. reps.)
- Suppose that there are N_1 l.h. fermions in the rep. r_1, N_2 in r_2 etc. and, for the moment, let's not give them masses
- The only fermionic terms in S_{cl} will be of the type

$$\sum_{f_1=1}^{N_1} \bar{\psi}_{f_1}^{(1)} i\gamma^\mu D_\mu^{(r_1)} \psi_{f_1}^{(1)} + \sum_{f_2=1}^{N_2} \bar{\psi}_{f_2}^{(2)} i\gamma^\mu D_\mu^{(r_2)} \psi_{f_2}^{(2)} + \dots$$

$$\sum_{f_1=1}^{N_1} \bar{\psi}_{f_1}^{(1)} i\gamma^\mu D_\mu^{(r_1)} \psi_{f_1}^{(1)} + \sum_{f_2=1}^{N_2} \bar{\psi}_{f_2}^{(2)} i\gamma^\mu D_\mu^{(r_2)} \psi_{f_2}^{(2)} + \dots$$

$$(D_\mu^{(r)})^i_j = \partial_\mu \delta^i_j + g A^a_\mu (T^a_{(r)})^i_j$$

An independent phase rotation of each fermion (corresponding to the abelian group $U(1)^{N_1+N_2+\dots}$) clearly leaves the action unchanged. Actually, any transformation of the form:

$$\psi_f^{(1)} \rightarrow U_{ff'} \psi_{f'}^{(1)} ; \psi_f^{(2)} \rightarrow V_{ff'} \psi_{f'}^{(2)} , UU^\dagger = VV^\dagger = 1$$

leaves L_{class} unchanged!

We have thus identified a large global-symmetry group:

$$G_{global} = U(N_1) \otimes U(N_2) \otimes U(N_3) \otimes \dots U(N_k)$$

Note that this large global symmetry group is a (empoisoned?) gift from having demanded gauge invariance!

What is G_{global} for QCD with N_f massless quarks?
By its definition, QCD has N_f l.h. quarks in the F-rep. and N_f antiquarks in its c.c. rep. F*. Thus its G_{global} is simply:

$$G^{QCD}_{global} \equiv G_F = U(N_f)_F \otimes U(N_f)_{\bar{F}}$$

more traditionally called

$$U(N_f)_L \otimes U(N_f)_R$$

1.2 Conserved Currents

A general theorem (due to Mme Noether) says that we can associate to any symmetry a conserved current $J_\mu^{(i)}$:

$$\partial_\mu J_\mu^{(i)} = 0 \Rightarrow \frac{d}{dt}\int d^3x J_0^{(i)} = \dot{Q}^{(i)} = 0$$

In our case these currents are easily identified:

$$J_\mu^{(L,R)a} = \bar{\Psi}_f \gamma_\mu (1 \pm \gamma_5) T^a_{ff'} \Psi_{f'}$$

where $T^a_{ff'}$ are the generators of U(N) in the rep. N

We now split the N_f^2 matrices $T^a_{ff'}$ into (N_f^2-1) traceless ones and the unit matrix. This amounts to the decomposition:

$$G_F = U(N_f)_L \otimes U(N_f)_R = SU(N_f)_L \otimes U(1)_L \otimes SU(N_f)_R \otimes U(1)_R$$

2.1 Spontaneous symmetry breaking (SSB)

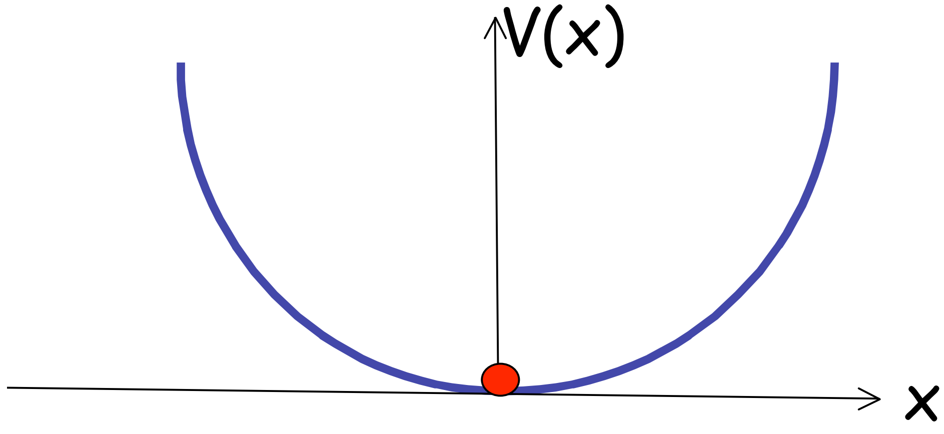
• A very basic distinction has to be made between what one calls explicit breaking and spontaneous breaking of a symmetry

•The former case is the most familiar one. The gravitational field, for instance, breaks rotation invariance in this room by making the vertical direction special. O(3) is broken to O(2) We will see later what breaks explicitly G_F in QCD, and how

•In the case of SSB the symmetry is not really broken: it is simply hidden. The apparent «breaking» is due to the non-invariance of the ground state (the so-called vacuum) under the symmetry's transformations.

Some illustrative examples

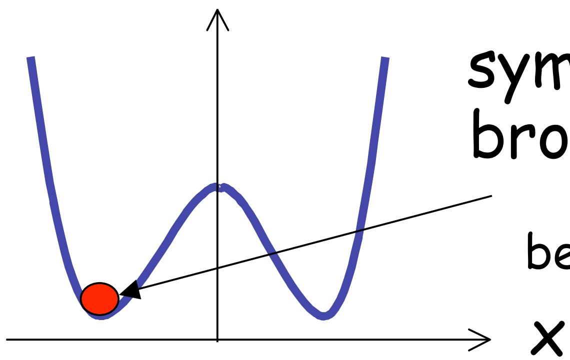
1. Symmetric potential with one global minimum



symmetry is x -> -x
(discrete = Z_2)

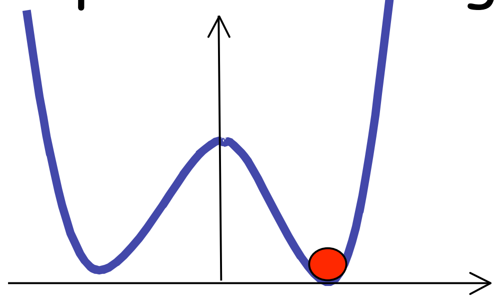
Neither explicit nor spontaneous SB

2. A double-well potential in Classical and Quantum Mechanics

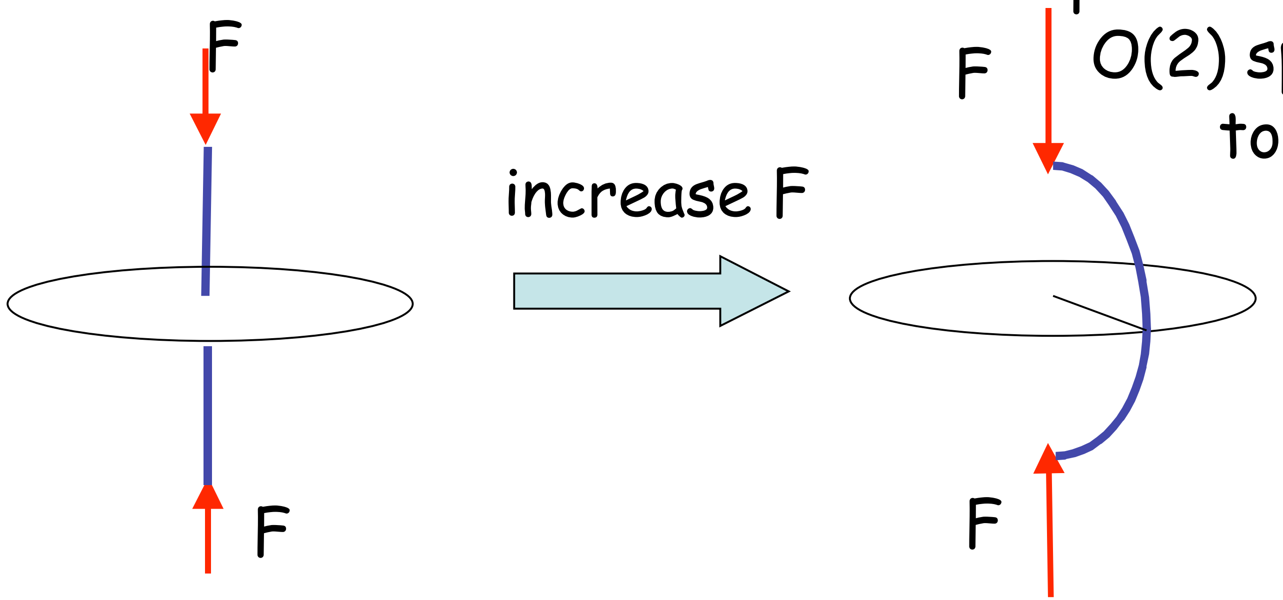


symmetry is x -> -x
broken spont. in CM
(unbroken in QM
because of tunneling)

explicit breaking



3. Continuous symmetry: O(2) ~ U(1)



the stick picks up a particular direction: O(2) spont. broken to nothing

4. A larger continuous symmetry: SO(3) ~ SU(2)

A ferromagnet: the rotation symmetry is broken by the direction along which the magnet's magnetic field aligns. An O(2) survives = rotations around the axis of that (arbitrarily) chosen direction

Spontaneous Symmetry Breaking <=> Degenerate Ground State

In a QM framework states are vectors in a Hilbert space. The ground state , |0> , is no exception; A symmetry transformation *g* becomes a unitary operator U(*g*) (*g* is a particular element of the symmetry group) acting on these states. Furthermore, [H, U(*g*)]=0.

If we apply a symmetry transformation to |0> we get U(*g*)|0> and there are two possibilities:

1. U(*g*) |0> = |0,*g*> = |0> => no SSB
2. U(*g*) |0> = |0,*g*> ≠ |0> => SSB w/ <0|H|0> = <0,*g*|H|0,*g*>

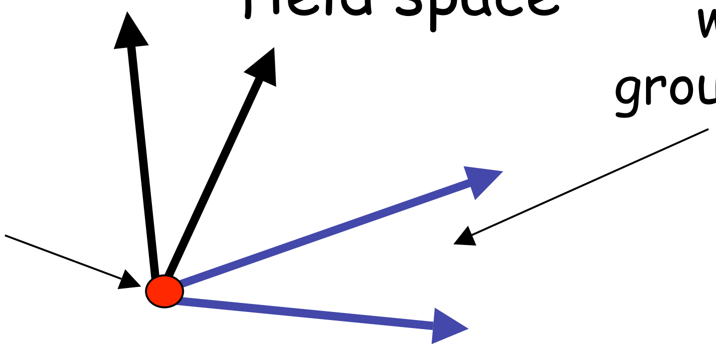
In latter case the subspace |0,*g*>, the so-called vacuum manifold, provides a non-trivial rep. of *G*

2.2 SSB and Nambu-Goldstone(NG) bosons

Goldstone's theorem: If a continuous global symmetry G is spontaneously broken down to its subgroup S (meaning that $S|0\rangle = |0\rangle$) there must a massless (Nambu-Goldstone) boson for each generators of G that is not a generator of S.

In formulae: (number of NG-bosons) = dimG - dimS

The formal proof is a bit techical, but the physics behind is quite clear:



Incidentally..

Goldstone's theorem is evaded if the broken symmetry is local. This is actually the only known consistent way to break a gauge symmetry and to give mass to gauge bosons & chiral fermions The Glashow-Salam-Weinberg theory of ElectroWeak (EW) interactions is very much based on this (Higgs et al.) mechanism

SSB in QCD

There is both theoretical and experimental evidence that, at zero temperature, the ground state of massless QCD is degenerate as a result of a Bose-Einstein condensate (BEC):

$$\langle \bar{\Psi}_f \, \Psi_{f'} \rangle = c \, \delta_{ff'} \, \Lambda^3_{QCD}$$

(cf. BCS theory of superconductivity, Cooper pairs)

What is S? We easily find S= $U(N_f)_V$, the «diagonal» subgroup of $U(N_f)_F \times U(N_f)_{F^*}$ defined by V = U. Indeed:

$$U^\dagger_{ff''} \langle \bar{\Psi}_{f''} \Psi_{f'''} \rangle U_{f'''f'} = c \, \delta_{ff'} \Lambda^3_{QCD} = \langle \bar{\Psi}_f \Psi_{f'} \rangle$$

By the Goldstone theorem we would expect $2N_f^2 - N_f^2 = N_f^2$ massless pseudoscalar (J=0, P=-1) bosons. This is NOT what we observe but we have still to take into account explicit breaking

3.1 Explicit classical symmetry breaking and accidental symmetries

What happens when we add (quark) «masses»*)? In QCD the most general mass term (using reality of the action) is:

$$L_m = -\sum_{ff'} \bar{\Psi}_f m_{ff'} \Psi_{f'} + c.c.$$

In general this terms breaks badly the global symmetry G_F
If $m_{ff'} = m\, \delta_{ff'}$ the full $U(N_f)_V$ subgroup of G_F is preserved.
This is just isospin symmetry (N_f =2) or Gell-Mann-Ne'eman's $SU(3)_{flavour}$ symmetry (for N_f =3).

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*) For the moment these are just parameters appearing in the Lagrangian. We will see later how to relate them to something we can actually measure.

These symmetries looked very fundamental in the sixties.

At first sight their existence in QCD looks very unnatural:

What forces $m_{ff'} = m\delta_{ff'}$?

One can show, however, that even if $m_f/m_{f'} \neq 1$, the $U(N_f)_V$ symmetry becomes very good as $m_f/\Lambda \to 0$. (e.g. $m_d \sim 1.8\ m_u$)

Thus the existence of an approximate $U(N_f)_{flavour}$ symmetry is a mere consequence of having N_f quarks that are light on the scale of Λ_{QCD} . This is a very good (decent) approx. for u,d (s).

One talks about accidental symmetries of the strong interactions, i.e. of symmetries which are there because of the terms we can possibly write down in the lagrangian

A side remark on quark «masses»

$$L_m = -\sum_{ff'} \bar{\Psi}_f m_{ff'} \Psi_{f'} + c.c.$$

A "theory" of quark masses needs the full SM, the already mentioned Higgs mechanism etc.

Within QCD alone these are just parameters, put-in from the outside, to be determined experimentally.

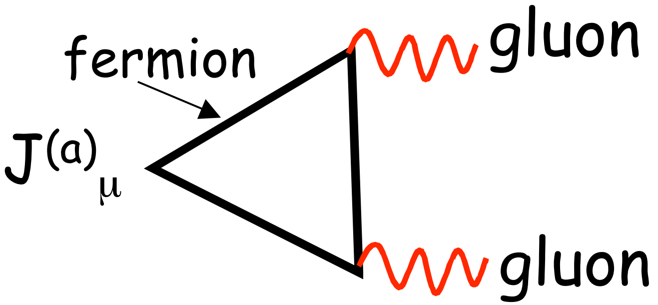
They turn out to cover an enormous range: the lightest quarks (u,d) have masses of a few MeV; the heaviest (t) has a mass of about 170 GeV (5 orders of magnitude!). Why? A big mystery.

(NB: Λ_{QCD} ~ few hundred MeV lies in the middle..)

Decoupling theorem of Appelquist-Carazzone: for $m_{qi} \gg \Lambda_{QCD}$, the i^{th} quark becomes irrelevant at $E \ll m_{qi}$ This theorem allows us to neglect heavy quarks (c,b,t) in many interesting cases

3.2 Explicit quantum symmetry breaking: anomalies

In the late 60's Adler, and then Bell and Jackiw found a very puzzling result: even in the absence of masses some global classical symmetries are broken at the quantum (loop) level. The effect is due to a «triangle» graph with fermions circulating in the loop. The one relevant to QCD is as shown in the figure (the original one was in QED: it is crucial to account for $\pi^0 \to 2\gamma$)



Since the gluon is blind to flavour this diagram does not contribute to SU(N) currents. It also cancels if we take the $U(1)_V$ combination of $U(1)_L \times U(1)_R$, corresponding to total baryon number $(q\text{-}q^*)_L + (q\text{-}q^*)_R$. But $(q+q^*)_L - (q+q^*)_R$ is no longer conserved

An anomaly is a symmetry of S_{class} which is not a symmetry of the effective action S_{eff} (the one that includes quantum corrections). Under a $U(1)_A$ transformation:

$$\psi_f^{(N)} \to e^{i\beta}\psi_f^{(N)}\,;\,\psi_f^{(N^*)} \to e^{i\beta}\psi_f^{(N^*)}$$

the effective action changes by:

$$\delta S_{eff} = 2N_f\beta \int d^4x\, Q(x)\,;\, Q(x) \equiv \frac{\alpha}{8\pi}\varepsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

At the level of the currents it can be expressed as a quantum-non-conservation of the classically-conserved $U(1)_A$ current:

$$\partial_\mu J^\mu_A \equiv \partial_\mu\Big(\sum_f \bar{\Psi}_f\gamma^\mu\gamma_5\Psi_f\Big) = 2N_f Q(x)$$

This will be discussed in more detail in the seminar that follows