Particules Élémentaires, Gravitation et Cosmologie

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Interactions fortes et chromodynamique quantique I: Aspects perturbatifs

Cours VIII: 19 avril 2005

Symmetries & Anomalies

1. Global vs. local symmetries
2. Classical global symmetries, conserved currents
3. Explicit breaking, accidental symmetries, anomalies
4. Explicit vs. spontaneous symmetry breaking
5. CP & U(1) problems in QCD

1. Global vs. local symmetries

- In QFT we have to make the important distinction between global (rigid) and local symmetries
- Example of former is Lorentz(Poincaré) invariance
- Example of latter are the gauge symmetries (NB. General Relativity can be seen as a gauge theory with G = local Poincaré-group: x -->x'(x))
- Recall: local symmetries mean that some d.o.f. are unphysical => they are sacred (have to be broken « carefully »)
- Global symmetries are not sacred: we are allowed to break them explicitly

- Occasionally symmetries of S_{cl} are broken at the quantum level (i.e. in S_{eff}). One talks about an «anomaly»
- We have seen an example all along: Classical-CD contains no dimensionful parameter, but QCD contains a scale, Λ: one talks indeed about a scale-invariance (or trace) anomaly, anomalous dimensions etc.
- Anomalies are acceptable in global symmetries
- Anomalies unacceptable in local symmetries (an important constraint on the standard model and its possible extensions)

2.1 Classical global symmetries

- Recall (again from lecture 1) the general definition of a gauge theory (without scalar fields for simplicity)
- Besides specifying the gauge group G we had to assign all our l.h. fermions to some reps. of G (the r.h. antiparticles will be automatically in the c.c. reps.)
- Suppose that there are N_1 l.h. fermions in the rep. r_1, N_2 in r_2 etc. and, for the moment, let's not give them masses
- The only fermionic terms in S_{cl} will be of the type

$$\sum_{f_1=1}^{N_1} \bar{\psi}_{f_1}^{(1)} i\sigma^\mu D_\mu^{(r_1)} \psi_{f_1}^{(1)} + \sum_{f_2=1}^{N_2} \bar{\psi}_{f_2}^{(2)} i\sigma^\mu D_\mu^{(r_2)} \psi_{f_2}^{(2)} + \dots$$

(actually $\psi*$ in the notation of lect. 1)

$$\sum_{f_1=1}^{N_1} \bar{\psi}^{(1)}_{f_1} i\sigma^\mu D^{(r_1)}_\mu \psi^{(1)}_{f_1} + \sum_{f_2=1}^{N_2} \bar{\psi}^{(2)}_{f_2} i\sigma^\mu D^{(r_2)}_\mu \psi^{(2)}_{f_2} + \dots$$

A global phase rotation of any fermion leaves the lagrangian invariant. Even better, any transformation of the form:

$$\psi^{(1)}_f \to U_{ff'}\psi^{(1)}_{f'}\,;\, \psi^{(2)}_f \to V_{ff'}\psi^{(2)}_{f'}\,,\, UU^\dagger = VV^\dagger = 1$$

leaves L_{class} unchanged.

=> We have identified a large global-symmetry group:

$G_{global} = U(N_1)xU(N_2)xU(N_3)x\dots$

What is G_{global} for QCD (w/ gauge group SU(N)) with N_f massless quarks? It has N_f l.h. quarks in the F-rep. and N_f in its c.c. rep. F*. Thus the classical symmetry is

$G= U(N_f)_F xU(N_f)_{F^*}$ (more traditionally called $U(N_f)_L xU(N_f)_R$)

2.2 Conserved Currents

A general theorem by (Mme)Noether tells us that we can associate to any symmetry a conserved current $J^{(i)}_\mu$:

$$\partial_\mu J^{(i)}_\mu = 0 \Rightarrow \frac{d}{dt}\int d^3x J^{(i)}_0 = \dot{Q}^{(i)} = 0$$

In our case these currents are easily identified. In QCD:

$$J^{(L)a}_\mu \equiv J^{(F)a}_\mu = \bar{\psi}_f \sigma_\mu T^a_{ff'} \psi_{f'}$$

where $T^a_{ff'}$ is hermitian,

and similarly for $J^{(R)}_\mu = J^{(F^*)}_\mu$ In Dirac notation they are:

$$J^{(L,R)a}_\mu = \bar{\Psi}_f \gamma_\mu (1 \pm \gamma_5) T^a_{ff'} \Psi_{f'}$$

Let us split the N_f^2 matrices $T^a_{ff'}$ into the (N_f^2-1) traceless ones and the unit matrix, corresponding to the decomposition:

$G = U(N_f)_L x U(N_f)_R = [SU(N_f)_L x\ U(1)_L] x [SU(N_f)_R\ x\ U(1)_R]$

3.1 Explicit classical symmetry breaking and accidental symmetries

- What happens when we add (quark) masses?
- Recall that mass terms involve two fermions of the same handness (and cc reps.). In QCD the most general mass term (using reality of the action) is (sum over f, f' !)

$$L_m = -\psi_f^{(N)} m_{ff'} \psi_{f'}^{(N^*)} + c.c$$

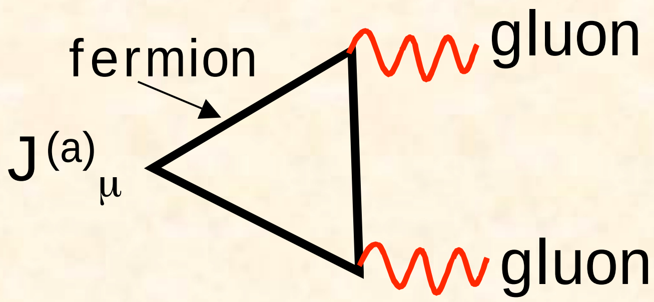
- In general this terms breaks badly the global symmetry leaving just a small unbroken subgroup (see below)
- If, however, $m_{ff'} = m\delta_{ff'}$ it is quite clear that a full $U(N_f)_V$ subgroup of $G = U(N_f)_F \times U(N_f)_{F^*}$ is preserved ($V = U^+$)
- This is nothing but the isospin symmetry ($N_f = 2$) or the Gell-Mann-Ne'eman $SU(3)_{flavour}$ symmetry for $N_f = 3$.

- These symmetries looked very fundamental in the sixties.
- At first sight their presence in QCD seems very unlikely. Why should $m_{ff'} = m\delta_{ff}$?
- One can show, however, that even if $m_f/m_{f'} \neq 1$, the $U(N_f)_V$ symmetry becomes very good as $m_f \to 0$ ($m_f/\Lambda \to 0$)
- Thus the existence of an approximate $U(N_f)_{flavour}$ symmetry is a mere consequence of having N_f light quarks on the scale of QCD, Λ.
- One talks about accidental (i.e. not at all fundamental) symmetries of the strong interactions, i.e. of symmetries which are there because of the terms we can possibly write down in the lagrangian, but do not require any precise fine-tuning of parameters.

3.2 Explicit quantum symmetry breaking: anomalies

In the late 60's Adler Bell and Jackiw found a very puzzling result, known today as the ABJ anomaly:

Even in the absence of masses some global classical symmetries are broken at the quantum (loop) level. The effect is due to a «triangle» graph with fermions circulating in the loop. The one relevant to QCD is as shown in the figure (the original one was in QED and it's crucial to account for $\pi^0 \to 2\gamma$)



Since the gluon is blind to flavour this diagram is proportional to $Tr(T^a_{ff'})$ and does not contribute to SU(N) currents. It also cancels if we take the right combination of $U(1)_L x U(1)_R$ called $U(1)_V$, corresponding to $(q\text{-}q^*)_L+(q\text{-}q^*)_R$ (i.e. baryon) number. But $(q+q^*)_L-(q+q^*)_R$ is no longer conserved (e.g. creation of $(q+q^*)_L$ or $(q+q^*)_R$ from the vacuum)

The anomaly is best defined as a symmetry of S_{class} which is not a symmetry of the effective action S_{eff} (the one that includes quantum corrections). Under a $U(1)_A$ transformation:

$$\psi_f^{(N)} \to e^{i\beta}\psi_f^{(N)}\,;\,\psi_f^{(N^*)} \to e^{i\beta}\psi_f^{(N^*)}$$

the effective action changes by:

$$\delta S_{eff} = 2N_f\beta \int d^4x Q(x)\,;\, Q(x) \equiv \frac{\alpha}{8\pi}\varepsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

At the level of the currents it can be expressed as a quantum-non-conservation of the classically conserved $U(1)_A$ current:

$$\partial_\mu J^\mu_A \equiv \partial_\mu\left(\bar{\Psi}_f\gamma^\mu\gamma_5\Psi_f\right) = 2N_f Q(x)$$

The symmetry and the conservation of the corresponding current are apparently lost…but the story is not yet over.

4.1 Explicit vs. spontaneous symmetry breaking

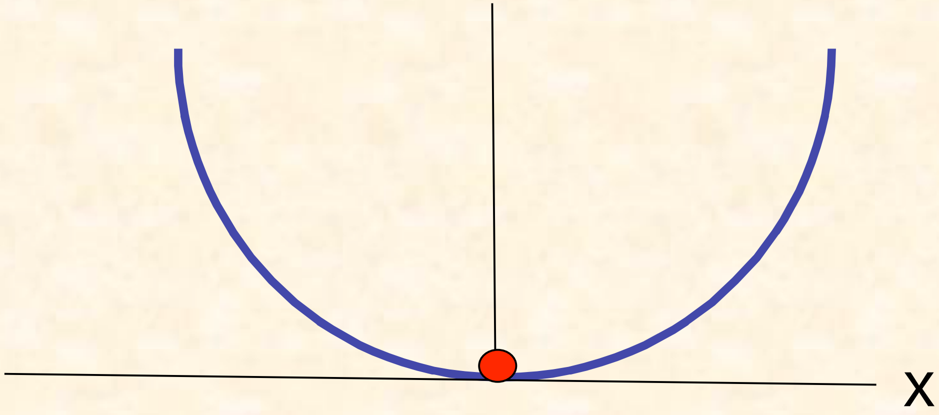
- Another very basic distinction is that of explicit breaking (what we have discussed so far both at the classical and at the quantum level) and spontaneous breaking

- In the former case the (effective) action is not invariant and the current is not conserved.

- In the latter case the opposite is true. One should not talk about symmetry breaking but of a hidden (or secret) symmetry. The « breaking » is due to the non-invariance of the ground state under the spontaneously broken symmetry transformations.

Some illustrative examples

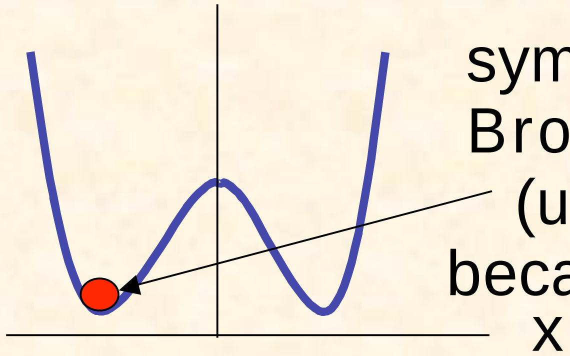
1. Symmetric potential with one global minimum



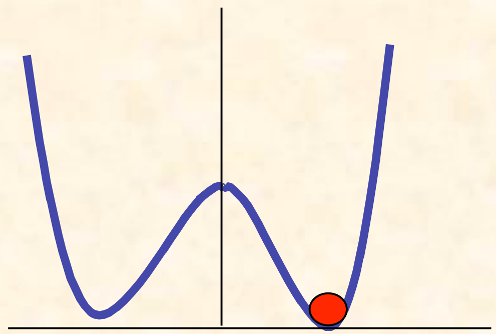
symmetry is x -> -x
(discrete = Z_2)

Neither explicit nor spontaneous SB

2. A double well potential in Classical and Quantum Mechanics



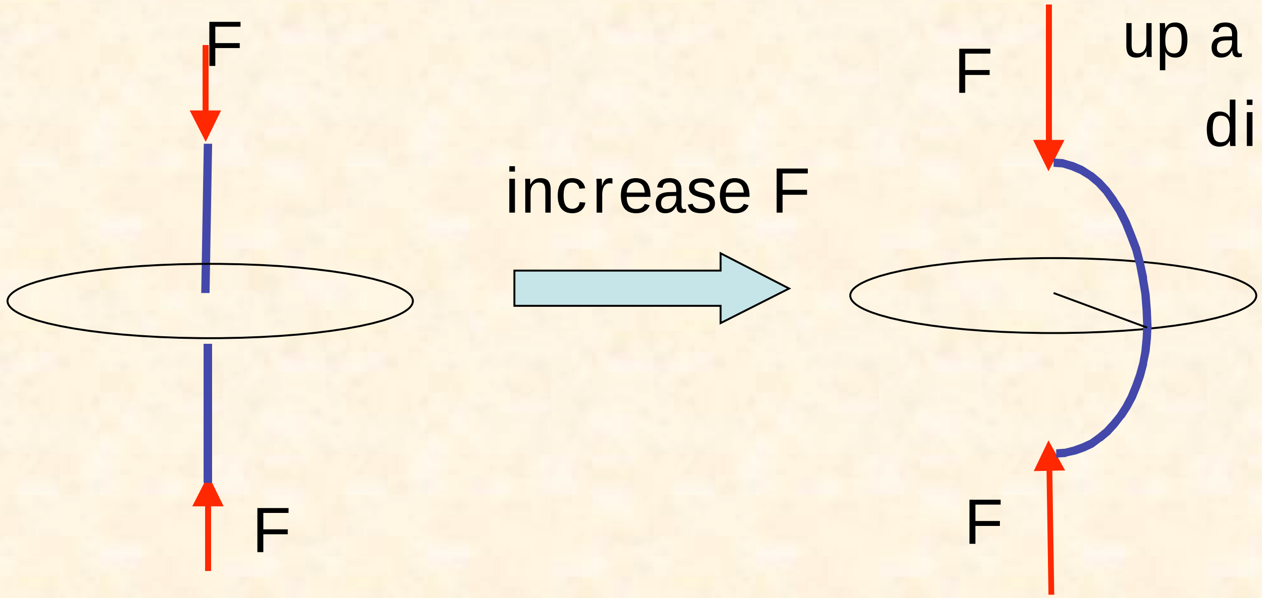
symmetry is x -> -x
Broken spont. in CM
(unbroken in QM because of tunneling)



explicit breaking

3. Continuous symmetry: O(2) ~ U(1)

SSB: the stick picks up a particular direction



4. A larger continuous symmetry: SO(3) ~ SU(2)

Material making a phase transition from antiferromagnet to ferromagnet: in the latter situation the rotation symmetry is broken by the direction of the magnet's magnetic field.
An SO(2) survives = rotation around the axis of the (arbitrarily) chosen direction

4.2 SSB and NG-bosons

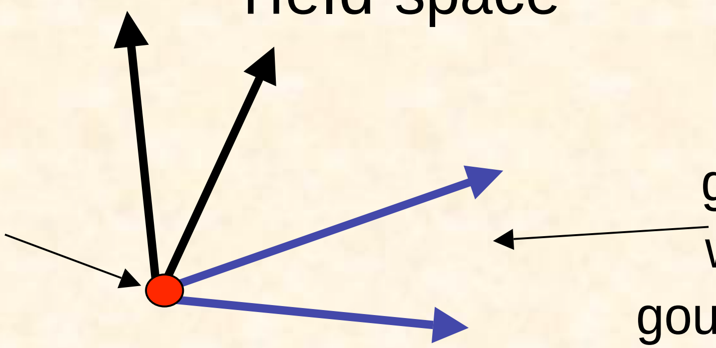
Goldstone's theorem: If a continuous global symmetry G is spontaneously broken to a subgroup H, there must be as many massless bosons (called Nambu-Goldstone bosons) as there are generators of G that are not in H.

In formulae: (Number of NG-bosons) = dimG-dimH

The formal proof is a bit techical, but the physics behind is quite clear:

field space

A particular gound state



The directions in which the ground state moves under G/H while remaining a degenerate gound state: each one corresponds to a massless excitation

Goldstone's thrm evaded if broken symmetry is local. This is actually the only consistent way to break a gauge symmetry & to give mass to gauge bosons & chiral fermions (EW theory)

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NG-bosons in QCD

There is both theoretical and experimental evidence that, at T=0, the ground state of QCD, for N_f massless quarks, is degenerate. It is characterized by a BEC consisting of qq* pairs (Cf. BCS theory of superconductivity):

$$\langle 0|q_f q_{f'}^*|0\rangle = c\,\Lambda^3\delta_{ff'}$$

=> We know already what H is: our $U(N_f)_V$ of the equal mass case (NB: math. Is the same, physics very different!)

By the Goldstone theorem we would expect $2N_f^2 - N_f^2 = N_f^2$ massless pseudoscalar (J=0, P=-1) bosons. In practice, for non-zero quark masses, they should be light (pseudo NG bosons).

But what about the anomaly?

5. Strong-CP & U(1) problems in QCD

Apparently unrelated, actually connected as we shall see

1. Strong CP problem

It's the question of whether QCD has another «accidental» symmetry, CP (changing each l.h. particle in its r.h. antiparticle). The mass term appears to break CP unless $m_{ff'}$ is real. The problem, experimentally, is that even a very small CP violation would induce an unacceptably large electric dipole moment of the neutron.

2. U(1) problem

Naively one would expect either 4 very light NG bosons (if only u & d quarks are considered light) or 9 (if also the s quark is considered light). Experimentally one observes either 3 or 8…

5.2 Strong-CP problem

Since physics is invariant under field redefinitions we can ask whether we can bring the mass matrix to a real form by redefining the fermionic fields while keeping the kinetic terms unchanged (they already conserve CP).

Let us start by performing an $SU(N_f)_F \times SU(N_f)_{F^*} \times U(1)_V$ transformation on the fermion fields.

It is easy to show that such a transformation can bring $m_{ff'}$ to a diagonal form, $m_{ff'} = m_f \, \delta_{ff'}$ but with complex m_f. Further transformations can eliminate all but one phase $\beta = \arg(\det m)$, (e.g. we can give to each mass the same phase) since β is invariant under an $SU(N_f)_F \times SU(N_f)_{F^*} \times U(1)_V$ transformation

- In order to get rid of this last «complexity» of $m_{ff'}$ we would need to perform a $U(1)_A$ transformation, but that's where the ABJ anomaly stops us. If we insist on doing that, the classical action would become CP invariant, but the effective action picks us a term β Q as we have already mentioned
- This terms beaks CP. But does it really? And why did we not add it from the start to the QCD Lagrangian as a term θ Q? (ending with all CP violation concentrated in a $(\beta+\theta)$ Q term)
- The tricky point is that Q can be written itself as the divergence of a current K_μ and normally we neglect such total derivatives since they do not contribute to the field equations.
- If we do that, our strong CP problem is automatically solved in QCD (early claim by S. Weinberg).
- But then what happens to the U(1) problem?

5.3 U(1) problem

There is an obvious way out to the problem: the true symmetry is not G= $U(N_f)_F x U(N_f)_{F^*}$. Because of the ABJ anomaly it is:

$$SU(N_f)_L x SU(N_f)_R x U(1)_V$$

Since H = $U(N_f)_V$ this obviously reduces by 1 the number of NG bosons in agreement with the data

However we cannot have the cake and eat it!

•The ABJ anomaly only solves the U(1) problem if we «forget» that Q(x) is itself a divergence, otherwise we can redefine a new $U(1)_A$ conserved current, using K_μ, and the extra NG boson is needed.

•But then we cannot eliminate CP violation...

The end of this story (as it's believed by most people at present) is interestingly complicated:
The current K_μ is not gauge invariant => the integral of $Q(x)$ over space-time is not trivial (topological charge, instantons..)

Full discussion needs tools and ideas that belong to non-perturbative QCD and that we have not developed yet.

The bottom line appears to be the naive one:

- The ABJ anomaly does solve the U(1) problem & explains the masses and mixing angles of the 9 known pseudoscalars;
- The absence of CP violation is either an accident, or is due to $m_u=0$ (which looks phenomenologically excluded), or implies a new very weakly coupled particle, the axion (not yet discovered but one of the leading candidates for dark matter!)

One of the subjects to be discussed next year!