Particules Élémentaires, Gravitation et Cosmologie

Année 2005-2006

Interactions fortes et chromodynamique quantique

II: aspects non-perturbatifs

Cours VII: 28 mars 2006

Confinement et spectra «quenched»

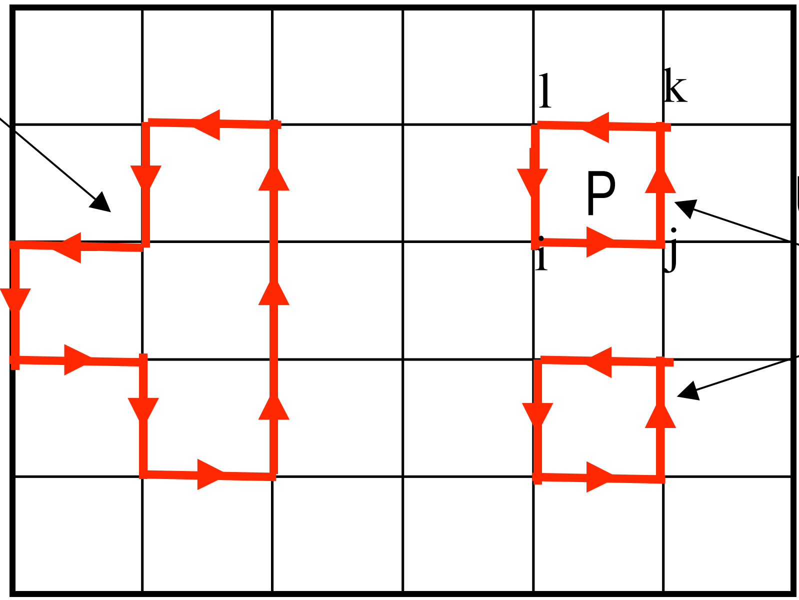
- Short reminder of previous lecture
- Weak & strong-coupling expansions:
  - Confinement at strong coupling
- Continuum limit, phase transitions (U(1) vs SU(N))
- Evidence for confinement, string tension
- Quenched correlators and spectra:
  - Glueballs, mesons, baryons

Short reminder of previous lecture

- We have introduced the idea of discretizing space and (possibly Euclidean) time accepting to lose some global symmetries while preserving gauge invariance
- We have introduced some basic lattice quantities: links, plaquettes, and Wilson's action

$$-\frac{1}{g^2}\sum_P \left(Tr\left[U_P + U_P^\dagger\right] - 2N\right) \rightarrow \frac{a^4}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

- We have introduced the Wilson loop and discussed a confinement criterion based on it
- We mentioned weak & strong coupling expansions



$U_P = Tr[U_{ij} U_{jk} U_{kl} U_{li}]$

plaquettes

Weak and strong coupling expansions

Starting from Wilson's action we can consider two limits where we can perform analytic expansions/calculations :

1. Weak-coupling expansion

Very similar to the loop expansion (perturbation theory) of the continuum and, of course, with the same limitations. Actually more complicated, since it does not respect many of the continuum symmetries: as a result one gets all sorts of terms that are forbidden by Lorentz and/or traslation invariance. Its main (and important) use is that it allows to connect the lattice parameters to those in the continuum and thus to observables that can be computed perturbatively (jets, DIS etc.). Example: relate Λ_{PQCD} to Λ_{LQCD}

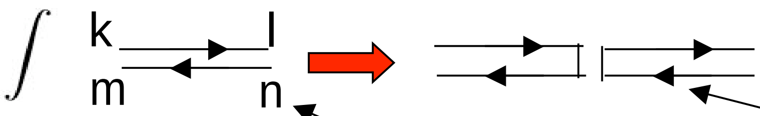
Strong coupling expansion

This has no continuum analogue. The surprise is that it automatically leads to colour confinement (even for a U(1) gauge symmetry!).

Let us discuss how and compute the string tension in this limit
At strong coupling the functional integral can be performed by expanding exp(-S_W) in powers of $1/g^2$ and by then doing the (elementary) group integrals.

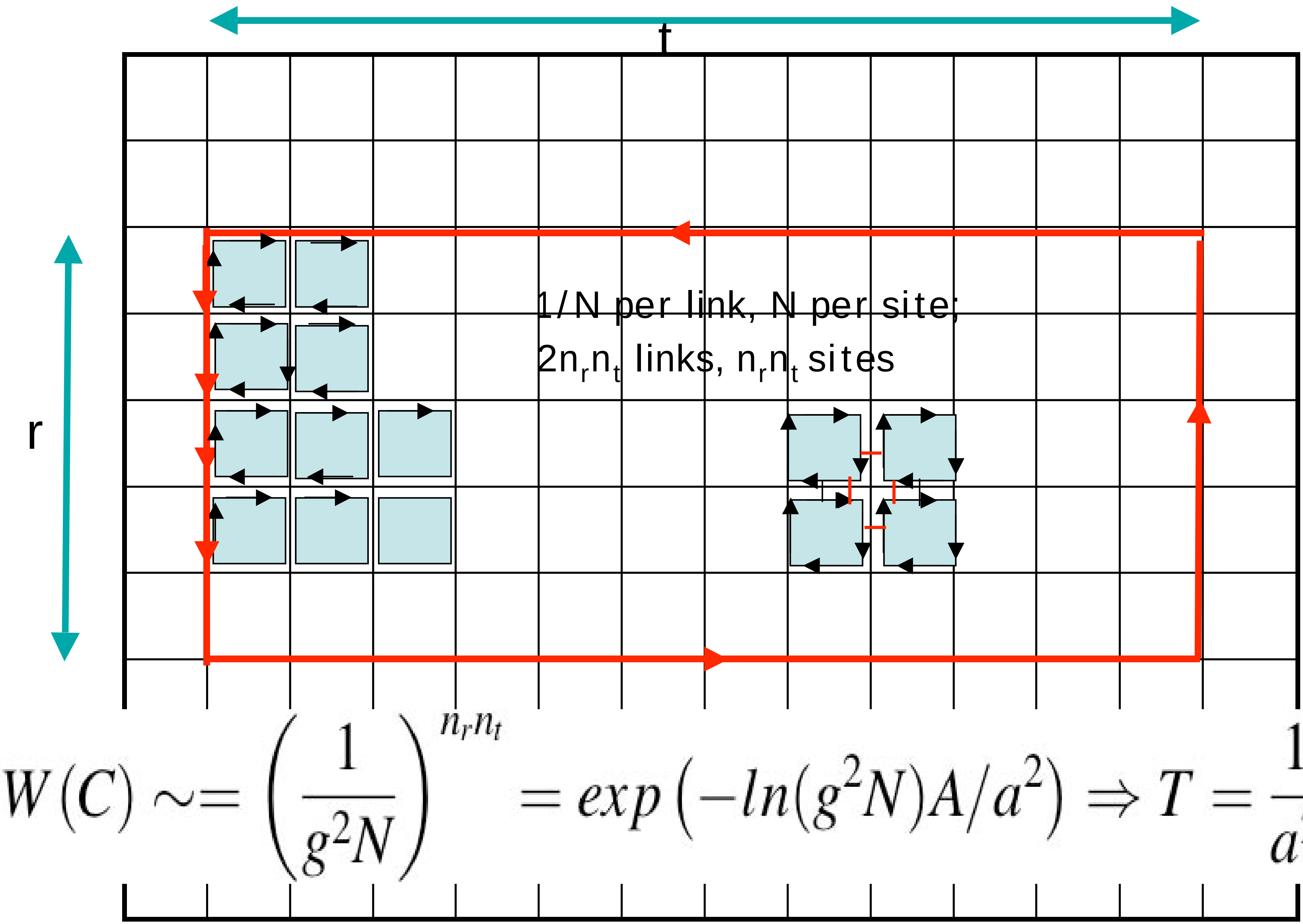
$$e^{\frac{1}{g^2}\sum_P \left(Tr\left[U_P+U_P^\dagger\right]-2N\right)} = \prod_P \left(1+\frac{1}{g^2}\left(Tr\left[U_P+U_P^\dagger\right]-2N\right)\right)$$

For a given link, and denoting by k, l, m, n the colour indices:



$$\int dU = 1\ ,\ \int dU\, U_{kl} = 0\ ,\ \int dU\, U_{kl} U^\dagger_{nm} \sim \frac{1}{N}\delta_{ln}\delta_{km}$$

Strong-coupling expansion for a planar Wilson loop



$$W(C) \sim= \left(\frac{1}{g^2N}\right)^{n_r n_t} = exp\left(-ln(g^2N)A/a^2\right) \Rightarrow T = \frac{1}{a^2} ln(g^2N)$$

Are there phase transitions on the way?

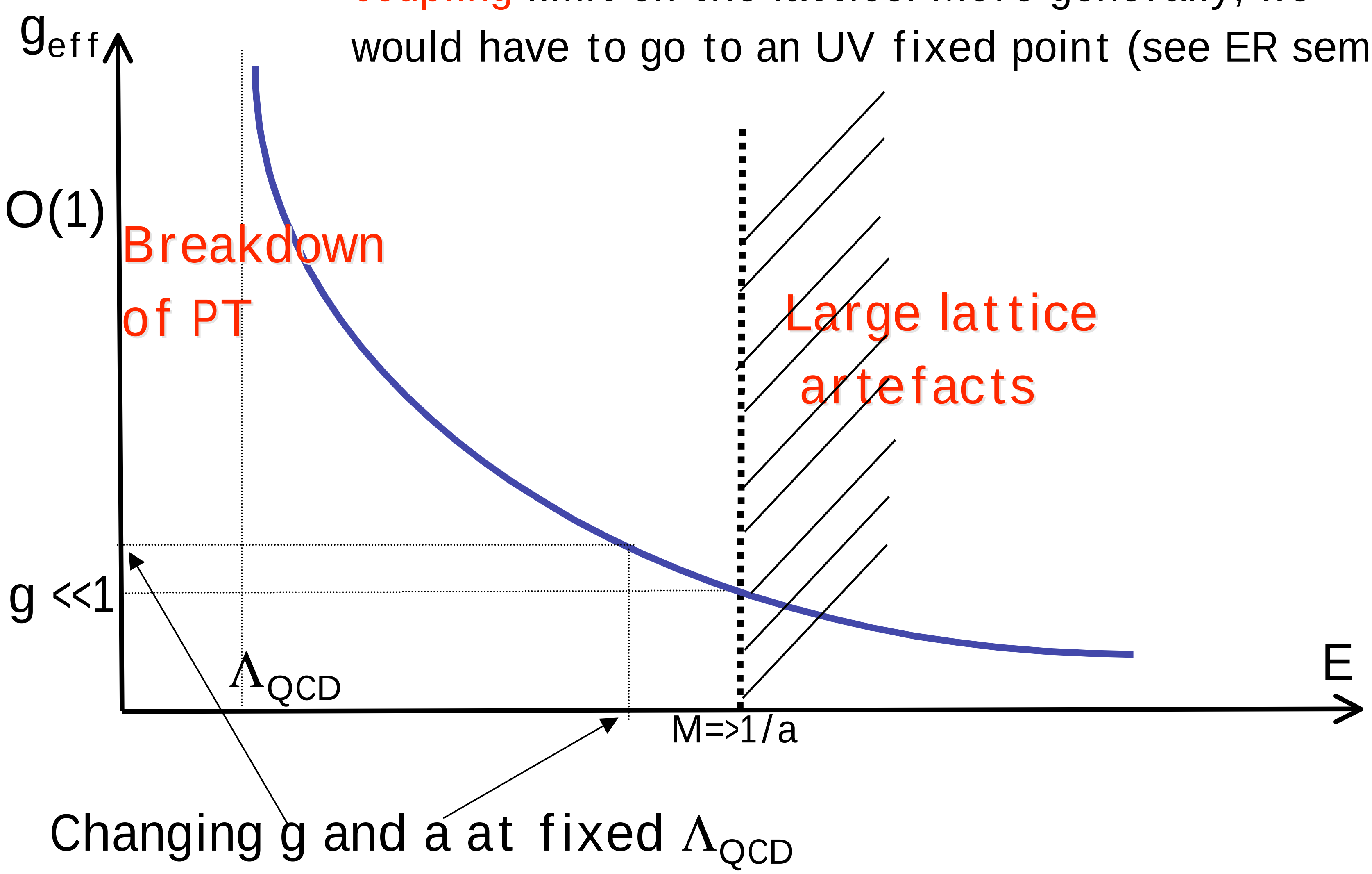
The strong coupling expansion automatically gives confinement. Can we then consider the problem as being solved? Obviously not! Confinement can be an artefact of the lattice regularization or of the particular choice of the action. Indeed, if we believed the s.c. expansion, we would conclude that even a QED-like theory confines! We need to understand what happens in the continuum limit..

The continuum limit, asymptotic freedom

- How does one actually take the continuum limit in LQCD?
- Physically one would like to make the mesh as fine as possible in some physical length unit, say Λ_{QCD}^{-1} (~ 1 Fermi = 10^{-13} cm.). In other words there should be many lattice points inside a 4-volume $V_4 \sim \Lambda_{QCD}^{-4}$
- On dimensional grounds, a physical (glueball) mass (or Λ_{QCD} itself) should be of the form (1/a) f(g) where g^{-2} is the constant we have put in front of the Wilson action. That constant is nothing but the physical coupling at distances of order a, the lattice spacing. Dimensional transmutation relates Λ_{QCD} to the UV cutoff 1/a and to g:

$$\Lambda_{QCD} = a^{-1} exp\left(\frac{2\pi}{\beta_0 g^2}\right)(g^2)^{-\beta_1/\beta_0^2}, \; \beta_0 = -\frac{1}{12\pi}(11N - 2N_f)$$

We want to study QCD at strong coupling and large distance but we need to take a small coupling limit on the lattice! More generally, we would have to go to an UV fixed point (see ER sem.)



$$\Lambda_{QCD} = a^{-1} exp\left(\frac{2\pi}{\beta_0 g^2}\right)(g^2)^{-\beta_1/\beta_0^2}, \ \beta_0 = -\frac{1}{12\pi}(11N - 2N_f)$$

Neglecting β_1, we see that, if we want to keep Λ_{QCD} fixed as we change the cutoff, we have to let the coupling g go to zero according to

$$g^2 \sim \frac{2\pi}{\beta_0 ln(a\Lambda_{QCD})}$$

Thus the continuum limit of LQCD corresponds to taking the small coupling limit in a very precise way. A check that we are approaching the continuum limit is that, at small g, physical quantities depend on g in the non-perturbative way predicted by AF («scaling»). In that case we can associate, to each value of g, a value for the lattice spacing and go to $(a\,\Lambda_{QCD}) \ll 1$.

Let us see what happens to our «proof» of confinement at large-N. Since

$$(-\beta_0)g^2 \sim \frac{2\pi}{ln(1/a^2)} \,, \; -\beta_0 = \frac{11N}{12\pi} + \ldots$$

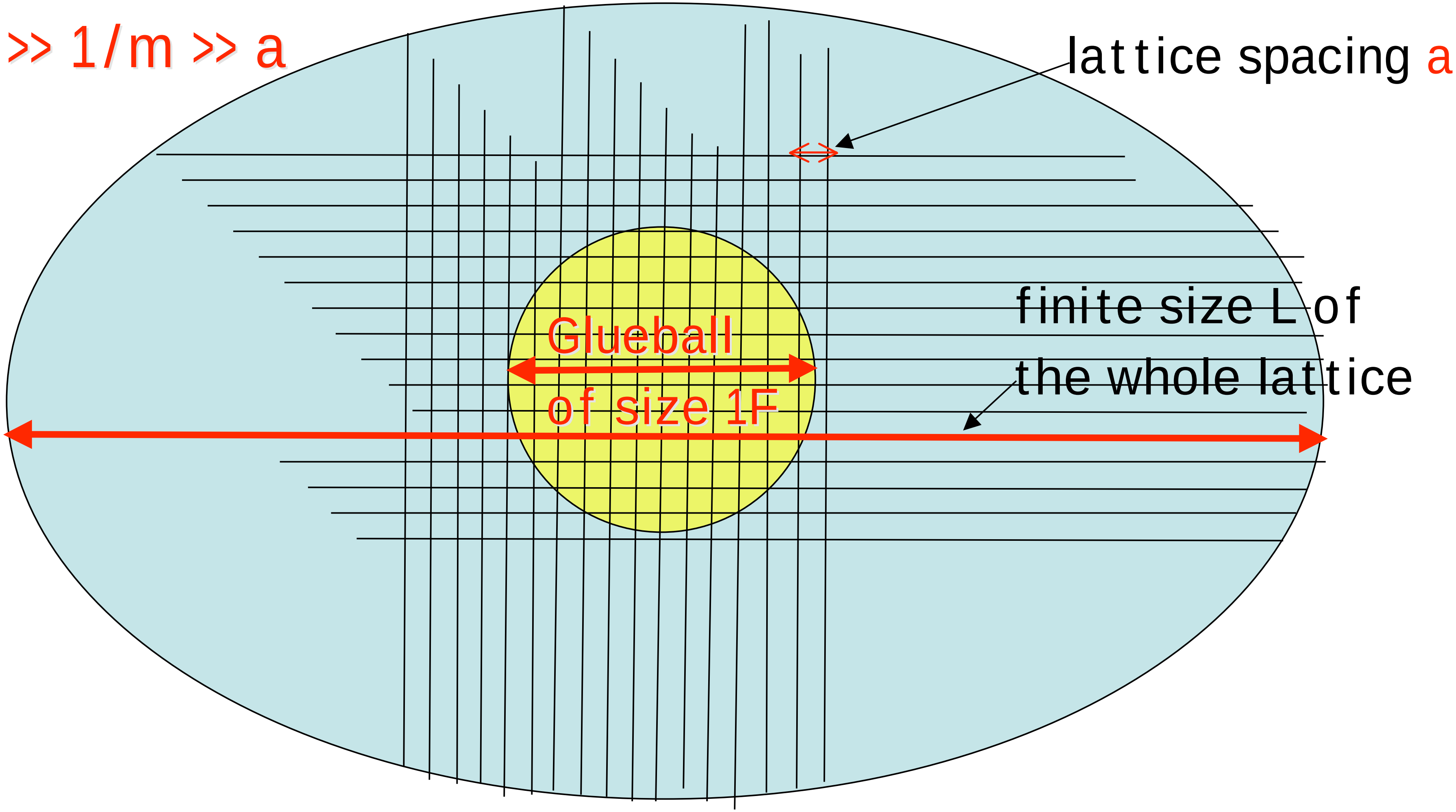
g^2N itself has to be taken small. Thus, even in the large N limit, we cannot rely on the SC expansion!

As we move from the strong coupling limit towards smaller and smaller coupling, surfaces of non-minimal area start contributing to the Wilson loop. They are penalized by a larger action (S ~ A) but they are favoured by a larger entropy (more non minimal than minimal-area surfaces). The questions are:

1. Is the area law holding true all the way as g approaches 0?
2. Is the tension T changing its g-dependence from log g^2 to $\exp(-cg^{-2})$ (with $c = -4\pi/\beta_0$) as required by AF?

Since we also want the total volume to be >> Λ_{QCD}^{-4} this implies many lattice points, i.e. a lot of variables to be integrated over. That's where LQCD meets its biggest challenges (large dedicated computers and ingenious algorithms needed, progress in computing power helps!)

L >> 1/m >> a

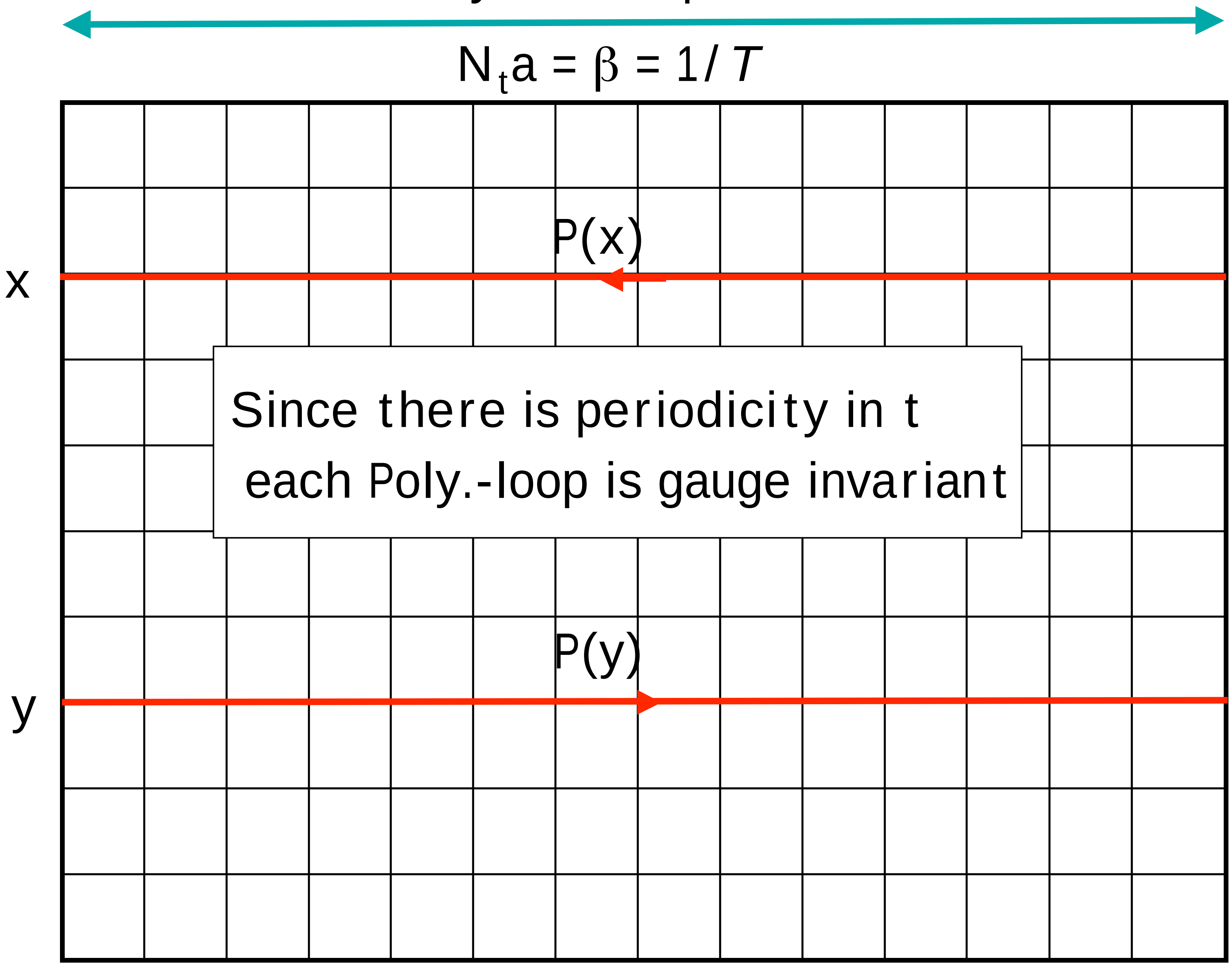


Evidence for confinement, string tension

The first evidence that, in the non-abelian case, there is confinement in the continuum limit goes back to M. Creutz's work (1979)
He showed that, indeed, the area law remains valid throughout the small coupling regime with a string tension that scales according to AF. Since then Creutz's result has been confirmed and improved

Today, the best determination of the quark potential is obtained by considering not a Wilson loop but an interesting variant of it, called the Polyakov loop.

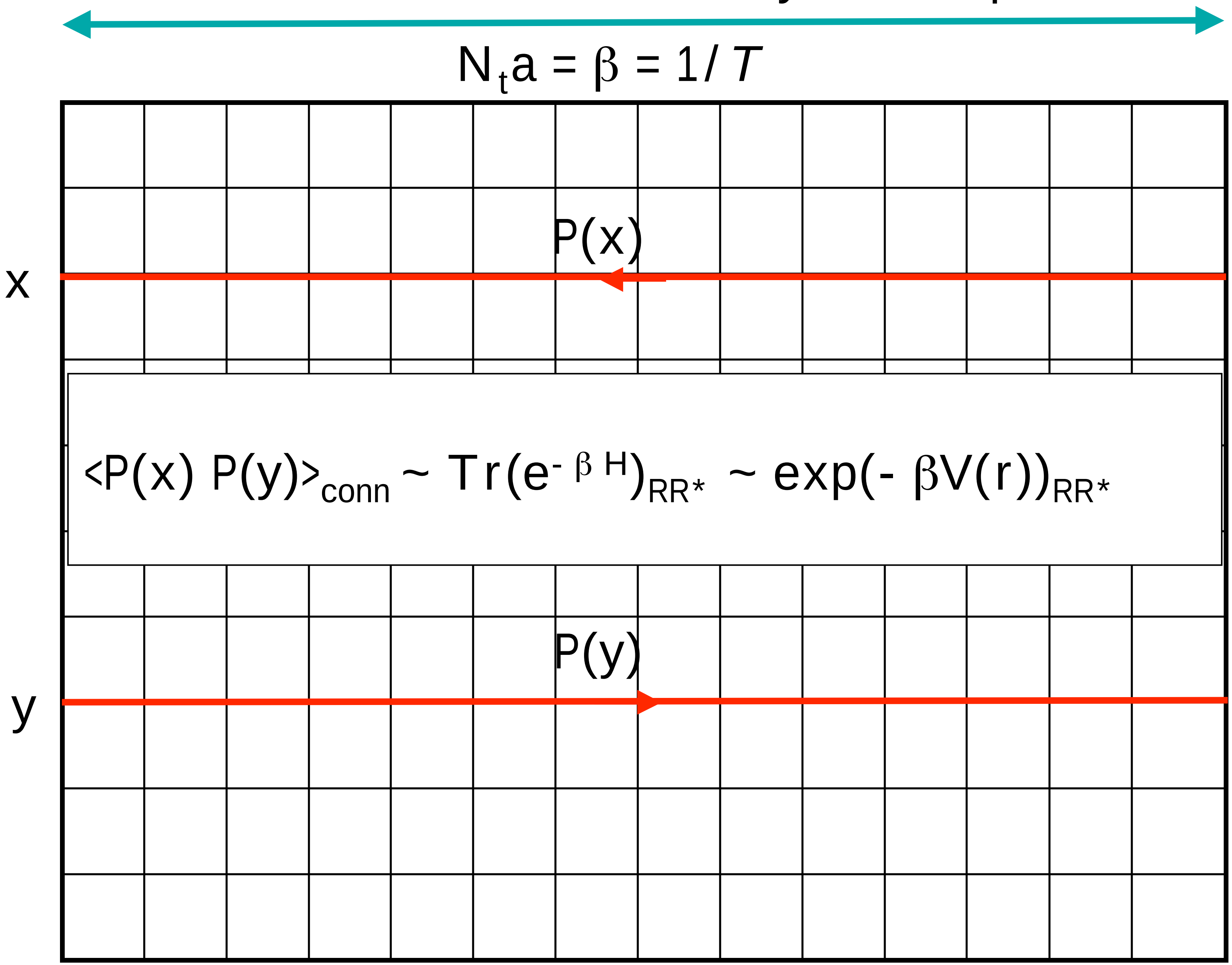
Polyakov loops



Recall that imposing (anti)periodic boundary conditions in compactified imaginary time over a distance β is equivalent to considering the theory at finite temperature $T = 1/\beta$.

$$\exp(-iEt) \dashrightarrow \mathrm{tr}(\exp(-\beta E))$$

Correlator of two Polyakov loops

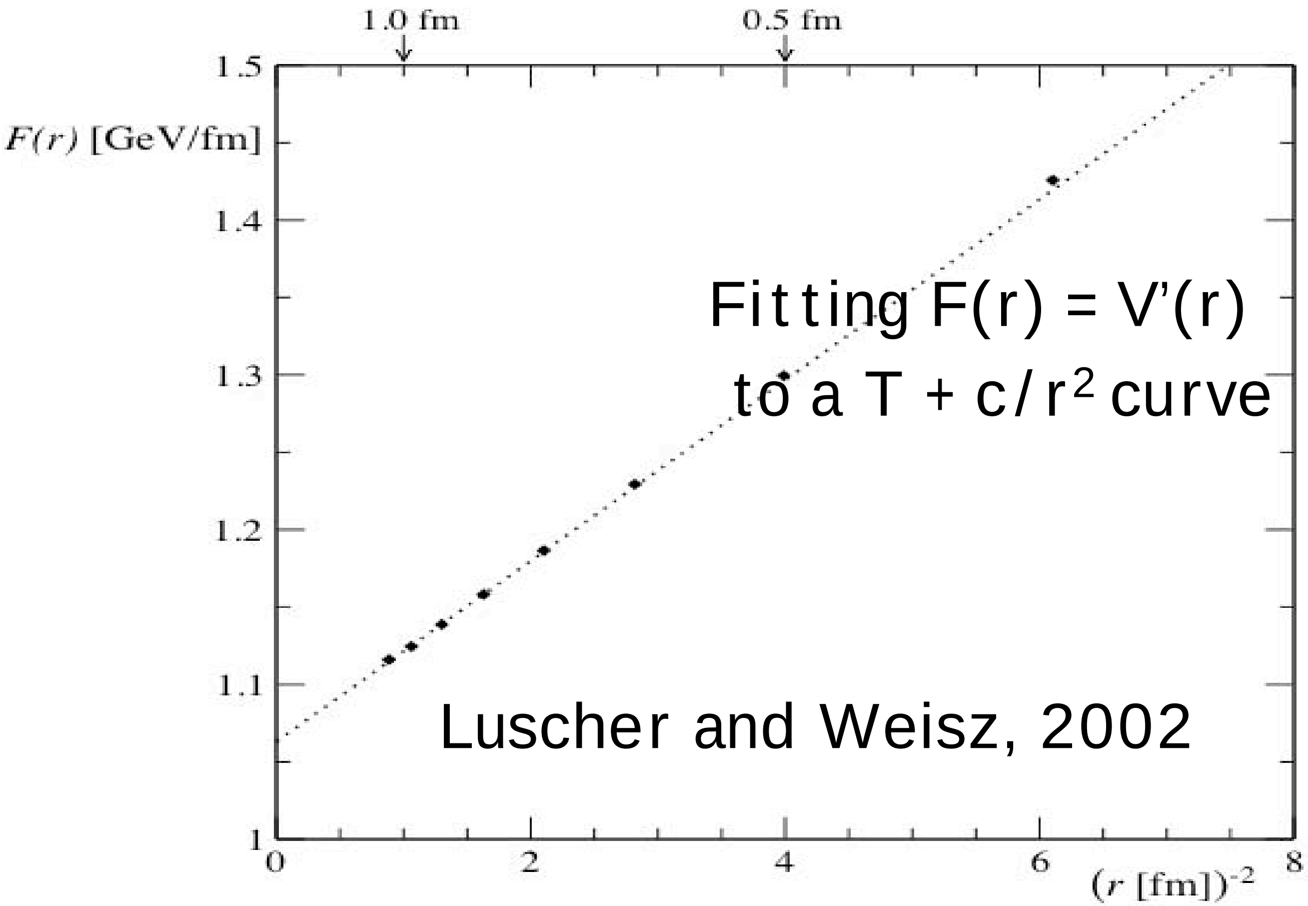


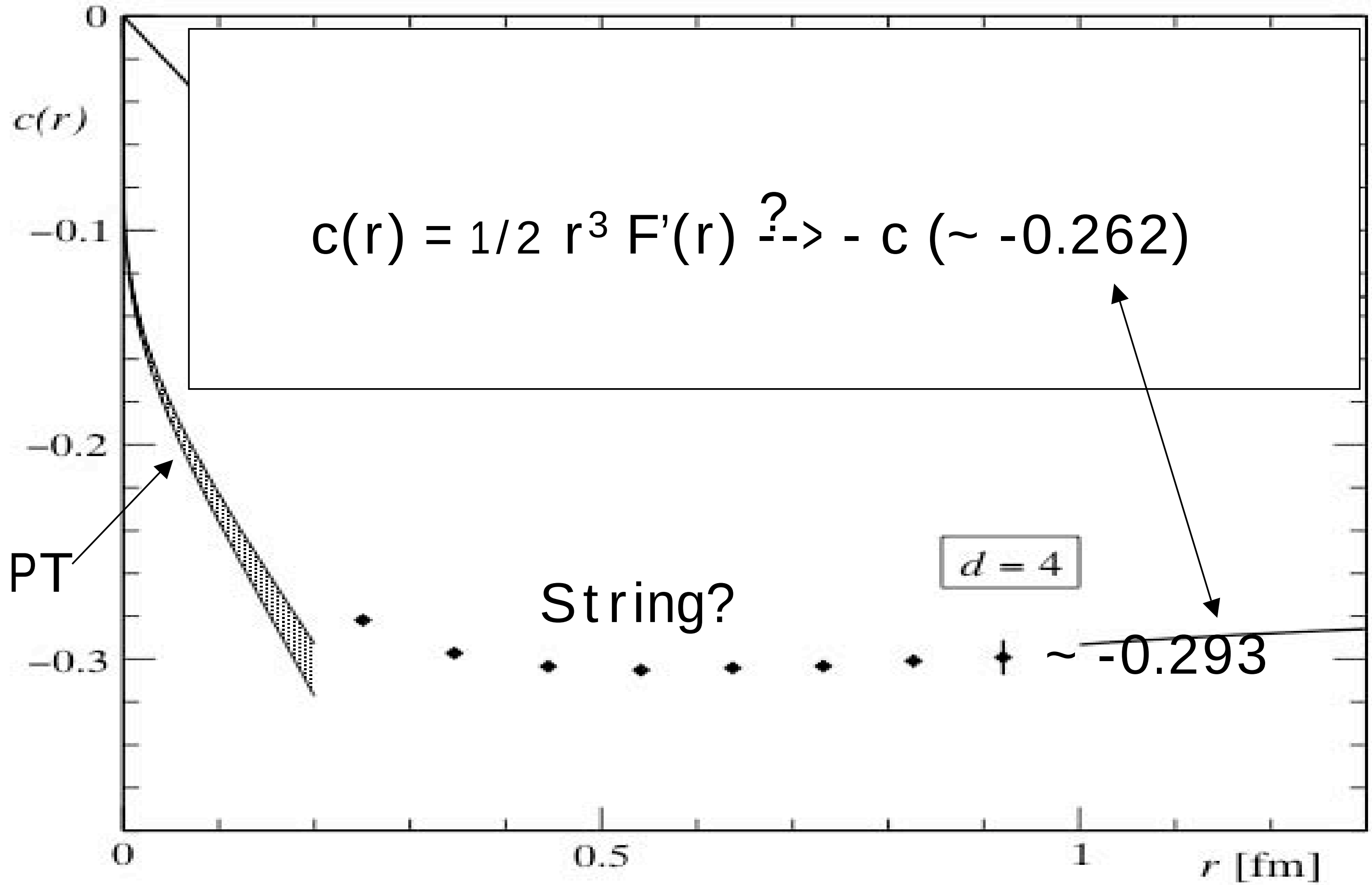
$$\langle P(x)\, P(y) \rangle_{conn} \sim Tr(e^{-\beta H})_{RR^*} \sim \exp(-\beta V(r))_{RR^*}$$

Large-r expectation for a (naive) string theory:

$V(r) = T\,r - c/r$ (with $c = (d-2)\pi/24 \rightarrow \pi/12$)

Luscher's term





c(r) = 1/2 r³ F'(r) ?--> - c (~ -0.262)
PT
String?
d = 4
~ -0.293
c(r)
r [fm]
0
−0.1
−0.2
−0.3
0
0.5
1

Quenched correlations and spectra

In order to compute hadronic masses the standard procedure consists in considering the large-time behaviour of gauge-invariant correlators, in particular two-point functions (higher-point functions will then give couplings etc.)

Consider in fact a 2-point function: $\langle 0|O_i(x,0)\, O_j(y,\tau)|0\rangle$. Using the Hamiltonian to make a time-traslation:

$$\langle 0|O_i(0)e^{-\tau H}O_j(0)|0\rangle = \sum_n \langle 0|O_i(0)|n\rangle e^{-\tau E_n}\langle n|O_j(0)|0\rangle$$

$$= \langle O_i\rangle \cdot \langle O_j\rangle + \sum_{n\neq 0} f_{in} f^*_{jn} e^{-\tau E_n}$$

This is the connected part containing info. about spectrum and coupling to sources

In order to be sensitive to a particular hadron we must make sure that:

1. There is a sizeable coupling between the operator and the hadron
2. Take a large time-interval in order to isolate the lightest states (otherwise the information is blurred). NB very light states need very long times...

Finally, in order to find the masses of particles we have to put them to rest. This implies summing over the spatial location of the operators

Pure Yang-Mills theory

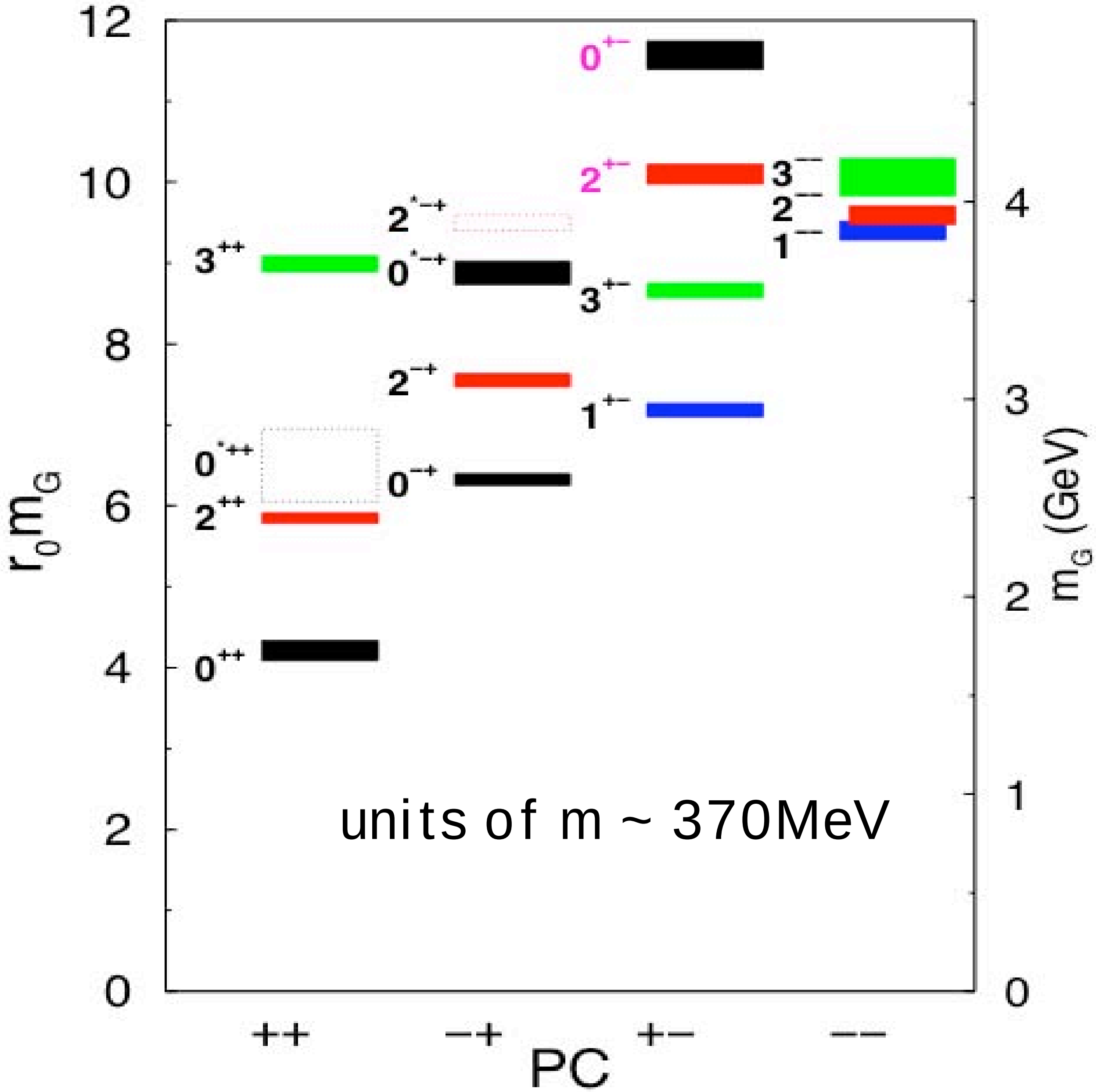
In the case of YM theory (i.e. in absence of quarks) the interesting objects are colour-singlet states made out only of gluons. We have already seen the gauge invariant operators in this case: Wilson-loops, plaquettes, Polyakov loops..

In particular, the analogue of F^2 (a sum over plaquettes) can be used to find the spectrum of glueballs with J=0, C=+1, P=+1 While correlators of some lattice version of FF* can be used to find the spectrum of glueballs with J=0, C=+1, P=-1

In order to extract J=0 states we have to sum over different spatial orientations of the plaquettes (or W-loops). If we do not we are also sensitive to J>0 states

From C. Michael
hep-lat/0101287

The lightest glueball in YM theory is a J^{PC} = 0^{++} state and its mass is about 1.6 GeV. Next comes a 2^{++} state at about 2GeV. Note that the spectrum is very different from the one expected in the qq* sector where, by Golstone's thrm, we expect the lightest states to be the 0^{-+} NG bosons



Quenched QCD

When going from YM to QCD things change in many respects. Not only we expect a much richer spectrum of colour-singlet states (mesons, baryons) but also glueball masses and stability will be affected.

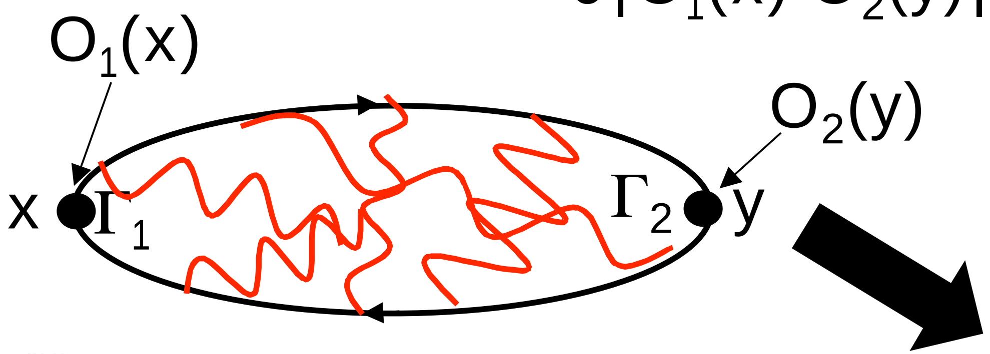
Glueballs will mix to qq* states though diagrams where gluonic states and flavour-singlet qq* states transform into each other. The eigenstates of the Hamiltonian will be linear combinations of qq* and gg states. Furthermore, the lowest $J^{PC} = 0^{++}$ glueball will no longer be stable if its mass is higher that 2 m_{π}.

An interesting limit, called the quenched approximation, is the one in which quark loops are neglected. For glueballs one gets back to YM. And in QCD?

In order to study qq* mesons, for instance, we have to introduce fermions on the lattice. This is a highly non-trivial problem to be discussed in the rest of the course.

However, in the quenched approximation, all we need is the quark propagator from x to y (NB: q.app. is indep. of large N!)

$\langle 0|O_1(x)\, O_2(y)|0\rangle$ $(O_i(x) = \psi^*\Gamma_i\psi\ (x)..)$



In the continuum if

$L \sim \psi^*\ [D(A)+m]\ \psi$

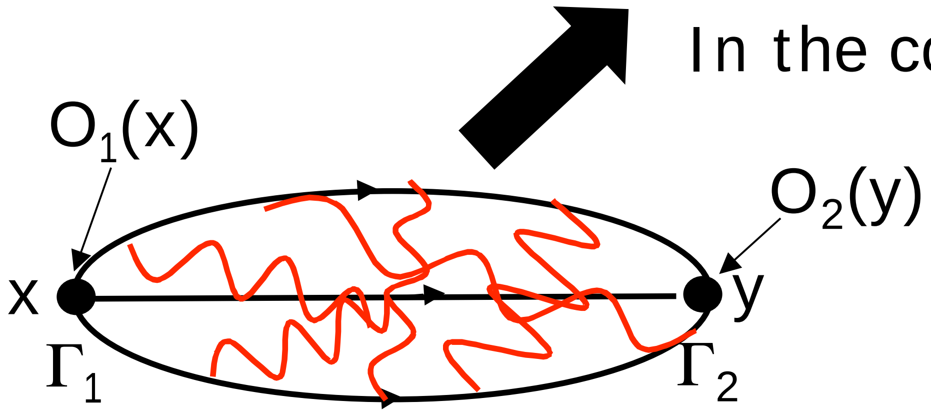
$$\int d[A_\mu] e^{iS_{YM}(A)} Tr\left(\Gamma_1 (D(A)+m)^{-1}_{xy} \Gamma_2 (D(A)+m)^{-1}_{yx}\right)$$

All we need is a lattice analogue of the fermion propagator in an external gauge field

Similarly for qqq baryons

$\langle 0|O_1(x)\ O_2(y)|0\rangle$ ($O_i(x) = \psi\psi\psi\Gamma_i(x)..$)

$$\int d[A_\mu]e^{iS_{YM}(A)}Tr\left(\Gamma_1(D(A)+m)^{-1}_{xy}(D(A)+m)^{-1}_{xy}(D(A)+m)^{-1}_{xy}\Gamma_2\right)$$

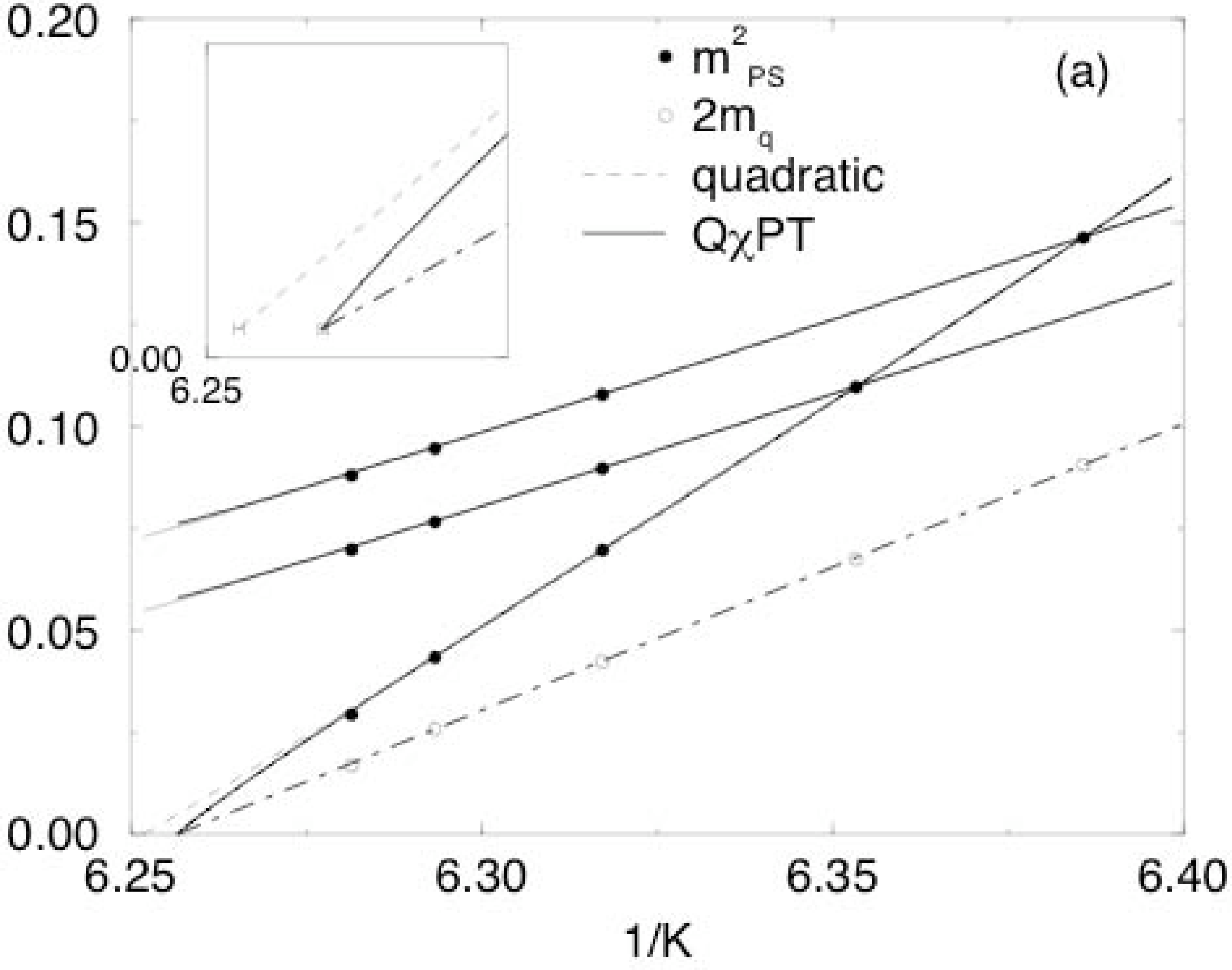


In the continuum

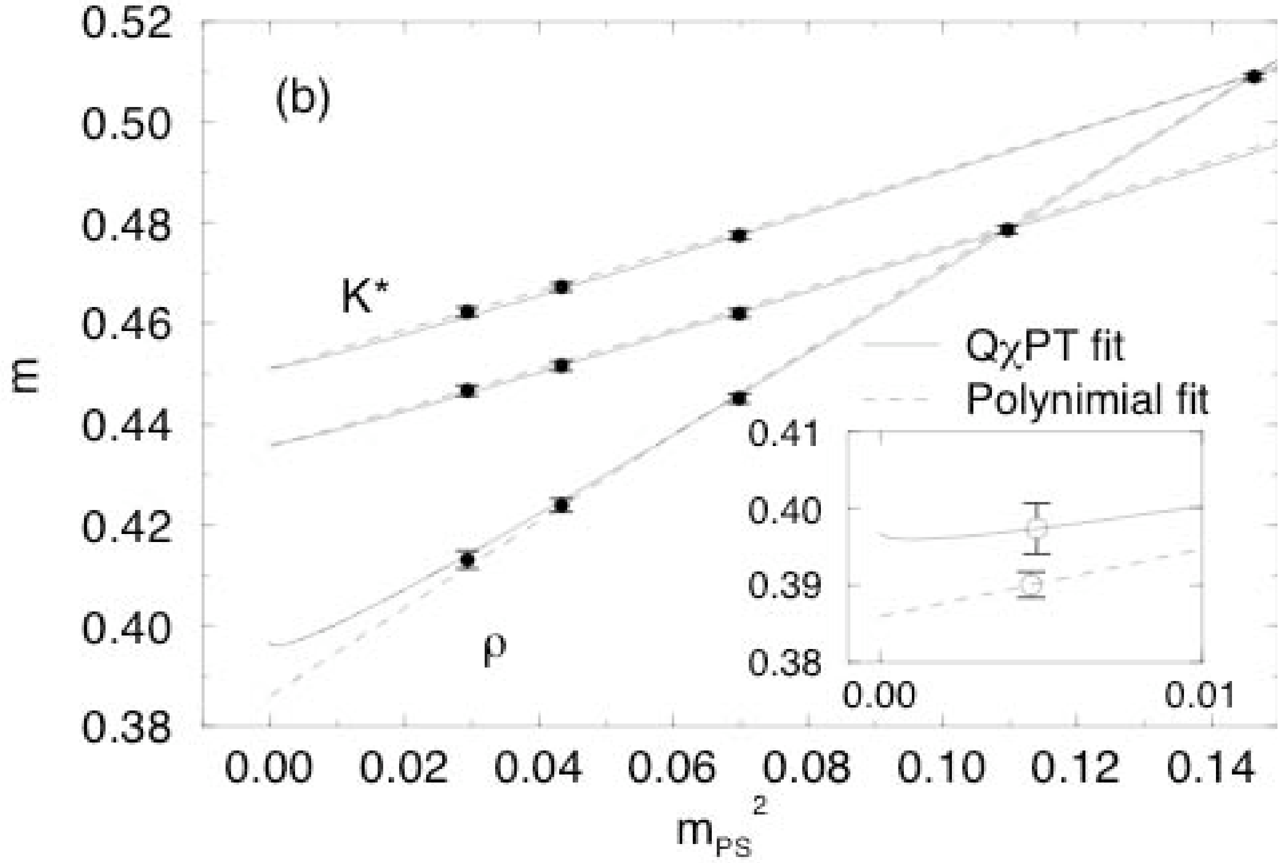
By looking at these correlators at large time separations we can extract the masses of the lightest mesons and baryons.

The following pictures are from: S. Aoki et al.hep lat/0206009

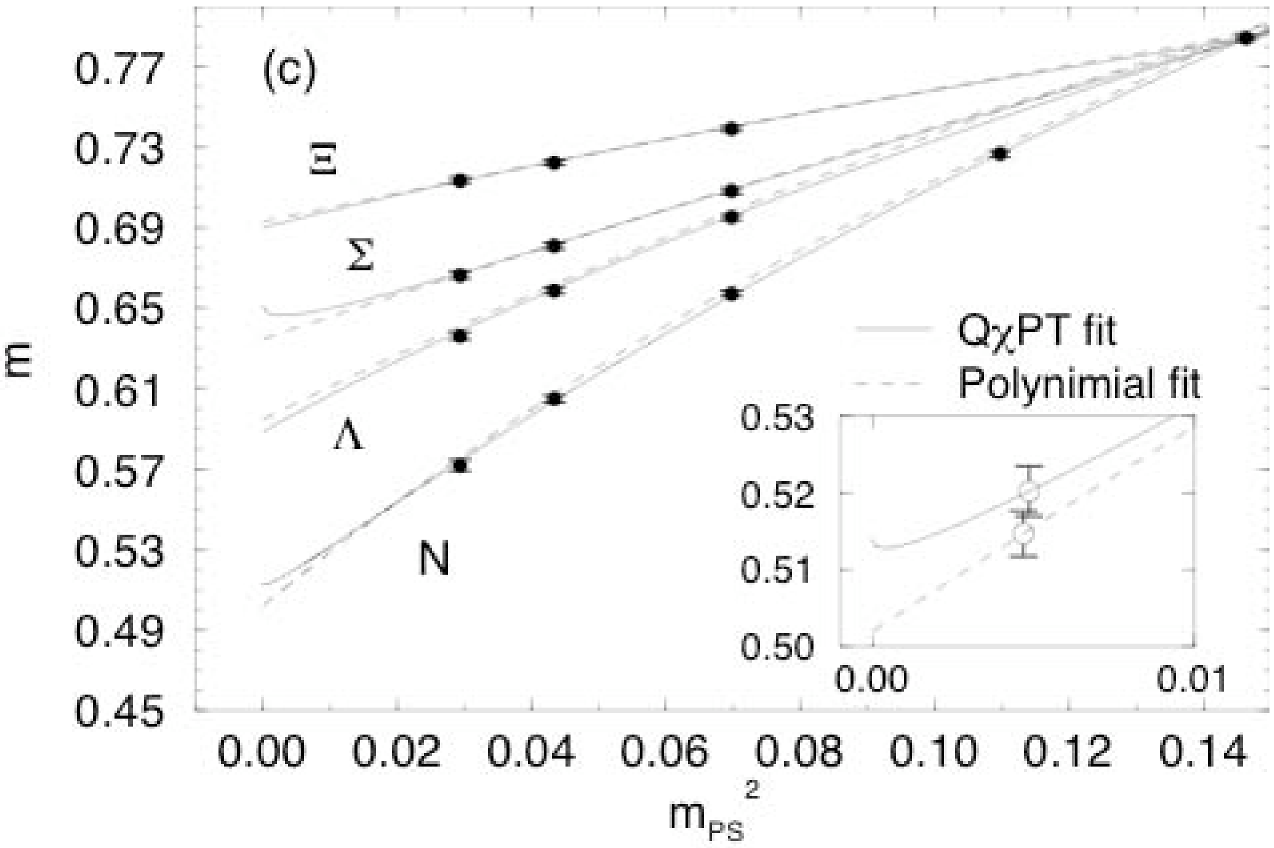
pseudoscalar meson masses



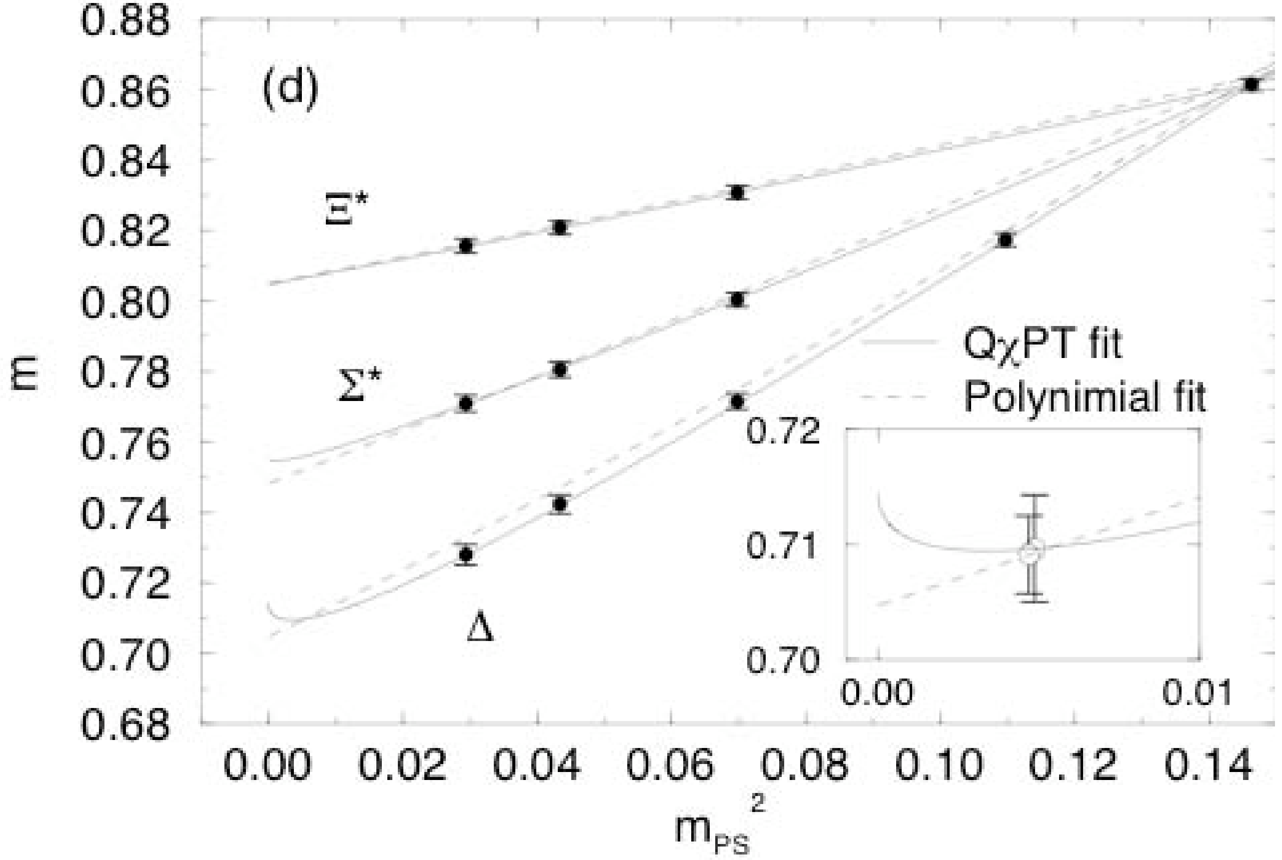
vector meson masses



baryon octet masses



baryon decuplet masses



overall comparison with data