Particules Élémentaires, Gravitation et Cosmologie

Année 2004-2005

Interactions fortes et chromodynamique quantique I: Aspects perturbatifs

Cours VI: 5 avril 2005

1. Summary of previous lecture
2. Polarized DIS: an instructive OPE exercise
3. Small-x physics vs. soft physics
4. Double-scaling limit and BFKL: an introduction

1. Summary of lecture no. 5

- OPE in a generic (renormalizable) QFT:

$O_i(x)\, O_j(y) \dashrightarrow \Sigma_k C_{ij}^{\ k}(z)\, O_k(X);\ z = x-y,\ X = (x+y)/2$

- Need to renormalize composite operators => anomalous dimensions, RGE, power violations of naive scaling become logarithmic in AF theories (QCD)
- OPE in e^+e^---> hadrons (only one operator for leading power of Q^2)
- OPE in DIS (whole towers of operators @ leading order)

$$O_f^{(n)} = S\left[\bar{\Psi}_f \gamma_{\mu_1} D_{\mu_2} \ldots D_{\mu_n} \Psi_f\right],\ f = 1, 2 \ldots N_f$$

n = 1, 2..

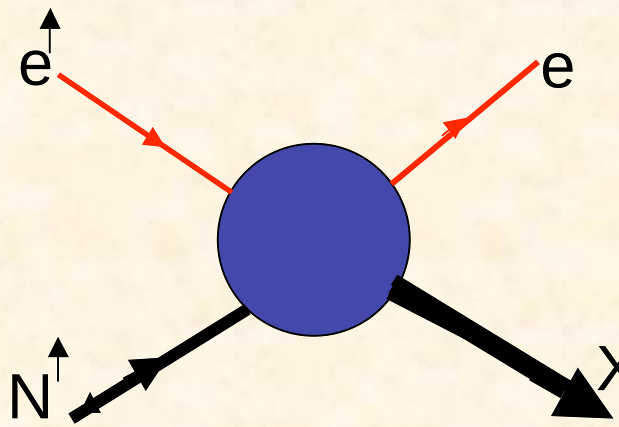
$$O_g^{(n)} = S\left[F_{\alpha\mu_1} D_{\mu_3} \ldots D_{\mu_n} F^{\alpha}_{\mu_2}\right]$$

n= 2, 3,..

=> nth moment of (unpolarized) quark and gluon distribution: DGLAP evolution <=> anomalous dimension (matrix)

- Heavy-quarks and their (non)-decoupling in (EWT)-QCD

2. Polarized DIS: an instructive OPE exercise



Consider the combinations

$$\sigma^{\pm} = \sigma^{\uparrow\downarrow}(eN \to eX) \pm \sigma^{\uparrow\uparrow}(eN \to eX) \equiv \sigma(e^{\uparrow}N^{\downarrow} \to eX) \pm \sigma(e^{\uparrow}N^{\uparrow} \to eX)$$

$\sigma^{(+)}$ is the unpolarized x-section discussed in previous lectures.

It is described in terms of unpolarized PDF: $q_f(x,Q^2)$, $g(x,Q^2)$

$\sigma^{(-)}$ is the polarized x-section described by the polarized PDF:

$\Delta q_f\,(x,Q^2)$, $\Delta g(x,Q^2)$ i.e.

$$\sigma^{(-)} \sim \frac{1}{2}\left[\frac{4}{9}\Delta u + \frac{1}{9}(\Delta d + \Delta s)\right]$$

NB: q and q* contribute with same sign

What can OPE say about $\sigma^{(-)}$? Recall:

$$J_\mu^{(el)} = \sum_f e_f \bar{\Psi}_f \gamma_\mu \Psi_f \equiv \sum_f e_f J_\mu^{(f)}$$

Last time we discussed the term of minimal d_k

$$J_\mu^{(el)}(x) J_\nu^{(el)}(0) \sim \sum_f e_f^2 \sigma_{\mu\lambda\nu}^{\rho} \frac{x^\lambda}{(x^2 - i\varepsilon)^2} J_\rho^{(f)}(0)$$

where $$\sigma_{\mu\lambda\nu\rho} = \eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\rho}$$

Does not contribute to $\sigma^{(-)}$ (actually not even to $\sigma^{(+)}$, one needs νN scattering). For $\sigma^{(-)}$ the term of minimal d_k is

$$J_\mu^{(el)}(x) J_\nu^{(el)}(0) \sim \sum_f e_f^2 \varepsilon_{\mu\nu\lambda}^{\rho} \frac{x^\lambda}{(x^2 - i\varepsilon)^2} J_{5\rho}^{(f)}(0)\, ;\, J_{5\rho}^{(f)} \equiv \bar{\Psi}_f \gamma_\mu \gamma_5 \Psi_f$$

where $\varepsilon_{\mu\lambda\nu\rho}$ is the completely antisymmetric tensor

These are the only relevant operators of dimension 3: they control the n=1 moments (= total number) of pol. PDF's. How?

(NB: absence of operator associated with n=1 moment of Δg!)

Let us denote the n=1 moments of $\Delta q_f(x,Q^2)$, $\Delta g(x,Q^2)$ by the same symbols $\Delta q_f(Q^2)$, $\Delta g(Q^2)$. One finds the following simple relations between partonic distributions and axial currents (NS = non-singlet, S= singlet, $s_\mu = \bar{N}\gamma_\mu\gamma_5 N$)

$$2C_{NS}(Q^2)\langle N|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|N\rangle = (\Delta u - \Delta d)s_\mu$$

$$2C_{NS}(Q^2)\langle N|\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s|N\rangle = (\Delta u + \Delta d - 2\Delta s)s_\mu$$

$$2C_{S}(Q^2)\langle N|\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s|N\rangle_Q = (\Delta u + \Delta d + \Delta s)_Q\, s_\mu$$

C_{NS} & $C_S \sim 1$, can be computed as a series expansion in $\alpha(Q^2) \ll 1$.

For the NS currents we need not specify a ren. scale (they have no an. dim.), while for the third we do (it is Q), since the non-conserved (anomalous) singlet current does have a γ !

What can we do with these results?

After all we only measure the combination $4\Delta u+\Delta d+\Delta s$!

A: We can do a lot of interesting physics!

First of all, as we have seen in GS's talk for the unpolarized case, we can combine proton and neutron (deuteron) data and, using isospin symmetry (which can be fully justified in QCD), extract the (Δu-Δd) combination.

Not only: we can check this against a theoretical prediction since, for once, we do know the matrix element from the neutron beta decay (using isospin again). This highly non-trivial check is called the Bjorken sum rule and has been verified with quite good accuracy.

One finds (Δu-Δd) ~ 1.257

For the second combination things are more difficult. We cannot measure it from experimentally accessible x-sections. But we can predict it (à la BjSR) in terms of hyperon (Σ, Λ) decays provided we use $SU(3)_F$ (which is reasonably good though not as good as isospin)

One finds ($\Delta u+\Delta d-2\Delta s$) ~ 0.58

At this point, if we also use the «proton» combination ($4\Delta u+\Delta d+\Delta s$), we get a result for the «singlet» combination ($\Delta u+\Delta d+\Delta s$) and can check it against theoretical expectations (prejudices). Here are the two most popular:

1. $\Delta u+\Delta d+\Delta s = J_{tot} = 1/2$ (the «spin-crisis» people)
2. $\Delta u+\Delta d+\Delta s = \Delta u+\Delta d-2\Delta s \sim 0.58$ (Ellis-Jaffe, 1974)

The data give instead $(\Delta u+\Delta d+\Delta s)_{10GeV^2} < 0.3$
(although one needs some extrapolation for small x).

...**the famous « spin crisis »!**

Many papers suggested that the situation is similar to the one of the momentum sum rule that lead to the conclusion that about half of the momentum is carried by gluons. Here also (at least) half of the « spin » is missing and people suggested that it is carried by gluons..

A spin crisis ?

1. There is a solid momentum sum rule but no spin sum rule!
2. There was no gluonic-spin operator…
3. How can something scale-dependent be related to J=1/2 or to a scale-independent quantity?

The solution to the puzzle is probably much more prosaic and related to a similar problem in hadron spectroscopy.

There are three neutral pseduscalar mesons, π, η and η'.

The first two are much lighter than the 3rd.

Their quark content is that of our three currents.

This is the well-known U(1) problem whose solution is now clear: it has to do with the fact that the S-axial current is not conserved at the quantum level (« ABJ anomaly + instantons »)

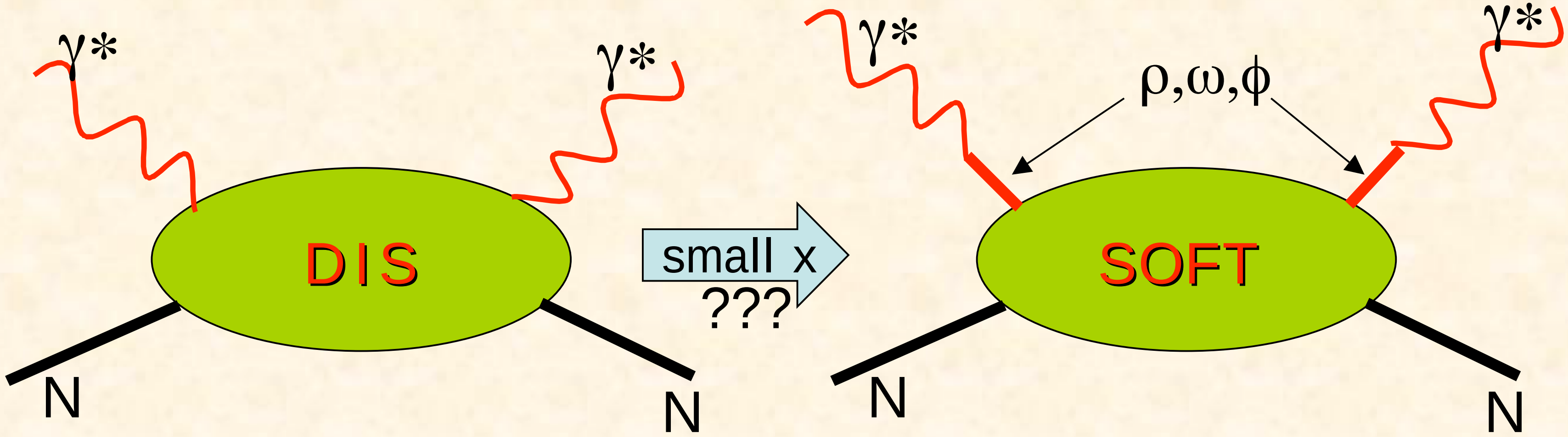
The smallness of $\Delta u+\Delta d+\Delta s$ thus gets related to the small nucleon matrix element of QCD's «topological charge»..

Moral: hard processes as a window into non perturbative QCD!

3. Small-x vs. soft QCD

3.1 Small-x DIS & soft high energy hadron scattering

Remember kinematics, $x = Q^2/2pQ \sim Q^2/s(\gamma^* N)$. Limiting case is Q^2 fixed and $O(\Lambda^2)$ hence $x \sim \Lambda^2/s$ => Vector-meson dominance



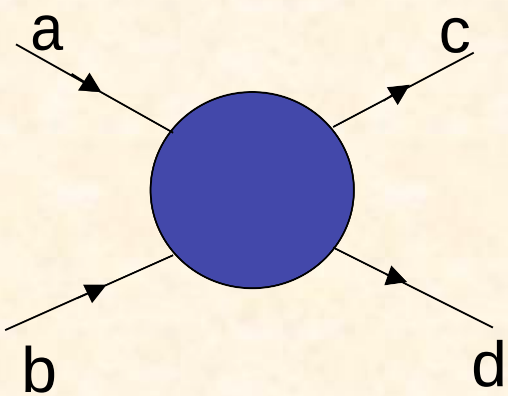
N.B. Small x means Q^2 going to infinity while keeping Q^2/s fixed and very small: one cannot usually interchange limits!!

Nevertheless, Q^2 can be fixed and large => α is small…

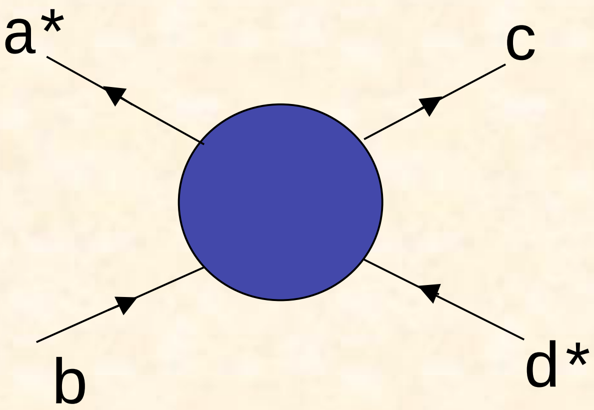
The process $(\rho,\omega,\phi)N \to (\rho,\omega,\phi)N$ at high c.m. energy s and small t (here t=0) can be studied using Regge (et al.'s) theory.

3.2 Regge theory in a nutshell

Analyticity and crossing (two very general principles) tell us that one and the same analytic function $A(s,t)$ describes various processes ($s = (p_a + p_b)^2$, $t = (p_a - p_c)^2$) e.g.(neglecting m)



if $s > 0$, $-s < t < 0$ ($\cos\theta_s = 1+2t/s$)



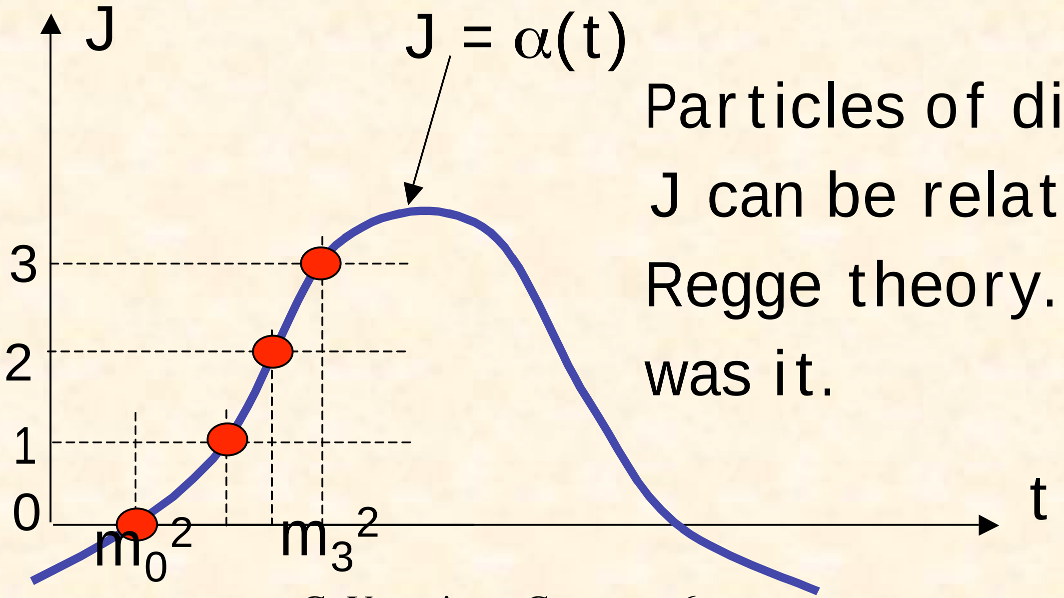
if $-t < s < 0$, $t > 0$ ($\cos\theta_t = 1+2s/t$)

Let us start from the 2nd case and perform a partial wave analysis of the scattering amplitude:

$$A(t,\theta_t) = \sum_{J=0}^{\infty} A_J(t) P_J(cos\theta_t)$$

Remember: $\cos\theta_t = 1+2s/t$

A particle of spin J exchanged in the t-clannel gives a pole in t in $A_J(t)$ at $t = m_J^2$. In 1959 T. Regge discovered that, at least in NR potential scattering, $A_J(t)$ is an analytic function of J with poles at $J=\alpha(t)$, the so-called Regge trajectory



Particles of different J can be related through Regge theory. But that was it.

This expansion converges for small cos θ_t .

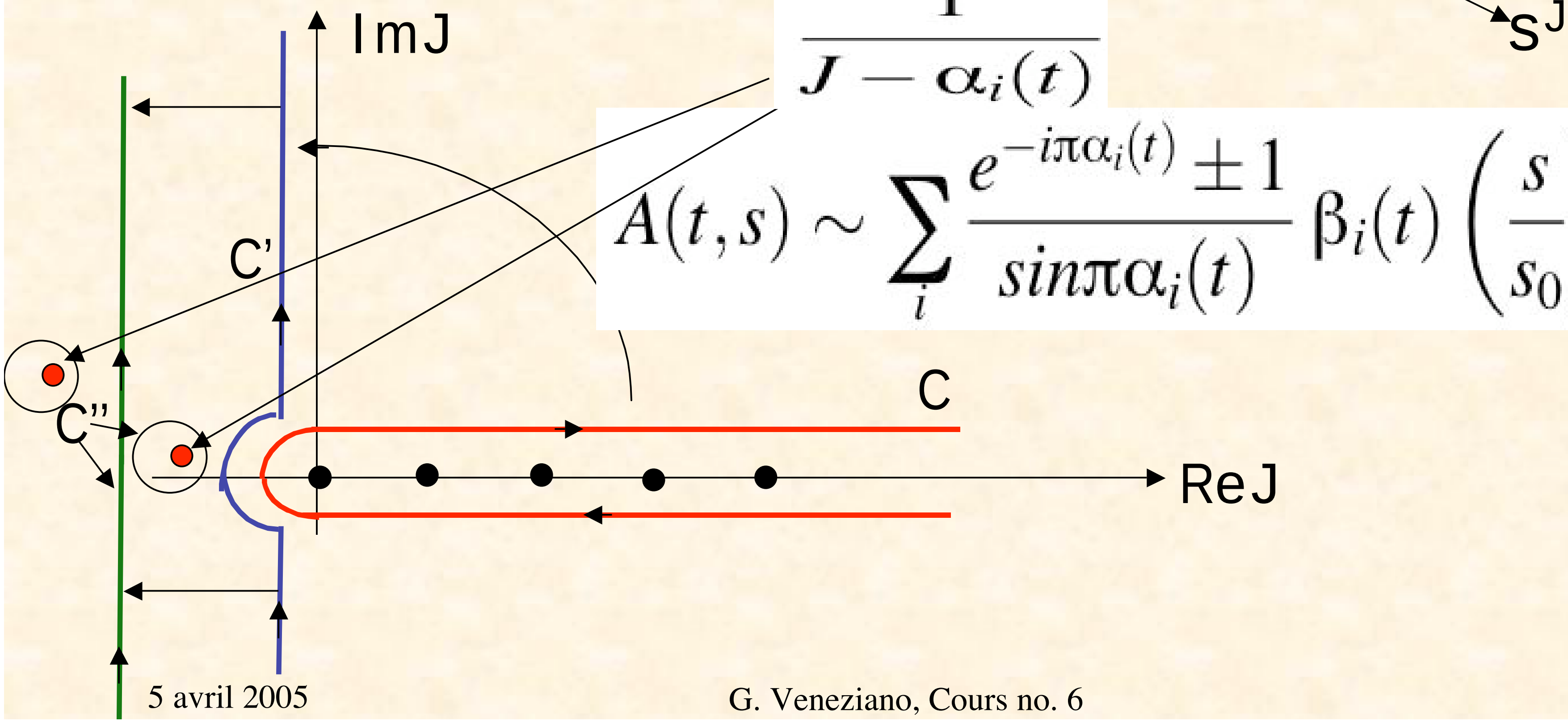
Q: can we use it also for large s, fixed t where it diverges?

A(Chew & Maldelstam): yes, provided we perform an analytic continuation (Sommerfeld-Watson transform). Basic idea:

$$A(t,s) \sim \int_C dJ \, \frac{e^{-i\pi J}}{sin\pi J} A(J,t) P_J(1+2s/t)$$

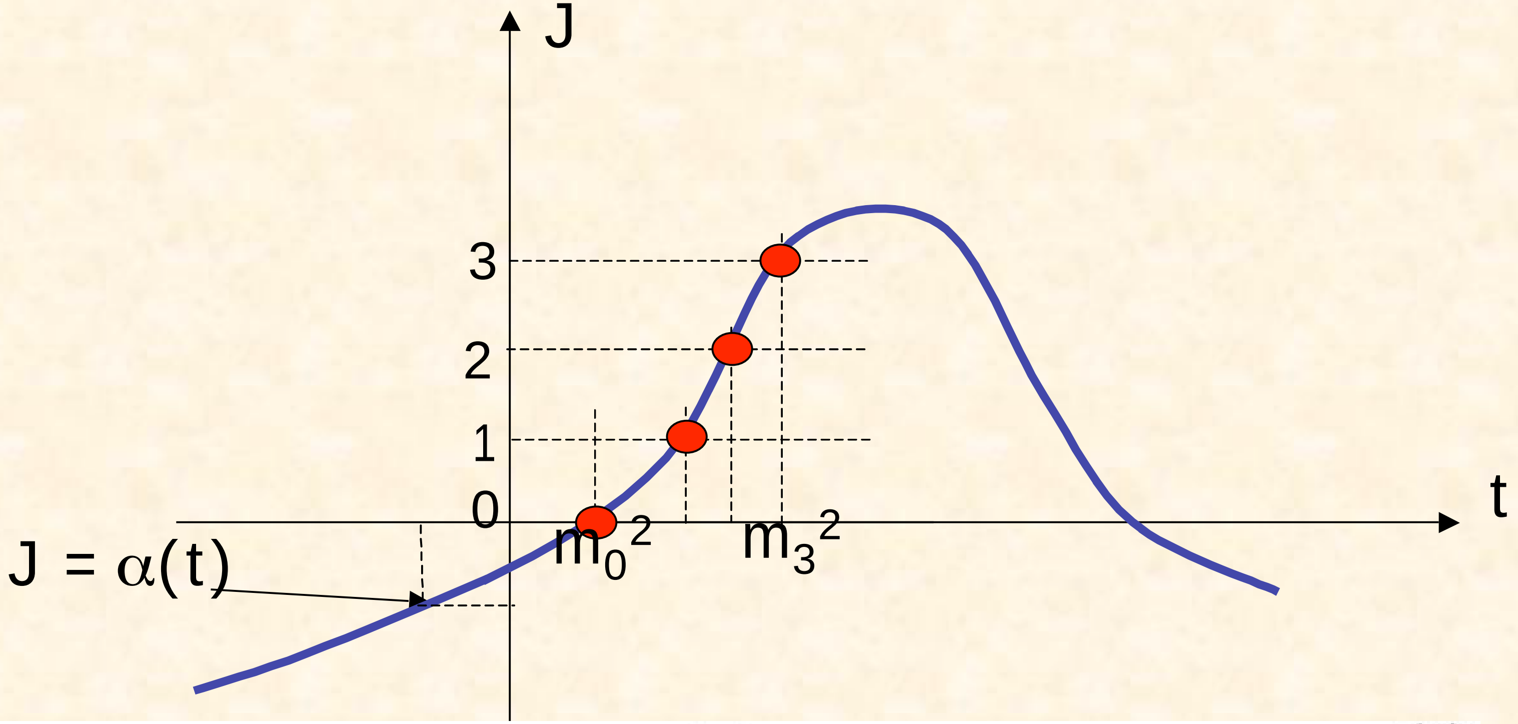
$$\frac{1}{J-\alpha_i(t)}$$

$$A(t,s) \sim \sum_i \frac{e^{-i\pi\alpha_i(t)} \pm 1}{sin\pi\alpha_i(t)} \beta_i(t) \left(\frac{s}{s_0}\right)^{\alpha_i(t)}$$



If there are various trajectories the highest one wins!

The Regge trajectory finds a new welcome use at t <0!



$$A(t,s) \sim \sum_i \frac{e^{-i\pi\alpha_i(t)} \pm 1}{\sin\pi\alpha_i(t)} \beta_i(t) \left(\frac{s}{s_0}\right)^{\alpha_i(t)}$$

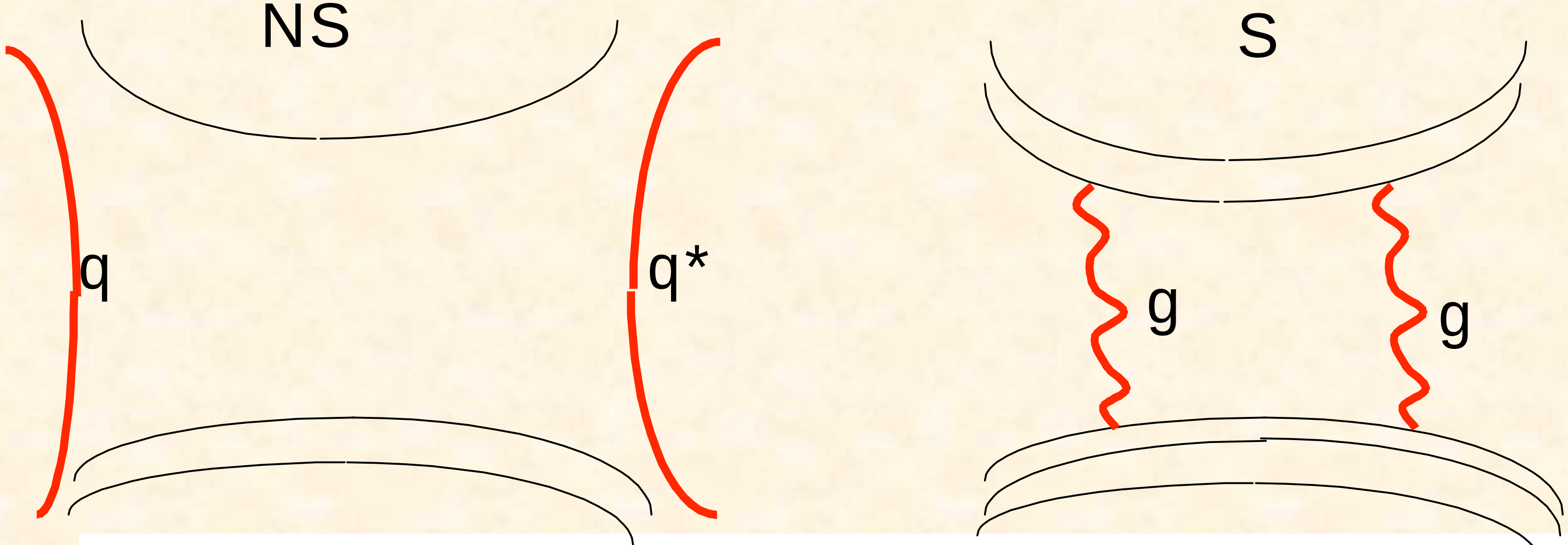
In the case of relevance to DIS, t=0 (optical theorem) and the leading trajectories that contribute are of two types:

1. Those that carry the q.n. of a qq* system and have $\alpha(0) \sim 0.5$: $A(s, t=0) \sim (s/s_0)^{0.5}$
2. Those that carry the q.n. of a gluonic system, i.e. the vacuum q.n. from the point of view of flavour (NB. all these trajectories correspond to hadrons, i.e. to colour singlets) and have $\alpha(0) \sim 1.0$: $A(s, t=0) \sim (s/s_0)^{1.0}$.
3. The latter does not contribute to differences such as (p-n)

It is thus very natural to associate the first kind of RP to valence quark distributions and the second to gluon or « sea » distributions. This can also be illustrated through so-called duality diagrams.

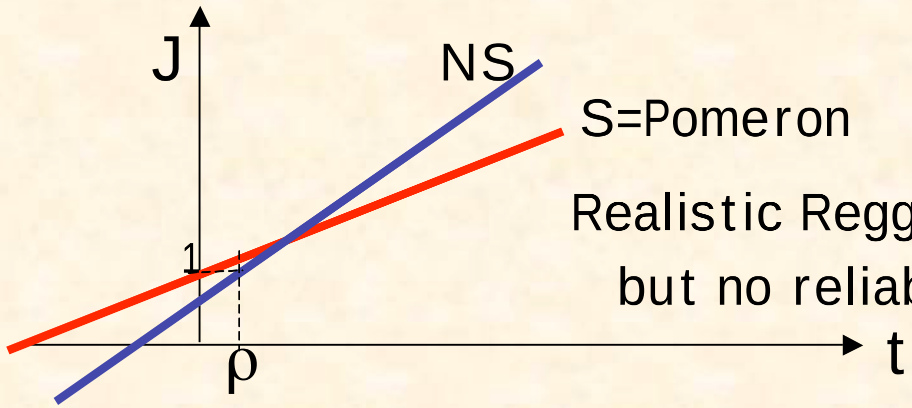
N.B. A famous bound (Froissart), imposes: $A(s, t=0) < s (\log s)^2$

Duality diagrams



$$A(t,s) \sim \sum_i \beta_i(t)\left(e^{-i\pi\alpha_i(t)} \pm 1\right)\Gamma(-\alpha_i(t))\left(\frac{s}{s_0}\right)^{\alpha_i(t)}$$

Regge behaviour in string theory (but trajectories are wrong)

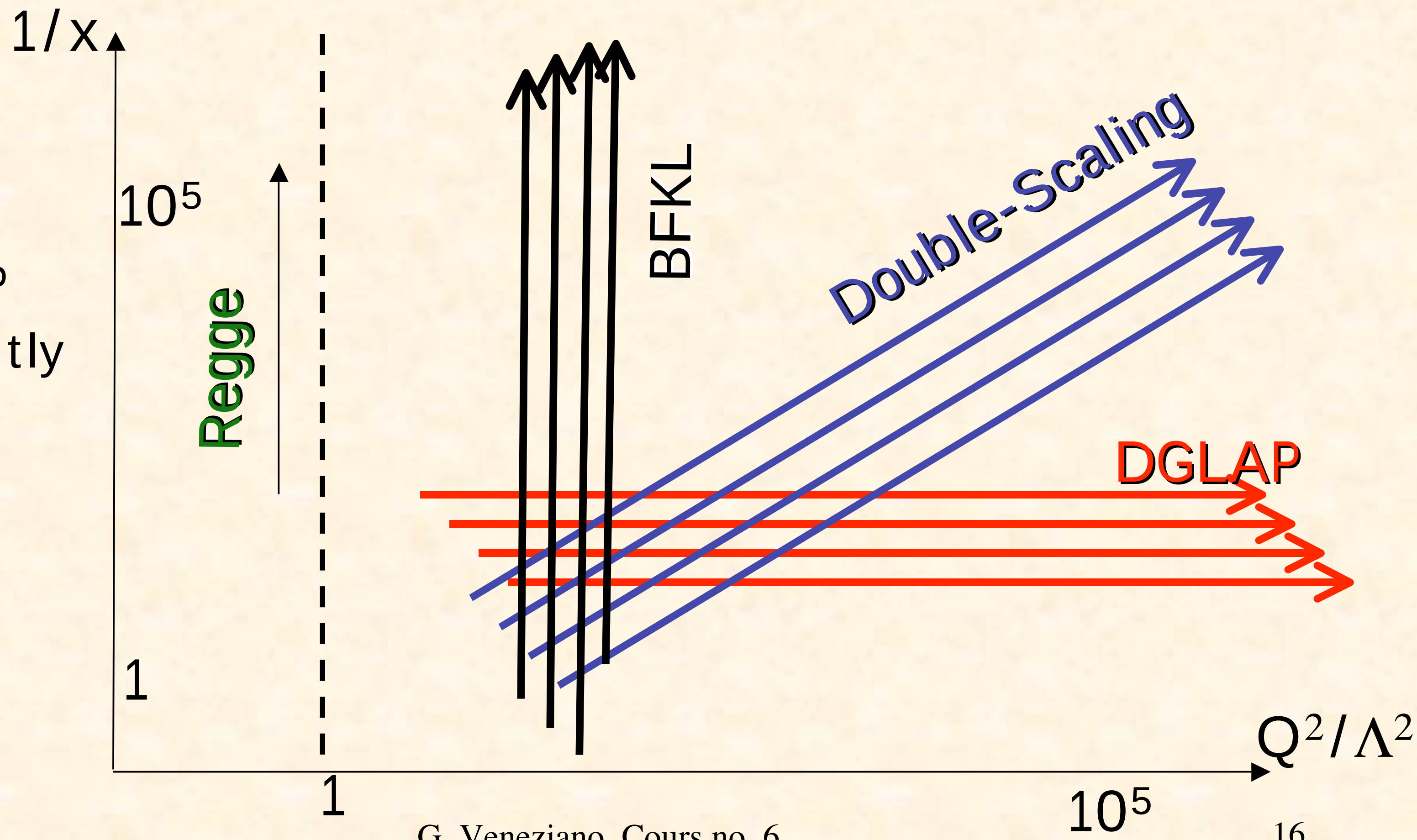


Realistic Regge trajectories, but no reliable calculation

4. Double-scaling limit & BFKL: an introduction

Let us go now a little bit more into the kinematics of DIS. It is most convenient to use a log-log plot with Q^2/Λ^2 on one axis and $1/x$ on the other

We have discussed at length DGLAP and very shortly Regge. What about the regimes in between?



Most naive guess: since, for fixed Q^2, $x \sim 1/s$:

$(s/s_0)^{\alpha(0)}$ <--> $x^{-\alpha(0)}$

This would imply $q_V(x) \sim x^{-0.5}$, $q_S(x) \sim g(x) \sim x^{-1.0}$

We can ask whether such a small-x behaviour, when imposed at some « initial » Q_0^2 is preserved by DGLAP evolution

$$Q^2 \frac{\partial}{\partial Q^2} F_h^i(x, Q^2) = \frac{\alpha(Q^2)}{2\pi} \sum_j \int_x^1 dz \frac{P_{j\to i}(z)}{z} F_h^j(x/z, Q^2)$$

Remember that the parton-splitting functions can be singular at z=0 (z=1 regularized by virtual effects). The leading small-x behaviour originates from such singularities.

At leading-order q->g and g->g splitting functions have c^2/z singularity. If we keep just those we get a very simple equation for (a suitable combination of) $f(x,Q^2) = x\, F\, (x,Q^2)$

$$Q^2\frac{\partial}{\partial Q^2}f(Y,Q^2)=\frac{\alpha(Q^2)}{2\pi}\int_0^Y dY' c^2 f(Y',Q^2)\ ,Y\equiv log(1/x)$$

Y is the so-called rapidity.
Let us also introduce a «time» variable T by

$$T=\int_{Q_0^2}^{Q^2}dq^2\frac{\alpha(q^2)}{2\pi q^2}\sim log\frac{logQ^2/\lambda^2}{logQ_0^2/\lambda^2}$$

==> $$\frac{\partial}{\partial T}f(Y,T)=c^2\int_0^Y dY' f(Y',T)$$ i.e.

$$\frac{\partial^2}{\partial T\partial Y}f(Y,T)=c^2 f(Y,T)$$ which is easily solved by

$$f(Y,T)=\int_{-i\infty}^{+i\infty}dj\, e^{jY+\frac{c^2}{j}T}\ \phi(j)\sim exp(2c\sqrt{TY})$$

where we have used a saddle point approx. Going back to F:

$$xF(x,Q^2)=exp\left(2\tilde{c}\sqrt{log(1/x)log\frac{logQ^2/\Lambda^2}{logQ_0^2/\Lambda^2}}\right)\ ;\tilde{c}\equiv\frac{c}{\sqrt{2\pi\beta_0}}$$

$$F(x,Q^2) = \frac{1}{x} exp\left(2\tilde{c}\sqrt{log(1/x) log\frac{logQ^2/\Lambda^2}{logQ_0^2/\Lambda^2}}\right)$$

This is the so-called DLLA (double-leading-log-approximation) formula showing that, in general, a 1/x behaviour gets modified by the evolution. In fact this formula, taken literally, gives a small-x behaviour that is more singular than 1/x $(\log x)^n$ but less than $(1/x)^{1+\varepsilon}$

It is not clear up to where in our log-log plane this result should be trusted, in particular one should worry about higher order corrections to the splitting functions:

One finds that, to (n+1) order in α, zP(z) ~ $(\alpha \log(1/z))^n$

Therefore we cannot stop at leading order if $\alpha(Q^2)\log(1/x) \sim 1$, i.e. $\log(s/Q^2) \sim k \log(Q^2/\Lambda^2)$. We need Q^2 to grow as some (possibly small) power of s. Here the DLLA result looks compatible with the data (« double asymptotic scaling »)

When $\alpha(Q^2)\log(1/x) = O(1)$ one would guess that a better approximation is to resum all terms of the type $(\alpha \log(1/z))^n$

This is the new approximation that leads to the BFKL (Balitsky-Fadin-Kuraev-Lipatov) equation and to its solution, an even more singular behaviour in x, $(1/x)^{1+\varepsilon}$ with a rather large ε and thus even more incompatible with the Froissart bound.

Q_1: Have we been too bold in pushing pQCD?
Q_2: Or simply are there new perturbative phenomena that come into play as we move towards smaller and smaller x?

Today's seminar may provide some answer to these crucial questions which are at the edge of our present understanding of pQCD!