

Particules Élémentaires, Gravitation et Cosmologie  
Année 2008-'09

Gravitation et Cosmologie: le Modèle Standard

Séminaires 1 et 2: 23 janvier 2009

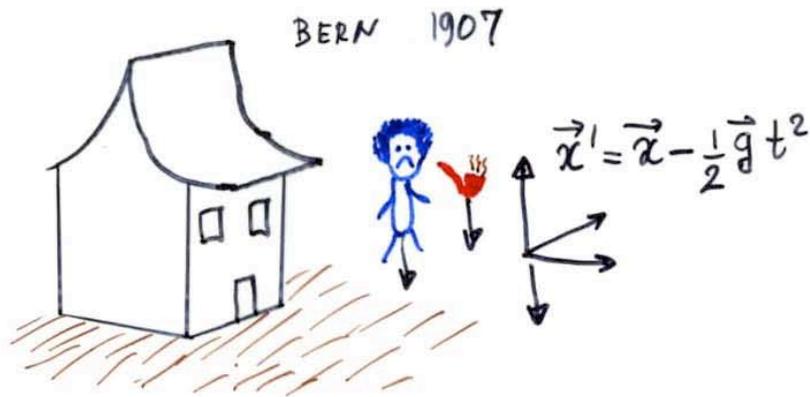
Thibault Damour  
(IHES)

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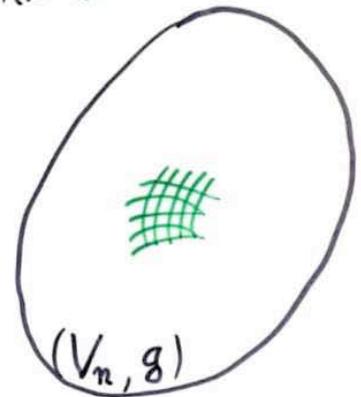
Experimental tests of  
relativistic gravity

2

# EINSTEIN'S VISION



RIEMANN ~ 1856



LOCAL EFFACEMENT OF  $\vec{g}$

$$\exists x'^{\mu}; g'_{\mu\nu}(x'^{\lambda}) = \eta_{\mu\nu} + O(|x' - x'_0|^2)$$

LOCAL EFFACEMENT OF  $\Gamma_{\mu\nu}^{\lambda} \sim \partial g$

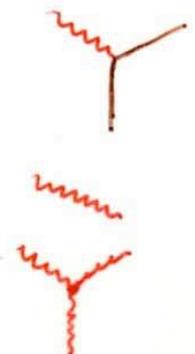
UNIVERSALITY OF FREE FALL  $\longleftrightarrow$  UNIVERSAL COUPLING OF MATTER  
 TO ONE  $g_{\mu\nu}(x^{\lambda})$   
 ('HYPOTHESIS OF EQUIVALENCE'  
 ['EQUIVALENCE PRINCIPLE'])

$$S_{\text{TOT}} = \frac{c^4}{16\pi G} \int \sqrt{g} \frac{d^4x}{c} R(g) + S_{\text{MATTER}}[\psi, A, H; g_{\mu\nu}]$$

TWO SORTS OF EXPERIMENTAL TESTS

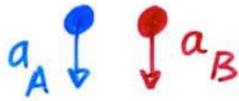
• MATTER-COUPLING TESTS (RHS)

• TESTS OF THE DYNAMICS OF  $g_{\mu\nu}$  (LHS)



# TESTS OF THE COUPLING MATTER & GRAVITY

"EQUIVALENCE PRINCIPLE"  $S_{\text{MATTER}}[\psi, A, H; g_{\mu\nu}]$

• UNIVERSALITY OF FREE FALL 

Adelberger's group  $\left(\frac{\Delta a}{a}\right)_{\text{Fe-Si}} = (3.6 \pm 5.0_{\text{STAT}} \pm 0.7_{\text{SYST}}) \times 10^{-13}$

Lunar Laser Ranging's group  $\left(\frac{\Delta a}{a}\right)_{\oplus \text{Moon}} = (-1.0 \pm 1.4) \times 10^{-13}$

• CONSTANCY OF "CONSTANTS"  $\alpha_{\text{EM}} \equiv \frac{e^2}{\hbar c}$

Atomic Clock Tests  
Marion '03; Bize '03; Fischer '05

$$\frac{\dot{\alpha}_{\text{EM}}}{\alpha_{\text{EM}}} = (-0.9 \pm 2.9) \times 10^{-15} \text{ yr}^{-1}$$

Oklo's natural fission reactor  
Shlyakhter '76, Damon & Dyson '96, Fujii '00

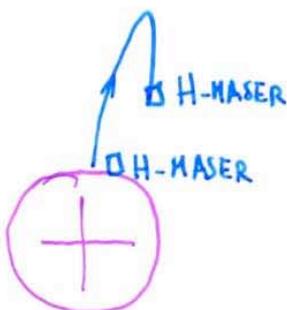
$$\left\langle \frac{\dot{\alpha}_{\text{EM}}}{\alpha_{\text{EM}}} \right\rangle = (-0.9 \pm 5.9) \times 10^{-17} \text{ yr}^{-1}$$

Quasar absorption lines  
Quast '04; Srianand '04

$$\left\langle \frac{\dot{\alpha}_{\text{EM}}}{\alpha_{\text{EM}}} \right\rangle = (-0.7 \pm 1.9) \times 10^{-16} \text{ yr}^{-1}$$

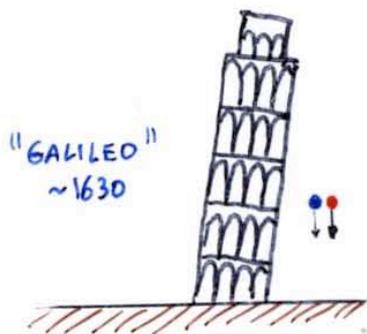
• GRAVITATIONAL REDSHIFT

Vessot, Levine '79

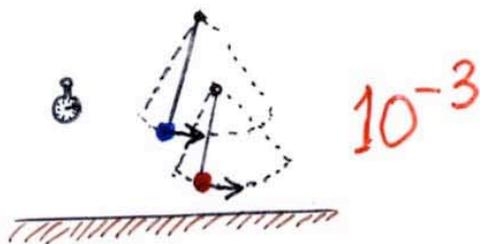


$$\frac{\Delta \nu}{\nu} = (1 \pm 10^{-4}) \frac{\Delta U}{c^2}$$

# UNIVERSALITY OF FREE FALL



NEWTON  
≤ 1686



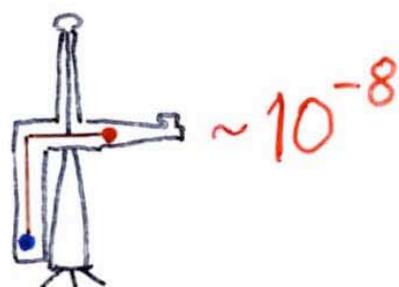
LAPLACE  
≤ 1805



[ POINCARÉ 1906 ]

~ 10<sup>-8</sup>

EÖTVÖS  
1896



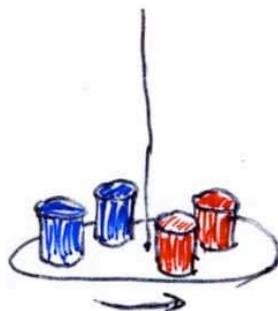
DICKE ET AL.



~ 3 × 10<sup>-11</sup>

1962

BRAGINSKY AND PANOV 1972



ADELBERGER ET AL.  
1989-1994, 2001-

~ 10<sup>-12</sup>

LUNAR  
LASER  
RANGING



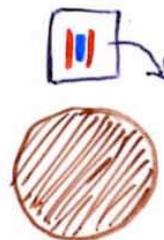
~ 10<sup>-13</sup>

1969 - NOW

DICKEY ET AL. '94  
WILLIAMS ET AL. '96



SPACE  
TESTS  
OF THE  
EQUIVALENCE PRINCIPLE



MICROSCOPE 2009 ~ 10<sup>-15</sup> ?  
STEP 200? ~ 10<sup>-18</sup> ?

## EXPLICIT FORM OF EINSTEIN'S FIELD EQUATIONS (IN HARMONIC COORDINATES)

$$\begin{aligned}
 & -g^{\alpha\beta} \partial_{\alpha\beta} g_{\mu\nu} + g^{\alpha\beta} g^{\gamma\delta} \left( \partial_\gamma g_{\mu\alpha} \partial_\delta g_{\nu\beta} - \partial_\gamma g_{\mu\alpha} \partial_\beta g_{\nu\delta} \right. \\
 & \left. + \partial_\gamma g_{\mu\alpha} \partial_\nu g_{\beta\delta} + \partial_\gamma g_{\nu\alpha} \partial_\mu g_{\beta\delta} - \frac{1}{2} \partial_\mu g_{\alpha\gamma} \partial_\nu g_{\beta\delta} \right) \\
 & = \frac{16\pi G}{c^4} \left( T_{\mu\nu} - \frac{1}{D-2} T g_{\mu\nu} \right)
 \end{aligned}$$

- WEAK-FIELD REGIME :  $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$  ;  $h_{\mu\nu} \ll 1$

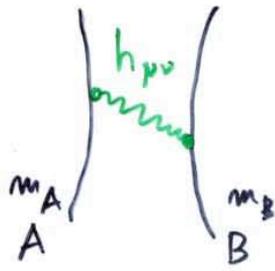
$$\rightarrow -\square h_{\mu\nu} = \frac{16\pi G}{c^4} \left( T_{\mu\nu} - \frac{1}{D-2} \eta_{\mu\nu} T \right)$$

- SOLAR-SYSTEM :

$$(D=4) \quad -\square \left( h_{00} + \frac{1}{2} h_{00}^2 \right) = \frac{8\pi G}{c^4} (T^{00} + T^{ii})$$

- STRONG-FIELD REGIME : NEED EXACT FORM

# ONE-GRAVITON EXCHANGE INTERACTION



$$S = -m_A \int ds_A - m_B \int ds_B + \int \frac{\sqrt{g} R}{16\pi G}$$

$$\sqrt{-g_{\mu\nu}(x_A) dx_A^\mu dx_A^\nu}$$

EXPAND  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

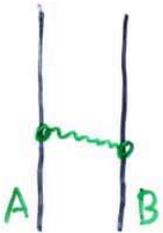
$$S \approx -m_A \int \sqrt{-\eta_{\mu\nu} dx_A^\mu dx_A^\nu} - m_B \int \sqrt{-\eta_{\mu\nu} dx_B^\mu dx_B^\nu} + \frac{1}{32\pi G} \int h^{\mu\nu} \square (h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu})$$

$$+ \frac{1}{2} \int h_{\mu\nu} T_A^{\mu\nu} + \frac{1}{2} \int h_{\mu\nu} T_B^{\mu\nu}$$

$$T_A^{\mu\nu}(x) = \int ds_0 m_A u_A^\mu u_A^\nu \delta(x - x_A) \quad u_A^\mu = \frac{dx_A^\mu}{ds_0}$$

ELIMINATE  $h_{\mu\nu}$  BY SOLVING  $\frac{\delta S}{\delta h_{\mu\nu}} = 0$ , i.e.

$$\square h_{\mu\nu} = -\frac{16\pi G}{c^4} (T_{\mu\nu} - \frac{1}{D-2} T \eta_{\mu\nu})$$



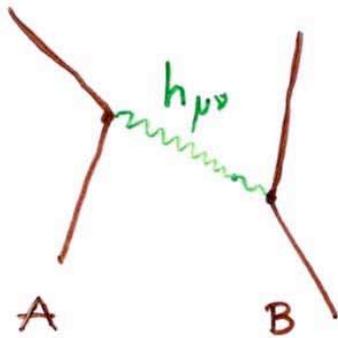
$$S^{\text{int}} = -\frac{8\pi G}{c^4} \int d^D x T_A^{\mu\nu} \square^{-1} (T_{\mu\nu}^B - \frac{1}{D-2} T^B \eta_{\mu\nu})$$

$$\square^{-1} P_{\mu\nu}^{\rho\sigma} T_{\rho\sigma}^B$$

$$S^{\text{int}} = 2G \iint ds_A^0 ds_B^0 m_A u_A^\mu u_A^\nu P_{\mu\nu}^{\rho\sigma} G(x_A^\mu - x_B^\nu) m_B u_B^\rho u_B^\sigma$$

Green's function  $\square G(x) = -4\pi \delta^D(x)$

# DYNAMICS OF THE GRAVITATIONAL FIELD: WEAK FIELD REGIME



"ONE - GRAVITON EXCHANGE"

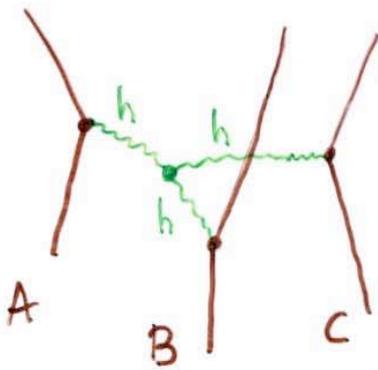


LINEARIZED EINSTEIN'S EQUATIONS

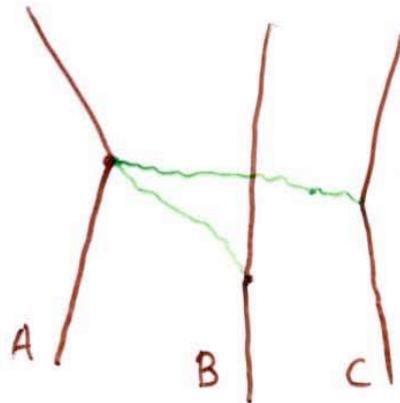
$$\square h_{\mu\nu} = -\frac{16\pi G}{c^4} \left( T_{\mu\nu} - \frac{1}{2} T \eta_{\mu\nu} \right)$$

$$S_{INT} = 2G \iint ds_A ds_B m_A^{\mu\nu} P_{\mu\nu}^{\rho\sigma} D[x_A(s_A) - x_B(s_B)] m_B^{\rho\sigma}$$

$$L^{2\text{-BODY}} = \frac{1}{2} \sum_{A \neq B} \frac{G m_A m_B}{r_{AB}} \left[ 1 + \frac{3}{2c^2} (\vec{v}_A^2 + \vec{v}_B^2) - \frac{7}{2c^2} \vec{v}_A \cdot \vec{v}_B - \frac{1}{2c^2} (\vec{n}_{AB} \cdot \vec{v}_A)(\vec{n}_{AB} \cdot \vec{v}_B) + O\left(\frac{1}{c^4}\right) \right]$$



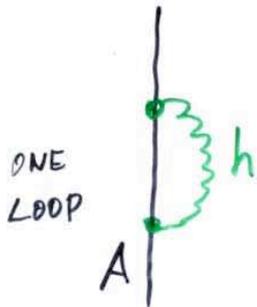
+



$$L^{3\text{-BODY}} = -\frac{1}{2} \sum_{B \neq A \neq C} \frac{G^2 m_A m_B m_C}{r_{AB} r_{AC} c^2} + O\left(\frac{1}{c^4}\right)$$

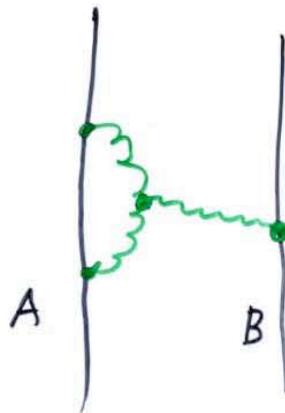
# CLASSICAL LOOPS

CLASSICAL CALCULATION ALSO CONTAINS 'SELF' DIAGRAMS

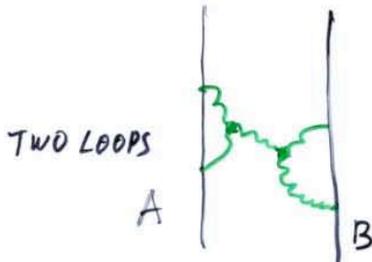


i.e.  $T_A \Pi^{-1} P T_A \propto G m_A^2$

AND, AT HIGHER ORDERS,

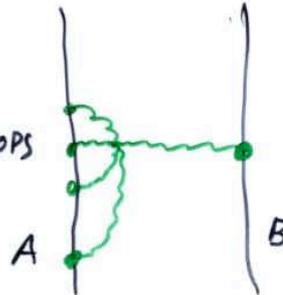


$\propto G^2 m_A^2 m_B$



$G^3 m_A^2 m_B^2$

--- THREE LOOPS



$G^4 m_A^4 m_B$

→ ASSOCIATED DIVERGENCES (WHEN LOOPS SHRINK)

→ BECOME NASTY AT THREE LOOPS

→ RENORMALIZABLE OR FINITE IN DIMENSIONAL REGULARIZATION

EXPECT NON-RENORMALIZABLE AMBIGUITIES ONLY AT  $\geq 5$  LOOPS

# TESTS OF THE DYNAMICS OF THE GRAV. FIELD

## SOLAR-SYSTEM TESTS :

WEAK ( $h_{\mu\nu} < 10^{-6}$ ) AND QUASI-STATIC ( $\frac{\partial h}{c \partial x} \sim \frac{v}{c} \lesssim 10^{-4}$ ) FIELDS

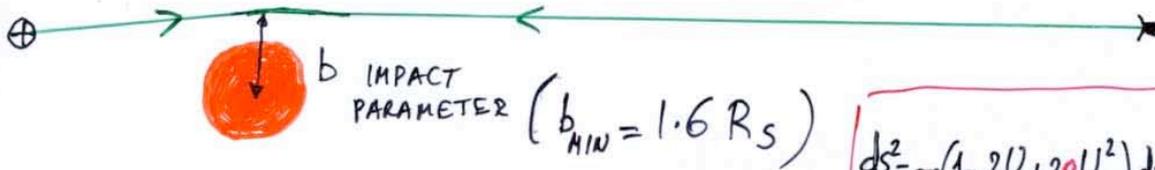
- PERIHELION ADVANCE OF MERCURY  
I. Shapiro '90; ASSUMING  $J_{2\oplus} \sim 2 \times 10^{-7}$   
$$\Delta \dot{\omega} = \overset{\dot{\omega}^{\text{GR}}}{\parallel} 42.98'' (1.000 \pm 0.001)$$
- LIGHT DEFLECTION (VLBI)  
S.S. Shapiro... '04  
$$\Delta \theta = \Delta \theta^{\text{GR}} (1 + (-0.9 \pm 2.2) \times 10^{-4})$$
- ORBITAL MOTION OF THE MOON  
(Nordtvedt '68) LUNAR LASER RANGING  
Williams... '04  
$$(\Delta r_{\oplus\text{M}})_{\text{SYNODIC}} = (3 \pm 4) \text{ mm } \cos D$$
- VARYING FREQUENCY SHIFT OF  
RADIO LINKS: CASSINI SPACECRAFT  
(Bertotti, Iess, Tortora '03)  
$$\frac{\Delta \nu/\nu}{(\Delta \nu/\nu)^{\text{GR}}} = 1 + (1.0 \pm 1.1) \times 10^{-5}$$

QUASI-STATIC, WEAK-FIELD EINSTEIN GRAVITY OK AT

$10^{-5}$  LEVEL

# VARYING FREQUENCY SHIFT OF RADIO LINKS WITH THE CASSINI SPACECRAFT

(Bertotti, Iess, Tortora '03)



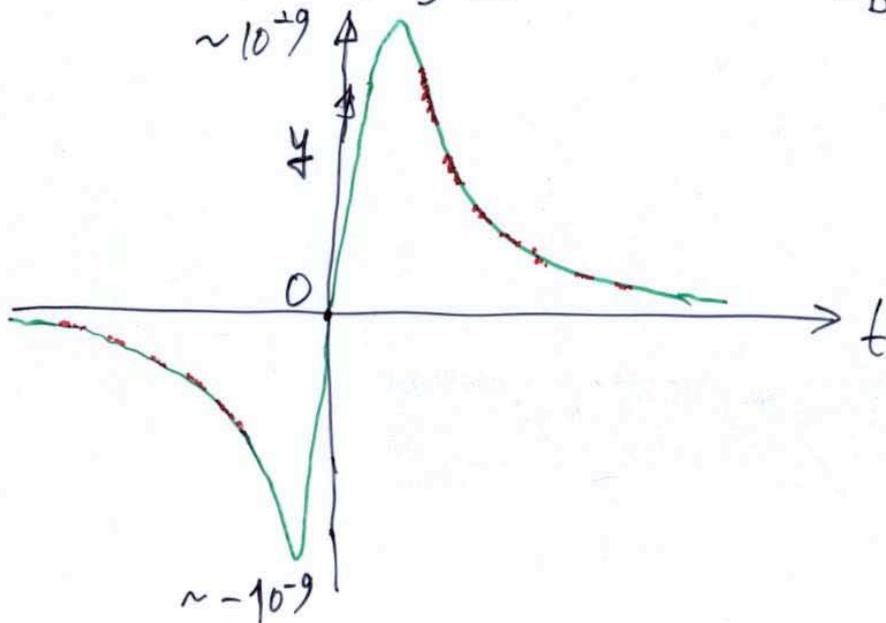
TWO-WAY RELATIVISTIC FREQUENCY SHIFT

$$ds^2 = -\left(1 - \frac{2U}{c^2} + 2\beta \frac{U^2}{c^4}\right) dt^2 + \left(1 + 2\gamma \frac{U}{c^2}\right) d\vec{x}^2$$

$$\gamma_{GR} \equiv 1$$

$$\gamma \equiv \left(\frac{\Delta\nu}{\nu}\right)_{\text{TWO-WAY}}^{\text{GRAV.}} = -4(1+\gamma) \frac{GM_{\text{SUN}}}{c^3 b} \frac{db}{dt} = -0.2 \mu\text{sec} (1+\gamma) \frac{1}{b} \frac{db}{dt}$$

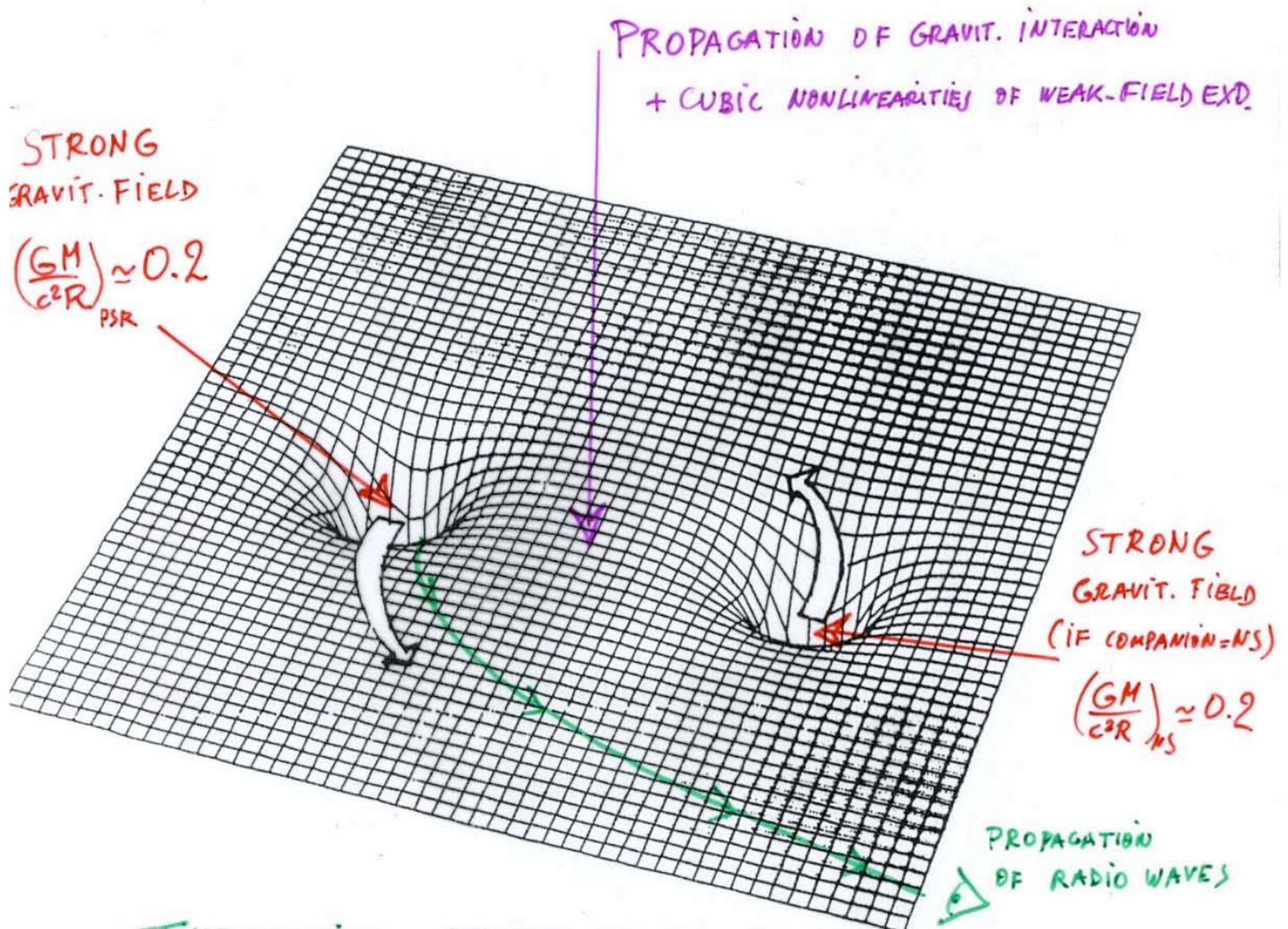
AS EARTH MOVES



$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

# BINARY PULSARS: FIRST POSSIBILITY OF PROBING THE FULL STRUCTURE OF RELATIVISTIC GRAVITY

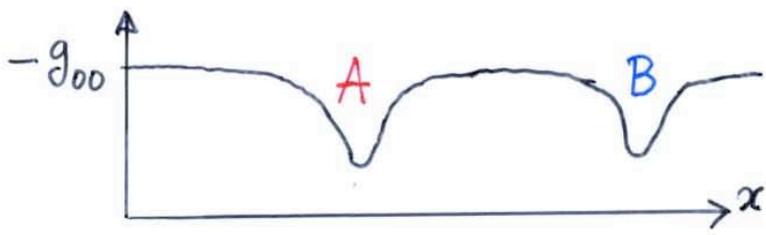
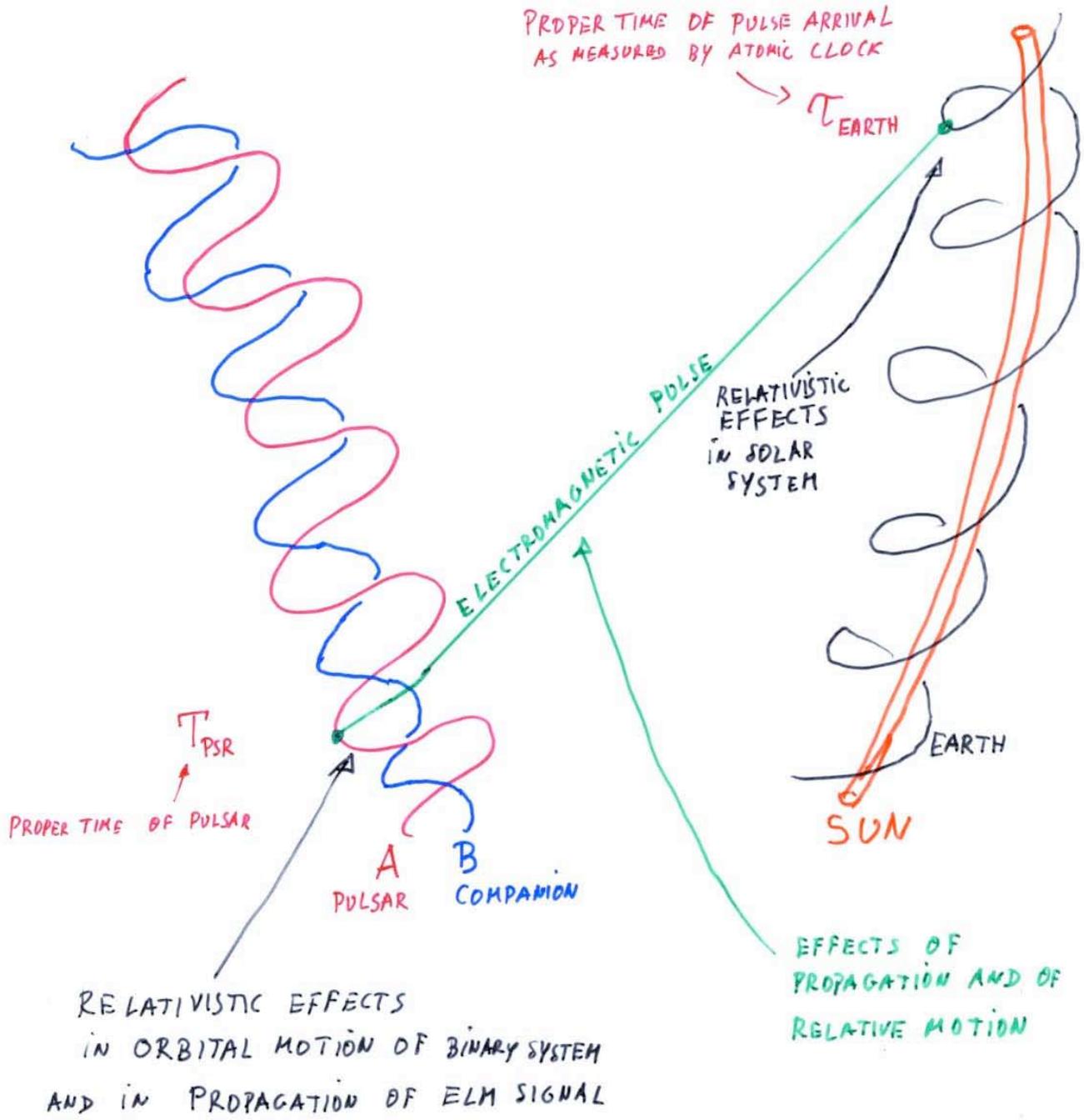
- RADIATIVE EFFECTS [ FIELD PROPAGATION ]
- HIGHLY NON-LINEAR EFFECTS [ STRONG FIELDS ]



## THEORETICAL ASPECTS OF BINARY PULSARS:

- ① MOTION OF TWO STRONGLY SELF-GRAVITATING BODIES  
 ( T.D. & DERUELLE '81, T.D. '82, '83 )
- ② RELATIVISTIC TIMING OF A BINARY PULSAR  
 ( BLANDFORD, TEUKOLSKY '76, T.D. & DERUELLE '85, '86 )
- ③ USE OF BINARY PULSARS AS PROBES OF RELATIVISTIC GRAVITY  
 ( EARDLEY '75, WILL, EARDLEY '77, T.D. '88, T.D. & TAYLOR '92 )

# RELATIVISTIC TIMING OF A BINARY PULSAR



$(V/c)^5$

# EQUATIONS OF MOTION IN GENERAL RELATIVITY

G6  
OL16

accelerations. Then each body must satisfy the following equation of motion (Damour and Deruelle, 1981a; Damour, 1982):

$$a^i = A_0^i(\ddot{z} - \ddot{z}') + c^{-2} A_2^i(\ddot{z} - \ddot{z}', \dot{v}, \dot{v}') + c^{-4} A_4^i(\ddot{z} - \ddot{z}', \dot{v}, \dot{v}', \dot{S}, \dot{S}') + c^{-5} A_5^i(\ddot{z} - \ddot{z}', \dot{v} - \dot{v}') + O(c^{-6}), \quad (154)$$

with

$$A_0^i = -Gm'R^{-2}N^i, \quad (155)$$

$$A_2^i = Gm'R^{-2}\{N^i[-v^2 - 2v'^2 + 4(vv') + \frac{3}{2}(Nv')^2 + 5(Gm/R) + 4(Gm'/R)] + (v^i - v'^i)[4(Nv) - 3(Nv')]\}, \quad (156)$$

$$A_4^i = B_4^i + C_4^i + D_4^i, \quad (157)$$

$$B_4^i = Gm'R^{-2}\{N^i[-2v'^4 + 4v'^2(vv') - 2(vv')^2 + \frac{3}{2}v^2(Nv')^2 + \frac{9}{2}v'^2(Nv')^2 - 6(vv')(Nv')^2 - \frac{1}{8}v^5(Nv')^4 + (Gm/R)(-\frac{1}{4}v^2 + \frac{5}{4}v'^2 - \frac{5}{2}vv') + \frac{39}{2}(Nv)^2 - 39(Nv)(Nv') + \frac{1}{2}(Nv')^2] + (Gm'/R)(4v'^2 - 8(vv') + 2(Nv)^2 - 4(Nv)(Nv') - 6(Nv')^2) + (v^i - v'^i)[v^2(Nv') + 4v'^2(Nv) - 5v'^2(Nv') - 4(vv')(Nv) + 4(vv')(Nv') - 6(Nv)(Nv')^2 + \frac{9}{2}(Nv')^3] + (Gm/R)(-\frac{6}{4}(Nv) + \frac{5}{4}(Nv')) + (Gm'/R)(-2(Nv) - 2(Nv'))\}, \quad (158)$$

$$C_4^i = G^3 m' R^{-4} N^i [-\frac{5}{4}m^2 - 9m'^2 - \frac{69}{2}mm'], \quad (159)$$

$$D_4^i = \left(\frac{S^{ik}}{m} + 2\frac{S'^{ik}}{m'}\right)(v^i - v'^i)\left(\frac{Gm'}{R}\right)_{,kl} + \left(2\frac{S^{kl}}{m} + 2\frac{S'^{kl}}{m'}\right)(v^l - v'^l)\left(\frac{Gm'}{R}\right)_{,ik}, \quad (160)$$

and

$$A_5^i = \frac{4}{3}G^2 mm'R^{-3}\{V^i[-V^2 + 2(Gm/R) - 8(Gm'/R)] + N^i(NV)[3V^2 - 6(Gm/R) + \frac{5}{3}(Gm'/R)]\}. \quad (161)$$

The two parameters  $m$  and  $m'$  appearing in eqs. (154)–(161) are the ‘Schwarzschild masses’ of the condensed bodies. They are two constants which appear in the external gravitational field, in which are hidden many internal structure effects (see the discussion of the ‘effacement of internal structure’ in Section 6.14). On the other hand, the spin tensors undergo a slow evolution (on the post-Newtonian time scale, i.e.  $\beta_c^{-2}$  times the orbital period) which is also obtained in the Einstein–Infeld–Hoffmann–Kerr-type approach (Damour, 1982, and references therein). Introducing, à la Schiff, a suitable spin-vector,  $\vec{S}$ , associated with  $S_{\mu\nu}$ , the law of evolution (‘spin precession’) reads for the first body (see also references in Section 6.13.2)

$$\frac{d\vec{S}}{dt} = \left[\frac{Gm'}{c^2 R^2} \vec{N} \times \left(\frac{3}{2}\dot{v} - 2\dot{v}'\right)\right] \times \vec{S} + O\left(\frac{1}{c^4}\right). \quad (162)$$

“DRESSED MASSES”  
IN CORPORATING STRONG-SELF-FIELD EFFECTS

GRAVITATIONAL RADIATION DAMPING

↑  
DIRECT EFFECT OF PROPAGATION OF GRAVITY AT SPEED C

# RELATIVISTIC TIMING FORMULA

Damour and Deruelle [36, 47] proved that it is possible to describe all of the independent  $O(v^2/c^2)$  timing effects in a simple mathematical way common to a wide class of alternative theories. This made it possible to revert to a theory-independent analysis of timing data, and led to the possibility of working within a strong-field analog of the PPN formalism, the so-called [37] "parametrized post-Keplerian" approach. The part of the Damour-Deruelle phenomenological timing model describing orbital effects reads

$$t_b - t_0 = F[T; \{p^K\}; \{p^{PK}\}; \{q^{PK}\}] , \quad (2.1a)$$

where  $t_b$  denotes the solar-system barycentric (infinite frequency) arrival time,  $T$  the pulsar proper time (corrected for aberration, see below),

$$\{p^K\} = \{P_b, T_0, e_0, \omega_0, x_0\} \quad (2.1b)$$

is the set of Keplerian parameters,

$$\{p^{PK}\} = \{k, \gamma, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\} \quad (2.1c)$$

the set of separately measurable post-Keplerian parameters, and

$$\{q^{PK}\} = \{\delta_r, A, B, D\} \quad (2.1d)$$

the set of not separately measurable post-Keplerian parameters. The right hand side of Eq. (2.1a) is given by

$$F(T) = D^{-1}[T + \Delta_R(T) + \Delta_E(T) + \Delta_S(T) + \Delta_A(T)] , \quad (2.2a)$$

$$\Delta_R = x \sin \omega [\cos u - e(1 + \delta_r)] + x[1 - e^2(1 + \delta_\theta)^2]^{1/2} \cos \omega \sin u , \quad (2.2b)$$

$$\Delta_E = \gamma \sin u , \quad (2.2c)$$

$$\Delta_S = -2r \ln \{1 - e \cos u - s[\sin \omega (\cos u - e) + (1 - e^2)^{1/2} \cos \omega \sin u]\} , \quad (2.2d)$$

$$\Delta_A = A \{\sin[\omega + A_e(u)] + e \sin \omega\} + B \{\cos[\omega + A_e(u)] + e \cos \omega\} , \quad (2.2e)$$

where

$$x = x_0 + \dot{x}(T - T_0) , \quad (2.3a)$$

$$e = e_0 + \dot{e}(T - T_0) , \quad (2.3b)$$

and where  $A_e(u)$  and  $\omega$  are the following functions of  $u$ ,

$$A_e(u) = 2 \arctan \left[ \left( \frac{1+e}{1-e} \right)^{1/2} \tan \frac{u}{2} \right] , \quad (2.3c)$$

$$\omega = \omega_0 + k A_e(u) , \quad (2.3d)$$

and  $u$  is the function of  $T$  defined by solving the Kepler equation

$$u - e \sin u = 2\pi \left[ \left( \frac{T - T_0}{P_b} \right) - \frac{1}{2} \dot{P}_b \left( \frac{T - T_0}{P_b} \right)^2 \right] . \quad (2.3e)$$

# TESTING RELATIVISTIC GRAVITY WITH BINARY PULSAR DATA

T4

## TWO APPROACHES

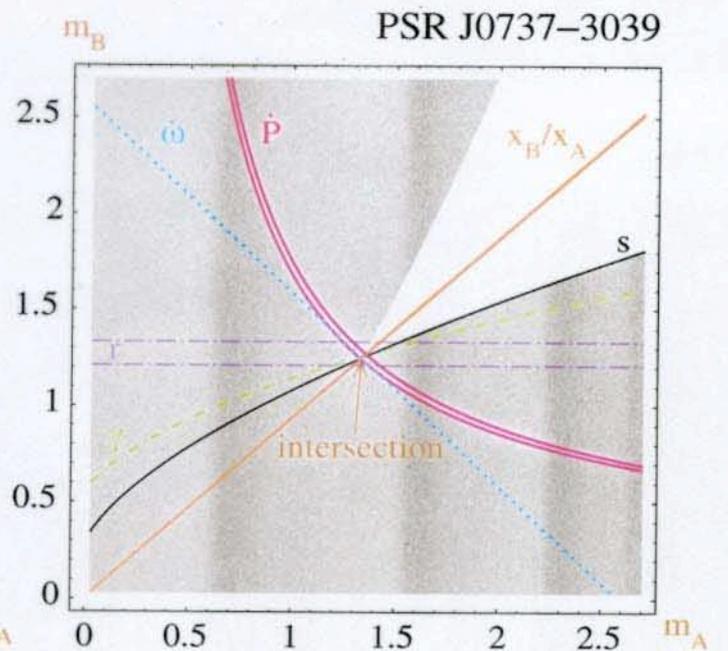
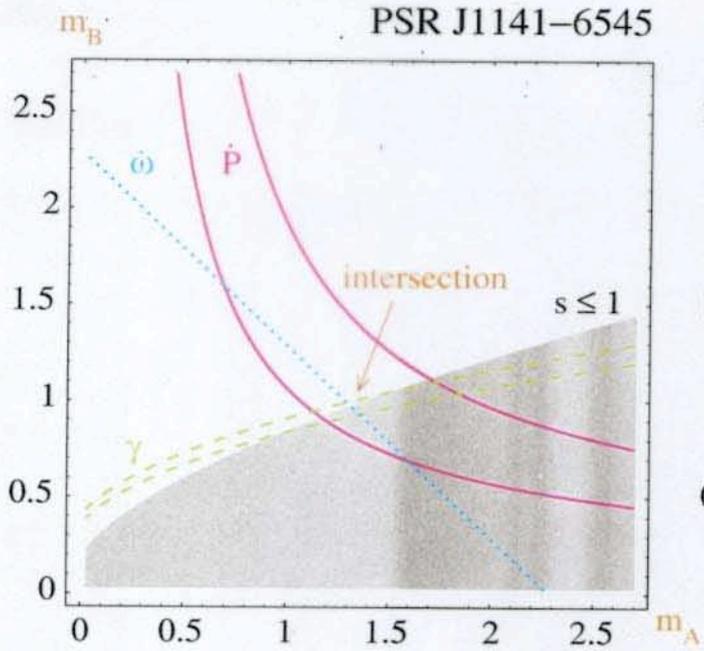
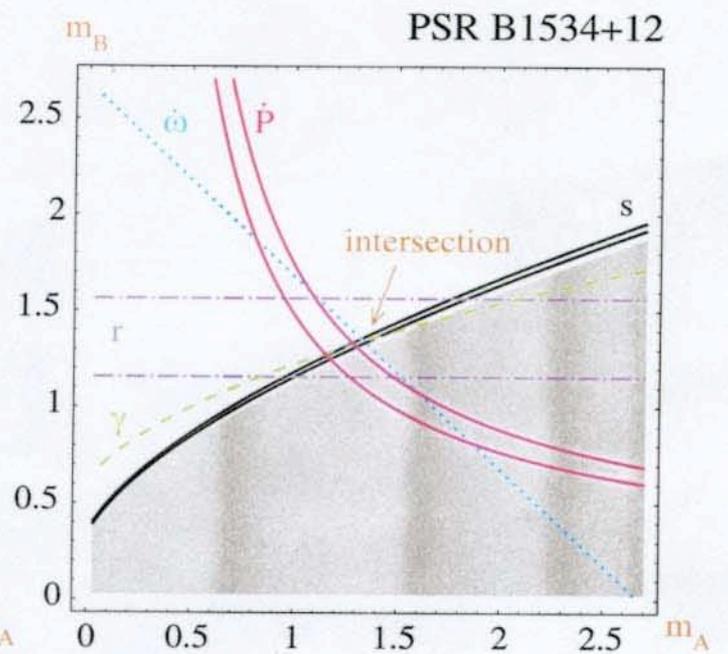
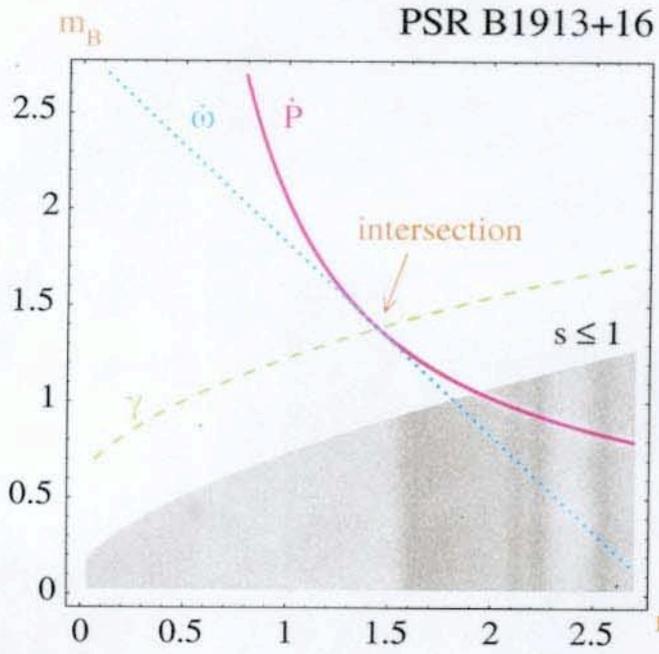
- "THEORY-INDEPENDENT" OR "PHENOMENOLOGICAL"

PARAMETRIZED POST-KEPLERIAN

- "THEORY-DEPENDENT"

- BEYOND USUAL POST-NEWTONIAN PARAMETERS

- CLASSES OF TENSOR-SCALAR THEORIES



# SOME HIGH-PRECISION BINARY-PULSAR TESTS

1913+16:

Weisberg, Taylor '04

Damour Taylor '91  $14\sigma$  CORRECTION

$$\frac{\dot{P}_b^{\text{OBS}} - \dot{P}_b^{\text{GALACTIC}}}{\dot{P}_b^{\text{GR}}[k^{\text{OBS}}, \gamma_{\text{TIMING}}^{\text{OBS}}]} = 1.0013 \pm 0.0021$$

1534+12:

Taylor, Wolszczan, Damour, Weisberg '92  
Stairs et al. '02

$$\frac{s^{\text{OBS}}}{s^{\text{GR}}[k^{\text{OBS}}, \gamma_{\text{TIMING}}^{\text{OBS}}]} = 1.000 \pm 0.007$$

0737-3039

Lyne et al. '04, Kramer et al '04, '06

$$\frac{s^{\text{OBS}}}{s^{\text{GR}}[k^{\text{OBS}}, R^{\text{OBS}}]} = 0.99987 \pm 0.00050$$

RADIATIVE AND STRONG-FIELD EINSTEIN GRAVITY OK<sub>AT</sub>

$10^{-3}$  LEVEL

# A CLASS OF ALTERNATIVES TO GR:

TENSOR-SCALAR GRAVITY

$$S = \frac{1}{16\pi G_0} \int \sqrt{g} [R(g) - 2(\partial\phi)^2] + S_{\text{MATTER}}[\text{matter}; \tilde{g}_{\mu\nu} = e^{2\alpha(\phi)} g_{\mu\nu}]$$

"EINSTEIN METRIC"  $\nearrow$   $R(g)$   
 SCALAR FIELD  $\phi$   $\nearrow$   $(\partial\phi)^2$   
 "MATTER METRIC"  $\nearrow$   $\tilde{g}_{\mu\nu}$

e.g.  $-\int \tilde{m}_A d\tilde{s} = -\int \tilde{m}_A e^{\alpha(\phi)} ds$

HERE, ONE HAS KEPT THE 'UNIVERSAL COUPLING' MATTER/GRAVITY

[CAN BE AN APPROXIMATION OF EP-VIOLATING DILATON-LIKE COUPLINGS]

EXAMPLES:

- IN KALUZA-KLEIN, AND JORDAN-FIERZ-BRANS-DICKE ONE NATURALLY GETS

$$\tilde{g}_{\mu\nu}^{\text{matter}} = e^{\alpha\phi} g_{\mu\nu}^{\text{Einstein}}$$

- NON-MINIMAL COUPLING  $(\square + \xi R)\phi = 0$

$$S = \int -(\partial\phi)^2 + \xi \phi^2 R + \int \frac{\sqrt{g} R}{16\pi G_0} + S_{\text{matter}}[g_{\mu\nu}]$$

NATURALLY LEADS TO

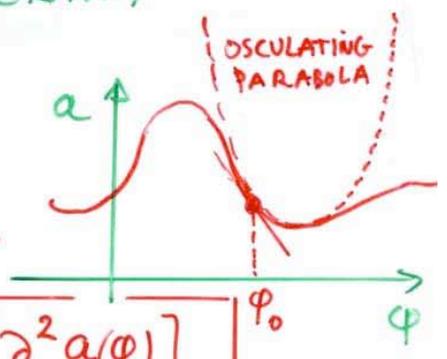
$$g_{\mu\nu}^{\text{matter}} = f(\phi) g_{\mu\nu}^{\text{Einstein}}$$

# THE TWO BASIC COUPLING PARAMETERS OF WEAK-FIELD TENSOR-SCALAR GRAVITY

$$m_A(\varphi) = \exp[a(\varphi)] m_A^{(0)}$$

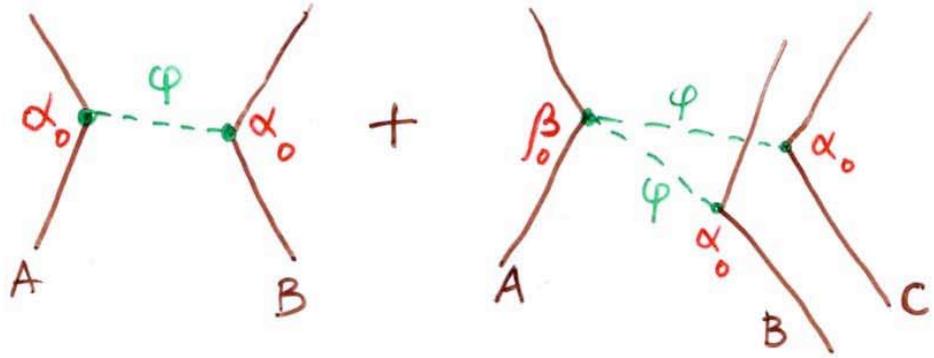
$$\varphi = \varphi_0 + \delta^{local} \varphi$$

↑  
COUPLING FUNCTION



$$\alpha_0 \equiv \left[ \frac{\partial a(\varphi)}{\partial \varphi} \right]_{\varphi_0} \quad \beta_0 \equiv \left[ \frac{\partial^2 a(\varphi)}{\partial \varphi^2} \right]_{\varphi_0}$$

NEW INTERACTIONS:



$$\Rightarrow \mathcal{L}^{2\text{-body}} = \frac{1}{2} \sum_{A \neq B} G_{AB} \frac{m_A m_B}{r_{AB}} \left[ 1 + \frac{G \text{Reffects}}{c^2} + \bar{\gamma} \frac{(\vec{v}_A - \vec{v}_B)^2}{c^2} \right]$$

$$\mathcal{L}^{3\text{-body}} = -\frac{1}{2} \sum_{B \neq A \neq C} (1 + 2\bar{\beta}) \frac{G_{AB} G_{AC} m_A m_B m_C}{r_{AB} r_{AC} c^2}$$

MORE GENERALLY

$$\bar{\gamma} = \frac{\langle \text{spin}^2 \rangle - 4}{2}$$

OBSERVABLE  
"POST-EINSTEIN"  
PARAMETERS

$$\bar{\gamma} \equiv \gamma_{PPN} - 1 = -2 \frac{\alpha_0^2}{1 + \alpha_0^2}$$

$$\bar{\beta} \equiv \beta_{PPN} - 1 = +\frac{1}{2} \beta_0 \frac{\alpha_0^2}{(1 + \alpha_0^2)^2}$$

LIGHT-DEFLECTION EXPTS  
MEASURE DIRECTLY  
THE BASIC  $\alpha_0$

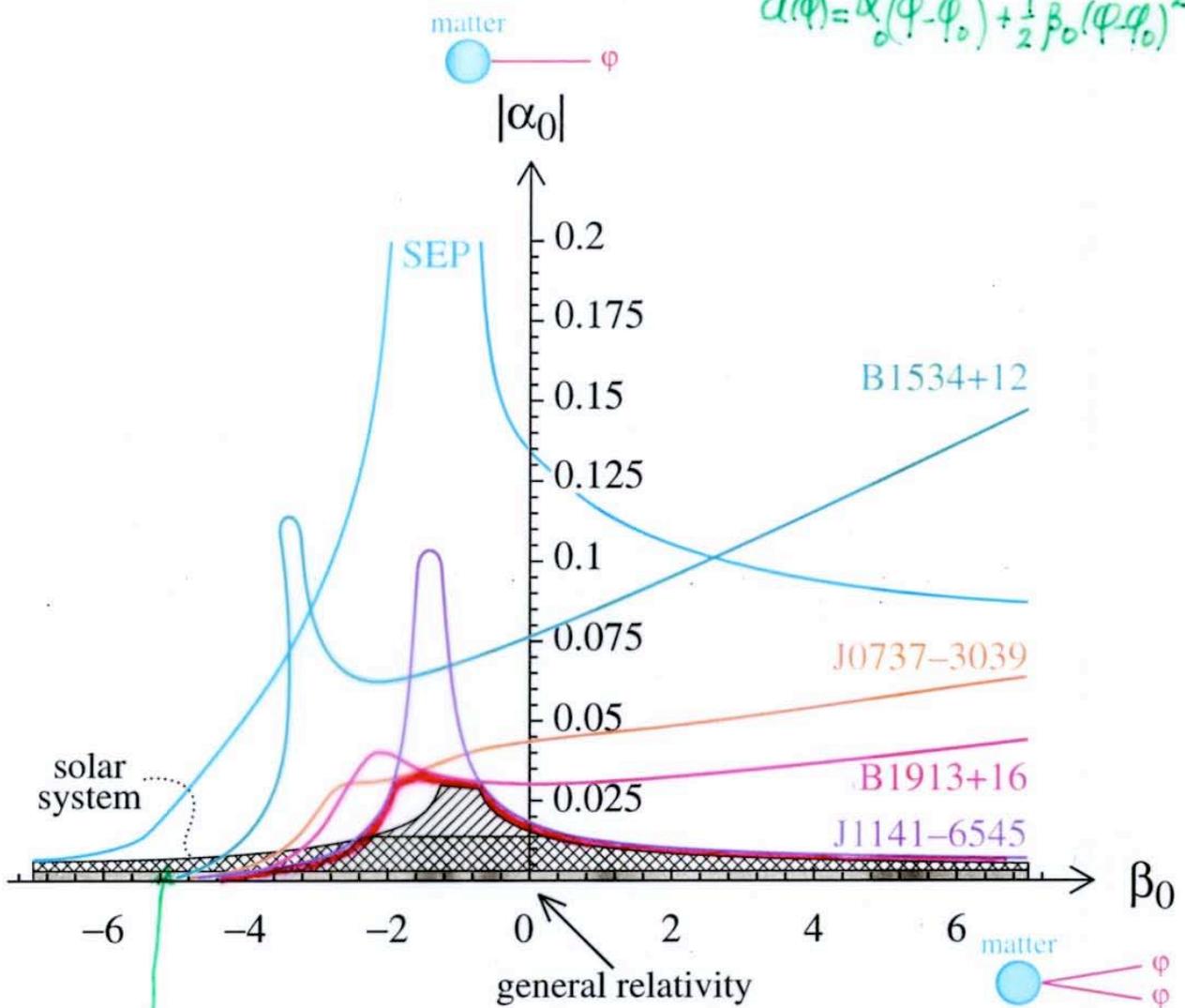
PPN PARAMETERS OF EDDINGTON-NORDTVEED-WILL

$$G_{AB} = G e^{2a(\varphi_0)} [1 + \alpha_0^2] \left[ 1 + (4\bar{\beta} - \bar{\gamma}) \left( \frac{E_A^{grav}}{m_A c^2} + \frac{E_B^{grav}}{m_B c^2} \right) \right]$$

# THEORY-SPACE ANALYSIS OF PULSAR AND SOLAR-SYSTEM DATA

$$T(\alpha_0, \beta_0): S = \frac{1}{16\pi G_*} \int \sqrt{g} [R(g) - 2(\partial\varphi)^2] + S_{\text{MATTER}}[\text{matter}; \tilde{g}_{\mu\nu} = e^{2\alpha(\varphi)} g_{\mu\nu}]$$

$$\alpha(\varphi) = \alpha_0(\varphi - \varphi_0) + \frac{1}{2}\beta_0(\varphi - \varphi_0)^2$$

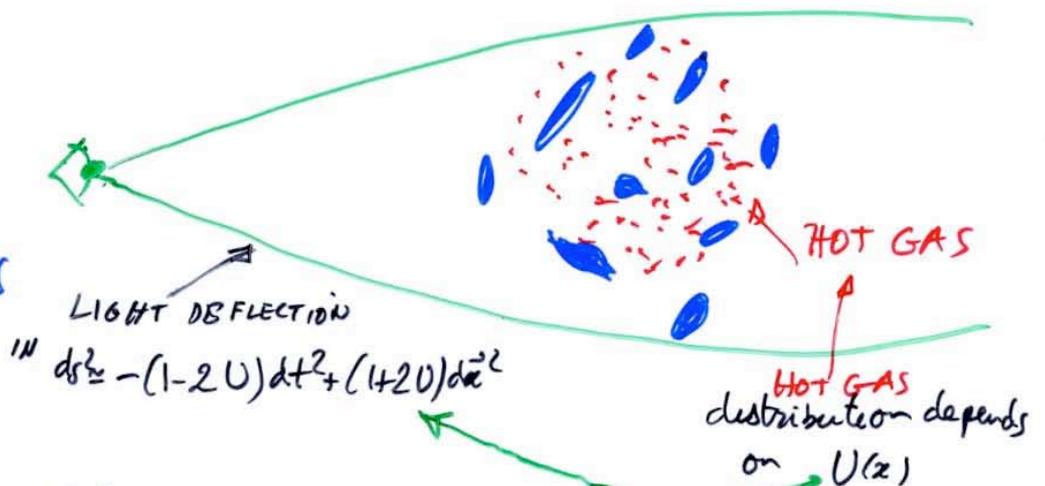


DOMAIN OF THEORY-SPACE  
 ALLOWED BY SOLAR-SYSTEM EXPTS  
 BUT EXCLUDED BY BINARY PSR ONES

(Domenico Esposito-Farise '06)

# TESTS OF GRAVITY ON VERY LARGE SCALES

GRAVITATIONAL LENSING BY GALAXY CLUSTERS



→ INDEPENDENTLY OF 'DARK MATTER ISSUE' ONE CHECKS CONSISTENCY OF GR ON LENGTH SCALES  $\sim 100$  kpc

PRIMORDIAL NUCLEOSYNTHESIS OF LIGHT ELEMENTS (He, Li, D)

CHECK THAT

$$\frac{G_{\text{BBN}}}{G_{\text{NOW}}} \approx 1 + \mathcal{O}(10\%)$$

ALSO: TESTS OF GRAVITY ON SMALL SCALES

SEVERAL EXPERIMENTS HAVE RECENTLY PROBED NEWTON'S  $1/r^2$  LAW ON SMALL DISTANCES.

NO DEVIATION HAS BEEN FOUND DOWN TO  $\sim 0.2$  mm