Twistors and Gauge Theory

Collège de France

April 19, 2005

The Storyline

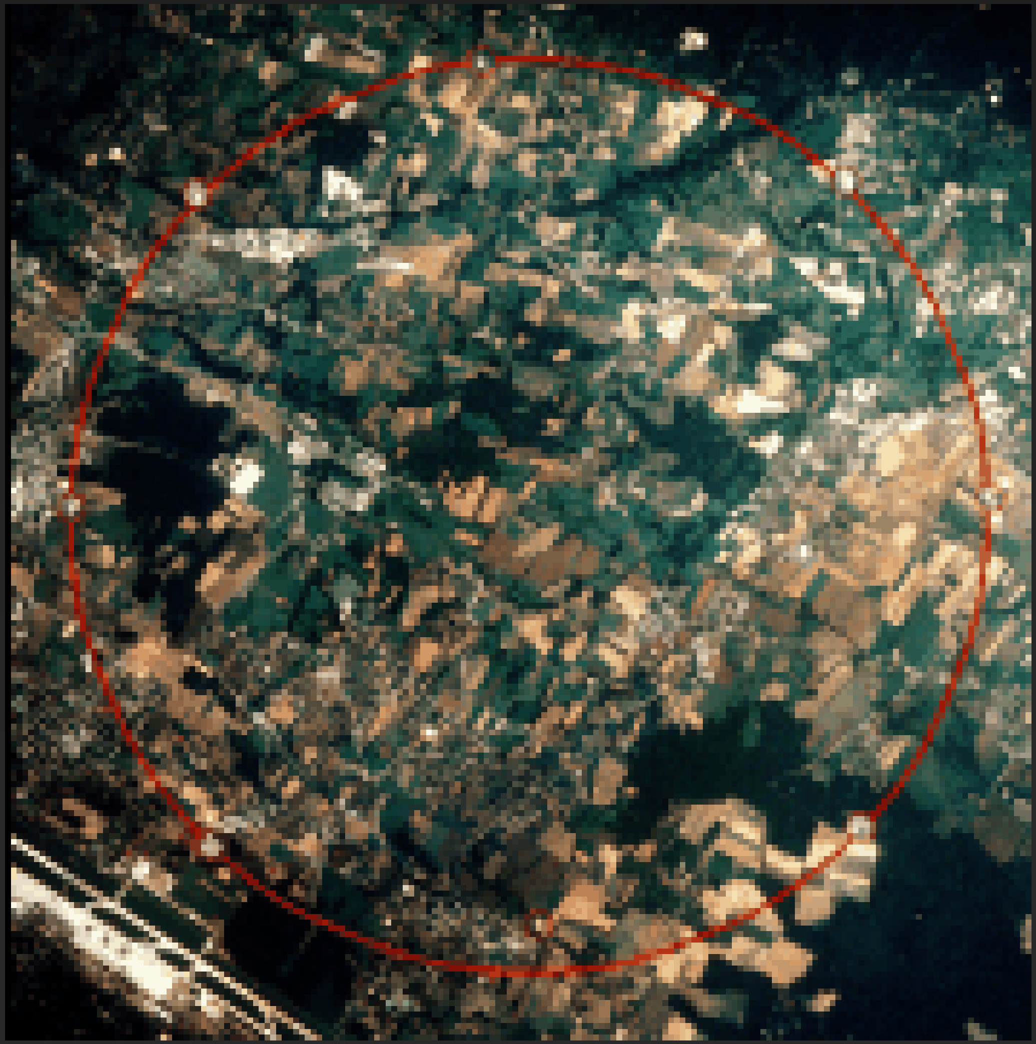
- We're in the midst of exciting advances in calculating amplitudes in gauge theories
- Motivation for hard calculations
- Twistor-space ideas originating with Nair and Witten
- Explicit calculations led to uncovering simple twistor-space structure
- Twistor-space structure led — and is leading — to new representations and techniques for calculations
- Powerful combination with another class of nonconventional techniques, the unitarity-based method for loop calculations

Why Calculate Amplitudes?

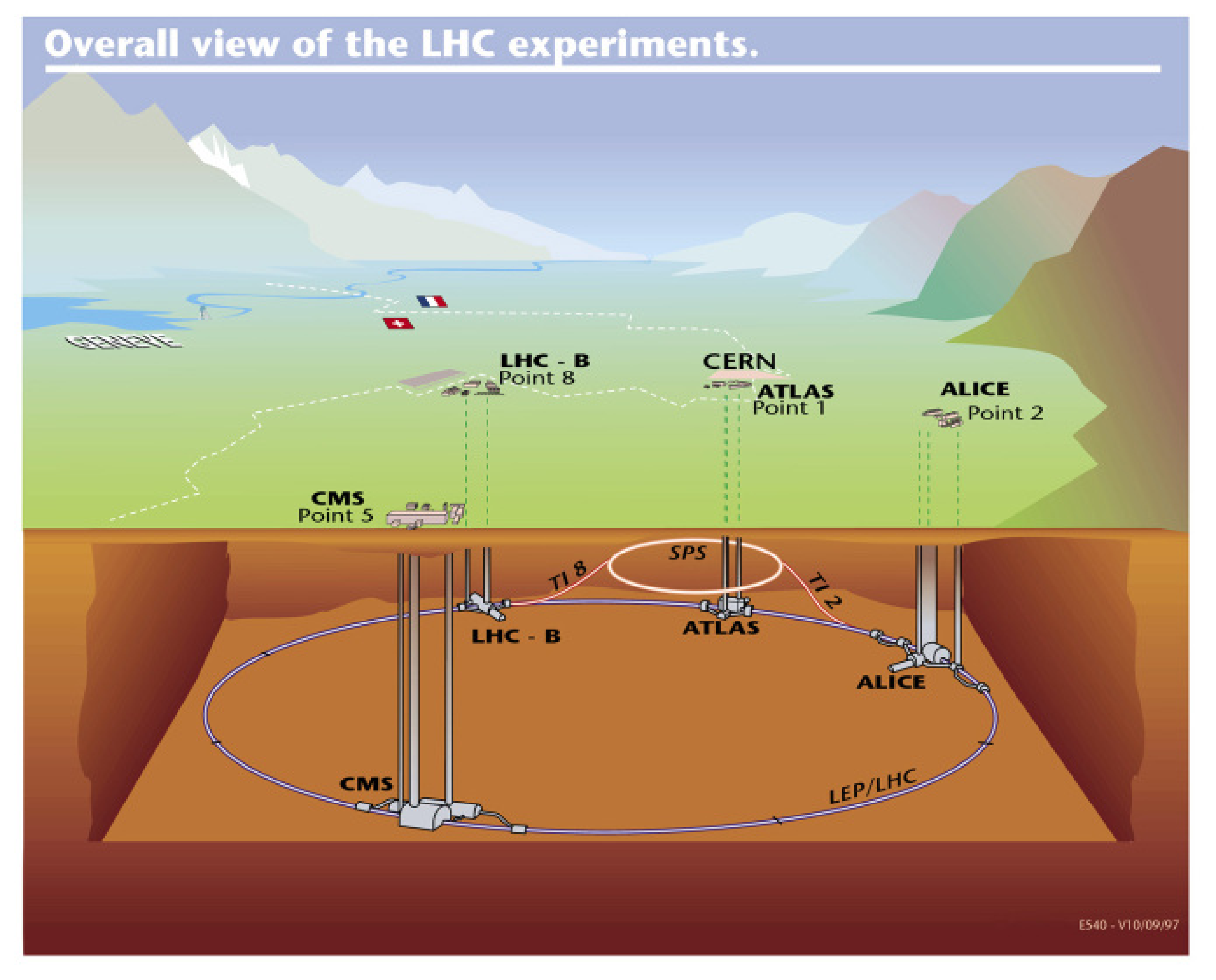
- ~~It's an easy way to while away your professional time~~
- ~~It's a popular activity~~

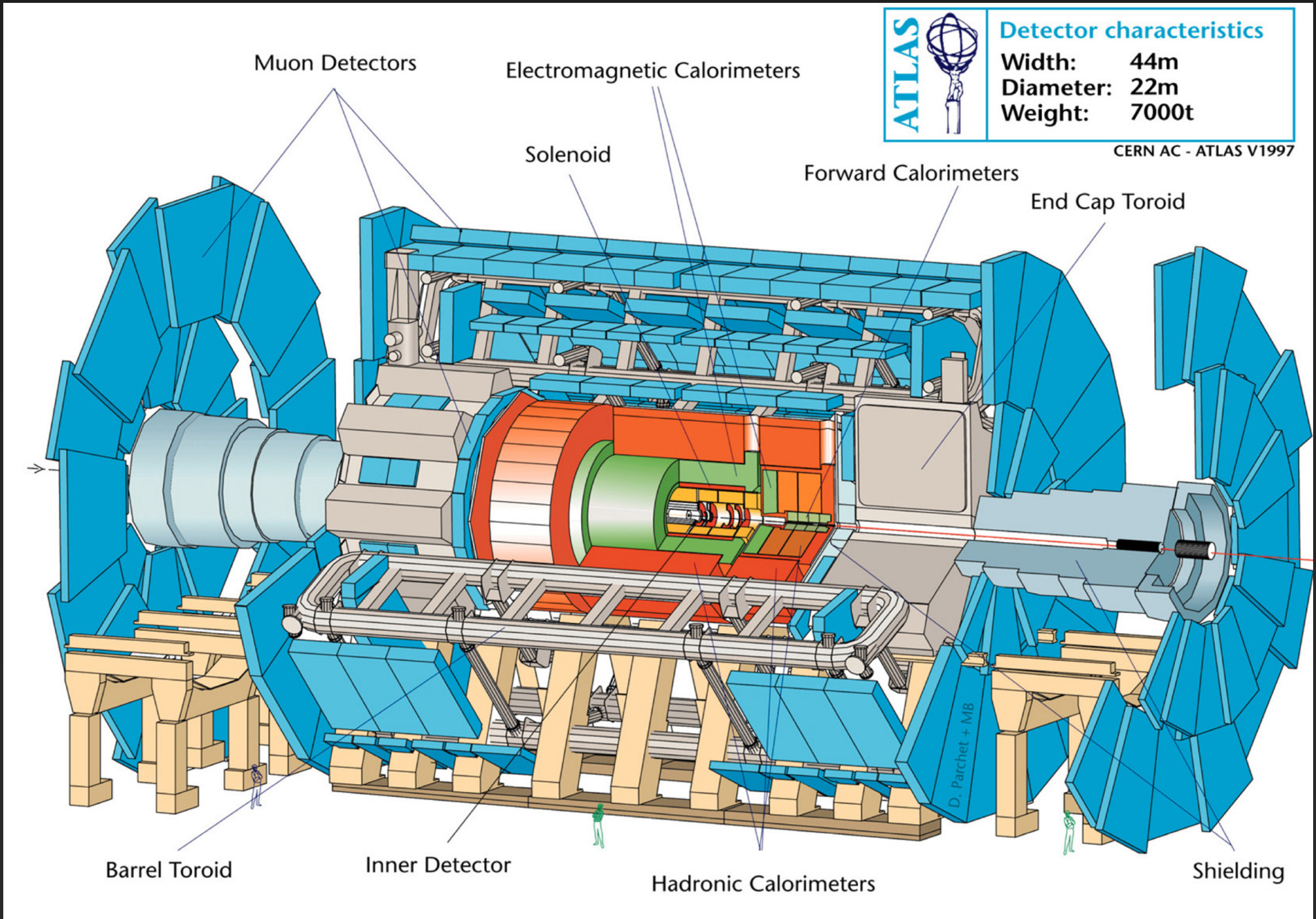
- There are strong physics motivations: LHC physics
- There are strong mathematical physics motivations: study of AdS/CFT duality

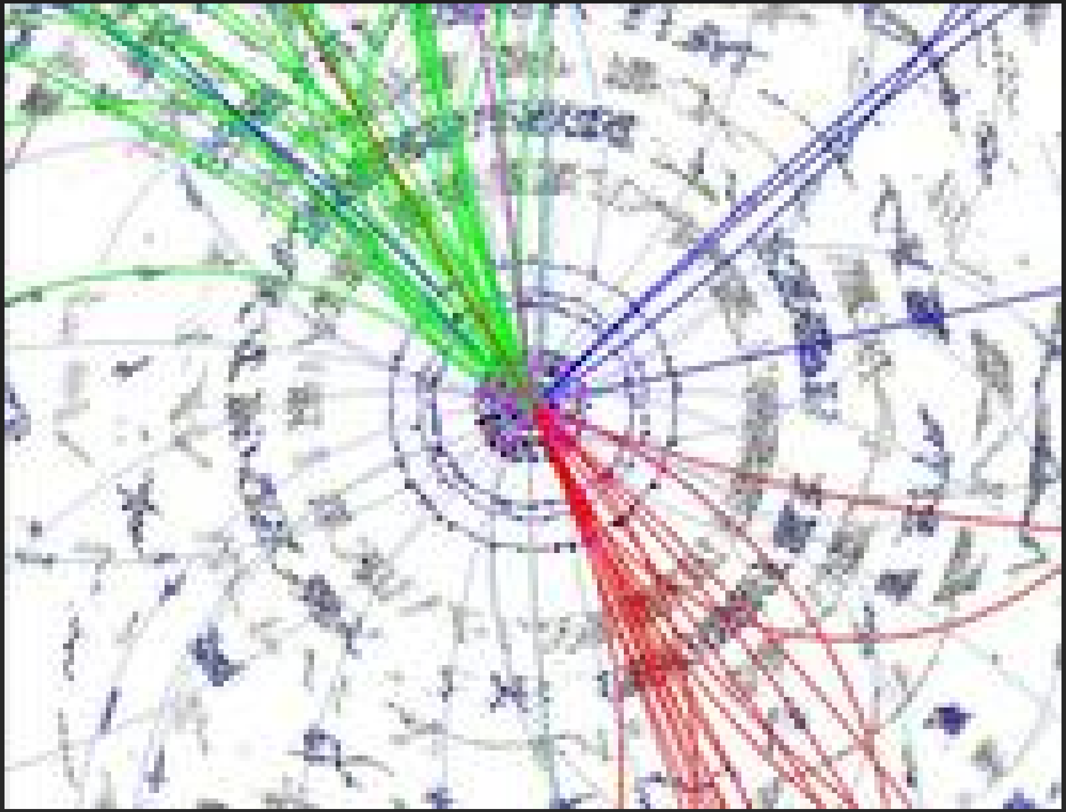
LHC Is Coming, LHC Is Coming!



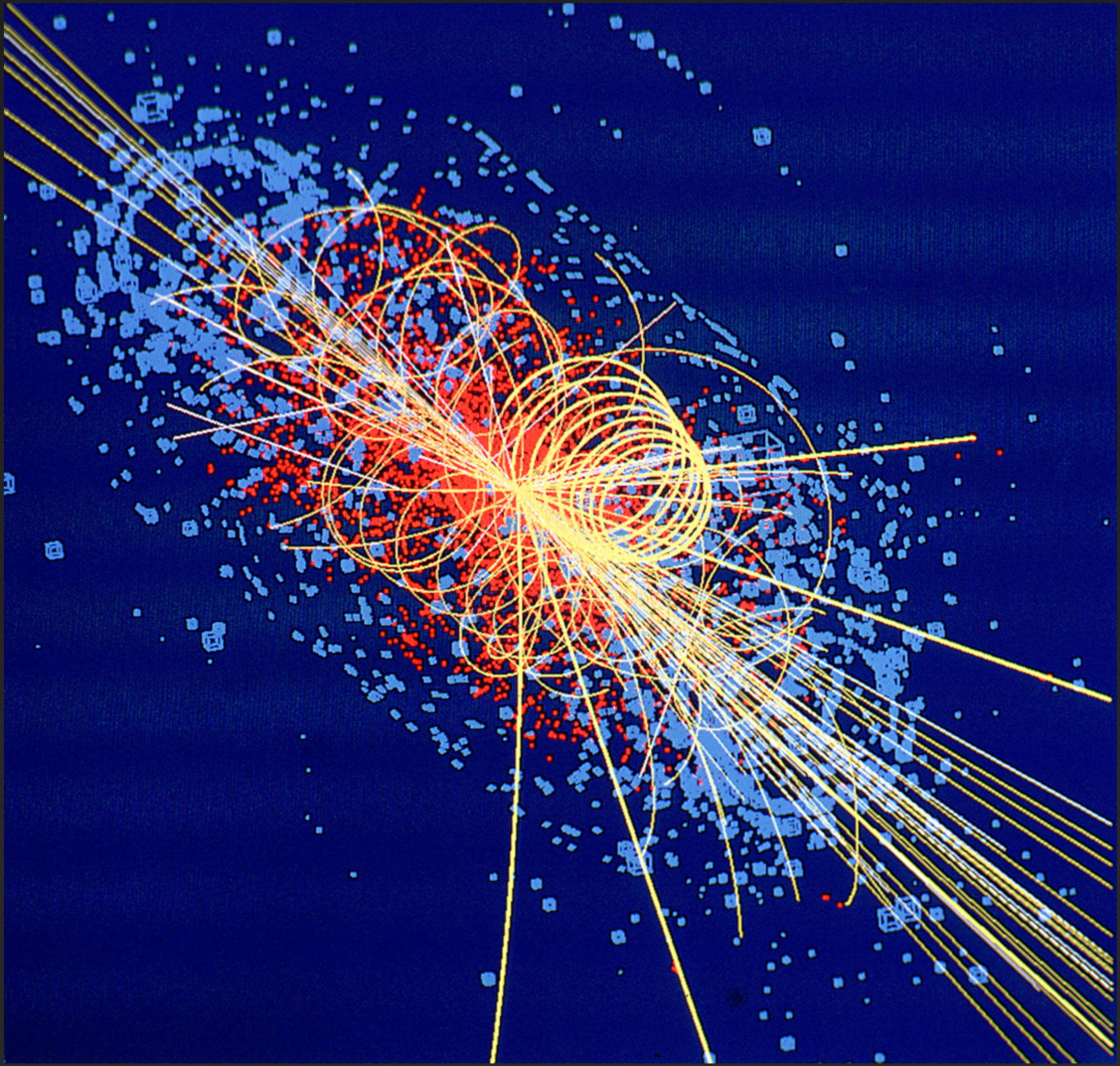
Less than O(1000) days away


Overall view of the LHC experiments.
LHC - B
Point 8
CERN
ATLAS
Point 1
ALICE
Point 2
CMS
Point 5
SPS
TI 8
TI 2
LHC - B
ATLAS
ALICE
CMS
LEP/LHC
E540 - V10/09/97

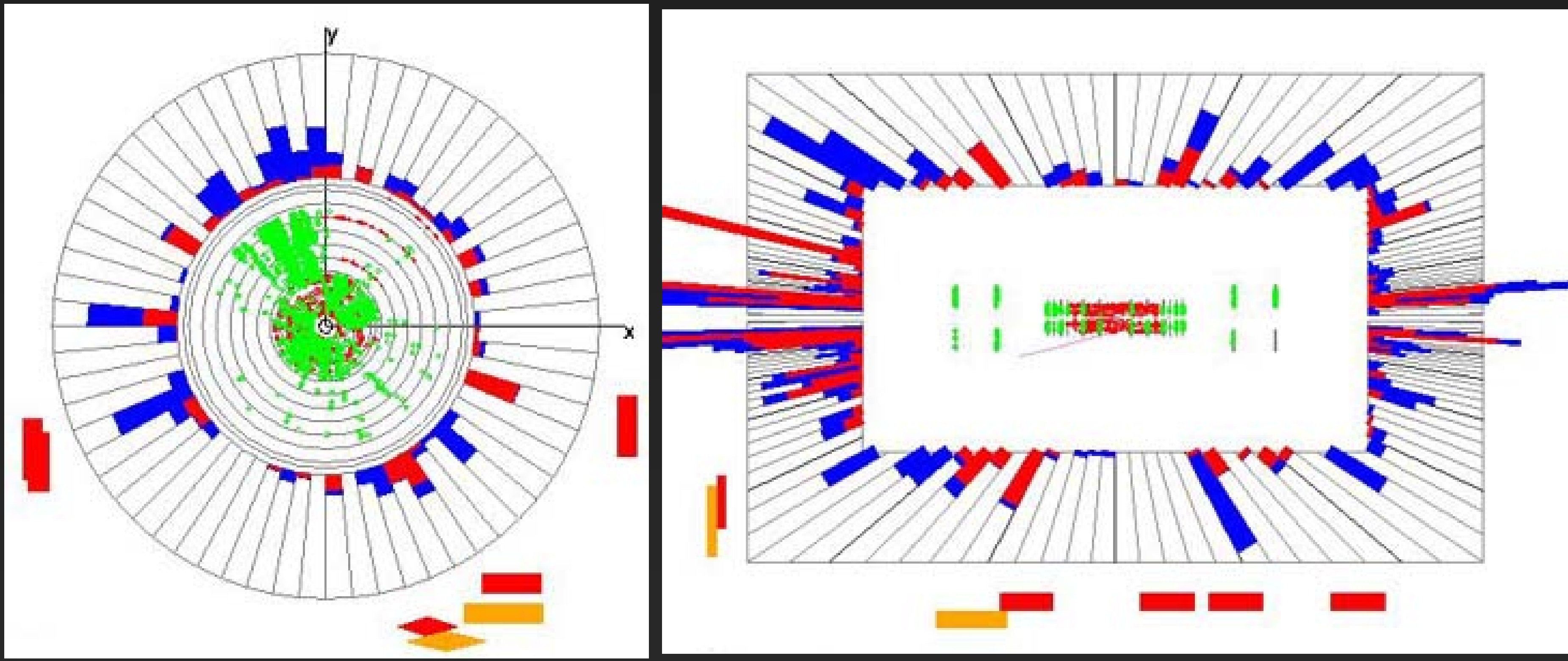

ATLAS
Detector characteristics
Width: 44m
Diameter: 22m
Weight: 7000t
CERN AC - ATLAS V1997
Muon Detectors
Electromagnetic Calorimeters
Solenoid
Forward Calorimeters
End Cap Toroid
Barrel Toroid
Inner Detector
Hadronic Calorimeters
Shielding
D. Parchet + MB



CDF event



CMS Higgs event simulation



D0 event

Event rates

Event production rates at L=10^{33} cm^{-2} s^{-1} and statistics to tape

Process	Events/s	Evts on tape, 10 fb^{-1}
$W \to e\nu$	15	10^8
$Z \to ee$	1	10^7
$t\bar{t}$	1	10^6
gluinos, m=1 TeV	0.001	10^3
Higgs, m=130 GeV	0.02	10^4
Minimum bias	10^8	10^7
$b\bar{b} \to \mu X$	10^3	10^7
QCD jets p_T>150 GeV/c	10^2	10^7

assuming 1% of trigger bandwidth (applies to the last three rows)

⇒ statistical error negligible after few days!

⇒ dominated by systematic errors (detector understanding, luminosity, theory)

G. Dissertori
ETH Zürich

KITP, Collider Physics, Jan 04

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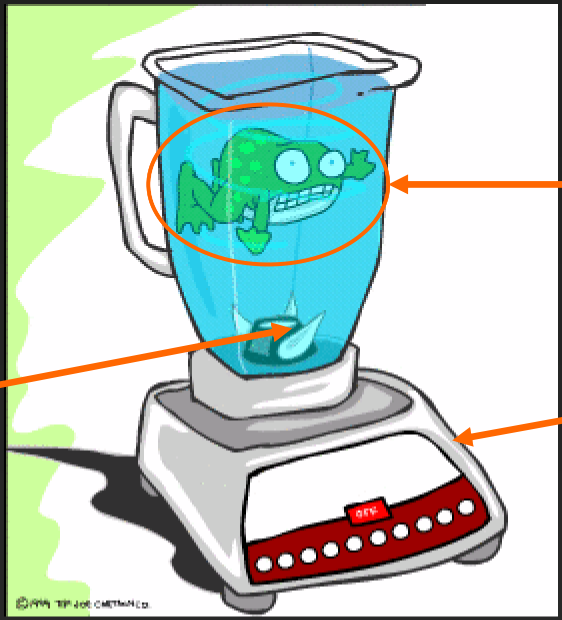
Guenther Dissertori, ETH Zurich (KITP Collider Conf 1-12-04) Issues in

Guenther Dissertori (Jan '04)

- In the modern era, knowledge comes from the confrontation of theory and experiment



Theory

Cutting edge:
precision calculations

Experiment

- In physics, that confrontation is quantitative
- And most powerful when it is systematic

Precision Perturbative QCD

- Predictions of signals, signals+jets
- Predictions of backgrounds
- Measurement of luminosity
- Measurement of fundamental parameters (α_s, m_t)
- Measurement of electroweak parameters
- Extraction of parton distributions — ingredients in any theoretical prediction

Everything at a hadron collider involves QCD

An experimenter's wishlist

- Hadron collider cross-sections one would like to know at NLO

Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

AdS/CFT Duality

- Strong-coupling $\mathcal{N}=4$ supersymmetric gauge theory
  $\Leftrightarrow$ String Theory on $AdS_5 \times S^5$
  for large N_c, $g^2 N_c$ $\Rightarrow$ perturbative supergravity

Maldacena (1997)
Gubser, Klebanov, & Polyakov; Witten (1998)

Strong–weak duality

- Many tests on quantities protected by supersymmetry

D'Hoker, Freedman, Mathur, Matusis, Rastelli, Liu, Tseytlin, Lee, Minwalla, Rangamani, Seiberg, Gubser, Klebanov, Polyakov & others

- More recently, tests on unprotected quantities

Berenstein, Maldacena, Nastase; Beisert, Frolov, Staudacher & Tseytlin; Minahan & Zarembo; & others (2002–4)

A New Duality

Topological B-model string theory

$\Leftrightarrow$

$\mathcal{N}$ =4 supersymmetric gauge theory

Weak–weak duality

- Computation of scattering amplitudes
- Novel differential equations

Witten (2003)

Roiban, Spradlin, & Volovich; Berkovits & Motl; Vafa & Neitzke; Siegel (2004)

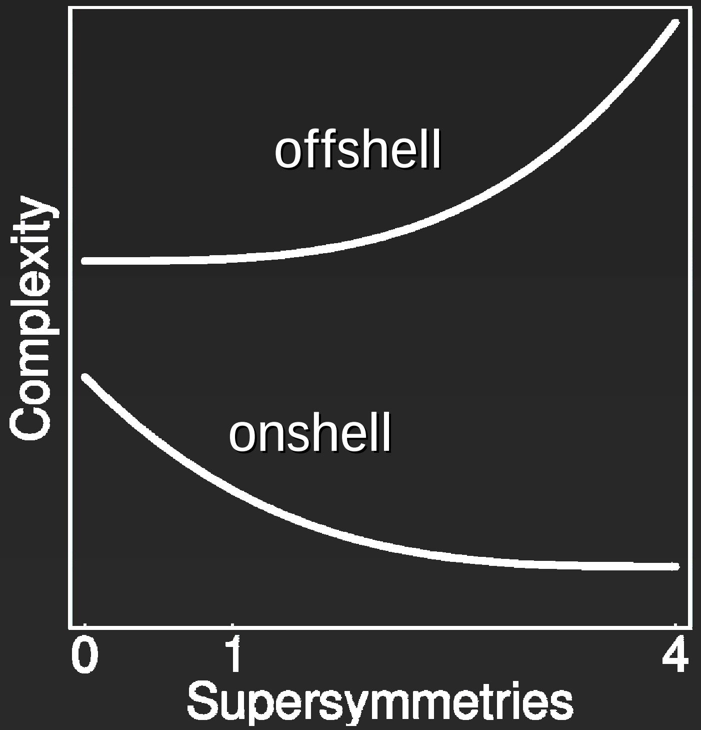
- Novel factorizations of amplitudes

Cachazo, Svrcek, & Witten (2003)

Witten, hep-th/0312171
Cachazo, Svrček & Witten, hep-th/0403047, th/0406177, th/0409245
Bena, Bern & DAK, hep-th/0406133
DAK, hep-th/0406175
Georgiou, Glover, & Khoze, hep-th/0407027; Wu & Zhu, hep-th/0406146
Brandhuber, Spence, & Travaglini, hep-th/0407214; Bedford, Brandhuber, Spence, & Travaglini, hep-th/0410280, th/0412108, hep-th/0502146
Bena, Bern, DAK & Roiban, hep-th/0410054
Cachazo, hep-th/0410077; Britto, Cachazo, & Feng, hep-th/0410179, th/0411107, th/0412103, th/0412308, th/0503132; Britto, Cachazo, Feng & Witten, hep-th/0501052; Cachazo & Svrček, hep-th/0502160
Bern, Del Duca, Dixon, DAK, hep-th/0410224; Bern, Dixon, DAK, th/0412210, th/0501240
Bidder, Bjerrum-Bohr, Dixon, & Dunbar, hep-th/0410296; Bidder, Bjerrum-Bohr, Dunbar, & Perkins, hep-th/0412023, th/0502028
Dixon, Glover, & Khoze, hep-th/0411092; Badger, Glover, Khoze, th/0412275
Bern, Forde, Mastrolia, DAK, hep-ph/0412167
Roiban, Spradlin, & Volovich hep-th/0412265; Britto, Feng, RSV, th/0503198
Luo & Wen, hep-th/0501121

The Amazing Simplicity of N=4 Perturbation Theory

- Manifestly N=4 supersymmetric calculations are very hard off-shell — much harder than ordinary gauge theory



- But on-shell calculations are much simpler than in nonsupersymmetric theories:
  - 4-pt one-loop = tree × one-loop scalar box

    Green & Schwartz (1982)

  - 5, 6, 7-pt one-loop known & simpler than QCD

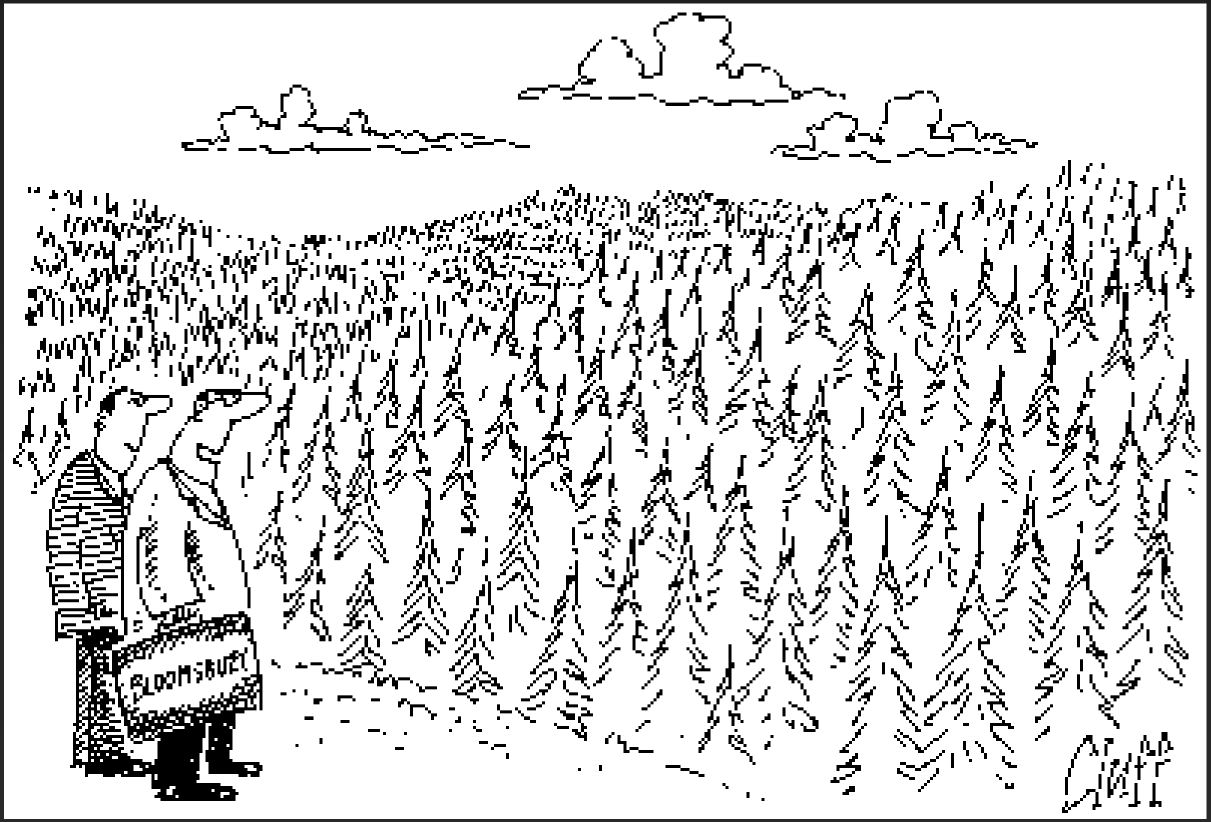
    Bern, Dixon, Dunbar, DAK (1994);
    Britto, Cachazo, Feng (2004);
    Bern, Del Duca, Dixon, DAK (2004)

  - all-n one-loop known for special helicities

    Bern, Dixon, Dunbar, DAK (1994);
    Bern, Dixon, DAK (2004)

Supersymmetry

Most often pursued in broken form as low-energy phenomenology



"One day, all of these will be supersymmetric phenomenology papers."

Exact Supersymmetry As a Computational Tool

- All-gluon amplitudes are the same at tree level in N=4 and QCD
- Fermion amplitudes obtained through Supersymmetry Ward Identities   Grisaru, Pendleton, van Nieuwenhuizen (1977); Kunszt, Mangano, Parke, Taylor (1980s)
- At loop level, N=4 amplitudes are one contribution to QCD amplitudes; N=1 multiplets give another

Gauge-theory amplitude

↓ Color decomposition & stripping

Color-ordered amplitude: function of k_i and ε_i

↓ Spinor-helicity basis

Helicity amplitude: function of spinor products and helicities ± 1

↓

Function of spinor variables and helicities ± 1

↓ Half-Fourier transform

Conjectured support on simple curves in twistor space

Spinors

- Want square root of Lorentz vector ⇒ need spin ½
- Spinors λ_a, conjugate spinors $\tilde{\lambda}_{\dot{a}}$
- Spinor product $\langle\lambda_1\lambda_2\rangle = -\langle\lambda_2\lambda_1\rangle = \epsilon^{ab}\lambda_{1a}\lambda_{2b}$
  $[\tilde{\lambda}_1\tilde{\lambda}_2] = -[\tilde{\lambda}_2\tilde{\lambda}_1] = \epsilon_{\dot{a}\dot{b}}\tilde{\lambda}_1^{\dot{a}}\tilde{\lambda}_2^{\dot{b}}$
- (½,0) ⊗ (0, ½) = vector $p_{a\dot{a}} = \sigma^{\mu}_{a\dot{a}}p_{\mu}$

$$p^2 = 0 \Leftrightarrow \det(p_{a\dot{a}}) = 0 \Leftrightarrow p_{a\dot{a}} = \lambda_a\tilde{\lambda}_{\dot{a}}$$

- Helicity ±1: $\epsilon_{a\dot{a}} = \dfrac{\lambda_a\tilde{\kappa}_{\dot{a}}}{[\tilde{\lambda}\tilde{\kappa}]}$

  ⇒ Amplitudes as pure functions of spinor variables

Parke–Taylor Amplitudes

- Pure gluon amplitudes
- All gluon helicities + ⇒ amplitude = 0
- Gluon helicities +–+…+ ⇒ amplitude = 0
- Gluon helicities +–+…+–+ ⇒ MHV amplitude

$$A_n(1^+,\ldots,m_1^-,(m_1+1)^+,\ldots,m_2^-,(m_2+1)^+,\ldots,n^+) =$$

$$i\frac{\langle m_1\, m_2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\cdots\langle (n-1)\,n\rangle\,\langle n\,1\rangle}$$

Parke & Taylor (1986)

- Holomorphic in spinor variables
- Proved via recurrence relations

Berends & Giele (1988)

Correlators in Projective Space

- Null cones $\Leftrightarrow$ points in twistor space

Penrose (1972)

- Spinors are homogeneous coordinates on complex projective space CP^1
- Current algebra on $\mathrm{CP}^1 \Rightarrow$ amplitudes
- Reproduce maximally helicity violating (MHV) amplitudes

Nair (1988)

Let's Travel to Twistor Space!

It turns out that the natural setting for amplitudes is not exactly spinor space, but something similar. The motivation comes from studying the representation of the conformal algebra.

Half-Fourier transform of spinors: transform $\tilde{\lambda}_{\dot{a}}$, leave alone λ_a
⇒ Penrose's original twistor space, real or complex

$$\tilde{\lambda}_{\dot{a}} \to i\frac{\partial}{\partial \mu^{\dot{a}}}, \qquad -i\frac{\partial}{\partial \tilde{\lambda}^{\dot{a}}} \to \mu_{\dot{a}}$$

Study amplitudes of definite helicity: introduce homogeneous coordinates $Z_I = (\lambda_a, \mu_{\dot{a}}) \equiv \tau(\lambda_a, \mu_{\dot{a}})$

⇒ CP^3 or RP^3 (projective) twistor space

Back to momentum space by Fourier-transforming μ