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Heavy Quarks

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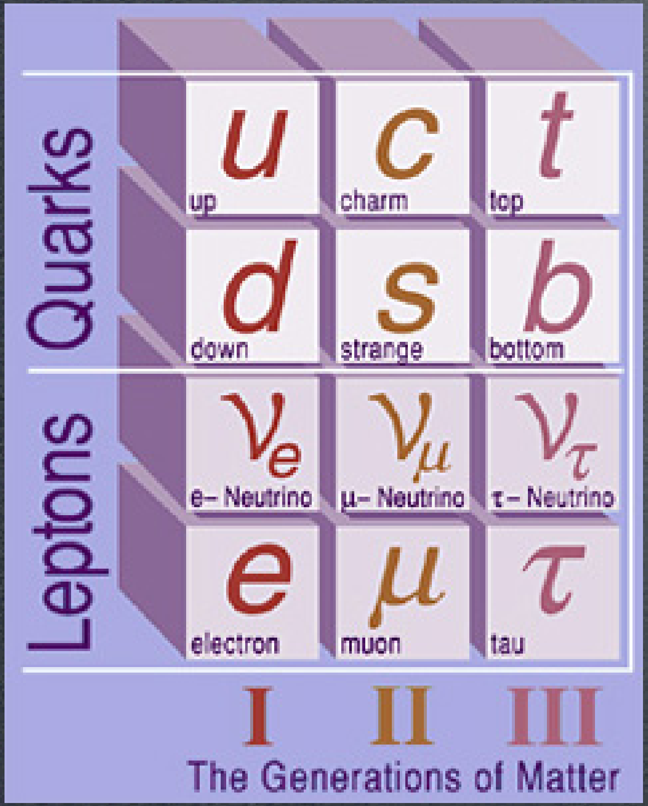
- Historical introduction
- Quarkonium
- Heavy quarks in QCD and pQCD
- Some phenomenological examples

The Standard Model



Three replicas of a family composed of two weak isospin doublets, one of leptons and one of quarks

The three families are roughly ordered with increasing mass. Because of this, of course, we started discovering them from the first (the common matter constituents, electrons, protons and neutrons) onwards.

In QCD, however, ``heavy quark" has a very specific meaning, which we shall detail later on.

In particular, we shall concentrate on the **charm**, **bottom** and **top** quarks

In the beginning, it was just $SU(3)_{flavour}$: Gell-Mann and the `Eightfold way'

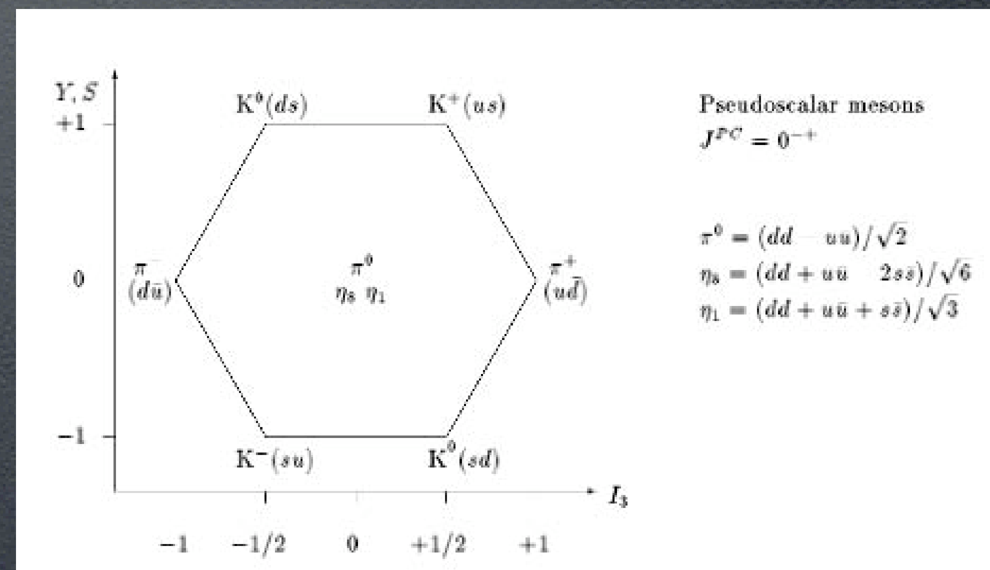
Fig. 6.35 Murray Gell-Mann (b.1929).

Murray Gell-Mann

> "*Three quarks for Muster Mark*"
> J. Joyce, Finnegan's Wake

Only three almost-degenerate light quarks, forming the observed mesons and baryons

$$3 \otimes \bar{3} = 8 \oplus 1$$
$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$



No need for further (heavier) quarks at this point: the three light ones were properly accounting for the observed hadron spectra

Enter GIM, and electroweak interactions

In 1970 Glashow, Iliopoulos and Maiani postulated he existence of a fourth quark, slightly heavier than the light ones, and called it **charm**

Purpose: suppress unobserved strangeness-violating Flavour Changing Neutral Currents (FCNCs) and restore lepton-hadron symmetry

Three quarks:
neutral current $J^0_{wk} \sim (\bar{u}, \bar{d}_C)\tau_3 \begin{pmatrix} u \\ d_C \end{pmatrix}$

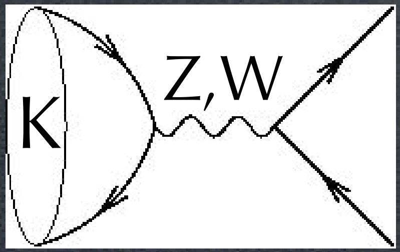
with $d_C = d\cos\theta_C + s\sin\theta_C$

This generates the strangeness-changing term

$$\bar{d}_C d_C = \bar{d}d\cos^2\theta_C + \bar{s}s\sin^2\theta_C + \bar{d}s\sin\theta_C\cos\theta_C + \bar{s}d\sin\theta_C\cos\theta_C$$

which is not observed:

$$K^0_L = (s\bar{d})$$
$$K^+ = (s\bar{u})$$



$$BR(K^0_L \to \mu^+\mu^-) \simeq 7\times 10^{-9}$$

$$BR(K^+ \to \mu^+\nu_\mu) \simeq .635$$

Four quarks and lepton-hadron symmetry imply

$$J^0_{wk} \sim (\bar{u}, \bar{d}_C)\tau_3 \begin{pmatrix} u \\ d_C \end{pmatrix} + (\bar{c}, \bar{s}_C)\tau_3 \begin{pmatrix} c \\ s_C \end{pmatrix}$$

with $\begin{pmatrix} d_C \\ s_C \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$

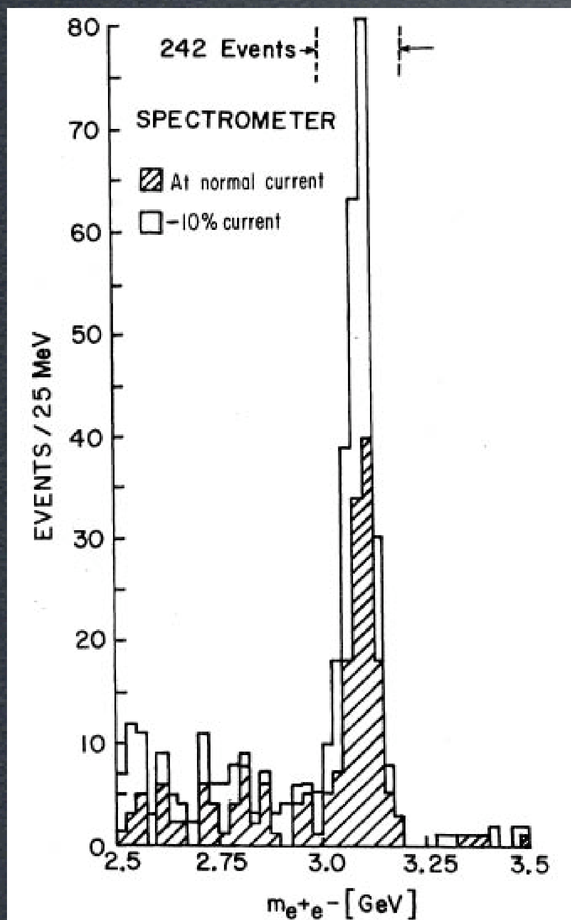
The symmetry eliminates the strangeness-changing terms bit. However, experimental data show that, while heavily suppressed, they are still present.

Hence, the symmetry cannot be exact.
The charm must be slightly heavier than u,d,s

Estimate: mass ~ 2 GeV

The discovery, and the surprise

In November 1974 a HUGE resonance with **EXTREMELY NARROW WIDTH** was simultaneously observed in $p\bar{p}$ collisions at BNL and e^+e^- at SLAC
[Phys. Rev. Lett. 33 , 1406 (1974), Phys. Rev. Lett. 33 , 1404 (1974)]



$M \simeq 3.1$ GeV $\quad \Gamma_h \simeq 70$ keV $\quad \Gamma_l \simeq 3$ keV

The `November revolution'

This was soon interpreted as a charm-anticharm bound state, a strong-interaction analogue of an ortho-positronium state

If we assume that the recently announced resonance[5] with mass ≈ 3 GeV is orthocharmonium, Eq. (5) fixes α_s. Preliminary estimates give[6] $\Gamma_l \approx 3$ keV and $\Gamma_h \approx 75$ keV. Their ratio gives $\alpha_s \approx 0.26$. This, along with $m_{\varphi'} \approx 1.5$ GeV, implies $\Gamma_l \approx 0.8$ keV and $\Gamma_h \approx 20$ keV, surely low estimates

[Appelquist, Politzer, PRL 34]

We say that it is a hadron with $J^P = 1^-$, $I^G = 0^-$: a ${}^3S_1$ bound state of a charmed[7,8] quark $\mathcal{P}'$ and its antiquark with mass below twice the mass of the lightest charmed hadron: "orthocharmonium."[9] Thus,[10]

[De Rujula, Glashow PRL 34]

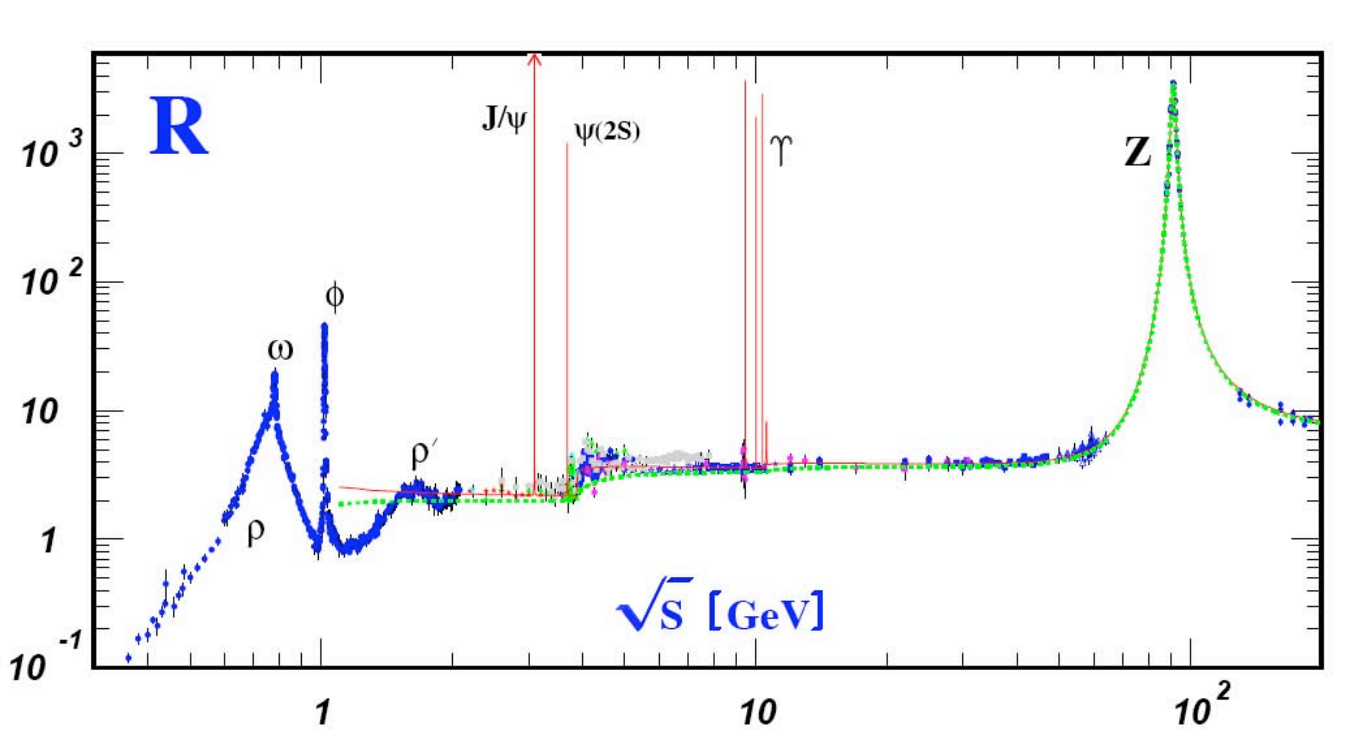
The particle will then take the modern name of J/ψ

Why a surprise

Hadronic resonances are normally LARGE, since they decay by strong interaction and have therefore very short lifetime:

$$10^{-22} - 10^{-23}\ \mathrm{s} \simeq \tau = \frac{1}{\Gamma} \simeq 10 - 200\ \mathrm{M}eV$$

Recall Heisenberg principle: Γ corresponds to the uncertainty on the mass (hence energy) of the particle



$$R \equiv \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$\Gamma_\rho \simeq 150\ \mathrm{M}eV$ $\Gamma_\omega \simeq 8.5\ \mathrm{M}eV$ $\Gamma_\phi \simeq 4.3\ \mathrm{M}eV$ $\Gamma_{J/\psi} \simeq 0.1\ \mathrm{M}eV$

How could the new resonance have a width a factor of 100/1000 smaller, and yet be a strongly interacting particle?

Asymptotic Freedom (Nobel prize 2004, Gross, Wilczek and Politzer)

$$\frac{d\alpha_S(\mu^2)}{d\ln\mu^2} = \beta(\alpha_S)$$

$$\beta(\alpha_S) = -b_0(1 + b_1\alpha_S + b_2\alpha_S^2 + \cdots)$$

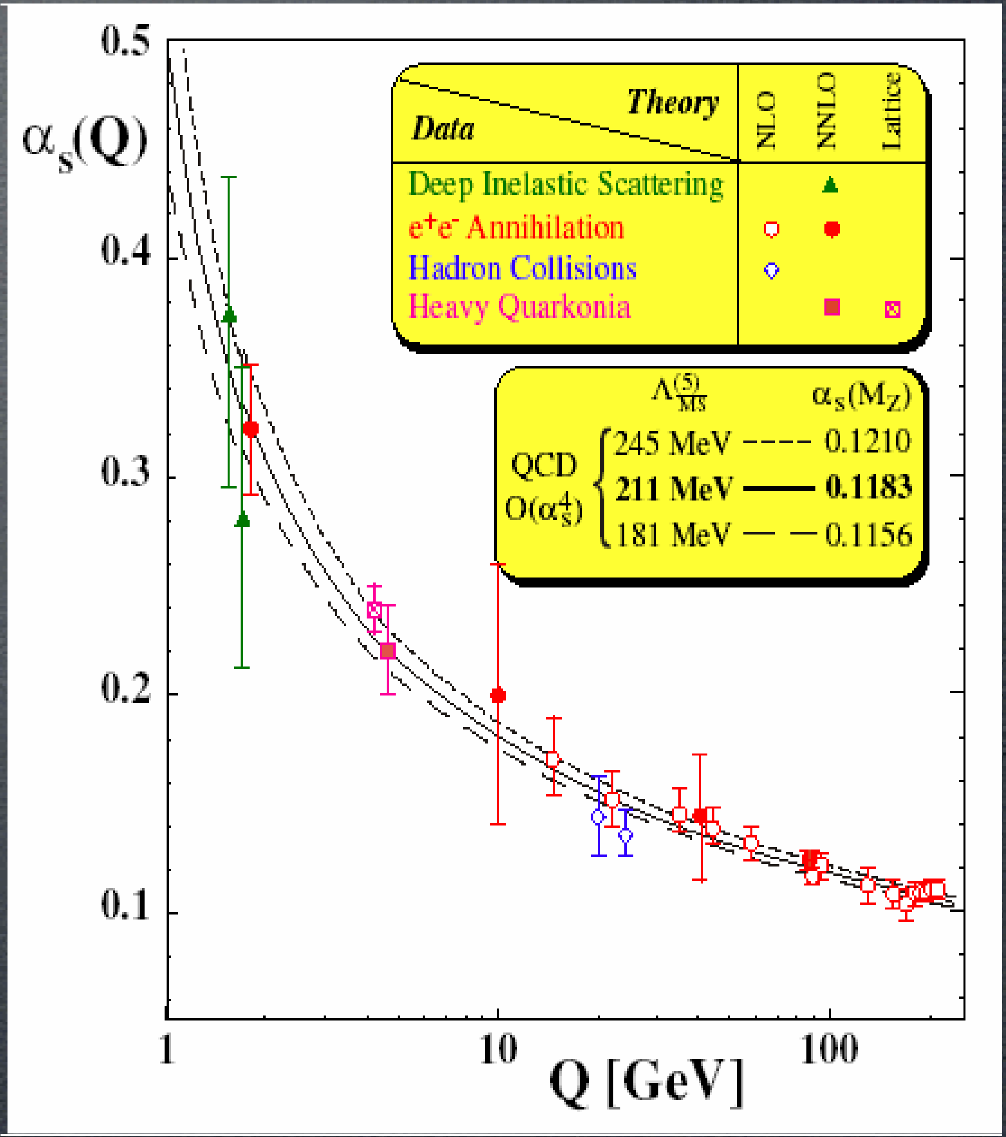
$$b_0 = \frac{33 - 2n_f}{12\pi} > 0 \text{ for } n_f < 16$$

The solution of the Renormalization Group Equation gives (leading order):

$$\alpha_S(\mu^2) = \frac{\alpha_S(\mu_0^2)}{1 + b_0\alpha_S(\mu_0^2)\log(\mu^2/\mu_0^2)}$$

which can also be rewritten as

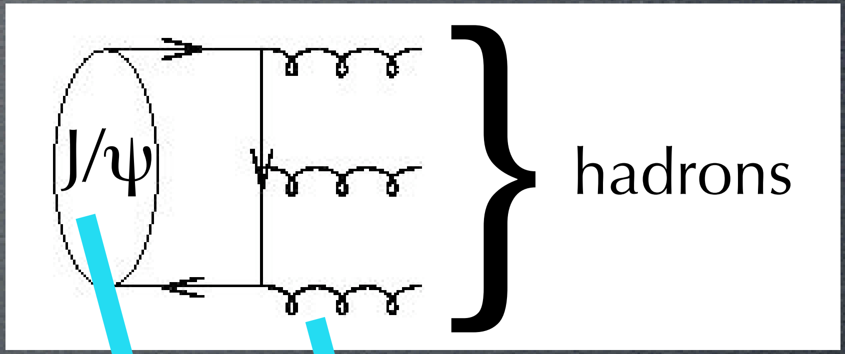
$$\alpha_S(\mu^2) = \frac{1}{b_0\log(\mu^2/\Lambda^2)} \text{ where } \Lambda \simeq 200 \text{ MeV}$$



The J/ψ width

If J/ψ is produced in the interaction of an electron and a positron via a photon it must therefore have the **same quantum numbers as the photon: $J^P = 1^-$**

If we assume that its decay into hadrons goes via gluons, the Landau-Yang theorem (a vector particle cannot decay into two vector states) implies there must be at least **three of them** in the final state



We write the decay width as: $\Gamma(^3S_1 \rightarrow 3\text{ gluons}) = |R(0)|^2 |M(q\bar{q} \rightarrow 3\text{ gluons})|^2$

Probability of finding the two quarks at the same point

annihilation probability at rest

We now need the **tools** to perform the calculations of the two terms.
We shall use a **Coulomb approximation** for the first term and the **QCD Feynman rules** for the second

The J/ψ width

Coulomb potential: $V(r) \sim -\frac{4}{3}\frac{\alpha_S}{r}$

Solving the Schroedinger equation we find $|R(0)|^2 = \frac{4}{(\text{Bohr radius})^3} = 4\left(\frac{4}{3}\alpha_S\right)^3\left(\frac{m}{2}\right)^3$

Colour factors

The QCD probability for annihilation into 3 gluons will also be proportional to the cube of the strong coupling: $|M(q\bar{q} \to 3 \text{ gluons})|^2 = \frac{\alpha_S^3}{m^2}\left(\frac{5}{18}\right)\frac{4(\pi^2-9)}{9\pi}$

Finally: $\Gamma(^3S_1 \to 3 \text{ gluons}) \propto \alpha_S^6$

The strong coupling runs with the scale. At what scale should I take it?

The renormalization group fixes it:

$$\Gamma(Q, g, \mu) = \Gamma(Q, \bar{g}(Q), Q)$$

so that $\Gamma(^3S_1 \to 3 \text{ gluons}) \propto [\alpha_S(4m^2)]^6$

In 1974, however, we had no measurement for the strong coupling at a scale around 3 GeV. We did not even know if such a perturbative coupling existed!

The J/ψ width

Two options for checking the consistency of the picture

1. - Try to rescale a lower energy decay width

From $\Gamma(\phi \to 3\pi) \simeq 600$ keV one can extract $\alpha_S((1\ \mathrm{GeV})^2) \simeq 0.53$

Asymptotic freedom scales this to $\alpha_S((3\ \mathrm{GeV})^2) \simeq 0.29$

$$\Gamma(J/\psi \to \mathrm{hadrons}) = \frac{3}{2}\frac{M_{J/\psi}}{M_\phi}\left(\frac{\alpha_S(M^2_{J/\psi})}{\alpha_S(M^2_\phi)}\right)^6 \Gamma(\phi \to 3\pi) \simeq 73\ \mathrm{keV}$$

OK!

2. - Use leptonic width to eliminate wavefunction and extract value of strong coupling

From $\Gamma(J/\psi \to \mathrm{leptons}) = |R(0)|^2 |M(q\bar{q} \to e^+e^-)|^2 = \frac{1}{m^2}\left(\frac{2}{3}\alpha_{em}\right)^2 |R(0)|^2 \simeq 3\ \mathrm{keV}$

and $$\frac{\Gamma(J/\psi \to \mathrm{leptons})}{\Gamma(J/\psi \to \mathrm{hadrons})} = \frac{18\pi\alpha_{em}^2}{5(\pi^2-9)\alpha_S^3} \simeq 0.04$$

we get $\alpha_S((3\ \mathrm{GeV})^2) \simeq 0.26$ OK!

Good consistency between strong coupling values. Good estimate of hadronic width.

Heavy Quarkonium Numerology

Is charmonium really a Coulomb bound state?
How can a hadronic system not be (much) sensitive to long distance effects?

NB. Proton radius ~ 1 fm ~ 1/(200 MeV) ~ 1/Λ

Obvious answer: it's a **small** system!

Two masses m orbiting each other +
Heisenberg uncertainty principle:

$$r \sim \frac{1}{2p} \sim \frac{1}{2}\frac{1}{(m/2)v} = \frac{1}{mv}$$

To estimate v, consider the Virial Theorem

$$\langle T \rangle = -\frac{1}{2}\langle V \rangle$$

and the energy of the first level $E_1 = -\frac{1}{2}\frac{m}{2}\left(\frac{4}{3}\alpha_S\right)^2$

$$==> \quad \left(\frac{m}{2}\right)v^2 = 2\langle T \rangle = -2E_1 = \frac{m}{2}\left(\frac{4}{3}\alpha_S\right)^2$$

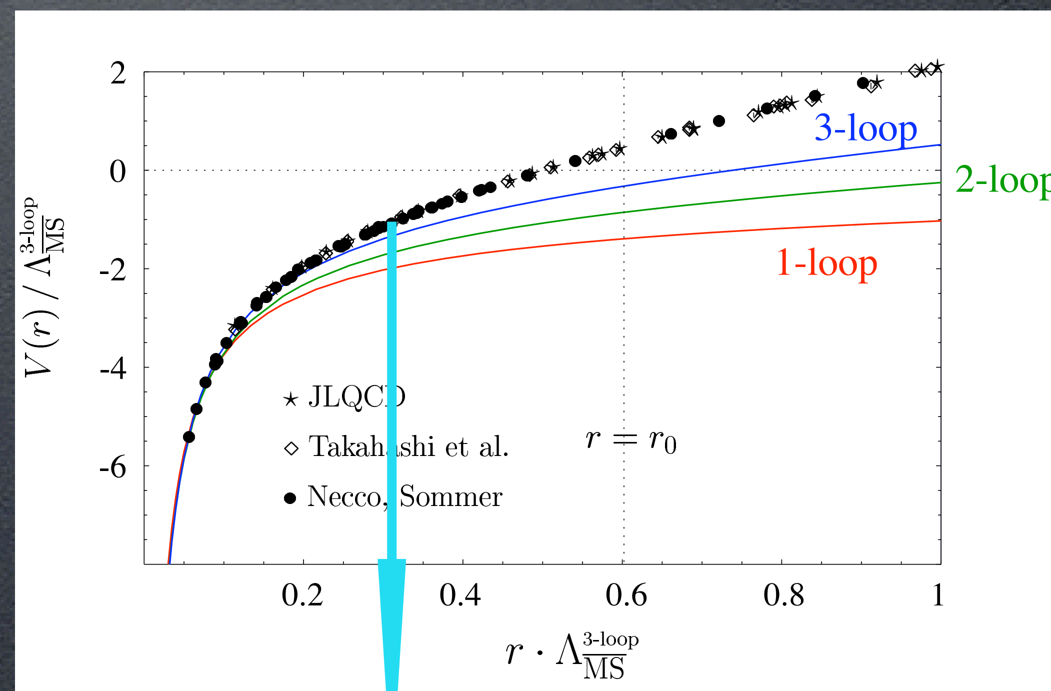
$$==> \quad v \simeq \frac{4}{3}\alpha_S((1/mv)^2) \quad ==> \quad v \simeq 0.5c$$

and consequently

$$r \simeq \frac{1}{750\ \mathrm{MeV}} \simeq \frac{1}{3\Lambda} \simeq 0.3\,fm$$

QCD potential: Coulomb + linear

$$V(r) \sim -\frac{4}{3}\frac{\alpha_S(1/r^2)}{r} + Kr$$



J/ψ

NB. Tight bound system below threshold for DDbar decay
(on the contrary, phi -> KKbar)
=> further explanation for small width

The third family

In 1977 a system of resonances similar to the charmonium was observed at M ~ 10 GeV. This was quickly interpreted as a bound state of **bottom-antibottom**, a fifth quark with a mass ~ 5 GeV and charge -1/3

Electroweak now requires a sixth quark to complete the third family, the **top quark.** Shall we see a toponium and, in general, hadrons composed of a top and a light quark?

Not necessarily: hadronization takes a certain time, namely the time for gluons to propagate the distance of a typical hadron radius R ~ 1 fm:

$$t_{\text{hadr}} \sim R/c \sim 1/\Lambda \sim 10^{-24}\ s$$

On the other hand, as member of a weak isospin doublet, a heavy top can decay weakly:

$$t \longrightarrow bW^+ \qquad t_{\text{decay}} = \frac{1}{\Gamma_{bW}} \simeq 1/\left(\frac{G_F m_t^3}{8\pi\sqrt{2}}\right) \sim 1/(G_F m_t^3) \sim \frac{M_W^2}{m_t^3} = \frac{1}{\Lambda}\frac{M_W^2\Lambda}{m_t^3}$$

so that $t_{\text{decay}} < t_{\text{hadr}}$ if $m_t > (M_W^2\Lambda)^{1/3} \simeq 10\ \text{GeV}$

NB. Neglected pretty big numerical factors. Real limit larger.

A heavy top quark with mass larger than the W boson will therefore decay before hadronizing

Heavy quarks are different

The time a coloured particle takes to hadronize is that taken by the colour field to travel a distance of the order of the typical hadron size: t′ ~ R ~ 1/Λ.

Boosting to the lab frame we find

$$t_q^{\rm hadr} = t'\gamma = R\frac{E}{\Lambda} = ER^2 = \frac{E}{\Lambda^2}$$ light quarks

$$t_Q^{\rm hadr} = t'\gamma = R\frac{E}{m}$$ heavy quarks

Consider now 'shaking' (i.e. accelerating) a quark. The regeneration time of a gluon field of momentum k around it is given by

$$t_g^{\rm regen}(k) = \frac{k_\parallel}{k_\perp^2}$$

For gluons such that $k_\perp \sim \Lambda,\ k_\parallel \sim E$ we have $t_g^{\rm regen}(k) \simeq t_q^{\rm hadr}$

A heavy quark will therefore behave like a light one only if

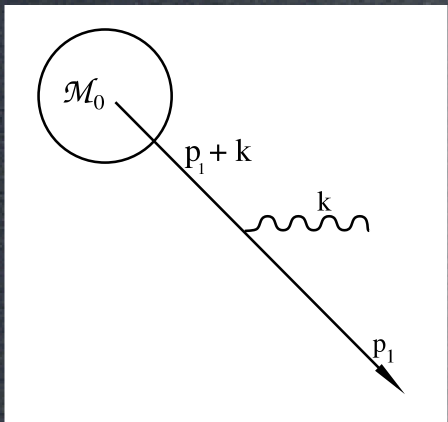
$$t_Q^{\rm hadr} > t_g^{\rm regen}(k) \Leftrightarrow \frac{E}{m}\frac{1}{\Lambda} > \frac{k_\parallel}{k_\perp^2} \simeq \frac{1}{\Theta}\frac{1}{\Lambda} \Leftrightarrow \Theta > \frac{m}{E} \equiv \Theta_0$$

Gluon transverse momenta leading to longer regeneration times will instead be suppressed (as the heavy quark is not there any more!!)

$\Theta < \Theta_0$ is called the 'dead cone' (no radiation from the heavy quark in a collinear region close to the quark)

The 'Dead Cone' in perturbative QCD

Consider gluon emission off a heavy quark using perturbation theory:

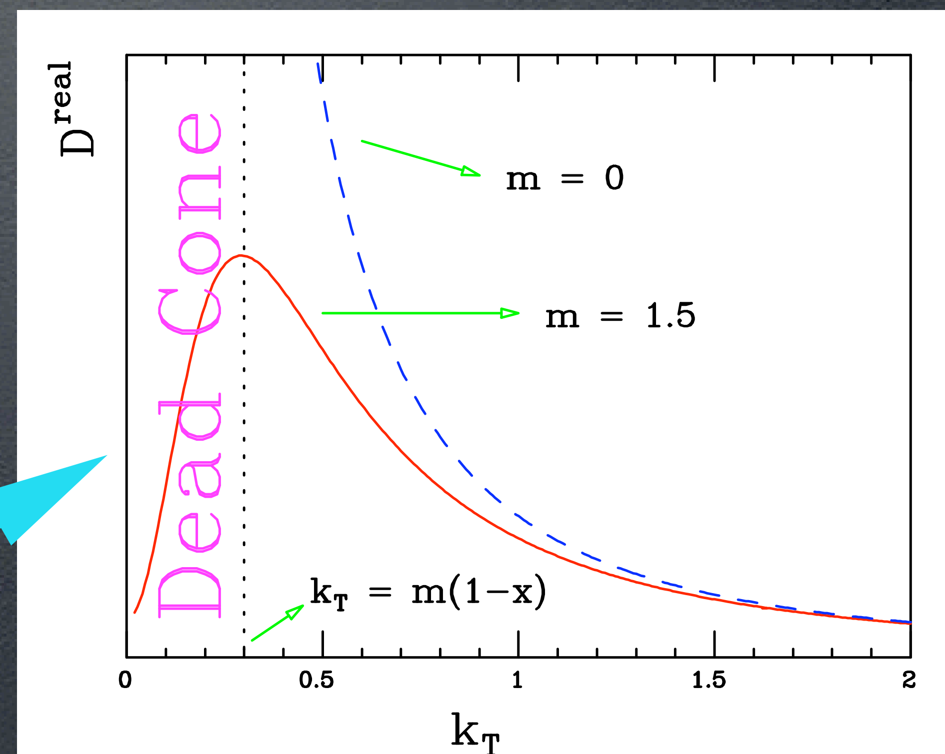


$$D^{real}(x,k_\perp^2,m^2) = \frac{C_F\alpha_S}{2\pi}\left[\frac{1+x^2}{1-x}\,\frac{1}{k_\perp^2+(1-x)^2m^2} - x(1-x)\,\frac{2m^2}{(k_\perp^2+(1-x)^2m^2)^2}\right]$$

In the **massless case** (m=0) we have a non-integrable collinear singularity:

$$\int_0 D(x,k_\perp^2)dk_\perp^2 = \frac{1+x^2}{1-x}\int_0 \frac{dk_\perp^2}{k_\perp^2} = \infty$$

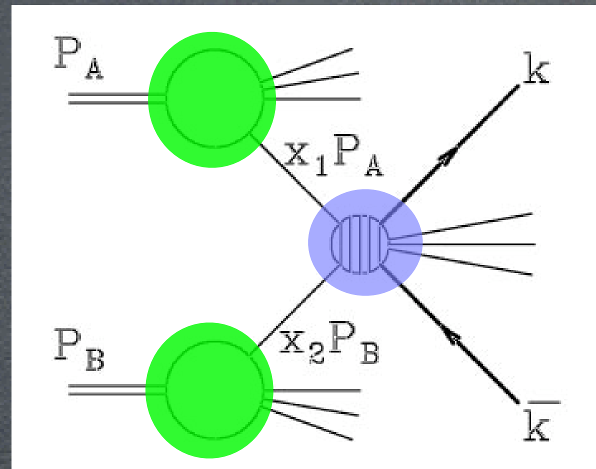
The presence of the heavy quark mass suppresses instead the radiation at small transverse momenta and allows the integration down to zero



=> We can calculate in pQCD heavy quark total cross sections and momentum distributions

Total cross sections

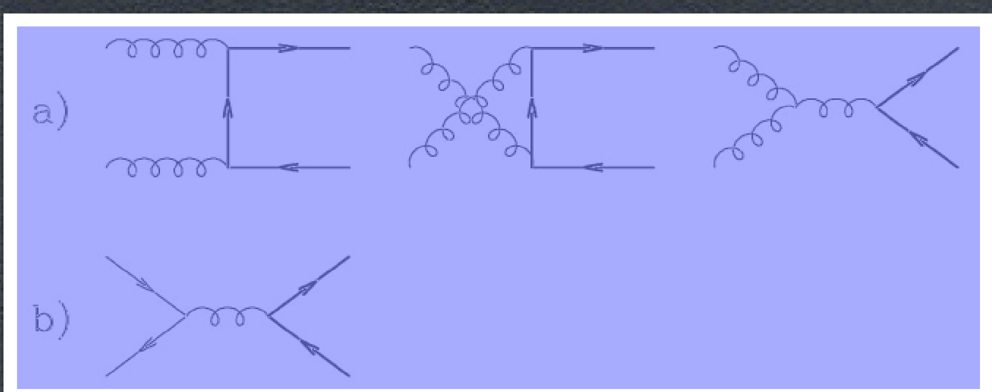
A case study: heavy quark production in $p\bar{p}$ collisions



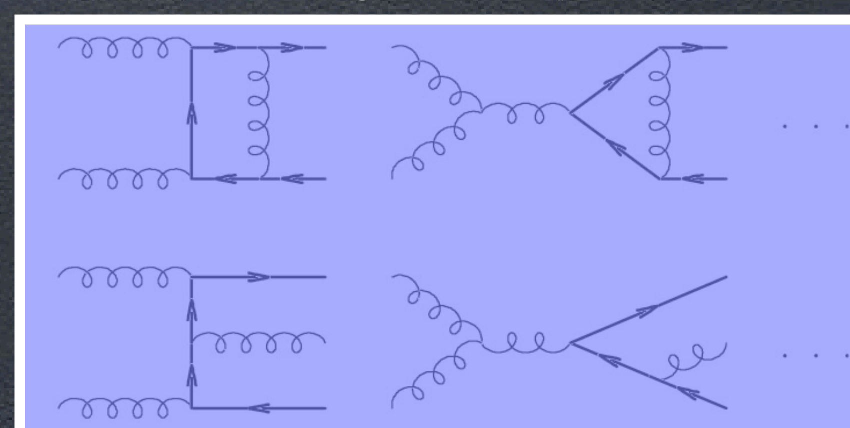
Parton distribution functions

$$d^3\sigma = \sum_{i,j} \int dx_1\, dx_2\, d^3\hat{\sigma}_{ij}(x_1 P_A, x_2 P_B, k, \bar{k}, m, \mu)\, F_i^A(x_1, \mu)\, F_j^B(x_2, \mu).$$

Leading Order diagrams, proportional to α_S^2



Next-to-Leading Order diagrams, proportional to α_S^3, starts compensating scale dep. of PDFs



Virtual corrections

Real corrections

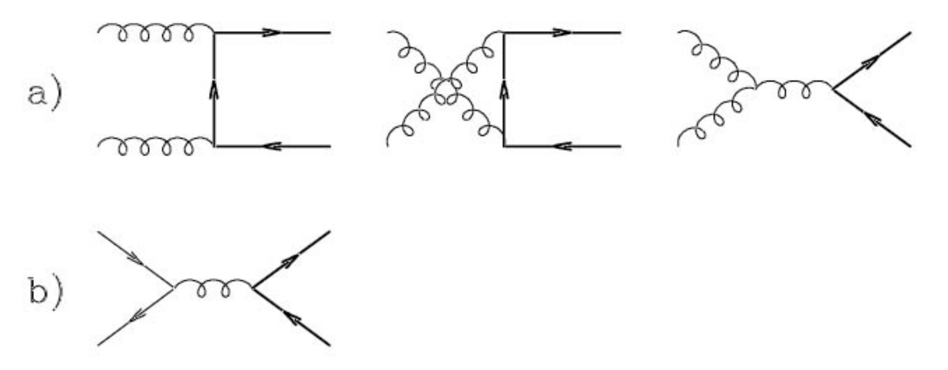
A glimpse of the LO result

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2\, \hat{\sigma}_{ij}(x_1 x_2 S, m^2, \mu^2)\; F_i^A(x_1,\mu) F_j^B(x_2,\mu)$$

$$\hat{\sigma}_{ij}(s, m^2, \mu^2) = \frac{\alpha_S^2(\mu^2)}{m^2} f_{ij}\left(\rho, \frac{\mu^2}{m^2}\right)$$

$$f_{ij}\left(\rho, \frac{\mu^2}{m^2}\right) = f_{ij}^{(0)}(\rho) + g^2(\mu^2)\left[f_{ij}^{(1)}(\rho) + \overline{f}_{ij}^{(1)}(\rho)\ln\left(\frac{\mu^2}{m^2}\right)\right] + O(g^4)$$

$$\rho = \frac{4m^2}{s}, \quad \beta = \sqrt{1-\rho}.$$



$$f_{gg}^{(0)}(\rho) = \frac{\pi\beta\rho}{192}\left[\frac{1}{\beta}(\rho^2 + 16\rho + 16)\ln\left(\frac{1+\beta}{1-\beta}\right) - 28 - 31\rho\right]$$

$$f_{q\bar{q}}^{(0)}(\rho) = \frac{\pi\beta\rho}{27}\left[2+\rho\right]$$

NB: **no heavy quarks** among the initial state partons.
They are only produced by QCD dynamics via gluon splitting

Theoretical uncertainties

Theoretical uncertainties in a NLO (or higher order) QCD prediction can come from:

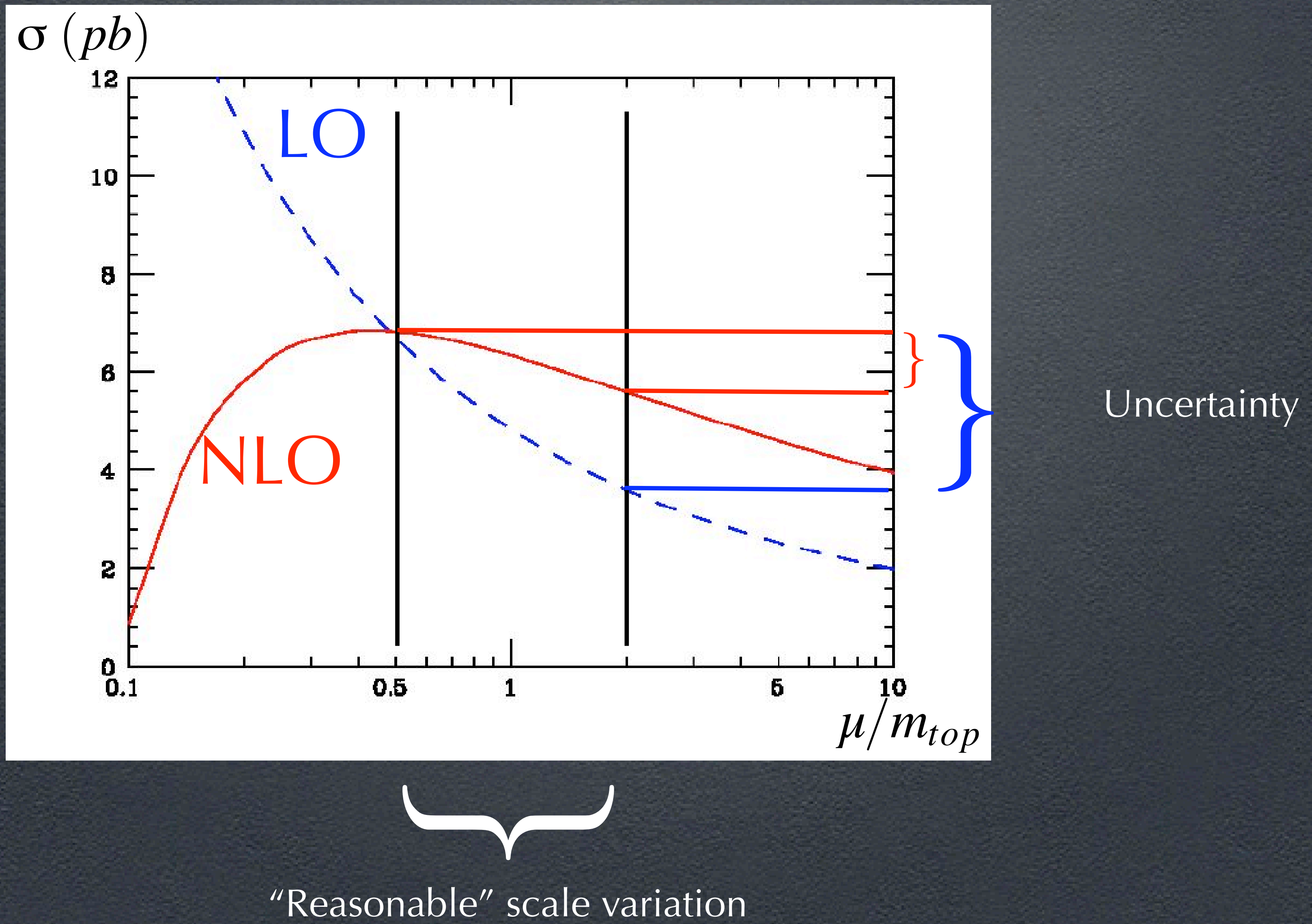
1. Imperfect knowledge of needed external inputs (i.e. Parton Distribution Functions, strong coupling value, heavy quark mass..)

2. Internal shortcomings, i.e. necessarily approximated PERTURBATIVE calculation. This is reflected in the dependence of the PHYSICAL cross section on the UNPHYSICAL renormalization and factorization scales

$$d^3\sigma = \sum_{i,j} \int dx_1\, dx_2\, d^3\hat{\sigma}_{ij}(x_1 P_A, x_2 P_B, k, \bar{k}, m, \mu)\, F_i^A(x_1, \mu)\, F_j^B(x_2, \mu).$$

Would compensate exactly only in a FULL calculation

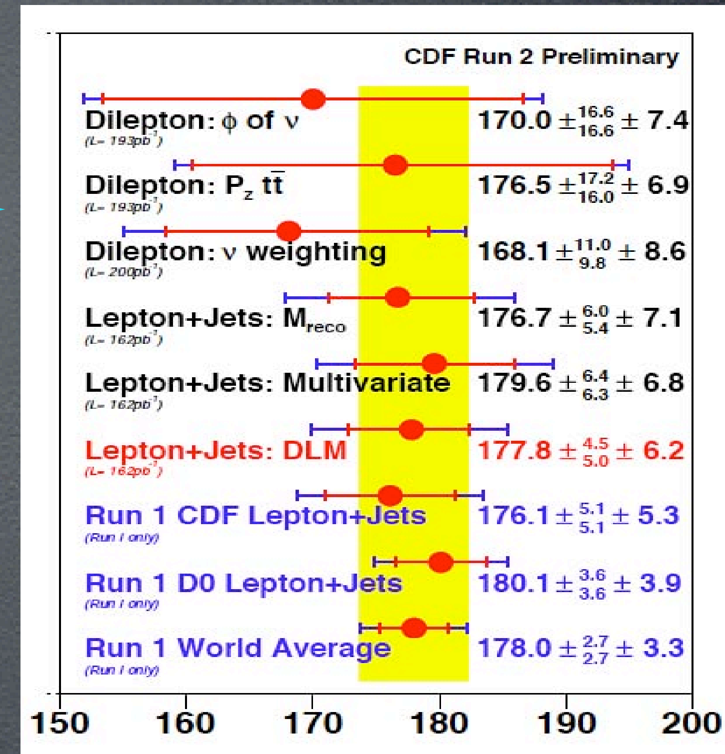
“Typical” behaviour of a cross-section w.r.t. scale variations:



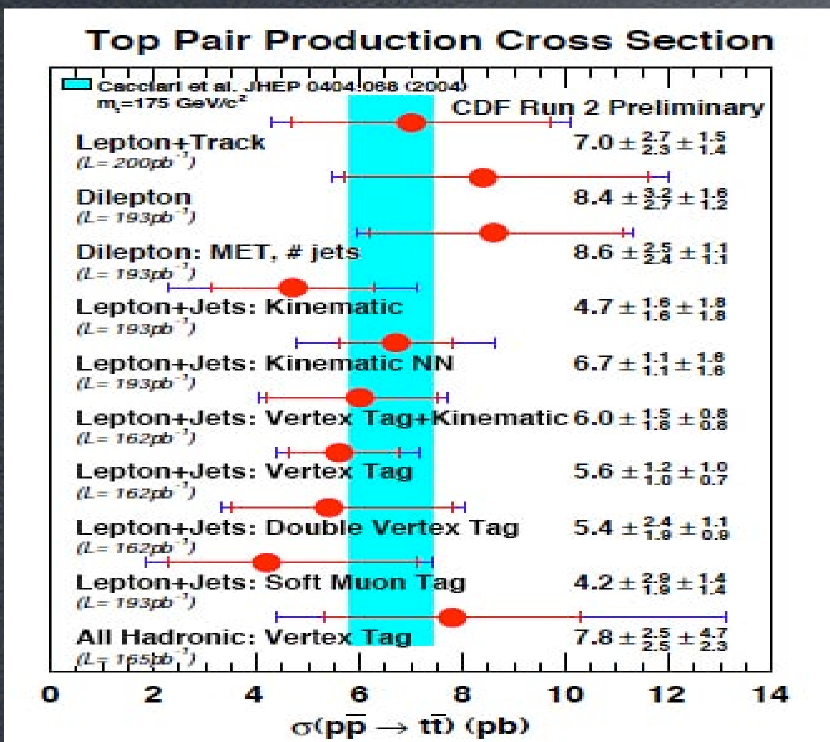
=> we want to go to N....LO to decrease the theoretical uncertainties

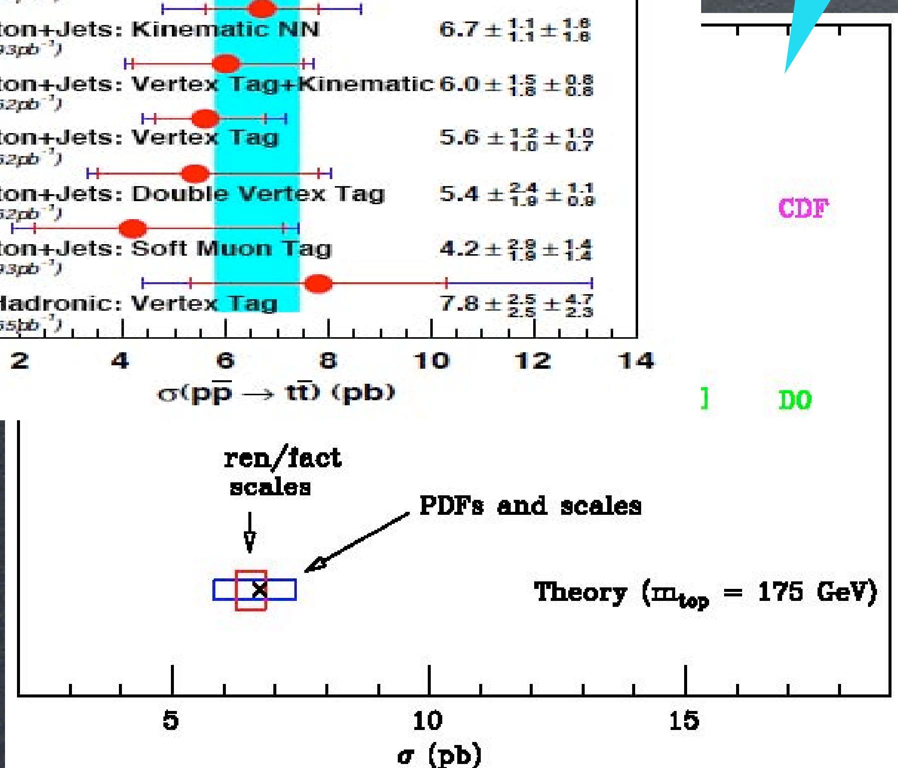
Selected phenomenological results: top production

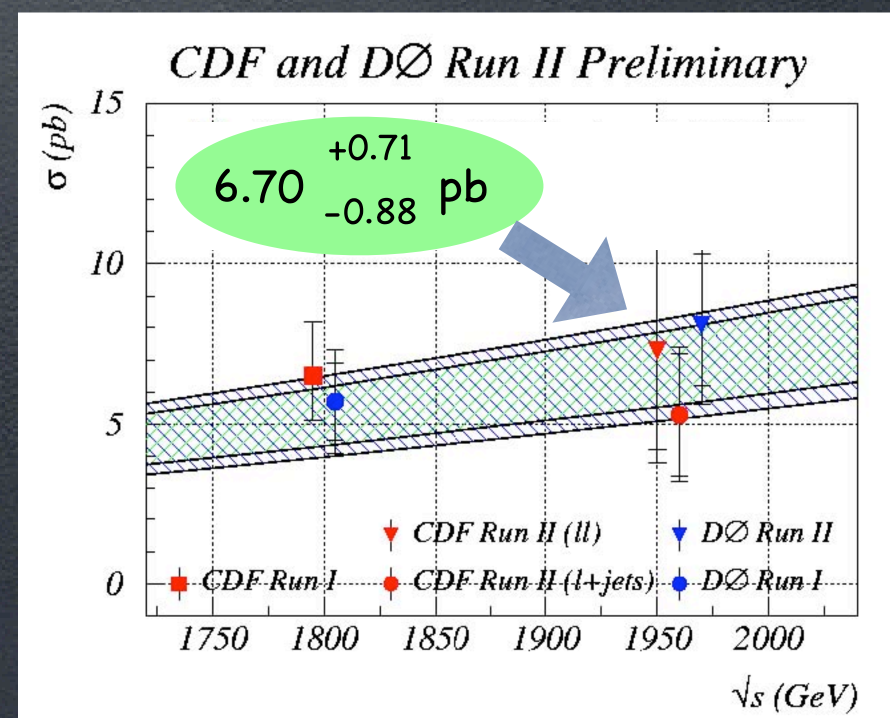
m ~ 175 GeV



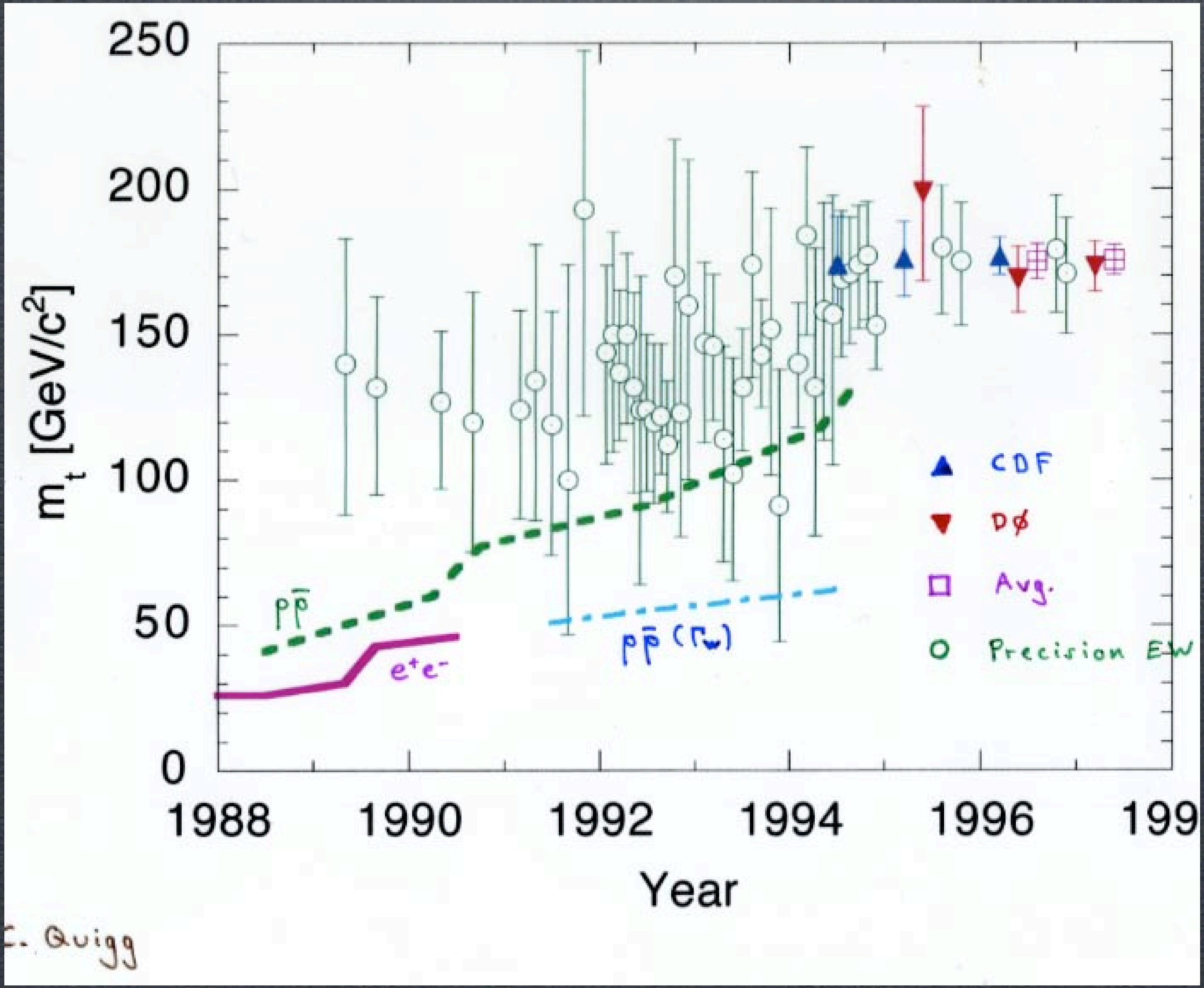
$\sigma_{p\bar{p}}(\sqrt{S} \simeq 2 \text{ TeV}) \simeq 6\ pb$







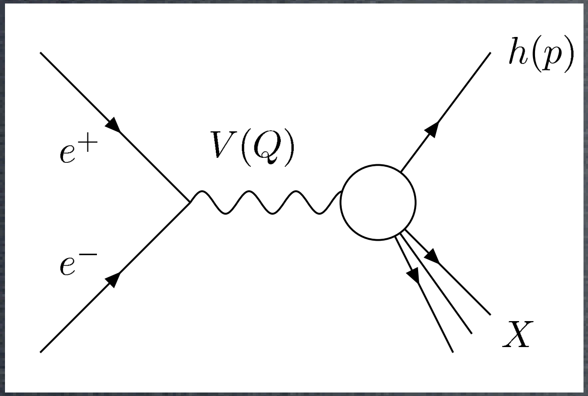
The prediction of the top mass



Sudden improvement in accuracy and central value after direct discovery perhaps a little suspicious, but large mass (m > 100 GeV) well predicted from one-loop analysis of electroweak LEP data

Momentum distributions

Consider the simplest case of single particle inclusive distribution



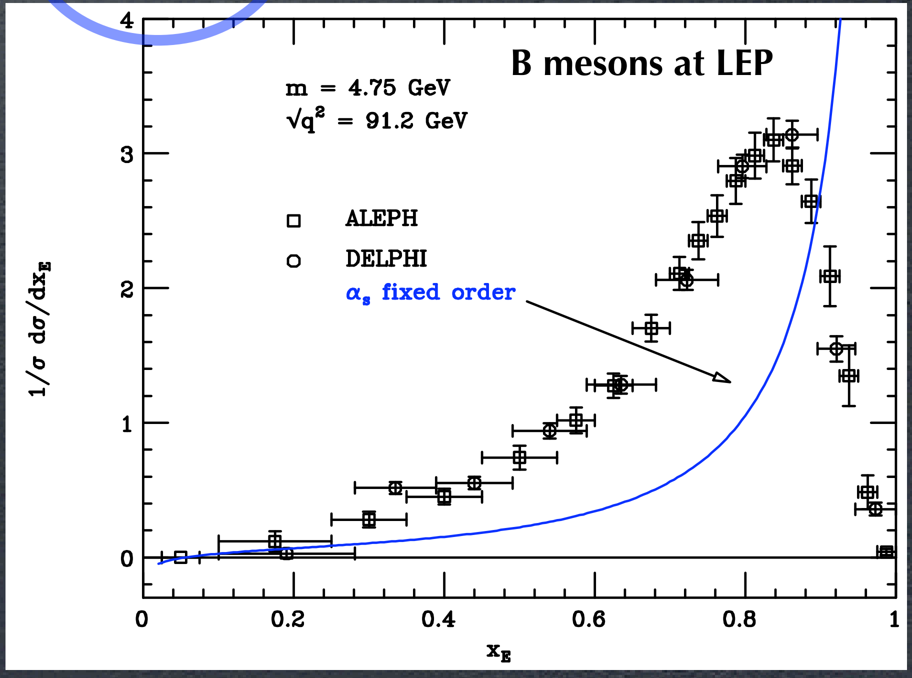
$$e^+ + e^- \to V(Q) \to h(p) + X$$

$$x = x_E = \frac{E_h}{E_{beam}}$$

If the quark is **heavy** this observable is **collinear safe**: I can calculate it in pQCD

$$\frac{1}{\sigma}\frac{d\sigma}{dx} = \delta(1-x) + \frac{(Q^2)}{2\pi}\left\{C_F + C_F\left[\ln\frac{Q^2}{m^2}\left(\frac{1+x^2}{1-x}\right)_+ + 2\frac{1+x^2}{1-x}\log x - \left(\frac{\ln(1-x)}{1-x}\right)_+(1+x^2) + \frac{1}{2}\left(\frac{1}{1-x}\right)_+(x^2-6x-2) + \left(\frac{2}{3}\pi^2 - \frac{5}{2}\right)\delta(1-x)\right]\right\} + O\left(\frac{m}{Q}\right)$$

Finite, but hardly accurate description of nature!



Two issues:

1. Large logarithms spoil converge of pQCD
2. Non-perturbative corrections: we observe hadrons, not quarks

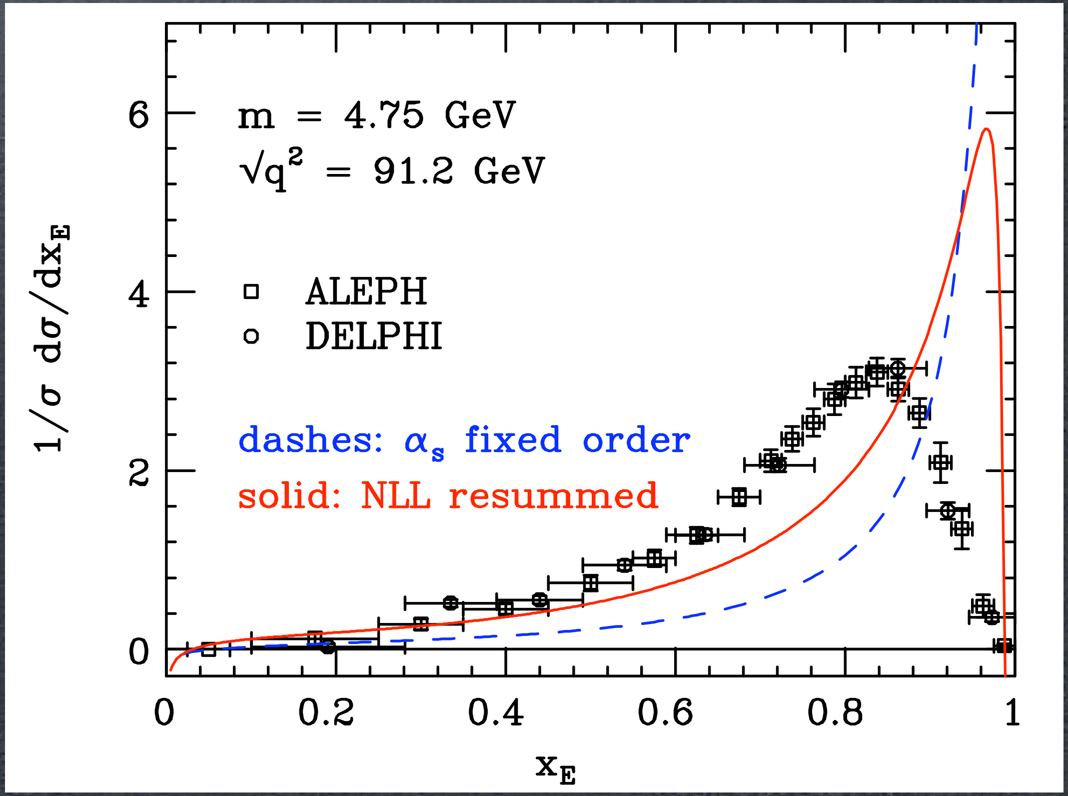
Resummation

Perturbative techniques can take care of resumming large logarithmic terms which appear in perturbative expansion:

$$\log \frac{Q^2}{m^2}$$ collinear logs -> DGLAP evolution eqs. (same as for PDFs)

$$\left(\frac{1}{1-x}\right)_+$$ soft gluon logs -> Sudakov resummation

Improved result is however still not up to the task:

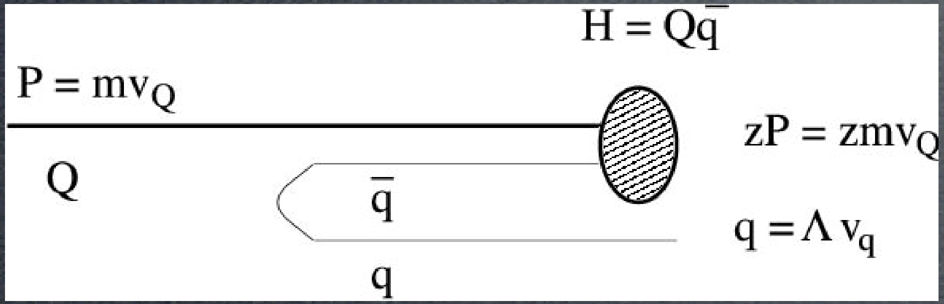


Still missing are the **non-perturbative contributions**

Non-perturbative contributions

We calculate **quarks**, but we observe **hadrons.**
The transition from the former to the latter cannot be calculated from first principles in QCD, but it can of course be modeled (and QCD can still give information on its behaviour)

In a very simple model, hadronization causes a degradation of the quark momentum:



Heavy quark momentum: $P = mv_Q$

Light quark momentum: $q = mv_q$

For the binding we need $v_Q \sim v_q \equiv v$

This leads to $P = zP + q$ $\qquad mv = zmv + \Lambda v$

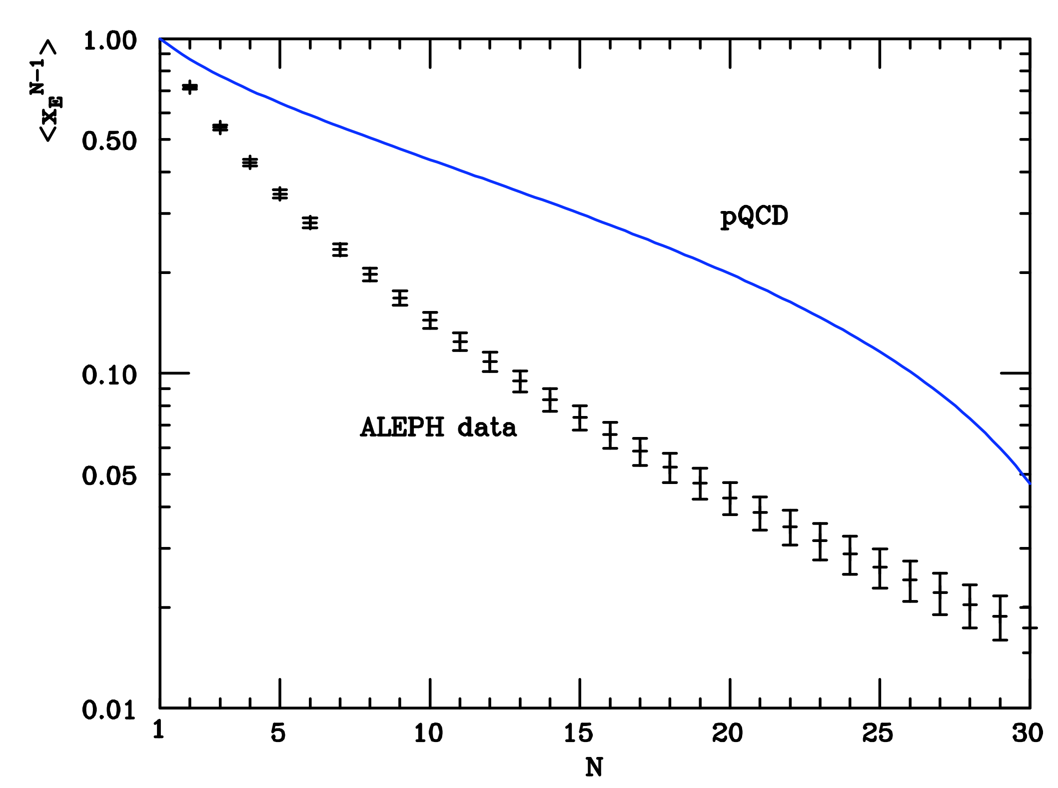
and therefore

$$\langle z \rangle \simeq 1 - \frac{\Lambda}{m}$$

Hence, hadronization effects scale **linearly** with the heavy quark mass.
This power correction is also predicted by more refined analyses in QCD

NB. This is a parametrically small correction: a heavy quark is not easily slowed down when dragging a light one out of the vacuum. Still, its consequences are evidently clearly measurable.

Test of non-perturbative scaling



LEP B meson data translated to Mellin space:

$$f_N \equiv \int_0^1 x^{N-1} f(x)\, dx = \langle x^{N-1} \rangle$$

In this space **convolutions become products**

Hence

$$\langle x \rangle_{expt} = \langle x \rangle_{pQCD} \langle x \rangle_{np}$$

From the measurements at LEP: $\langle x \rangle_{expt} \simeq 0.71$

From resummed pQCD: $\langle x \rangle_{pQCD} \simeq 0.76$

Their ratio: $$\frac{\langle x \rangle_{expt}}{\langle x \rangle_{pQCD}} = 0.93 \sim 1 - \frac{\Lambda}{m}$$

Summary

- Heavy quarks (charm, bottom, top) are special because **m > Λ**

- The large mass parameter can be used to construct effective theories which facilitate the calculations. Examples (not mentioned in the seminar) are

  - Heavy Quark Effective Theory (HQET), expansion in 1/m
  - Non Relativistic QCD (NRQCD), expansion in the small velocity v

- Three issues facilitate the calculation of heavy quark-related processes in QCD:

  - the coupling is small at larger scales
  - collinear divergences are screened by the large quark mass
  - non-perturbative corrections are power suppressed

- (Almost) predictive power at the few per cent level in a (in principle) strongly coupled theory. Not bad as a challenge/achievement